Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/1.2.2.4-f-x-^m-d+e-x^2-^q-a+b-x^2+c-x^4-^p

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September 20, 2021 Compiled on September 20, 2021 at 2:34pm

Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [300]. This is test number [28].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (300)	0.00 (0)
Mathematica	100.00 (300)	0.00 (0)
Maple	100.00 (300)	0.00 (0)
Giac	88.00 (264)	12.00 (36)
Fricas	85.33 (256)	14.67 (44)
Mupad	71.33 (214)	28.67 (86)
Maxima	57.67 (173)	42.33 (127)
IntegrateAlgebraic	49.67 (149)	50.33 (151)
Sympy	35.33 (106)	% 64.67 (194)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

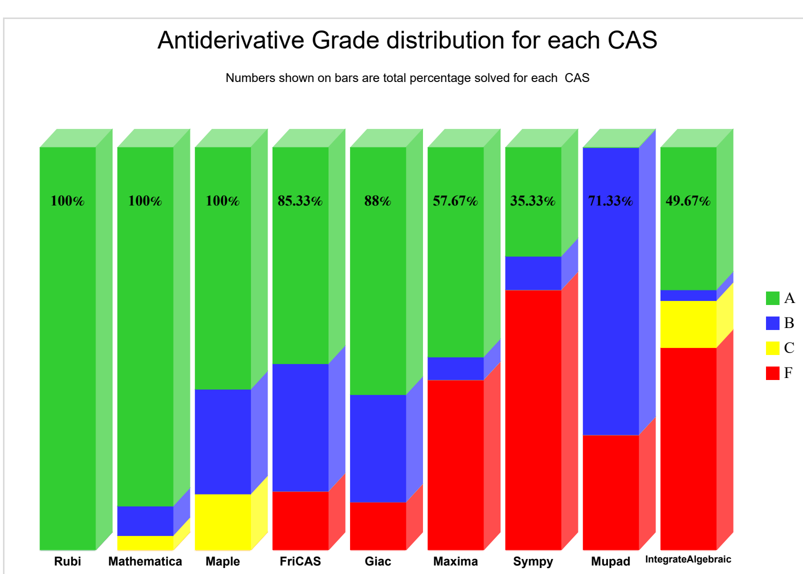
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

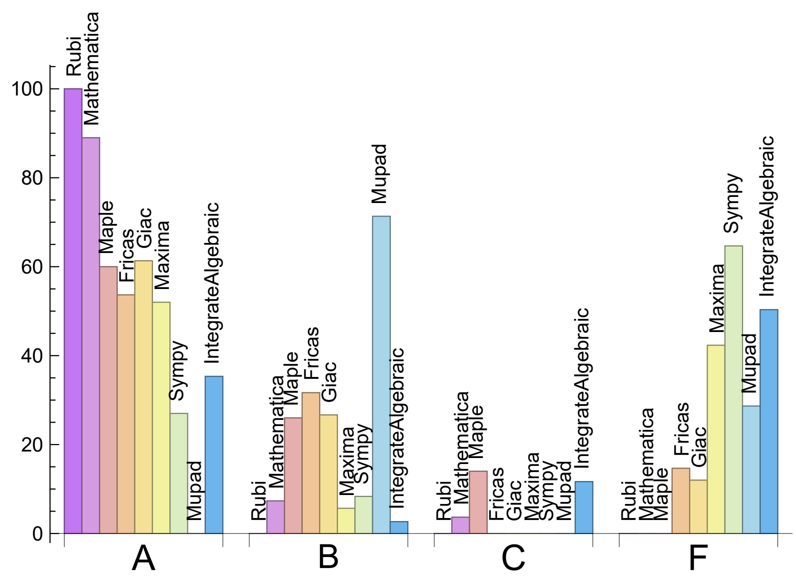
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	89.00	7.33	3.67	0.00
Giac	61.33	26.67	0.00	12.00
Maple	60.00	26.00	14.00	0.00
Fricas	53.67	31.67	0.00	14.67
Maxima	52.00	5.67	0.00	42.33
IntegrateAlgebraic	35.33	2.67	11.67	50.33
Sympy	27.00	8.33	0.00	64.67
Mupad	N/A	71.33	0.00	28.67

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Fricas	44	0.00 %	100.00 %	0.00 %
IntegrateAlgebraic	151	99.34 %	0.66 %	0.00 %
Giac	36	11.11 %	30.56 %	58.33 %
Maxima	127	68.50 %	0.00 %	31.50 %
Sympy	194	56.70 %	43.30 %	0.00 %
Mupad	86	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

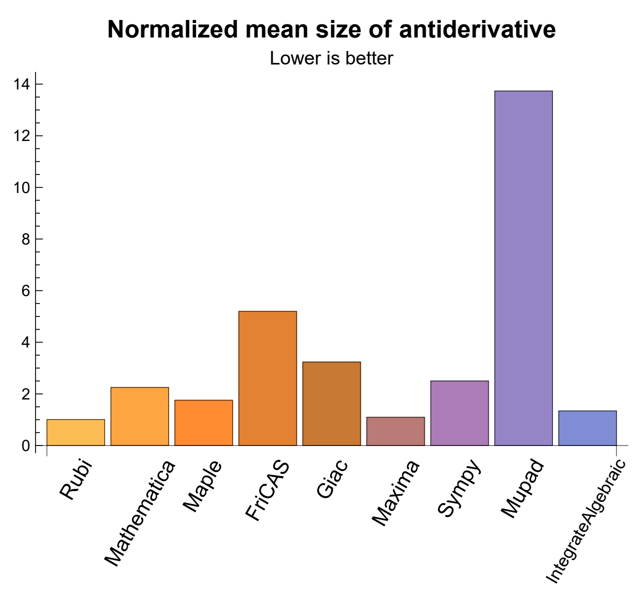
1.3 Performance

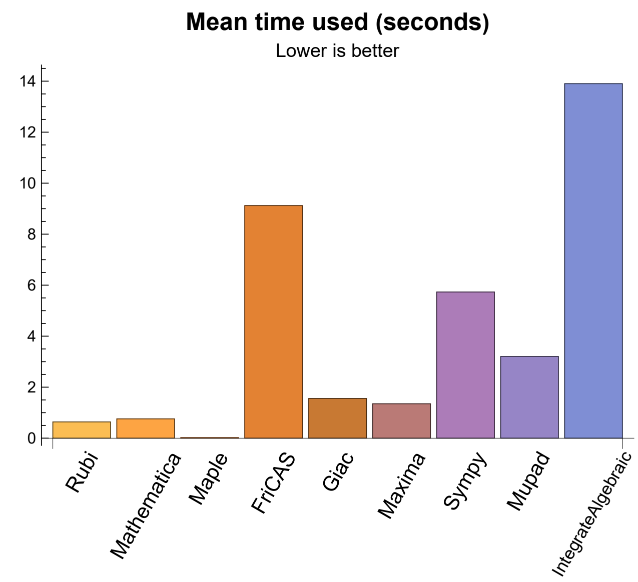
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.64	203.02	1.01	153.00	1.00
Mathematica	0.76	663.39	2.25	142.50	0.99
Maple	0.02	411.12	1.75	187.00	1.16
Maxima	1.35	134.41	1.09	107.00	0.98
Fricas	9.12	1503.34	5.20	215.50	2.06
Sympy	5.73	316.00	2.50	132.00	1.33
Giac	1.56	834.20	3.24	144.00	1.08
Mupad	3.20	4618.84	13.73	175.00	2.20
IntegrateAlgebraic	13.90	315.46	1.34	141.00	1.04

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




Mean time used (seconds)
Lower is better
0
2
4
6
8
10
12
14
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {254, 261, 262, 264, 265, 266, 271, 272, 273, 274, 275, 281, 283, 284, 285, 294, 295, 296, 297, 298, 299}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

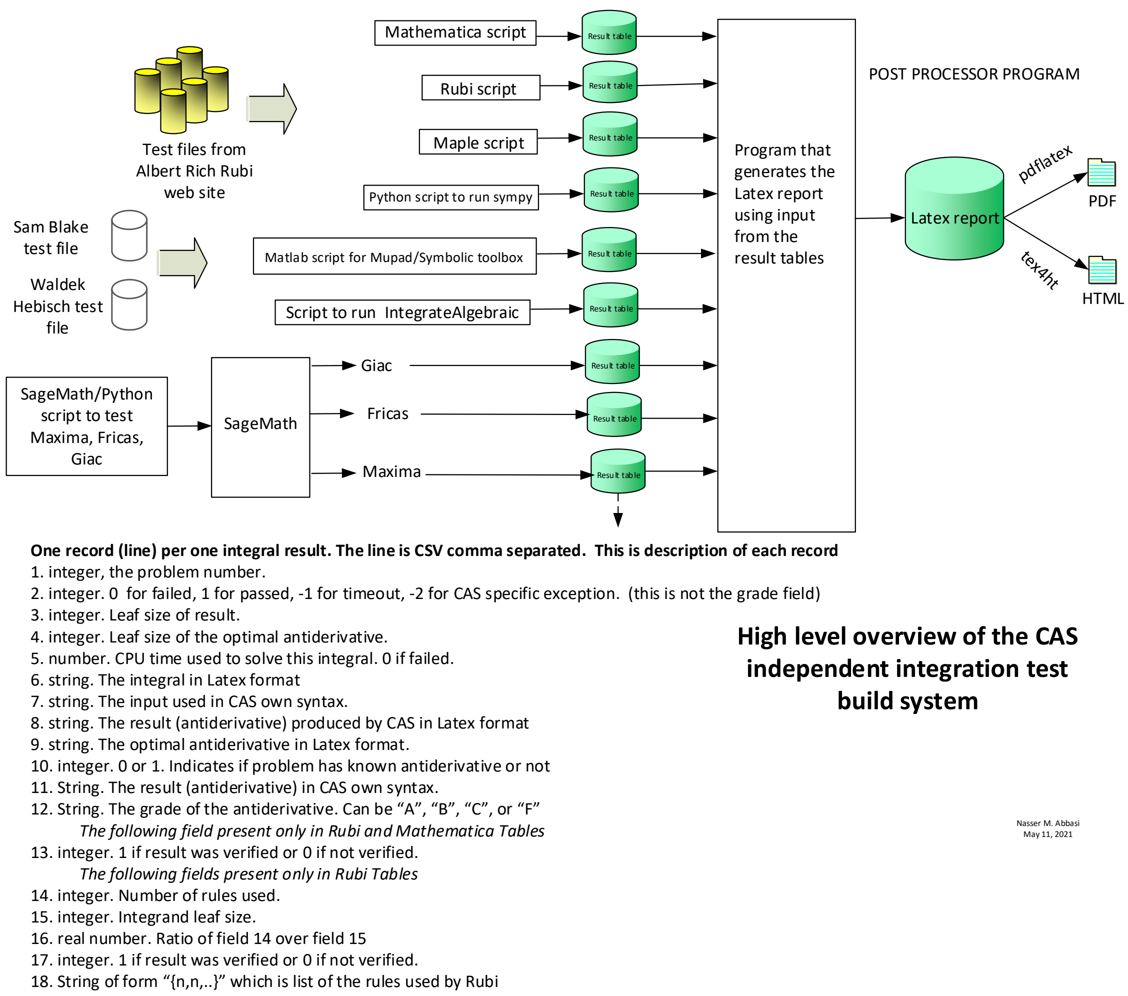
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 37, 39, 41, 42, 43, 44, 45, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 267, 268, 269, 270, 276, 277, 278, 279, 280, 286, 287, 288, 289, 290, 291, 292, 293, 300 }

B grade: { 36, 38, 40, 46, 48, 254, 261, 262, 263, 264, 265, 266, 271, 272, 273, 274, 275, 281, 282, 283, 284, 294 }

C grade: { 19, 20, 21, 34, 232, 285, 295, 296, 297, 298, 299 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 39, 41, 42, 43, 44, 47, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 92, 93, 107, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 219, 220, 221, 222, 243, 244, 246, 247 }

B grade: { 35, 36, 38, 40, 45, 46, 48, 50, 67, 68, 80, 85, 86, 87, 88, 89, 90, 91, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 136, 137, 152, 156, 157, 158, 217, 218, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 248, 249, 250, 251, 252, 253, 276, 277, 278, 279, 280, 285 }

C grade: { 232, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 281, 282, 283, 284, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

F grade: { }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 12, 13, 14, 15, 19, 20, 21, 22, 23, 27, 28, 29, 30, 31, 32, 33, 34, 35, 37, 39, 41, 42, 43, 44, 45, 47, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216 }

B grade: { 8, 9, 10, 11, 16, 17, 18, 24, 25, 26, 36, 38, 40, 46, 48, 50, 70 }

C grade: { }

F grade: { 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 119, 136, 137, 138, 139, 140, 141, 142, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 39, 41, 42, 43, 44, 47, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 155, 159, 160, 161, 162, 163, 164, 165, 174, 175, 176, 177, 178, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 219, 220, 221, 234, 235, 237, 247 }

B grade: { 35, 36, 38, 40, 45, 46, 48, 50, 67, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 153, 154, 156, 157, 158, 167, 168, 169, 170, 171, 172, 173, 182, 183, 184, 185, 186, 187, 227, 228, 243, 244, 245, 246, 248, 249, 250, 251, 252, 253, 256, 257, 261, 262, 263, 264, 265, 266, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 288, 289, 290, 291, 300 }

C grade: { }

F grade: { 166, 179, 180, 181, 188, 217, 218, 222, 223, 224, 225, 226, 229, 230, 231, 232, 233, 236, 238, 239, 240, 241, 242, 254, 255, 258, 259, 260, 267, 268, 269, 270, 271, 272, 286, 287, 292, 293, 294, 295, 296, 297, 298, 299 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 37, 39, 41, 42, 43, 44, 47, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 70, 73, 74, 75, 76, 77, 78, 79, 87, 115, 116, 117, 118, 119, 156, 157, 158, 195, 196, 197, 198, 199, 200, 201, 203, 204, 206, 207, 210, 211, 212, 213, 214, 215 }

B grade: { 16, 17, 32, 33, 34, 36, 38, 40, 46, 48, 50, 80, 81, 82, 92, 93, 104, 105, 106, 107, 202, 205, 208, 209, 216 }

C grade: { }

F grade: { 35, 45, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 83, 84, 85, 86, 88, 89, 90, 91, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 108, 109, 110, 111, 112, 113, 114, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 22, 23, 24, 25, 29, 30, 31, 32, 33, 34, 37, 39, 41, 42, 43, 44, 47, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 64, 66, 70, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 90, 91, 92, 93, 94, 95, 102, 103, 105, 106, 107, 108, 109, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 129, 131, 132, 133, 134, 136, 137, 138, 140, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 237, 245, 247, 257, 261, 262, 270, 271, 272, 273, 286, 287, 288, 294, 300 }

B grade: { 13, 14, 20, 21, 26, 27, 28, 35, 36, 38, 40, 45, 46, 48, 50, 67, 68, 69, 71, 72, 85, 86, 87, 88, 89, 96, 97, 98, 99, 100, 101, 104, 110, 111, 112, 113, 114, 125, 126, 127, 128, 130, 135, 141, 142, 148, 149, 150, 156, 157, 158, 225, 226, 227, 228, 229, 230, 231, 249, 250, 251, 253, 254, 255, 256, 258, 260, 267, 268, 269, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285 }

C grade: { }

F grade: { 61, 63, 65, 139, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 244, 246, 248, 252, 259, 263, 264, 265, 266, 274, 275, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 58, 60, 62, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 131, 138, 139, 140, 146, 147, 148, 152, 153, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 254, 255, 256, 257, 258, 259, 260, 267, 268, 269, 270, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285 }

C grade: { }

F grade: { 55, 57, 59, 61, 63, 64, 65, 66, 67, 68, 69, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 137, 141, 142, 143, 144, 145, 149, 150, 151, 154, 155, 189, 190, 191, 192, 193, 194, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 261, 262, 263, 264, 265, 266, 271, 272, 273, 274, 275, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

2.1.9 IntegrateAlgebraic

A grade: { 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 55, 57, 59, 61, 63, 64, 65, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 189, 190, 191, 192, 193, 194, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 258, 259, 260, 276, 277, 278, 279, 280 }

B grade: { 56, 58, 60, 62, 66, 238, 285, 300 }

C grade: { 232, 256, 257, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 272, 273, 274, 275, 281, 282, 283, 284, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 271 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA` and `IntegrateAlgebraic` to `I.A.`

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	149	126	125	125	151	131	125	0
N.S.	1	1.00	1.00	0.85	0.84	0.84	1.01	0.88	0.84	0.00
time (sec)	N/A	0.220	0.016	0.010	0.431	0.554	0.145	0.261	0.306	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	149	126	125	125	155	131	125	0
N.S.	1	1.00	1.00	0.85	0.84	0.84	1.04	0.88	0.84	0.00
time (sec)	N/A	0.098	0.004	0.000	0.463	0.571	0.088	0.193	0.069	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	146	125	124	124	150	130	124	0
N.S.	1	1.00	1.64	1.40	1.39	1.39	1.69	1.46	1.39	0.00
time (sec)	N/A	0.077	0.003	0.003	0.441	0.790	0.089	0.180	0.071	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	141	122	121	121	148	127	121	0
N.S.	1	1.00	1.00	0.87	0.86	0.86	1.05	0.90	0.86	0.00
time (sec)	N/A	0.082	0.003	0.001	0.465	0.567	0.089	0.221	0.070	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	142	123	125	122	150	131	122	0
N.S.	1	1.00	1.00	0.87	0.88	0.86	1.06	0.92	0.86	0.00
time (sec)	N/A	0.111	0.012	0.014	0.518	0.504	0.250	0.197	0.109	0.001

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	139	122	121	127	143	127	121	0
N.S.	1	1.00	1.00	0.88	0.87	0.91	1.03	0.91	0.87	0.00
time (sec)	N/A	0.082	0.007	0.017	0.522	0.527	0.244	0.195	0.094	0.001

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	142	123	125	129	150	142	122	0
N.S.	1	1.00	1.00	0.87	0.88	0.91	1.06	1.00	0.86	0.00
time (sec)	N/A	0.121	0.008	0.009	0.504	0.646	0.279	0.208	0.072	0.001

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	50	53	102	48	97	54	42	56
N.S.	1	1.00	0.75	0.79	1.52	0.72	1.45	0.81	0.63	0.84
time (sec)	N/A	0.050	0.053	0.046	1.012	0.611	6.015	0.233	0.370	0.131

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	44	46	93	43	70	45	37	51
N.S.	1	1.00	0.86	0.90	1.82	0.84	1.37	0.88	0.73	1.00
time (sec)	N/A	0.031	0.026	0.006	1.016	0.746	4.307	0.233	0.392	0.111

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	36	34	67	34	53	38	32	42
N.S.	1	1.00	0.82	0.77	1.52	0.77	1.20	0.86	0.73	0.95
time (sec)	N/A	0.021	0.030	0.008	1.325	0.639	3.083	0.197	0.137	0.112

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	57	49	99	56	83	76	45	75
N.S.	1	1.00	0.98	0.84	1.71	0.97	1.43	1.31	0.78	1.29
time (sec)	N/A	0.055	0.059	0.016	1.561	0.630	15.588	0.213	0.146	0.157

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	59	61	88	72	83	91	51	76
N.S.	1	1.00	1.00	1.03	1.49	1.22	1.41	1.54	0.86	1.29
time (sec)	N/A	0.056	0.044	0.013	1.408	0.665	7.460	0.245	0.787	0.175

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	59	75	91	72	76	129	56	77
N.S.	1	1.00	0.94	1.19	1.44	1.14	1.21	2.05	0.89	1.22
time (sec)	N/A	0.055	0.067	0.016	1.237	0.614	6.042	0.244	0.423	0.261

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	72	52	59	59	63	116	43	65
N.S.	1	1.00	1.24	0.90	1.02	1.02	1.09	2.00	0.74	1.12
time (sec)	N/A	0.047	0.034	0.013	1.612	0.728	5.956	0.232	0.681	0.248

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	72	73	127	58	131	80	52	66
N.S.	1	1.00	0.87	0.88	1.53	0.70	1.58	0.96	0.63	0.80
time (sec)	N/A	0.059	0.049	0.025	1.349	0.631	14.314	0.247	0.319	0.184

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	54	58	118	53	124	71	47	61
N.S.	1	1.00	0.81	0.87	1.76	0.79	1.85	1.06	0.70	0.91
time (sec)	N/A	0.040	0.036	0.009	1.133	0.648	11.572	0.205	0.430	0.168

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	56	46	95	48	109	57	42	56
N.S.	1	1.00	0.93	0.77	1.58	0.80	1.82	0.95	0.70	0.93
time (sec)	N/A	0.030	0.027	0.014	1.597	0.824	8.193	0.242	0.177	0.170

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	67	75	138	67	114	90	55	85
N.S.	1	1.00	0.86	0.96	1.77	0.86	1.46	1.15	0.71	1.09
time (sec)	N/A	0.075	0.034	0.020	1.187	0.537	33.672	0.261	0.180	0.220

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	71	75	122	78	114	102	64	84
N.S.	1	1.00	0.88	0.93	1.51	0.96	1.41	1.26	0.79	1.04
time (sec)	N/A	0.075	0.053	0.016	1.330	0.604	11.391	0.258	0.765	0.252

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	60	73	123	82	133	146	71	88
N.S.	1	1.00	0.70	0.85	1.43	0.95	1.55	1.70	0.83	1.02
time (sec)	N/A	0.077	0.027	0.020	1.372	0.586	12.755	0.255	0.552	0.305

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	60	73	112	82	148	158	82	88
N.S.	1	1.00	0.73	0.89	1.37	1.00	1.80	1.93	1.00	1.07
time (sec)	N/A	0.075	0.025	0.023	1.346	0.486	12.558	0.270	0.946	0.332

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	44	51	104	43	85	46	38	51
N.S.	1	1.00	0.66	0.76	1.55	0.64	1.27	0.69	0.57	0.76
time (sec)	N/A	0.058	0.028	0.018	1.314	0.548	7.072	0.189	0.590	0.129

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	35	39	76	34	66	37	32	42
N.S.	1	1.00	0.69	0.76	1.49	0.67	1.29	0.73	0.63	0.82
time (sec)	N/A	0.042	0.022	0.012	1.218	0.742	5.519	0.216	0.309	0.148

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	34	32	65	33	53	33	27	41
N.S.	1	1.00	0.97	0.91	1.86	0.94	1.51	0.94	0.77	1.17
time (sec)	N/A	0.026	0.017	0.007	1.158	0.616	4.041	0.216	0.491	0.131

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	20	42	26	22	26	19	32
N.S.	1	1.00	1.00	0.83	1.75	1.08	0.92	1.08	0.79	1.33
time (sec)	N/A	0.017	0.019	0.010	1.118	0.547	2.131	0.196	0.294	0.131

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	38	30	67	41	31	61	30	55
N.S.	1	1.00	1.00	0.79	1.76	1.08	0.82	1.61	0.79	1.45
time (sec)	N/A	0.038	0.016	0.013	1.173	0.540	6.008	0.205	0.615	0.151

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	31	47	47	31	66	31	51
N.S.	1	1.00	1.00	0.74	1.12	1.12	0.74	1.57	0.74	1.21
time (sec)	N/A	0.037	0.021	0.014	1.257	0.612	3.597	0.216	0.329	0.166

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	49	43	59	50	88	114	43	60
N.S.	1	1.00	0.84	0.74	1.02	0.86	1.52	1.97	0.74	1.03
time (sec)	N/A	0.051	0.023	0.019	1.118	0.724	14.329	0.217	0.694	0.235

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	51	50	89	62	66	45	97	51
N.S.	1	1.00	0.88	0.86	1.53	1.07	1.14	0.78	1.67	0.88
time (sec)	N/A	0.046	0.024	0.021	1.211	0.873	14.277	0.207	1.108	0.179

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	46	37	63	58	48	39	89	44
N.S.	1	1.00	1.02	0.82	1.40	1.29	1.07	0.87	1.98	0.98
time (sec)	N/A	0.039	0.021	0.014	1.414	0.750	12.336	0.232	0.894	0.208

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	41	34	54	52	39	33	82	41
N.S.	1	1.00	1.17	0.97	1.54	1.49	1.11	0.94	2.34	1.17
time (sec)	N/A	0.027	0.017	0.007	1.227	0.697	10.675	0.254	0.841	0.195

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	17	22	31	31	16	16	20
N.S.	1	1.00	1.00	0.85	1.10	1.55	1.55	0.80	0.80	1.00
time (sec)	N/A	0.016	0.009	0.003	1.486	0.536	7.856	0.245	0.164	0.183

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	40	56	61	212	61	40	57
N.S.	1	1.00	1.00	0.87	1.22	1.33	4.61	1.33	0.87	1.24
time (sec)	N/A	0.043	0.032	0.020	1.405	0.540	19.618	0.253	0.475	0.210

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	45	47	68	77	228	82	47	65
N.S.	1	1.00	0.69	0.72	1.05	1.18	3.51	1.26	0.72	1.00
time (sec)	N/A	0.055	0.025	0.014	1.098	0.629	12.982	0.243	0.537	0.224

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	269	269	189	2295	372	1571	0	3752	1539	0
N.S.	1	1.00	0.70	8.53	1.38	5.84	0.00	13.95	5.72	0.00
time (sec)	N/A	0.161	0.557	0.027	0.885	0.819	0.000	0.459	1.777	1.202

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	153	130	129	132	134	143	121	0
N.S.	1	1.00	2.43	2.06	2.05	2.10	2.13	2.27	1.92	0.00
time (sec)	N/A	0.197	0.022	0.002	0.591	0.506	0.096	0.278	0.094	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	153	130	129	133	141	144	123	0
N.S.	1	1.00	1.00	0.85	0.84	0.87	0.92	0.94	0.80	0.00
time (sec)	N/A	0.117	0.023	0.000	0.667	0.865	0.095	0.296	0.121	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	151	130	129	133	136	144	123	0
N.S.	1	1.00	3.36	2.89	2.87	2.96	3.02	3.20	2.73	0.00
time (sec)	N/A	0.123	0.017	0.000	0.654	0.482	0.098	0.400	0.078	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	153	130	129	133	139	144	123	0
N.S.	1	1.00	1.00	0.85	0.84	0.87	0.91	0.94	0.80	0.00
time (sec)	N/A	0.085	0.017	0.002	0.499	0.813	0.097	0.288	0.081	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	149	130	129	133	133	144	123	0
N.S.	1	1.00	5.14	4.48	4.45	4.59	4.59	4.97	4.24	0.00
time (sec)	N/A	0.049	0.012	0.002	0.645	0.836	0.096	0.314	0.077	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	143	127	125	130	134	141	120	0
N.S.	1	1.00	1.00	0.89	0.87	0.91	0.94	0.99	0.84	0.00
time (sec)	N/A	0.074	0.016	0.001	0.576	0.564	0.098	0.362	0.079	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	149	132	130	127	131	145	121	0
N.S.	1	1.00	1.60	1.42	1.40	1.37	1.41	1.56	1.30	0.00
time (sec)	N/A	0.055	0.025	0.003	0.478	0.758	0.328	0.267	0.129	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	141	129	125	131	124	139	119	0
N.S.	1	1.00	1.00	0.91	0.89	0.93	0.88	0.99	0.84	0.00
time (sec)	N/A	0.082	0.026	0.005	0.504	0.880	0.321	0.361	0.081	0.001

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	147	131	130	133	131	156	120	0
N.S.	1	1.00	1.00	0.89	0.88	0.90	0.89	1.06	0.82	0.00
time (sec)	N/A	0.136	0.034	0.007	0.698	0.784	0.386	0.263	0.084	0.001

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	203	203	122	1121	192	759	0	1848	1483	0
N.S.	1	1.00	0.60	5.52	0.95	3.74	0.00	9.10	7.31	0.00
time (sec)	N/A	0.073	0.044	0.013	0.972	0.819	0.000	0.619	1.251	0.559

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	85	62	61	61	76	61	61	0
N.S.	1	1.00	2.50	1.82	1.79	1.79	2.24	1.79	1.79	0.00
time (sec)	N/A	0.047	0.002	0.001	0.637	0.560	0.072	0.339	0.062	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	83	62	61	61	75	61	61	0
N.S.	1	1.00	1.00	0.75	0.73	0.73	0.90	0.73	0.73	0.00
time (sec)	N/A	0.031	0.002	0.003	0.762	0.765	0.071	0.414	0.059	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	83	62	61	61	75	61	61	0
N.S.	1	1.00	3.61	2.70	2.65	2.65	3.26	2.65	2.65	0.00
time (sec)	N/A	0.022	0.002	0.001	0.563	0.859	0.073	0.362	0.060	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	83	62	61	61	75	61	61	0
N.S.	1	1.00	1.00	0.75	0.73	0.73	0.90	0.73	0.73	0.00
time (sec)	N/A	0.027	0.001	0.000	0.880	0.566	0.071	0.304	0.059	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	62	61	61	71	76	61	0
N.S.	1	1.00	1.00	5.64	5.55	5.55	6.45	6.91	5.55	0.00
time (sec)	N/A	0.002	0.002	0.001	0.782	0.512	0.070	0.314	0.058	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	73	58	57	57	68	57	57	0
N.S.	1	1.00	1.00	0.79	0.78	0.78	0.93	0.78	0.78	0.00
time (sec)	N/A	0.022	0.001	0.001	0.977	1.103	0.074	0.274	0.058	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	80	59	62	58	75	62	58	0
N.S.	1	1.00	1.00	0.74	0.78	0.72	0.94	0.78	0.72	0.00
time (sec)	N/A	0.033	0.003	0.003	0.821	0.846	0.107	0.230	0.061	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	73	60	59	62	66	59	59	0
N.S.	1	1.00	1.00	0.82	0.81	0.85	0.90	0.81	0.81	0.00
time (sec)	N/A	0.025	0.003	0.004	0.826	0.786	0.101	0.297	0.060	0.001

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	80	61	62	64	75	69	60	0
N.S.	1	1.00	1.00	0.76	0.78	0.80	0.94	0.86	0.75	0.00
time (sec)	N/A	0.040	0.003	0.005	0.736	0.504	0.112	0.357	0.061	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	80	90	54	129	90	101	-1	81
N.S.	1	1.00	0.55	0.62	0.37	0.89	0.62	0.70	-0.01	0.56
time (sec)	N/A	0.090	0.083	0.039	1.519	0.770	0.365	0.430	0.000	9.872

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	51	55	31	29	27	42	103	208
N.S.	1	1.00	0.61	0.66	0.37	0.35	0.33	0.51	1.24	2.51
time (sec)	N/A	0.072	0.021	0.010	0.779	0.834	0.277	0.383	0.876	0.461

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	69	62	33	98	82	59	-1	60
N.S.	1	1.00	0.71	0.64	0.34	1.01	0.85	0.61	-0.01	0.62
time (sec)	N/A	0.047	0.027	0.007	1.507	0.593	0.317	0.265	0.000	8.951

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	54	57	35	33	26	61	83	198
N.S.	1	1.00	0.59	0.62	0.38	0.36	0.28	0.66	0.90	2.15
time (sec)	N/A	0.072	0.021	0.010	0.734	0.780	0.709	0.381	0.766	0.442

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	72	67	37	105	82	62	-1	63
N.S.	1	1.00	0.71	0.66	0.37	1.04	0.81	0.61	-0.01	0.62
time (sec)	N/A	0.063	0.030	0.010	1.216	0.702	0.372	0.343	0.000	6.376

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	70	79	48	48	41	131	125	713
N.S.	1	1.00	0.51	0.58	0.35	0.35	0.30	0.96	0.91	5.20
time (sec)	N/A	0.098	0.033	0.013	0.784	0.712	0.726	0.386	0.810	1.170

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	108	188	125	300	0	0	-1	105
N.S.	1	1.00	0.71	1.23	0.82	1.96	0.00	0.00	-0.01	0.69
time (sec)	N/A	0.119	0.060	0.019	1.489	0.934	0.000	0.000	0.000	15.923

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	45	38	65	42	0	40	48	192
N.S.	1	1.00	0.58	0.49	0.84	0.55	0.00	0.52	0.62	2.49
time (sec)	N/A	0.066	0.019	0.007	0.636	0.847	0.000	0.507	0.179	0.851

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	156	156	108	186	124	301	0	0	-1	105
N.S.	1	1.00	0.69	1.19	0.79	1.93	0.00	0.00	-0.01	0.67
time (sec)	N/A	0.089	0.051	0.014	1.670	0.782	0.000	0.000	0.000	15.777

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	92	133	88	119	0	96	-1	258
N.S.	1	1.00	0.57	0.83	0.55	0.74	0.00	0.60	-0.01	1.60
time (sec)	N/A	0.118	0.041	0.021	0.888	0.645	0.000	0.528	0.000	0.885

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	190	190	124	206	134	334	0	0	-1	117
N.S.	1	1.00	0.65	1.08	0.71	1.76	0.00	0.00	-0.01	0.62
time (sec)	N/A	0.185	0.069	0.023	1.403	0.964	0.000	0.000	0.000	19.381

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	130	249	138	205	0	144	-1	1473
N.S.	1	1.00	0.58	1.12	0.62	0.92	0.00	0.65	-0.00	6.61
time (sec)	N/A	0.184	0.070	0.025	0.841	0.666	0.000	0.403	0.000	6.044

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	400	400	160	1099	491	853	0	2213	-1	0
N.S.	1	1.00	0.40	2.75	1.23	2.13	0.00	5.53	-0.00	0.00
time (sec)	N/A	0.243	0.220	0.010	0.815	0.859	0.000	0.687	0.000	5.116

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	276	276	112	495	243	381	0	1013	-1	0
N.S.	1	1.00	0.41	1.79	0.88	1.38	0.00	3.67	-0.00	0.00
time (sec)	N/A	0.153	0.110	0.009	0.964	0.693	0.000	0.524	0.000	3.138

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	86	131	75	94	0	269	-1	0
N.S.	1	1.00	0.56	0.86	0.49	0.61	0.00	1.76	-0.01	0.00
time (sec)	N/A	0.076	0.055	0.006	0.811	0.726	0.000	0.322	0.000	1.600

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	25	40	86	47	165	32	59	0
N.S.	1	1.00	0.74	1.18	2.53	1.38	4.85	0.94	1.74	0.00
time (sec)	N/A	0.029	0.008	0.002	0.722	0.709	9.597	0.289	0.142	0.132

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	45	62	135	92	0	196	108	0
N.S.	1	1.00	0.52	0.72	1.57	1.07	0.00	2.28	1.26	0.00
time (sec)	N/A	0.089	0.024	0.007	0.737	0.664	0.000	0.427	0.171	0.285

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	68	99	196	140	0	331	169	0
N.S.	1	1.00	0.53	0.77	1.53	1.09	0.00	2.59	1.32	0.00
time (sec)	N/A	0.129	0.035	0.006	0.698	0.774	0.000	0.385	0.202	0.420

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	166	226	166	193	202	193	169	0
N.S.	1	1.00	1.00	1.36	1.00	1.16	1.22	1.16	1.02	0.00
time (sec)	N/A	0.393	0.050	0.000	0.604	0.714	0.102	0.338	0.076	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	166	226	166	193	204	193	169	0
N.S.	1	1.00	1.00	1.36	1.00	1.16	1.23	1.16	1.02	0.00
time (sec)	N/A	0.154	0.054	0.000	0.726	0.615	0.101	0.270	0.096	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	154	226	166	193	199	193	169	0
N.S.	1	1.00	0.93	1.36	1.00	1.16	1.20	1.16	1.02	0.00
time (sec)	N/A	0.287	0.056	0.000	0.768	0.596	0.102	0.287	0.048	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	161	223	163	189	199	189	165	0
N.S.	1	1.00	1.00	1.39	1.01	1.17	1.24	1.17	1.02	0.00
time (sec)	N/A	0.118	0.046	0.000	0.730	0.764	0.102	0.264	0.049	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	162	191	167	164	199	193	166	0
N.S.	1	1.00	1.00	1.18	1.03	1.01	1.23	1.19	1.02	0.00
time (sec)	N/A	0.227	0.056	0.005	0.787	0.818	0.309	0.335	0.103	0.001

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	156	156	156	186	162	168	185	185	163	0
N.S.	1	1.00	1.00	1.19	1.04	1.08	1.19	1.19	1.04	0.00
time (sec)	N/A	0.108	0.082	0.004	0.707	0.955	0.307	0.306	0.051	0.001

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	162	190	167	170	197	212	166	0
N.S.	1	1.00	1.00	1.17	1.03	1.05	1.22	1.31	1.02	0.00
time (sec)	N/A	0.226	0.071	0.007	0.785	0.842	0.401	0.404	0.056	0.001

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	126	261	0	421	620	126	1343	0
N.S.	1	1.00	0.95	1.96	0.00	3.17	4.66	0.95	10.10	0.00
time (sec)	N/A	0.207	0.064	0.007	0.000	1.586	43.892	1.873	0.458	0.001

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	93	175	0	312	434	91	979	0
N.S.	1	1.00	0.96	1.80	0.00	3.22	4.47	0.94	10.09	0.00
time (sec)	N/A	0.116	0.068	0.004	0.000	0.772	10.545	1.812	0.646	0.001

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	71	98	0	219	287	67	606	0
N.S.	1	1.00	1.00	1.38	0.00	3.08	4.04	0.94	8.54	0.00
time (sec)	N/A	0.070	0.046	0.003	0.000	0.827	3.589	1.715	0.498	0.001

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	128	105	0	249	0	78	2424	0
N.S.	1	1.00	1.64	1.35	0.00	3.19	0.00	1.00	31.08	0.00
time (sec)	N/A	0.139	0.103	0.007	0.000	1.145	0.000	1.610	4.476	0.001

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	186	191	0	385	0	124	3729	0
N.S.	1	1.00	1.66	1.71	0.00	3.44	0.00	1.11	33.29	0.00
time (sec)	N/A	0.245	0.218	0.010	0.000	1.075	0.000	1.874	4.855	0.001

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	261	261	327	825	0	5140	0	4391	10177	0
N.S.	1	1.00	1.25	3.16	0.00	19.69	0.00	16.82	38.99	0.00
time (sec)	N/A	1.489	0.405	0.053	0.000	3.519	0.000	3.789	1.594	0.001

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	208	208	251	560	0	2632	0	3179	6366	0
N.S.	1	1.00	1.21	2.69	0.00	12.65	0.00	15.28	30.61	0.00
time (sec)	N/A	0.528	0.163	0.026	0.000	1.296	0.000	3.498	1.256	0.001

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	173	328	0	1569	314	1400	4109	0
N.S.	1	1.00	1.01	1.91	0.00	9.12	1.83	8.14	23.89	0.00
time (sec)	N/A	0.201	0.094	0.020	0.000	1.327	16.959	2.416	0.997	0.001

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	206	353	0	2914	0	2805	6335	0
N.S.	1	1.00	1.09	1.87	0.00	15.42	0.00	14.84	33.52	0.00
time (sec)	N/A	0.400	0.291	0.030	0.000	1.475	0.000	3.568	1.354	0.001

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	271	271	267	611	0	5442	0	2870	10101	0
N.S.	1	1.00	0.99	2.25	0.00	20.08	0.00	10.59	37.27	0.00
time (sec)	N/A	0.653	0.316	0.033	0.000	3.554	0.000	3.346	2.187	0.001

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	212	212	208	689	0	1323	0	239	2282	0
N.S.	1	1.00	0.98	3.25	0.00	6.24	0.00	1.13	10.76	0.00
time (sec)	N/A	0.381	0.282	0.020	0.000	0.779	0.000	1.649	0.834	0.001

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	160	286	0	849	0	194	1527	0
N.S.	1	1.00	1.09	1.95	0.00	5.78	0.00	1.32	10.39	0.00
time (sec)	N/A	0.175	0.186	0.014	0.000	1.008	0.000	1.617	1.217	0.001

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	111	158	0	538	394	120	283	0
N.S.	1	1.00	1.04	1.48	0.00	5.03	3.68	1.12	2.64	0.00
time (sec)	N/A	0.113	0.080	0.012	0.000	0.800	5.380	1.723	0.323	0.001

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	101	127	0	474	374	102	264	0
N.S.	1	1.00	1.07	1.35	0.00	5.04	3.98	1.09	2.81	0.00
time (sec)	N/A	0.088	0.074	0.007	0.000	0.691	3.363	1.367	0.301	0.001

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	243	361	0	1014	0	201	7119	0
N.S.	1	1.00	1.62	2.41	0.00	6.76	0.00	1.34	47.46	0.00
time (sec)	N/A	0.331	0.330	0.018	0.000	1.817	0.000	1.710	7.884	0.001

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	379	622	0	1635	0	250	10034	0
N.S.	1	1.00	1.70	2.79	0.00	7.33	0.00	1.12	45.00	0.00
time (sec)	N/A	0.419	0.564	0.023	0.000	3.863	0.000	1.647	9.088	0.001

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	425	425	455	1507	0	7252	0	5681	16604	0
N.S.	1	1.00	1.07	3.55	0.00	17.06	0.00	13.37	39.07	0.00
time (sec)	N/A	3.675	1.202	0.043	0.000	8.558	0.000	6.490	4.358	0.001

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	336	336	362	1030	0	4658	0	4538	12396	0
N.S.	1	1.00	1.08	3.07	0.00	13.86	0.00	13.51	36.89	0.00
time (sec)	N/A	1.717	0.853	0.036	0.000	2.817	0.000	6.288	5.184	0.001

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	276	276	298	733	0	3467	0	3776	9444	0
N.S.	1	1.00	1.08	2.66	0.00	12.56	0.00	13.68	34.22	0.00
time (sec)	N/A	0.553	0.664	0.029	0.000	1.916	0.000	4.777	4.411	0.001

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	293	293	304	1761	0	4885	0	4426	12349	0
N.S.	1	1.00	1.04	6.01	0.00	16.67	0.00	15.11	42.15	0.00
time (sec)	N/A	0.846	0.786	0.110	0.000	3.721	0.000	5.965	4.839	0.001

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	389	389	382	1252	0	7583	0	5408	17591	0
N.S.	1	1.00	0.98	3.22	0.00	19.49	0.00	13.90	45.22	0.00
time (sec)	N/A	1.220	1.039	0.040	0.000	9.541	0.000	6.160	5.380	0.001

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	522	522	487	1653	0	10190	0	6327	21554	0
N.S.	1	1.00	0.93	3.17	0.00	19.52	0.00	12.12	41.29	0.00
time (sec)	N/A	1.365	1.199	0.047	0.000	22.200	0.000	8.150	5.698	0.001

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	365	365	435	2054	0	3196	0	598	4501	0
N.S.	1	1.00	1.19	5.63	0.00	8.76	0.00	1.64	12.33	0.00
time (sec)	N/A	1.455	0.663	0.031	0.000	1.408	0.000	5.883	4.660	0.001

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	254	254	354	723	0	2167	0	466	3062	0
N.S.	1	1.00	1.39	2.85	0.00	8.53	0.00	1.83	12.06	0.00
time (sec)	N/A	0.404	0.479	0.026	0.000	1.163	0.000	6.311	5.151	0.001

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	261	398	0	1378	775	318	593	0
N.S.	1	1.00	1.79	2.73	0.00	9.44	5.31	2.18	4.06	0.00
time (sec)	N/A	0.139	0.268	0.017	0.000	0.948	102.043	6.463	0.694	0.001

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	185	233	411	0	1369	833	268	625	0
N.S.	1	1.00	1.26	2.22	0.00	7.40	4.50	1.45	3.38	0.00
time (sec)	N/A	0.262	0.236	0.020	0.000	0.953	44.844	6.587	0.680	0.001

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	170	170	172	379	0	1226	789	228	587	0
N.S.	1	1.00	1.01	2.23	0.00	7.21	4.64	1.34	3.45	0.00
time (sec)	N/A	0.163	0.204	0.017	0.000	0.928	21.367	6.057	0.660	0.001

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	142	262	0	1109	661	208	517	0
N.S.	1	1.00	1.02	1.88	0.00	7.98	4.76	1.50	3.72	0.00
time (sec)	N/A	0.124	0.135	0.010	0.000	0.688	12.404	5.570	0.587	0.001

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	396	1161	0	2494	0	421	11674	0
N.S.	1	1.00	1.57	4.61	0.00	9.90	0.00	1.67	46.33	0.00
time (sec)	N/A	0.543	0.691	0.029	0.000	7.448	0.000	6.445	11.572	0.001

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	363	363	642	1862	0	3956	0	648	16265	0
N.S.	1	1.00	1.77	5.13	0.00	10.90	0.00	1.79	44.81	0.00
time (sec)	N/A	0.771	1.500	0.039	0.000	14.441	0.000	6.371	15.906	0.001

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	554	554	644	2015	0	9636	0	3987	22911	0
N.S.	1	1.00	1.16	3.64	0.00	17.39	0.00	7.20	41.36	0.00
time (sec)	N/A	11.195	2.398	0.069	0.000	20.777	0.000	8.280	5.047	0.001

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	461	461	543	1631	0	7060	0	7578	19041	0
N.S.	1	1.00	1.18	3.54	0.00	15.31	0.00	16.44	41.30	0.00
time (sec)	N/A	4.622	2.021	0.048	0.000	6.935	0.000	11.662	3.951	0.001

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	380	380	447	1283	0	5650	0	3162	16688	0
N.S.	1	1.00	1.18	3.38	0.00	14.87	0.00	8.32	43.92	0.00
time (sec)	N/A	1.415	1.696	0.049	0.000	5.874	0.000	8.261	3.487	0.001

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	438	438	436	1335	0	7270	0	7267	18992	0
N.S.	1	1.00	1.00	3.05	0.00	16.60	0.00	16.59	43.36	0.00
time (sec)	N/A	1.090	1.646	0.046	0.000	9.419	0.000	11.773	3.916	0.001

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	460	460	516	11936	0	9909	0	4609	22914	0
N.S.	1	1.00	1.12	25.95	0.00	21.54	0.00	10.02	49.81	0.00
time (sec)	N/A	1.353	2.189	0.281	0.000	23.684	0.000	8.537	4.614	0.001

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	18	17	17	17	19	17	0
N.S.	1	1.00	1.00	0.72	0.68	0.68	0.68	0.76	0.68	0.00
time (sec)	N/A	0.018	0.009	0.006	0.718	1.144	0.121	0.305	0.057	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	18	25	17	17	19	17	0
N.S.	1	1.00	1.00	0.72	1.00	0.68	0.68	0.76	0.68	0.00
time (sec)	N/A	0.029	0.006	0.004	0.739	0.665	0.115	0.283	0.029	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	31	30	30	37	30	32	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	1.00	0.81	0.86	0.00
time (sec)	N/A	0.035	0.013	0.003	1.401	0.944	0.120	0.282	0.212	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	31	53	30	37	30	32	0
N.S.	1	1.00	1.00	0.84	1.43	0.81	1.00	0.81	0.86	0.00
time (sec)	N/A	0.042	0.006	0.003	1.597	1.189	0.121	0.372	0.034	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	45	41	0	47	44	38	41	0
N.S.	1	1.00	1.00	0.91	0.00	1.04	0.98	0.84	0.91	0.00
time (sec)	N/A	0.049	0.026	0.006	0.000	0.830	0.152	0.944	0.052	0.001

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	71	91	104	61	0	102	102	69
N.S.	1	1.00	0.70	0.89	1.02	0.60	0.00	1.00	1.00	0.68
time (sec)	N/A	0.079	0.029	0.029	0.672	1.361	0.000	0.465	0.490	0.178

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	66	74	87	56	0	88	85	64
N.S.	1	1.00	0.81	0.91	1.07	0.69	0.00	1.09	1.05	0.79
time (sec)	N/A	0.056	0.021	0.015	0.593	1.283	0.000	0.363	0.433	0.146

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	61	57	70	51	0	74	67	59
N.S.	1	1.00	0.82	0.77	0.95	0.69	0.00	1.00	0.91	0.80
time (sec)	N/A	0.044	0.017	0.011	0.653	1.283	0.000	0.342	0.287	0.148

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	92	85	89	95	0	98	86	93
N.S.	1	1.00	0.98	0.90	0.95	1.01	0.00	1.04	0.91	0.99
time (sec)	N/A	0.083	0.038	0.015	1.496	1.261	0.000	0.478	0.426	0.204

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	97	104	89	112	0	138	84	96
N.S.	1	1.00	1.00	1.07	0.92	1.15	0.00	1.42	0.87	0.99
time (sec)	N/A	0.083	0.041	0.016	1.350	0.711	0.000	0.515	0.878	0.224

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	97	121	106	112	0	169	-1	98
N.S.	1	1.00	0.98	1.22	1.07	1.13	0.00	1.71	-0.01	0.99
time (sec)	N/A	0.083	0.039	0.016	1.426	0.924	0.000	0.572	0.000	0.272

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	74	118	99	90	0	189	-1	75
N.S.	1	1.00	0.82	1.31	1.10	1.00	0.00	2.10	-0.01	0.83
time (sec)	N/A	0.068	0.026	0.015	1.453	0.754	0.000	0.532	0.000	0.326

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	82	135	116	95	0	233	-1	80
N.S.	1	1.00	0.74	1.22	1.05	0.86	0.00	2.10	-0.01	0.72
time (sec)	N/A	0.086	0.027	0.017	1.736	1.217	0.000	0.457	0.000	0.368

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	84	152	133	100	0	255	-1	85
N.S.	1	1.00	0.64	1.15	1.01	0.76	0.00	1.93	-0.01	0.64
time (sec)	N/A	0.109	0.037	0.019	1.730	0.932	0.000	0.563	0.000	0.448

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	81	138	135	71	0	207	-1	79
N.S.	1	1.00	0.64	1.09	1.06	0.56	0.00	1.63	-0.01	0.62
time (sec)	N/A	0.096	0.035	0.033	0.585	0.996	0.000	0.541	0.000	0.303

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	76	121	118	66	0	179	-1	74
N.S.	1	1.00	0.72	1.14	1.11	0.62	0.00	1.69	-0.01	0.70
time (sec)	N/A	0.072	0.031	0.016	0.878	1.022	0.000	0.587	0.000	0.245

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	71	104	101	61	0	151	127	69
N.S.	1	1.00	0.72	1.05	1.02	0.62	0.00	1.53	1.28	0.70
time (sec)	N/A	0.058	0.025	0.016	0.666	0.765	0.000	0.452	0.530	0.242

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	104	117	120	106	0	113	-1	103
N.S.	1	1.00	0.87	0.98	1.01	0.89	0.00	0.95	-0.01	0.87
time (sec)	N/A	0.106	0.058	0.015	1.396	1.232	0.000	0.498	0.000	0.320

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	107	117	120	122	0	153	-1	106
N.S.	1	1.00	0.88	0.96	0.98	1.00	0.00	1.25	-0.01	0.87
time (sec)	N/A	0.107	0.054	0.021	1.746	1.035	0.000	0.559	0.000	0.346

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	107	117	137	122	0	190	-1	108
N.S.	1	1.00	0.84	0.92	1.08	0.96	0.00	1.50	-0.01	0.85
time (sec)	N/A	0.109	0.062	0.020	1.461	1.155	0.000	0.654	0.000	0.392

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	107	117	154	122	0	227	-1	108
N.S.	1	1.00	0.84	0.92	1.21	0.96	0.00	1.79	-0.01	0.85
time (sec)	N/A	0.107	0.051	0.022	1.585	1.021	0.000	0.681	0.000	0.534

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	139	286	0	315	0	138	-1	135
N.S.	1	1.00	0.91	1.87	0.00	2.06	0.00	0.90	-0.01	0.88
time (sec)	N/A	0.203	0.111	0.030	0.000	0.727	0.000	0.544	0.000	0.540

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	101	176	0	233	0	98	-1	104
N.S.	1	1.00	1.01	1.76	0.00	2.33	0.00	0.98	-0.01	1.04
time (sec)	N/A	0.093	0.047	0.015	0.000	1.093	0.000	0.449	0.000	0.473

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	78	93	0	178	0	69	92	78
N.S.	1	1.00	1.03	1.22	0.00	2.34	0.00	0.91	1.21	1.03
time (sec)	N/A	0.064	0.025	0.012	0.000	1.300	0.000	0.461	1.046	0.385

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	89	76	0	517	0	0	81	89
N.S.	1	1.00	0.99	0.84	0.00	5.74	0.00	0.00	0.90	0.99
time (sec)	N/A	0.094	0.028	0.013	0.000	1.250	0.000	0.000	0.760	0.335

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	82	104	0	197	0	124	103	81
N.S.	1	1.00	1.02	1.30	0.00	2.46	0.00	1.55	1.29	1.01
time (sec)	N/A	0.083	0.035	0.018	0.000	1.330	0.000	0.538	0.776	0.392

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	107	194	0	255	0	339	-1	148
N.S.	1	1.00	0.86	1.56	0.00	2.06	0.00	2.73	-0.01	1.19
time (sec)	N/A	0.145	0.075	0.019	0.000	1.592	0.000	0.516	0.000	0.637

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	177	177	148	311	0	339	0	571	-1	188
N.S.	1	1.00	0.84	1.76	0.00	1.92	0.00	3.23	-0.01	1.06
time (sec)	N/A	0.239	0.108	0.020	0.000	1.884	0.000	0.639	0.000	0.905

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	66	87	90	56	0	60	-1	64
N.S.	1	1.00	0.67	0.89	0.92	0.57	0.00	0.61	-0.01	0.65
time (sec)	N/A	0.087	0.028	0.019	0.926	0.620	0.000	0.370	0.000	0.201

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	61	70	73	51	0	53	-1	59
N.S.	1	1.00	0.79	0.91	0.95	0.66	0.00	0.69	-0.01	0.77
time (sec)	N/A	0.066	0.020	0.014	0.988	0.968	0.000	0.344	0.000	0.185

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	56	53	56	46	0	46	-1	54
N.S.	1	1.00	1.00	0.95	1.00	0.82	0.00	0.82	-0.02	0.96
time (sec)	N/A	0.045	0.016	0.013	1.040	0.984	0.000	0.348	0.000	0.182

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	36	39	39	0	39	35	47
N.S.	1	1.00	1.00	0.73	0.80	0.80	0.00	0.80	0.71	0.96
time (sec)	N/A	0.032	0.009	0.012	0.923	0.748	0.000	0.468	0.523	0.170

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	69	52	58	75	0	78	56	68
N.S.	1	1.00	1.00	0.75	0.84	1.09	0.00	1.13	0.81	0.99
time (sec)	N/A	0.062	0.013	0.012	1.929	0.804	0.000	0.435	1.009	0.193

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	62	49	51	78	0	101	83	63
N.S.	1	1.00	1.00	0.79	0.82	1.26	0.00	1.63	1.34	1.02
time (sec)	N/A	0.050	0.015	0.015	2.021	0.782	0.000	0.386	0.663	0.251

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	67	66	68	83	0	145	-1	70
N.S.	1	1.00	0.81	0.80	0.82	1.00	0.00	1.75	-0.01	0.84
time (sec)	N/A	0.070	0.022	0.013	2.000	0.812	0.000	0.521	0.000	0.284

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	77	83	85	90	0	167	-1	75
N.S.	1	1.00	0.74	0.80	0.82	0.87	0.00	1.61	-0.01	0.72
time (sec)	N/A	0.088	0.024	0.017	1.981	0.860	0.000	0.507	0.000	0.339

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	72	91	73	86	0	52	-1	59
N.S.	1	1.00	0.94	1.18	0.95	1.12	0.00	0.68	-0.01	0.77
time (sec)	N/A	0.058	0.018	0.022	0.923	1.007	0.000	0.383	0.000	0.338

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	54	95	56	81	0	46	52	54
N.S.	1	1.00	0.96	1.70	1.00	1.45	0.00	0.82	0.93	0.96
time (sec)	N/A	0.043	0.121	0.013	0.933	0.990	0.000	0.512	0.312	0.273

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	22	32	46	0	21	21	25
N.S.	1	1.00	1.00	0.88	1.28	1.84	0.00	0.84	0.84	1.00
time (sec)	N/A	0.019	0.092	0.006	0.938	0.904	0.000	0.363	0.238	0.285

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	66	67	65	107	0	78	-1	67
N.S.	1	1.00	1.00	1.02	0.98	1.62	0.00	1.18	-0.02	1.02
time (sec)	N/A	0.057	0.024	0.019	2.006	0.935	0.000	0.556	0.000	0.364

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	88	84	82	124	0	122	-1	75
N.S.	1	1.00	0.98	0.93	0.91	1.38	0.00	1.36	-0.01	0.83
time (sec)	N/A	0.071	0.018	0.016	2.088	1.094	0.000	0.492	0.000	0.418

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	243	243	191	1935	408	1357	11538	2816	769	0
N.S.	1	1.00	0.79	7.96	1.68	5.58	47.48	11.59	3.16	0.00
time (sec)	N/A	0.176	0.298	0.009	1.388	0.860	12.378	0.750	1.058	1.208

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	155	155	117	783	230	573	4190	1178	429	0
N.S.	1	1.00	0.75	5.05	1.48	3.70	27.03	7.60	2.77	0.00
time (sec)	N/A	0.099	0.123	0.008	1.221	0.704	5.444	0.398	0.598	0.817

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	59	221	104	171	1056	350	171	0
N.S.	1	1.00	0.71	2.66	1.25	2.06	12.72	4.22	2.06	0.00
time (sec)	N/A	0.047	0.049	0.004	1.058	0.717	1.862	0.419	0.339	0.118

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	134	122	120	277	0	121	181	0
N.S.	1	1.00	1.00	0.91	0.90	2.07	0.00	0.90	1.35	0.00
time (sec)	N/A	0.179	0.063	0.014	1.998	11.001	0.000	0.309	0.873	0.001

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	99	108	107	212	0	105	166	0
N.S.	1	1.00	0.84	0.92	0.91	1.80	0.00	0.89	1.41	0.00
time (sec)	N/A	0.152	0.094	0.010	2.018	5.176	0.000	0.307	0.722	0.001

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	77	92	89	170	0	90	138	0
N.S.	1	1.00	0.73	0.88	0.85	1.62	0.00	0.86	1.31	0.00
time (sec)	N/A	0.137	0.034	0.009	1.998	2.920	0.000	0.356	0.989	0.001

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	66	83	82	145	0	86	944	0
N.S.	1	1.00	0.69	0.86	0.85	1.51	0.00	0.90	9.83	0.00
time (sec)	N/A	0.094	0.040	0.007	1.985	1.332	0.000	0.393	1.936	0.001

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	67	83	82	146	0	85	328	0
N.S.	1	1.00	0.70	0.86	0.85	1.52	0.00	0.89	3.42	0.00
time (sec)	N/A	0.064	0.037	0.007	1.985	1.500	0.000	0.354	1.020	0.001

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	134	101	101	201	0	102	527	0
N.S.	1	1.00	1.18	0.89	0.89	1.76	0.00	0.89	4.62	0.00
time (sec)	N/A	0.125	0.071	0.010	1.966	16.718	0.000	0.289	0.961	0.001

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	169	119	120	265	0	132	820	0
N.S.	1	1.00	1.31	0.92	0.93	2.05	0.00	1.02	6.36	0.00
time (sec)	N/A	0.150	0.101	0.013	1.997	101.424	0.000	0.302	1.381	0.001

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	156	156	209	145	145	0	0	168	1017	0
N.S.	1	1.00	1.34	0.93	0.93	0.00	0.00	1.08	6.52	0.00
time (sec)	N/A	0.183	0.093	0.016	2.050	0.000	0.000	0.342	1.868	0.001

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	359	359	344	405	294	4414	0	363	6097	0
N.S.	1	1.00	0.96	1.13	0.82	12.30	0.00	1.01	16.98	0.00
time (sec)	N/A	0.346	0.360	0.017	2.048	32.470	0.000	0.541	2.056	0.001

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	345	345	373	387	289	4354	0	333	5908	0
N.S.	1	1.00	1.08	1.12	0.84	12.62	0.00	0.97	17.12	0.00
time (sec)	N/A	0.302	0.238	0.008	2.089	4.864	0.000	0.444	1.827	0.001

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	336	336	233	363	268	4040	0	327	5111	0
N.S.	1	1.00	0.69	1.08	0.80	12.02	0.00	0.97	15.21	0.00
time (sec)	N/A	0.274	0.151	0.009	2.627	1.764	0.000	0.434	2.199	0.001

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	337	337	232	351	275	3892	0	336	4720	0
N.S.	1	1.00	0.69	1.04	0.82	11.55	0.00	1.00	14.01	0.00
time (sec)	N/A	0.267	0.126	0.009	1.427	1.776	0.000	0.377	1.589	0.001

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	336	336	234	363	268	4084	0	339	4802	0
N.S.	1	1.00	0.70	1.08	0.80	12.15	0.00	1.01	14.29	0.00
time (sec)	N/A	0.273	0.145	0.008	1.090	3.200	0.000	0.416	1.672	0.001

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	348	348	389	390	292	4362	0	348	5761	0
N.S.	1	1.00	1.12	1.12	0.84	12.53	0.00	1.00	16.55	0.00
time (sec)	N/A	0.300	0.253	0.012	1.274	9.080	0.000	0.398	1.999	0.001

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	360	360	367	406	297	4442	0	364	5972	0
N.S.	1	1.00	1.02	1.13	0.82	12.34	0.00	1.01	16.59	0.00
time (sec)	N/A	0.301	0.401	0.012	2.123	26.625	0.000	0.395	2.257	0.001

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	135	305	220	555	0	251	305	0
N.S.	1	1.00	0.80	1.80	1.30	3.28	0.00	1.49	1.80	0.00
time (sec)	N/A	0.367	0.210	0.020	2.075	35.277	0.000	0.364	1.302	0.001

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	142	260	197	457	0	223	647	0
N.S.	1	1.00	0.95	1.73	1.31	3.05	0.00	1.49	4.31	0.00
time (sec)	N/A	0.247	0.112	0.019	2.047	17.039	0.000	0.351	1.487	0.001

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	155	153	120	252	192	487	0	220	528	0
N.S.	1	0.99	0.77	1.63	1.24	3.14	0.00	1.42	3.41	0.00
time (sec)	N/A	0.248	0.153	0.016	2.077	6.195	0.000	0.331	1.520	0.001

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	148	114	247	186	492	0	188	527	0
N.S.	1	0.99	0.77	1.66	1.25	3.30	0.00	1.26	3.54	0.00
time (sec)	N/A	0.187	0.141	0.015	2.027	7.088	0.000	0.281	1.411	0.001

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	151	151	117	255	196	458	0	199	649	0
N.S.	1	1.00	0.77	1.69	1.30	3.03	0.00	1.32	4.30	0.00
time (sec)	N/A	0.181	0.133	0.017	2.030	16.308	0.000	0.383	1.493	0.001

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	209	209	241	309	228	0	0	279	1082	0
N.S.	1	1.00	1.15	1.48	1.09	0.00	0.00	1.33	5.18	0.00
time (sec)	N/A	0.239	0.183	0.023	2.054	0.000	0.000	0.368	2.577	0.001

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	236	236	248	332	278	0	0	344	1337	0
N.S.	1	1.00	1.05	1.41	1.18	0.00	0.00	1.46	5.67	0.00
time (sec)	N/A	0.261	0.427	0.025	2.007	0.000	0.000	0.353	2.942	0.001

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	265	265	278	363	332	0	0	350	1545	0
N.S.	1	1.00	1.05	1.37	1.25	0.00	0.00	1.32	5.83	0.00
time (sec)	N/A	0.327	0.391	0.025	2.082	0.000	0.000	0.367	3.477	0.001

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	712	712	431	873	504	9856	0	581	18343	0
N.S.	1	1.00	0.61	1.23	0.71	13.84	0.00	0.82	25.76	0.00
time (sec)	N/A	0.671	0.304	0.017	2.064	43.130	0.000	0.573	2.864	0.001

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	687	687	428	852	476	9822	0	595	17909	0
N.S.	1	1.00	0.62	1.24	0.69	14.30	0.00	0.87	26.07	0.00
time (sec)	N/A	0.602	0.369	0.019	2.079	27.277	0.000	0.587	2.825	0.001

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	685	685	423	848	490	9678	0	586	17180	0
N.S.	1	1.00	0.62	1.24	0.72	14.13	0.00	0.86	25.08	0.00
time (sec)	N/A	0.609	0.272	0.016	2.095	24.280	0.000	0.466	4.873	0.001

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	685	685	428	852	472	9774	0	603	17812	0
N.S.	1	1.00	0.62	1.24	0.69	14.27	0.00	0.88	26.00	0.00
time (sec)	N/A	0.560	0.283	0.017	2.148	26.340	0.000	0.502	2.867	0.001

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	689	689	429	873	506	9892	0	603	17945	0
N.S.	1	1.00	0.62	1.27	0.73	14.36	0.00	0.88	26.04	0.00
time (sec)	N/A	0.601	0.291	0.018	2.060	41.284	0.000	0.466	2.732	0.001

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	745	745	499	911	521	10188	0	639	24015	0
N.S.	1	1.00	0.67	1.22	0.70	13.68	0.00	0.86	32.23	0.00
time (sec)	N/A	0.772	0.370	0.020	2.111	111.346	0.000	0.452	5.161	0.001

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	751	751	513	932	543	0	0	628	20828	0
N.S.	1	1.00	0.68	1.24	0.72	0.00	0.00	0.84	27.73	0.00
time (sec)	N/A	0.686	0.386	0.021	2.110	0.000	0.000	0.497	5.219	0.001

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	243	243	142	159	124	206	0	156	-1	126
N.S.	1	1.00	0.58	0.65	0.51	0.85	0.00	0.64	-0.00	0.52
time (sec)	N/A	0.130	0.184	0.011	0.918	0.855	0.000	0.376	0.000	0.197

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	56	51	50	50	0	68	-1	56
N.S.	1	1.00	0.52	0.47	0.46	0.46	0.00	0.63	-0.01	0.52
time (sec)	N/A	0.101	0.030	0.004	0.898	0.604	0.000	0.336	0.000	0.086

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	178	178	121	119	81	155	0	109	-1	100
N.S.	1	1.00	0.68	0.67	0.46	0.87	0.00	0.61	-0.01	0.56
time (sec)	N/A	0.076	0.113	0.007	1.080	0.942	0.000	0.461	0.000	0.157

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	83	80	45	123	0	84	-1	82
N.S.	1	1.00	0.55	0.53	0.30	0.81	0.00	0.55	-0.01	0.54
time (sec)	N/A	0.095	0.046	0.007	1.453	0.950	0.000	0.445	0.000	0.109

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	177	177	122	128	59	134	0	116	-1	90
N.S.	1	1.00	0.69	0.72	0.33	0.76	0.00	0.66	-0.01	0.51
time (sec)	N/A	0.091	0.109	0.013	1.080	0.883	0.000	0.471	0.000	0.198

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	177	177	90	133	83	141	0	100	-1	88
N.S.	1	1.00	0.51	0.75	0.47	0.80	0.00	0.56	-0.01	0.50
time (sec)	N/A	0.119	0.045	0.012	1.225	0.869	0.000	0.443	0.000	0.153

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	72	73	72	79	76	79	73	0
N.S.	1	1.00	0.92	0.94	0.92	1.01	0.97	1.01	0.94	0.00
time (sec)	N/A	0.135	0.026	0.001	1.106	0.831	0.079	0.267	0.038	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	78	73	72	79	82	79	73	0
N.S.	1	1.00	1.00	0.94	0.92	1.01	1.05	1.01	0.94	0.00
time (sec)	N/A	0.064	0.016	0.001	1.189	0.790	0.079	0.388	0.028	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	72	73	72	79	76	79	73	0
N.S.	1	1.00	0.96	0.97	0.96	1.05	1.01	1.05	0.97	0.00
time (sec)	N/A	0.131	0.022	0.000	1.211	0.809	0.077	0.269	0.029	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	73	70	69	76	78	76	70	0
N.S.	1	1.00	1.00	0.96	0.95	1.04	1.07	1.04	0.96	0.00
time (sec)	N/A	0.044	0.015	0.002	1.173	0.518	0.077	0.354	0.029	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	74	77	73	70	73	79	70	0
N.S.	1	1.00	1.00	1.04	0.99	0.95	0.99	1.07	0.95	0.00
time (sec)	N/A	0.090	0.020	0.003	1.120	0.897	0.171	0.263	0.034	0.001

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	71	75	69	74	73	74	70	0
N.S.	1	1.00	1.00	1.06	0.97	1.04	1.03	1.04	0.99	0.00
time (sec)	N/A	0.048	0.032	0.003	1.070	0.845	0.164	0.361	0.033	0.001

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	71	76	73	76	71	97	70	0
N.S.	1	1.00	0.96	1.03	0.99	1.03	0.96	1.31	0.95	0.00
time (sec)	N/A	0.096	0.042	0.007	1.094	0.891	0.263	0.379	0.038	0.001

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	168	168	165	214	165	426	320	160	251	0
N.S.	1	1.00	0.98	1.27	0.98	2.54	1.90	0.95	1.49	0.00
time (sec)	N/A	0.234	0.138	0.013	2.487	0.986	1.177	0.321	0.325	0.001

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	133	176	130	350	189	125	179	0
N.S.	1	1.00	0.99	1.30	0.96	2.59	1.40	0.93	1.33	0.00
time (sec)	N/A	0.159	0.078	0.012	2.451	0.993	1.084	0.277	0.322	0.001

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	102	141	95	302	162	91	95	0
N.S.	1	1.00	0.96	1.33	0.90	2.85	1.53	0.86	0.90	0.00
time (sec)	N/A	0.106	0.065	0.010	2.517	1.029	0.966	0.335	0.342	0.001

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	88	118	84	268	153	75	77	0
N.S.	1	1.00	1.06	1.42	1.01	3.23	1.84	0.90	0.93	0.00
time (sec)	N/A	0.093	0.054	0.010	2.335	1.085	0.770	0.406	0.357	0.001

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	86	89	121	87	267	155	83	81	0
N.S.	1	0.97	1.00	1.36	0.98	3.00	1.74	0.93	0.91	0.00
time (sec)	N/A	0.118	0.061	0.013	2.462	0.899	1.118	0.293	0.367	0.001

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	105	146	103	316	167	94	98	0
N.S.	1	1.00	0.99	1.38	0.97	2.98	1.58	0.89	0.92	0.00
time (sec)	N/A	0.137	0.063	0.013	2.385	0.882	1.527	0.261	0.357	0.001

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	135	183	139	360	284	131	128	0
N.S.	1	1.00	0.99	1.35	1.02	2.65	2.09	0.96	0.94	0.00
time (sec)	N/A	0.252	0.088	0.015	2.463	0.858	2.133	0.327	0.383	0.001

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	166	221	174	436	328	164	156	0
N.S.	1	1.00	0.99	1.32	1.04	2.61	1.96	0.98	0.93	0.00
time (sec)	N/A	0.330	0.095	0.016	2.570	0.917	2.679	0.417	0.403	0.001

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	170	239	175	504	235	160	223	0
N.S.	1	1.00	0.98	1.38	1.01	2.91	1.36	0.92	1.29	0.00
time (sec)	N/A	0.321	0.111	0.015	2.466	0.727	3.583	0.359	0.354	0.001

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	141	202	139	462	212	125	137	0
N.S.	1	1.00	0.99	1.41	0.97	3.23	1.48	0.87	0.96	0.00
time (sec)	N/A	0.210	0.087	0.013	2.509	0.608	3.370	0.471	0.344	0.001

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	122	179	126	421	201	107	118	0
N.S.	1	1.00	0.98	1.44	1.02	3.40	1.62	0.86	0.95	0.00
time (sec)	N/A	0.138	0.104	0.011	2.461	0.579	2.621	0.263	0.386	0.001

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	110	131	121	391	196	101	112	0
N.S.	1	1.00	0.96	1.14	1.05	3.40	1.70	0.88	0.97	0.00
time (sec)	N/A	0.116	0.096	0.010	2.462	0.647	1.501	0.312	0.379	0.001

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	124	124	182	129	421	202	110	118	0
N.S.	1	0.98	0.98	1.43	1.02	3.31	1.59	0.87	0.93	0.00
time (sec)	N/A	0.204	0.135	0.013	2.650	0.666	2.142	0.316	0.390	0.001

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	141	207	147	476	214	128	138	0
N.S.	1	1.00	0.99	1.46	1.04	3.35	1.51	0.90	0.97	0.00
time (sec)	N/A	0.217	0.085	0.016	2.594	0.781	2.924	0.340	0.398	0.001

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	171	173	245	183	514	330	164	168	0
N.S.	1	1.00	1.01	1.43	1.07	3.01	1.93	0.96	0.98	0.00
time (sec)	N/A	0.373	0.114	0.016	2.453	0.703	3.757	0.347	0.412	0.001

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	230	230	228	538	0	0	0	236	7024	0
N.S.	1	1.00	0.99	2.34	0.00	0.00	0.00	1.03	30.54	0.00
time (sec)	N/A	0.487	0.243	0.016	0.000	0.000	0.000	1.728	69.941	0.001

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	186	408	0	0	0	194	2304	0
N.S.	1	1.00	0.98	2.16	0.00	0.00	0.00	1.03	12.19	0.00
time (sec)	N/A	0.329	0.187	0.010	0.000	0.000	0.000	2.185	15.207	0.001

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	139	289	0	421	0	157	1853	0
N.S.	1	1.00	0.88	1.83	0.00	2.66	0.00	0.99	11.73	0.00
time (sec)	N/A	0.261	0.106	0.010	0.000	128.564	0.000	1.843	11.051	0.001

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	114	176	0	321	0	133	3704	0
N.S.	1	1.00	0.86	1.33	0.00	2.43	0.00	1.01	28.06	0.00
time (sec)	N/A	0.157	0.070	0.008	0.000	37.369	0.000	1.720	9.751	0.001

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	112	176	0	321	0	134	2434	0
N.S.	1	1.00	0.84	1.32	0.00	2.41	0.00	1.01	18.30	0.00
time (sec)	N/A	0.123	0.068	0.010	0.000	26.755	0.000	1.908	8.707	0.001

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	242	298	0	0	0	172	6285	0
N.S.	1	1.00	1.45	1.78	0.00	0.00	0.00	1.03	37.63	0.00
time (sec)	N/A	0.306	0.321	0.011	0.000	0.000	0.000	1.917	17.199	0.001

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	205	205	331	430	0	0	0	237	5368	0
N.S.	1	1.00	1.61	2.10	0.00	0.00	0.00	1.16	26.19	0.00
time (sec)	N/A	0.470	0.343	0.019	0.000	0.000	0.000	1.962	62.948	0.001

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	268	268	426	584	0	0	0	332	10300	0
N.S.	1	1.00	1.59	2.18	0.00	0.00	0.00	1.24	38.43	0.00
time (sec)	N/A	0.597	0.434	0.020	0.000	0.000	0.000	1.462	144.755	0.001

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	387	387	463	1449	0	0	0	12506	41755	0
N.S.	1	1.00	1.20	3.74	0.00	0.00	0.00	32.32	107.89	0.00
time (sec)	N/A	4.032	0.614	0.042	0.000	0.000	0.000	12.649	7.134	0.001

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	323	323	385	1098	0	0	0	11030	33892	0
N.S.	1	1.00	1.19	3.40	0.00	0.00	0.00	34.15	104.93	0.00
time (sec)	N/A	1.366	0.520	0.036	0.000	0.000	0.000	13.868	6.446	0.001

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	280	280	323	764	0	15553	0	8658	25202	0
N.S.	1	1.00	1.15	2.73	0.00	55.55	0.00	30.92	90.01	0.00
time (sec)	N/A	0.894	0.334	0.026	0.000	9.248	0.000	11.236	5.800	0.001

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	277	478	0	12269	0	6921	19401	0
N.S.	1	1.00	1.10	1.90	0.00	48.88	0.00	27.57	77.29	0.00
time (sec)	N/A	0.451	0.498	0.024	0.000	3.760	0.000	8.453	4.959	0.001

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	254	254	274	480	0	0	0	7650	23640	0
N.S.	1	1.00	1.08	1.89	0.00	0.00	0.00	30.12	93.07	0.00
time (sec)	N/A	0.520	0.248	0.027	0.000	0.000	0.000	9.134	5.614	0.001

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	298	298	340	817	0	0	0	10058	33644	0
N.S.	1	1.00	1.14	2.74	0.00	0.00	0.00	33.75	112.90	0.00
time (sec)	N/A	0.960	0.399	0.030	0.000	0.000	0.000	12.818	5.890	0.001

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	348	348	410	1160	0	0	0	12268	42882	0
N.S.	1	1.00	1.18	3.33	0.00	0.00	0.00	35.25	123.22	0.00
time (sec)	N/A	1.549	0.525	0.040	0.000	0.000	0.000	12.000	6.726	0.001

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	866	866	267	336	0	0	0	0	43112	308
N.S.	1	1.00	0.31	0.39	0.00	0.00	0.00	0.00	49.78	0.36
time (sec)	N/A	2.509	0.377	0.096	0.000	0.000	0.000	0.000	6.836	0.397

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	272	272	267	1049	0	0	0	0	-1	316
N.S.	1	1.00	0.98	3.86	0.00	0.00	0.00	0.00	-0.00	1.16
time (sec)	N/A	0.573	0.435	0.043	0.000	0.000	0.000	0.000	0.000	1.331

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	208	208	205	887	0	1231	0	0	-1	247
N.S.	1	1.00	0.99	4.26	0.00	5.92	0.00	0.00	-0.00	1.19
time (sec)	N/A	0.315	0.262	0.010	0.000	56.150	0.000	0.000	0.000	0.835

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	168	168	167	757	0	1050	0	0	-1	203
N.S.	1	1.00	0.99	4.51	0.00	6.25	0.00	0.00	-0.01	1.21
time (sec)	N/A	0.217	0.128	0.005	0.000	4.093	0.000	0.000	0.000	0.581

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	186	186	179	851	0	0	0	0	-1	227
N.S.	1	1.00	0.96	4.58	0.00	0.00	0.00	0.00	-0.01	1.22
time (sec)	N/A	0.256	0.149	0.017	0.000	0.000	0.000	0.000	0.000	0.703

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	361	361	165	1009	0	1094	0	216	-1	212
N.S.	1	1.00	0.46	2.80	0.00	3.03	0.00	0.60	-0.00	0.59
time (sec)	N/A	0.509	0.315	0.020	0.000	2.363	0.000	0.523	0.000	0.775

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	482	482	545	2068	0	0	0	0	-1	13888
N.S.	1	1.00	1.13	4.29	0.00	0.00	0.00	0.00	-0.00	28.81
time (sec)	N/A	1.103	0.976	0.062	0.000	0.000	0.000	0.000	0.000	30.303

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	360	360	344	1696	0	0	0	0	-1	410
N.S.	1	1.00	0.96	4.71	0.00	0.00	0.00	0.00	-0.00	1.14
time (sec)	N/A	0.697	0.600	0.014	0.000	0.000	0.000	0.000	0.000	2.778

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	269	269	255	1411	0	0	0	0	-1	291
N.S.	1	1.00	0.95	5.25	0.00	0.00	0.00	0.00	-0.00	1.08
time (sec)	N/A	0.456	0.351	0.007	0.000	0.000	0.000	0.000	0.000	1.422

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	350	350	251	1270	0	0	0	0	-1	267
N.S.	1	1.00	0.72	3.63	0.00	0.00	0.00	0.00	-0.00	0.76
time (sec)	N/A	0.573	0.534	0.036	0.000	0.000	0.000	0.000	0.000	1.284

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	562	562	240	1207	0	0	0	0	-1	261
N.S.	1	1.00	0.43	2.15	0.00	0.00	0.00	0.00	-0.00	0.46
time (sec)	N/A	0.924	0.503	0.033	0.000	0.000	0.000	0.000	0.000	1.285

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	171	267	0	1364	0	0	-1	230
N.S.	1	1.00	0.99	1.54	0.00	7.88	0.00	0.00	-0.01	1.33
time (sec)	N/A	0.314	0.367	0.020	0.000	58.184	0.000	0.000	0.000	0.697

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	133	204	0	1084	0	0	-1	195
N.S.	1	1.00	0.97	1.49	0.00	7.91	0.00	0.00	-0.01	1.42
time (sec)	N/A	0.157	0.118	0.010	0.000	4.113	0.000	0.000	0.000	0.541

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	87	165	0	357	0	75	-1	141
N.S.	1	1.00	1.01	1.92	0.00	4.15	0.00	0.87	-0.01	1.64
time (sec)	N/A	0.084	0.018	0.007	0.000	1.190	0.000	0.486	0.000	0.382

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	134	207	0	1097	0	0	-1	195
N.S.	1	1.00	0.97	1.50	0.00	7.95	0.00	0.00	-0.01	1.41
time (sec)	N/A	0.195	0.140	0.010	0.000	1.742	0.000	0.000	0.000	0.518

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	218	218	175	276	0	1414	0	208	-1	237
N.S.	1	1.00	0.80	1.27	0.00	6.49	0.00	0.95	-0.00	1.09
time (sec)	N/A	0.266	0.367	0.011	0.000	2.781	0.000	0.488	0.000	0.815

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	236	236	271	720	0	4901	0	0	-1	308
N.S.	1	1.00	1.15	3.05	0.00	20.77	0.00	0.00	-0.00	1.31
time (sec)	N/A	0.474	0.821	0.063	0.000	172.624	0.000	0.000	0.000	1.207

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	204	613	0	1381	0	458	-1	226
N.S.	1	1.00	1.22	3.67	0.00	8.27	0.00	2.74	-0.01	1.35
time (sec)	N/A	0.294	0.616	0.015	0.000	2.201	0.000	0.720	0.000	0.784

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	162	506	0	1349	0	441	-1	218
N.S.	1	1.00	1.02	3.18	0.00	8.48	0.00	2.77	-0.01	1.37
time (sec)	N/A	0.191	0.172	0.011	0.000	2.245	0.000	0.641	0.000	0.753

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	167	454	0	1379	0	454	-1	224
N.S.	1	1.00	1.01	2.73	0.00	8.31	0.00	2.73	-0.01	1.35
time (sec)	N/A	0.171	0.149	0.008	0.000	2.491	0.000	0.618	0.000	0.746

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	266	266	236	612	0	4909	0	0	-1	324
N.S.	1	1.00	0.89	2.30	0.00	18.45	0.00	0.00	-0.00	1.22
time (sec)	N/A	0.388	0.731	0.024	0.000	10.494	0.000	0.000	0.000	1.491

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	419	419	350	863	0	6486	0	762	-1	479
N.S.	1	1.00	0.84	2.06	0.00	15.48	0.00	1.82	-0.00	1.14
time (sec)	N/A	0.564	1.477	0.028	0.000	22.941	0.000	2.736	0.000	2.074

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	F(-1)	F(-1)	B	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	406	406	943	496	0	0	0	928	11195	502
N.S.	1	1.00	2.32	1.22	0.00	0.00	0.00	2.29	27.57	1.24
time (sec)	N/A	8.593	10.842	0.059	0.000	0.000	0.000	0.650	2.476	1.650

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	324	324	591	332	0	0	0	745	8222	454
N.S.	1	1.00	1.82	1.02	0.00	0.00	0.00	2.30	25.38	1.40
time (sec)	N/A	3.529	7.251	0.033	0.000	0.000	0.000	0.819	1.994	1.233

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	292	292	308	275	0	2435	0	619	5705	352
N.S.	1	1.00	1.05	0.94	0.00	8.34	0.00	2.12	19.54	1.21
time (sec)	N/A	3.600	0.547	0.026	0.000	138.367	0.000	0.782	2.342	1.112

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	179	177	0	1085	0	228	717	279
N.S.	1	1.00	0.89	0.88	0.00	5.37	0.00	1.13	3.55	1.38
time (sec)	N/A	0.363	0.328	0.016	0.000	21.449	0.000	0.555	1.723	0.733

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	281	281	241	294	0	0	0	717	10964	280
N.S.	1	1.00	0.86	1.05	0.00	0.00	0.00	2.55	39.02	1.00
time (sec)	N/A	1.350	0.814	0.030	0.000	0.000	0.000	0.675	6.875	0.735

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	382	370	349	401	0	0	0	0	19959	398
N.S.	1	0.97	0.91	1.05	0.00	0.00	0.00	0.00	52.25	1.04
time (sec)	N/A	4.134	1.389	0.030	0.000	0.000	0.000	0.000	5.460	1.500

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	552	552	466	655	0	0	0	1055	33925	579
N.S.	1	1.00	0.84	1.19	0.00	0.00	0.00	1.91	61.46	1.05
time (sec)	N/A	4.244	1.967	0.035	0.000	0.000	0.000	0.905	7.300	2.392

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	A	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	390	390	10915	290	0	6534	0	53	-1	566
N.S.	1	1.00	27.99	0.74	0.00	16.75	0.00	0.14	-0.00	1.45
time (sec)	N/A	2.919	6.397	0.040	0.000	104.324	0.000	2.038	0.000	91.937

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	A	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	324	324	7768	224	0	3260	0	27	-1	425
N.S.	1	1.00	23.98	0.69	0.00	10.06	0.00	0.08	-0.00	1.31
time (sec)	N/A	1.517	6.171	0.027	0.000	13.844	0.000	1.832	0.000	57.344

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	240	240	2585	161	0	985	0	0	-1	250
N.S.	1	1.00	10.77	0.67	0.00	4.10	0.00	0.00	-0.00	1.04
time (sec)	N/A	0.318	5.125	0.018	0.000	3.343	0.000	0.000	0.000	17.048

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	F(-1)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	291	291	4644	272	0	2402	0	0	-1	348
N.S.	1	1.00	15.96	0.93	0.00	8.25	0.00	0.00	-0.00	1.20
time (sec)	N/A	0.671	6.322	0.029	0.000	7.224	0.000	0.000	0.000	70.513

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	F(-1)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	373	373	7777	322	0	4095	0	0	-1	479
N.S.	1	1.00	20.85	0.86	0.00	10.98	0.00	0.00	-0.00	1.28
time (sec)	N/A	2.527	6.391	0.038	0.000	31.157	0.000	0.000	0.000	111.742

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	F(-1)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	512	512	10933	503	0	5773	0	0	-1	687
N.S.	1	1.00	21.35	0.98	0.00	11.28	0.00	0.00	-0.00	1.34
time (sec)	N/A	4.943	6.586	0.038	0.000	77.192	0.000	0.000	0.000	160.945

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F(-1)	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	460	460	457	490	0	0	0	857	16951	524
N.S.	1	1.00	0.99	1.07	0.00	0.00	0.00	1.86	36.85	1.14
time (sec)	N/A	5.084	0.966	0.033	0.000	0.000	0.000	1.246	3.413	2.124

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F(-1)	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	327	327	324	279	0	0	0	649	12392	447
N.S.	1	1.00	0.99	0.85	0.00	0.00	0.00	1.98	37.90	1.37
time (sec)	N/A	1.456	0.568	0.021	0.000	0.000	0.000	1.089	4.129	1.553

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F(-1)	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	346	346	333	388	0	0	0	827	28434	389
N.S.	1	1.00	0.96	1.12	0.00	0.00	0.00	2.39	82.18	1.12
time (sec)	N/A	1.744	1.379	0.029	0.000	0.000	0.000	1.024	7.673	1.531

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F(-1)	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	417	416	380	555	0	0	0	433	35855	548
N.S.	1	1.00	0.91	1.33	0.00	0.00	0.00	1.04	85.98	1.31
time (sec)	N/A	3.243	1.600	0.043	0.000	0.000	0.000	0.725	6.097	2.117

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	F(-1)	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	595	595	18689	516	0	0	0	104	-1	0
N.S.	1	1.00	31.41	0.87	0.00	0.00	0.00	0.17	-0.00	0.00
time (sec)	N/A	3.285	6.489	0.039	0.000	0.000	0.000	1.869	0.000	180.173

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	F(-1)	F	A	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	491	491	14032	382	0	0	0	58	-1	692
N.S.	1	1.00	28.58	0.78	0.00	0.00	0.00	0.12	-0.00	1.41
time (sec)	N/A	1.802	6.261	0.033	0.000	0.000	0.000	1.975	0.000	154.730

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	A	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	487	487	9290	217	0	7721	0	27	-1	392
N.S.	1	1.00	19.08	0.45	0.00	15.85	0.00	0.06	-0.00	0.80
time (sec)	N/A	1.572	6.160	0.027	0.000	99.051	0.000	1.968	0.000	121.925

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	F(-1)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	260	432	7789	360	0	4059	0	0	-1	423
N.S.	1	1.66	29.96	1.38	0.00	15.61	0.00	0.00	-0.00	1.63
time (sec)	N/A	0.855	6.302	0.036	0.000	37.774	0.000	0.000	0.000	121.559

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F(-1)	F(-1)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	523	523	9321	511	0	7830	0	0	-1	484
N.S.	1	1.00	17.82	0.98	0.00	14.97	0.00	0.00	-0.00	0.93
time (sec)	N/A	2.617	6.408	0.036	0.000	149.195	0.000	0.000	0.000	179.824

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	281	281	354	2134	0	3615	0	4637	917	425
N.S.	1	1.00	1.26	7.59	0.00	12.86	0.00	16.50	3.26	1.51
time (sec)	N/A	7.336	0.542	0.099	0.000	16.821	0.000	4.484	1.450	0.975

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	229	229	276	1223	0	2053	0	4060	776	285
N.S.	1	1.00	1.21	5.34	0.00	8.97	0.00	17.73	3.39	1.24
time (sec)	N/A	1.751	0.387	0.063	0.000	6.457	0.000	4.104	1.311	0.677

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	182	182	169	1167	0	871	0	591	649	190
N.S.	1	1.00	0.93	6.41	0.00	4.79	0.00	3.25	3.57	1.04
time (sec)	N/A	0.268	0.243	0.051	0.000	3.123	0.000	4.242	1.288	0.311

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	241	241	212	2099	0	1232	0	3639	669	277
N.S.	1	1.00	0.88	8.71	0.00	5.11	0.00	15.10	2.78	1.15
time (sec)	N/A	1.645	0.432	0.064	0.000	11.648	0.000	3.674	1.298	0.645

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	290	290	292	2770	0	2799	0	1675	825	360
N.S.	1	1.00	1.01	9.55	0.00	9.65	0.00	5.78	2.84	1.24
time (sec)	N/A	2.362	0.762	0.085	0.000	32.217	0.000	7.120	1.410	2.087

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	B	B	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	325	325	10606	222	0	2860	0	1710	1024	594
N.S.	1	1.00	32.63	0.68	0.00	8.80	0.00	5.26	3.15	1.83
time (sec)	N/A	5.391	6.307	0.042	0.000	4.252	0.000	6.686	1.300	0.966

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	263	263	7543	175	0	1430	0	3580	870	415
N.S.	1	1.00	28.68	0.67	0.00	5.44	0.00	13.61	3.31	1.58
time (sec)	N/A	2.135	6.140	0.023	0.000	1.982	0.000	4.734	1.272	0.631

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	B	B	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	220	220	2266	130	0	759	0	641	989	155
N.S.	1	1.00	10.30	0.59	0.00	3.45	0.00	2.91	4.50	0.70
time (sec)	N/A	0.294	5.418	0.013	0.000	0.925	0.000	5.122	1.268	0.171

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	B	B	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	265	265	2661	217	0	1998	0	3965	1234	471
N.S.	1	1.00	10.04	0.82	0.00	7.54	0.00	14.96	4.66	1.78
time (sec)	N/A	0.780	4.946	0.027	0.000	1.418	0.000	5.037	1.205	0.690

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	B	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	96	96	743	160	0	290	0	209	383	202
N.S.	1	1.00	7.74	1.67	0.00	3.02	0.00	2.18	3.99	2.10
time (sec)	N/A	0.200	0.496	0.088	0.000	1.072	0.000	0.718	1.500	0.591

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	479	479	461	377	0	0	0	105	-1	538
N.S.	1	1.00	0.96	0.79	0.00	0.00	0.00	0.22	-0.00	1.12
time (sec)	N/A	1.858	1.868	0.034	0.000	0.000	0.000	2.027	0.000	60.185

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	366	366	355	269	0	0	0	55	-1	458
N.S.	1	1.00	0.97	0.73	0.00	0.00	0.00	0.15	-0.00	1.25
time (sec)	N/A	1.173	1.030	0.026	0.000	0.000	0.000	2.140	0.000	45.098

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	298	298	292	200	0	11094	0	27	-1	320
N.S.	1	1.00	0.98	0.67	0.00	37.23	0.00	0.09	-0.00	1.07
time (sec)	N/A	0.724	0.588	0.022	0.000	51.774	0.000	2.049	0.000	30.969

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	240	240	227	161	0	3395	0	0	-1	250
N.S.	1	1.00	0.95	0.67	0.00	14.15	0.00	0.00	-0.00	1.04
time (sec)	N/A	0.304	0.478	0.017	0.000	12.713	0.000	0.000	0.000	9.822

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F(-1)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	243	243	229	151	0	4557	0	0	-1	179
N.S.	1	1.00	0.94	0.62	0.00	18.75	0.00	0.00	-0.00	0.74
time (sec)	N/A	0.178	0.406	0.016	0.000	30.483	0.000	0.000	0.000	0.195

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F(-1)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	280	280	271	197	0	6431	0	0	-1	312
N.S.	1	1.00	0.97	0.70	0.00	22.97	0.00	0.00	-0.00	1.11
time (sec)	N/A	0.602	1.013	0.025	0.000	59.583	0.000	0.000	0.000	36.312

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F	F(-1)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	341	341	320	248	0	0	0	0	-1	371
N.S.	1	1.00	0.94	0.73	0.00	0.00	0.00	0.00	-0.00	1.09
time (sec)	N/A	0.741	0.686	0.029	0.000	0.000	0.000	0.000	0.000	49.091

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F	F(-1)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	443	443	383	350	0	0	0	0	-1	534
N.S.	1	1.00	0.86	0.79	0.00	0.00	0.00	0.00	-0.00	1.21
time (sec)	N/A	1.432	1.636	0.036	0.000	0.000	0.000	0.000	0.000	65.035

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	F(-1)	F	A	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	350	507	10968	480	0	0	0	75	-1	593
N.S.	1	1.45	31.34	1.37	0.00	0.00	0.00	0.21	-0.00	1.69
time (sec)	N/A	4.328	11.262	0.040	0.000	0.000	0.000	2.254	0.000	84.947

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F	F(-2)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	360	360	2162	338	0	0	0	0	-1	405
N.S.	1	1.00	6.01	0.94	0.00	0.00	0.00	0.00	-0.00	1.12
time (sec)	N/A	1.267	7.897	0.034	0.000	0.000	0.000	0.000	0.000	55.591

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F	F(-2)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	333	333	2119	252	0	0	0	0	-1	343
N.S.	1	1.00	6.36	0.76	0.00	0.00	0.00	0.00	-0.00	1.03
time (sec)	N/A	0.658	6.719	0.029	0.000	0.000	0.000	0.000	0.000	49.218

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F	F(-2)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	341	341	2112	246	0	0	0	0	-1	342
N.S.	1	1.00	6.19	0.72	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.774	7.152	0.024	0.000	0.000	0.000	0.000	0.000	62.476

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F	F(-1)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	339	462	2158	387	0	0	0	0	-1	546
N.S.	1	1.36	6.37	1.14	0.00	0.00	0.00	0.00	-0.00	1.61
time (sec)	N/A	2.837	6.774	0.037	0.000	0.000	0.000	0.000	0.000	109.355

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F	F(-1)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	419	647	2218	541	0	0	0	0	-1	762
N.S.	1	1.54	5.29	1.29	0.00	0.00	0.00	0.00	-0.00	1.82
time (sec)	N/A	5.566	6.796	0.042	0.000	0.000	0.000	0.000	0.000	144.272

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	44	44	101	0	120	0	42	-1	107
N.S.	1	1.10	1.10	2.52	0.00	3.00	0.00	1.05	-0.02	2.68
time (sec)	N/A	0.072	0.052	0.059	0.000	0.937	0.000	0.222	0.000	1.087

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [182] had the largest ratio of [.5000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	20	0.100
2	A	2	1	1.00	20	0.050
3	A	4	3	1.00	18	0.167
4	A	2	1	1.00	17	0.059
5	A	3	2	1.00	20	0.100
6	A	2	1	1.00	20	0.050
7	A	3	2	1.00	20	0.100
8	A	5	5	1.00	20	0.250
9	A	4	4	1.00	20	0.200
10	A	4	4	1.00	18	0.222
11	A	7	7	1.00	20	0.350
12	A	7	7	1.00	20	0.350
13	A	7	7	1.00	20	0.350
14	A	6	6	1.00	20	0.300
15	A	6	5	1.00	20	0.250
16	A	5	4	1.00	20	0.200
17	A	5	4	1.00	18	0.222
18	A	8	7	1.00	20	0.350
19	A	8	8	1.00	20	0.400
20	A	8	7	1.00	20	0.350
21	A	8	8	1.00	20	0.400
22	A	5	4	1.00	20	0.200
23	A	4	4	1.00	20	0.200
24	A	3	3	1.00	20	0.150
25	A	3	3	1.00	18	0.167
26	A	6	6	1.00	20	0.300
27	A	5	5	1.00	20	0.250
28	A	6	6	1.00	20	0.300
29	A	4	4	1.00	20	0.200
30	A	4	4	1.00	20	0.200
31	A	3	3	1.00	20	0.150
32	A	2	2	1.00	18	0.111
33	A	6	6	1.00	20	0.300
34	A	6	6	1.00	20	0.300
35	A	3	2	1.00	25	0.080
36	A	4	3	1.00	23	0.130
37	A	3	2	1.00	23	0.087
38	A	4	3	1.00	23	0.130
39	A	3	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
40	A	4	3	1.00	21	0.143
41	A	3	2	1.00	20	0.100
42	A	5	4	1.00	23	0.174
43	A	3	2	1.00	23	0.087
44	A	4	3	1.00	23	0.130
45	A	3	2	1.00	23	0.087
46	A	4	3	1.00	21	0.143
47	A	3	2	1.00	21	0.095
48	A	4	3	1.00	21	0.143
49	A	3	2	1.00	21	0.095
50	A	2	2	1.00	19	0.105
51	A	3	2	1.00	18	0.111
52	A	4	3	1.00	21	0.143
53	A	3	2	1.00	21	0.095
54	A	4	3	1.00	21	0.143
55	A	4	4	1.00	33	0.121
56	A	4	4	1.00	31	0.129
57	A	3	3	1.00	30	0.100
58	A	4	3	1.00	33	0.091
59	A	3	3	1.00	33	0.091
60	A	4	3	1.00	33	0.091
61	A	4	4	1.00	33	0.121
62	A	3	3	1.00	31	0.097
63	A	4	4	1.00	30	0.133
64	A	4	3	1.00	33	0.091
65	A	5	4	1.00	33	0.121
66	A	4	3	1.00	33	0.091
67	A	3	2	1.00	35	0.057
68	A	3	2	1.00	35	0.057
69	A	3	2	1.00	35	0.057
70	A	2	2	1.00	29	0.069
71	A	5	4	1.00	31	0.129
72	A	5	4	1.00	31	0.129
73	A	3	2	1.00	25	0.080
74	A	2	1	1.00	25	0.040
75	A	3	2	1.00	23	0.087
76	A	2	1	1.00	22	0.045
77	A	3	2	1.00	25	0.080
78	A	2	1	1.00	25	0.040
79	A	3	2	1.00	25	0.080
80	A	7	6	1.00	25	0.240
81	A	6	6	1.00	25	0.240
82	A	5	5	1.00	23	0.217
83	A	7	6	1.00	25	0.240
84	A	7	6	1.00	25	0.240
85	A	5	3	1.00	25	0.120
86	A	4	3	1.00	25	0.120
87	A	3	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
88	A	4	3	1.00	25	0.120
89	A	5	3	1.00	25	0.120
90	A	7	7	1.00	25	0.280
91	A	6	6	1.00	25	0.240
92	A	4	4	1.00	25	0.160
93	A	4	4	1.00	23	0.174
94	A	8	7	1.00	25	0.280
95	A	8	7	1.00	25	0.280
96	A	6	4	1.00	25	0.160
97	A	5	4	1.00	25	0.160
98	A	4	3	1.00	25	0.120
99	A	4	3	1.00	22	0.136
100	A	5	4	1.00	25	0.160
101	A	6	4	1.00	25	0.160
102	A	8	7	1.00	25	0.280
103	A	7	6	1.00	25	0.240
104	A	5	5	1.00	25	0.200
105	A	5	5	1.00	25	0.200
106	A	5	5	1.00	25	0.200
107	A	5	5	1.00	23	0.217
108	A	9	7	1.00	25	0.280
109	A	9	7	1.00	25	0.280
110	A	7	4	1.00	25	0.160
111	A	6	4	1.00	25	0.160
112	A	5	3	1.00	25	0.120
113	A	5	4	1.00	25	0.160
114	A	5	3	1.00	22	0.136
115	A	4	3	1.00	21	0.143
116	A	5	4	1.00	22	0.182
117	A	5	5	1.00	17	0.294
118	A	6	6	1.00	18	0.333
119	A	5	5	1.00	22	0.227
120	A	6	6	1.00	25	0.240
121	A	5	5	1.00	25	0.200
122	A	5	5	1.00	23	0.217
123	A	7	6	1.00	25	0.240
124	A	7	6	1.00	25	0.240
125	A	7	6	1.00	25	0.240
126	A	5	5	1.00	25	0.200
127	A	6	6	1.00	25	0.240
128	A	7	6	1.00	25	0.240
129	A	7	6	1.00	25	0.240
130	A	6	5	1.00	25	0.200
131	A	6	5	1.00	23	0.217
132	A	8	6	1.00	25	0.240
133	A	8	7	1.00	25	0.280
134	A	8	6	1.00	25	0.240
135	A	8	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
136	A	5	5	1.00	27	0.185
137	A	4	4	1.00	27	0.148
138	A	4	4	1.00	25	0.160
139	A	6	5	1.00	27	0.185
140	A	4	4	1.00	27	0.148
141	A	5	5	1.00	27	0.185
142	A	6	5	1.00	27	0.185
143	A	6	5	1.00	25	0.200
144	A	5	5	1.00	25	0.200
145	A	4	4	1.00	25	0.160
146	A	4	4	1.00	23	0.174
147	A	6	5	1.00	25	0.200
148	A	4	4	1.00	25	0.160
149	A	5	5	1.00	25	0.200
150	A	6	5	1.00	25	0.200
151	A	5	5	1.00	25	0.200
152	A	4	4	1.00	25	0.160
153	A	2	2	1.00	23	0.087
154	A	5	5	1.00	25	0.200
155	A	5	5	1.00	25	0.200
156	A	2	1	1.00	27	0.037
157	A	2	1	1.00	27	0.037
158	A	2	1	1.00	25	0.040
159	A	6	5	1.00	22	0.227
160	A	6	5	1.00	22	0.227
161	A	6	5	1.00	22	0.227
162	A	6	5	1.00	22	0.227
163	A	6	6	1.00	20	0.300
164	A	6	5	1.00	22	0.227
165	A	6	5	1.00	22	0.227
166	A	6	5	1.00	22	0.227
167	A	12	8	1.00	22	0.364
168	A	12	8	1.00	22	0.364
169	A	12	8	1.00	22	0.364
170	A	12	8	1.00	22	0.364
171	A	12	8	1.00	19	0.421
172	A	12	8	1.00	22	0.364
173	A	12	8	1.00	22	0.364
174	A	7	6	1.00	22	0.273
175	A	7	6	1.00	22	0.273
176	A	7	6	0.99	22	0.273
177	A	7	6	0.99	22	0.273
178	A	7	6	1.00	20	0.300
179	A	8	6	1.00	22	0.273
180	A	8	6	1.00	22	0.273
181	A	8	6	1.00	22	0.273
182	A	24	11	1.00	22	0.500
183	A	23	10	1.00	22	0.454

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
184	A	23	10	1.00	22	0.454
185	A	23	10	1.00	22	0.454
186	A	22	9	1.00	19	0.474
187	A	22	9	1.00	22	0.409
188	A	22	9	1.00	22	0.409
189	A	6	6	1.00	37	0.162
190	A	4	3	1.00	35	0.086
191	A	5	5	1.00	34	0.147
192	A	6	6	1.00	37	0.162
193	A	5	5	1.00	37	0.135
194	A	6	6	1.00	37	0.162
195	A	3	2	1.00	25	0.080
196	A	2	1	1.00	25	0.040
197	A	3	2	1.00	23	0.087
198	A	2	1	1.00	22	0.045
199	A	3	2	1.00	25	0.080
200	A	2	1	1.00	25	0.040
201	A	3	2	1.00	25	0.080
202	A	4	3	1.00	25	0.120
203	A	4	3	1.00	25	0.120
204	A	4	3	1.00	25	0.120
205	A	3	3	1.00	22	0.136
206	A	3	3	0.97	25	0.120
207	A	4	3	1.00	25	0.120
208	A	4	3	1.00	25	0.120
209	A	4	3	1.00	25	0.120
210	A	5	4	1.00	25	0.160
211	A	5	4	1.00	25	0.160
212	A	4	4	1.00	25	0.160
213	A	3	3	1.00	22	0.136
214	A	4	4	0.98	25	0.160
215	A	5	3	1.00	25	0.120
216	A	5	4	1.00	25	0.160
217	A	7	6	1.00	27	0.222
218	A	7	6	1.00	27	0.222
219	A	7	6	1.00	27	0.222
220	A	7	6	1.00	27	0.222
221	A	7	7	1.00	25	0.280
222	A	7	6	1.00	27	0.222
223	A	7	6	1.00	27	0.222
224	A	7	6	1.00	27	0.222
225	A	6	3	1.00	27	0.111
226	A	6	3	1.00	27	0.111
227	A	6	3	1.00	27	0.111
228	A	6	3	1.00	27	0.111
229	A	6	3	1.00	24	0.125
230	A	6	3	1.00	27	0.111
231	A	6	3	1.00	27	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
232	A	19	12	1.00	31	0.387
233	A	8	7	1.00	29	0.241
234	A	7	6	1.00	29	0.207
235	A	7	6	1.00	27	0.222
236	A	9	6	1.00	29	0.207
237	A	21	8	1.00	29	0.276
238	A	9	7	1.00	29	0.241
239	A	8	6	1.00	29	0.207
240	A	8	7	1.00	27	0.259
241	A	14	8	1.00	29	0.276
242	A	24	9	1.00	29	0.310
243	A	7	6	1.00	29	0.207
244	A	6	5	1.00	29	0.172
245	A	3	3	1.00	27	0.111
246	A	7	4	1.00	29	0.138
247	A	10	5	1.00	29	0.172
248	A	7	6	1.00	29	0.207
249	A	5	5	1.00	29	0.172
250	A	5	5	1.00	29	0.172
251	A	5	5	1.00	27	0.185
252	A	11	6	1.00	29	0.207
253	A	15	7	1.00	29	0.241
254	A	7	5	1.00	29	0.172
255	A	7	5	1.00	29	0.172
256	A	6	5	1.00	29	0.172
257	A	5	4	1.00	27	0.148
258	A	8	6	1.00	29	0.207
259	A	10	7	0.97	29	0.241
260	A	13	7	1.00	29	0.241
261	A	10	7	1.00	29	0.241
262	A	9	6	1.00	29	0.207
263	A	11	6	1.00	26	0.231
264	A	8	5	1.00	29	0.172
265	A	12	7	1.00	29	0.241
266	A	15	7	1.00	29	0.241
267	A	7	5	1.00	29	0.172
268	A	6	5	1.00	27	0.185
269	A	8	6	1.00	29	0.207
270	A	10	7	1.00	29	0.241
271	A	17	9	1.00	29	0.310
272	A	16	8	1.00	29	0.276
273	A	13	7	1.00	26	0.269
274	A	16	8	1.66	29	0.276
275	A	19	10	1.00	29	0.345
276	A	7	5	1.00	29	0.172
277	A	6	5	1.00	29	0.172
278	A	5	4	1.00	27	0.148
279	A	8	6	1.00	29	0.207

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
280	A	8	6	1.00	29	0.207
281	A	9	6	1.00	29	0.207
282	A	8	5	1.00	29	0.172
283	A	9	5	1.00	26	0.192
284	A	8	5	1.00	29	0.172
285	A	8	6	1.00	25	0.240
286	A	17	7	1.00	29	0.241
287	A	13	7	1.00	29	0.241
288	A	10	6	1.00	29	0.207
289	A	6	3	1.00	29	0.103
290	A	5	3	1.00	26	0.115
291	A	9	5	1.00	29	0.172
292	A	11	6	1.00	29	0.207
293	A	14	6	1.00	29	0.207
294	A	14	7	1.45	29	0.241
295	A	8	5	1.00	29	0.172
296	A	8	5	1.00	29	0.172
297	A	8	5	1.00	26	0.192
298	A	12	8	1.36	29	0.276
299	A	15	8	1.54	29	0.276
300	A	5	5	1.10	28	0.179

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^3 \left(d+ex^2\right) \left(a+cx^4\right)^5 \, dx$

Optimal. Leaf size=149

$$\frac{1}{4}a^5dx^4+\frac{1}{6}a^5ex^6+\frac{5}{8}a^4cdx^8+\frac{1}{2}a^4cex^{10}+\frac{5}{6}a^3c^2dx^{12}+\frac{5}{7}a^3c^2ex^{14}+\frac{5}{8}a^2c^3dx^{16}+\frac{5}{9}a^2c^3ex^{18}+\frac{1}{4}ac^4dx^{20}+\frac{5}{22}ac^4ex^{22}+$$

Rubi [A] time = 0.22, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1252, 766}

$$\frac{5}{8}a^2c^3dx^{16}+\frac{5}{6}a^3c^2dx^{12}+\frac{5}{9}a^2c^3ex^{18}+\frac{5}{7}a^3c^2ex^{14}+\frac{5}{8}a^4cdx^8+\frac{1}{2}a^4cex^{10}+\frac{1}{4}a^5dx^4+\frac{1}{6}a^5ex^6+\frac{1}{4}ac^4dx^{20}+\frac{5}{22}ac^4ex^{22}+\frac{1}{24}c^5dx^{24}+\frac{1}{26}c^5ex^{26}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^4)/4 + (a^5*e*x^6)/6 + (5*a^4*c*d*x^8)/8 + (a^4*c*e*x^10)/2 + (5*a
^3*c^2*d*x^12)/6 + (5*a^3*c^2*e*x^14)/7 + (5*a^2*c^3*d*x^16)/8 + (5*a^2*c^3
*e*x^18)/9 + (a*c^4*d*x^20)/4 + (5*a*c^4*e*x^22)/22 + (c^5*d*x^24)/24 + (c^
5*e*x^26)/26
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(d+ex^2\right)\left(a+cx^4\right)^5 \, dx &= \frac{1}{2}\text{Subst}\left(\int x(d+ex)\left(a+cx^2\right)^5 \, dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(a^5dx+a^5ex^2+5a^4cdx^3+5a^4cex^4+10a^3c^2dx^5+10a^3c^2ex^6+10\right.\right.\\ &= \frac{1}{4}a^5dx^4+\frac{1}{6}a^5ex^6+\frac{5}{8}a^4cdx^8+\frac{1}{2}a^4cex^{10}+\frac{5}{6}a^3c^2dx^{12}+\frac{5}{7}a^3c^2ex^{14}+\frac{5}{8}a^2c^3dx\end{aligned}$$

Mathematica [A] time = 0.02, size = 149, normalized size = 1.00

$$\frac{1}{4}a^5dx^4+\frac{1}{6}a^5ex^6+\frac{5}{8}a^4cdx^8+\frac{1}{2}a^4cex^{10}+\frac{5}{6}a^3c^2dx^{12}+\frac{5}{7}a^3c^2ex^{14}+\frac{5}{8}a^2c^3dx^{16}+\frac{5}{9}a^2c^3ex^{18}+\frac{1}{4}ac^4dx^{20}+\frac{5}{22}ac^4ex^{22}+\frac{1}{24}c^5dx^{24}+\frac{1}{26}c^5ex^{26}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^4)/4 + (a^5*e*x^6)/6 + (5*a^4*c*d*x^8)/8 + (a^4*c*e*x^10)/2 + (5*a
^3*c^2*d*x^12)/6 + (5*a^3*c^2*e*x^14)/7 + (5*a^2*c^3*d*x^16)/8 + (5*a^2*c^3
*e*x^18)/9 + (a*c^4*d*x^20)/4 + (5*a*c^4*e*x^22)/22 + (c^5*d*x^24)/24 + (c^
5*e*x^26)/26
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(d+ex^2\right)\left(a+cx^4\right)^5 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] IntegrateAlgebraic[x^3*(d + e*x^2)*(a + c*x^4)^5, x]
```

fricas [A] time = 0.55, size = 125, normalized size = 0.84

$$\frac{1}{26}x^{26}ec^5 + \frac{1}{24}x^{24}dc^5 + \frac{5}{22}x^{22}ec^4a + \frac{1}{4}x^{20}dc^4a + \frac{5}{9}x^{18}ec^3a^2 + \frac{5}{8}x^{16}dc^3a^2 + \frac{5}{7}x^{14}ec^2a^3 + \frac{5}{6}x^{12}dc^2a^3 + \frac{1}{2}x^{10}eca^4 + \frac{5}{8}x^8dca^4 + \frac{1}{6}x^6ea^5 + \frac{1}{4}x^4da^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

```
[Out] 1/26*x^26*e*c^5 + 1/24*x^24*d*c^5 + 5/22*x^22*e*c^4*a + 1/4*x^20*d*c^4*a +
5/9*x^18*e*c^3*a^2 + 5/8*x^16*d*c^3*a^2 + 5/7*x^14*e*c^2*a^3 + 5/6*x^12*d*c
^2*a^3 + 1/2*x^10*e*c*a^4 + 5/8*x^8*d*c*a^4 + 1/6*x^6*e*a^5 + 1/4*x^4*d*a^5
```

giac [A] time = 0.26, size = 131, normalized size = 0.88

$$\frac{1}{26}c^5x^{26}e + \frac{1}{24}c^5dx^{24} + \frac{5}{22}ac^4x^{22}e + \frac{1}{4}ac^4dx^{20} + \frac{5}{9}a^2c^3x^{18}e + \frac{5}{8}a^2c^3dx^{16} + \frac{5}{7}a^3c^2x^{14}e + \frac{5}{6}a^3c^2dx^{12} + \frac{1}{2}a^4cx^{10}e + \frac{5}{8}a^4cdx^8 + \frac{1}{6}a^5x^6e + \frac{1}{4}a^5dx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

```
[Out] 1/26*c^5*x^26*e + 1/24*c^5*d*x^24 + 5/22*a*c^4*x^22*e + 1/4*a*c^4*d*x^20 +
5/9*a^2*c^3*x^18*e + 5/8*a^2*c^3*d*x^16 + 5/7*a^3*c^2*x^14*e + 5/6*a^3*c^2*
d*x^12 + 1/2*a^4*c*x^10*e + 5/8*a^4*c*d*x^8 + 1/6*a^5*x^6*e + 1/4*a^5*d*x^4
```

maple [A] time = 0.01, size = 126, normalized size = 0.85

$$\frac{1}{26}c^5e\,x^{26} + \frac{1}{24}c^5d\,x^{24} + \frac{5}{22}a\,c^4e\,x^{22} + \frac{1}{4}a\,c^4d\,x^{20} + \frac{5}{9}a^2c^3e\,x^{18} + \frac{5}{8}a^2c^3d\,x^{16} + \frac{5}{7}a^3c^2e\,x^{14} + \frac{5}{6}a^3c^2d\,x^{12} + \frac{1}{2}a^4ce\,x^{10} + \frac{5}{8}a^4cd\,x^8 + \frac{1}{6}a^5e\,x^6 + \frac{1}{4}a^5d\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)*(c*x^4+a)^5,x)
```

```
[Out] 1/4*a^5*d*x^4+1/6*a^5*e*x^6+5/8*a^4*c*d*x^8+1/2*a^4*c*e*x^10+5/6*a^3*c^2*d*
x^12+5/7*a^3*c^2*e*x^14+5/8*a^2*c^3*d*x^16+5/9*a^2*c^3*e*x^18+1/4*a*c^4*d*x
^20+5/22*a*c^4*e*x^22+1/24*c^5*d*x^24+1/26*c^5*e*x^26
```

maxima [A] time = 0.43, size = 125, normalized size = 0.84

$$\frac{1}{26}c^5ex^{26} + \frac{1}{24}c^5dx^{24} + \frac{5}{22}ac^4ex^{22} + \frac{1}{4}ac^4dx^{20} + \frac{5}{9}a^2c^3ex^{18} + \frac{5}{8}a^2c^3dx^{16} + \frac{5}{7}a^3c^2ex^{14} + \frac{5}{6}a^3c^2dx^{12} + \frac{1}{2}a^4cex^{10} + \frac{5}{8}a^4cdx^8 + \frac{1}{6}a^5ex^6 + \frac{1}{4}a^5dx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
```

```
[Out] 1/26*c^5*e*x^26 + 1/24*c^5*d*x^24 + 5/22*a*c^4*e*x^22 + 1/4*a*c^4*d*x^20 +
5/9*a^2*c^3*e*x^18 + 5/8*a^2*c^3*d*x^16 + 5/7*a^3*c^2*e*x^14 + 5/6*a^3*c^2*
d*x^12 + 1/2*a^4*c*e*x^10 + 5/8*a^4*c*d*x^8 + 1/6*a^5*e*x^6 + 1/4*a^5*d*x^4
```

mupad [B] time = 0.31, size = 125, normalized size = 0.84

$$\frac{e\,a^5\,x^6}{6} + \frac{d\,a^5\,x^4}{4} + \frac{e\,a^4\,c\,x^{10}}{2} + \frac{5\,d\,a^4\,c\,x^8}{8} + \frac{5\,e\,a^3\,c^2\,x^{14}}{7} + \frac{5\,d\,a^3\,c^2\,x^{12}}{6} + \frac{5\,e\,a^2\,c^3\,x^{18}}{9} + \frac{5\,d\,a^2\,c^3\,x^{16}}{8} + \frac{5\,e\,a\,c^4\,x^{22}}{22} + \frac{d\,a\,c^4\,x^{20}}{4} + \frac{e\,c^5\,x^{26}}{26} + \frac{d\,c^5\,x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + c*x^4)^5*(d + e*x^2),x)
```

```
[Out] (a^5*d*x^4)/4 + (a^5*e*x^6)/6 + (c^5*d*x^24)/24 + (c^5*e*x^26)/26 + (5*a^3*
c^2*d*x^12)/6 + (5*a^2*c^3*d*x^16)/8 + (5*a^3*c^2*e*x^14)/7 + (5*a^2*c^3*e*
```

```
x^18)/9 + (5*a^4*c*d*x^8)/8 + (a*c^4*d*x^20)/4 + (a^4*c*e*x^10)/2 + (5*a*c^
4*e*x^22)/22
```

sympy [A] time = 0.15, size = 151, normalized size = 1.01

$$\frac{a^5dx^4}{4}+\frac{a^5ex^6}{6}+\frac{5a^4cdx^8}{8}+\frac{a^4cex^{10}}{2}+\frac{5a^3c^2dx^{12}}{6}+\frac{5a^3c^2ex^{14}}{7}+\frac{5a^2c^3dx^{16}}{8}+\frac{5a^2c^3ex^{18}}{9}+\frac{ac^4dx^{20}}{4}+\frac{5ac^4ex^{22}}{22}+\frac{c^5dx^{24}}{24}+\frac{c^5ex^{26}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x**4/4 + a**5*e*x**6/6 + 5*a**4*c*d*x**8/8 + a**4*c*e*x**10/2 + 5*a*
*3*c**2*d*x**12/6 + 5*a**3*c**2*e*x**14/7 + 5*a**2*c**3*d*x**16/8 + 5*a**2*
c**3*e*x**18/9 + a*c**4*d*x**20/4 + 5*a*c**4*e*x**22/22 + c**5*d*x**24/24 +
 c**5*e*x**26/26
```

3.2 $\int x^2 \left(d+ex^2\right) \left(a+cx^4\right)^5 \, dx$

Optimal. Leaf size=149

$$\frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{10}{11}a^3c^2dx^{11}+\frac{10}{13}a^3c^2ex^{13}+\frac{2}{3}a^2c^3dx^{15}+\frac{10}{17}a^2c^3ex^{17}+\frac{5}{19}ac^4dx^{19}+\frac{5}{21}ac^4ex^{21}+$$

Rubi [A] time = 0.10, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1262}

$$\frac{2}{3}a^2c^3dx^{15}+\frac{10}{11}a^3c^2dx^{11}+\frac{10}{17}a^2c^3ex^{17}+\frac{10}{13}a^3c^2ex^{13}+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{19}ac^4dx^{19}+\frac{5}{21}ac^4ex^{21}+\frac{1}{23}c^5dx^{23}+\frac{1}{25}c^5ex^{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^3)/3 + (a^5*e*x^5)/5 + (5*a^4*c*d*x^7)/7 + (5*a^4*c*e*x^9)/9 + (10
*a^3*c^2*d*x^11)/11 + (10*a^3*c^2*e*x^13)/13 + (2*a^2*c^3*d*x^15)/3 + (10*a
^2*c^3*e*x^17)/17 + (5*a*c^4*d*x^19)/19 + (5*a*c^4*e*x^21)/21 + (c^5*d*x^23
)/23 + (c^5*e*x^25)/25
```

Rule 1262

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(d+ex^2\right) \left(a+cx^4\right)^5 \, dx &= \int \left(a^5dx^2+a^5ex^4+5a^4cdx^6+5a^4cex^8+10a^3c^2dx^{10}+10a^3c^2ex^{12}+10a^2c^3dx^{14}\right.\\ &= \frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{10}{11}a^3c^2dx^{11}+\frac{10}{13}a^3c^2ex^{13}+\frac{2}{3}a^2c^3dx^{15}\end{aligned}$$

Mathematica [A] time = 0.00, size = 149, normalized size = 1.00

$$\frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{10}{11}a^3c^2dx^{11}+\frac{10}{13}a^3c^2ex^{13}+\frac{2}{3}a^2c^3dx^{15}+\frac{10}{17}a^2c^3ex^{17}+\frac{5}{19}ac^4dx^{19}+\frac{5}{21}ac^4ex^{21}+\frac{1}{23}c^5dx^{23}+\frac{1}{25}c^5ex^{25}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^3)/3 + (a^5*e*x^5)/5 + (5*a^4*c*d*x^7)/7 + (5*a^4*c*e*x^9)/9 + (10
*a^3*c^2*d*x^11)/11 + (10*a^3*c^2*e*x^13)/13 + (2*a^2*c^3*d*x^15)/3 + (10*a
^2*c^3*e*x^17)/17 + (5*a*c^4*d*x^19)/19 + (5*a*c^4*e*x^21)/21 + (c^5*d*x^23
)/23 + (c^5*e*x^25)/25
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(d+ex^2\right) \left(a+cx^4\right)^5 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] IntegrateAlgebraic[x^2*(d + e*x^2)*(a + c*x^4)^5, x]
```

fricas [A] time = 0.57, size = 125, normalized size = 0.84

$$\frac{1}{25}x^{25}ec^5 + \frac{1}{23}x^{23}dc^5 + \frac{5}{21}x^{21}ec^4a + \frac{5}{19}x^{19}dc^4a + \frac{10}{17}x^{17}ec^3a^2 + \frac{2}{3}x^{15}dc^3a^2 + \frac{10}{13}x^{13}ec^2a^3 + \frac{10}{11}x^{11}dc^2a^3 + \frac{5}{9}x^9eca^4 + \frac{5}{7}x^7dca^4 + \frac{1}{5}x^5ea^5 + \frac{1}{3}x^3da^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

```
[Out] 1/25*x^25*e*c^5 + 1/23*x^23*d*c^5 + 5/21*x^21*e*c^4*a + 5/19*x^19*d*c^4*a +
 10/17*x^17*e*c^3*a^2 + 2/3*x^15*d*c^3*a^2 + 10/13*x^13*e*c^2*a^3 + 10/11*x
^11*d*c^2*a^3 + 5/9*x^9*e*c*a^4 + 5/7*x^7*d*c*a^4 + 1/5*x^5*e*a^5 + 1/3*x^3
*d*a^5
```

giac [A] time = 0.19, size = 131, normalized size = 0.88

$$\frac{1}{25}c^5x^{25}e + \frac{1}{23}c^5dx^{23} + \frac{5}{21}ac^4x^{21}e + \frac{5}{19}ac^4dx^{19} + \frac{10}{17}a^2c^3x^{17}e + \frac{2}{3}a^2c^3dx^{15} + \frac{10}{13}a^3c^2x^{13}e + \frac{10}{11}a^3c^2dx^{11} + \frac{5}{9}a^4cx^9e + \frac{5}{7}a^4cdx^7 + \frac{1}{5}a^5x^5e + \frac{1}{3}a^5dx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

```
[Out] 1/25*c^5*x^25*e + 1/23*c^5*d*x^23 + 5/21*a*c^4*x^21*e + 5/19*a*c^4*d*x^19 +
 10/17*a^2*c^3*x^17*e + 2/3*a^2*c^3*d*x^15 + 10/13*a^3*c^2*x^13*e + 10/11*a
^3*c^2*d*x^11 + 5/9*a^4*c*x^9*e + 5/7*a^4*c*d*x^7 + 1/5*a^5*x^5*e + 1/3*a^5
*d*x^3
```

maple [A] time = 0.00, size = 126, normalized size = 0.85

$$\frac{1}{25}c^5e\,x^{25} + \frac{1}{23}c^5d\,x^{23} + \frac{5}{21}a\,c^4e\,x^{21} + \frac{5}{19}a\,c^4d\,x^{19} + \frac{10}{17}a^2c^3e\,x^{17} + \frac{2}{3}a^2c^3d\,x^{15} + \frac{10}{13}a^3c^2e\,x^{13} + \frac{10}{11}a^3c^2d\,x^{11} + \frac{5}{9}a^4ce\,x^9 + \frac{5}{7}a^4cd\,x^7 + \frac{1}{5}a^5e\,x^5 + \frac{1}{3}a^5d\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)*(c*x^4+a)^5,x)
```

```
[Out] 1/3*a^5*d*x^3+1/5*a^5*e*x^5+5/7*a^4*c*d*x^7+5/9*a^4*c*e*x^9+10/11*a^3*c^2*d
*x^11+10/13*a^3*c^2*e*x^13+2/3*a^2*c^3*d*x^15+10/17*a^2*c^3*e*x^17+5/19*a*c
^4*d*x^19+5/21*a*c^4*e*x^21+1/23*c^5*d*x^23+1/25*c^5*e*x^25
```

maxima [A] time = 0.46, size = 125, normalized size = 0.84

$$\frac{1}{25}c^5ex^{25} + \frac{1}{23}c^5dx^{23} + \frac{5}{21}ac^4ex^{21} + \frac{5}{19}ac^4dx^{19} + \frac{10}{17}a^2c^3ex^{17} + \frac{2}{3}a^2c^3dx^{15} + \frac{10}{13}a^3c^2ex^{13} + \frac{10}{11}a^3c^2dx^{11} + \frac{5}{9}a^4cex^9 + \frac{5}{7}a^4cdx^7 + \frac{1}{5}a^5ex^5 + \frac{1}{3}a^5dx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
```

```
[Out] 1/25*c^5*e*x^25 + 1/23*c^5*d*x^23 + 5/21*a*c^4*e*x^21 + 5/19*a*c^4*d*x^19 +
 10/17*a^2*c^3*e*x^17 + 2/3*a^2*c^3*d*x^15 + 10/13*a^3*c^2*e*x^13 + 10/11*a
^3*c^2*d*x^11 + 5/9*a^4*c*e*x^9 + 5/7*a^4*c*d*x^7 + 1/5*a^5*e*x^5 + 1/3*a^5
*d*x^3
```

mupad [B] time = 0.07, size = 125, normalized size = 0.84

$$\frac{e\,a^5\,x^5}{5} + \frac{d\,a^5\,x^3}{3} + \frac{5\,e\,a^4\,c\,x^9}{9} + \frac{5\,d\,a^4\,c\,x^7}{7} + \frac{10\,e\,a^3\,c^2\,x^{13}}{13} + \frac{10\,d\,a^3\,c^2\,x^{11}}{11} + \frac{10\,e\,a^2\,c^3\,x^{17}}{17} + \frac{2\,d\,a^2\,c^3\,x^{15}}{3} + \frac{5\,e\,a\,c^4\,x^{21}}{21} + \frac{5\,d\,a\,c^4\,x^{19}}{19} + \frac{e\,c^5\,x^{25}}{25} + \frac{d\,c^5\,x^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + c*x^4)^5*(d + e*x^2),x)
```

```
[Out] (a^5*d*x^3)/3 + (a^5*e*x^5)/5 + (c^5*d*x^23)/23 + (c^5*e*x^25)/25 + (10*a^3
*c^2*d*x^11)/11 + (2*a^2*c^3*d*x^15)/3 + (10*a^3*c^2*e*x^13)/13 + (10*a^2*c
^3*e*x^17)/17 + (5*a^4*c*d*x^7)/7 + (5*a*c^4*d*x^19)/19 + (5*a^4*c*e*x^9)/9
 + (5*a*c^4*e*x^21)/21
```

sympy [A] time = 0.09, size = 155, normalized size = 1.04

$$\frac{a^5dx^3}{3}+\frac{a^5ex^5}{5}+\frac{5a^4cdx^7}{7}+\frac{5a^4cex^9}{9}+\frac{10a^3c^2dx^{11}}{11}+\frac{10a^3c^2ex^{13}}{13}+\frac{2a^2c^3dx^{15}}{3}+\frac{10a^2c^3ex^{17}}{17}+\frac{5ac^4dx^{19}}{19}+\frac{5ac^4ex^{21}}{21}+\frac{c^5dx^{23}}{23}+\frac{c^5ex^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x**3/3 + a**5*e*x**5/5 + 5*a**4*c*d*x**7/7 + 5*a**4*c*e*x**9/9 + 10*
a**3*c**2*d*x**11/11 + 10*a**3*c**2*e*x**13/13 + 2*a**2*c**3*d*x**15/3 + 10
*a**2*c**3*e*x**17/17 + 5*a*c**4*d*x**19/19 + 5*a*c**4*e*x**21/21 + c**5*d*
x**23/23 + c**5*e*x**25/25
```

3.3 $\int x\left(d+ex^2\right)\left(a+cx^4\right)^5\,dx$

Optimal. Leaf size=89

$$\frac{1}{2}a^5dx^2+\frac{5}{6}a^4cdx^6+a^3c^2dx^{10}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{18}ac^4dx^{18}+\frac{e\left(a+cx^4\right)^6}{24c}+\frac{1}{22}c^5dx^{22}$$

Rubi [A] time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1248, 641, 194}

$$\frac{5}{7}a^2c^3dx^{14}+a^3c^2dx^{10}+\frac{5}{6}a^4cdx^6+\frac{1}{2}a^5dx^2+\frac{5}{18}ac^4dx^{18}+\frac{e\left(a+cx^4\right)^6}{24c}+\frac{1}{22}c^5dx^{22}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^2)/2 + (5*a^4*c*d*x^6)/6 + a^3*c^2*d*x^10 + (5*a^2*c^3*d*x^14)/7 +
 (5*a*c^4*d*x^18)/18 + (c^5*d*x^22)/22 + (e*(a + c*x^4)^6)/(24*c)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(d+ex^2\right)\left(a+cx^4\right)^5\,dx&=\frac{1}{2}\text{Subst}\left(\int(d+ex)\left(a+cx^2\right)^5\,dx,x,x^2\right)\\&=\frac{e\left(a+cx^4\right)^6}{24c}+\frac{1}{2}d\,\text{Subst}\left(\int\left(a+cx^2\right)^5\,dx,x,x^2\right)\\&=\frac{e\left(a+cx^4\right)^6}{24c}+\frac{1}{2}d\,\text{Subst}\left(\int\left(a^5+5a^4cx^2+10a^3c^2x^4+10a^2c^3x^6+5ac^4x^8+c^5\right.\right.\\&=\frac{1}{2}a^5dx^2+\frac{5}{6}a^4cdx^6+a^3c^2dx^{10}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{18}ac^4dx^{18}+\frac{1}{22}c^5dx^{22}+\frac{e\left(a+c\right.}{24c}\end{aligned}$$

Mathematica [A] time = 0.00, size = 146, normalized size = 1.64

$$\frac{1}{2}a^5dx^2+\frac{1}{4}a^5ex^4+\frac{5}{6}a^4cdx^6+\frac{5}{8}a^4cex^8+a^3c^2dx^{10}+\frac{5}{6}a^3c^2ex^{12}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{8}a^2c^3ex^{16}+\frac{5}{18}ac^4dx^{18}+\frac{1}{4}ac^4ex^{20}+\frac{1}{22}c^5dx^{22}+\frac{1}{24}c^5ex^{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^2)/2 + (a^5*e*x^4)/4 + (5*a^4*c*d*x^6)/6 + (5*a^4*c*e*x^8)/8 + a^3
*c^2*d*x^10 + (5*a^3*c^2*e*x^12)/6 + (5*a^2*c^3*d*x^14)/7 + (5*a^2*c^3*e*x^
16)/8 + (5*a*c^4*d*x^18)/18 + (a*c^4*e*x^20)/4 + (c^5*d*x^22)/22 + (c^5*e*x
^24)/24
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(d+ex^2\right)\left(a+cx^4\right)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] IntegrateAlgebraic[x*(d + e*x^2)*(a + c*x^4)^5, x]
```

fricas [A] time = 0.79, size = 124, normalized size = 1.39

$$\frac{1}{24}x^{24}ec^5+\frac{1}{22}x^{22}dc^5+\frac{1}{4}x^{20}ec^4a+\frac{5}{18}x^{18}dc^4a+\frac{5}{8}x^{16}ec^3a^2+\frac{5}{7}x^{14}dc^3a^2+\frac{5}{6}x^{12}ec^2a^3+x^{10}dc^2a^3+\frac{5}{8}x^8eca^4+\frac{5}{6}x^6dca^4+\frac{1}{4}x^4ea^5+\frac{1}{2}x^2da^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

```
[Out] 1/24*x^24*e*c^5 + 1/22*x^22*d*c^5 + 1/4*x^20*e*c^4*a + 5/18*x^18*d*c^4*a +
5/8*x^16*e*c^3*a^2 + 5/7*x^14*d*c^3*a^2 + 5/6*x^12*e*c^2*a^3 + x^10*d*c^2*a
^3 + 5/8*x^8*e*c*a^4 + 5/6*x^6*d*c*a^4 + 1/4*x^4*e*a^5 + 1/2*x^2*d*a^5
```

giac [A] time = 0.18, size = 130, normalized size = 1.46

$$\frac{1}{24}c^5x^{24}e+\frac{1}{22}c^5dx^{22}+\frac{1}{4}ac^4x^{20}e+\frac{5}{18}ac^4dx^{18}+\frac{5}{8}a^2c^3x^{16}e+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{6}a^3c^2x^{12}e+a^3c^2dx^{10}+\frac{5}{8}a^4cx^8e+\frac{5}{6}a^4cdx^6+\frac{1}{4}a^5x^4e+\frac{1}{2}a^5dx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

```
[Out] 1/24*c^5*x^24*e + 1/22*c^5*d*x^22 + 1/4*a*c^4*x^20*e + 5/18*a*c^4*d*x^18 +
5/8*a^2*c^3*x^16*e + 5/7*a^2*c^3*d*x^14 + 5/6*a^3*c^2*x^12*e + a^3*c^2*d*x^
10 + 5/8*a^4*c*x^8*e + 5/6*a^4*c*d*x^6 + 1/4*a^5*x^4*e + 1/2*a^5*d*x^2
```

maple [A] time = 0.00, size = 125, normalized size = 1.40

$$\frac{1}{24}c^5e\,x^{24}+\frac{1}{22}c^5d\,x^{22}+\frac{1}{4}a\,c^4e\,x^{20}+\frac{5}{18}a\,c^4d\,x^{18}+\frac{5}{8}a^2c^3e\,x^{16}+\frac{5}{7}a^2c^3d\,x^{14}+\frac{5}{6}a^3c^2e\,x^{12}+a^3c^2d\,x^{10}+\frac{5}{8}a^4ce\,x^8+\frac{5}{6}a^4cd\,x^6+\frac{1}{4}a^5e\,x^4+\frac{1}{2}a^5d\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)*(c*x^4+a)^5,x)
```

```
[Out] 1/24*c^5*e*x^24+1/22*c^5*d*x^22+1/4*a*c^4*e*x^20+5/18*a*c^4*d*x^18+5/8*a^2*
c^3*e*x^16+5/7*a^2*c^3*d*x^14+5/6*a^3*c^2*e*x^12+a^3*c^2*d*x^10+5/8*a^4*c*e
*x^8+5/6*a^4*c*d*x^6+1/4*a^5*e*x^4+1/2*a^5*d*x^2
```

maxima [A] time = 0.44, size = 124, normalized size = 1.39

$$\frac{1}{24}c^5ex^{24}+\frac{1}{22}c^5dx^{22}+\frac{1}{4}ac^4ex^{20}+\frac{5}{18}ac^4dx^{18}+\frac{5}{8}a^2c^3ex^{16}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{6}a^3c^2ex^{12}+a^3c^2dx^{10}+\frac{5}{8}a^4cex^8+\frac{5}{6}a^4cdx^6+\frac{1}{4}a^5ex^4+\frac{1}{2}a^5dx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
```

```
[Out] 1/24*c^5*e*x^24 + 1/22*c^5*d*x^22 + 1/4*a*c^4*e*x^20 + 5/18*a*c^4*d*x^18 +
5/8*a^2*c^3*e*x^16 + 5/7*a^2*c^3*d*x^14 + 5/6*a^3*c^2*e*x^12 + a^3*c^2*d*x^
10 + 5/8*a^4*c*e*x^8 + 5/6*a^4*c*d*x^6 + 1/4*a^5*e*x^4 + 1/2*a^5*d*x^2
```

mupad [B] time = 0.07, size = 124, normalized size = 1.39

$$\frac{e\,a^5\,x^4}{4}+\frac{d\,a^5\,x^2}{2}+\frac{5\,e\,a^4\,c\,x^8}{8}+\frac{5\,d\,a^4\,c\,x^6}{6}+\frac{5\,e\,a^3\,c^2\,x^{12}}{6}+d\,a^3\,c^2\,x^{10}+\frac{5\,e\,a^2\,c^3\,x^{16}}{8}+\frac{5\,d\,a^2\,c^3\,x^{14}}{7}+\frac{e\,a\,c^4\,x^{20}}{4}+\frac{5\,d\,a\,c^4\,x^{18}}{18}+\frac{e\,c^5\,x^{24}}{24}+\frac{d\,c^5\,x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + c*x^4)^5*(d + e*x^2),x)
```

```
[Out] (a^5*d*x^2)/2 + (a^5*e*x^4)/4 + (c^5*d*x^22)/22 + (c^5*e*x^24)/24 + a^3*c^2
*d*x^10 + (5*a^2*c^3*d*x^14)/7 + (5*a^3*c^2*e*x^12)/6 + (5*a^2*c^3*e*x^16)/
8 + (5*a^4*c*d*x^6)/6 + (5*a*c^4*d*x^18)/18 + (5*a^4*c*e*x^8)/8 + (a*c^4*e*
x^20)/4
```

sympy [A] time = 0.09, size = 150, normalized size = 1.69

$$\frac{a^5dx^2}{2}+\frac{a^5ex^4}{4}+\frac{5a^4cdx^6}{6}+\frac{5a^4cex^8}{8}+a^3c^2dx^{10}+\frac{5a^3c^2ex^{12}}{6}+\frac{5a^2c^3dx^{14}}{7}+\frac{5a^2c^3ex^{16}}{8}+\frac{5ac^4dx^{18}}{18}+\frac{ac^4ex^{20}}{4}+\frac{c^5dx^{22}}{22}+\frac{c^5ex^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x**2/2 + a**5*e*x**4/4 + 5*a**4*c*d*x**6/6 + 5*a**4*c*e*x**8/8 + a**
3*c**2*d*x**10 + 5*a**3*c**2*e*x**12/6 + 5*a**2*c**3*d*x**14/7 + 5*a**2*c**
3*e*x**16/8 + 5*a*c**4*d*x**18/18 + a*c**4*e*x**20/4 + c**5*d*x**22/22 + c*
*5*e*x**24/24
```

3.4 $\int \left(d+ex^2\right)\left(a+cx^4\right)^5\,dx$

Optimal. Leaf size=141

$$a^5dx+\frac{1}{3}a^5ex^3+a^4cdx^5+\frac{5}{7}a^4cex^7+\frac{10}{9}a^3c^2dx^9+\frac{10}{11}a^3c^2ex^{11}+\frac{10}{13}a^2c^3dx^{13}+\frac{2}{3}a^2c^3ex^{15}+\frac{5}{17}ac^4dx^{17}+\frac{5}{19}ac^4ex^{19}+\frac{1}{21}c^5$$

Rubi [A] time = 0.08, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1154}

$$\frac{10}{13}a^2c^3dx^{13}+\frac{10}{9}a^3c^2dx^9+\frac{2}{3}a^2c^3ex^{15}+\frac{10}{11}a^3c^2ex^{11}+a^4cdx^5+\frac{5}{7}a^4cex^7+a^5dx+\frac{1}{3}a^5ex^3+\frac{5}{17}ac^4dx^{17}+\frac{5}{19}ac^4ex^{19}+\frac{1}{21}c^5dx^{21}+\frac{1}{23}c^5ex^{23}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] a^5*d*x + (a^5*e*x^3)/3 + a^4*c*d*x^5 + (5*a^4*c*e*x^7)/7 + (10*a^3*c^2*d*x
^9)/9 + (10*a^3*c^2*e*x^11)/11 + (10*a^2*c^3*d*x^13)/13 + (2*a^2*c^3*e*x^15
)/3 + (5*a*c^4*d*x^17)/17 + (5*a*c^4*e*x^19)/19 + (c^5*d*x^21)/21 + (c^5*e*
x^23)/23
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \left(d+ex^2\right)\left(a+cx^4\right)^5\,dx=\int \left(a^5d+a^5ex^2+5a^4cdx^4+5a^4cex^6+10a^3c^2dx^8+10a^3c^2ex^{10}+10a^2c^3dx^{12}+10a\right.$$
$$=a^5dx+\frac{1}{3}a^5ex^3+a^4cdx^5+\frac{5}{7}a^4cex^7+\frac{10}{9}a^3c^2dx^9+\frac{10}{11}a^3c^2ex^{11}+\frac{10}{13}a^2c^3dx^{13}+\frac{2}{3}a^2$$

Mathematica [A] time = 0.00, size = 141, normalized size = 1.00

$$a^5dx+\frac{1}{3}a^5ex^3+a^4cdx^5+\frac{5}{7}a^4cex^7+\frac{10}{9}a^3c^2dx^9+\frac{10}{11}a^3c^2ex^{11}+\frac{10}{13}a^2c^3dx^{13}+\frac{2}{3}a^2c^3ex^{15}+\frac{5}{17}ac^4dx^{17}+\frac{5}{19}ac^4ex^{19}+\frac{1}{21}c^5dx^{21}+\frac{1}{23}c^5ex^{23}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] a^5*d*x + (a^5*e*x^3)/3 + a^4*c*d*x^5 + (5*a^4*c*e*x^7)/7 + (10*a^3*c^2*d*x
^9)/9 + (10*a^3*c^2*e*x^11)/11 + (10*a^2*c^3*d*x^13)/13 + (2*a^2*c^3*e*x^15
)/3 + (5*a*c^4*d*x^17)/17 + (5*a*c^4*e*x^19)/19 + (c^5*d*x^21)/21 + (c^5*e*
x^23)/23
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d+ex^2\right)\left(a+cx^4\right)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^2)*(a + c*x^4)^5, x]
```

fricas [A] time = 0.57, size = 121, normalized size = 0.86

$$\frac{1}{23}x^{23}ec^5+\frac{1}{21}x^{21}dc^5+\frac{5}{19}x^{19}ec^4a+\frac{5}{17}x^{17}dc^4a+\frac{2}{3}x^{15}ec^3a^2+\frac{10}{13}x^{13}dc^3a^2+\frac{10}{11}x^{11}ec^2a^3+\frac{10}{9}x^9dc^2a^3+\frac{5}{7}x^7eca^4+x^5dca^4+\frac{1}{3}x^3ea^5+xda^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

```
[Out] 1/23*x^23*e*c^5 + 1/21*x^21*d*c^5 + 5/19*x^19*e*c^4*a + 5/17*x^17*d*c^4*a +
 2/3*x^15*e*c^3*a^2 + 10/13*x^13*d*c^3*a^2 + 10/11*x^11*e*c^2*a^3 + 10/9*x^
9*d*c^2*a^3 + 5/7*x^7*e*c*a^4 + x^5*d*c*a^4 + 1/3*x^3*e*a^5 + x*d*a^5
```

giac [A] time = 0.22, size = 127, normalized size = 0.90

$$\frac{1}{23}c^5x^{23}e+\frac{1}{21}c^5dx^{21}+\frac{5}{19}ac^4x^{19}e+\frac{5}{17}ac^4dx^{17}+\frac{2}{3}a^2c^3x^{15}e+\frac{10}{13}a^2c^3dx^{13}+\frac{10}{11}a^3c^2x^{11}e+\frac{10}{9}a^3c^2dx^9+\frac{5}{7}a^4cx^7e+a^4cdx^5+\frac{1}{3}a^5x^3e+a^5dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

```
[Out] 1/23*c^5*x^23*e + 1/21*c^5*d*x^21 + 5/19*a*c^4*x^19*e + 5/17*a*c^4*d*x^17 +
 2/3*a^2*c^3*x^15*e + 10/13*a^2*c^3*d*x^13 + 10/11*a^3*c^2*x^11*e + 10/9*a^
3*c^2*d*x^9 + 5/7*a^4*c*x^7*e + a^4*c*d*x^5 + 1/3*a^5*x^3*e + a^5*d*x
```

maple [A] time = 0.00, size = 122, normalized size = 0.87

$$\frac{1}{23}c^5e\,x^{23}+\frac{1}{21}c^5d\,x^{21}+\frac{5}{19}a\,c^4e\,x^{19}+\frac{5}{17}a\,c^4d\,x^{17}+\frac{2}{3}a^2c^3e\,x^{15}+\frac{10}{13}a^2c^3d\,x^{13}+\frac{10}{11}a^3c^2e\,x^{11}+\frac{10}{9}a^3c^2d\,x^9+\frac{5}{7}a^4ce\,x^7+a^4cd\,x^5+\frac{1}{3}a^5e\,x^3+a^5dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+a)^5,x)
```

```
[Out] a^5*d*x+1/3*a^5*e*x^3+a^4*c*d*x^5+5/7*a^4*c*e*x^7+10/9*a^3*c^2*d*x^9+10/11*
a^3*c^2*e*x^11+10/13*a^2*c^3*d*x^13+2/3*a^2*c^3*e*x^15+5/17*a*c^4*d*x^17+5/
19*a*c^4*e*x^19+1/21*c^5*d*x^21+1/23*c^5*e*x^23
```

maxima [A] time = 0.46, size = 121, normalized size = 0.86

$$\frac{1}{23}c^5ex^{23}+\frac{1}{21}c^5dx^{21}+\frac{5}{19}ac^4ex^{19}+\frac{5}{17}ac^4dx^{17}+\frac{2}{3}a^2c^3ex^{15}+\frac{10}{13}a^2c^3dx^{13}+\frac{10}{11}a^3c^2ex^{11}+\frac{10}{9}a^3c^2dx^9+\frac{5}{7}a^4cex^7+a^4cdx^5+\frac{1}{3}a^5ex^3+a^5dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
```

```
[Out] 1/23*c^5*e*x^23 + 1/21*c^5*d*x^21 + 5/19*a*c^4*e*x^19 + 5/17*a*c^4*d*x^17 +
 2/3*a^2*c^3*e*x^15 + 10/13*a^2*c^3*d*x^13 + 10/11*a^3*c^2*e*x^11 + 10/9*a^
3*c^2*d*x^9 + 5/7*a^4*c*e*x^7 + a^4*c*d*x^5 + 1/3*a^5*e*x^3 + a^5*d*x
```

mupad [B] time = 0.07, size = 121, normalized size = 0.86

$$\frac{e\,a^5\,x^3}{3}+d\,a^5\,x+\frac{5\,e\,a^4\,c\,x^7}{7}+d\,a^4\,c\,x^5+\frac{10\,e\,a^3\,c^2\,x^{11}}{11}+\frac{10\,d\,a^3\,c^2\,x^9}{9}+\frac{2\,e\,a^2\,c^3\,x^{15}}{3}+\frac{10\,d\,a^2\,c^3\,x^{13}}{13}+\frac{5\,e\,a\,c^4\,x^{19}}{19}+\frac{5\,d\,a\,c^4\,x^{17}}{17}+\frac{e\,c^5\,x^{23}}{23}+\frac{d\,c^5\,x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^5*(d + e*x^2),x)
```

```
[Out] (a^5*e*x^3)/3 + (c^5*d*x^21)/21 + (c^5*e*x^23)/23 + a^5*d*x + (10*a^3*c^2*d
*x^9)/9 + (10*a^2*c^3*d*x^13)/13 + (10*a^3*c^2*e*x^11)/11 + (2*a^2*c^3*e*x^
15)/3 + a^4*c*d*x^5 + (5*a*c^4*d*x^17)/17 + (5*a^4*c*e*x^7)/7 + (5*a*c^4*e*
x^19)/19
```

sympy [A] time = 0.09, size = 148, normalized size = 1.05

$$a^5dx+\frac{a^5ex^3}{3}+a^4cdx^5+\frac{5a^4cex^7}{7}+\frac{10a^3c^2dx^9}{9}+\frac{10a^3c^2ex^{11}}{11}+\frac{10a^2c^3dx^{13}}{13}+\frac{2a^2c^3ex^{15}}{3}+\frac{5ac^4dx^{17}}{17}+\frac{5ac^4ex^{19}}{19}+\frac{c^5dx^{21}}{21}+\frac{c^5ex^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x + a**5*e*x**3/3 + a**4*c*d*x**5 + 5*a**4*c*e*x**7/7 + 10*a**3*c**2
*d*x**9/9 + 10*a**3*c**2*e*x**11/11 + 10*a**2*c**3*d*x**13/13 + 2*a**2*c**3
*e*x**15/3 + 5*a*c**4*d*x**17/17 + 5*a*c**4*e*x**19/19 + c**5*d*x**21/21 +
c**5*e*x**23/23
```

3.5 $\int \frac{(d+ex^2)(a+cx^4)^5}{x}\,dx$

Optimal. Leaf size=142

$$a^5d\log(x)+\frac{1}{2}a^5ex^2+\frac{5}{4}a^4cdx^4+\frac{5}{6}a^4cex^6+\frac{5}{4}a^3c^2dx^8+a^3c^2ex^{10}+\frac{5}{6}a^2c^3dx^{12}+\frac{5}{7}a^2c^3ex^{14}+\frac{5}{16}ac^4dx^{16}+\frac{5}{18}ac^4ex^{18}+$$

Rubi [A] time = 0.11, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1252, 766}

$$\frac{5}{6}a^2c^3dx^{12}+\frac{5}{4}a^3c^2dx^8+\frac{5}{7}a^2c^3ex^{14}+a^3c^2ex^{10}+\frac{5}{4}a^4cdx^4+\frac{5}{6}a^4cex^6+a^5d\log(x)+\frac{1}{2}a^5ex^2+\frac{5}{16}ac^4dx^{16}+\frac{5}{18}ac^4ex^{18}+\frac{1}{20}c^5dx^{20}+\frac{1}{22}c^5ex^{22}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + c*x^4)^5)/x,x]
```

```
[Out] (a^5*e*x^2)/2 + (5*a^4*c*d*x^4)/4 + (5*a^4*c*e*x^6)/6 + (5*a^3*c^2*d*x^8)/4
 + a^3*c^2*e*x^10 + (5*a^2*c^3*d*x^12)/6 + (5*a^2*c^3*e*x^14)/7 + (5*a*c^4*
d*x^16)/16 + (5*a*c^4*e*x^18)/18 + (c^5*d*x^20)/20 + (c^5*e*x^22)/22 + a^5*
d*Log[x]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)(a+cx^4)^5}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)(a+cx^2)^5}{x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(a^5e+\frac{a^5d}{x}+5a^4cdx+5a^4cex^2+10a^3c^2dx^3+10a^3c^2ex^4+10a^2c^3dx\right.\right.\\ &= \frac{1}{2}a^5ex^2+\frac{5}{4}a^4cdx^4+\frac{5}{6}a^4cex^6+\frac{5}{4}a^3c^2dx^8+a^3c^2ex^{10}+\frac{5}{6}a^2c^3dx^{12}+\frac{5}{7}a^2c^3ex^{14}+\end{aligned}$$

Mathematica [A] time = 0.01, size = 142, normalized size = 1.00

$$a^5d\log(x)+\frac{1}{2}a^5ex^2+\frac{5}{4}a^4cdx^4+\frac{5}{6}a^4cex^6+\frac{5}{4}a^3c^2dx^8+a^3c^2ex^{10}+\frac{5}{6}a^2c^3dx^{12}+\frac{5}{7}a^2c^3ex^{14}+\frac{5}{16}ac^4dx^{16}+\frac{5}{18}ac^4ex^{18}+\frac{1}{20}c^5dx^{20}+\frac{1}{22}c^5ex^{22}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + c*x^4)^5)/x,x]
```

```
[Out] (a^5*e*x^2)/2 + (5*a^4*c*d*x^4)/4 + (5*a^4*c*e*x^6)/6 + (5*a^3*c^2*d*x^8)/4
 + a^3*c^2*e*x^10 + (5*a^2*c^3*d*x^12)/6 + (5*a^2*c^3*e*x^14)/7 + (5*a*c^4*
d*x^16)/16 + (5*a*c^4*e*x^18)/18 + (c^5*d*x^20)/20 + (c^5*e*x^22)/22 + a^5*
d*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)\left(a+cx^4\right)^5}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x^2)*(a + c*x^4)^5)/x,x]
```

```
[Out] IntegrateAlgebraic[((d + e*x^2)*(a + c*x^4)^5)/x, x]
```

fricas [A] time = 0.50, size = 122, normalized size = 0.86

$$\frac{1}{22}c^5ex^{22}+\frac{1}{20}c^5dx^{20}+\frac{5}{18}ac^4ex^{18}+\frac{5}{16}ac^4dx^{16}+\frac{5}{7}a^2c^3ex^{14}+\frac{5}{6}a^2c^3dx^{12}+a^3c^2ex^{10}+\frac{5}{4}a^3c^2dx^8+\frac{5}{6}a^4cex^6+\frac{5}{4}a^4cdx^4+\frac{1}{2}a^5ex^2+a^5d\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x,x, algorithm="fricas")
```

```
[Out] 1/22*c^5*e*x^22 + 1/20*c^5*d*x^20 + 5/18*a*c^4*e*x^18 + 5/16*a*c^4*d*x^16 +
 5/7*a^2*c^3*e*x^14 + 5/6*a^2*c^3*d*x^12 + a^3*c^2*e*x^10 + 5/4*a^3*c^2*d*x
^8 + 5/6*a^4*c*e*x^6 + 5/4*a^4*c*d*x^4 + 1/2*a^5*e*x^2 + a^5*d*log(x)
```

giac [A] time = 0.20, size = 131, normalized size = 0.92

$$\frac{1}{22}c^5x^{22}e+\frac{1}{20}c^5dx^{20}+\frac{5}{18}ac^4x^{18}e+\frac{5}{16}ac^4dx^{16}+\frac{5}{7}a^2c^3x^{14}e+\frac{5}{6}a^2c^3dx^{12}+a^3c^2x^{10}e+\frac{5}{4}a^3c^2dx^8+\frac{5}{6}a^4cx^6e+\frac{5}{4}a^4cdx^4+\frac{1}{2}a^5x^2e+\frac{1}{2}a^5d\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x,x, algorithm="giac")
```

```
[Out] 1/22*c^5*x^22*e + 1/20*c^5*d*x^20 + 5/18*a*c^4*x^18*e + 5/16*a*c^4*d*x^16 +
 5/7*a^2*c^3*x^14*e + 5/6*a^2*c^3*d*x^12 + a^3*c^2*x^10*e + 5/4*a^3*c^2*d*x
^8 + 5/6*a^4*c*x^6*e + 5/4*a^4*c*d*x^4 + 1/2*a^5*x^2*e + 1/2*a^5*d*log(x^2)
```

maple [A] time = 0.01, size = 123, normalized size = 0.87

$$\frac{c^5e\,x^{22}}{22}+\frac{c^5d\,x^{20}}{20}+\frac{5a\,c^4e\,x^{18}}{18}+\frac{5a\,c^4d\,x^{16}}{16}+\frac{5a^2c^3e\,x^{14}}{7}+\frac{5a^2c^3d\,x^{12}}{6}+a^3c^2e\,x^{10}+\frac{5a^3c^2d\,x^8}{4}+\frac{5a^4ce\,x^6}{6}+\frac{5a^4cd\,x^4}{4}+\frac{a^5e\,x^2}{2}+a^5d\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+a)^5/x,x)
```

```
[Out] 1/2*a^5*e*x^2+5/4*a^4*c*d*x^4+5/6*a^4*c*e*x^6+5/4*a^3*c^2*d*x^8+a^3*c^2*e*x
^10+5/6*a^2*c^3*d*x^12+5/7*a^2*c^3*e*x^14+5/16*a*c^4*d*x^16+5/18*a*c^4*e*x^
18+1/20*c^5*d*x^20+1/22*c^5*e*x^22+a^5*d*ln(x)
```

maxima [A] time = 0.52, size = 125, normalized size = 0.88

$$\frac{1}{22}c^5ex^{22}+\frac{1}{20}c^5dx^{20}+\frac{5}{18}ac^4ex^{18}+\frac{5}{16}ac^4dx^{16}+\frac{5}{7}a^2c^3ex^{14}+\frac{5}{6}a^2c^3dx^{12}+a^3c^2ex^{10}+\frac{5}{4}a^3c^2dx^8+\frac{5}{6}a^4cex^6+\frac{5}{4}a^4cdx^4+\frac{1}{2}a^5ex^2+\frac{1}{2}a^5d\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x,x, algorithm="maxima")
```

```
[Out] 1/22*c^5*e*x^22 + 1/20*c^5*d*x^20 + 5/18*a*c^4*e*x^18 + 5/16*a*c^4*d*x^16 +
 5/7*a^2*c^3*e*x^14 + 5/6*a^2*c^3*d*x^12 + a^3*c^2*e*x^10 + 5/4*a^3*c^2*d*x
^8 + 5/6*a^4*c*e*x^6 + 5/4*a^4*c*d*x^4 + 1/2*a^5*e*x^2 + 1/2*a^5*d*log(x^2)
```

mupad [B] time = 0.11, size = 122, normalized size = 0.86

$$\frac{a^5\,e\,x^2}{2}+\frac{c^5\,d\,x^{20}}{20}+\frac{c^5\,e\,x^{22}}{22}+a^5\,d\,\ln(x)+\frac{5\,a^3\,c^2\,d\,x^8}{4}+\frac{5\,a^2\,c^3\,d\,x^{12}}{6}+a^3\,c^2\,e\,x^{10}+\frac{5\,a^2\,c^3\,e\,x^{14}}{7}+\frac{5\,a^4\,c\,d\,x^4}{4}+\frac{5\,a\,c^4\,d\,x^{16}}{16}+\frac{5\,a^4\,c\,e\,x^6}{6}+\frac{5\,a\,c^4\,e\,x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^4)^5*(d + e*x^2))/x,x)
```

```
[Out] (a^5*e*x^2)/2 + (c^5*d*x^20)/20 + (c^5*e*x^22)/22 + a^5*d*log(x) + (5*a^3*c
^2*d*x^8)/4 + (5*a^2*c^3*d*x^12)/6 + a^3*c^2*e*x^10 + (5*a^2*c^3*e*x^14)/7
+ (5*a^4*c*d*x^4)/4 + (5*a*c^4*d*x^16)/16 + (5*a^4*c*e*x^6)/6 + (5*a*c^4*e*
x^18)/18
```

sympy [A] time = 0.25, size = 150, normalized size = 1.06

$$a^5 d \log(x) + \frac{a^5 e x^2}{2} + \frac{5a^4 c d x^4}{4} + \frac{5a^4 c e x^6}{6} + \frac{5a^3 c^2 d x^8}{4} + a^3 c^2 e x^{10} + \frac{5a^2 c^3 d x^{12}}{6} + \frac{5a^2 c^3 e x^{14}}{7} + \frac{5a c^4 d x^{16}}{16} + \frac{5a c^4 e x^{18}}{18} + \frac{c^5 d x^{20}}{20} + \frac{c^5 e x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+a)**5/x,x)
```

```
[Out] a**5*d*log(x) + a**5*e*x**2/2 + 5*a**4*c*d*x**4/4 + 5*a**4*c*e*x**6/6 + 5*a
**3*c**2*d*x**8/4 + a**3*c**2*e*x**10 + 5*a**2*c**3*d*x**12/6 + 5*a**2*c**3
*e*x**14/7 + 5*a*c**4*d*x**16/16 + 5*a*c**4*e*x**18/18 + c**5*d*x**20/20 +
c**5*e*x**22/22
```

3.6 $\int \frac{(d+ex^2)(a+cx^4)^5}{x^2}\,dx$

Optimal. Leaf size=139

$$-\frac{a^5d}{x}+a^5ex+\frac{5}{3}a^4cdx^3+a^4cex^5+\frac{10}{7}a^3c^2dx^7+\frac{10}{9}a^3c^2ex^9+\frac{10}{11}a^2c^3dx^{11}+\frac{10}{13}a^2c^3ex^{13}+\frac{1}{3}ac^4dx^{15}+\frac{5}{17}ac^4ex^{17}+\frac{1}{19}c^5dx^1$$

Rubi [A] time = 0.08, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1262}

$$\frac{10}{11}a^2c^3dx^{11}+\frac{10}{7}a^3c^2dx^7+\frac{10}{13}a^2c^3ex^{13}+\frac{10}{9}a^3c^2ex^9+\frac{5}{3}a^4cdx^3+a^4cex^5-\frac{a^5d}{x}+a^5ex+\frac{1}{3}ac^4dx^{15}+\frac{5}{17}ac^4ex^{17}+\frac{1}{19}c^5dx^{19}+\frac{1}{21}c^5ex^{21}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + c*x^4)^5)/x^2,x]
```

```
[Out] -((a^5*d)/x) + a^5*e*x + (5*a^4*c*d*x^3)/3 + a^4*c*e*x^5 + (10*a^3*c^2*d*x^
7)/7 + (10*a^3*c^2*e*x^9)/9 + (10*a^2*c^3*d*x^11)/11 + (10*a^2*c^3*e*x^13)/
13 + (a*c^4*d*x^15)/3 + (5*a*c^4*e*x^17)/17 + (c^5*d*x^19)/19 + (c^5*e*x^21
)/21
```

Rule 1262

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^2}\,dx = \int \left(a^5e+\frac{a^5d}{x^2}+5a^4cdx^2+5a^4cex^4+10a^3c^2dx^6+10a^3c^2ex^8+10a^2c^3dx^{10}+10a^2c^3\right.$$
$$= -\frac{a^5d}{x}+a^5ex+\frac{5}{3}a^4cdx^3+a^4cex^5+\frac{10}{7}a^3c^2dx^7+\frac{10}{9}a^3c^2ex^9+\frac{10}{11}a^2c^3dx^{11}+\frac{10}{13}a^2c^3$$

Mathematica [A] time = 0.01, size = 139, normalized size = 1.00

$$-\frac{a^5d}{x}+a^5ex+\frac{5}{3}a^4cdx^3+a^4cex^5+\frac{10}{7}a^3c^2dx^7+\frac{10}{9}a^3c^2ex^9+\frac{10}{11}a^2c^3dx^{11}+\frac{10}{13}a^2c^3ex^{13}+\frac{1}{3}ac^4dx^{15}+\frac{5}{17}ac^4ex^{17}+\frac{1}{19}c^5dx^{19}+\frac{1}{21}c^5ex^{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + c*x^4)^5)/x^2,x]
```

```
[Out] -((a^5*d)/x) + a^5*e*x + (5*a^4*c*d*x^3)/3 + a^4*c*e*x^5 + (10*a^3*c^2*d*x^
7)/7 + (10*a^3*c^2*e*x^9)/9 + (10*a^2*c^3*d*x^11)/11 + (10*a^2*c^3*e*x^13)/
13 + (a*c^4*d*x^15)/3 + (5*a*c^4*e*x^17)/17 + (c^5*d*x^19)/19 + (c^5*e*x^21
)/21
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x^2)*(a + c*x^4)^5)/x^2,x]
```

```
[Out] IntegrateAlgebraic[((d + e*x^2)*(a + c*x^4)^5)/x^2, x]
```

fricas [A] time = 0.53, size = 127, normalized size = 0.91

$$\frac{138567\,c^5ex^{22} + 153153\,c^5dx^{20} + 855855\,ac^4ex^{18} + 969969\,ac^4dx^{16} + 2238390\,a^2c^3ex^{14} + 2645370\,a^2c^3dx^{12} + 3233230\,a^3c^2ex^{10} + 4157010\,a^3c^2dx^8 + 2909907\,a^4cex^6 + 4849845\,a^4cdx^4 + 2909907\,a^5ex^2 - 2909907\,a^5d}{2909907\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/2909907*(138567*c^5*e*x^22 + 153153*c^5*d*x^20 + 855855*a*c^4*e*x^18 + 96
9969*a*c^4*d*x^16 + 2238390*a^2*c^3*e*x^14 + 2645370*a^2*c^3*d*x^12 + 32332
30*a^3*c^2*e*x^10 + 4157010*a^3*c^2*d*x^8 + 2909907*a^4*c*e*x^6 + 4849845*a
^4*c*d*x^4 + 2909907*a^5*e*x^2 - 2909907*a^5*d)/x
```

giac [A] time = 0.20, size = 127, normalized size = 0.91

$$\frac{1}{21}\,c^5x^{21}e + \frac{1}{19}\,c^5dx^{19} + \frac{5}{17}\,ac^4x^{17}e + \frac{1}{3}\,ac^4dx^{15} + \frac{10}{13}\,a^2c^3x^{13}e + \frac{10}{11}\,a^2c^3dx^{11} + \frac{10}{9}\,a^3c^2x^9e + \frac{10}{7}\,a^3c^2dx^7 + a^4cx^5e + \frac{5}{3}\,a^4cdx^3 + a^5xe - \frac{a^5d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/21*c^5*x^21*e + 1/19*c^5*d*x^19 + 5/17*a*c^4*x^17*e + 1/3*a*c^4*d*x^15 +
10/13*a^2*c^3*x^13*e + 10/11*a^2*c^3*d*x^11 + 10/9*a^3*c^2*x^9*e + 10/7*a^3
*c^2*d*x^7 + a^4*c*x^5*e + 5/3*a^4*c*d*x^3 + a^5*x*e - a^5*d/x
```

maple [A] time = 0.02, size = 122, normalized size = 0.88

$$\frac{c^5e\,x^{21}}{21} + \frac{c^5d\,x^{19}}{19} + \frac{5a\,c^4e\,x^{17}}{17} + \frac{a\,c^4d\,x^{15}}{3} + \frac{10a^2c^3e\,x^{13}}{13} + \frac{10a^2c^3d\,x^{11}}{11} + \frac{10a^3c^2e\,x^9}{9} + \frac{10a^3c^2d\,x^7}{7} + a^4ce\,x^5 + \frac{5a^4cd\,x^3}{3} + a^5ex - \frac{a^5d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+a)^5/x^2,x)
```

```
[Out] -a^5*d/x+a^5*e*x+5/3*a^4*c*d*x^3+a^4*c*e*x^5+10/7*a^3*c^2*d*x^7+10/9*a^3*c^
2*e*x^9+10/11*a^2*c^3*d*x^11+10/13*a^2*c^3*e*x^13+1/3*a*c^4*d*x^15+5/17*a*c
^4*e*x^17+1/19*c^5*d*x^19+1/21*c^5*e*x^21
```

maxima [A] time = 0.52, size = 121, normalized size = 0.87

$$\frac{1}{21}\,c^5ex^{21} + \frac{1}{19}\,c^5dx^{19} + \frac{5}{17}\,ac^4ex^{17} + \frac{1}{3}\,ac^4dx^{15} + \frac{10}{13}\,a^2c^3ex^{13} + \frac{10}{11}\,a^2c^3dx^{11} + \frac{10}{9}\,a^3c^2ex^9 + \frac{10}{7}\,a^3c^2dx^7 + a^4cex^5 + \frac{5}{3}\,a^4cdx^3 + a^5ex - \frac{a^5d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/21*c^5*e*x^21 + 1/19*c^5*d*x^19 + 5/17*a*c^4*e*x^17 + 1/3*a*c^4*d*x^15 +
10/13*a^2*c^3*e*x^13 + 10/11*a^2*c^3*d*x^11 + 10/9*a^3*c^2*e*x^9 + 10/7*a^3
*c^2*d*x^7 + a^4*c*e*x^5 + 5/3*a^4*c*d*x^3 + a^5*e*x - a^5*d/x
```

mupad [B] time = 0.09, size = 121, normalized size = 0.87

$$\frac{c^5\,d\,x^{19}}{19} - \frac{a^5\,d}{x} + \frac{c^5\,e\,x^{21}}{21} + a^5\,e\,x + \frac{10\,a^3\,c^2\,d\,x^7}{7} + \frac{10\,a^2\,c^3\,d\,x^{11}}{11} + \frac{10\,a^3\,c^2\,e\,x^9}{9} + \frac{10\,a^2\,c^3\,e\,x^{13}}{13} + \frac{5\,a^4\,c\,d\,x^3}{3} + \frac{a\,c^4\,d\,x^{15}}{3} + a^4\,c\,e\,x^5 + \frac{5\,a\,c^4\,e\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^4)^5*(d + e*x^2))/x^2,x)
```

```
[Out] (c^5*d*x^19)/19 - (a^5*d)/x + (c^5*e*x^21)/21 + a^5*e*x + (10*a^3*c^2*d*x^7
)/7 + (10*a^2*c^3*d*x^11)/11 + (10*a^3*c^2*e*x^9)/9 + (10*a^2*c^3*e*x^13)/1
3 + (5*a^4*c*d*x^3)/3 + (a*c^4*d*x^15)/3 + a^4*c*e*x^5 + (5*a*c^4*e*x^17)/1
7
```

sympy [A] time = 0.24, size = 143, normalized size = 1.03

$$-\frac{a^5d}{x}+a^5ex+\frac{5a^4cdx^3}{3}+a^4cex^5+\frac{10a^3c^2dx^7}{7}+\frac{10a^3c^2ex^9}{9}+\frac{10a^2c^3dx^{11}}{11}+\frac{10a^2c^3ex^{13}}{13}+\frac{ac^4dx^{15}}{3}+\frac{5ac^4ex^{17}}{17}+\frac{c^5dx^{19}}{19}+\frac{c^5ex^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+a)**5/x**2,x)
```

```
[Out] -a**5*d/x + a**5*e*x + 5*a**4*c*d*x**3/3 + a**4*c*e*x**5 + 10*a**3*c**2*d*x
**7/7 + 10*a**3*c**2*e*x**9/9 + 10*a**2*c**3*d*x**11/11 + 10*a**2*c**3*e*x*
*13/13 + a*c**4*d*x**15/3 + 5*a*c**4*e*x**17/17 + c**5*d*x**19/19 + c**5*e*
x**21/21
```

3.7 $\int \frac{(d+ex^2)(a+cx^4)^5}{x^3} dx$

Optimal. Leaf size=142

$$-\frac{a^5d}{2x^2}+a^5e\log(x)+\frac{5}{2}a^4cdx^2+\frac{5}{4}a^4cex^4+\frac{5}{3}a^3c^2dx^6+\frac{5}{4}a^3c^2ex^8+a^2c^3dx^{10}+\frac{5}{6}a^2c^3ex^{12}+\frac{5}{14}ac^4dx^{14}+\frac{5}{16}ac^4ex^{16}+\frac{1}{18}$$

Rubi [A] time = 0.12, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1252, 766}

$$a^2c^3dx^{10}+\frac{5}{3}a^3c^2dx^6+\frac{5}{6}a^2c^3ex^{12}+\frac{5}{4}a^3c^2ex^8+\frac{5}{2}a^4cdx^2+\frac{5}{4}a^4cex^4-\frac{a^5d}{2x^2}+a^5e\log(x)+\frac{5}{14}ac^4dx^{14}+\frac{5}{16}ac^4ex^{16}+\frac{1}{18}c^5dx^{18}+\frac{1}{20}c^5ex^{20}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + c*x^4)^5)/x^3,x]
```

```
[Out] -(a^5*d)/(2*x^2) + (5*a^4*c*d*x^2)/2 + (5*a^4*c*e*x^4)/4 + (5*a^3*c^2*d*x^6
)/3 + (5*a^3*c^2*e*x^8)/4 + a^2*c^3*d*x^10 + (5*a^2*c^3*e*x^12)/6 + (5*a*c^
4*d*x^14)/14 + (5*a*c^4*e*x^16)/16 + (c^5*d*x^18)/18 + (c^5*e*x^20)/20 + a^
5*e*Log[x]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)(a+cx^4)^5}{x^3} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)(a+cx^2)^5}{x^2} dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(5a^4cd+\frac{a^5d}{x^2}+\frac{a^5e}{x}+5a^4cex+10a^3c^2dx^2+10a^3c^2ex^3+10a^2c^3dx^4\right.\right.\\ &= -\frac{a^5d}{2x^2}+\frac{5}{2}a^4cdx^2+\frac{5}{4}a^4cex^4+\frac{5}{3}a^3c^2dx^6+\frac{5}{4}a^3c^2ex^8+a^2c^3dx^{10}+\frac{5}{6}a^2c^3ex^{12}+\frac{5}{1}\end{aligned}$$

Mathematica [A] time = 0.01, size = 142, normalized size = 1.00

$$-\frac{a^5d}{2x^2}+a^5e\log(x)+\frac{5}{2}a^4cdx^2+\frac{5}{4}a^4cex^4+\frac{5}{3}a^3c^2dx^6+\frac{5}{4}a^3c^2ex^8+a^2c^3dx^{10}+\frac{5}{6}a^2c^3ex^{12}+\frac{5}{14}ac^4dx^{14}+\frac{5}{16}ac^4ex^{16}+\frac{1}{18}c^5dx^{18}+\frac{1}{20}c^5ex^{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + c*x^4)^5)/x^3,x]
```

```
[Out] -1/2*(a^5*d)/x^2 + (5*a^4*c*d*x^2)/2 + (5*a^4*c*e*x^4)/4 + (5*a^3*c^2*d*x^6
)/3 + (5*a^3*c^2*e*x^8)/4 + a^2*c^3*d*x^10 + (5*a^2*c^3*e*x^12)/6 + (5*a*c^
4*d*x^14)/14 + (5*a*c^4*e*x^16)/16 + (c^5*d*x^18)/18 + (c^5*e*x^20)/20 + a^
5*e*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x^2)*(a + c*x^4)^5)/x^3,x]
```

```
[Out] IntegrateAlgebraic[((d + e*x^2)*(a + c*x^4)^5)/x^3, x]
```

fricas [A] time = 0.65, size = 129, normalized size = 0.91

$$\frac{252\,c^5ex^{22}+280\,c^5dx^{20}+1575\,ac^4ex^{18}+1800\,ac^4dx^{16}+4200\,a^2c^3ex^{14}+5040\,a^2c^3dx^{12}+6300\,a^3c^2ex^{10}+8400\,a^3c^2dx^8+6300\,a^4cex^6+12600\,a^4cdx^4+5040\,a^5ex^2\log(x)-2520\,a^5d}{5040\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/5040*(252*c^5*e*x^22 + 280*c^5*d*x^20 + 1575*a*c^4*e*x^18 + 1800*a*c^4*d*
x^16 + 4200*a^2*c^3*e*x^14 + 5040*a^2*c^3*d*x^12 + 6300*a^3*c^2*e*x^10 + 84
00*a^3*c^2*d*x^8 + 6300*a^4*c*e*x^6 + 12600*a^4*c*d*x^4 + 5040*a^5*e*x^2*lo
g(x) - 2520*a^5*d)/x^2
```

giac [A] time = 0.21, size = 142, normalized size = 1.00

$$\frac{1}{20}\,c^5x^{20}e+\frac{1}{18}\,c^5dx^{18}+\frac{5}{16}\,ac^4x^{16}e+\frac{5}{14}\,ac^4dx^{14}+\frac{5}{6}\,a^2c^3x^{12}e+a^2c^3dx^{10}+\frac{5}{4}\,a^3c^2x^8e+\frac{5}{3}\,a^3c^2dx^6+\frac{5}{4}\,a^4cx^4e+\frac{5}{2}\,a^4cdx^2+\frac{1}{2}\,a^5e\log\left(x^2\right)-\frac{a^5x^2e+a^5d}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/20*c^5*x^20*e + 1/18*c^5*d*x^18 + 5/16*a*c^4*x^16*e + 5/14*a*c^4*d*x^14 +
 5/6*a^2*c^3*x^12*e + a^2*c^3*d*x^10 + 5/4*a^3*c^2*x^8*e + 5/3*a^3*c^2*d*x^
6 + 5/4*a^4*c*x^4*e + 5/2*a^4*c*d*x^2 + 1/2*a^5*e*log(x^2) - 1/2*(a^5*x^2*e
 + a^5*d)/x^2
```

maple [A] time = 0.01, size = 123, normalized size = 0.87

$$\frac{c^5e\,x^{20}}{20}+\frac{c^5d\,x^{18}}{18}+\frac{5a\,c^4e\,x^{16}}{16}+\frac{5a\,c^4d\,x^{14}}{14}+\frac{5a^2c^3e\,x^{12}}{6}+a^2c^3d\,x^{10}+\frac{5a^3c^2e\,x^8}{4}+\frac{5a^3c^2d\,x^6}{3}+\frac{5a^4ce\,x^4}{4}+\frac{5a^4cd\,x^2}{2}+a^5e\ln(x)-\frac{a^5d}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+a)^5/x^3,x)
```

```
[Out] -1/2*a^5*d/x^2+5/2*a^4*c*d*x^2+5/4*a^4*c*e*x^4+5/3*a^3*c^2*d*x^6+5/4*a^3*c^
2*e*x^8+a^2*c^3*d*x^10+5/6*a^2*c^3*e*x^12+5/14*a*c^4*d*x^14+5/16*a*c^4*e*x^
16+1/18*c^5*d*x^18+1/20*c^5*e*x^20+a^5*e*ln(x)
```

maxima [A] time = 0.50, size = 125, normalized size = 0.88

$$\frac{1}{20}\,c^5ex^{20}+\frac{1}{18}\,c^5dx^{18}+\frac{5}{16}\,ac^4ex^{16}+\frac{5}{14}\,ac^4dx^{14}+\frac{5}{6}\,a^2c^3ex^{12}+a^2c^3dx^{10}+\frac{5}{4}\,a^3c^2ex^8+\frac{5}{3}\,a^3c^2dx^6+\frac{5}{4}\,a^4cex^4+\frac{5}{2}\,a^4cdx^2+\frac{1}{2}\,a^5e\log\left(x^2\right)-\frac{a^5d}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/20*c^5*e*x^20 + 1/18*c^5*d*x^18 + 5/16*a*c^4*e*x^16 + 5/14*a*c^4*d*x^14 +
 5/6*a^2*c^3*e*x^12 + a^2*c^3*d*x^10 + 5/4*a^3*c^2*e*x^8 + 5/3*a^3*c^2*d*x^
6 + 5/4*a^4*c*e*x^4 + 5/2*a^4*c*d*x^2 + 1/2*a^5*e*log(x^2) - 1/2*a^5*d/x^2
```

mupad [B] time = 0.07, size = 122, normalized size = 0.86

$$\frac{c^5\,d\,x^{18}}{18}-\frac{a^5\,d}{2\,x^2}+\frac{c^5\,e\,x^{20}}{20}+a^5\,e\,\ln(x)+\frac{5\,a^3\,c^2\,d\,x^6}{3}+a^2\,c^3\,d\,x^{10}+\frac{5\,a^3\,c^2\,e\,x^8}{4}+\frac{5\,a^2\,c^3\,e\,x^{12}}{6}+\frac{5\,a^4\,c\,d\,x^2}{2}+\frac{5\,a\,c^4\,d\,x^{14}}{14}+\frac{5\,a^4\,c\,e\,x^4}{4}+\frac{5\,a\,c^4\,e\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^4)^5*(d + e*x^2))/x^3,x)
```

```
[Out] (c^5*d*x^18)/18 - (a^5*d)/(2*x^2) + (c^5*e*x^20)/20 + a^5*e*log(x) + (5*a^3
*c^2*d*x^6)/3 + a^2*c^3*d*x^10 + (5*a^3*c^2*e*x^8)/4 + (5*a^2*c^3*e*x^12)/6
 + (5*a^4*c*d*x^2)/2 + (5*a*c^4*d*x^14)/14 + (5*a^4*c*e*x^4)/4 + (5*a*c^4*e
*x^16)/16
```

sympy [A] time = 0.28, size = 150, normalized size = 1.06

$$-\frac{a^5d}{2x^2}+a^5e\log(x)+\frac{5a^4cdx^2}{2}+\frac{5a^4cex^4}{4}+\frac{5a^3c^2dx^6}{3}+\frac{5a^3c^2ex^8}{4}+a^2c^3dx^{10}+\frac{5a^2c^3ex^{12}}{6}+\frac{5ac^4dx^{14}}{14}+\frac{5ac^4ex^{16}}{16}+\frac{c^5dx^{18}}{18}+\frac{c^5ex^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+a)**5/x**3,x)
```

```
[Out] -a**5*d/(2*x**2) + a**5*e*log(x) + 5*a**4*c*d*x**2/2 + 5*a**4*c*e*x**4/4 +
5*a**3*c**2*d*x**6/3 + 5*a**3*c**2*e*x**8/4 + a**2*c**3*d*x**10 + 5*a**2*c*
*3*e*x**12/6 + 5*a*c**4*d*x**14/14 + 5*a*c**4*e*x**16/16 + c**5*d*x**18/18
+ c**5*e*x**20/20
```

3.8 $\int x^5 \left(2+3x^2\right) \sqrt{5+x^4}\, dx$

Optimal. Leaf size=67

$$\frac{3}{10}\left(x^4+5\right)^{3/2}x^4-\frac{25}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{5}{8}\sqrt{x^4+5}\,x^2-\frac{1}{4}\left(4-x^2\right)\left(x^4+5\right)^{3/2}$$

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1252, 833, 780, 195, 215}

$$\frac{3}{10}\left(x^4+5\right)^{3/2}x^4-\frac{5}{8}\sqrt{x^4+5}\,x^2-\frac{1}{4}\left(4-x^2\right)\left(x^4+5\right)^{3/2}-\frac{25}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (-5*x^2*Sqrt[5 + x^4])/8 + (3*x^4*(5 + x^4)^(3/2))/10 - ((4 - x^2)*(5 + x^4
)^(3/2))/4 - (25*ArcSinh[x^2/Sqrt[5]])/8
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^5\left(2+3x^2\right)\sqrt{5+x^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2(2+3x)\sqrt{5+x^2}\,dx,x,x^2\right)\\ &= \frac{3}{10}x^4\left(5+x^4\right)^{3/2}+\frac{1}{10}\operatorname{Subst}\left(\int x(-30+10x)\sqrt{5+x^2}\,dx,x,x^2\right)\\ &= \frac{3}{10}x^4\left(5+x^4\right)^{3/2}-\frac{1}{4}\left(4-x^2\right)\left(5+x^4\right)^{3/2}-\frac{5}{4}\operatorname{Subst}\left(\int\sqrt{5+x^2}\,dx,x,x^2\right)\\ &= -\frac{5}{8}x^2\sqrt{5+x^4}+\frac{3}{10}x^4\left(5+x^4\right)^{3/2}-\frac{1}{4}\left(4-x^2\right)\left(5+x^4\right)^{3/2}-\frac{25}{8}\operatorname{Subst}\left(\int\frac{1}{\sqrt{5+}}\right.\\ &= -\frac{5}{8}x^2\sqrt{5+x^4}+\frac{3}{10}x^4\left(5+x^4\right)^{3/2}-\frac{1}{4}\left(4-x^2\right)\left(5+x^4\right)^{3/2}-\frac{25}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 0.75

$$\frac{1}{40}\sqrt{x^4+5}\left(12x^8+10x^6+20x^4+25x^2-200\right)-\frac{25}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (Sqrt[5 + x^4]*(-200 + 25*x^2 + 20*x^4 + 10*x^6 + 12*x^8))/40 - (25*ArcSinh
[x^2/Sqrt[5]])/8
```

IntegrateAlgebraic [A] time = 0.13, size = 56, normalized size = 0.84

$$\frac{25}{8}\log\left(\sqrt{x^4+5}-x^2\right)+\frac{1}{40}\sqrt{x^4+5}\left(12x^8+10x^6+20x^4+25x^2-200\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (Sqrt[5 + x^4]*(-200 + 25*x^2 + 20*x^4 + 10*x^6 + 12*x^8))/40 + (25*Log[-x^
2 + Sqrt[5 + x^4]])/8
```

fricas [A] time = 0.61, size = 48, normalized size = 0.72

$$\frac{1}{40}\left(12\,x^8+10\,x^6+20\,x^4+25\,x^2-200\right)\sqrt{x^4+5}+\frac{25}{8}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(12*x^8 + 10*x^6 + 20*x^4 + 25*x^2 - 200)*sqrt(x^4 + 5) + 25/8*log(-x^
2 + sqrt(x^4 + 5))
```

giac [A] time = 0.23, size = 54, normalized size = 0.81

$$\frac{1}{8}\left(2\,x^4+5\right)\sqrt{x^4+5}\,x^2+\frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}}-\frac{5}{2}\left(x^4+5\right)^{\frac{3}{2}}+\frac{25}{8}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 + 3/10*(x^4 + 5)^(5/2) - 5/2*(x^4 + 5)^(3
/2) + 25/8*log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.05, size = 53, normalized size = 0.79

$$\frac{\left(x^{4}+5\right)^{\frac{3}{2}} x^{2}}{4}-\frac{5\sqrt{x^{4}+5}\, x^{2}}{8}-\frac{25\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\, x^{2}}{5}\right)}{8}+\frac{\left(x^{4}+5\right)^{\frac{3}{2}}\left(3x^{4}-10\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)*(x^4+5)^(1/2),x)
```

```
[Out] 1/10*(x^4+5)^(3/2)*(3*x^4-10)+1/4*x^2*(x^4+5)^(3/2)-5/8*x^2*(x^4+5)^(1/2)-2
5/8*arcsinh(1/5*x^2*5^(1/2))
```

maxima [B] time = 1.01, size = 102, normalized size = 1.52

$$\frac{3}{10}\left(x^{4}+5\right)^{\frac{5}{2}}-\frac{5}{2}\left(x^{4}+5\right)^{\frac{3}{2}}-\frac{25\left(\frac{\sqrt{x^{4}+5}}{x^{2}}+\frac{\left(x^{4}+5\right)^{\frac{3}{2}}}{x^{6}}\right)}{8\left(\frac{2\left(x^{4}+5\right)}{x^{4}}-\frac{\left(x^{4}+5\right)^{2}}{x^{8}}-1\right)}-\frac{25}{16}\log\left(\frac{\sqrt{x^{4}+5}}{x^{2}}+1\right)+\frac{25}{16}\log\left(\frac{\sqrt{x^{4}+5}}{x^{2}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/10*(x^4 + 5)^(5/2) - 5/2*(x^4 + 5)^(3/2) - 25/8*(sqrt(x^4 + 5)/x^2 + (x^4
 + 5)^(3/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) - 25/16*log(sqrt(x
^4 + 5)/x^2 + 1) + 25/16*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.37, size = 42, normalized size = 0.63

$$\sqrt{x^{4}+5}\left(\frac{3\,x^{8}}{10}+\frac{x^{6}}{4}+\frac{x^{4}}{2}+\frac{5\,x^{2}}{8}-5\right)-\frac{25\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^{2}}{5}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)
```

```
[Out] (x^4 + 5)^(1/2)*((5*x^2)/8 + x^4/2 + x^6/4 + (3*x^8)/10 - 5) - (25*asinh((5
^(1/2)*x^2)/5))/8
```

sympy [A] time = 6.01, size = 97, normalized size = 1.45

$$\frac{x^{10}}{4\sqrt{x^{4}+5}}+\frac{3x^{8}\sqrt{x^{4}+5}}{10}+\frac{15x^{6}}{8\sqrt{x^{4}+5}}+\frac{x^{4}\sqrt{x^{4}+5}}{2}+\frac{25x^{2}}{8\sqrt{x^{4}+5}}-5\sqrt{x^{4}+5}-\frac{25\,\mathrm{asinh}\left(\frac{\sqrt{5}x^{2}}{5}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] x**10/(4*sqrt(x**4 + 5)) + 3*x**8*sqrt(x**4 + 5)/10 + 15*x**6/(8*sqrt(x**4
+ 5)) + x**4*sqrt(x**4 + 5)/2 + 25*x**2/(8*sqrt(x**4 + 5)) - 5*sqrt(x**4 +
5) - 25*asinh(sqrt(5)*x**2/5)/8
```

3.9 $\int x^3 \left(2+3x^2\right) \sqrt{5+x^4} \, dx$

Optimal. Leaf size=51

$$-\frac{75}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{15}{16}\sqrt{x^4+5}\,x^2+\frac{1}{24}\left(9x^2+8\right)\left(x^4+5\right)^{3/2}$$

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1252, 780, 195, 215}

$$-\frac{15}{16}\sqrt{x^4+5}\,x^2+\frac{1}{24}\left(9x^2+8\right)\left(x^4+5\right)^{3/2}-\frac{75}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (-15*x^2*Sqrt[5 + x^4])/16 + ((8 + 9*x^2)*(5 + x^4)^(3/2))/24 - (75*ArcSinh
[x^2/Sqrt[5]])/16
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(2+3x^2\right) \sqrt{5+x^4} \, dx &= \frac{1}{2}\text{Subst}\left(\int x(2+3x)\sqrt{5+x^2} \, dx,x,x^2\right)\\
&= \frac{1}{24}\left(8+9x^2\right)\left(5+x^4\right)^{3/2}-\frac{15}{8}\text{Subst}\left(\int \sqrt{5+x^2} \, dx,x,x^2\right)\\
&= -\frac{15}{16}x^2\sqrt{5+x^4}+\frac{1}{24}\left(8+9x^2\right)\left(5+x^4\right)^{3/2}-\frac{75}{16}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}} \, dx,x,x^2\right)\\
&= -\frac{15}{16}x^2\sqrt{5+x^4}+\frac{1}{24}\left(8+9x^2\right)\left(5+x^4\right)^{3/2}-\frac{75}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.86

$$\frac{1}{48}\left(\sqrt{x^4+5}\left(18x^6+16x^4+45x^2+80\right)-225\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (Sqrt[5 + x^4]*(80 + 45*x^2 + 16*x^4 + 18*x^6) - 225*ArcSinh[x^2/Sqrt[5]])/
48
```

IntegrateAlgebraic [A] time = 0.11, size = 51, normalized size = 1.00

$$\frac{75}{16}\log\left(\sqrt{x^4+5}-x^2\right)+\frac{1}{48}\sqrt{x^4+5}\left(18x^6+16x^4+45x^2+80\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (Sqrt[5 + x^4]*(80 + 45*x^2 + 16*x^4 + 18*x^6))/48 + (75*Log[-x^2 + Sqrt[5
+ x^4]])/16
```

fricas [A] time = 0.75, size = 43, normalized size = 0.84

$$\frac{1}{48}\left(18\,x^6+16\,x^4+45\,x^2+80\right)\sqrt{x^4+5}+\frac{75}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*(18*x^6 + 16*x^4 + 45*x^2 + 80)*sqrt(x^4 + 5) + 75/16*log(-x^2 + sqrt(
x^4 + 5))
```

giac [A] time = 0.23, size = 45, normalized size = 0.88

$$\frac{3}{16}\left(2\,x^4+5\right)\sqrt{x^4+5}\,x^2+\frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}}+\frac{75}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 3/16*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 + 1/3*(x^4 + 5)^(3/2) + 75/16*log(-x^2 +
 sqrt(x^4 + 5))
```

maple [A] time = 0.01, size = 46, normalized size = 0.90

$$\frac{3\left(x^4+5\right)^{\frac{3}{2}}x^2}{8}-\frac{15\sqrt{x^4+5}\,x^2}{16}-\frac{75\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16}+\frac{\left(x^4+5\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)*(x^4+5)^(1/2),x)
```

```
[Out] 3/8*(x^4+5)^(3/2)*x^2-15/16*(x^4+5)^(1/2)*x^2-75/16*arcsinh(1/5*5^(1/2)*x^2
)+1/3*(x^4+5)^(3/2)
```

maxima [B] time = 1.02, size = 93, normalized size = 1.82

$$\frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}} - \frac{75\left(\frac{\sqrt{x^4+5}}{x^2} + \frac{\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{16\left(\frac{2\left(x^4+5\right)}{x^4} - \frac{\left(x^4+5\right)^2}{x^8} - 1\right)} - \frac{75}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2} + 1\right) + \frac{75}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^4 + 5)^(3/2) - 75/16*(sqrt(x^4 + 5)/x^2 + (x^4 + 5)^(3/2)/x^6)/(2*(x
^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) - 75/32*log(sqrt(x^4 + 5)/x^2 + 1) + 75/
32*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.39, size = 37, normalized size = 0.73

$$\sqrt{x^4+5}\,\left(\frac{3\,x^6}{8} + \frac{x^4}{3} + \frac{15\,x^2}{16} + \frac{5}{3}\right) - \frac{75\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)
```

```
[Out] (x^4 + 5)^(1/2)*((15*x^2)/16 + x^4/3 + (3*x^6)/8 + 5/3) - (75*asinh((5^(1/2
)*x^2)/5))/16
```

sympy [A] time = 4.31, size = 70, normalized size = 1.37

$$\frac{3x^{10}}{8\sqrt{x^4+5}} + \frac{45x^6}{16\sqrt{x^4+5}} + \frac{75x^2}{16\sqrt{x^4+5}} + \frac{\left(x^4+5\right)^{\frac{3}{2}}}{3} - \frac{75\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] 3*x**10/(8*sqrt(x**4 + 5)) + 45*x**6/(16*sqrt(x**4 + 5)) + 75*x**2/(16*sqrt
(x**4 + 5)) + (x**4 + 5)**(3/2)/3 - 75*asinh(sqrt(5)*x**2/5)/16
```

3.10 $\int x\left(2+3x^2\right)\sqrt{5+x^4}\,dx$

Optimal. Leaf size=44

$$\frac{1}{2}\left(x^4+5\right)^{3/2}+\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{2}\sqrt{x^4+5}\,x^2$$

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1248, 641, 195, 215}

$$\frac{1}{2}\sqrt{x^4+5}\,x^2+\frac{1}{2}\left(x^4+5\right)^{3/2}+\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (x^2*Sqrt[5 + x^4])/2 + (5 + x^4)^(3/2)/2 + (5*ArcSinh[x^2/Sqrt[5]])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int x\left(2+3x^2\right)\sqrt{5+x^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int (2+3x)\sqrt{5+x^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\left(5+x^4\right)^{3/2}+\text{Subst}\left(\int \sqrt{5+x^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}x^2\sqrt{5+x^4}+\frac{1}{2}\left(5+x^4\right)^{3/2}+\frac{5}{2}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}x^2\sqrt{5+x^4}+\frac{1}{2}\left(5+x^4\right)^{3/2}+\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.82

$$\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{2}\sqrt{x^4+5}\left(x^4+x^2+5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2 + 3*x^2)*Sqrt[5 + x^4],x]
[Out] (Sqrt[5 + x^4]*(5 + x^2 + x^4))/2 + (5*ArcSinh[x^2/Sqrt[5]])/2
```

IntegrateAlgebraic [A] time = 0.11, size = 42, normalized size = 0.95

$$\frac{1}{2}\sqrt{x^4+5}\left(x^4+x^2+5\right)-\frac{5}{2}\log\left(\sqrt{x^4+5}-x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(2 + 3*x^2)*Sqrt[5 + x^4],x]
[Out] (Sqrt[5 + x^4]*(5 + x^2 + x^4))/2 - (5*Log[-x^2 + Sqrt[5 + x^4]])/2
```

fricas [A] time = 0.64, size = 34, normalized size = 0.77

$$\frac{1}{2}\left(x^4+x^2+5\right)\sqrt{x^4+5}-\frac{5}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
[Out] 1/2*(x^4 + x^2 + 5)*sqrt(x^4 + 5) - 5/2*log(-x^2 + sqrt(x^4 + 5))
```

giac [A] time = 0.20, size = 38, normalized size = 0.86

$$\frac{1}{2}\sqrt{x^4+5}\,x^2+\frac{1}{2}\left(x^4+5\right)^{\frac{3}{2}}-\frac{5}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^4 + 5)*x^2 + 1/2*(x^4 + 5)^(3/2) - 5/2*log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.01, size = 34, normalized size = 0.77

$$\frac{\sqrt{x^4+5}\,x^2}{2}+\frac{5\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}+\frac{\left(x^4+5\right)^{\frac{3}{2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)*(x^4+5)^(1/2),x)
[Out] 1/2*(x^4+5)^(3/2)+5/2*arcsinh(1/5*5^(1/2)*x^2)+1/2*(x^4+5)^(1/2)*x^2
```

maxima [B] time = 1.32, size = 67, normalized size = 1.52

$$\frac{1}{2}\left(x^4+5\right)^{\frac{3}{2}}+\frac{5\sqrt{x^4+5}}{2\,x^2\left(\frac{x^4+5}{x^4}-1\right)}+\frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(x^4 + 5)^(3/2) + 5/2*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) + 5/4*log
(sqrt(x^4 + 5)/x^2 + 1) - 5/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.14, size = 32, normalized size = 0.73

$$\frac{5\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}+\sqrt{x^4+5}\,\left(\frac{x^4}{2}+\frac{x^2}{2}+\frac{5}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)
```

```
[Out] (5*asinh((5^(1/2)*x^2)/5))/2 + (x^4 + 5)^(1/2)*(x^2/2 + x^4/2 + 5/2)
```

sympy [A] time = 3.08, size = 53, normalized size = 1.20

$$\frac{x^6}{2\sqrt{x^4+5}}+\frac{5x^2}{2\sqrt{x^4+5}}+\frac{\left(x^4+5\right)^{\frac{3}{2}}}{2}+\frac{5\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] x**6/(2*sqrt(x**4 + 5)) + 5*x**2/(2*sqrt(x**4 + 5)) + (x**4 + 5)**(3/2)/2 +
 5*asinh(sqrt(5)*x**2/5)/2
```

3.11 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx$

Optimal. Leaf size=58

$$-\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{4}\sqrt{x^4+5}\left(3x^2+4\right)$$

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1252, 815, 844, 215, 266, 63, 207}

$$\frac{1}{4}\sqrt{x^4+5}\left(3x^2+4\right)+\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x,x]
```

```
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 + (15*ArcSinh[x^2/Sqrt[5]])/4 - Sqrt[5]*ArcTa
nh[Sqrt[5 + x^4]/Sqrt[5]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(2+3x^2\right)\sqrt{5+x^4}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(2+3x)\sqrt{5+x^2}}{x}\,dx, x, x^2\right)\\
&= \frac{1}{4}\left(4+3x^2\right)\sqrt{5+x^4}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{20+15x}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{4}\left(4+3x^2\right)\sqrt{5+x^4}+\frac{15}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)+5\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx,\right.\\
&= \frac{1}{4}\left(4+3x^2\right)\sqrt{5+x^4}+\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{5}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)\\
&= \frac{1}{4}\left(4+3x^2\right)\sqrt{5+x^4}+\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+5\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)\\
&= \frac{1}{4}\left(4+3x^2\right)\sqrt{5+x^4}+\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] time = 0.06, size = 57, normalized size = 0.98

$$\frac{1}{4}\left(-4\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+15\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\sqrt{x^4+5}\left(3x^2+4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x,x]
```

```
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4] + 15*ArcSinh[x^2/Sqrt[5]] - 4*Sqrt[5]*ArcTanh[Sq
rt[5 + x^4]/Sqrt[5]])/4
```

IntegrateAlgebraic [A] time = 0.16, size = 75, normalized size = 1.29

$$\frac{1}{4}\sqrt{x^4+5}\left(3x^2+4\right)-\frac{15}{4}\log\left(\sqrt{x^4+5}-x^2\right)+2\sqrt{5}\tanh^{-1}\left(\frac{x^2}{\sqrt{5}}-\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + 3*x^2)*Sqrt[5 + x^4])/x,x]
```

```
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 + 2*Sqrt[5]*ArcTanh[x^2/Sqrt[5] - Sqrt[5 + x^
4]/Sqrt[5]] - (15*Log[-x^2 + Sqrt[5 + x^4]])/4
```

fricas [A] time = 0.63, size = 56, normalized size = 0.97

$$\frac{1}{4}\sqrt{x^4+5}\left(3\,x^2+4\right)+\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-\frac{15}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2)
 - 15/4*log(-x^2 + sqrt(x^4 + 5))
```

giac [A] time = 0.21, size = 76, normalized size = 1.31

$$\frac{1}{4}\sqrt{x^4+5}\left(3x^2+4\right)+\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)-\frac{15}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5)
)/(x^2 - sqrt(5) - sqrt(x^4 + 5))) - 15/4*log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.02, size = 49, normalized size = 0.84

$$\frac{3\sqrt{x^4+5}\,x^2}{4}+\frac{15\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}-\sqrt{5}\,\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)+\sqrt{x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x,x)
```

```
[Out] 3/4*(x^4+5)^(1/2)*x^2+15/4*arcsinh(1/5*5^(1/2)*x^2)+(x^4+5)^(1/2)-5^(1/2)*a
rctanh(5^(1/2)/(x^4+5)^(1/2))
```

maxima [B] time = 1.56, size = 99, normalized size = 1.71

$$\frac{1}{2}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\sqrt{x^4+5}+\frac{15\sqrt{x^4+5}}{4x^2\left(\frac{x^4+5}{x^4}-1\right)}+\frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + sqr
t(x^4 + 5) + 15/4*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) + 15/8*log(sqrt(x
^4 + 5)/x^2 + 1) - 15/8*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.15, size = 45, normalized size = 0.78

$$\frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}-\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)+\sqrt{x^4+5}\,\left(\frac{3x^2}{4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x,x)
```

```
[Out] (15*asinh((5^(1/2)*x^2)/5))/4 - 5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5)
+ (x^4 + 5)^(1/2)*((3*x^2)/4 + 1)
```

sympy [A] time = 15.59, size = 83, normalized size = 1.43

$$\frac{3x^6}{4\sqrt{x^4+5}}+\frac{15x^2}{4\sqrt{x^4+5}}+\sqrt{x^4+5}+\frac{\sqrt{5}\log\left(x^4\right)}{2}-\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)+\frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x,x)
```

```
[Out] 3*x**6/(4*sqrt(x**4 + 5)) + 15*x**2/(4*sqrt(x**4 + 5)) + sqrt(x**4 + 5) + s
qrt(5)*log(x**4)/2 - sqrt(5)*log(sqrt(x**4/5 + 1) + 1) + 15*asinh(sqrt(5)*x
**2/5)/4
```

3.12 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx$

Optimal. Leaf size=59

$$-\frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\sqrt{x^4+5}\,(2-3x^2)}{2x^2}$$

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1252, 813, 844, 215, 266, 63, 207}

$$-\frac{\sqrt{x^4+5}\,(2-3x^2)}{2x^2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^3,x]
```

```
[Out] -((2 - 3*x^2)*Sqrt[5 + x^4])/(2*x^2) + ArcSinh[x^2/Sqrt[5]] - (3*Sqrt[5]*Ar
cTanh[Sqrt[5 + x^4]/Sqrt[5]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(2+3x)\sqrt{5+x^2}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} - \frac{1}{4}\operatorname{Subst}\left(\int \frac{-30-4x}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} + \frac{15}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right) + \operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x,\right.\\
&= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{15}{4}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)\\
&= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{15}{2}\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)\\
&= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 59, normalized size = 1.00

$$\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{1}{2}\left(\frac{(3x^2-2)\sqrt{x^4+5}}{x^2} - 3\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^3,x]
```

```
[Out] ArcSinh[x^2/Sqrt[5]] + (((-2 + 3*x^2)*Sqrt[5 + x^4])/x^2 - 3*Sqrt[5]*ArcTan
h[Sqrt[5 + x^4]/Sqrt[5]])/2
```

IntegrateAlgebraic [A] time = 0.17, size = 76, normalized size = 1.29

$$\frac{\sqrt{x^4+5}\,(3x^2-2)}{2x^2} - \log\left(\sqrt{x^4+5}-x^2\right) + 3\sqrt{5}\tanh^{-1}\left(\frac{x^2}{\sqrt{5}} - \frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + 3*x^2)*Sqrt[5 + x^4])/x^3,x]
```

```
[Out] ((-2 + 3*x^2)*Sqrt[5 + x^4])/(2*x^2) + 3*Sqrt[5]*ArcTanh[x^2/Sqrt[5] - Sqrt
[5 + x^4]/Sqrt[5]] - Log[-x^2 + Sqrt[5 + x^4]]
```

fricas [A] time = 0.66, size = 72, normalized size = 1.22

$$\frac{3\sqrt{5}\,x^2\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-2\,x^2\log\left(-x^2+\sqrt{x^4+5}\right)-2\,x^2+\sqrt{x^4+5}\left(3\,x^2-2\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*sqrt(5)*x^2*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 2*x^2*log(-x^2 + s
qrt(x^4 + 5)) - 2*x^2 + sqrt(x^4 + 5)*(3*x^2 - 2))/x^2
```

giac [A] time = 0.25, size = 91, normalized size = 1.54

$$\frac{3}{2}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)+\frac{3}{2}\sqrt{x^4+5}+\frac{10}{\left(x^2-\sqrt{x^4+5}\right)^2-5}-\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 3/2*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
+ 5))) + 3/2*sqrt(x^4 + 5) + 10/((x^2 - sqrt(x^4 + 5))^2 - 5) - log(-x^2 +
sqrt(x^4 + 5))
```

maple [A] time = 0.01, size = 61, normalized size = 1.03

$$\frac{\sqrt{x^4+5}\,x^2}{5}+\operatorname{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}-\frac{\left(x^4+5\right)^{\frac{3}{2}}}{5x^2}+\frac{3\sqrt{x^4+5}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^3,x)
```

```
[Out] 3/2*(x^4+5)^(1/2)-3/2*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-1/5/x^2*(x^4+5
)^(3/2)+1/5*(x^4+5)^(1/2)*x^2+arcsinh(1/5*5^(1/2)*x^2)
```

maxima [A] time = 1.41, size = 88, normalized size = 1.49

$$\frac{3}{4}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{3}{2}\sqrt{x^4+5}-\frac{\sqrt{x^4+5}}{x^2}+\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + 3/2
*sqrt(x^4 + 5) - sqrt(x^4 + 5)/x^2 + 1/2*log(sqrt(x^4 + 5)/x^2 + 1) - 1/2*l
og(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.79, size = 51, normalized size = 0.86

$$\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)+\frac{3\,\sqrt{x^4+5}}{2}-\frac{\sqrt{x^4+5}}{x^2}+\frac{\sqrt{5}\,\operatorname{atan}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}\,1{}\mathrm{i}}{5}\right)\,3{}\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^3,x)
```

```
[Out] asinh((5^(1/2)*x^2)/5) + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/5)*3i)/
2 + (3*(x^4 + 5)^(1/2))/2 - (x^4 + 5)^(1/2)/x^2
```

sympy [A] time = 7.46, size = 83, normalized size = 1.41

$$-\frac{x^2}{\sqrt{x^4+5}}+\frac{3\sqrt{x^4+5}}{2}+\frac{3\sqrt{5}\log\left(x^4\right)}{4}-\frac{3\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2}+\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)-\frac{5}{x^2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**3,x)
```

```
[Out] -x**2/sqrt(x**4 + 5) + 3*sqrt(x**4 + 5)/2 + 3*sqrt(5)*log(x**4)/4 - 3*sqrt(
5)*log(sqrt(x**4/5 + 1) + 1)/2 + asinh(sqrt(5)*x**2/5) - 5/(x**2*sqrt(x**4
+ 5))
```

3.13 $\int \frac{\left(2+3x^2\right)\sqrt{5+x^4}}{x^5}\,dx$

Optimal. Leaf size=63

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}+\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\sqrt{x^4+5}\,\left(3x^2+1\right)}{2x^4}$$

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1252, 811, 844, 215, 266, 63, 207}

$$-\frac{\sqrt{x^4+5}\,\left(3x^2+1\right)}{2x^4}+\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^5,x]
```

```
[Out] -((1 + 3*x^2)*Sqrt[5 + x^4])/(2*x^4) + (3*ArcSinh[x^2/Sqrt[5]])/2 - ArcTanh
[Sqrt[5 + x^4]/Sqrt[5]]/(2*Sqrt[5])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
```

```
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(2+3x^2\right)\sqrt{5+x^4}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(2+3x)\sqrt{5+x^2}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{\left(1+3x^2\right)\sqrt{5+x^4}}{2x^4}-\frac{1}{40}\operatorname{Subst}\left(\int \frac{-20-60x}{x\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= -\frac{\left(1+3x^2\right)\sqrt{5+x^4}}{2x^4}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx,x,x^2\right)+\frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx,x\right.\\
&= -\frac{\left(1+3x^2\right)\sqrt{5+x^4}}{2x^4}+\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx,x,x^4\right)\\
&= -\frac{\left(1+3x^2\right)\sqrt{5+x^4}}{2x^4}+\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx,x,\sqrt{5+x^4}\right)\\
&= -\frac{\left(1+3x^2\right)\sqrt{5+x^4}}{2x^4}+\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{2\sqrt{5}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 59, normalized size = 0.94

$$\frac{1}{10}\left(-\sqrt{5}\tanh^{-1}\left(\sqrt{\frac{x^4}{5}+1}\right)+15\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{5\sqrt{x^4+5}\left(3x^2+1\right)}{x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^5,x]
```

```
[Out] ((-5*(1 + 3*x^2)*Sqrt[5 + x^4])/x^4 + 15*ArcSinh[x^2/Sqrt[5]] - Sqrt[5]*Arc
Tanh[Sqrt[1 + x^4/5]])/10
```

IntegrateAlgebraic [A] time = 0.26, size = 77, normalized size = 1.22

$$\frac{\sqrt{x^4+5}\left(-3x^2-1\right)}{2x^4}-\frac{3}{2}\log\left(\sqrt{x^4+5}-x^2\right)+\frac{\tanh^{-1}\left(\frac{x^2}{\sqrt{5}}-\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + 3*x^2)*Sqrt[5 + x^4])/x^5,x]
```

```
[Out] ((-1 - 3*x^2)*Sqrt[5 + x^4])/(2*x^4) + ArcTanh[x^2/Sqrt[5] - Sqrt[5 + x^4]/
Sqrt[5]]/Sqrt[5] - (3*Log[-x^2 + Sqrt[5 + x^4]])/2
```

fricas [A] time = 0.61, size = 72, normalized size = 1.14

$$\frac{\sqrt{5}\,x^4\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-15\,x^4\log\left(-x^2+\sqrt{x^4+5}\right)-15\,x^4-5\,\sqrt{x^4+5}\left(3\,x^2+1\right)}{10\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/10*(sqrt(5)*x^4*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 15*x^4*log(-x^2 + s
qrt(x^4 + 5)) - 15*x^4 - 5*sqrt(x^4 + 5)*(3*x^2 + 1))/x^4
```

giac [B] time = 0.24, size = 129, normalized size = 2.05

$$\frac{1}{10}\,\sqrt{5}\,\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)+\frac{\left(x^2-\sqrt{x^4+5}\right)^3+15\left(x^2-\sqrt{x^4+5}\right)^2+5\,x^2-5\,\sqrt{x^4+5}-75}{\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^2}-\frac{3}{2}\,\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/10*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
 + 5))) + ((x^2 - sqrt(x^4 + 5))^3 + 15*(x^2 - sqrt(x^4 + 5))^2 + 5*x^2 - 5
*sqrt(x^4 + 5) - 75)/((x^2 - sqrt(x^4 + 5))^2 - 5)^2 - 3/2*log(-x^2 + sqrt(
x^4 + 5))
```

maple [A] time = 0.02, size = 75, normalized size = 1.19

$$\frac{3\sqrt{x^4+5}\;x^2}{10}+\frac{3\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}-\frac{\sqrt{5}\;\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{10}-\frac{3\left(x^4+5\right)^{\frac{3}{2}}}{10x^2}-\frac{\left(x^4+5\right)^{\frac{3}{2}}}{10x^4}+\frac{\sqrt{x^4+5}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^5,x)
```

```
[Out] -1/10/x^4*(x^4+5)^(3/2)+1/10*(x^4+5)^(1/2)-1/10*5^(1/2)*arctanh(5^(1/2)/(x^
4+5)^(1/2))-3/10*(x^4+5)^(3/2)/x^2+3/10*(x^4+5)^(1/2)*x^2+3/2*arcsinh(1/5*5
^(1/2)*x^2)
```

maxima [A] time = 1.24, size = 91, normalized size = 1.44

$$\frac{1}{20}\,\sqrt{5}\,\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)-\frac{3\,\sqrt{x^4+5}}{2\,x^2}-\frac{\sqrt{x^4+5}}{2\,x^4}+\frac{3}{4}\,\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{3}{4}\,\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) - 3/
2*sqrt(x^4 + 5)/x^2 - 1/2*sqrt(x^4 + 5)/x^4 + 3/4*log(sqrt(x^4 + 5)/x^2 + 1
) - 3/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.42, size = 56, normalized size = 0.89

$$\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}-\frac{3\,\sqrt{x^4+5}}{2\,x^2}-\frac{\sqrt{x^4+5}}{2\,x^4}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}\,1{}\mathrm{i}}{5}\right)\,1{}\mathrm{i}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^5,x)
```

```
[Out] (3*asinh((5^(1/2)*x^2)/5))/2 + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/5
)*1i)/10 - (3*(x^4 + 5)^(1/2))/(2*x^2) - (x^4 + 5)^(1/2)/(2*x^4)
```

sympy [A] time = 6.04, size = 76, normalized size = 1.21

$$-\frac{3x^2}{2\sqrt{x^4+5}}-\frac{\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{10}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}-\frac{\sqrt{1+\frac{5}{x^4}}}{2x^2}-\frac{15}{2x^2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**5,x)
```

```
[Out] -3*x**2/(2*sqrt(x**4 + 5)) - sqrt(5)*asinh(sqrt(5)/x**2)/10 + 3*asinh(sqrt(
5)*x**2/5)/2 - sqrt(1 + 5/x**4)/(2*x**2) - 15/(2*x**2*sqrt(x**4 + 5))
```

3.14 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sqrt{x^4+5}}{4x^4}-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{4\sqrt{5}}-\frac{\left(x^4+5\right)^{3/2}}{15x^6}$$

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1252, 807, 266, 47, 63, 207}

$$-\frac{\left(x^4+5\right)^{3/2}}{15x^6}-\frac{3\sqrt{x^4+5}}{4x^4}-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^7,x]
```

```
[Out] (-3*Sqrt[5 + x^4])/(4*x^4) - (5 + x^4)^(3/2)/(15*x^6) - (3*ArcTanh[Sqrt[5 +
 x^4]/Sqrt[5]])/(4*Sqrt[5])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(2+3x)\sqrt{5+x^2}}{x^4}\,dx, x, x^2\right)\\
&= -\frac{(5+x^4)^{3/2}}{15x^6} + \frac{3}{2}\operatorname{Subst}\left(\int \frac{\sqrt{5+x^2}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{(5+x^4)^{3/2}}{15x^6} + \frac{3}{4}\operatorname{Subst}\left(\int \frac{\sqrt{5+x}}{x^2}\,dx, x, x^4\right)\\
&= -\frac{3\sqrt{5+x^4}}{4x^4} - \frac{(5+x^4)^{3/2}}{15x^6} + \frac{3}{8}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)\\
&= -\frac{3\sqrt{5+x^4}}{4x^4} - \frac{(5+x^4)^{3/2}}{15x^6} + \frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)\\
&= -\frac{3\sqrt{5+x^4}}{4x^4} - \frac{(5+x^4)^{3/2}}{15x^6} - \frac{3\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{4\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 1.24

$$-\frac{3\left(5x^4+\sqrt{5}\sqrt{x^4+5}\,x^4\tanh^{-1}\left(\sqrt{\frac{x^4}{5}+1}\right)+25\right)}{20x^4\sqrt{x^4+5}} - \frac{(x^4+5)^{3/2}}{15x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^7,x]
```

```
[Out] -1/15*(5 + x^4)^(3/2)/x^6 - (3*(25 + 5*x^4 + Sqrt[5]*x^4*Sqrt[5 + x^4]*ArcT
anh[Sqrt[1 + x^4/5]]))/(20*x^4*Sqrt[5 + x^4])
```

IntegrateAlgebraic [A] time = 0.25, size = 65, normalized size = 1.12

$$\frac{3\tanh^{-1}\left(\frac{x^2}{\sqrt{5}}-\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}} + \frac{\sqrt{x^4+5}\,(-4x^4-45x^2-20)}{60x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + 3*x^2)*Sqrt[5 + x^4])/x^7,x]
```

```
[Out] ((-20 - 45*x^2 - 4*x^4)*Sqrt[5 + x^4])/(60*x^6) + (3*ArcTanh[x^2/Sqrt[5] -
Sqrt[5 + x^4]/Sqrt[5]])/(2*Sqrt[5])
```

fricas [A] time = 0.73, size = 59, normalized size = 1.02

$$\frac{9\sqrt{5}\,x^6\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-4\,x^6-\left(4\,x^4+45\,x^2+20\right)\sqrt{x^4+5}}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/60*(9*sqrt(5)*x^6*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 4*x^6 - (4*x^4 +
45*x^2 + 20)*sqrt(x^4 + 5))/x^6
```

giac [B] time = 0.23, size = 116, normalized size = 2.00

$$\frac{3}{20}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)+\frac{9\left(x^2-\sqrt{x^4+5}\right)^5+12\left(x^2-\sqrt{x^4+5}\right)^4-225\,x^2+225\sqrt{x^4+5}+100}{6\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 3/20*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
 + 5))) + 1/6*(9*(x^2 - sqrt(x^4 + 5))^5 + 12*(x^2 - sqrt(x^4 + 5))^4 - 225
*x^2 + 225*sqrt(x^4 + 5) + 100)/((x^2 - sqrt(x^4 + 5))^2 - 5)^3
```

maple [A] time = 0.01, size = 52, normalized size = 0.90

$$-\frac{3\sqrt{5}\ \operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{20}-\frac{3\left(x^4+5\right)^{\frac{3}{2}}}{20x^4}-\frac{\left(x^4+5\right)^{\frac{3}{2}}}{15x^6}+\frac{3\sqrt{x^4+5}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^7,x)
```

```
[Out] -1/15*(x^4+5)^(3/2)/x^6-3/20*(x^4+5)^(3/2)/x^4+3/20*(x^4+5)^(1/2)-3/20*5^(1
/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))
```

maxima [A] time = 1.61, size = 59, normalized size = 1.02

$$\frac{3}{40}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)-\frac{3\sqrt{x^4+5}}{4\,x^4}-\frac{\left(x^4+5\right)^{\frac{3}{2}}}{15\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] 3/40*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) - 3/
4*sqrt(x^4 + 5)/x^4 - 1/15*(x^4 + 5)^(3/2)/x^6
```

mupad [B] time = 0.68, size = 43, normalized size = 0.74

$$-\frac{3\sqrt{5}\ \operatorname{atanh}\left(\frac{\sqrt{5}\ \sqrt{x^4+5}}{5}\right)}{20}-\frac{3\sqrt{x^4+5}}{4\,x^4}-\frac{\left(x^4+5\right)^{3/2}}{15\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^7,x)
```

```
[Out] - (3*5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/20 - (3*(x^4 + 5)^(1/2))/(
4*x^4) - (x^4 + 5)^(3/2)/(15*x^6)
```

sympy [A] time = 5.96, size = 63, normalized size = 1.09

$$-\frac{\sqrt{1+\frac{5}{x^4}}}{15}-\frac{3\sqrt{5}\ \operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{20}-\frac{3\sqrt{1+\frac{5}{x^4}}}{4x^2}-\frac{\sqrt{1+\frac{5}{x^4}}}{3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**7,x)
```

```
[Out] -sqrt(1 + 5/x**4)/15 - 3*sqrt(5)*asinh(sqrt(5)/x**2)/20 - 3*sqrt(1 + 5/x**4
)/(4*x**2) - sqrt(1 + 5/x**4)/(3*x**4)
```

3.15 $\int x^5 \left(2+3x^2\right) \left(5+x^4\right)^{3/2} \, dx$

Optimal. Leaf size=83

$$\frac{3}{14}\left(x^4+5\right)^{5/2}x^4-\frac{125}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{5}{24}\left(x^4+5\right)^{3/2}x^2-\frac{25}{16}\sqrt{x^4+5}\,x^2-\frac{1}{42}\left(18-7x^2\right)\left(x^4+5\right)^{5/2}$$

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1252, 833, 780, 195, 215}

$$\frac{3}{14}\left(x^4+5\right)^{5/2}x^4-\frac{5}{24}\left(x^4+5\right)^{3/2}x^2-\frac{25}{16}\sqrt{x^4+5}\,x^2-\frac{1}{42}\left(18-7x^2\right)\left(x^4+5\right)^{5/2}-\frac{125}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (-25*x^2*Sqrt[5 + x^4])/16 - (5*x^2*(5 + x^4)^(3/2))/24 + (3*x^4*(5 + x^4)^
(5/2))/14 - ((18 - 7*x^2)*(5 + x^4)^(5/2))/42 - (125*ArcSinh[x^2/Sqrt[5]])/
16
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int x^5 \left(2+3x^2\right)\left(5+x^4\right)^{3/2} dx = \frac{1}{2}\text{Subst}\left(\int x^2(2+3x)\left(5+x^2\right)^{3/2} dx, x, x^2\right)$$
$$= \frac{3}{14}x^4\left(5+x^4\right)^{5/2} + \frac{1}{14}\text{Subst}\left(\int x(-30+14x)\left(5+x^2\right)^{3/2} dx, x, x^2\right)$$
$$= \frac{3}{14}x^4\left(5+x^4\right)^{5/2} - \frac{1}{42}\left(18-7x^2\right)\left(5+x^4\right)^{5/2} - \frac{5}{6}\text{Subst}\left(\int \left(5+x^2\right)^{3/2} dx, x, x^2\right)$$
$$= -\frac{5}{24}x^2\left(5+x^4\right)^{3/2} + \frac{3}{14}x^4\left(5+x^4\right)^{5/2} - \frac{1}{42}\left(18-7x^2\right)\left(5+x^4\right)^{5/2} - \frac{25}{8}\text{Subst}\Big(\int$$
$$= -\frac{25}{16}x^2\sqrt{5+x^4} - \frac{5}{24}x^2\left(5+x^4\right)^{3/2} + \frac{3}{14}x^4\left(5+x^4\right)^{5/2} - \frac{1}{42}\left(18-7x^2\right)\left(5+x^4\right)^{5/}$$
$$= -\frac{25}{16}x^2\sqrt{5+x^4} - \frac{5}{24}x^2\left(5+x^4\right)^{3/2} + \frac{3}{14}x^4\left(5+x^4\right)^{5/2} - \frac{1}{42}\left(18-7x^2\right)\left(5+x^4\right)^{5/}$$

Mathematica [A] time = 0.05, size = 72, normalized size = 0.87

$$\frac{3}{14}\left(x^4-2\right)\left(x^4+5\right)^{5/2} + \frac{1}{6}x^2\left(x^4+5\right)^{5/2} - \frac{5}{48}\left(75\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \sqrt{x^4+5}\left(2x^4+25\right)x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (x^2*(5 + x^4)^(5/2))/6 + (3*(-2 + x^4)*(5 + x^4)^(5/2))/14 - (5*(x^2*Sqrt[
5 + x^4]*(25 + 2*x^4) + 75*ArcSinh[x^2/Sqrt[5]]))/48
```

IntegrateAlgebraic [A] time = 0.18, size = 66, normalized size = 0.80

$$\frac{125}{16}\log\left(\sqrt{x^4+5}-x^2\right) + \frac{1}{336}\sqrt{x^4+5}\left(72x^{12}+56x^{10}+576x^8+490x^6+360x^4+525x^2-3600\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[5 + x^4]*(-3600 + 525*x^2 + 360*x^4 + 490*x^6 + 576*x^8 + 56*x^10 + 7
2*x^12))/336 + (125*Log[-x^2 + Sqrt[5 + x^4]])/16
```

fricas [A] time = 0.63, size = 58, normalized size = 0.70

$$\frac{1}{336}\left(72x^{12}+56x^{10}+576x^8+490x^6+360x^4+525x^2-3600\right)\sqrt{x^4+5} + \frac{125}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/336*(72*x^12 + 56*x^10 + 576*x^8 + 490*x^6 + 360*x^4 + 525*x^2 - 3600)*sq
rt(x^4 + 5) + 125/16*log(-x^2 + sqrt(x^4 + 5))
```

giac [A] time = 0.25, size = 80, normalized size = 0.96

$$\frac{3}{14}\left(x^4+5\right)^{\frac{7}{2}} + \frac{1}{48}\left(2\left(4x^4+5\right)x^4-75\right)\sqrt{x^4+5}\,x^2 + \frac{5}{8}\left(2x^4+5\right)\sqrt{x^4+5}\,x^2 - \frac{3}{2}\left(x^4+5\right)^{\frac{5}{2}} + \frac{125}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 3/14*(x^4 + 5)^(7/2) + 1/48*(2*(4*x^4 + 5)*x^4 - 75)*sqrt(x^4 + 5)*x^2 + 5/
8*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 - 3/2*(x^4 + 5)^(5/2) + 125/16*log(-x^2 + s
qrt(x^4 + 5))
```

maple [A] time = 0.02, size = 73, normalized size = 0.88

$$\frac{\sqrt{x^4+5}\,x^{10}}{6}+\frac{35\sqrt{x^4+5}\,x^6}{24}+\frac{25\sqrt{x^4+5}\,x^2}{16}-\frac{125\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16}+\frac{3\sqrt{x^4+5}\,\left(x^4-2\right)\left(x^8+10x^4+25\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)*(x^4+5)^(3/2),x)
```

```
[Out] 3/14*(x^4+5)^(1/2)*(x^4-2)*(x^8+10*x^4+25)+1/6*x^10*(x^4+5)^(1/2)+35/24*x^6
*(x^4+5)^(1/2)+25/16*(x^4+5)^(1/2)*x^2-125/16*arcsinh(1/5*5^(1/2)*x^2)
```

maxima [A] time = 1.35, size = 127, normalized size = 1.53

$$\frac{3}{14}\left(x^4+5\right)^{\frac{7}{2}}-\frac{3}{2}\left(x^4+5\right)^{\frac{5}{2}}-\frac{125\left(\frac{3\sqrt{x^4+5}}{x^2}-\frac{8\left(x^4+5\right)^{\frac{3}{2}}}{x^6}-\frac{3\left(x^4+5\right)^{\frac{5}{2}}}{x^{10}}\right)}{48\left(\frac{3\left(x^4+5\right)}{x^4}-\frac{3\left(x^4+5\right)^2}{x^8}+\frac{\left(x^4+5\right)^3}{x^{12}}-1\right)}-\frac{125}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{125}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/14*(x^4 + 5)^(7/2) - 3/2*(x^4 + 5)^(5/2) - 125/48*(3*sqrt(x^4 + 5)/x^2 -
8*(x^4 + 5)^(3/2)/x^6 - 3*(x^4 + 5)^(5/2)/x^10)/(3*(x^4 + 5)/x^4 - 3*(x^4 +
 5)^2/x^8 + (x^4 + 5)^3/x^12 - 1) - 125/32*log(sqrt(x^4 + 5)/x^2 + 1) + 125
/32*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.32, size = 52, normalized size = 0.63

$$\sqrt{x^4+5}\,\left(\frac{3\,x^{12}}{14}+\frac{x^{10}}{6}+\frac{12\,x^8}{7}+\frac{35\,x^6}{24}+\frac{15\,x^4}{14}+\frac{25\,x^2}{16}-\frac{75}{7}\right)-\frac{125\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)
```

```
[Out] (x^4 + 5)^(1/2)*((25*x^2)/16 + (15*x^4)/14 + (35*x^6)/24 + (12*x^8)/7 + x^1
0/6 + (3*x^12)/14 - 75/7) - (125*asinh((5^(1/2)*x^2)/5))/16
```

sympy [A] time = 14.31, size = 131, normalized size = 1.58

$$\frac{x^{14}}{6\sqrt{x^4+5}}+\frac{3x^{12}\sqrt{x^4+5}}{14}+\frac{55x^{10}}{24\sqrt{x^4+5}}+\frac{12x^8\sqrt{x^4+5}}{7}+\frac{425x^6}{48\sqrt{x^4+5}}+\frac{15x^4\sqrt{x^4+5}}{14}+\frac{125x^2}{16\sqrt{x^4+5}}-\frac{75\sqrt{x^4+5}}{7}-\frac{125\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] x**14/(6*sqrt(x**4 + 5)) + 3*x**12*sqrt(x**4 + 5)/14 + 55*x**10/(24*sqrt(x*
*4 + 5)) + 12*x**8*sqrt(x**4 + 5)/7 + 425*x**6/(48*sqrt(x**4 + 5)) + 15*x**
4*sqrt(x**4 + 5)/14 + 125*x**2/(16*sqrt(x**4 + 5)) - 75*sqrt(x**4 + 5)/7 -
125*asinh(sqrt(5)*x**2/5)/16
```

3.16 $\int x^3 \left(2+3x^2\right) \left(5+x^4\right)^{3/2} \, dx$

Optimal. Leaf size=67

$$-\frac{375}{32} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{20} \left(5x^2+4\right) \left(x^4+5\right)^{5/2}-\frac{5}{16}x^2 \left(x^4+5\right)^{3/2}-\frac{75}{32}x^2\sqrt{x^4+5}$$

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1252, 780, 195, 215}

$$\frac{1}{20} \left(5x^2+4\right) \left(x^4+5\right)^{5/2}-\frac{5}{16}x^2 \left(x^4+5\right)^{3/2}-\frac{75}{32}x^2\sqrt{x^4+5}-\frac{375}{32} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (-75*x^2*Sqrt[5 + x^4])/32 - (5*x^2*(5 + x^4)^(3/2))/16 + ((4 + 5*x^2)*(5 +
 x^4)^(5/2))/20 - (375*ArcSinh[x^2/Sqrt[5]])/32
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int x^3\left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx = \frac{1}{2}\text{Subst}\left(\int x(2+3x)\left(5+x^2\right)^{3/2}\,dx,x,x^2\right)$$
$$= \frac{1}{20}\left(4+5x^2\right)\left(5+x^4\right)^{5/2} - \frac{5}{4}\text{Subst}\left(\int\left(5+x^2\right)^{3/2}\,dx,x,x^2\right)$$
$$= -\frac{5}{16}x^2\left(5+x^4\right)^{3/2} + \frac{1}{20}\left(4+5x^2\right)\left(5+x^4\right)^{5/2} - \frac{75}{16}\text{Subst}\left(\int\sqrt{5+x^2}\,dx,x,x\right.$$
$$= -\frac{75}{32}x^2\sqrt{5+x^4} - \frac{5}{16}x^2\left(5+x^4\right)^{3/2} + \frac{1}{20}\left(4+5x^2\right)\left(5+x^4\right)^{5/2} - \frac{375}{32}\text{Subst}\left(\int\right.$$
$$= -\frac{75}{32}x^2\sqrt{5+x^4} - \frac{5}{16}x^2\left(5+x^4\right)^{3/2} + \frac{1}{20}\left(4+5x^2\right)\left(5+x^4\right)^{5/2} - \frac{375}{32}\sinh^{-1}\left(\right.$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.81

$$\frac{1}{160}\left(\sqrt{x^4+5}\left(40x^{10}+32x^8+350x^6+320x^4+375x^2+800\right)-1875\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[5 + x^4]*(800 + 375*x^2 + 320*x^4 + 350*x^6 + 32*x^8 + 40*x^10) - 187
5*ArcSinh[x^2/Sqrt[5]])/160
```

IntegrateAlgebraic [A] time = 0.17, size = 61, normalized size = 0.91

$$\frac{375}{32}\log\left(\sqrt{x^4+5}-x^2\right)+\frac{1}{160}\sqrt{x^4+5}\left(40x^{10}+32x^8+350x^6+320x^4+375x^2+800\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[5 + x^4]*(800 + 375*x^2 + 320*x^4 + 350*x^6 + 32*x^8 + 40*x^10))/160
+ (375*Log[-x^2 + Sqrt[5 + x^4]])/32
```

fricas [A] time = 0.65, size = 53, normalized size = 0.79

$$\frac{1}{160}\left(40\,x^{10}+32\,x^8+350\,x^6+320\,x^4+375\,x^2+800\right)\sqrt{x^4+5}+\frac{375}{32}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/160*(40*x^10 + 32*x^8 + 350*x^6 + 320*x^4 + 375*x^2 + 800)*sqrt(x^4 + 5)
+ 375/32*log(-x^2 + sqrt(x^4 + 5))
```

giac [A] time = 0.20, size = 71, normalized size = 1.06

$$\frac{1}{32}\left(2\left(4\,x^4+5\right)x^4-75\right)\sqrt{x^4+5}\,x^2+\frac{15}{16}\left(2\,x^4+5\right)\sqrt{x^4+5}\,x^2+\frac{1}{5}\left(x^4+5\right)^{\frac{5}{2}}+\frac{375}{32}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(2*(4*x^4 + 5)*x^4 - 75)*sqrt(x^4 + 5)*x^2 + 15/16*(2*x^4 + 5)*sqrt(x^
4 + 5)*x^2 + 1/5*(x^4 + 5)^(5/2) + 375/32*log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.01, size = 58, normalized size = 0.87

$$\frac{\sqrt{x^4+5}\,x^{10}}{4}+\frac{35\sqrt{x^4+5}\,x^6}{16}+\frac{75\sqrt{x^4+5}\,x^2}{32}-\frac{375\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{32}+\frac{\left(x^4+5\right)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)*(x^4+5)^(3/2),x)
```

```
[Out] 1/4*(x^4+5)^(1/2)*x^10+35/16*(x^4+5)^(1/2)*x^6+75/32*(x^4+5)^(1/2)*x^2-375/
32*arcsinh(1/5*5^(1/2)*x^2)+1/5*(x^4+5)^(5/2)
```

maxima [B] time = 1.13, size = 118, normalized size = 1.76

$$\frac{1}{5}\left(x^4+5\right)^{\frac{5}{2}}-\frac{125\left(\frac{3\sqrt{x^4+5}}{x^2}-\frac{8\left(x^4+5\right)^{\frac{3}{2}}}{x^6}-\frac{3\left(x^4+5\right)^{\frac{5}{2}}}{x^{10}}\right)}{32\left(\frac{3\left(x^4+5\right)}{x^4}-\frac{3\left(x^4+5\right)^2}{x^8}+\frac{\left(x^4+5\right)^3}{x^{12}}-1\right)}-\frac{375}{64}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{375}{64}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*(x^4 + 5)^(5/2) - 125/32*(3*sqrt(x^4 + 5)/x^2 - 8*(x^4 + 5)^(3/2)/x^6 -
 3*(x^4 + 5)^(5/2)/x^10)/(3*(x^4 + 5)/x^4 - 3*(x^4 + 5)^2/x^8 + (x^4 + 5)^3
/x^12 - 1) - 375/64*log(sqrt(x^4 + 5)/x^2 + 1) + 375/64*log(sqrt(x^4 + 5)/x
^2 - 1)
```

mupad [B] time = 0.43, size = 47, normalized size = 0.70

$$\sqrt{x^4+5}\,\left(\frac{x^{10}}{4}+\frac{x^8}{5}+\frac{35\,x^6}{16}+2\,x^4+\frac{75\,x^2}{32}+5\right)-\frac{375\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)
```

```
[Out] (x^4 + 5)^(1/2)*((75*x^2)/32 + 2*x^4 + (35*x^6)/16 + x^8/5 + x^10/4 + 5) -
(375*asinh((5^(1/2)*x^2)/5))/32
```

sympy [B] time = 11.57, size = 124, normalized size = 1.85

$$\frac{x^{14}}{4\sqrt{x^4+5}}+\frac{55x^{10}}{16\sqrt{x^4+5}}+\frac{x^8\sqrt{x^4+5}}{5}+\frac{425x^6}{32\sqrt{x^4+5}}+\frac{x^4\sqrt{x^4+5}}{3}+\frac{375x^2}{32\sqrt{x^4+5}}+\frac{5\left(x^4+5\right)^{\frac{3}{2}}}{3}-\frac{10\sqrt{x^4+5}}{3}-\frac{375\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] x**14/(4*sqrt(x**4 + 5)) + 55*x**10/(16*sqrt(x**4 + 5)) + x**8*sqrt(x**4 +
5)/5 + 425*x**6/(32*sqrt(x**4 + 5)) + x**4*sqrt(x**4 + 5)/3 + 375*x**2/(32*
sqrt(x**4 + 5)) + 5*(x**4 + 5)**(3/2)/3 - 10*sqrt(x**4 + 5)/3 - 375*asinh(s
qrt(5)*x**2/5)/32
```

3.17 $\int x\left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=60

$$\frac{3}{10}\left(x^4+5\right)^{5/2}+\frac{75}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{4}x^2\left(x^4+5\right)^{3/2}+\frac{15}{8}x^2\sqrt{x^4+5}$$

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1248, 641, 195, 215}

$$\frac{3}{10}\left(x^4+5\right)^{5/2}+\frac{1}{4}x^2\left(x^4+5\right)^{3/2}+\frac{15}{8}x^2\sqrt{x^4+5}+\frac{75}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (15*x^2*Sqrt[5 + x^4])/8 + (x^2*(5 + x^4)^(3/2))/4 + (3*(5 + x^4)^(5/2))/10
 + (75*ArcSinh[x^2/Sqrt[5]])/8
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int x\left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int(2+3x)\left(5+x^2\right)^{3/2}\,dx,x,x^2\right)\\
&= \frac{3}{10}\left(5+x^4\right)^{5/2}+\operatorname{Subst}\left(\int\left(5+x^2\right)^{3/2}\,dx,x,x^2\right)\\
&= \frac{1}{4}x^2\left(5+x^4\right)^{3/2}+\frac{3}{10}\left(5+x^4\right)^{5/2}+\frac{15}{4}\operatorname{Subst}\left(\int\sqrt{5+x^2}\,dx,x,x^2\right)\\
&= \frac{15}{8}x^2\sqrt{5+x^4}+\frac{1}{4}x^2\left(5+x^4\right)^{3/2}+\frac{3}{10}\left(5+x^4\right)^{5/2}+\frac{75}{8}\operatorname{Subst}\left(\int\frac{1}{\sqrt{5+x^2}}\,dx,\right.\\
&= \frac{15}{8}x^2\sqrt{5+x^4}+\frac{1}{4}x^2\left(5+x^4\right)^{3/2}+\frac{3}{10}\left(5+x^4\right)^{5/2}+\frac{75}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.93

$$\frac{75}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{2}\sqrt{x^4+5}\left(\frac{3x^8}{5}+\frac{x^6}{2}+6x^4+\frac{25x^2}{4}+15\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[5 + x^4]*(15 + (25*x^2)/4 + 6*x^4 + x^6/2 + (3*x^8)/5))/2 + (75*ArcSi
nh[x^2/Sqrt[5]])/8
```

IntegrateAlgebraic [A] time = 0.17, size = 56, normalized size = 0.93

$$\frac{1}{40}\sqrt{x^4+5}\left(12x^8+10x^6+120x^4+125x^2+300\right)-\frac{75}{8}\log\left(\sqrt{x^4+5}-x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[5 + x^4]*(300 + 125*x^2 + 120*x^4 + 10*x^6 + 12*x^8))/40 - (75*Log[-x
^2 + Sqrt[5 + x^4]])/8
```

fricas [A] time = 0.82, size = 48, normalized size = 0.80

$$\frac{1}{40}\left(12\,x^8+10\,x^6+120\,x^4+125\,x^2+300\right)\sqrt{x^4+5}-\frac{75}{8}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*(12*x^8 + 10*x^6 + 120*x^4 + 125*x^2 + 300)*sqrt(x^4 + 5) - 75/8*log(-
x^2 + sqrt(x^4 + 5))
```

giac [A] time = 0.24, size = 57, normalized size = 0.95

$$\frac{1}{8}\left(2\,x^4+5\right)\sqrt{x^4+5}\,x^2+\frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}}+\frac{5}{2}\sqrt{x^4+5}\,x^2-\frac{75}{8}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 + 3/10*(x^4 + 5)^(5/2) + 5/2*sqrt(x^4 + 5
)*x^2 - 75/8*log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.01, size = 46, normalized size = 0.77

$$\frac{\sqrt{x^4+5}\,x^6}{4}+\frac{25\sqrt{x^4+5}\,x^2}{8}+\frac{75\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{8}+\frac{3\left(x^4+5\right)^{\frac{5}{2}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)*(x^4+5)^(3/2),x)
```

```
[Out] 3/10*(x^4+5)^(5/2)+1/4*(x^4+5)^(1/2)*x^6+25/8*(x^4+5)^(1/2)*x^2+75/8*arcsin
h(1/5*5^(1/2)*x^2)
```

maxima [B] time = 1.60, size = 95, normalized size = 1.58

$$\frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}}+\frac{25\left(\frac{3\sqrt{x^4+5}}{x^2}-\frac{5\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{8\left(\frac{2\left(x^4+5\right)}{x^4}-\frac{\left(x^4+5\right)^2}{x^8}-1\right)}+\frac{75}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{75}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/10*(x^4 + 5)^(5/2) + 25/8*(3*sqrt(x^4 + 5)/x^2 - 5*(x^4 + 5)^(3/2)/x^6)/(
2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) + 75/16*log(sqrt(x^4 + 5)/x^2 + 1) -
 75/16*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.18, size = 42, normalized size = 0.70

$$\frac{75\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{8}+\sqrt{x^4+5}\,\left(\frac{3\,x^8}{10}+\frac{x^6}{4}+3\,x^4+\frac{25\,x^2}{8}+\frac{15}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)
```

```
[Out] (75*asinh((5^(1/2)*x^2)/5))/8 + (x^4 + 5)^(1/2)*((25*x^2)/8 + 3*x^4 + x^6/4
 + (3*x^8)/10 + 15/2)
```

sympy [B] time = 8.19, size = 109, normalized size = 1.82

$$\frac{x^{10}}{4\sqrt{x^4+5}}+\frac{3x^8\sqrt{x^4+5}}{10}+\frac{35x^6}{8\sqrt{x^4+5}}+\frac{x^4\sqrt{x^4+5}}{2}+\frac{125x^2}{8\sqrt{x^4+5}}+\frac{5\left(x^4+5\right)^{\frac{3}{2}}}{2}-5\sqrt{x^4+5}+\frac{75\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] x**10/(4*sqrt(x**4 + 5)) + 3*x**8*sqrt(x**4 + 5)/10 + 35*x**6/(8*sqrt(x**4
+ 5)) + x**4*sqrt(x**4 + 5)/2 + 125*x**2/(8*sqrt(x**4 + 5)) + 5*(x**4 + 5)*
*(3/2)/2 - 5*sqrt(x**4 + 5) + 75*asinh(sqrt(5)*x**2/5)/8
```

3.18 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x}\,dx$

Optimal. Leaf size=78

$$-5\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{24}\left(9x^2+8\right)\left(x^4+5\right)^{3/2}+\frac{5}{16}\left(9x^2+16\right)\sqrt{x^4+5}$$

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1252, 815, 844, 215, 266, 63, 207}

$$\frac{1}{24}\left(9x^2+8\right)\left(x^4+5\right)^{3/2}+\frac{5}{16}\left(9x^2+16\right)\sqrt{x^4+5}+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-5\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x,x]
```

```
[Out] (5*(16 + 9*x^2)*Sqrt[5 + x^4])/16 + ((8 + 9*x^2)*(5 + x^4)^(3/2))/24 + (225
*ArcSinh[x^2/Sqrt[5]])/16 - 5*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)(5+x^2)^{3/2}}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{24}(8+9x^2)(5+x^4)^{3/2} + \frac{1}{8}\text{Subst}\left(\int \frac{(40+45x)\sqrt{5+x^2}}{x}\,dx, x, x^2\right)$$
$$= \frac{5}{16}(16+9x^2)\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)(5+x^4)^{3/2} + \frac{1}{16}\text{Subst}\left(\int \frac{400+225x}{x\sqrt{5+x^2}}\,d\right.$$
$$= \frac{5}{16}(16+9x^2)\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)(5+x^4)^{3/2} + \frac{225}{16}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx\right.$$
$$= \frac{5}{16}(16+9x^2)\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)(5+x^4)^{3/2} + \frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{25}{2}\text{Su}$$
$$= \frac{5}{16}(16+9x^2)\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)(5+x^4)^{3/2} + \frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + 25\,\text{Sub}$$
$$= \frac{5}{16}(16+9x^2)\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)(5+x^4)^{3/2} + \frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - 5\sqrt{5}\,\text{ta}$$

Mathematica [A] time = 0.03, size = 67, normalized size = 0.86

$$\frac{1}{48}\left(-240\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right) + 675\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \sqrt{x^4+5}\,(18x^6+16x^4+225x^2+320)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x,x]
[Out] (Sqrt[5 + x^4]*(320 + 225*x^2 + 16*x^4 + 18*x^6) + 675*ArcSinh[x^2/Sqrt[5]]
 - 240*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/48
```

IntegrateAlgebraic [A] time = 0.22, size = 85, normalized size = 1.09

$$-\frac{225}{16}\log\left(\sqrt{x^4+5}-x^2\right) + 10\sqrt{5}\tanh^{-1}\left(\frac{x^2}{\sqrt{5}} - \frac{\sqrt{x^4+5}}{\sqrt{5}}\right) + \frac{1}{48}\sqrt{x^4+5}\,(18x^6+16x^4+225x^2+320)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + 3*x^2)*(5 + x^4)^(3/2))/x,x]
[Out] (Sqrt[5 + x^4]*(320 + 225*x^2 + 16*x^4 + 18*x^6))/48 + 10*Sqrt[5]*ArcTanh[x
^2/Sqrt[5] - Sqrt[5 + x^4]/Sqrt[5]] - (225*Log[-x^2 + Sqrt[5 + x^4]])/16
```

fricas [A] time = 0.54, size = 67, normalized size = 0.86

$$\frac{1}{48}\left(18\,x^{6}+16\,x^{4}+225\,x^{2}+320\right)\sqrt{x^{4}+5}+5\,\sqrt{5}\,\log\left(-\frac{\sqrt{5}-\sqrt{x^{4}+5}}{x^{2}}\right)-\frac{225}{16}\,\log\left(-x^{2}+\sqrt{x^{4}+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/48*(18*x^6 + 16*x^4 + 225*x^2 + 320)*sqrt(x^4 + 5) + 5*sqrt(5)*log(-(sqrt
(5) - sqrt(x^4 + 5))/x^2) - 225/16*log(-x^2 + sqrt(x^4 + 5))
```

giac [A] time = 0.26, size = 90, normalized size = 1.15

$$\frac{1}{48}\,\sqrt{x^{4}+5}\left(\left(2\left(9\,x^{2}+8\right)x^{2}+225\right)x^{2}+320\right)+5\,\sqrt{5}\,\log\left(-\frac{x^{2}+\sqrt{5}-\sqrt{x^{4}+5}}{x^{2}-\sqrt{5}-\sqrt{x^{4}+5}}\right)-\frac{225}{16}\,\log\left(-x^{2}+\sqrt{x^{4}+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/48*sqrt(x^4 + 5)*((2*(9*x^2 + 8)*x^2 + 225)*x^2 + 320) + 5*sqrt(5)*log(-(
x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) - 225/16*lo
g(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.02, size = 75, normalized size = 0.96

$$\frac{3\sqrt{x^{4}+5}\,x^{6}}{8}+\frac{\sqrt{x^{4}+5}\,x^{4}}{3}+\frac{75\sqrt{x^{4}+5}\,x^{2}}{16}+\frac{225\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^{2}}{5}\right)}{16}-5\sqrt{5}\,\;\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^{4}+5}}\right)+\frac{20\sqrt{x^{4}+5}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x,x)
```

```
[Out] 3/8*(x^4+5)^(1/2)*x^6+75/16*(x^4+5)^(1/2)*x^2+225/16*arcsinh(1/5*5^(1/2)*x^
2)+1/3*x^4*(x^4+5)^(1/2)+20/3*(x^4+5)^(1/2)-5*5^(1/2)*arctanh(5^(1/2)/(x^4+
5)^(1/2))
```

maxima [B] time = 1.19, size = 138, normalized size = 1.77

$$\frac{1}{3}\left(x^{4}+5\right)^{\frac{3}{2}}+\frac{5}{2}\,\sqrt{5}\,\log\left(-\frac{\sqrt{5}-\sqrt{x^{4}+5}}{\sqrt{5}+\sqrt{x^{4}+5}}\right)+5\,\sqrt{x^{4}+5}+\frac{75\left(\frac{3\,\sqrt{x^{4}+5}}{x^{2}}-\frac{5\left(x^{4}+5\right)^{\frac{3}{2}}}{x^{6}}\right)}{16\left(\frac{2\left(x^{4}+5\right)}{x^{4}}-\frac{\left(x^{4}+5\right)^{2}}{x^{8}}-1\right)}+\frac{225}{32}\,\log\left(\frac{\sqrt{x^{4}+5}}{x^{2}}+1\right)-\frac{225}{32}\,\log\left(\frac{\sqrt{x^{4}+5}}{x^{2}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*(x^4 + 5)^(3/2) + 5/2*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) +
 sqrt(x^4 + 5))) + 5*sqrt(x^4 + 5) + 75/16*(3*sqrt(x^4 + 5)/x^2 - 5*(x^4 +
5)^(3/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) + 225/32*log(sqrt(x^4
 + 5)/x^2 + 1) - 225/32*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.18, size = 55, normalized size = 0.71

$$\frac{225\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^{2}}{5}\right)}{16}-5\,\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^{4}+5}}{5}\right)+\sqrt{x^{4}+5}\,\left(\frac{3\,x^{6}}{8}+\frac{x^{4}}{3}+\frac{75\,x^{2}}{16}+\frac{20}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x,x)
```

```
[Out] (225*asinh((5^(1/2)*x^2)/5))/16 - 5*5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))
/5) + (x^4 + 5)^(1/2)*((75*x^2)/16 + x^4/3 + (3*x^6)/8 + 20/3)
```

sympy [A] time = 33.67, size = 114, normalized size = 1.46

$$\frac{3x^{10}}{8\sqrt{x^4+5}}+\frac{105x^6}{16\sqrt{x^4+5}}+\frac{375x^2}{16\sqrt{x^4+5}}+\frac{\left(x^4+5\right)^{\frac{3}{2}}}{3}+5\sqrt{x^4+5}+\frac{5\sqrt{5}\log\left(x^4\right)}{2}-5\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)+\frac{225\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x,x)
```

```
[Out] 3*x**10/(8*sqrt(x**4 + 5)) + 105*x**6/(16*sqrt(x**4 + 5)) + 375*x**2/(16*sq
rt(x**4 + 5)) + (x**4 + 5)**(3/2)/3 + 5*sqrt(x**4 + 5) + 5*sqrt(5)*log(x**4
)/2 - 5*sqrt(5)*log(sqrt(x**4/5 + 1) + 1) + 225*asinh(sqrt(5)*x**2/5)/16
```

3.19 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=81

$$-\frac{15}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{(2-x^2)(x^4+5)^{3/2}}{2x^2}+\frac{3}{2}(x^2+5)\sqrt{x^4+5}$$

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1252, 813, 815, 844, 215, 266, 63, 207}

$$-\frac{(2-x^2)(x^4+5)^{3/2}}{2x^2}+\frac{3}{2}(x^2+5)\sqrt{x^4+5}+\frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{15}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^3,x]
```

```
[Out] (3*(5 + x^2)*Sqrt[5 + x^4])/2 - ((2 - x^2)*(5 + x^4)^(3/2))/(2*x^2) + (15*A
rcSinh[x^2/Sqrt[5]])/2 - (15*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)(5+x^2)^{3/2}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} - \frac{1}{4}\text{Subst}\left(\int \frac{(-30-12x)\sqrt{5+x^2}}{x}\,dx, x, x^2\right)$$

$$= \frac{3}{2}(5+x^2)\sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} - \frac{1}{8}\text{Subst}\left(\int \frac{-300-60x}{x\sqrt{5+x^2}}\,dx, x, x^2\right)$$

$$= \frac{3}{2}(5+x^2)\sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} + \frac{15}{2}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right) +$$

$$= \frac{3}{2}(5+x^2)\sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} + \frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{75}{4}\text{Subst}\left(\int \frac{}{x}\right.$$

$$= \frac{3}{2}(5+x^2)\sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} + \frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{75}{2}\text{Subst}\left(\int \right.$$

$$= \frac{3}{2}(5+x^2)\sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} + \frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{15}{2}\sqrt{5}\tanh^{-1}\left(\right.$$

Mathematica [C] time = 0.05, size = 71, normalized size = 0.88

$$\frac{1}{2}\left(\sqrt{x^4+5}\,(x^4+20) - 15\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)\right) - \frac{5\sqrt{5}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{x^4}{5}\right)}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^3,x]
```

```
[Out] (Sqrt[5 + x^4]*(20 + x^4) - 15*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/2 -
(5*Sqrt[5]*Hypergeometric2F1[-3/2, -1/2, 1/2, -1/5*x^4])/x^2
```

IntegrateAlgebraic [A] time = 0.25, size = 84, normalized size = 1.04

$$-\frac{15}{2}\log\left(\sqrt{x^4+5}-x^2\right)+15\sqrt{5}\tanh^{-1}\left(\frac{x^2}{\sqrt{5}}-\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{\sqrt{x^4+5}\left(x^6+x^4+20x^2-10\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^3,x]
```

```
[Out] (Sqrt[5 + x^4]*(-10 + 20*x^2 + x^4 + x^6))/(2*x^2) + 15*Sqrt[5]*ArcTanh[x^2
/Sqrt[5] - Sqrt[5 + x^4]/Sqrt[5]] - (15*Log[-x^2 + Sqrt[5 + x^4]])/2
```

fricas [A] time = 0.60, size = 78, normalized size = 0.96

$$\frac{15\sqrt{5}\,x^2\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-15\,x^2\log\left(-x^2+\sqrt{x^4+5}\right)-10\,x^2+\left(x^6+x^4+20\,x^2-10\right)\sqrt{x^4+5}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(15*sqrt(5)*x^2*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 15*x^2*log(-x^2 +
 sqrt(x^4 + 5)) - 10*x^2 + (x^6 + x^4 + 20*x^2 - 10)*sqrt(x^4 + 5))/x^2
```

giac [A] time = 0.26, size = 102, normalized size = 1.26

$$\frac{1}{2}\sqrt{x^4+5}\left(\left(x^2+1\right)x^2+20\right)+\frac{15}{2}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)+\frac{50}{\left(x^2-\sqrt{x^4+5}\right)^2-5}-\frac{15}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^4 + 5)*((x^2 + 1)*x^2 + 20) + 15/2*sqrt(5)*log(-(x^2 + sqrt(5) -
 sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) + 50/((x^2 - sqrt(x^4 + 5)
)^2 - 5) - 15/2*log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.02, size = 75, normalized size = 0.93

$$\frac{\sqrt{x^4+5}\,x^4}{2}+\frac{\sqrt{x^4+5}\,x^2}{2}+\frac{15\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}-\frac{15\sqrt{5}\,\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}-\frac{5\sqrt{x^4+5}}{x^2}+10\sqrt{x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^3,x)
```

```
[Out] 1/2*(x^4+5)^(1/2)*x^4+10*(x^4+5)^(1/2)-15/2*5^(1/2)*arctanh(5^(1/2)/(x^4+5)
^(1/2))+1/2*(x^4+5)^(1/2)*x^2+15/2*arcsinh(1/5*5^(1/2)*x^2)-5*(x^4+5)^(1/2)
/x^2
```

maxima [A] time = 1.33, size = 122, normalized size = 1.51

$$\frac{1}{2}\left(x^4+5\right)^{\frac{3}{2}}+\frac{15}{4}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{15}{2}\sqrt{x^4+5}-\frac{5\sqrt{x^4+5}}{x^2}+\frac{5\sqrt{x^4+5}}{2x^2\left(\frac{x^4+5}{x^4}-1\right)}+\frac{15}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{15}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(x^4 + 5)^(3/2) + 15/4*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5)
+ sqrt(x^4 + 5))) + 15/2*sqrt(x^4 + 5) - 5*sqrt(x^4 + 5)/x^2 + 5/2*sqrt(x^4
 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) + 15/4*log(sqrt(x^4 + 5)/x^2 + 1) - 15/4*lo
g(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.76, size = 64, normalized size = 0.79

$$\frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}+\sqrt{x^4+5}\,\left(\frac{x^4}{2}+\frac{x^2}{2}+10\right)-\frac{5\,\sqrt{x^4+5}}{x^2}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}\,1\mathrm{i}}{5}\right)\,15\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^3,x)
```

```
[Out] (15*asinh((5^(1/2)*x^2)/5))/2 + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/
5)*15i)/2 + (x^4 + 5)^(1/2)*(x^2/2 + x^4/2 + 10) - (5*(x^4 + 5)^(1/2))/x^2
```

sympy [A] time = 11.39, size = 114, normalized size = 1.41

$$\frac{x^6}{2\sqrt{x^4+5}}-\frac{5x^2}{2\sqrt{x^4+5}}+\frac{\left(x^4+5\right)^{\frac{3}{2}}}{2}+\frac{15\sqrt{x^4+5}}{2}+\frac{15\sqrt{5}\,\log\left(x^4\right)}{4}-\frac{15\sqrt{5}\,\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2}+\frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}-\frac{25}{x^2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**3,x)
```

```
[Out] x**6/(2*sqrt(x**4 + 5)) - 5*x**2/(2*sqrt(x**4 + 5)) + (x**4 + 5)**(3/2)/2 +
 15*sqrt(x**4 + 5)/2 + 15*sqrt(5)*log(x**4)/4 - 15*sqrt(5)*log(sqrt(x**4/5
+ 1) + 1)/2 + 15*asinh(sqrt(5)*x**2/5)/2 - 25/(x**2*sqrt(x**4 + 5))
```

3.20 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=86

$$-\frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{(2-3x^2)(x^4+5)^{3/2}}{4x^4}-\frac{3(15-2x^2)\sqrt{x^4+5}}{4x^2}$$

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1252, 813, 844, 215, 266, 63, 207}

$$-\frac{(2-3x^2)(x^4+5)^{3/2}}{4x^4}-\frac{3(15-2x^2)\sqrt{x^4+5}}{4x^2}+\frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^5,x]
```

```
[Out] (-3*(15 - 2*x^2)*Sqrt[5 + x^4])/(4*x^2) - ((2 - 3*x^2)*(5 + x^4)^(3/2))/(4*
x^4) + (45*ArcSinh[x^2/Sqrt[5]])/4 - (3*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[
5]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
```

```
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)(5+x^2)^{3/2}}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} - \frac{3}{16}\text{Subst}\left(\int \frac{(-60-8x)\sqrt{5+x^2}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} + \frac{3}{32}\text{Subst}\left(\int \frac{80+120x}{x\sqrt{5+x^2}}\,dx, x,\right.$$

$$= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} + \frac{15}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x,\right.$$

$$= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} + \frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{15}{4}\text{Subst}\left(\right.$$

$$= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} + \frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{15}{2}\text{Subst}\left(\right.$$

$$= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} + \frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{3}{2}\sqrt{5}\tanh$$

Mathematica [C] time = 0.03, size = 60, normalized size = 0.70

$$\frac{1}{125}\left(x^4+5\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{x^4}{5}+1\right)-\frac{15\sqrt{5}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{x^4}{5}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^5,x]
```

```
[Out] (-15*Sqrt[5]*Hypergeometric2F1[-3/2, -1/2, 1/2, -1/5*x^4])/(2*x^2) + ((5 +
x^4)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + x^4/5])/125
```

IntegrateAlgebraic [A] time = 0.31, size = 88, normalized size = 1.02

$$-\frac{45}{4}\log\left(\sqrt{x^4+5}-x^2\right)+3\sqrt{5}\tanh^{-1}\left(\frac{x^2}{\sqrt{5}}-\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{\sqrt{x^4+5}\left(3x^6+4x^4-30x^2-10\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^5,x]
```

```
[Out] (Sqrt[5 + x^4]*(-10 - 30*x^2 + 4*x^4 + 3*x^6))/(4*x^4) + 3*Sqrt[5]*ArcTanh[
x^2/Sqrt[5] - Sqrt[5 + x^4]/Sqrt[5]] - (45*Log[-x^2 + Sqrt[5 + x^4]])/4
```

fricas [A] time = 0.59, size = 82, normalized size = 0.95

$$\frac{6\sqrt{5}\,x^4\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-45\,x^4\log\left(-x^2+\sqrt{x^4+5}\right)-30\,x^4+\left(3\,x^6+4\,x^4-30\,x^2-10\right)\sqrt{x^4+5}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(6*sqrt(5)*x^4*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 45*x^4*log(-x^2 +
sqrt(x^4 + 5)) - 30*x^4 + (3*x^6 + 4*x^4 - 30*x^2 - 10)*sqrt(x^4 + 5))/x^4
```

giac [B] time = 0.25, size = 146, normalized size = 1.70

$$\frac{1}{4}\sqrt{x^4+5}\left(3\,x^2+4\right)+\frac{3}{2}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)+\frac{5\left(\left(x^2-\sqrt{x^4+5}\right)^3+15\left(x^2-\sqrt{x^4+5}\right)^2+5\,x^2-5\sqrt{x^4+5}-75\right)}{\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^2}-\frac{45}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + 3/2*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4
+ 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) + 5*((x^2 - sqrt(x^4 + 5))^3 + 15*(x
^2 - sqrt(x^4 + 5))^2 + 5*x^2 - 5*sqrt(x^4 + 5) - 75)/((x^2 - sqrt(x^4 + 5)
)^2 - 5)^2 - 45/4*log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.02, size = 73, normalized size = 0.85

$$\frac{3\sqrt{x^4+5}\;x^2}{4}+\frac{45\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}-\frac{3\sqrt{5}\;\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}-\frac{15\sqrt{x^4+5}}{2x^2}-\frac{5\sqrt{x^4+5}}{2x^4}+\sqrt{x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^5,x)
```

```
[Out] (x^4+5)^(1/2)-3/2*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-5/2*(x^4+5)^(1/2)/
x^4+3/4*(x^4+5)^(1/2)*x^2+45/4*arcsinh(1/5*5^(1/2)*x^2)-15/2*(x^4+5)^(1/2)/
x^2
```

maxima [A] time = 1.37, size = 123, normalized size = 1.43

$$\frac{3}{4}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\sqrt{x^4+5}-\frac{15\sqrt{x^4+5}}{2\,x^2}+\frac{15\sqrt{x^4+5}}{4\,x^2\left(\frac{x^4+5}{x^4}-1\right)}-\frac{5\sqrt{x^4+5}}{2\,x^4}+\frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + sqr
t(x^4 + 5) - 15/2*sqrt(x^4 + 5)/x^2 + 15/4*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^
4 - 1)) - 5/2*sqrt(x^4 + 5)/x^4 + 45/8*log(sqrt(x^4 + 5)/x^2 + 1) - 45/8*lo
g(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.55, size = 71, normalized size = 0.83

$$\frac{45\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}+\sqrt{x^4+5}\,\left(\frac{3\,x^2}{4}+1\right)-\frac{15\,\sqrt{x^4+5}}{2\,x^2}-\frac{5\,\sqrt{x^4+5}}{2\,x^4}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}\,1\mathrm{i}}{5}\right)\,3\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^5,x)
```

```
[Out] (45*asinh((5^(1/2)*x^2)/5))/4 + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/
5)*3i)/2 + (x^4 + 5)^(1/2)*((3*x^2)/4 + 1) - (15*(x^4 + 5)^(1/2))/(2*x^2) -
 (5*(x^4 + 5)^(1/2))/(2*x^4)
```

sympy [A] time = 12.75, size = 133, normalized size = 1.55

$$\frac{3x^6}{4\sqrt{x^4+5}}-\frac{15x^2}{4\sqrt{x^4+5}}+\sqrt{x^4+5}+\frac{\sqrt{5}\log\left(x^4\right)}{2}-\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)-\frac{\sqrt{5}\,\mathrm{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{2}+\frac{45\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4}-\frac{5\sqrt{1+\frac{5}{x^4}}}{2x^2}-\frac{75}{2x^2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**5,x)
```

```
[Out] 3*x**6/(4*sqrt(x**4 + 5)) - 15*x**2/(4*sqrt(x**4 + 5)) + sqrt(x**4 + 5) + s
qrt(5)*log(x**4)/2 - sqrt(5)*log(sqrt(x**4/5 + 1) + 1) - sqrt(5)*asinh(sqrt
(5)/x**2)/2 + 45*asinh(sqrt(5)*x**2/5)/4 - 5*sqrt(1 + 5/x**4)/(2*x**2) - 75
/(2*x**2*sqrt(x**4 + 5))
```

3.21 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=82

$$-\frac{9}{4}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{(4-9x^2)\sqrt{x^4+5}}{4x^2}-\frac{(9x^2+4)(x^4+5)^{3/2}}{12x^6}$$

Rubi [A] time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1252, 811, 813, 844, 215, 266, 63, 207}

$$-\frac{(9x^2+4)(x^4+5)^{3/2}}{12x^6}-\frac{(4-9x^2)\sqrt{x^4+5}}{4x^2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{9}{4}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^7,x]
```

```
[Out] -((4 - 9*x^2)*Sqrt[5 + x^4])/(4*x^2) - ((4 + 9*x^2)*(5 + x^4)^(3/2))/(12*x^
6) + ArcSinh[x^2/Sqrt[5]] - (9*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/4
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 && !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{\left(2+3x^2\right)\left(5+x^4\right)^{3/2}}{x^7}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(2+3x)\left(5+x^2\right)^{3/2}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{\left(4+9x^2\right)\left(5+x^4\right)^{3/2}}{12x^6} - \frac{1}{40}\operatorname{Subst}\left(\int \frac{(-40-90x)\sqrt{5+x^2}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{\left(4-9x^2\right)\sqrt{5+x^4}}{4x^2} - \frac{\left(4+9x^2\right)\left(5+x^4\right)^{3/2}}{12x^6} + \frac{1}{80}\operatorname{Subst}\left(\int \frac{900+80x}{x\sqrt{5+x^2}}\,dx, x, x^2 \right.$$

$$= -\frac{\left(4-9x^2\right)\sqrt{5+x^4}}{4x^2} - \frac{\left(4+9x^2\right)\left(5+x^4\right)^{3/2}}{12x^6} + \frac{45}{4}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2 \right.$$

$$= -\frac{\left(4-9x^2\right)\sqrt{5+x^4}}{4x^2} - \frac{\left(4+9x^2\right)\left(5+x^4\right)^{3/2}}{12x^6} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{45}{8}\operatorname{Subst}\left(\int \right.$$

$$= -\frac{\left(4-9x^2\right)\sqrt{5+x^4}}{4x^2} - \frac{\left(4+9x^2\right)\left(5+x^4\right)^{3/2}}{12x^6} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{45}{4}\operatorname{Subst}\left(\int \right.$$

$$= -\frac{\left(4-9x^2\right)\sqrt{5+x^4}}{4x^2} - \frac{\left(4+9x^2\right)\left(5+x^4\right)^{3/2}}{12x^6} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{9}{4}\sqrt{5}\tanh^{-1}\left(\right.$$

Mathematica [C] time = 0.02, size = 60, normalized size = 0.73

$$\frac{3}{250}\left(x^4+5\right)^{5/2}{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{x^4}{5}+1\right)-\frac{5\sqrt{5}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{x^4}{5}\right)}{3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^7,x]
```

```
[Out] (-5*Sqrt[5]*Hypergeometric2F1[-3/2, -3/2, -1/2, -1/5*x^4])/(3*x^6) + (3*(5
+ x^4)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + x^4/5])/250
```

IntegrateAlgebraic [A] time = 0.33, size = 88, normalized size = 1.07

$$-\log\left(\sqrt{x^4+5}-x^2\right)+\frac{9}{2}\sqrt{5}\tanh^{-1}\left(\frac{x^2}{\sqrt{5}}-\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{\sqrt{x^4+5}\left(18x^6-16x^4-45x^2-20\right)}{12x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^7,x]
```

```
[Out] (Sqrt[5 + x^4]*(-20 - 45*x^2 - 16*x^4 + 18*x^6))/(12*x^6) + (9*Sqrt[5]*ArcT
anh[x^2/Sqrt[5] - Sqrt[5 + x^4]/Sqrt[5]])/2 - Log[-x^2 + Sqrt[5 + x^4]]
```

fricas [A] time = 0.49, size = 82, normalized size = 1.00

$$\frac{27\sqrt{5}x^6\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-12x^6\log\left(-x^2+\sqrt{x^4+5}\right)-16x^6+\left(18x^6-16x^4-45x^2-20\right)\sqrt{x^4+5}}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(27*sqrt(5)*x^6*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 12*x^6*log(-x^2
+ sqrt(x^4 + 5)) - 16*x^6 + (18*x^6 - 16*x^4 - 45*x^2 - 20)*sqrt(x^4 + 5))/
x^6
```

giac [B] time = 0.27, size = 158, normalized size = 1.93

$$\frac{9}{4}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)+\frac{3}{2}\sqrt{x^4+5}+\frac{5\left(9\left(x^2-\sqrt{x^4+5}\right)^5+24\left(x^2-\sqrt{x^4+5}\right)^4-120\left(x^2-\sqrt{x^4+5}\right)^2-225x^2+225\sqrt{x^4+5}+400\right)}{6\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^3}-\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 9/4*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
+ 5))) + 3/2*sqrt(x^4 + 5) + 5/6*(9*(x^2 - sqrt(x^4 + 5))^5 + 24*(x^2 - sqr
t(x^4 + 5))^4 - 120*(x^2 - sqrt(x^4 + 5))^2 - 225*x^2 + 225*sqrt(x^4 + 5) +
 400)/((x^2 - sqrt(x^4 + 5))^2 - 5)^3 - log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.02, size = 73, normalized size = 0.89

$$\operatorname{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)-\frac{9\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{4}-\frac{4\sqrt{x^4+5}}{3x^2}-\frac{15\sqrt{x^4+5}}{4x^4}-\frac{5\sqrt{x^4+5}}{3x^6}+\frac{3\sqrt{x^4+5}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^7,x)
```

```
[Out] 3/2*(x^4+5)^(1/2)-9/4*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-15/4*(x^4+5)^(
1/2)/x^4+arcsinh(1/5*5^(1/2)*x^2)-4/3*(x^4+5)^(1/2)/x^2-5/3*(x^4+5)^(1/2)/x
^6
```

maxima [A] time = 1.35, size = 112, normalized size = 1.37

$$\frac{9}{8}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{3}{2}\sqrt{x^4+5}-\frac{\sqrt{x^4+5}}{x^2}-\frac{15\sqrt{x^4+5}}{4x^4}-\frac{\left(x^4+5\right)^{\frac{3}{2}}}{3x^6}+\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 9/8*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + 3/2
*sqrt(x^4 + 5) - sqrt(x^4 + 5)/x^2 - 15/4*sqrt(x^4 + 5)/x^4 - 1/3*(x^4 + 5)
^(3/2)/x^6 + 1/2*log(sqrt(x^4 + 5)/x^2 + 1) - 1/2*log(sqrt(x^4 + 5)/x^2 - 1
)
```

mupad [B] time = 0.95, size = 82, normalized size = 1.00

$$\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)+\frac{3\,\sqrt{x^4+5}}{2}+\sqrt{x^4+5}\,\left(\frac{2}{3\,x^2}-\frac{5}{3\,x^6}\right)-\frac{2\,\sqrt{x^4+5}}{x^2}-\frac{15\,\sqrt{x^4+5}}{4\,x^4}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}\,1{}\mathrm{i}}{5}\right)\,9{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^7,x)
```

```
[Out] asinh((5^(1/2)*x^2)/5) + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/5)*9i)/
4 + (3*(x^4 + 5)^(1/2))/2 + (x^4 + 5)^(1/2)*(2/(3*x^2) - 5/(3*x^6)) - (2*(x
^4 + 5)^(1/2))/x^2 - (15*(x^4 + 5)^(1/2))/(4*x^4)
```

sympy [A] time = 12.56, size = 148, normalized size = 1.80

$$-\frac{x^2}{\sqrt{x^4+5}}-\frac{\sqrt{1+\frac{5}{x^4}}}{3}+\frac{3\sqrt{x^4+5}}{2}+\frac{3\sqrt{5}\log\left(x^4\right)}{4}-\frac{3\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2}-\frac{3\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{4}+\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)-\frac{15\sqrt{1+\frac{5}{x^4}}}{4x^2}-\frac{5}{x^2\sqrt{x^4+5}}-\frac{5\sqrt{1+\frac{5}{x^4}}}{3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**7,x)
```

```
[Out] -x**2/sqrt(x**4 + 5) - sqrt(1 + 5/x**4)/3 + 3*sqrt(x**4 + 5)/2 + 3*sqrt(5)*
log(x**4)/4 - 3*sqrt(5)*log(sqrt(x**4/5 + 1) + 1)/2 - 3*sqrt(5)*asinh(sqrt(
5)/x**2)/4 + asinh(sqrt(5)*x**2/5) - 15*sqrt(1 + 5/x**4)/(4*x**2) - 5/(x**2
*sqrt(x**4 + 5)) - 5*sqrt(1 + 5/x**4)/(3*x**4)
```

3.22 $\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=67

$$\frac{1}{3}\sqrt{x^4+5}\,x^4+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{3}{8}\sqrt{x^4+5}\,x^6-\frac{5}{48}\left(27x^2+32\right)\sqrt{x^4+5}$$

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1252, 833, 780, 215}

$$\frac{3}{8}\sqrt{x^4+5}\,x^6+\frac{1}{3}\sqrt{x^4+5}\,x^4-\frac{5}{48}\left(27x^2+32\right)\sqrt{x^4+5}+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (x^4*Sqrt[5 + x^4])/3 + (3*x^6*Sqrt[5 + x^4])/8 - (5*(32 + 27*x^2)*Sqrt[5 +
 x^4])/48 + (225*ArcSinh[x^2/Sqrt[5]])/16
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7\left(2+3x^2\right)}{\sqrt{5+x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3(2+3x)}{\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= \frac{3}{8}x^6\sqrt{5+x^4}+\frac{1}{8}\operatorname{Subst}\left(\int \frac{x^2(-45+8x)}{\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= \frac{1}{3}x^4\sqrt{5+x^4}+\frac{3}{8}x^6\sqrt{5+x^4}+\frac{1}{24}\operatorname{Subst}\left(\int \frac{(-80-135x)x}{\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= \frac{1}{3}x^4\sqrt{5+x^4}+\frac{3}{8}x^6\sqrt{5+x^4}-\frac{5}{48}\left(32+27x^2\right)\sqrt{5+x^4}+\frac{225}{16}\operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx,x,\right.\\
&= \frac{1}{3}x^4\sqrt{5+x^4}+\frac{3}{8}x^6\sqrt{5+x^4}-\frac{5}{48}\left(32+27x^2\right)\sqrt{5+x^4}+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.66

$$\frac{1}{48}\left(675\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\sqrt{x^4+5}\left(18x^6+16x^4-135x^2-160\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (Sqrt[5 + x^4]*(-160 - 135*x^2 + 16*x^4 + 18*x^6) + 675*ArcSinh[x^2/Sqrt[5]
])/48
```

IntegrateAlgebraic [A] time = 0.13, size = 51, normalized size = 0.76

$$\frac{1}{48}\sqrt{x^4+5}\left(18x^6+16x^4-135x^2-160\right)-\frac{225}{16}\log\left(\sqrt{x^4+5}-x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^7*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (Sqrt[5 + x^4]*(-160 - 135*x^2 + 16*x^4 + 18*x^6))/48 - (225*Log[-x^2 + Sqr
t[5 + x^4]])/16
```

fricas [A] time = 0.55, size = 43, normalized size = 0.64

$$\frac{1}{48}\left(18\,x^6+16\,x^4-135\,x^2-160\right)\sqrt{x^4+5}-\frac{225}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*(18*x^6 + 16*x^4 - 135*x^2 - 160)*sqrt(x^4 + 5) - 225/16*log(-x^2 + sq
rt(x^4 + 5))
```

giac [A] time = 0.19, size = 46, normalized size = 0.69

$$\frac{1}{48}\sqrt{x^4+5}\left(\left(2\left(9\,x^2+8\right)x^2-135\right)x^2-160\right)-\frac{225}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(x^4 + 5)*((2*(9*x^2 + 8)*x^2 - 135)*x^2 - 160) - 225/16*log(-x^2
+ sqrt(x^4 + 5))
```

maple [A] time = 0.02, size = 51, normalized size = 0.76

$$\frac{3\sqrt{x^4+5}\,x^6}{8}-\frac{45\sqrt{x^4+5}\,x^2}{16}+\frac{225\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16}+\frac{\sqrt{x^4+5}\,\left(x^4-10\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(3*x^2+2)/(x^4+5)^(1/2),x)
```

```
[Out] 3/8*(x^4+5)^(1/2)*x^6-45/16*(x^4+5)^(1/2)*x^2+225/16*arcsinh(1/5*5^(1/2)*x^
2)+1/3*(x^4+5)^(1/2)*(x^4-10)
```

maxima [A] time = 1.31, size = 104, normalized size = 1.55

$$\frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}}-5\sqrt{x^4+5}-\frac{75\left(\frac{5\sqrt{x^4+5}}{x^2}-\frac{3\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{16\left(\frac{2\left(x^4+5\right)}{x^4}-\frac{\left(x^4+5\right)^2}{x^8}-1\right)}+\frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^4 + 5)^(3/2) - 5*sqrt(x^4 + 5) - 75/16*(5*sqrt(x^4 + 5)/x^2 - 3*(x^4
 + 5)^(3/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) + 225/32*log(sqrt(
x^4 + 5)/x^2 + 1) - 225/32*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.59, size = 38, normalized size = 0.57

$$\frac{225\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16}-\sqrt{x^4+5}\,\left(-\frac{3\,x^6}{8}-\frac{x^4}{3}+\frac{45\,x^2}{16}+\frac{10}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
```

```
[Out] (225*asinh((5^(1/2)*x^2)/5))/16 - (x^4 + 5)^(1/2)*((45*x^2)/16 - x^4/3 - (3
*x^6)/8 + 10/3)
```

sympy [A] time = 7.07, size = 85, normalized size = 1.27

$$\frac{3x^{10}}{8\sqrt{x^4+5}}-\frac{15x^6}{16\sqrt{x^4+5}}+\frac{x^4\sqrt{x^4+5}}{3}-\frac{225x^2}{16\sqrt{x^4+5}}-\frac{10\sqrt{x^4+5}}{3}+\frac{225\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*x**10/(8*sqrt(x**4 + 5)) - 15*x**6/(16*sqrt(x**4 + 5)) + x**4*sqrt(x**4 +
 5)/3 - 225*x**2/(16*sqrt(x**4 + 5)) - 10*sqrt(x**4 + 5)/3 + 225*asinh(sqrt
(5)*x**2/5)/16
```

3.23 $\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=51

$$\frac{1}{2}\sqrt{x^4+5}\,x^4-\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{1}{2}\left(10-x^2\right)\sqrt{x^4+5}$$

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1252, 833, 780, 215}

$$\frac{1}{2}\sqrt{x^4+5}\,x^4-\frac{1}{2}\left(10-x^2\right)\sqrt{x^4+5}-\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (x^4*Sqrt[5 + x^4])/2 - ((10 - x^2)*Sqrt[5 + x^4])/2 - (5*ArcSinh[x^2/Sqrt[
5]])/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{x^5\left(2+3x^2\right)}{\sqrt{5+x^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(2+3x)}{\sqrt{5+x^2}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}x^4\sqrt{5+x^4} + \frac{1}{6}\operatorname{Subst}\left(\int \frac{x(-30+6x)}{\sqrt{5+x^2}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}x^4\sqrt{5+x^4} - \frac{1}{2}\left(10-x^2\right)\sqrt{5+x^4} - \frac{5}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}x^4\sqrt{5+x^4} - \frac{1}{2}\left(10-x^2\right)\sqrt{5+x^4} - \frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.69

$$\frac{1}{2}\left(\sqrt{x^4+5}\left(x^4+x^2-10\right) - 5\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(2 + 3*x^2))/Sqrt[5 + x^4],x]
[Out] (Sqrt[5 + x^4]*(-10 + x^2 + x^4) - 5*ArcSinh[x^2/Sqrt[5]])/2
```

IntegrateAlgebraic [A] time = 0.15, size = 42, normalized size = 0.82

$$\frac{1}{2}\sqrt{x^4+5}\left(x^4+x^2-10\right) + \frac{5}{2}\log\left(\sqrt{x^4+5}-x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(2 + 3*x^2))/Sqrt[5 + x^4],x]
[Out] (Sqrt[5 + x^4]*(-10 + x^2 + x^4))/2 + (5*Log[-x^2 + Sqrt[5 + x^4]])/2
```

fricas [A] time = 0.74, size = 34, normalized size = 0.67

$$\frac{1}{2}\left(x^4+x^2-10\right)\sqrt{x^4+5} + \frac{5}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
[Out] 1/2*(x^4 + x^2 - 10)*sqrt(x^4 + 5) + 5/2*log(-x^2 + sqrt(x^4 + 5))
```

giac [A] time = 0.22, size = 37, normalized size = 0.73

$$\frac{1}{2}\sqrt{x^4+5}\left(\left(x^2+1\right)x^2-10\right) + \frac{5}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^4 + 5)*((x^2 + 1)*x^2 - 10) + 5/2*log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.01, size = 39, normalized size = 0.76

$$\frac{\sqrt{x^4+5}\,x^2}{2} - \frac{5\operatorname{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2} + \frac{\sqrt{x^4+5}\,\left(x^4-10\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)/(x^4+5)^(1/2),x)
```

```
[Out] 1/2*(x^4+5)^(1/2)*(x^4-10)+1/2*(x^4+5)^(1/2)*x^2-5/2*arcsinh(1/5*5^(1/2)*x^
2)
```

maxima [A] time = 1.22, size = 76, normalized size = 1.49

$$\frac{1}{2}\left(x^4+5\right)^{\frac{3}{2}}-\frac{15}{2}\sqrt{x^4+5}+\frac{5\sqrt{x^4+5}}{2\,x^2\left(\frac{x^4+5}{x^4}-1\right)}-\frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(x^4 + 5)^(3/2) - 15/2*sqrt(x^4 + 5) + 5/2*sqrt(x^4 + 5)/(x^2*((x^4 + 5
)/x^4 - 1)) - 5/4*log(sqrt(x^4 + 5)/x^2 + 1) + 5/4*log(sqrt(x^4 + 5)/x^2 -
1)
```

mupad [B] time = 0.31, size = 32, normalized size = 0.63

$$\sqrt{x^4+5}\,\left(\frac{x^4}{2}+\frac{x^2}{2}-5\right)-\frac{5\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
```

```
[Out] (x^4 + 5)^(1/2)*(x^2/2 + x^4/2 - 5) - (5*asinh((5^(1/2)*x^2)/5))/2
```

sympy [A] time = 5.52, size = 66, normalized size = 1.29

$$\frac{x^6}{2\sqrt{x^4+5}}+\frac{x^4\sqrt{x^4+5}}{2}+\frac{5x^2}{2\sqrt{x^4+5}}-5\sqrt{x^4+5}-\frac{5\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] x**6/(2*sqrt(x**4 + 5)) + x**4*sqrt(x**4 + 5)/2 + 5*x**2/(2*sqrt(x**4 + 5))
 - 5*sqrt(x**4 + 5) - 5*asinh(sqrt(5)*x**2/5)/2
```

3.24 $\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}} \, dx$

Optimal. Leaf size=35

$$\frac{1}{4}\left(3x^2+4\right)\sqrt{x^4+5}-\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1252, 780, 215}

$$\frac{1}{4}\left(3x^2+4\right)\sqrt{x^4+5}-\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 - (15*ArcSinh[x^2/Sqrt[5]])/4
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(2+3x^2\right)}{\sqrt{5+x^4}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x(2+3x)}{\sqrt{5+x^2}} \, dx, x, x^2\right)\\
&= \frac{1}{4}\left(4+3x^2\right)\sqrt{5+x^4}-\frac{15}{4}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}} \, dx, x, x^2\right)\\
&= \frac{1}{4}\left(4+3x^2\right)\sqrt{5+x^4}-\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.97

$$\frac{1}{4}\left(\left(3x^2+4\right)\sqrt{x^4+5}-15\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(2 + 3*x^2))/Sqrt[5 + x^4],x]
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4] - 15*ArcSinh[x^2/Sqrt[5]])/4
```

IntegrateAlgebraic [A] time = 0.13, size = 41, normalized size = 1.17

$$\frac{1}{4}\sqrt{x^4+5}\left(3x^2+4\right)+\frac{15}{4}\log\left(\sqrt{x^4+5}-x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(2 + 3*x^2))/Sqrt[5 + x^4],x]
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 + (15*Log[-x^2 + Sqrt[5 + x^4]])/4
```

fricas [A] time = 0.62, size = 33, normalized size = 0.94

$$\frac{1}{4}\sqrt{x^4+5}\left(3x^2+4\right)+\frac{15}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + 15/4*log(-x^2 + sqrt(x^4 + 5))
```

giac [A] time = 0.22, size = 33, normalized size = 0.94

$$\frac{1}{4}\sqrt{x^4+5}\left(3x^2+4\right)+\frac{15}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + 15/4*log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.01, size = 32, normalized size = 0.91

$$\frac{3\sqrt{x^4+5}\,x^2}{4}-\frac{15\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}+\sqrt{x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)/(x^4+5)^(1/2),x)
[Out] 3/4*(x^4+5)^(1/2)*x^2-15/4*arcsinh(1/5*5^(1/2)*x^2)+(x^4+5)^(1/2)
```

maxima [B] time = 1.16, size = 65, normalized size = 1.86

$$\sqrt{x^4+5}+\frac{15\sqrt{x^4+5}}{4\,x^2\left(\frac{x^4+5}{x^4}-1\right)}-\frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
[Out] sqrt(x^4 + 5) + 15/4*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) - 15/8*log(sqr
t(x^4 + 5)/x^2 + 1) + 15/8*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.49, size = 27, normalized size = 0.77

$$\sqrt{x^4+5}\left(\frac{3\,x^2}{4}+1\right)-\frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
```

```
[Out] (x^4 + 5)^(1/2)*((3*x^2)/4 + 1) - (15*asinh((5^(1/2)*x^2)/5))/4
```

sympy [A] time = 4.04, size = 53, normalized size = 1.51

$$\frac{3x^6}{4\sqrt{x^4+5}} + \frac{15x^2}{4\sqrt{x^4+5}} + \sqrt{x^4+5} - \frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*x**6/(4*sqrt(x**4 + 5)) + 15*x**2/(4*sqrt(x**4 + 5)) + sqrt(x**4 + 5) - 1
5*asinh(sqrt(5)*x**2/5)/4
```

3.25 $\int \frac{x(2+3x^2)}{\sqrt{5+x^4}} \, dx$

Optimal. Leaf size=24

$$\frac{3\sqrt{x^4+5}}{2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1248, 641, 215}

$$\frac{3\sqrt{x^4+5}}{2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (3*Sqrt[5 + x^4])/2 + ArcSinh[x^2/Sqrt[5]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(2+3x^2\right)}{\sqrt{5+x^4}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{\sqrt{5+x^2}} \, dx, x, x^2\right)\\
&= \frac{3\sqrt{5+x^4}}{2}+\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}} \, dx, x, x^2\right)\\
&= \frac{3\sqrt{5+x^4}}{2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 1.00

$$\frac{3\sqrt{x^4+5}}{2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (3*Sqrt[5 + x^4])/2 + ArcSinh[x^2/Sqrt[5]]
```

IntegrateAlgebraic [A]   time = 0.13, size = 32, normalized size = 1.33

$$\frac{3\sqrt{x^4+5}}{2} - \log\left(\sqrt{x^4+5} - x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(2 + 3*x^2))/Sqrt[5 + x^4],x]
[Out] (3*Sqrt[5 + x^4])/2 - Log[-x^2 + Sqrt[5 + x^4]]
```

fricas [A]   time = 0.55, size = 26, normalized size = 1.08

$$\frac{3}{2}\sqrt{x^4+5} - \log\left(-x^2 + \sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
[Out] 3/2*sqrt(x^4 + 5) - log(-x^2 + sqrt(x^4 + 5))
```

giac [A]   time = 0.20, size = 26, normalized size = 1.08

$$\frac{3}{2}\sqrt{x^4+5} - \log\left(-x^2 + \sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
[Out] 3/2*sqrt(x^4 + 5) - log(-x^2 + sqrt(x^4 + 5))
```

maple [A]   time = 0.01, size = 20, normalized size = 0.83

$$\operatorname{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right) + \frac{3\sqrt{x^4+5}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)/(x^4+5)^(1/2),x)
[Out] arcsinh(1/5*5^(1/2)*x^2)+3/2*(x^4+5)^(1/2)
```

maxima [B]   time = 1.12, size = 42, normalized size = 1.75

$$\frac{3}{2}\sqrt{x^4+5} + \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2} + 1\right) - \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
[Out] 3/2*sqrt(x^4 + 5) + 1/2*log(sqrt(x^4 + 5)/x^2 + 1) - 1/2*log(sqrt(x^4 + 5)/
x^2 - 1)
```

mupad [B]   time = 0.29, size = 19, normalized size = 0.79

$$\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right) + \frac{3\,\sqrt{x^4+5}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
[Out] asinh((5^(1/2)*x^2)/5) + (3*(x^4 + 5)^(1/2))/2
```

sympy [A] time = 2.13, size = 22, normalized size = 0.92

$$\frac{3\sqrt{x^4+5}}{2}+\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)/(x**4+5)**(1/2),x)
[Out] 3*sqrt(x**4 + 5)/2 + asinh(sqrt(5)*x**2/5)
```

3.26 $\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=38

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1252, 844, 215, 266, 63, 207}

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x*Sqrt[5 + x^4]),x]
```

```
[Out] (3*ArcSinh[x^2/Sqrt[5]])/2 - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/Sqrt[5]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{2+3x}{x\sqrt{5+x^2}}\,dx,x,x^2\right)\\ &= \frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx,x,x^2\right)+\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx,x,x^2\right)\\ &= \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx,x,x^4\right)\\ &= \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx,x,\sqrt{5+x^4}\right)\\ &= \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{\sqrt{5}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.00

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x*Sqrt[5 + x^4]),x]
```

```
[Out] (3*ArcSinh[x^2/Sqrt[5]])/2 - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/Sqrt[5]
```

IntegrateAlgebraic [A] time = 0.15, size = 55, normalized size = 1.45

$$\frac{2\tanh^{-1}\left(\frac{x^2}{\sqrt{5}}-\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}-\frac{3}{2}\log\left(\sqrt{x^4+5}-x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x^2)/(x*Sqrt[5 + x^4]),x]
```

```
[Out] (2*ArcTanh[x^2/Sqrt[5] - Sqrt[5 + x^4]/Sqrt[5]])/Sqrt[5] - (3*Log[-x^2 + Sq
rt[5 + x^4]])/2
```

fricas [A] time = 0.54, size = 41, normalized size = 1.08

$$\frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-\frac{3}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 3/2*log(-x^2 + sqrt(x^4 +
 5))
```

giac [B] time = 0.20, size = 61, normalized size = 1.61

$$\frac{1}{5}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)-\frac{3}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
+ 5))) - 3/2*log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.01, size = 30, normalized size = 0.79

$$\frac{3 \operatorname{arcsinh}\left(\frac{\sqrt{5}\, x^{2}}{5}\right)}{2}-\frac{\sqrt{5}\, \operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^{4}+5}}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x/(x^4+5)^(1/2),x)
```

```
[Out] 3/2*arcsinh(1/5*5^(1/2)*x^2)-1/5*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))
```

maxima [B] time = 1.17, size = 67, normalized size = 1.76

$$\frac{1}{10} \sqrt{5} \log\left(-\frac{\sqrt{5}-\sqrt{x^{4}+5}}{\sqrt{5}+\sqrt{x^{4}+5}}\right)+\frac{3}{4} \log\left(\frac{\sqrt{x^{4}+5}}{x^{2}}+1\right)-\frac{3}{4} \log\left(\frac{\sqrt{x^{4}+5}}{x^{2}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + 3/
4*log(sqrt(x^4 + 5)/x^2 + 1) - 3/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.61, size = 30, normalized size = 0.79

$$\frac{3 \operatorname{asinh}\left(\frac{\sqrt{5}\, x^{2}}{5}\right)}{2}-\frac{\sqrt{5}\, \operatorname{atanh}\left(\frac{\sqrt{5}\, \sqrt{x^{4}+5}}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x*(x^4 + 5)^(1/2)),x)
```

```
[Out] (3*asinh((5^(1/2)*x^2)/5))/2 - (5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))
/5
```

sympy [A] time = 6.01, size = 31, normalized size = 0.82

$$-\frac{\sqrt{5}\, \operatorname{asinh}\left(\frac{\sqrt{5}}{x^{2}}\right)}{5}+\frac{3 \operatorname{asinh}\left(\frac{\sqrt{5}\, x^{2}}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x/(x**4+5)**(1/2),x)
```

```
[Out] -sqrt(5)*asinh(sqrt(5)/x**2)/5 + 3*asinh(sqrt(5)*x**2/5)/2
```

3.27 $\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=42

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}-\frac{\sqrt{x^4+5}}{5x^2}$$

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1252, 807, 266, 63, 207}

$$-\frac{\sqrt{x^4+5}}{5x^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^3*Sqrt[5 + x^4]),x]
```

```
[Out] -Sqrt[5 + x^4]/(5*x^2) - (3*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/(2*Sqrt[5])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x^2\sqrt{5+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{5+x^4}}{5x^2} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{5+x^4}}{5x^2} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)$$
$$= -\frac{\sqrt{5+x^4}}{5x^2} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)$$
$$= -\frac{\sqrt{5+x^4}}{5x^2} - \frac{3\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}} - \frac{\sqrt{x^4+5}}{5x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^3*Sqrt[5 + x^4]),x]
[Out] -1/5*Sqrt[5 + x^4]/x^2 - (3*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/(2*Sqrt[5])
```

IntegrateAlgebraic [A] time = 0.17, size = 51, normalized size = 1.21

$$\frac{3\tanh^{-1}\left(\frac{x^2}{\sqrt{5}} - \frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}} - \frac{\sqrt{x^4+5}}{5x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x^2)/(x^3*Sqrt[5 + x^4]),x]
[Out] -1/5*Sqrt[5 + x^4]/x^2 + (3*ArcTanh[x^2/Sqrt[5] - Sqrt[5 + x^4]/Sqrt[5]])/S
qrt[5]
```

fricas [A] time = 0.61, size = 47, normalized size = 1.12

$$\frac{3\sqrt{5}\,x^2\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) - 2\,x^2 - 2\sqrt{x^4+5}}{10\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(1/2),x, algorithm="fricas")
[Out] 1/10*(3*sqrt(5)*x^2*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 2*x^2 - 2*sqrt(x^
4 + 5))/x^2
```

giac [B] time = 0.22, size = 66, normalized size = 1.57

$$\frac{3}{10}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) + \frac{2}{\left(x^2-\sqrt{x^4+5}\right)^2-5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 3/10*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
 + 5))) + 2/((x^2 - sqrt(x^4 + 5))^2 - 5)
```

maple [A] time = 0.01, size = 31, normalized size = 0.74

$$-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{10}-\frac{\sqrt{x^4+5}}{5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^3/(x^4+5)^(1/2),x)
```

```
[Out] -3/10*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-1/5*(x^4+5)^(1/2)/x^2
```

maxima [A] time = 1.26, size = 47, normalized size = 1.12

$$\frac{3}{20}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)-\frac{\sqrt{x^4+5}}{5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/20*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) - 1/
5*sqrt(x^4 + 5)/x^2
```

mupad [B] time = 0.33, size = 31, normalized size = 0.74

$$-\frac{3\,\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{10}-\frac{\sqrt{x^4+5}}{5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^3*(x^4 + 5)^(1/2)),x)
```

```
[Out] - (3*5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/10 - (x^4 + 5)^(1/2)/(5*x^
2)
```

sympy [A] time = 3.60, size = 31, normalized size = 0.74

$$-\frac{\sqrt{1+\frac{5}{x^4}}}{5}-\frac{3\sqrt{5}\,\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**3/(x**4+5)**(1/2),x)
```

```
[Out] -sqrt(1 + 5/x**4)/5 - 3*sqrt(5)*asinh(sqrt(5)/x**2)/10
```

3.28 $\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=58

$$-\frac{\sqrt{x^4+5}}{10x^4}+\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}}-\frac{3\sqrt{x^4+5}}{10x^2}$$

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1252, 835, 807, 266, 63, 207}

$$-\frac{3\sqrt{x^4+5}}{10x^2}-\frac{\sqrt{x^4+5}}{10x^4}+\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^5*Sqrt[5 + x^4]),x]
```

```
[Out] -Sqrt[5 + x^4]/(10*x^4) - (3*Sqrt[5 + x^4])/(10*x^2) + ArcTanh[Sqrt[5 + x^4
]/Sqrt[5]]/(10*Sqrt[5])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{2+3x}{x^3\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{1}{20}\operatorname{Subst}\left(\int \frac{-30+2x}{x^2\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{3\sqrt{5+x^4}}{10x^2} - \frac{1}{10}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{3\sqrt{5+x^4}}{10x^2} - \frac{1}{20}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)\\
&= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{3\sqrt{5+x^4}}{10x^2} - \frac{1}{10}\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)\\
&= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{3\sqrt{5+x^4}}{10x^2} + \frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{10\sqrt{5}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.84

$$\frac{\sqrt{5}\,x^4\tanh^{-1}\left(\sqrt{\frac{x^4}{5}+1}\right) - 5\left(3x^2+1\right)\sqrt{x^4+5}}{50x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^5*Sqrt[5 + x^4]),x]
```

```
[Out] (-5*(1 + 3*x^2)*Sqrt[5 + x^4] + Sqrt[5]*x^4*ArcTanh[Sqrt[1 + x^4/5]])/(50*x
^4)
```

IntegrateAlgebraic [A] time = 0.23, size = 60, normalized size = 1.03

$$\frac{\left(-3x^2-1\right)\sqrt{x^4+5}}{10x^4} - \frac{\tanh^{-1}\left(\frac{x^2}{\sqrt{5}} - \frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x^2)/(x^5*Sqrt[5 + x^4]),x]
```

```
[Out] ((-1 - 3*x^2)*Sqrt[5 + x^4])/(10*x^4) - ArcTanh[x^2/Sqrt[5] - Sqrt[5 + x^4]
/Sqrt[5]]/(5*Sqrt[5])
```

fricas [A] time = 0.72, size = 50, normalized size = 0.86

$$\frac{\sqrt{5}\,x^4\log\left(\frac{\sqrt{5}+\sqrt{x^4+5}}{x^2}\right) - 15\,x^4 - 5\,\sqrt{x^4+5}\left(3\,x^2+1\right)}{50\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/50*(sqrt(5)*x^4*log((sqrt(5) + sqrt(x^4 + 5))/x^2) - 15*x^4 - 5*sqrt(x^4
+ 5)*(3*x^2 + 1))/x^4
```

giac [B] time = 0.22, size = 114, normalized size = 1.97

$$-\frac{1}{50}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)+\frac{\left(x^2-\sqrt{x^4+5}\right)^3+15\left(x^2-\sqrt{x^4+5}\right)^2+5x^2-5\sqrt{x^4+5}-75}{5\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/50*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^
4 + 5))) + 1/5*((x^2 - sqrt(x^4 + 5))^3 + 15*(x^2 - sqrt(x^4 + 5))^2 + 5*x^
2 - 5*sqrt(x^4 + 5) - 75)/((x^2 - sqrt(x^4 + 5))^2 - 5)^2
```

maple [A] time = 0.02, size = 43, normalized size = 0.74

$$\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{50}-\frac{3\sqrt{x^4+5}}{10x^2}-\frac{\sqrt{x^4+5}}{10x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^5/(x^4+5)^(1/2),x)
```

```
[Out] -1/10*(x^4+5)^(1/2)/x^4+1/50*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-3/10*(x
^4+5)^(1/2)/x^2
```

maxima [A] time = 1.12, size = 59, normalized size = 1.02

$$-\frac{1}{100}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)-\frac{3\sqrt{x^4+5}}{10\,x^2}-\frac{\sqrt{x^4+5}}{10\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/100*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) -
3/10*sqrt(x^4 + 5)/x^2 - 1/10*sqrt(x^4 + 5)/x^4
```

mupad [B] time = 0.69, size = 43, normalized size = 0.74

$$\frac{\sqrt{5}\,\operatorname{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{50}-\frac{3\,\sqrt{x^4+5}}{10\,x^2}-\frac{\sqrt{x^4+5}}{10\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^5*(x^4 + 5)^(1/2)),x)
```

```
[Out] (5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/50 - (3*(x^4 + 5)^(1/2))/(10*x
^2) - (x^4 + 5)^(1/2)/(10*x^4)
```

sympy [A] time = 14.33, size = 88, normalized size = 1.52

$$\frac{\sqrt{5}\left(-\frac{\log\left(\sqrt{\frac{x^4}{5}+1}-1\right)}{4}+\frac{\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{4}-\frac{1}{4\left(\sqrt{\frac{x^4}{5}+1}+1\right)}-\frac{1}{4\left(\sqrt{\frac{x^4}{5}+1}-1\right)}\right)}{25}-\frac{3\sqrt{5}\,\sqrt{5x^4+25}}{50x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**5/(x**4+5)**(1/2),x)
```

```
[Out] sqrt(5)*(-log(sqrt(x**4/5 + 1) - 1)/4 + log(sqrt(x**4/5 + 1) + 1)/4 - 1/(4*
(sqrt(x**4/5 + 1) + 1)) - 1/(4*(sqrt(x**4/5 + 1) - 1)))/25 - 3*sqrt(5)*sqrt
(5*x**4 + 25)/(50*x**2)
```

3.29 $\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}} \, dx$

Optimal. Leaf size=58

$$-\frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{(3x^2+2)\,x^4}{2\sqrt{x^4+5}}+\frac{1}{4}\left(9x^2+8\right)\sqrt{x^4+5}$$

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1252, 819, 780, 215}

$$-\frac{(3x^2+2)\,x^4}{2\sqrt{x^4+5}}+\frac{1}{4}\left(9x^2+8\right)\sqrt{x^4+5}-\frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -(x^4*(2 + 3*x^2))/(2*Sqrt[5 + x^4]) + ((8 + 9*x^2)*Sqrt[5 + x^4])/4 - (45*
ArcSinh[x^2/Sqrt[5]])/4
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7 \left(2+3x^2\right)}{\left(5+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3(2+3x)}{\left(5+x^2\right)^{3/2}}\,dx,x,x^2\right)\\ &= -\frac{x^4\left(2+3x^2\right)}{2\sqrt{5+x^4}}+\frac{1}{10}\,\text{Subst}\left(\int \frac{x(20+45x)}{\sqrt{5+x^2}}\,dx,x,x^2\right)\\ &= -\frac{x^4\left(2+3x^2\right)}{2\sqrt{5+x^4}}+\frac{1}{4}\left(8+9x^2\right)\sqrt{5+x^4}-\frac{45}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx,x,x^2\right)\\ &= -\frac{x^4\left(2+3x^2\right)}{2\sqrt{5+x^4}}+\frac{1}{4}\left(8+9x^2\right)\sqrt{5+x^4}-\frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.88

$$\frac{3x^6+4x^4+45x^2-45\sqrt{x^4+5}\,\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+40}{4\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] (40 + 45*x^2 + 4*x^4 + 3*x^6 - 45*Sqrt[5 + x^4]*ArcSinh[x^2/Sqrt[5]])/(4*Sq
rt[5 + x^4])
```

IntegrateAlgebraic [A] time = 0.18, size = 51, normalized size = 0.88

$$\frac{45}{4}\log\left(\sqrt{x^4+5}-x^2\right)+\frac{3x^6+4x^4+45x^2+40}{4\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^7*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] (40 + 45*x^2 + 4*x^4 + 3*x^6)/(4*Sqrt[5 + x^4]) + (45*Log[-x^2 + Sqrt[5 + x
^4]])/4
```

fricas [A] time = 0.87, size = 62, normalized size = 1.07

$$\frac{30\,x^4+45\left(x^4+5\right)\log\left(-x^2+\sqrt{x^4+5}\right)+\left(3\,x^6+4\,x^4+45\,x^2+40\right)\sqrt{x^4+5}+150}{4\left(x^4+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(30*x^4 + 45*(x^4 + 5)*log(-x^2 + sqrt(x^4 + 5)) + (3*x^6 + 4*x^4 + 45*
x^2 + 40)*sqrt(x^4 + 5) + 150)/(x^4 + 5)
```

giac [A] time = 0.21, size = 45, normalized size = 0.78

$$\frac{\left(\left(3\,x^2+4\right)x^2+45\right)x^2+40}{4\,\sqrt{x^4+5}}+\frac{45}{4}\,\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(((3*x^2 + 4)*x^2 + 45)*x^2 + 40)/sqrt(x^4 + 5) + 45/4*log(-x^2 + sqrt(
x^4 + 5))
```

maple [A] time = 0.02, size = 50, normalized size = 0.86

$$\frac{3x^6}{4\sqrt{x^4+5}}+\frac{45x^2}{4\sqrt{x^4+5}}-\frac{45\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}+\frac{x^4+10}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(3*x^2+2)/(x^4+5)^(3/2),x)
```

```
[Out] 3/4*x^6/(x^4+5)^(1/2)+45/4*x^2/(x^4+5)^(1/2)-45/4*arcsinh(1/5*5^(1/2)*x^2)+
1/(x^4+5)^(1/2)*(x^4+10)
```

maxima [A] time = 1.21, size = 89, normalized size = 1.53

$$\sqrt{x^4+5}-\frac{15\left(\frac{3\left(x^4+5\right)}{x^4}-2\right)}{4\left(\frac{\sqrt{x^4+5}}{x^2}-\frac{\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}+\frac{5}{\sqrt{x^4+5}}-\frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^4 + 5) - 15/4*(3*(x^4 + 5)/x^4 - 2)/(sqrt(x^4 + 5)/x^2 - (x^4 + 5)^(
3/2)/x^6) + 5/sqrt(x^4 + 5) - 45/8*log(sqrt(x^4 + 5)/x^2 + 1) + 45/8*log(sq
rt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 1.11, size = 97, normalized size = 1.67

$$\sqrt{x^4+5}\,\left(\frac{3\,x^2}{4}+1\right)-\frac{45\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}+\frac{\sqrt{5}\,\left(10+\sqrt{5}\,15\mathrm{i}\right)\,\sqrt{x^4+5}\,1\mathrm{i}}{20\,\left(-x^2+\sqrt{5}\,1\mathrm{i}\right)}-\frac{\sqrt{5}\,\left(-10+\sqrt{5}\,15\mathrm{i}\right)\,\sqrt{x^4+5}\,1\mathrm{i}}{20\,\left(x^2+\sqrt{5}\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
```

```
[Out] (x^4 + 5)^(1/2)*((3*x^2)/4 + 1) - (45*asinh((5^(1/2)*x^2)/5))/4 + (5^(1/2)*
(5^(1/2)*15i + 10)*(x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i - x^2)) - (5^(1/2)*(
5^(1/2)*15i - 10)*(x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i + x^2))
```

sympy [A] time = 14.28, size = 66, normalized size = 1.14

$$\frac{3x^6}{4\sqrt{x^4+5}}+\frac{x^4}{\sqrt{x^4+5}}+\frac{45x^2}{4\sqrt{x^4+5}}-\frac{45\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}+\frac{10}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] 3*x**6/(4*sqrt(x**4 + 5)) + x**4/sqrt(x**4 + 5) + 45*x**2/(4*sqrt(x**4 + 5)
) - 45*asinh(sqrt(5)*x**2/5)/4 + 10/sqrt(x**4 + 5)
```

3.30 $\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=45

$$3\sqrt{x^4+5}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{(3x^2+2)\,x^2}{2\sqrt{x^4+5}}$$

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1252, 819, 641, 215}

$$-\frac{(3x^2+2)\,x^2}{2\sqrt{x^4+5}}+3\sqrt{x^4+5}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -(x^2*(2 + 3*x^2))/(2*Sqrt[5 + x^4]) + 3*Sqrt[5 + x^4] + ArcSinh[x^2/Sqrt[5
]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\left(2+3x^2\right)}{\left(5+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(2+3x)}{\left(5+x^2\right)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{x^2\left(2+3x^2\right)}{2\sqrt{5+x^4}} + \frac{1}{10}\operatorname{Subst}\left(\int \frac{10+30x}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{x^2\left(2+3x^2\right)}{2\sqrt{5+x^4}} + 3\sqrt{5+x^4} + \operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{x^2\left(2+3x^2\right)}{2\sqrt{5+x^4}} + 3\sqrt{5+x^4} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.02

$$\frac{3x^4-2x^2+2\sqrt{x^4+5}\,\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+30}{2\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
[Out] (30 - 2*x^2 + 3*x^4 + 2*Sqrt[5 + x^4]*ArcSinh[x^2/Sqrt[5]])/(2*Sqrt[5 + x^4
])
```

IntegrateAlgebraic [A] time = 0.21, size = 44, normalized size = 0.98

$$\frac{3x^4-2x^2+30}{2\sqrt{x^4+5}} - \log\left(\sqrt{x^4+5}-x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
[Out] (30 - 2*x^2 + 3*x^4)/(2*Sqrt[5 + x^4]) - Log[-x^2 + Sqrt[5 + x^4]]
```

fricas [A] time = 0.75, size = 58, normalized size = 1.29

$$-\frac{2\,x^4+2\left(x^4+5\right)\log\left(-x^2+\sqrt{x^4+5}\right)-\left(3\,x^4-2\,x^2+30\right)\sqrt{x^4+5}+10}{2\left(x^4+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
[Out] -1/2*(2*x^4 + 2*(x^4 + 5)*log(-x^2 + sqrt(x^4 + 5)) - (3*x^4 - 2*x^2 + 30)*
sqrt(x^4 + 5) + 10)/(x^4 + 5)
```

giac [A] time = 0.23, size = 39, normalized size = 0.87

$$\frac{\left(3\,x^2-2\right)x^2+30}{2\,\sqrt{x^4+5}} - \log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
[Out] 1/2*((3*x^2 - 2)*x^2 + 30)/sqrt(x^4 + 5) - log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.01, size = 37, normalized size = 0.82

$$-\frac{x^2}{\sqrt{x^4+5}}+\operatorname{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)+\frac{\frac{3x^4}{2}+15}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)/(x^4+5)^(3/2),x)
```

```
[Out] 3/2/(x^4+5)^(1/2)*(x^4+10)-1/(x^4+5)^(1/2)*x^2+arcsinh(1/5*5^(1/2)*x^2)
```

maxima [A] time = 1.41, size = 63, normalized size = 1.40

$$-\frac{x^2}{\sqrt{x^4+5}}+\frac{3}{2}\sqrt{x^4+5}+\frac{15}{2\sqrt{x^4+5}}+\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] -x^2/sqrt(x^4 + 5) + 3/2*sqrt(x^4 + 5) + 15/2/sqrt(x^4 + 5) + 1/2*log(sqrt(
x^4 + 5)/x^2 + 1) - 1/2*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.89, size = 89, normalized size = 1.98

$$\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)+\frac{3\sqrt{x^4+5}}{2}-\frac{\sqrt{5}\,\left(-15+\sqrt{5}\,2\mathrm{i}\right)\sqrt{x^4+5}\,1\mathrm{i}}{20\left(-x^2+\sqrt{5}\,1\mathrm{i}\right)}+\frac{\sqrt{5}\,\left(15+\sqrt{5}\,2\mathrm{i}\right)\sqrt{x^4+5}\,1\mathrm{i}}{20\left(x^2+\sqrt{5}\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
```

```
[Out] asinh((5^(1/2)*x^2)/5) + (3*(x^4 + 5)^(1/2))/2 - (5^(1/2)*(5^(1/2)*2i - 15)
*(x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i - x^2)) + (5^(1/2)*(5^(1/2)*2i + 15)*(
x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i + x^2))
```

sympy [A] time = 12.34, size = 48, normalized size = 1.07

$$\frac{3x^4}{2\sqrt{x^4+5}}-\frac{x^2}{\sqrt{x^4+5}}+\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)+\frac{15}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] 3*x**4/(2*sqrt(x**4 + 5)) - x**2/sqrt(x**4 + 5) + asinh(sqrt(5)*x**2/5) + 1
5/sqrt(x**4 + 5)
```

3.31 $\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}}\, dx$

Optimal. Leaf size=35

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{-3x^2-2}{2\sqrt{x^4+5}}$$

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1252, 778, 215}

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{3x^2+2}{2\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -(2 + 3*x^2)/(2*Sqrt[5 + x^4]) + (3*ArcSinh[x^2/Sqrt[5]])/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(2+3x^2\right)}{\left(5+x^4\right)^{3/2}}\, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x(2+3x)}{\left(5+x^2\right)^{3/2}}\, dx, x, x^2\right)\\ &= -\frac{2+3x^2}{2\sqrt{5+x^4}}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\, dx, x, x^2\right)\\ &= -\frac{2+3x^2}{2\sqrt{5+x^4}}+\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 1.17

$$\frac{-3x^2+3\sqrt{x^4+5}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-2}{2\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] (-2 - 3*x^2 + 3*Sqrt[5 + x^4]*ArcSinh[x^2/Sqrt[5]])/(2*Sqrt[5 + x^4])
```

IntegrateAlgebraic [A] time = 0.19, size = 41, normalized size = 1.17

$$\frac{-3x^2-2}{2\sqrt{x^4+5}}-\frac{3}{2}\log\left(\sqrt{x^4+5}-x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] (-2 - 3*x^2)/(2*Sqrt[5 + x^4]) - (3*Log[-x^2 + Sqrt[5 + x^4]])/2
```

fricas [A] time = 0.70, size = 52, normalized size = 1.49

$$-\frac{3\,x^4+3\left(x^4+5\right)\log\left(-x^2+\sqrt{x^4+5}\right)+\sqrt{x^4+5}\left(3\,x^2+2\right)+15}{2\left(x^4+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(3*x^4 + 3*(x^4 + 5)*log(-x^2 + sqrt(x^4 + 5)) + sqrt(x^4 + 5)*(3*x^2
+ 2) + 15)/(x^4 + 5)
```

giac [A] time = 0.25, size = 33, normalized size = 0.94

$$-\frac{3\,x^2+2}{2\,\sqrt{x^4+5}}-\frac{3}{2}\,\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(3*x^2 + 2)/sqrt(x^4 + 5) - 3/2*log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.01, size = 34, normalized size = 0.97

$$-\frac{3x^2}{2\sqrt{x^4+5}}+\frac{3\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}-\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)/(x^4+5)^(3/2),x)
```

```
[Out] -3/2/(x^4+5)^(1/2)*x^2+3/2*arcsinh(1/5*5^(1/2)*x^2)-1/(x^4+5)^(1/2)
```

maxima [A] time = 1.23, size = 54, normalized size = 1.54

$$-\frac{3\,x^2}{2\,\sqrt{x^4+5}}-\frac{1}{\sqrt{x^4+5}}+\frac{3}{4}\,\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{3}{4}\,\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/2*x^2/sqrt(x^4 + 5) - 1/sqrt(x^4 + 5) + 3/4*log(sqrt(x^4 + 5)/x^2 + 1) -
 3/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.84, size = 82, normalized size = 2.34

$$\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}-\frac{\sqrt{5}\,\left(2+\sqrt{5}\,3\mathrm{i}\right)\,\sqrt{x^4+5}\,1\mathrm{i}}{20\,\left(-x^2+\sqrt{5}\,1\mathrm{i}\right)}+\frac{\sqrt{5}\,\left(-2+\sqrt{5}\,3\mathrm{i}\right)\,\sqrt{x^4+5}\,1\mathrm{i}}{20\,\left(x^2+\sqrt{5}\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
```

```
[Out] (3*asinh((5^(1/2)*x^2)/5))/2 - (5^(1/2)*(5^(1/2)*3i + 2)*(x^4 + 5)^(1/2)*1i
)/(20*(5^(1/2)*1i - x^2)) + (5^(1/2)*(5^(1/2)*3i - 2)*(x^4 + 5)^(1/2)*1i)/(
20*(5^(1/2)*1i + x^2))
```

sympy [A] time = 10.68, size = 39, normalized size = 1.11

$$-\frac{3x^2}{2\sqrt{x^4+5}}+\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}-\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] -3*x**2/(2*sqrt(x**4 + 5)) + 3*asinh(sqrt(5)*x**2/5)/2 - 1/sqrt(x**4 + 5)
```

3.32 $\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}} \, dx$

Optimal. Leaf size=20

$$\frac{2x^2-15}{10\sqrt{x^4+5}}$$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1248, 637}

$$-\frac{15-2x^2}{10\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
[Out] -(15 - 2*x^2)/(10*Sqrt[5 + x^4])
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{x(2+3x^2)}{(5+x^4)^{3/2}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{(5+x^2)^{3/2}} \, dx, x, x^2\right) \\ &= -\frac{15-2x^2}{10\sqrt{5+x^4}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{2x^2-15}{10\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
[Out] (-15 + 2*x^2)/(10*Sqrt[5 + x^4])
```

IntegrateAlgebraic [A] time = 0.18, size = 20, normalized size = 1.00

$$\frac{2x^2-15}{10\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
[Out] (-15 + 2*x^2)/(10*Sqrt[5 + x^4])
```

fricas [A] time = 0.54, size = 31, normalized size = 1.55

$$\frac{2\,x^4+\sqrt{x^4+5}\,\left(2\,x^2-15\right)+10}{10\,\left(x^4+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
[Out] 1/10*(2*x^4 + sqrt(x^4 + 5)*(2*x^2 - 15) + 10)/(x^4 + 5)
```

giac [A] time = 0.24, size = 16, normalized size = 0.80

$$\frac{2\,x^2-15}{10\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
[Out] 1/10*(2*x^2 - 15)/sqrt(x^4 + 5)
```

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$\frac{2x^2-15}{10\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)/(x^4+5)^(3/2),x)
[Out] 1/10*(2*x^2-15)/(x^4+5)^(1/2)
```

maxima [A] time = 1.49, size = 22, normalized size = 1.10

$$\frac{x^2}{5\,\sqrt{x^4+5}}-\frac{3}{2\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
[Out] 1/5*x^2/sqrt(x^4 + 5) - 3/2/sqrt(x^4 + 5)
```

mupad [B] time = 0.16, size = 16, normalized size = 0.80

$$\frac{2\,x^2-15}{10\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
[Out] (2*x^2 - 15)/(10*(x^4 + 5)^(1/2))
```

sympy [B] time = 7.86, size = 31, normalized size = 1.55

$$\frac{\sqrt{5}\,x^2}{5\sqrt{5x^4+25}}-\frac{3}{2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] sqrt(5)*x**2/(5*sqrt(5*x**4 + 25)) - 3/(2*sqrt(x**4 + 5))
```

3.33 $\int \frac{2+3x^2}{x(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{3x^2+2}{10\sqrt{x^4+5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1252, 823, 12, 266, 63, 207}

$$\frac{3x^2+2}{10\sqrt{x^4+5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x*(5 + x^4)^(3/2)),x]
```

```
[Out] (2 + 3*x^2)/(10*Sqrt[5 + x^4]) - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/(5*Sqrt[5])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x\left(5+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x\left(5+x^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{2+3x^2}{10\sqrt{5+x^4}}-\frac{1}{50}\text{Subst}\left(\int -\frac{10}{x\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= \frac{2+3x^2}{10\sqrt{5+x^4}}+\frac{1}{5}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= \frac{2+3x^2}{10\sqrt{5+x^4}}+\frac{1}{10}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx,x,x^4\right)\\
&= \frac{2+3x^2}{10\sqrt{5+x^4}}+\frac{1}{5}\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx,x,\sqrt{5+x^4}\right)\\
&= \frac{2+3x^2}{10\sqrt{5+x^4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{5\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.00

$$\frac{1}{50}\left(\frac{5\left(3x^2+2\right)}{\sqrt{x^4+5}}-2\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x*(5 + x^4)^(3/2)),x]
```

```
[Out] ((5*(2 + 3*x^2))/Sqrt[5 + x^4] - 2*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/
50
```

IntegrateAlgebraic [A] time = 0.21, size = 57, normalized size = 1.24

$$\frac{3x^2+2}{10\sqrt{x^4+5}}+\frac{2\tanh^{-1}\left(\frac{x^2}{\sqrt{5}}-\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x^2)/(x*(5 + x^4)^(3/2)),x]
```

```
[Out] (2 + 3*x^2)/(10*Sqrt[5 + x^4]) + (2*ArcTanh[x^2/Sqrt[5] - Sqrt[5 + x^4]/Sqr
t[5]])/(5*Sqrt[5])
```

fricas [A] time = 0.54, size = 61, normalized size = 1.33

$$\frac{15\,x^4+2\,\sqrt{5}\left(x^4+5\right)\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)+5\,\sqrt{x^4+5}\left(3\,x^2+2\right)+75}{50\left(x^4+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/50*(15*x^4 + 2*sqrt(5)*(x^4 + 5)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) + 5*
sqrt(x^4 + 5)*(3*x^2 + 2) + 75)/(x^4 + 5)
```

giac [A] time = 0.25, size = 61, normalized size = 1.33

$$\frac{1}{25}\,\sqrt{5}\log\left(x^2+\sqrt{5}-\sqrt{x^4+5}\right)-\frac{1}{25}\,\sqrt{5}\log\left(-x^2+\sqrt{5}+\sqrt{x^4+5}\right)+\frac{3\,x^2+2}{10\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/25*sqrt(5)*log(x^2 + sqrt(5) - sqrt(x^4 + 5)) - 1/25*sqrt(5)*log(-x^2 + s
qrt(5) + sqrt(x^4 + 5)) + 1/10*(3*x^2 + 2)/sqrt(x^4 + 5)
```

maple [A] time = 0.02, size = 40, normalized size = 0.87

$$\frac{3x^2}{10\sqrt{x^4+5}}-\frac{\sqrt{5}\ \mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{25}+\frac{1}{5\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x/(x^4+5)^(3/2),x)
```

```
[Out] 3/10/(x^4+5)^(1/2)*x^2+1/5/(x^4+5)^(1/2)-1/25*5^(1/2)*arctanh(5^(1/2)/(x^4+
5)^(1/2))
```

maxima [A] time = 1.41, size = 56, normalized size = 1.22

$$\frac{3\,x^2}{10\,\sqrt{x^4+5}}+\frac{1}{50}\,\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{1}{5\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/10*x^2/sqrt(x^4 + 5) + 1/50*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(
5) + sqrt(x^4 + 5))) + 1/5/sqrt(x^4 + 5)
```

mupad [B] time = 0.48, size = 40, normalized size = 0.87

$$\frac{1}{5\,\sqrt{x^4+5}}-\frac{\sqrt{5}\ \mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{25}+\frac{3\,x^2}{10\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x*(x^4 + 5)^(3/2)),x)
```

```
[Out] 1/(5*(x^4 + 5)^(1/2)) - (5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/25 + (
3*x^2)/(10*(x^4 + 5)^(1/2))
```

sympy [B] time = 19.62, size = 212, normalized size = 4.61

$$\frac{2x^4\log\left(x^4\right)}{20\sqrt{5}\,x^4+100\sqrt{5}}-\frac{4x^4\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{20\sqrt{5}\,x^4+100\sqrt{5}}-\frac{2x^4\log\left(5\right)}{20\sqrt{5}\,x^4+100\sqrt{5}}+\frac{3x^2}{10\sqrt{x^4+5}}+\frac{4\sqrt{5}\,\sqrt{x^4+5}}{20\sqrt{5}\,x^4+100\sqrt{5}}+\frac{10\log\left(x^4\right)}{20\sqrt{5}\,x^4+100\sqrt{5}}-\frac{20\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{20\sqrt{5}\,x^4+100\sqrt{5}}-\frac{10\log\left(5\right)}{20\sqrt{5}\,x^4+100\sqrt{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x/(x**4+5)**(3/2),x)
```

```
[Out] 2*x**4*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 4*x**4*log(sqrt(x**4/5 +
 1) + 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 2*x**4*log(5)/(20*sqrt(5)*x**4 +
 100*sqrt(5)) + 3*x**2/(10*sqrt(x**4 + 5)) + 4*sqrt(5)*sqrt(x**4 + 5)/(20*s
qrt(5)*x**4 + 100*sqrt(5)) + 10*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) -
 20*log(sqrt(x**4/5 + 1) + 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 10*log(5)/(
20*sqrt(5)*x**4 + 100*sqrt(5))
```

3.34 $\int \frac{2+3x^2}{x^3\left(5+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}}+\frac{3x^2+2}{10x^2\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{25x^2}$$

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1252, 823, 807, 266, 63, 207}

$$\frac{3x^2+2}{10x^2\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{25x^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^3*(5 + x^4)^(3/2)),x]
```

```
[Out] (2 + 3*x^2)/(10*x^2*Sqrt[5 + x^4]) - (2*Sqrt[5 + x^4])/(25*x^2) - (3*ArcTan
h[Sqrt[5 + x^4]/Sqrt[5]])/(10*Sqrt[5])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
```

```
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x^3\left(5+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{2+3x}{x^2\left(5+x^2\right)^{3/2}}\,dx, x, x^2\right)\\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}} - \frac{1}{50}\operatorname{Subst}\left(\int \frac{-20-15x}{x^2\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}} - \frac{2\sqrt{5+x^4}}{25x^2} + \frac{3}{10}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}} - \frac{2\sqrt{5+x^4}}{25x^2} + \frac{3}{20}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)\\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}} - \frac{2\sqrt{5+x^4}}{25x^2} + \frac{3}{10}\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)\\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}} - \frac{2\sqrt{5+x^4}}{25x^2} - \frac{3\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{10\sqrt{5}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.69

$$\frac{15x^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{x^4}{5}+1\right)-4x^4-10}{50x^2\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^3*(5 + x^4)^(3/2)),x]
```

```
[Out] (-10 - 4*x^4 + 15*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + x^4/5])/(50*x^2*S
qrt[5 + x^4])
```

IntegrateAlgebraic [A] time = 0.22, size = 65, normalized size = 1.00

$$\frac{-4x^4+15x^2-10}{50x^2\sqrt{x^4+5}}+\frac{3\tanh^{-1}\left(\frac{x^2}{\sqrt{5}}-\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x^2)/(x^3*(5 + x^4)^(3/2)),x]
```

```
[Out] (-10 + 15*x^2 - 4*x^4)/(50*x^2*Sqrt[5 + x^4]) + (3*ArcTanh[x^2/Sqrt[5] - Sq
rt[5 + x^4]/Sqrt[5]])/(5*Sqrt[5])
```

fricas [A] time = 0.63, size = 77, normalized size = 1.18

$$-\frac{4\,x^6-3\,\sqrt{5}\left(x^6+5\,x^2\right)\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)+20\,x^2+\left(4\,x^4-15\,x^2+10\right)\sqrt{x^4+5}}{50\left(x^6+5\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/50*(4*x^6 - 3*sqrt(5)*(x^6 + 5*x^2)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2)
+ 20*x^2 + (4*x^4 - 15*x^2 + 10)*sqrt(x^4 + 5))/(x^6 + 5*x^2)
```

giac [A] time = 0.24, size = 82, normalized size = 1.26

$$\frac{3}{50}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)-\frac{2x^2-15}{50\sqrt{x^4+5}}+\frac{2}{5\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 3/50*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
 + 5))) - 1/50*(2*x^2 - 15)/sqrt(x^4 + 5) + 2/5/((x^2 - sqrt(x^4 + 5))^2 -
5)
```

maple [A] time = 0.01, size = 47, normalized size = 0.72

$$-\frac{3\sqrt{5}\ \operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{50}-\frac{2x^4+5}{25\sqrt{x^4+5}\ x^2}+\frac{3}{10\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^3/(x^4+5)^(3/2),x)
```

```
[Out] 3/10/(x^4+5)^(1/2)-3/50*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-1/25/x^2*(2*
x^4+5)/(x^4+5)^(1/2)
```

maxima [A] time = 1.10, size = 68, normalized size = 1.05

$$-\frac{x^2}{25\sqrt{x^4+5}}+\frac{3}{100}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{3}{10\sqrt{x^4+5}}-\frac{\sqrt{x^4+5}}{25x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/25*x^2/sqrt(x^4 + 5) + 3/100*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqr
t(5) + sqrt(x^4 + 5))) + 3/10/sqrt(x^4 + 5) - 1/25*sqrt(x^4 + 5)/x^2
```

mupad [B] time = 0.54, size = 47, normalized size = 0.72

$$\frac{3}{10\sqrt{x^4+5}}-\frac{3\sqrt{5}\ \operatorname{atanh}\left(\frac{\sqrt{5}\ \sqrt{x^4+5}}{5}\right)}{50}-\frac{2x^4+5}{25x^2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^3*(x^4 + 5)^(3/2)),x)
```

```
[Out] 3/(10*(x^4 + 5)^(1/2)) - (3*5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/50
- (2*x^4 + 5)/(25*x^2*(x^4 + 5)^(1/2))
```

sympy [B] time = 12.98, size = 228, normalized size = 3.51

$$\frac{3x^4\log\left(x^4\right)}{20\sqrt{5}x^4+100\sqrt{5}}-\frac{6x^4\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{20\sqrt{5}x^4+100\sqrt{5}}-\frac{3x^4\log\left(5\right)}{20\sqrt{5}x^4+100\sqrt{5}}+\frac{6\sqrt{5}\sqrt{x^4+5}}{20\sqrt{5}x^4+100\sqrt{5}}+\frac{15\log\left(x^4\right)}{20\sqrt{5}x^4+100\sqrt{5}}-\frac{30\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{20\sqrt{5}x^4+100\sqrt{5}}-\frac{15\log\left(5\right)}{20\sqrt{5}x^4+100\sqrt{5}}-\frac{2}{25\sqrt{1+\frac{5}{x^4}}}-\frac{1}{5x^4\sqrt{1+\frac{5}{x^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**3/(x**4+5)**(3/2),x)
```

```
[Out] 3*x**4*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 6*x**4*log(sqrt(x**4/5 +
 1) + 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 3*x**4*log(5)/(20*sqrt(5)*x**4 +
 100*sqrt(5)) + 6*sqrt(5)*sqrt(x**4 + 5)/(20*sqrt(5)*x**4 + 100*sqrt(5)) +
15*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 30*log(sqrt(x**4/5 + 1) + 1)
/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 15*log(5)/(20*sqrt(5)*x**4 + 100*sqrt(5)
) - 2/(25*sqrt(1 + 5/x**4)) - 1/(5*x**4*sqrt(1 + 5/x**4))
```

3.35 $\int (fx)^m \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=269

$$\frac{(d+10e)(fx)^{m+21}}{f^{21}(m+21)}+\frac{5(2d+9e)(fx)^{m+19}}{f^{19}(m+19)}+\frac{15(3d+8e)(fx)^{m+17}}{f^{17}(m+17)}+\frac{30(4d+7e)(fx)^{m+15}}{f^{15}(m+15)}+\frac{42(5d+6e)(fx)^{m+13}}{f^{13}(m+13)}+\frac{42(}{}$$

Rubi [A] time = 0.16, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {28, 448}

$$\frac{(10d+e)(fx)^{m+3}}{f^3(m+3)}+\frac{5(9d+2e)(fx)^{m+5}}{f^5(m+5)}+\frac{15(8d+3e)(fx)^{m+7}}{f^7(m+7)}+\frac{30(7d+4e)(fx)^{m+9}}{f^9(m+9)}+\frac{42(6d+5e)(fx)^{m+11}}{f^{11}(m+11)}+\frac{42(5d+6e)(fx)^{m+13}}{f^{13}(m+13)}+\frac{30(4d+7e)(fx)^{m+15}}{f^{15}(m+15)}+\frac{15(3d+8e)(fx)^{m+17}}{f^{17}(m+17)}+\frac{5(2d+9e)(fx)^{m+19}}{f^{19}(m+19)}+\frac{(d+10e)(fx)^{m+21}}{f^{21}(m+21)}+\frac{d(fx)^{m+1}}{f(m+1)}+\frac{e(fx)^{m+23}}{f^{23}(m+23)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*(f*x)^(1 + m))/(f*(1 + m)) + ((10*d + e)*(f*x)^(3 + m))/(f^3*(3 + m)) +
(5*(9*d + 2*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + (15*(8*d + 3*e)*(f*x)^(7 + m)
)/(f^7*(7 + m)) + (30*(7*d + 4*e)*(f*x)^(9 + m))/(f^9*(9 + m)) + (42*(6*d +
 5*e)*(f*x)^(11 + m))/(f^11*(11 + m)) + (42*(5*d + 6*e)*(f*x)^(13 + m))/(f^
13*(13 + m)) + (30*(4*d + 7*e)*(f*x)^(15 + m))/(f^15*(15 + m)) + (15*(3*d +
 8*e)*(f*x)^(17 + m))/(f^17*(17 + m)) + (5*(2*d + 9*e)*(f*x)^(19 + m))/(f^1
9*(19 + m)) + ((d + 10*e)*(f*x)^(21 + m))/(f^21*(21 + m)) + (e*(f*x)^(23 +
m))/(f^23*(23 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int (fx)^m \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx &= \int (fx)^m \left(1+x^2\right)^{10} \left(d+ex^2\right) \, dx \\
&= \int \left(d(fx)^m+\frac{(10d+e)(fx)^{2+m}}{f^2}+\frac{5(9d+2e)(fx)^{4+m}}{f^4}+\frac{15(8d+3e)(fx)^6}{f^6}\right. \\
&= \frac{d(fx)^{1+m}}{f(1+m)}+\frac{(10d+e)(fx)^{3+m}}{f^3(3+m)}+\frac{5(9d+2e)(fx)^{5+m}}{f^5(5+m)}+\frac{15(8d+3e)(fx)^{7+}}{f^7(7+m)}
\end{aligned}$$

Mathematica [A] time = 0.56, size = 189, normalized size = 0.70

$$x(fx)^m \left(\frac{x^{20}(d+10e)}{m+21}+\frac{5x^{18}(2d+9e)}{m+19}+\frac{15x^{16}(3d+8e)}{m+17}+\frac{30x^{14}(4d+7e)}{m+15}+\frac{42x^{12}(5d+6e)}{m+13}+\frac{42x^{10}(6d+5e)}{m+11}+\frac{30x^8(7d+4e)}{m+9}+\frac{15x^6(8d+3e)}{m+7}+\frac{5x^4(9d+2e)}{m+5}+\frac{x^2(10d+e)}{m+3}+\frac{d}{m+1}+\frac{ex^{22}}{m+23}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x*(f*x)^m*(d/(1 + m) + ((10*d + e)*x^2)/(3 + m) + (5*(9*d + 2*e)*x^4)/(5 +
m) + (15*(8*d + 3*e)*x^6)/(7 + m) + (30*(7*d + 4*e)*x^8)/(9 + m) + (42*(6*d
 + 5*e)*x^10)/(11 + m) + (42*(5*d + 6*e)*x^12)/(13 + m) + (30*(4*d + 7*e)*x
^14)/(15 + m) + (15*(3*d + 8*e)*x^16)/(17 + m) + (5*(2*d + 9*e)*x^18)/(19 +
 m) + ((d + 10*e)*x^20)/(21 + m) + (e*x^22)/(23 + m))
```

IntegrateAlgebraic [F] time = 1.20, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+ex^2\right)\left(1+2x^2+x^4\right)^5 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f*x)^m*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] Defer[IntegrateAlgebraic][(f*x)^m*(d + e*x^2)*(1 + 2*x^2 + x^4)^5, x]
```

fricas [B] time = 0.82, size = 1571, normalized size = 5.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] ((e*m^11 + 121*e*m^10 + 6435*e*m^9 + 197835*e*m^8 + 3889578*e*m^7 + 5106901
8*e*m^6 + 453714470*e*m^5 + 2702025590*e*m^4 + 10431670821*e*m^3 + 24372200
061*e*m^2 + 29985521895*e*m + 13749310575*e)*x^23 + ((d + 10*e)*m^11 + 123*
(d + 10*e)*m^10 + 6635*(d + 10*e)*m^9 + 206505*(d + 10*e)*m^8 + 4103178*(d
+ 10*e)*m^7 + 54362574*(d + 10*e)*m^6 + 486687830*(d + 10*e)*m^5 + 29170139
70*(d + 10*e)*m^4 + 11320966021*(d + 10*e)*m^3 + 26560342503*(d + 10*e)*m^2
 + 32778930735*(d + 10*e)*m + 15058768725*d + 150587687250*e)*x^21 + 5*((2*
d + 9*e)*m^11 + 125*(2*d + 9*e)*m^10 + 6843*(2*d + 9*e)*m^9 + 215823*(2*d +
 9*e)*m^8 + 4339146*(2*d + 9*e)*m^7 + 58085538*(2*d + 9*e)*m^6 + 524676662*
(2*d + 9*e)*m^5 + 3168601822*(2*d + 9*e)*m^4 + 12374824773*(2*d + 9*e)*m^3
+ 29178958257*(2*d + 9*e)*m^2 + 36145916415*(2*d + 9*e)*m + 33287804550*d +
 149795120475*e)*x^19 + 15*((3*d + 8*e)*m^11 + 127*(3*d + 8*e)*m^10 + 7059*
(3*d + 8*e)*m^9 + 225837*(3*d + 8*e)*m^8 + 4600554*(3*d + 8*e)*m^7 + 623198
94*(3*d + 8*e)*m^6 + 568863686*(3*d + 8*e)*m^5 + 3466775738*(3*d + 8*e)*m^4
 + 13643071845*(3*d + 8*e)*m^3 + 32368407579*(3*d + 8*e)*m^2 + 40283194455*
(3*d + 8*e)*m + 55806025275*d + 148816067400*e)*x^17 + 30*((4*d + 7*e)*m^11
 + 129*(4*d + 7*e)*m^10 + 7283*(4*d + 7*e)*m^9 + 236595*(4*d + 7*e)*m^8 + 4
890858*(4*d + 7*e)*m^7 + 67166442*(4*d + 7*e)*m^6 + 620805254*(4*d + 7*e)*m
^5 + 3825379590*(4*d + 7*e)*m^4 + 15197565541*(4*d + 7*e)*m^3 + 36337145829
*(4*d + 7*e)*m^2 + 45488935863*(4*d + 7*e)*m + 84329104860*d + 147575933505
*e)*x^15 + 42*((5*d + 6*e)*m^11 + 131*(5*d + 6*e)*m^10 + 7515*(5*d + 6*e)*m
^9 + 248145*(5*d + 6*e)*m^8 + 5213898*(5*d + 6*e)*m^7 + 72748638*(5*d + 6*e
)*m^6 + 682569590*(5*d + 6*e)*m^5 + 4264053730*(5*d + 6*e)*m^4 + 1714556090
1*(5*d + 6*e)*m^3 + 41408337231*(5*d + 6*e)*m^2 + 52237739295*(5*d + 6*e)*m
 + 121628516625*d + 145954219950*e)*x^13 + 42*((6*d + 5*e)*m^11 + 133*(6*d
+ 5*e)*m^10 + 7755*(6*d + 5*e)*m^9 + 260535*(6*d + 5*e)*m^8 + 5573898*(6*d
+ 5*e)*m^7 + 79216434*(6*d + 5*e)*m^6 + 756921110*(6*d + 5*e)*m^5 + 4811326
190*(6*d + 5*e)*m^4 + 19653671301*(6*d + 5*e)*m^3 + 48110244633*(6*d + 5*e)
*m^2 + 61333432335*(6*d + 5*e)*m + 172491350850*d + 143742792375*e)*x^11 +
30*((7*d + 4*e)*m^11 + 135*(7*d + 4*e)*m^10 + 8003*(7*d + 4*e)*m^9 + 273813
*(7*d + 4*e)*m^8 + 5975466*(7*d + 4*e)*m^7 + 86750118*(7*d + 4*e)*m^6 + 847
550822*(7*d + 4*e)*m^5 + 5509501002*(7*d + 4*e)*m^4 + 22992750373*(7*d + 4*
e)*m^3 + 57365875587*(7*d + 4*e)*m^2 + 74253243015*(7*d + 4*e)*m + 24595988
9175*d + 140548508100*e)*x^9 + 15*((8*d + 3*e)*m^11 + 137*(8*d + 3*e)*m^10
+ 8259*(8*d + 3*e)*m^9 + 288027*(8*d + 3*e)*m^8 + 6423594*(8*d + 3*e)*m^7 +
 95564154*(8*d + 3*e)*m^6 + 959352806*(8*d + 3*e)*m^5 + 6421988758*(8*d + 3
*e)*m^4 + 27624338085*(8*d + 3*e)*m^3 + 70930262349*(8*d + 3*e)*m^2 + 94034
286855*(8*d + 3*e)*m + 361410449400*d + 135528918525*e)*x^7 + 5*((9*d + 2*e
```

```
)*m^11 + 139*(9*d + 2*e)*m^10 + 8523*(9*d + 2*e)*m^9 + 303225*(9*d + 2*e)*m
^8 + 6923658*(9*d + 2*e)*m^7 + 105911022*(9*d + 2*e)*m^6 + 1098746774*(9*d
+ 2*e)*m^5 + 7643724530*(9*d + 2*e)*m^4 + 34359636741*(9*d + 2*e)*m^3 + 925
02445239*(9*d + 2*e)*m^2 + 128033897103*(9*d + 2*e)*m + 569221457805*d + 12
6493657290*e)*x^5 + ((10*d + e)*m^11 + 141*(10*d + e)*m^10 + 8795*(10*d + e
)*m^9 + 319455*(10*d + e)*m^8 + 7481418*(10*d + e)*m^7 + 118085058*(10*d +
e)*m^6 + 1274046710*(10*d + e)*m^5 + 9315318270*(10*d + e)*m^4 + 4463230458
1*(10*d + e)*m^3 + 130403715201*(10*d + e)*m^2 + 199334977695*(10*d + e)*m
+ 105411381075 0*d + 105411381075*e)*x^3 + (d*m^11 + 143*d*m^10 + 9075*d*m^9
 + 336765*d*m^8 + 8103018*d*m^7 + 132426294*d*m^6 + 1495875590*d*m^5 + 1164
1582810*d*m^4 + 60936676581*d*m^3 + 203363952363*d*m^2 + 387182170935*d*m +
 316234143225*d)*x)*(f*x)^m/(m^12 + 144*m^11 + 9218*m^10 + 345840*m^9 + 843
9783*m^8 + 140529312*m^7 + 1628301884*m^6 + 13137458400*m^5 + 72578259391*m
^4 + 264300628944*m^3 + 590546123298*m^2 + 703416314160*m + 316234143225)
```

giac [B] time = 0.46, size = 3752, normalized size = 13.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] ((f*x)^m*m^11*x^23*e + 121*(f*x)^m*m^10*x^23*e + (f*x)^m*d*m^11*x^21 + 10*(
f*x)^m*m^11*x^21*e + 6435*(f*x)^m*m^9*x^23*e + 123*(f*x)^m*d*m^10*x^21 + 12
30*(f*x)^m*m^10*x^21*e + 197835*(f*x)^m*m^8*x^23*e + 10*(f*x)^m*d*m^11*x^19
 + 6635*(f*x)^m*d*m^9*x^21 + 45*(f*x)^m*m^11*x^19*e + 66350*(f*x)^m*m^9*x^2
1*e + 3889578*(f*x)^m*m^7*x^23*e + 1250*(f*x)^m*d*m^10*x^19 + 206505*(f*x)^
m*d*m^8*x^21 + 5625*(f*x)^m*m^10*x^19*e + 2065050*(f*x)^m*m^8*x^21*e + 5106
9018*(f*x)^m*m^6*x^23*e + 45*(f*x)^m*d*m^11*x^17 + 68430*(f*x)^m*d*m^9*x^19
 + 4103178*(f*x)^m*d*m^7*x^21 + 120*(f*x)^m*m^11*x^17*e + 307935*(f*x)^m*m^
9*x^19*e + 41031780*(f*x)^m*m^7*x^21*e + 453714470*(f*x)^m*m^5*x^23*e + 571
5*(f*x)^m*d*m^10*x^17 + 2158230*(f*x)^m*d*m^8*x^19 + 54362574*(f*x)^m*d*m^6
*x^21 + 15240*(f*x)^m*m^10*x^17*e + 9712035*(f*x)^m*m^8*x^19*e + 543625740*
(f*x)^m*m^6*x^21*e + 2702025590*(f*x)^m*m^4*x^23*e + 120*(f*x)^m*d*m^11*x^1
5 + 317655*(f*x)^m*d*m^9*x^17 + 43391460*(f*x)^m*d*m^7*x^19 + 486687830*(f*
x)^m*d*m^5*x^21 + 210*(f*x)^m*m^11*x^15*e + 847080*(f*x)^m*m^9*x^17*e + 195
261570*(f*x)^m*m^7*x^19*e + 4866878300*(f*x)^m*m^5*x^21*e + 10431670821*(f*
x)^m*m^3*x^23*e + 15480*(f*x)^m*d*m^10*x^15 + 10162665*(f*x)^m*d*m^8*x^17 +
 580855380*(f*x)^m*d*m^6*x^19 + 2917013970*(f*x)^m*d*m^4*x^21 + 27090*(f*x)
^m*m^10*x^15*e + 27100440*(f*x)^m*m^8*x^17*e + 2613849210*(f*x)^m*m^6*x^19*
e + 29170139700*(f*x)^m*m^4*x^21*e + 24372200061*(f*x)^m*m^2*x^23*e + 210*(
f*x)^m*d*m^11*x^13 + 873960*(f*x)^m*d*m^9*x^15 + 207024930*(f*x)^m*d*m^7*x^
17 + 5246766620*(f*x)^m*d*m^5*x^19 + 11320966021*(f*x)^m*d*m^3*x^21 + 252*(
f*x)^m*m^11*x^13*e + 1529430*(f*x)^m*m^9*x^15*e + 552066480*(f*x)^m*m^7*x^1
7*e + 23610449790*(f*x)^m*m^5*x^19*e + 113209660210*(f*x)^m*m^3*x^21*e + 29
985521895*(f*x)^m*m*x^23*e + 27510*(f*x)^m*d*m^10*x^13 + 28391400*(f*x)^m*d
*m^8*x^15 + 2804395230*(f*x)^m*d*m^6*x^17 + 31686018220*(f*x)^m*d*m^4*x^19
+ 26560342503*(f*x)^m*d*m^2*x^21 + 33012*(f*x)^m*m^10*x^13*e + 49684950*(f*
x)^m*m^8*x^15*e + 7478387280*(f*x)^m*m^6*x^17*e + 142587081990*(f*x)^m*m^4*
x^19*e + 265603425030*(f*x)^m*m^2*x^21*e + 13749310575*(f*x)^m*x^23*e + 252
*(f*x)^m*d*m^11*x^11 + 1578150*(f*x)^m*d*m^9*x^13 + 586902960*(f*x)^m*d*m^7
*x^15 + 25598865870*(f*x)^m*d*m^5*x^17 + 123748247730*(f*x)^m*d*m^3*x^19 +
32778930735*(f*x)^m*d*m*x^21 + 210*(f*x)^m*m^11*x^11*e + 1893780*(f*x)^m*m^
9*x^13*e + 1027080180*(f*x)^m*m^7*x^15*e + 68263642320*(f*x)^m*m^5*x^17*e +
 556867114785*(f*x)^m*m^3*x^19*e + 327789307350*(f*x)^m*m*x^21*e + 33516*(f
*x)^m*d*m^10*x^11 + 52110450*(f*x)^m*d*m^8*x^13 + 8059973040*(f*x)^m*d*m^6*
x^15 + 156004908210*(f*x)^m*d*m^4*x^17 + 291789582570*(f*x)^m*d*m^2*x^19 +
15058768725*(f*x)^m*d*x^21 + 27930*(f*x)^m*m^10*x^11*e + 62532540*(f*x)^m*m
^8*x^13*e + 14104952820*(f*x)^m*m^6*x^15*e + 416013088560*(f*x)^m*m^4*x^17*
e + 1313053121565*(f*x)^m*m^2*x^19*e + 150587687250*(f*x)^m*x^21*e + 210*(f
```

```
*x)^m*d*m^11*x^9 + 1954260*(f*x)^m*d*m^9*x^11 + 1094918580*(f*x)^m*d*m^7*x^
13 + 74496630480*(f*x)^m*d*m^5*x^15 + 613938233025*(f*x)^m*d*m^3*x^17 + 361
459164150*(f*x)^m*d*m*x^19 + 120*(f*x)^m*m^11*x^9*e + 1628550*(f*x)^m*m^9*x
^11*e + 1313902296*(f*x)^m*m^7*x^13*e + 130369103340*(f*x)^m*m^5*x^15*e + 1
637168621400*(f*x)^m*m^3*x^17*e + 162656238675*(f*x)^m*m*x^19*e + 28350*(f
*x)^m*d*m^10*x^9 + 65654820*(f*x)^m*d*m^8*x^11 + 15277213980*(f*x)^m*d*m^6*
x^13 + 459045550800*(f*x)^m*d*m^4*x^15 + 1456578341055*(f*x)^m*d*m^2*x^17 +
 166439022750*(f*x)^m*d*x^19 + 16200*(f*x)^m*m^10*x^9*e + 54712350*(f*x)^m*
m^8*x^11*e + 18332656776*(f*x)^m*m^6*x^13*e + 803329713900*(f*x)^m*m^4*x^15
*e + 3884208909480*(f*x)^m*m^2*x^17*e + 748975602375*(f*x)^m*x^19*e + 120*(
f*x)^m*d*m^11*x^7 + 1680630*(f*x)^m*d*m^9*x^9 + 1404622296*(f*x)^m*d*m^7*x^
11 + 143339613900*(f*x)^m*d*m^5*x^13 + 1823707864920*(f*x)^m*d*m^3*x^15 + 1
812743750475*(f*x)^m*d*m*x^17 + 45*(f*x)^m*m^11*x^7*e + 960360*(f*x)^m*m^9*
x^9*e + 1170518580*(f*x)^m*m^7*x^11*e + 172007536680*(f*x)^m*m^5*x^13*e + 3
191488763610*(f*x)^m*m^3*x^15*e + 4833983334600*(f*x)^m*m*x^17*e + 16440*(f
*x)^m*d*m^10*x^7 + 57500730*(f*x)^m*d*m^8*x^9 + 19962541368*(f*x)^m*d*m^6*x
^11 + 895451283300*(f*x)^m*d*m^4*x^13 + 4360457499480*(f*x)^m*d*m^2*x^15 +
837090379125*(f*x)^m*d*x^17 + 6165*(f*x)^m*m^10*x^7*e + 32857560*(f*x)^m*m^
8*x^9*e + 16635451140*(f*x)^m*m^6*x^11*e + 1074541539960*(f*x)^m*m^4*x^13*e
 + 7630800624090*(f*x)^m*m^2*x^15*e + 2232241011000*(f*x)^m*x^17*e + 45*(f*
x)^m*d*m^11*x^5 + 991080*(f*x)^m*d*m^9*x^7 + 1254847860*(f*x)^m*d*m^7*x^9 +
 190744119720*(f*x)^m*d*m^5*x^11 + 3600567789210*(f*x)^m*d*m^3*x^13 + 54586
72303560*(f*x)^m*d*m*x^15 + 10*(f*x)^m*m^11*x^5*e + 371655*(f*x)^m*m^9*x^7*
e + 717055920*(f*x)^m*m^7*x^9*e + 158953433100*(f*x)^m*m^5*x^11*e + 4320681
347052*(f*x)^m*m^3*x^13*e + 9552676531230*(f*x)^m*m*x^15*e + 6255*(f*x)^m*d
*m^10*x^5 + 34563240*(f*x)^m*d*m^8*x^7 + 18217524780*(f*x)^m*d*m^6*x^9 + 12
12454199880*(f*x)^m*d*m^4*x^11 + 8695750818510*(f*x)^m*d*m^2*x^13 + 2529873
145800*(f*x)^m*d*x^15 + 1390*(f*x)^m*m^10*x^5*e + 12961215*(f*x)^m*m^8*x^7*
e + 10410014160*(f*x)^m*m^6*x^9*e + 1010378499900*(f*x)^m*m^4*x^11*e + 1043
4900982212*(f*x)^m*m^2*x^13*e + 4427278005150*(f*x)^m*x^15*e + 10*(f*x)^m*d
*m^11*x^3 + 383535*(f*x)^m*d*m^9*x^5 + 770831280*(f*x)^m*d*m^7*x^7 + 177985
672620*(f*x)^m*d*m^5*x^9 + 4952725167852*(f*x)^m*d*m^3*x^11 + 1096992525195
0*(f*x)^m*d*m*x^13 + (f*x)^m*m^11*x^3*e + 85230*(f*x)^m*m^9*x^5*e + 2890617
30*(f*x)^m*m^7*x^7*e + 101706098640*(f*x)^m*m^5*x^9*e + 4127270973210*(f*x)
^m*m^3*x^11*e + 13163910302340*(f*x)^m*m*x^13*e + 1410*(f*x)^m*d*m^10*x^3 +
 13645125*(f*x)^m*d*m^8*x^5 + 11467698480*(f*x)^m*d*m^6*x^7 + 1156995210420
*(f*x)^m*d*m^4*x^9 + 12123781647516*(f*x)^m*d*m^2*x^11 + 5108397698250*(f*x
)^m*d*x^13 + 141*(f*x)^m*m^10*x^3*e + 3032250*(f*x)^m*m^8*x^5*e + 430038693
0*(f*x)^m*m^6*x^7*e + 661140120240*(f*x)^m*m^4*x^9*e + 10103151372930*(f*x)
^m*m^2*x^11*e + 6130077237900*(f*x)^m*x^13*e + (f*x)^m*d*m^11*x + 87950*(f*
x)^m*d*m^9*x^3 + 311564610*(f*x)^m*d*m^7*x^5 + 115122336720*(f*x)^m*d*m^5*x
^7 + 4828477578330*(f*x)^m*d*m^3*x^9 + 15456024948420*(f*x)^m*d*m*x^11 + 87
95*(f*x)^m*m^9*x^3*e + 69236580*(f*x)^m*m^7*x^5*e + 43170876270*(f*x)^m*m^5
*x^7*e + 2759130044760*(f*x)^m*m^3*x^9*e + 12880020790350*(f*x)^m*m*x^11*e
+ 143*(f*x)^m*d*m^10*x + 3194550*(f*x)^m*d*m^8*x^3 + 4765995990*(f*x)^m*d*m
^6*x^5 + 770638650960*(f*x)^m*d*m^4*x^7 + 12046833873270*(f*x)^m*d*m^2*x^9
+ 7244636735700*(f*x)^m*d*x^11 + 319455*(f*x)^m*m^8*x^3*e + 1059110220*(f*x
)^m*m^6*x^5*e + 288989494110*(f*x)^m*m^4*x^7*e + 6883905070440*(f*x)^m*m^2*
x^9*e + 6037197279750*(f*x)^m*x^11*e + 9075*(f*x)^m*d*m^9*x + 74814180*(f*x
)^m*d*m^7*x^3 + 49443604830*(f*x)^m*d*m^5*x^5 + 3314920570200*(f*x)^m*d*m^3
*x^7 + 15593181033150*(f*x)^m*d*m*x^9 + 7481418*(f*x)^m*m^7*x^3*e + 1098746
7740*(f*x)^m*m^5*x^5*e + 1243095213825*(f*x)^m*m^3*x^7*e + 8910389161800*(f
*x)^m*m*x^9*e + 336765*(f*x)^m*d*m^8*x + 1180850580*(f*x)^m*d*m^6*x^3 + 343
967603850*(f*x)^m*d*m^4*x^5 + 8511631481880*(f*x)^m*d*m^2*x^7 + 73787966752
50*(f*x)^m*d*x^9 + 118085058*(f*x)^m*m^6*x^3*e + 76437245300*(f*x)^m*m^4*x^
5*e + 3191861805705*(f*x)^m*m^2*x^7*e + 4216455243000*(f*x)^m*x^9*e + 81030
18*(f*x)^m*d*m^7*x + 12740467100*(f*x)^m*d*m^5*x^3 + 1546183653345*(f*x)^m*
d*m^3*x^5 + 11284114422600*(f*x)^m*d*m*x^7 + 1274046710*(f*x)^m*m^5*x^3*e +
 343596367410*(f*x)^m*m^3*x^5*e + 4231542908475*(f*x)^m*m*x^7*e + 132426294
```

```
*(f*x)^m*d*m^6*x + 93153182700*(f*x)^m*d*m^4*x^3 + 4162610035755*(f*x)^m*d*
m^2*x^5 + 5421156741000*(f*x)^m*d*x^7 + 9315318270*(f*x)^m*m^4*x^3*e + 9250
24452390*(f*x)^m*m^2*x^5*e + 2032933777875*(f*x)^m*x^7*e + 1495875590*(f*x)
^m*d*m^5*x + 446323045810*(f*x)^m*d*m^3*x^3 + 5761525369635*(f*x)^m*d*m*x^5
 + 44632304581*(f*x)^m*m^3*x^3*e + 1280338971030*(f*x)^m*m*x^5*e + 11641582
810*(f*x)^m*d*m^4*x + 1304037152010*(f*x)^m*d*m^2*x^3 + 2846107289025*(f*x)
^m*d*x^5 + 130403715201*(f*x)^m*m^2*x^3*e + 632468286450*(f*x)^m*x^5*e + 60
936676581*(f*x)^m*d*m^3*x + 1993349776950*(f*x)^m*d*m*x^3 + 199334977695*(f
*x)^m*m*x^3*e + 203363952363*(f*x)^m*d*m^2*x + 1054113810750*(f*x)^m*d*x^3
+ 105411381075*(f*x)^m*x^3*e + 387182170935*(f*x)^m*d*m*x + 316234143225*(f
*x)^m*d*x)/(m^12 + 144*m^11 + 9218*m^10 + 345840*m^9 + 8439783*m^8 + 140529
312*m^7 + 1628301884*m^6 + 13137458400*m^5 + 72578259391*m^4 + 264300628944
*m^3 + 590546123298*m^2 + 703416314160*m + 316234143225)
```

maple [B] time = 0.03, size = 2295, normalized size = 8.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] (f*x)^m*(e*m^11*x^22+121*e*m^10*x^22+d*m^11*x^20+10*e*m^11*x^20+6435*e*m^9*
x^22+123*d*m^10*x^20+1230*e*m^10*x^20+197835*e*m^8*x^22+10*d*m^11*x^18+6635
*d*m^9*x^20+45*e*m^11*x^18+66350*e*m^9*x^20+3889578*e*m^7*x^22+1250*d*m^10*
x^18+206505*d*m^8*x^20+5625*e*m^10*x^18+2065050*e*m^8*x^20+51069018*e*m^6*x
^22+45*d*m^11*x^16+68430*d*m^9*x^18+4103178*d*m^7*x^20+120*e*m^11*x^16+3079
35*e*m^9*x^18+41031780*e*m^7*x^20+453714470*e*m^5*x^22+5715*d*m^10*x^16+215
8230*d*m^8*x^18+54362574*d*m^6*x^20+15240*e*m^10*x^16+9712035*e*m^8*x^18+54
3625740*e*m^6*x^20+2702025590*e*m^4*x^22+120*d*m^11*x^14+317655*d*m^9*x^16+
43391460*d*m^7*x^18+486687830*d*m^5*x^20+210*e*m^11*x^14+847080*e*m^9*x^16+
195261570*e*m^7*x^18+4866878300*e*m^5*x^20+10431670821*e*m^3*x^22+15480*d*m
^10*x^14+10162665*d*m^8*x^16+580855380*d*m^6*x^18+2917013970*d*m^4*x^20+270
90*e*m^10*x^14+27100440*e*m^8*x^16+2613849210*e*m^6*x^18+29170139700*e*m^4*
x^20+24372200061*e*m^2*x^22+210*d*m^11*x^12+873960*d*m^9*x^14+207024930*d*m
^7*x^16+5246766620*d*m^5*x^18+11320966021*d*m^3*x^20+252*e*m^11*x^12+152943
0*e*m^9*x^14+552066480*e*m^7*x^16+23610449790*e*m^5*x^18+113209660210*e*m^3
*x^20+29985521895*e*m*x^22+27510*d*m^10*x^12+28391400*d*m^8*x^14+2804395230
*d*m^6*x^16+31686018220*d*m^4*x^18+26560342503*d*m^2*x^20+33012*e*m^10*x^12
+49684950*e*m^8*x^14+7478387280*e*m^6*x^16+142587081990*e*m^4*x^18+26560342
5030*e*m^2*x^20+13749310575*e*x^22+252*d*m^11*x^10+1578150*d*m^9*x^12+58690
2960*d*m^7*x^14+25598865870*d*m^5*x^16+123748247730*d*m^3*x^18+32778930735*
d*m*x^20+210*e*m^11*x^10+1893780*e*m^9*x^12+1027080180*e*m^7*x^14+682636423
20*e*m^5*x^16+556867114785*e*m^3*x^18+327789307350*e*m*x^20+33516*d*m^10*x^
10+52110450*d*m^8*x^12+8059973040*d*m^6*x^14+156004908210*d*m^4*x^16+291789
582570*d*m^2*x^18+15058768725*d*x^20+27930*e*m^10*x^10+62532540*e*m^8*x^12+
14104952820*e*m^6*x^14+416013088560*e*m^4*x^16+1313053121565*e*m^2*x^18+150
587687250*e*x^20+210*d*m^11*x^8+1954260*d*m^9*x^10+1094918580*d*m^7*x^12+74
496630480*d*m^5*x^14+613938233025*d*m^3*x^16+361459164150*d*m*x^18+120*e*m^
11*x^8+1628550*e*m^9*x^10+1313902296*e*m^7*x^12+130369103340*e*m^5*x^14+163
7168621400*e*m^3*x^16+1626566238675*e*m*x^18+28350*d*m^10*x^8+65654820*d*m^
8*x^10+15277213980*d*m^6*x^12+459045550800*d*m^4*x^14+1456578341055*d*m^2*x
^16+166439022750*d*x^18+16200*e*m^10*x^8+54712350*e*m^8*x^10+18332656776*e*
m^6*x^12+803329713900*e*m^4*x^14+3884208909480*e*m^2*x^16+748975602375*e*x^
18+120*d*m^11*x^6+1680630*d*m^9*x^8+1404622296*d*m^7*x^10+143339613900*d*m^
5*x^12+1823707864920*d*m^3*x^14+1812743750475*d*m*x^16+45*e*m^11*x^6+960360
*e*m^9*x^8+1170518580*e*m^7*x^10+172007536680*e*m^5*x^12+3191488763610*e*m^
3*x^14+4833983334600*e*m*x^16+16440*d*m^10*x^6+57500730*d*m^8*x^8+199625413
68*d*m^6*x^10+895451283300*d*m^4*x^12+4360457499480*d*m^2*x^14+837090379125
*d*x^16+6165*e*m^10*x^6+32857560*e*m^8*x^8+16635451140*e*m^6*x^10+107454153
9960*e*m^4*x^12+7630800624090*e*m^2*x^14+2232241011000*e*x^16+45*d*m^11*x^4
```

```
+991080*d*m^9*x^6+1254847860*d*m^7*x^8+190744119720*d*m^5*x^10+360056778921
0*d*m^3*x^12+5458672303560*d*m*x^14+10*e*m^11*x^4+371655*e*m^9*x^6+71705592
0*e*m^7*x^8+158953433100*e*m^5*x^10+4320681347052*e*m^3*x^12+9552676531230*
e*m*x^14+6255*d*m^10*x^4+34563240*d*m^8*x^6+18217524780*d*m^6*x^8+121245419
9880*d*m^4*x^10+8695750818510*d*m^2*x^12+2529873145800*d*x^14+1390*e*m^10*x
^4+12961215*e*m^8*x^6+10410014160*e*m^6*x^8+1010378499900*e*m^4*x^10+104349
00982212*e*m^2*x^12+4427278005150*e*x^14+10*d*m^11*x^2+383535*d*m^9*x^4+770
831280*d*m^7*x^6+177985672620*d*m^5*x^8+4952725167852*d*m^3*x^10+1096992525
1950*d*m*x^12+e*m^11*x^2+85230*e*m^9*x^4+289061730*e*m^7*x^6+101706098640*e
*m^5*x^8+4127270973210*e*m^3*x^10+13163910302340*e*m*x^12+1410*d*m^10*x^2+1
3645125*d*m^8*x^4+11467698480*d*m^6*x^6+1156995210420*d*m^4*x^8+12123781647
516*d*m^2*x^10+5108397698250*d*x^12+141*e*m^10*x^2+3032250*e*m^8*x^4+430038
6930*e*m^6*x^6+661140120240*e*m^4*x^8+10103151372930*e*m^2*x^10+61300772379
00*e*x^12+d*m^11+87950*d*m^9*x^2+311564610*d*m^7*x^4+115122336720*d*m^5*x^6
+4828477578330*d*m^3*x^8+15456024948420*d*m*x^10+8795*e*m^9*x^2+69236580*e*
m^7*x^4+43170876270*e*m^5*x^6+2759130044760*e*m^3*x^8+12880020790350*e*m*x^
10+143*d*m^10+3194550*d*m^8*x^2+4765995990*d*m^6*x^4+770638650960*d*m^4*x^6
+12046833873270*d*m^2*x^8+7244636735700*d*x^10+319455*e*m^8*x^2+1059110220*
e*m^6*x^4+288989494110*e*m^4*x^6+6883905070440*e*m^2*x^8+6037197279750*e*x^
10+9075*d*m^9+74814180*d*m^7*x^2+49443604830*d*m^5*x^4+3314920570200*d*m^3*
x^6+15593181033150*d*m*x^8+7481418*e*m^7*x^2+10987467740*e*m^5*x^4+12430952
13825*e*m^3*x^6+8910389161800*e*m*x^8+336765*d*m^8+1180850580*d*m^6*x^2+343
967603850*d*m^4*x^4+8511631481880*d*m^2*x^6+7378796675250*d*x^8+118085058*e
*m^6*x^2+76437245300*e*m^4*x^4+3191861805705*e*m^2*x^6+4216455243000*e*x^8+
8103018*d*m^7+12740467100*d*m^5*x^2+1546183653345*d*m^3*x^4+11284114422600*
d*m*x^6+1274046710*e*m^5*x^2+343596367410*e*m^3*x^4+4231542908475*e*m*x^6+1
32426294*d*m^6+93153182700*d*m^4*x^2+4162610035755*d*m^2*x^4+5421156741000*
d*x^6+9315318270*e*m^4*x^2+925024452390*e*m^2*x^4+2032933777875*e*x^6+14958
75590*d*m^5+446323045810*d*m^3*x^2+5761525369635*d*m*x^4+44632304581*e*m^3*
x^2+1280338971030*e*m*x^4+11641582810*d*m^4+1304037152010*d*m^2*x^2+2846107
289025*d*x^4+130403715201*e*m^2*x^2+632468286450*e*x^4+60936676581*d*m^3+19
93349776950*d*m*x^2+199334977695*e*m*x^2+203363952363*d*m^2+1054113810750*d
*x^2+105411381075*e*x^2+387182170935*d*m+316234143225*d)*x/(1+m)/(3+m)/(5+m
)/(7+m)/(9+m)/(11+m)/(13+m)/(15+m)/(17+m)/(19+m)/(21+m)/(23+m)
```

maxima [A] time = 0.89, size = 372, normalized size = 1.38

$$\frac{ef^m x^{23} x^m}{m+23} + \frac{df^m x^{21} x^m}{m+21} + \frac{10\,ef^m x^{21} x^m}{m+21} + \frac{10\,df^m x^{19} x^m}{m+19} + \frac{45\,ef^m x^{19} x^m}{m+19} + \frac{45\,df^m x^{17} x^m}{m+17} + \frac{120\,ef^m x^{17} x^m}{m+17} + \frac{120\,df^m x^{15} x^m}{m+15} + \frac{210\,ef^m x^{15} x^m}{m+15} + \frac{210\,df^m x^{13} x^m}{m+13} + \frac{252\,ef^m x^{13} x^m}{m+13} + \frac{252\,df^m x^{11} x^m}{m+11} + \frac{210\,ef^m x^{11} x^m}{m+11} + \frac{210\,df^m x^9 x^m}{m+9} + \frac{120\,ef^m x^9 x^m}{m+9} + \frac{120\,df^m x^7 x^m}{m+7} + \frac{45\,ef^m x^7 x^m}{m+7} + \frac{45\,df^m x^5 x^m}{m+5} + \frac{10\,ef^m x^5 x^m}{m+5} + \frac{10\,df^m x^3 x^m}{m+3} + \frac{ef^m x^3 x^m}{m+3} + \frac{(fx)^{m+1} d}{f(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] e*f^m*x^23*x^m/(m + 23) + d*f^m*x^21*x^m/(m + 21) + 10*e*f^m*x^21*x^m/(m +
21) + 10*d*f^m*x^19*x^m/(m + 19) + 45*e*f^m*x^19*x^m/(m + 19) + 45*d*f^m*x^
17*x^m/(m + 17) + 120*e*f^m*x^17*x^m/(m + 17) + 120*d*f^m*x^15*x^m/(m + 15)
 + 210*e*f^m*x^15*x^m/(m + 15) + 210*d*f^m*x^13*x^m/(m + 13) + 252*e*f^m*x^
13*x^m/(m + 13) + 252*d*f^m*x^11*x^m/(m + 11) + 210*e*f^m*x^11*x^m/(m + 11)
 + 210*d*f^m*x^9*x^m/(m + 9) + 120*e*f^m*x^9*x^m/(m + 9) + 120*d*f^m*x^7*x^
m/(m + 7) + 45*e*f^m*x^7*x^m/(m + 7) + 45*d*f^m*x^5*x^m/(m + 5) + 10*e*f^m*
x^5*x^m/(m + 5) + 10*d*f^m*x^3*x^m/(m + 3) + e*f^m*x^3*x^m/(m + 3) + (f*x)^
(m + 1)*d/(f*(m + 1))
```

mupad [B] time = 1.78, size = 1539, normalized size = 5.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] (d*x*(f*x)^m*(387182170935*m + 203363952363*m^2 + 60936676581*m^3 + 1164158
2810*m^4 + 1495875590*m^5 + 132426294*m^6 + 8103018*m^7 + 336765*m^8 + 9075
```

```
*m^9 + 143*m^10 + m^11 + 316234143225))/(703416314160*m + 590546123298*m^2
+ 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 1
40529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316
234143225) + (e*x^23*(f*x)^m*(29985521895*m + 24372200061*m^2 + 10431670821
*m^3 + 2702025590*m^4 + 453714470*m^5 + 51069018*m^6 + 3889578*m^7 + 197835
*m^8 + 6435*m^9 + 121*m^10 + m^11 + 13749310575))/(703416314160*m + 5905461
23298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 16283018
84*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 +
m^12 + 316234143225) + (30*x^15*(f*x)^m*(4*d + 7*e)*(45488935863*m + 363371
45829*m^2 + 15197565541*m^3 + 3825379590*m^4 + 620805254*m^5 + 67166442*m^6
 + 4890858*m^7 + 236595*m^8 + 7283*m^9 + 129*m^10 + m^11 + 21082276215))/(7
03416314160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 131
37458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 +
9218*m^10 + 144*m^11 + m^12 + 316234143225) + (42*x^13*(f*x)^m*(5*d + 6*e)*
(52237739295*m + 41408337231*m^2 + 17145560901*m^3 + 4264053730*m^4 + 68256
9590*m^5 + 72748638*m^6 + 5213898*m^7 + 248145*m^8 + 7515*m^9 + 131*m^10 +
m^11 + 24325703325))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3
+ 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439
783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (30*x^
9*(f*x)^m*(7*d + 4*e)*(74253243015*m + 57365875587*m^2 + 22992750373*m^3 +
5509501002*m^4 + 847550822*m^5 + 86750118*m^6 + 5975466*m^7 + 273813*m^8 +
8003*m^9 + 135*m^10 + m^11 + 35137127025))/(703416314160*m + 590546123298*m
^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6
+ 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 +
316234143225) + (x^3*(f*x)^m*(10*d + e)*(199334977695*m + 130403715201*m^2
+ 44632304581*m^3 + 9315318270*m^4 + 1274046710*m^5 + 118085058*m^6 + 74814
18*m^7 + 319455*m^8 + 8795*m^9 + 141*m^10 + m^11 + 105411381075))/(70341631
4160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 1313745840
0*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^
10 + 144*m^11 + m^12 + 316234143225) + (5*x^19*(f*x)^m*(2*d + 9*e)*(3614591
6415*m + 29178958257*m^2 + 12374824773*m^3 + 3168601822*m^4 + 524676662*m^5
 + 58085538*m^6 + 4339146*m^7 + 215823*m^8 + 6843*m^9 + 125*m^10 + m^11 + 1
6643902275))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 725782
59391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8
+ 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (42*x^11*(f*x)
^m*(6*d + 5*e)*(61333432335*m + 48110244633*m^2 + 19653671301*m^3 + 4811326
190*m^4 + 756921110*m^5 + 79216434*m^6 + 5573898*m^7 + 260535*m^8 + 7755*m^
9 + 133*m^10 + m^11 + 28748558475))/(703416314160*m + 590546123298*m^2 + 26
4300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 14052
9312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 3162341
43225) + (15*x^7*(f*x)^m*(8*d + 3*e)*(94034286855*m + 70930262349*m^2 + 276
24338085*m^3 + 6421988758*m^4 + 959352806*m^5 + 95564154*m^6 + 6423594*m^7
+ 288027*m^8 + 8259*m^9 + 137*m^10 + m^11 + 45176306175))/(703416314160*m +
 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 +
1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144
*m^11 + m^12 + 316234143225) + (5*x^5*(f*x)^m*(9*d + 2*e)*(128033897103*m +
 92502445239*m^2 + 34359636741*m^3 + 7643724530*m^4 + 1098746774*m^5 + 1059
11022*m^6 + 6923658*m^7 + 303225*m^8 + 8523*m^9 + 139*m^10 + m^11 + 6324682
8645))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*
m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 3458
40*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (15*x^17*(f*x)^m*(3*
d + 8*e)*(40283194455*m + 32368407579*m^2 + 13643071845*m^3 + 3466775738*m^
4 + 568863686*m^5 + 62319894*m^6 + 4600554*m^7 + 225837*m^8 + 7059*m^9 + 12
7*m^10 + m^11 + 18602008425))/(703416314160*m + 590546123298*m^2 + 26430062
8944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m
^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225)
 + (x^21*(f*x)^m*(d + 10*e)*(32778930735*m + 26560342503*m^2 + 11320966021*
m^3 + 2917013970*m^4 + 486687830*m^5 + 54362574*m^6 + 4103178*m^7 + 206505*
m^8 + 6635*m^9 + 123*m^10 + m^11 + 15058768725))/(703416314160*m + 59054612
```

```
3298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 162830188
4*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m
^12 + 316234143225)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] Timed out
```

3.36 $\int x^5 \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=63

$$\frac{1}{26}\left(x^2+1\right)^{13}(d-3e)-\frac{1}{24}\left(x^2+1\right)^{12}(2d-3e)+\frac{1}{22}\left(x^2+1\right)^{11}(d-e)+\frac{1}{28}e\left(x^2+1\right)^{14}$$

Rubi [A] time = 0.20, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {28, 446, 76}

$$\frac{1}{26}\left(x^2+1\right)^{13}(d-3e)-\frac{1}{24}\left(x^2+1\right)^{12}(2d-3e)+\frac{1}{22}\left(x^2+1\right)^{11}(d-e)+\frac{1}{28}e\left(x^2+1\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] ((d - e)*(1 + x^2)^11)/22 - ((2*d - 3*e)*(1 + x^2)^12)/24 + ((d - 3*e)*(1 +
 x^2)^13)/26 + (e*(1 + x^2)^14)/28
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(d+ex^2\right)\left(1+2x^2+x^4\right)^5 \, dx &= \int x^5 \left(1+x^2\right)^{10}\left(d+ex^2\right) \, dx \\
&= \frac{1}{2}\text{Subst}\left(\int x^2(1+x)^{10}(d+ex) \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left((d-e)(1+x)^{10}+(-2d+3e)(1+x)^{11}+(d-3e)(1+x)^{12}+e(1\right.\right. \\
&= \frac{1}{22}(d-e)\left(1+x^2\right)^{11}-\frac{1}{24}(2d-3e)\left(1+x^2\right)^{12}+\frac{1}{26}(d-3e)\left(1+x^2\right)^{13}+\frac{1}{28}e
\end{aligned}$$

Mathematica [B] time = 0.02, size = 153, normalized size = 2.43

$$\frac{1}{26}x^{26}(d+10e)+\frac{5}{24}x^{24}(2d+9e)+\frac{15}{22}x^{22}(3d+8e)+\frac{3}{2}x^{20}(4d+7e)+\frac{7}{3}x^{18}(5d+6e)+\frac{21}{8}x^{16}(6d+5e)+\frac{15}{7}x^{14}(7d+4e)+\frac{5}{4}x^{12}(8d+3e)+\frac{1}{2}x^{10}(9d+2e)+\frac{1}{8}x^8(10d+e)+\frac{dx^6}{6}+\frac{ex^{28}}{28}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^6)/6 + ((10*d + e)*x^8)/8 + ((9*d + 2*e)*x^10)/2 + (5*(8*d + 3*e)*x^12
)/4 + (15*(7*d + 4*e)*x^14)/7 + (21*(6*d + 5*e)*x^16)/8 + (7*(5*d + 6*e)*x^
18)/3 + (3*(4*d + 7*e)*x^20)/2 + (15*(3*d + 8*e)*x^22)/22 + (5*(2*d + 9*e)*
x^24)/24 + ((d + 10*e)*x^26)/26 + (e*x^28)/28
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \left(d + ex^2\right) \left(1 + 2x^2 + x^4\right)^5 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] IntegrateAlgebraic[x^5*(d + e*x^2)*(1 + 2*x^2 + x^4)^5, x]
```

fricas [B] time = 0.51, size = 132, normalized size = 2.10

$$\frac{1}{28}x^{28}e + \frac{5}{13}x^{26}e + \frac{1}{26}x^{26}d + \frac{15}{8}x^{24}e + \frac{5}{12}x^{24}d + \frac{60}{11}x^{22}e + \frac{45}{22}x^{22}d + \frac{21}{2}x^{20}e + 6x^{20}d + 14x^{18}e + \frac{35}{3}x^{18}d + \frac{105}{8}x^{16}e + \frac{63}{4}x^{16}d + \frac{60}{7}x^{14}e + 15x^{14}d + \frac{15}{4}x^{12}e + 10x^{12}d + x^{10}e + \frac{9}{2}x^{10}d + \frac{1}{8}x^{8}e + \frac{5}{4}x^{8}d + \frac{1}{6}x^{6}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/28*x^28*e + 5/13*x^26*e + 1/26*x^26*d + 15/8*x^24*e + 5/12*x^24*d + 60/11
*x^22*e + 45/22*x^22*d + 21/2*x^20*e + 6*x^20*d + 14*x^18*e + 35/3*x^18*d +
 105/8*x^16*e + 63/4*x^16*d + 60/7*x^14*e + 15*x^14*d + 15/4*x^12*e + 10*x^
12*d + x^10*e + 9/2*x^10*d + 1/8*x^8*e + 5/4*x^8*d + 1/6*x^6*d
```

giac [B] time = 0.28, size = 143, normalized size = 2.27

$$\frac{1}{28}x^{28}e + \frac{1}{26}dx^{26} + \frac{5}{13}x^{26}e + \frac{5}{12}dx^{24} + \frac{15}{8}x^{24}e + \frac{45}{22}dx^{22} + \frac{60}{11}x^{22}e + 6\,dx^{20} + \frac{21}{2}x^{20}e + \frac{35}{3}dx^{18} + 14\,x^{18}e + \frac{63}{4}dx^{16} + \frac{105}{8}x^{16}e + 15\,dx^{14} + \frac{60}{7}x^{14}e + 10\,dx^{12} + \frac{15}{4}x^{12}e + \frac{9}{2}dx^{10} + x^{10}e + \frac{5}{4}dx^{8} + \frac{1}{8}x^{8}e + \frac{1}{6}dx^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/28*x^28*e + 1/26*d*x^26 + 5/13*x^26*e + 5/12*d*x^24 + 15/8*x^24*e + 45/22
*d*x^22 + 60/11*x^22*e + 6*d*x^20 + 21/2*x^20*e + 35/3*d*x^18 + 14*x^18*e +
 63/4*d*x^16 + 105/8*x^16*e + 15*d*x^14 + 60/7*x^14*e + 10*d*x^12 + 15/4*x^
12*e + 9/2*d*x^10 + x^10*e + 5/4*d*x^8 + 1/8*x^8*e + 1/6*d*x^6
```

maple [B] time = 0.00, size = 130, normalized size = 2.06

$$\frac{e\,x^{28}}{28} + \frac{(d + 10e)\,x^{26}}{26} + \frac{(10d + 45e)\,x^{24}}{24} + \frac{(45d + 120e)\,x^{22}}{22} + \frac{(120d + 210e)\,x^{20}}{20} + \frac{(210d + 252e)\,x^{18}}{18} + \frac{(252d + 210e)\,x^{16}}{16} + \frac{(210d + 120e)\,x^{14}}{14} + \frac{(120d + 45e)\,x^{12}}{12} + \frac{(45d + 10e)\,x^{10}}{10} + \frac{(10d + e)\,x^{8}}{8} + \frac{d\,x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/28*e*x^28+1/26*(d+10*e)*x^26+1/24*(10*d+45*e)*x^24+1/22*(45*d+120*e)*x^22
+1/20*(120*d+210*e)*x^20+1/18*(210*d+252*e)*x^18+1/16*(252*d+210*e)*x^16+1/
14*(210*d+120*e)*x^14+1/12*(120*d+45*e)*x^12+1/10*(45*d+10*e)*x^10+1/8*(10*
d+e)*x^8+1/6*d*x^6
```

maxima [B] time = 0.59, size = 129, normalized size = 2.05

$$\frac{1}{28}\,ex^{28} + \frac{1}{26}\,(d + 10\,e)x^{26} + \frac{5}{24}\,(2\,d + 9\,e)x^{24} + \frac{15}{22}\,(3\,d + 8\,e)x^{22} + \frac{3}{2}\,(4\,d + 7\,e)x^{20} + \frac{7}{3}\,(5\,d + 6\,e)x^{18} + \frac{21}{8}\,(6\,d + 5\,e)x^{16} + \frac{15}{7}\,(7\,d + 4\,e)x^{14} + \frac{5}{4}\,(8\,d + 3\,e)x^{12} + \frac{1}{2}\,(9\,d + 2\,e)x^{10} + \frac{1}{8}\,(10\,d + e)x^{8} + \frac{1}{6}\,dx^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/28*e*x^28 + 1/26*(d + 10*e)*x^26 + 5/24*(2*d + 9*e)*x^24 + 15/22*(3*d + 8
*e)*x^22 + 3/2*(4*d + 7*e)*x^20 + 7/3*(5*d + 6*e)*x^18 + 21/8*(6*d + 5*e)*x
^16 + 15/7*(7*d + 4*e)*x^14 + 5/4*(8*d + 3*e)*x^12 + 1/2*(9*d + 2*e)*x^10 +
 1/8*(10*d + e)*x^8 + 1/6*d*x^6
```

mupad [B] time = 0.09, size = 121, normalized size = 1.92

$$\frac{e\,x^{28}}{28}+\left(\frac{d}{26}+\frac{5\,e}{13}\right)\,x^{26}+\left(\frac{5\,d}{12}+\frac{15\,e}{8}\right)\,x^{24}+\left(\frac{45\,d}{22}+\frac{60\,e}{11}\right)\,x^{22}+\left(6\,d+\frac{21\,e}{2}\right)\,x^{20}+\left(\frac{35\,d}{3}+14\,e\right)\,x^{18}+\left(\frac{63\,d}{4}+\frac{105\,e}{8}\right)\,x^{16}+\left(15\,d+\frac{60\,e}{7}\right)\,x^{14}+\left(10\,d+\frac{15\,e}{4}\right)\,x^{12}+\left(\frac{9\,d}{2}+e\right)\,x^{10}+\left(\frac{5\,d}{4}+\frac{e}{8}\right)\,x^8+\frac{d\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^8*((5*d)/4 + e/8) + x^12*(10*d + (15*e)/4) + x^20*(6*d + (21*e)/2) + x^24
*((5*d)/12 + (15*e)/8) + x^18*((35*d)/3 + 14*e) + x^26*(d/26 + (5*e)/13) +
x^14*(15*d + (60*e)/7) + x^22*((45*d)/22 + (60*e)/11) + x^16*((63*d)/4 + (1
05*e)/8) + (d*x^6)/6 + (e*x^28)/28 + x^10*((9*d)/2 + e)
```

sympy [B] time = 0.10, size = 134, normalized size = 2.13

$$\frac{dx^{6}}{6}+\frac{ex^{28}}{28}+x^{26}\left(\frac{d}{26}+\frac{5e}{13}\right)+x^{24}\left(\frac{5d}{12}+\frac{15e}{8}\right)+x^{22}\left(\frac{45d}{22}+\frac{60e}{11}\right)+x^{20}\left(6d+\frac{21e}{2}\right)+x^{18}\left(\frac{35d}{3}+14e\right)+x^{16}\left(\frac{63d}{4}+\frac{105e}{8}\right)+x^{14}\left(15d+\frac{60e}{7}\right)+x^{12}\left(10d+\frac{15e}{4}\right)+x^{10}\left(\frac{9d}{2}+e\right)+x^{8}\left(\frac{5d}{4}+\frac{e}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**6/6 + e*x**28/28 + x**26*(d/26 + 5*e/13) + x**24*(5*d/12 + 15*e/8) + x
**22*(45*d/22 + 60*e/11) + x**20*(6*d + 21*e/2) + x**18*(35*d/3 + 14*e) + x
**16*(63*d/4 + 105*e/8) + x**14*(15*d + 60*e/7) + x**12*(10*d + 15*e/4) + x
**10*(9*d/2 + e) + x**8*(5*d/4 + e/8)
```

3.37 $\int x^4 \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=153

$$\frac{1}{25}x^{25}(d+10e)+\frac{5}{23}x^{23}(2d+9e)+\frac{5}{7}x^{21}(3d+8e)+\frac{30}{19}x^{19}(4d+7e)+\frac{42}{17}x^{17}(5d+6e)+\frac{14}{5}x^{15}(6d+5e)+\frac{30}{13}x^{13}(7d+4e)+$$

Rubi [A] time = 0.12, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = $\{28, 448\}$

$$\frac{1}{25}x^{25}(d+10e)+\frac{5}{23}x^{23}(2d+9e)+\frac{5}{7}x^{21}(3d+8e)+\frac{30}{19}x^{19}(4d+7e)+\frac{42}{17}x^{17}(5d+6e)+\frac{14}{5}x^{15}(6d+5e)+\frac{30}{13}x^{13}(7d+4e)+\frac{15}{11}x^{11}(8d+3e)+\frac{5}{9}x^9(9d+2e)+\frac{1}{7}x^7(10d+e)+\frac{dx^5}{5}+\frac{ex^{27}}{27}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^5)/5 + ((10*d + e)*x^7)/7 + (5*(9*d + 2*e)*x^9)/9 + (15*(8*d + 3*e)*x^
11)/11 + (30*(7*d + 4*e)*x^13)/13 + (14*(6*d + 5*e)*x^15)/5 + (42*(5*d + 6*
e)*x^17)/17 + (30*(4*d + 7*e)*x^19)/19 + (5*(3*d + 8*e)*x^21)/7 + (5*(2*d +
 9*e)*x^23)/23 + ((d + 10*e)*x^25)/25 + (e*x^27)/27
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^4 \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx &= \int x^4 \left(1+x^2\right)^{10} \left(d+ex^2\right) \, dx\\ &= \int \left(dx^4+(10d+e)x^6+5(9d+2e)x^8+15(8d+3e)x^{10}+30(7d+4e)x^{12}\right.\\ &= \frac{dx^5}{5}+\frac{1}{7}(10d+e)x^7+\frac{5}{9}(9d+2e)x^9+\frac{15}{11}(8d+3e)x^{11}+\frac{30}{13}(7d+4e)x^{13}\end{aligned}$$

Mathematica [A] time = 0.02, size = 153, normalized size = 1.00

$$\frac{1}{25}x^{25}(d+10e)+\frac{5}{23}x^{23}(2d+9e)+\frac{5}{7}x^{21}(3d+8e)+\frac{30}{19}x^{19}(4d+7e)+\frac{42}{17}x^{17}(5d+6e)+\frac{14}{5}x^{15}(6d+5e)+\frac{30}{13}x^{13}(7d+4e)+\frac{15}{11}x^{11}(8d+3e)+\frac{5}{9}x^9(9d+2e)+\frac{1}{7}x^7(10d+e)+\frac{dx^5}{5}+\frac{ex^{27}}{27}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^5)/5 + ((10*d + e)*x^7)/7 + (5*(9*d + 2*e)*x^9)/9 + (15*(8*d + 3*e)*x^
11)/11 + (30*(7*d + 4*e)*x^13)/13 + (14*(6*d + 5*e)*x^15)/5 + (42*(5*d + 6*
e)*x^17)/17 + (30*(4*d + 7*e)*x^19)/19 + (5*(3*d + 8*e)*x^21)/7 + (5*(2*d +
 9*e)*x^23)/23 + ((d + 10*e)*x^25)/25 + (e*x^27)/27
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(d + ex^2\right) \left(1 + 2x^2 + x^4\right)^5 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] IntegrateAlgebraic[x^4*(d + e*x^2)*(1 + 2*x^2 + x^4)^5, x]
```

fricas [A] time = 0.87, size = 133, normalized size = 0.87

$$\frac{1}{27}x^{27}e + \frac{2}{5}x^{25}e + \frac{1}{25}x^{25}d + \frac{45}{23}x^{23}e + \frac{10}{23}x^{23}d + \frac{40}{7}x^{21}e + \frac{15}{7}x^{21}d + \frac{210}{19}x^{19}e + \frac{120}{19}x^{19}d + \frac{252}{17}x^{17}e + \frac{210}{17}x^{17}d + 14x^{15}e + \frac{84}{5}x^{15}d + \frac{120}{13}x^{13}e + \frac{210}{13}x^{13}d + \frac{45}{11}x^{11}e + \frac{120}{11}x^{11}d + \frac{10}{9}x^{9}e + 5x^{9}d + \frac{1}{7}x^{7}e + \frac{10}{7}x^{7}d + \frac{1}{5}x^{5}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/27*x^27*e + 2/5*x^25*e + 1/25*x^25*d + 45/23*x^23*e + 10/23*x^23*d + 40/7
*x^21*e + 15/7*x^21*d + 210/19*x^19*e + 120/19*x^19*d + 252/17*x^17*e + 210
/17*x^17*d + 14*x^15*e + 84/5*x^15*d + 120/13*x^13*e + 210/13*x^13*d + 45/1
1*x^11*e + 120/11*x^11*d + 10/9*x^9*e + 5*x^9*d + 1/7*x^7*e + 10/7*x^7*d +
1/5*x^5*d
```

giac [A] time = 0.30, size = 144, normalized size = 0.94

$$\frac{1}{27}x^{27}e + \frac{1}{25}dx^{25} + \frac{2}{5}x^{25}e + \frac{10}{23}dx^{23} + \frac{45}{23}x^{23}e + \frac{15}{7}dx^{21} + \frac{40}{7}x^{21}e + \frac{120}{19}dx^{19} + \frac{210}{19}x^{19}e + \frac{210}{17}dx^{17} + \frac{252}{17}x^{17}e + \frac{84}{5}dx^{15} + 14x^{15}e + \frac{210}{13}dx^{13} + \frac{120}{13}x^{13}e + \frac{120}{11}dx^{11} + \frac{45}{11}x^{11}e + 5dx^{9} + \frac{10}{9}x^{9}e + \frac{10}{7}dx^{7} + \frac{1}{7}x^{7}e + \frac{1}{5}dx^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/27*x^27*e + 1/25*d*x^25 + 2/5*x^25*e + 10/23*d*x^23 + 45/23*x^23*e + 15/7
*d*x^21 + 40/7*x^21*e + 120/19*d*x^19 + 210/19*x^19*e + 210/17*d*x^17 + 252
/17*x^17*e + 84/5*d*x^15 + 14*x^15*e + 210/13*d*x^13 + 120/13*x^13*e + 120/
11*d*x^11 + 45/11*x^11*e + 5*d*x^9 + 10/9*x^9*e + 10/7*d*x^7 + 1/7*x^7*e +
1/5*d*x^5
```

maple [A] time = 0.00, size = 130, normalized size = 0.85

$$\frac{e\,x^{27}}{27} + \frac{(d+10e)\,x^{25}}{25} + \frac{(10d+45e)\,x^{23}}{23} + \frac{(45d+120e)\,x^{21}}{21} + \frac{(120d+210e)\,x^{19}}{19} + \frac{(210d+252e)\,x^{17}}{17} + \frac{(252d+210e)\,x^{15}}{15} + \frac{(210d+120e)\,x^{13}}{13} + \frac{(120d+45e)\,x^{11}}{11} + \frac{(45d+10e)\,x^{9}}{9} + \frac{(10d+e)\,x^{7}}{7} + \frac{d\,x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/27*e*x^27+1/25*(d+10*e)*x^25+1/23*(10*d+45*e)*x^23+1/21*(45*d+120*e)*x^21
+1/19*(120*d+210*e)*x^19+1/17*(210*d+252*e)*x^17+1/15*(252*d+210*e)*x^15+1/
13*(210*d+120*e)*x^13+1/11*(120*d+45*e)*x^11+1/9*(45*d+10*e)*x^9+1/7*(10*d+
e)*x^7+1/5*d*x^5
```

maxima [A] time = 0.67, size = 129, normalized size = 0.84

$$\frac{1}{27}ex^{27} + \frac{1}{25}(d+10e)x^{25} + \frac{5}{23}(2d+9e)x^{23} + \frac{5}{7}(3d+8e)x^{21} + \frac{30}{19}(4d+7e)x^{19} + \frac{42}{17}(5d+6e)x^{17} + \frac{14}{5}(6d+5e)x^{15} + \frac{30}{13}(7d+4e)x^{13} + \frac{15}{11}(8d+3e)x^{11} + \frac{5}{9}(9d+2e)x^{9} + \frac{1}{7}(10d+e)x^{7} + \frac{1}{5}dx^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/27*e*x^27 + 1/25*(d + 10*e)*x^25 + 5/23*(2*d + 9*e)*x^23 + 5/7*(3*d + 8*e
)*x^21 + 30/19*(4*d + 7*e)*x^19 + 42/17*(5*d + 6*e)*x^17 + 14/5*(6*d + 5*e)
*x^15 + 30/13*(7*d + 4*e)*x^13 + 15/11*(8*d + 3*e)*x^11 + 5/9*(9*d + 2*e)*x
^9 + 1/7*(10*d + e)*x^7 + 1/5*d*x^5
```

mupad [B] time = 0.12, size = 123, normalized size = 0.80

$$\frac{e\,x^{27}}{27}+\left(\frac{d}{25}+\frac{2\,e}{5}\right)\,x^{25}+\left(\frac{10\,d}{23}+\frac{45\,e}{23}\right)\,x^{23}+\left(\frac{15\,d}{7}+\frac{40\,e}{7}\right)\,x^{21}+\left(\frac{120\,d}{19}+\frac{210\,e}{19}\right)\,x^{19}+\left(\frac{210\,d}{17}+\frac{252\,e}{17}\right)\,x^{17}+\left(\frac{84\,d}{5}+14\,e\right)\,x^{15}+\left(\frac{210\,d}{13}+\frac{120\,e}{13}\right)\,x^{13}+\left(\frac{120\,d}{11}+\frac{45\,e}{11}\right)\,x^{11}+\left(5\,d+\frac{10\,e}{9}\right)\,x^9+\left(\frac{10\,d}{7}+\frac{e}{7}\right)\,x^7+\frac{d\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^7*((10*d)/7 + e/7) + x^9*(5*d + (10*e)/9) + x^25*(d/25 + (2*e)/5) + x^21*
((15*d)/7 + (40*e)/7) + x^15*((84*d)/5 + 14*e) + x^23*((10*d)/23 + (45*e)/2
3) + x^11*((120*d)/11 + (45*e)/11) + x^13*((210*d)/13 + (120*e)/13) + x^19*
((120*d)/19 + (210*e)/19) + x^17*((210*d)/17 + (252*e)/17) + (d*x^5)/5 + (e
*x^27)/27
```

sympy [A] time = 0.10, size = 141, normalized size = 0.92

$$\frac{dx^{5}}{5}+\frac{ex^{27}}{27}+x^{25}\left(\frac{d}{25}+\frac{2e}{5}\right)+x^{23}\left(\frac{10d}{23}+\frac{45e}{23}\right)+x^{21}\left(\frac{15d}{7}+\frac{40e}{7}\right)+x^{19}\left(\frac{120d}{19}+\frac{210e}{19}\right)+x^{17}\left(\frac{210d}{17}+\frac{252e}{17}\right)+x^{15}\left(\frac{84d}{5}+14e\right)+x^{13}\left(\frac{210d}{13}+\frac{120e}{13}\right)+x^{11}\left(\frac{120d}{11}+\frac{45e}{11}\right)+x^{9}\left(5d+\frac{10e}{9}\right)+x^{7}\left(\frac{10d}{7}+\frac{e}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**5/5 + e*x**27/27 + x**25*(d/25 + 2*e/5) + x**23*(10*d/23 + 45*e/23) +
x**21*(15*d/7 + 40*e/7) + x**19*(120*d/19 + 210*e/19) + x**17*(210*d/17 + 2
52*e/17) + x**15*(84*d/5 + 14*e) + x**13*(210*d/13 + 120*e/13) + x**11*(120
*d/11 + 45*e/11) + x**9*(5*d + 10*e/9) + x**7*(10*d/7 + e/7)
```

3.38 $\int x^3 \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=45

$$\frac{1}{24}\left(x^2+1\right)^{12}(d-2e)-\frac{1}{22}\left(x^2+1\right)^{11}(d-e)+\frac{1}{26}e\left(x^2+1\right)^{13}$$

Rubi [A] time = 0.12, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {28, 446, 76}

$$\frac{1}{24}\left(x^2+1\right)^{12}(d-2e)-\frac{1}{22}\left(x^2+1\right)^{11}(d-e)+\frac{1}{26}e\left(x^2+1\right)^{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] -((d - e)*(1 + x^2)^11)/22 + ((d - 2*e)*(1 + x^2)^12)/24 + (e*(1 + x^2)^13)
/26
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(d+ex^2\right)\left(1+2x^2+x^4\right)^5 \, dx &= \int x^3\left(1+x^2\right)^{10}\left(d+ex^2\right) dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int x(1+x)^{10}(d+ex)\, dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left((-d+e)(1+x)^{10}+(d-2e)(1+x)^{11}+e(1+x)^{12}\right) dx, x, x^2\right)\\
&= -\frac{1}{22}(d-e)\left(1+x^2\right)^{11}+\frac{1}{24}(d-2e)\left(1+x^2\right)^{12}+\frac{1}{26}e\left(1+x^2\right)^{13}
\end{aligned}$$

Mathematica [B] time = 0.02, size = 151, normalized size = 3.36

$$\frac{1}{24}x^{24}(d+10e)+\frac{5}{22}x^{22}(2d+9e)+\frac{3}{4}x^{20}(3d+8e)+\frac{5}{3}x^{18}(4d+7e)+\frac{21}{8}x^{16}(5d+6e)+3x^{14}(6d+5e)+\frac{5}{2}x^{12}(7d+4e)+\frac{3}{2}x^{10}(8d+3e)+\frac{5}{8}x^8(9d+2e)+\frac{1}{6}x^6(10d+e)+\frac{dx^4}{4}+\frac{ex^{26}}{26}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^4)/4 + ((10*d + e)*x^6)/6 + (5*(9*d + 2*e)*x^8)/8 + (3*(8*d + 3*e)*x^1
0)/2 + (5*(7*d + 4*e)*x^12)/2 + 3*(6*d + 5*e)*x^14 + (21*(5*d + 6*e)*x^16)/
8 + (5*(4*d + 7*e)*x^18)/3 + (3*(3*d + 8*e)*x^20)/4 + (5*(2*d + 9*e)*x^22)/
22 + ((d + 10*e)*x^24)/24 + (e*x^26)/26
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] IntegrateAlgebraic[x^3*(d + e*x^2)*(1 + 2*x^2 + x^4)^5, x]
```

fricas [B] time = 0.48, size = 133, normalized size = 2.96

$$\frac{1}{26}x^{26}e+\frac{5}{12}x^{24}e+\frac{1}{24}x^{24}d+\frac{45}{22}x^{22}e+\frac{5}{11}x^{22}d+6x^{20}e+\frac{9}{4}x^{20}d+\frac{35}{3}x^{18}e+\frac{20}{3}x^{18}d+\frac{63}{4}x^{16}e+\frac{105}{8}x^{16}d+15x^{14}e+18x^{14}d+10x^{12}e+\frac{35}{2}x^{12}d+\frac{9}{2}x^{10}e+12x^{10}d+\frac{5}{4}x^{8}e+\frac{45}{8}x^{8}d+\frac{1}{6}x^{6}e+\frac{5}{3}x^{6}d+\frac{1}{4}x^{4}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/26*x^26*e + 5/12*x^24*e + 1/24*x^24*d + 45/22*x^22*e + 5/11*x^22*d + 6*x^
20*e + 9/4*x^20*d + 35/3*x^18*e + 20/3*x^18*d + 63/4*x^16*e + 105/8*x^16*d
+ 15*x^14*e + 18*x^14*d + 10*x^12*e + 35/2*x^12*d + 9/2*x^10*e + 12*x^10*d
+ 5/4*x^8*e + 45/8*x^8*d + 1/6*x^6*e + 5/3*x^6*d + 1/4*x^4*d
```

giac [B] time = 0.40, size = 144, normalized size = 3.20

$$\frac{1}{26}x^{26}e+\frac{1}{24}dx^{24}+\frac{5}{12}x^{24}e+\frac{5}{11}dx^{22}+\frac{45}{22}x^{22}e+\frac{9}{4}dx^{20}+6x^{20}e+\frac{20}{3}dx^{18}+\frac{35}{3}x^{18}e+\frac{105}{8}dx^{16}+\frac{63}{4}x^{16}e+18dx^{14}+15x^{14}e+\frac{35}{2}dx^{12}+10x^{12}e+12dx^{10}+\frac{9}{2}x^{10}e+\frac{45}{8}dx^{8}+\frac{5}{4}x^{8}e+\frac{5}{3}dx^{6}+\frac{1}{6}x^{6}e+\frac{1}{4}dx^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/26*x^26*e + 1/24*d*x^24 + 5/12*x^24*e + 5/11*d*x^22 + 45/22*x^22*e + 9/4*
d*x^20 + 6*x^20*e + 20/3*d*x^18 + 35/3*x^18*e + 105/8*d*x^16 + 63/4*x^16*e
+ 18*d*x^14 + 15*x^14*e + 35/2*d*x^12 + 10*x^12*e + 12*d*x^10 + 9/2*x^10*e
+ 45/8*d*x^8 + 5/4*x^8*e + 5/3*d*x^6 + 1/6*x^6*e + 1/4*d*x^4
```

maple [B] time = 0.00, size = 130, normalized size = 2.89

$$\frac{e\,x^{26}}{26}+\frac{(d+10e)\,x^{24}}{24}+\frac{(10d+45e)\,x^{22}}{22}+\frac{(45d+120e)\,x^{20}}{20}+\frac{(120d+210e)\,x^{18}}{18}+\frac{(210d+252e)\,x^{16}}{16}+\frac{(252d+210e)\,x^{14}}{14}+\frac{(210d+120e)\,x^{12}}{12}+\frac{(120d+45e)\,x^{10}}{10}+\frac{(45d+10e)\,x^{8}}{8}+\frac{(10d+e)\,x^{6}}{6}+\frac{d\,x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/26*e*x^26+1/24*(d+10*e)*x^24+1/22*(10*d+45*e)*x^22+1/20*(45*d+120*e)*x^20
+1/18*(120*d+210*e)*x^18+1/16*(210*d+252*e)*x^16+1/14*(252*d+210*e)*x^14+1/
12*(210*d+120*e)*x^12+1/10*(120*d+45*e)*x^10+1/8*(45*d+10*e)*x^8+1/6*(10*d+
e)*x^6+1/4*d*x^4
```

maxima [B] time = 0.65, size = 129, normalized size = 2.87

$$\frac{1}{26}ex^{26}+\frac{1}{24}(d+10e)x^{24}+\frac{5}{22}(2d+9e)x^{22}+\frac{3}{4}(3d+8e)x^{20}+\frac{5}{3}(4d+7e)x^{18}+\frac{21}{8}(5d+6e)x^{16}+3(6d+5e)x^{14}+\frac{5}{2}(7d+4e)x^{12}+\frac{3}{2}(8d+3e)x^{10}+\frac{5}{8}(9d+2e)x^{8}+\frac{1}{6}(10d+e)x^{6}+\frac{1}{4}dx^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/26*e*x^26 + 1/24*(d + 10*e)*x^24 + 5/22*(2*d + 9*e)*x^22 + 3/4*(3*d + 8*e
)*x^20 + 5/3*(4*d + 7*e)*x^18 + 21/8*(5*d + 6*e)*x^16 + 3*(6*d + 5*e)*x^14
+ 5/2*(7*d + 4*e)*x^12 + 3/2*(8*d + 3*e)*x^10 + 5/8*(9*d + 2*e)*x^8 + 1/6*(
10*d + e)*x^6 + 1/4*d*x^4
```

mupad [B] time = 0.08, size = 123, normalized size = 2.73

$$\frac{e\,x^{26}}{26}+\left(\frac{d}{24}+\frac{5\,e}{12}\right)\,x^{24}+\left(\frac{5\,d}{11}+\frac{45\,e}{22}\right)\,x^{22}+\left(\frac{9\,d}{4}+6\,e\right)\,x^{20}+\left(\frac{20\,d}{3}+\frac{35\,e}{3}\right)\,x^{18}+\left(\frac{105\,d}{8}+\frac{63\,e}{4}\right)\,x^{16}+\left(18\,d+15\,e\right)\,x^{14}+\left(\frac{35\,d}{2}+10\,e\right)\,x^{12}+\left(12\,d+\frac{9\,e}{2}\right)\,x^{10}+\left(\frac{45\,d}{8}+\frac{5\,e}{4}\right)\,x^8+\left(\frac{5\,d}{3}+\frac{e}{6}\right)\,x^6+\frac{d\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^6*((5*d)/3 + e/6) + x^10*(12*d + (9*e)/2) + x^20*((9*d)/4 + 6*e) + x^14*(
18*d + 15*e) + x^12*((35*d)/2 + 10*e) + x^24*(d/24 + (5*e)/12) + x^8*((45*d
)/8 + (5*e)/4) + x^18*((20*d)/3 + (35*e)/3) + x^22*((5*d)/11 + (45*e)/22) +
 x^16*((105*d)/8 + (63*e)/4) + (d*x^4)/4 + (e*x^26)/26
```

sympy [B] time = 0.10, size = 136, normalized size = 3.02

$$\frac{dx^4}{4}+\frac{ex^{26}}{26}+x^{24}\left(\frac{d}{24}+\frac{5e}{12}\right)+x^{22}\left(\frac{5d}{11}+\frac{45e}{22}\right)+x^{20}\left(\frac{9d}{4}+6e\right)+x^{18}\left(\frac{20d}{3}+\frac{35e}{3}\right)+x^{16}\left(\frac{105d}{8}+\frac{63e}{4}\right)+x^{14}\left(18d+15e\right)+x^{12}\left(\frac{35d}{2}+10e\right)+x^{10}\left(12d+\frac{9e}{2}\right)+x^8\left(\frac{45d}{8}+\frac{5e}{4}\right)+x^6\left(\frac{5d}{3}+\frac{e}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**4/4 + e*x**26/26 + x**24*(d/24 + 5*e/12) + x**22*(5*d/11 + 45*e/22) +
x**20*(9*d/4 + 6*e) + x**18*(20*d/3 + 35*e/3) + x**16*(105*d/8 + 63*e/4) +
x**14*(18*d + 15*e) + x**12*(35*d/2 + 10*e) + x**10*(12*d + 9*e/2) + x**8*(
45*d/8 + 5*e/4) + x**6*(5*d/3 + e/6)
```

3.39 $\int x^2 \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=153

$$\frac{1}{23}x^{23}(d+10e)+\frac{5}{21}x^{21}(2d+9e)+\frac{15}{19}x^{19}(3d+8e)+\frac{30}{17}x^{17}(4d+7e)+\frac{14}{5}x^{15}(5d+6e)+\frac{42}{13}x^{13}(6d+5e)+\frac{30}{11}x^{11}(7d+4e)+$$

Rubi [A] time = 0.08, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = $\{28, 448\}$

$$\frac{1}{23}x^{23}(d+10e)+\frac{5}{21}x^{21}(2d+9e)+\frac{15}{19}x^{19}(3d+8e)+\frac{30}{17}x^{17}(4d+7e)+\frac{14}{5}x^{15}(5d+6e)+\frac{42}{13}x^{13}(6d+5e)+\frac{30}{11}x^{11}(7d+4e)+\frac{5}{3}x^9(8d+3e)+\frac{5}{7}x^7(9d+2e)+\frac{1}{5}x^5(10d+e)+\frac{dx^3}{3}+\frac{ex^{25}}{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^3)/3 + ((10*d + e)*x^5)/5 + (5*(9*d + 2*e)*x^7)/7 + (5*(8*d + 3*e)*x^9
)/3 + (30*(7*d + 4*e)*x^11)/11 + (42*(6*d + 5*e)*x^13)/13 + (14*(5*d + 6*e)
*x^15)/5 + (30*(4*d + 7*e)*x^17)/17 + (15*(3*d + 8*e)*x^19)/19 + (5*(2*d +
9*e)*x^21)/21 + ((d + 10*e)*x^23)/23 + (e*x^25)/25
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \left(d+ex^2\right)\left(1+2x^2+x^4\right)^5 \, dx &= \int x^2 \left(1+x^2\right)^{10} \left(d+ex^2\right) \, dx \\
&= \int \left(dx^2+(10d+e)x^4+5(9d+2e)x^6+15(8d+3e)x^8+30(7d+4e)x^{10}+\right. \\
&= \frac{dx^3}{3}+\frac{1}{5}(10d+e)x^5+\frac{5}{7}(9d+2e)x^7+\frac{5}{3}(8d+3e)x^9+\frac{30}{11}(7d+4e)x^{11}+
\end{aligned}$$

Mathematica [A] time = 0.02, size = 153, normalized size = 1.00

$$\frac{1}{23}x^{23}(d+10e)+\frac{5}{21}x^{21}(2d+9e)+\frac{15}{19}x^{19}(3d+8e)+\frac{30}{17}x^{17}(4d+7e)+\frac{14}{5}x^{15}(5d+6e)+\frac{42}{13}x^{13}(6d+5e)+\frac{30}{11}x^{11}(7d+4e)+\frac{5}{3}x^9(8d+3e)+\frac{5}{7}x^7(9d+2e)+\frac{1}{5}x^5(10d+e)+\frac{dx^3}{3}+\frac{ex^{25}}{25}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^3)/3 + ((10*d + e)*x^5)/5 + (5*(9*d + 2*e)*x^7)/7 + (5*(8*d + 3*e)*x^9
)/3 + (30*(7*d + 4*e)*x^11)/11 + (42*(6*d + 5*e)*x^13)/13 + (14*(5*d + 6*e)
*x^15)/5 + (30*(4*d + 7*e)*x^17)/17 + (15*(3*d + 8*e)*x^19)/19 + (5*(2*d +
9*e)*x^21)/21 + ((d + 10*e)*x^23)/23 + (e*x^25)/25
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] IntegrateAlgebraic[x^2*(d + e*x^2)*(1 + 2*x^2 + x^4)^5, x]
```

fricas [A] time = 0.81, size = 133, normalized size = 0.87

$$\frac{1}{25}x^{25}e + \frac{10}{23}x^{23}e + \frac{1}{23}x^{23}d + \frac{15}{7}x^{21}e + \frac{10}{21}x^{21}d + \frac{120}{19}x^{19}e + \frac{45}{19}x^{19}d + \frac{210}{17}x^{17}e + \frac{120}{17}x^{17}d + \frac{84}{5}x^{15}e + 14x^{15}d + \frac{210}{13}x^{13}e + \frac{252}{13}x^{13}d + \frac{120}{11}x^{11}e + \frac{210}{11}x^{11}d + 5x^9e + \frac{40}{3}x^9d + \frac{10}{7}x^7e + \frac{45}{7}x^7d + \frac{1}{5}x^5e + 2x^5d + \frac{1}{3}x^3d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/25*x^25*e + 10/23*x^23*e + 1/23*x^23*d + 15/7*x^21*e + 10/21*x^21*d + 120
/19*x^19*e + 45/19*x^19*d + 210/17*x^17*e + 120/17*x^17*d + 84/5*x^15*e + 1
4*x^15*d + 210/13*x^13*e + 252/13*x^13*d + 120/11*x^11*e + 210/11*x^11*d +
5*x^9*e + 40/3*x^9*d + 10/7*x^7*e + 45/7*x^7*d + 1/5*x^5*e + 2*x^5*d + 1/3*
x^3*d
```

giac [A] time = 0.29, size = 144, normalized size = 0.94

$$\frac{1}{25}x^{25}e + \frac{1}{23}dx^{23} + \frac{10}{23}x^{23}e + \frac{10}{21}dx^{21} + \frac{15}{7}x^{21}e + \frac{45}{19}dx^{19} + \frac{120}{19}x^{19}e + \frac{120}{17}dx^{17} + \frac{210}{17}x^{17}e + 14dx^{15} + \frac{84}{5}x^{15}e + \frac{252}{13}dx^{13} + \frac{210}{13}x^{13}e + \frac{210}{11}dx^{11} + \frac{120}{11}x^{11}e + \frac{40}{3}dx^9 + 5x^9e + \frac{45}{7}dx^7 + \frac{10}{7}x^7e + 2dx^5 + \frac{1}{5}x^5e + \frac{1}{3}dx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/25*x^25*e + 1/23*d*x^23 + 10/23*x^23*e + 10/21*d*x^21 + 15/7*x^21*e + 45/
19*d*x^19 + 120/19*x^19*e + 120/17*d*x^17 + 210/17*x^17*e + 14*d*x^15 + 84/
5*x^15*e + 252/13*d*x^13 + 210/13*x^13*e + 210/11*d*x^11 + 120/11*x^11*e +
40/3*d*x^9 + 5*x^9*e + 45/7*d*x^7 + 10/7*x^7*e + 2*d*x^5 + 1/5*x^5*e + 1/3*
d*x^3
```

maple [A] time = 0.00, size = 130, normalized size = 0.85

$$\frac{e x^{25}}{25} + \frac{(d+10e)x^{23}}{23} + \frac{(10d+45e)x^{21}}{21} + \frac{(45d+120e)x^{19}}{19} + \frac{(120d+210e)x^{17}}{17} + \frac{(210d+252e)x^{15}}{15} + \frac{(252d+210e)x^{13}}{13} + \frac{(210d+120e)x^{11}}{11} + \frac{(120d+45e)x^9}{9} + \frac{(45d+10e)x^7}{7} + \frac{(10d+e)x^5}{5} + \frac{dx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/25*e*x^25+1/23*(d+10*e)*x^23+1/21*(10*d+45*e)*x^21+1/19*(45*d+120*e)*x^19
+1/17*(120*d+210*e)*x^17+1/15*(210*d+252*e)*x^15+1/13*(252*d+210*e)*x^13+1/
11*(210*d+120*e)*x^11+1/9*(120*d+45*e)*x^9+1/7*(45*d+10*e)*x^7+1/5*(10*d+e)
*x^5+1/3*d*x^3
```

maxima [A] time = 0.50, size = 129, normalized size = 0.84

$$\frac{1}{25}ex^{25} + \frac{1}{23}(d+10e)x^{23} + \frac{5}{21}(2d+9e)x^{21} + \frac{15}{19}(3d+8e)x^{19} + \frac{30}{17}(4d+7e)x^{17} + \frac{14}{5}(5d+6e)x^{15} + \frac{42}{13}(6d+5e)x^{13} + \frac{30}{11}(7d+4e)x^{11} + \frac{5}{3}(8d+3e)x^9 + \frac{5}{7}(9d+2e)x^7 + \frac{1}{5}(10d+e)x^5 + \frac{1}{3}dx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/25*e*x^25 + 1/23*(d + 10*e)*x^23 + 5/21*(2*d + 9*e)*x^21 + 15/19*(3*d + 8
*e)*x^19 + 30/17*(4*d + 7*e)*x^17 + 14/5*(5*d + 6*e)*x^15 + 42/13*(6*d + 5*
e)*x^13 + 30/11*(7*d + 4*e)*x^11 + 5/3*(8*d + 3*e)*x^9 + 5/7*(9*d + 2*e)*x^
7 + 1/5*(10*d + e)*x^5 + 1/3*d*x^3
```

mupad [B] time = 0.08, size = 123, normalized size = 0.80

$$\frac{e\,x^{25}}{25}+\left(\frac{d}{23}+\frac{10\,e}{23}\right)\,x^{23}+\left(\frac{10\,d}{21}+\frac{15\,e}{7}\right)\,x^{21}+\left(\frac{45\,d}{19}+\frac{120\,e}{19}\right)\,x^{19}+\left(\frac{120\,d}{17}+\frac{210\,e}{17}\right)\,x^{17}+\left(14\,d+\frac{84\,e}{5}\right)\,x^{15}+\left(\frac{252\,d}{13}+\frac{210\,e}{13}\right)\,x^{13}+\left(\frac{210\,d}{11}+\frac{120\,e}{11}\right)\,x^{11}+\left(\frac{40\,d}{3}+5\,e\right)\,x^9+\left(\frac{45\,d}{7}+\frac{10\,e}{7}\right)\,x^7+\left(2\,d+\frac{e}{5}\right)\,x^5+\frac{d\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^5*(2*d + e/5) + x^9*((40*d)/3 + 5*e) + x^21*((10*d)/21 + (15*e)/7) + x^7*
((45*d)/7 + (10*e)/7) + x^23*(d/23 + (10*e)/23) + x^15*(14*d + (84*e)/5) +
x^19*((45*d)/19 + (120*e)/19) + x^11*((210*d)/11 + (120*e)/11) + x^17*((120
*d)/17 + (210*e)/17) + x^13*((252*d)/13 + (210*e)/13) + (d*x^3)/3 + (e*x^25
)/25
```

sympy [A] time = 0.10, size = 139, normalized size = 0.91

$$\frac{dx^{3}}{3}+\frac{ex^{25}}{25}+x^{23}\left(\frac{d}{23}+\frac{10e}{23}\right)+x^{21}\left(\frac{10d}{21}+\frac{15e}{7}\right)+x^{19}\left(\frac{45d}{19}+\frac{120e}{19}\right)+x^{17}\left(\frac{120d}{17}+\frac{210e}{17}\right)+x^{15}\left(14d+\frac{84e}{5}\right)+x^{13}\left(\frac{252d}{13}+\frac{210e}{13}\right)+x^{11}\left(\frac{210d}{11}+\frac{120e}{11}\right)+x^{9}\left(\frac{40d}{3}+5e\right)+x^{7}\left(\frac{45d}{7}+\frac{10e}{7}\right)+x^{5}\left(2d+\frac{e}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**3/3 + e*x**25/25 + x**23*(d/23 + 10*e/23) + x**21*(10*d/21 + 15*e/7) +
 x**19*(45*d/19 + 120*e/19) + x**17*(120*d/17 + 210*e/17) + x**15*(14*d + 8
4*e/5) + x**13*(252*d/13 + 210*e/13) + x**11*(210*d/11 + 120*e/11) + x**9*(
40*d/3 + 5*e) + x**7*(45*d/7 + 10*e/7) + x**5*(2*d + e/5)
```

3.40 $\int x\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx$

Optimal. Leaf size=29

$$\frac{1}{22}\left(x^2+1\right)^{11}(d-e)+\frac{1}{24}e\left(x^2+1\right)^{12}$$

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 444, 43}

$$\frac{1}{22}\left(x^2+1\right)^{11}(d-e)+\frac{1}{24}e\left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] ((d - e)*(1 + x^2)^11)/22 + (e*(1 + x^2)^12)/24
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx &= \int x\left(1+x^2\right)^{10}\left(d+ex^2\right)\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int(1+x)^{10}(d+ex)\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left((d-e)(1+x)^{10}+e(1+x)^{11}\right)\,dx,x,x^2\right)\\ &= \frac{1}{22}(d-e)\left(1+x^2\right)^{11}+\frac{1}{24}e\left(1+x^2\right)^{12}\end{aligned}$$

Mathematica [B] time = 0.01, size = 149, normalized size = 5.14

$$\frac{1}{22}x^{22}(d+10e)+\frac{1}{4}x^{20}(2d+9e)+\frac{5}{6}x^{18}(3d+8e)+\frac{15}{8}x^{16}(4d+7e)+3x^{14}(5d+6e)+\frac{7}{2}x^{12}(6d+5e)+3x^{10}(7d+4e)+\frac{15}{8}x^8(8d+3e)+\frac{5}{6}x^6(9d+2e)+\frac{1}{4}x^4(10d+e)+\frac{dx^2}{2}+\frac{ex^{24}}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^2)/2 + ((10*d + e)*x^4)/4 + (5*(9*d + 2*e)*x^6)/6 + (15*(8*d + 3*e)*x^
8)/8 + 3*(7*d + 4*e)*x^10 + (7*(6*d + 5*e)*x^12)/2 + 3*(5*d + 6*e)*x^14 + (
15*(4*d + 7*e)*x^16)/8 + (5*(3*d + 8*e)*x^18)/6 + ((2*d + 9*e)*x^20)/4 + ((
d + 10*e)*x^22)/22 + (e*x^24)/24
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] IntegrateAlgebraic[x*(d + e*x^2)*(1 + 2*x^2 + x^4)^5, x]
```

fricas [B] time = 0.84, size = 133, normalized size = 4.59

$$\frac{1}{24}x^{24}e+\frac{5}{11}x^{22}e+\frac{1}{22}x^{22}d+\frac{9}{4}x^{20}e+\frac{1}{2}x^{20}d+\frac{20}{3}x^{18}e+\frac{5}{2}x^{18}d+\frac{105}{8}x^{16}e+\frac{15}{2}x^{16}d+18x^{14}e+15x^{14}d+\frac{35}{2}x^{12}e+21x^{12}d+12x^{10}e+21x^{10}d+\frac{45}{8}x^{8}e+15x^{8}d+\frac{5}{3}x^{6}e+\frac{15}{2}x^{6}d+\frac{1}{4}x^{4}e+\frac{5}{2}x^{4}d+\frac{1}{2}x^{2}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/24*x^24*e + 5/11*x^22*e + 1/22*x^22*d + 9/4*x^20*e + 1/2*x^20*d + 20/3*x^
18*e + 5/2*x^18*d + 105/8*x^16*e + 15/2*x^16*d + 18*x^14*e + 15*x^14*d + 35
/2*x^12*e + 21*x^12*d + 12*x^10*e + 21*x^10*d + 45/8*x^8*e + 15*x^8*d + 5/3
*x^6*e + 15/2*x^6*d + 1/4*x^4*e + 5/2*x^4*d + 1/2*x^2*d
```

giac [B] time = 0.31, size = 144, normalized size = 4.97

$$\frac{1}{24}x^{24}e+\frac{1}{22}dx^{22}+\frac{5}{11}x^{22}e+\frac{1}{2}dx^{20}+\frac{9}{4}x^{20}e+\frac{5}{2}dx^{18}+\frac{20}{3}x^{18}e+\frac{15}{2}dx^{16}+\frac{105}{8}x^{16}e+15\,dx^{14}+18\,x^{14}e+21\,dx^{12}+\frac{35}{2}x^{12}e+21\,dx^{10}+12\,x^{10}e+15\,dx^{8}+\frac{45}{8}x^{8}e+\frac{15}{2}dx^{6}+\frac{5}{3}x^{6}e+\frac{5}{2}dx^{4}+\frac{1}{4}x^{4}e+\frac{1}{2}dx^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/24*x^24*e + 1/22*d*x^22 + 5/11*x^22*e + 1/2*d*x^20 + 9/4*x^20*e + 5/2*d*x
^18 + 20/3*x^18*e + 15/2*d*x^16 + 105/8*x^16*e + 15*d*x^14 + 18*x^14*e + 21
*d*x^12 + 35/2*x^12*e + 21*d*x^10 + 12*x^10*e + 15*d*x^8 + 45/8*x^8*e + 15/
2*d*x^6 + 5/3*x^6*e + 5/2*d*x^4 + 1/4*x^4*e + 1/2*d*x^2
```

maple [B] time = 0.00, size = 130, normalized size = 4.48

$$\frac{e\,x^{24}}{24}+\frac{(d+10e)\,x^{22}}{22}+\frac{(10d+45e)\,x^{20}}{20}+\frac{(45d+120e)\,x^{18}}{18}+\frac{(120d+210e)\,x^{16}}{16}+\frac{(210d+252e)\,x^{14}}{14}+\frac{(252d+210e)\,x^{12}}{12}+\frac{(210d+120e)\,x^{10}}{10}+\frac{(120d+45e)\,x^{8}}{8}+\frac{(45d+10e)\,x^{6}}{6}+\frac{(10d+e)\,x^{4}}{4}+\frac{d\,x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/24*e*x^24+1/22*(d+10*e)*x^22+1/20*(10*d+45*e)*x^20+1/18*(45*d+120*e)*x^18
+1/16*(120*d+210*e)*x^16+1/14*(210*d+252*e)*x^14+1/12*(252*d+210*e)*x^12+1/
10*(210*d+120*e)*x^10+1/8*(120*d+45*e)*x^8+1/6*(45*d+10*e)*x^6+1/4*(10*d+e)
*x^4+1/2*d*x^2
```

maxima [B] time = 0.65, size = 129, normalized size = 4.45

$$\frac{1}{24}\,ex^{24}+\frac{1}{22}\,(d+10\,e)x^{22}+\frac{1}{4}\,(2\,d+9\,e)x^{20}+\frac{5}{6}\,(3\,d+8\,e)x^{18}+\frac{15}{8}\,(4\,d+7\,e)x^{16}+3\,(5\,d+6\,e)x^{14}+\frac{7}{2}\,(6\,d+5\,e)x^{12}+3\,(7\,d+4\,e)x^{10}+\frac{15}{8}\,(8\,d+3\,e)x^{8}+\frac{5}{6}\,(9\,d+2\,e)x^{6}+\frac{1}{4}\,(10\,d+e)x^{4}+\frac{1}{2}\,dx^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/24*e*x^24 + 1/22*(d + 10*e)*x^22 + 1/4*(2*d + 9*e)*x^20 + 5/6*(3*d + 8*e)
*x^18 + 15/8*(4*d + 7*e)*x^16 + 3*(5*d + 6*e)*x^14 + 7/2*(6*d + 5*e)*x^12 +
```

```
 3*(7*d + 4*e)*x^10 + 15/8*(8*d + 3*e)*x^8 + 5/6*(9*d + 2*e)*x^6 + 1/4*(10*
d + e)*x^4 + 1/2*d*x^2
```

mupad [B] time = 0.08, size = 123, normalized size = 4.24

$$\frac{e\,x^{24}}{24}+\left(\frac{d}{22}+\frac{5\,e}{11}\right)\,x^{22}+\left(\frac{d}{2}+\frac{9\,e}{4}\right)\,x^{20}+\left(\frac{5\,d}{2}+\frac{20\,e}{3}\right)\,x^{18}+\left(\frac{15\,d}{2}+\frac{105\,e}{8}\right)\,x^{16}+\left(15\,d+18\,e\right)\,x^{14}+\left(21\,d+\frac{35\,e}{2}\right)\,x^{12}+\left(21\,d+12\,e\right)\,x^{10}+\left(15\,d+\frac{45\,e}{8}\right)\,x^8+\left(\frac{15\,d}{2}+\frac{5\,e}{3}\right)\,x^6+\left(\frac{5\,d}{2}+\frac{e}{4}\right)\,x^4+\frac{d\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^4*((5*d)/2 + e/4) + x^6*((15*d)/2 + (5*e)/3) + x^20*(d/2 + (9*e)/4) + x^1
0*(21*d + 12*e) + x^14*(15*d + 18*e) + x^18*((5*d)/2 + (20*e)/3) + x^22*(d/
22 + (5*e)/11) + x^12*(21*d + (35*e)/2) + x^8*(15*d + (45*e)/8) + x^16*((15
*d)/2 + (105*e)/8) + (d*x^2)/2 + (e*x^24)/24
```

sympy [B] time = 0.10, size = 133, normalized size = 4.59

$$\frac{dx^2}{2}+\frac{ex^{24}}{24}+x^{22}\left(\frac{d}{22}+\frac{5e}{11}\right)+x^{20}\left(\frac{d}{2}+\frac{9e}{4}\right)+x^{18}\left(\frac{5d}{2}+\frac{20e}{3}\right)+x^{16}\left(\frac{15d}{2}+\frac{105e}{8}\right)+x^{14}\left(15d+18e\right)+x^{12}\left(21d+\frac{35e}{2}\right)+x^{10}\left(21d+12e\right)+x^8\left(15d+\frac{45e}{8}\right)+x^6\left(\frac{15d}{2}+\frac{5e}{3}\right)+x^4\left(\frac{5d}{2}+\frac{e}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**2/2 + e*x**24/24 + x**22*(d/22 + 5*e/11) + x**20*(d/2 + 9*e/4) + x**18
*(5*d/2 + 20*e/3) + x**16*(15*d/2 + 105*e/8) + x**14*(15*d + 18*e) + x**12*
(21*d + 35*e/2) + x**10*(21*d + 12*e) + x**8*(15*d + 45*e/8) + x**6*(15*d/2
 + 5*e/3) + x**4*(5*d/2 + e/4)
```

3.41 $\int \left(d+e x^2\right) \left(1+2 x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=143

$$\frac{1}{21}x^{21}(d+10e)+\frac{5}{19}x^{19}(2d+9e)+\frac{15}{17}x^{17}(3d+8e)+2x^{15}(4d+7e)+\frac{42}{13}x^{13}(5d+6e)+\frac{42}{11}x^{11}(6d+5e)+\frac{10}{3}x^9(7d+4e)+\frac{15}{7}$$

Rubi [A] time = 0.07, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {28, 373}

$$\frac{1}{21}x^{21}(d+10e)+\frac{5}{19}x^{19}(2d+9e)+\frac{15}{17}x^{17}(3d+8e)+2x^{15}(4d+7e)+\frac{42}{13}x^{13}(5d+6e)+\frac{42}{11}x^{11}(6d+5e)+\frac{10}{3}x^9(7d+4e)+\frac{15}{7}x^7(8d+3e)+x^5(9d+2e)+\frac{1}{3}x^3(10d+e)+dx+\frac{ex^{23}}{23}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] d*x + ((10*d + e)*x^3)/3 + (9*d + 2*e)*x^5 + (15*(8*d + 3*e)*x^7)/7 + (10*(
7*d + 4*e)*x^9)/3 + (42*(6*d + 5*e)*x^11)/11 + (42*(5*d + 6*e)*x^13)/13 + 2
*(4*d + 7*e)*x^15 + (15*(3*d + 8*e)*x^17)/17 + (5*(2*d + 9*e)*x^19)/19 + ((
d + 10*e)*x^21)/21 + (e*x^23)/23
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)\left(1+2x^2+x^4\right)^5 \, dx &= \int \left(1+x^2\right)^{10}\left(d+ex^2\right) dx\\ &= \int \left(d+(10d+e)x^2+5(9d+2e)x^4+15(8d+3e)x^6+30(7d+4e)x^8+42(6\right.\\ &= dx+\frac{1}{3}(10d+e)x^3+(9d+2e)x^5+\frac{15}{7}(8d+3e)x^7+\frac{10}{3}(7d+4e)x^9+\frac{42}{11}(6d\end{aligned}$$

Mathematica [A] time = 0.02, size = 143, normalized size = 1.00

$$\frac{1}{21}x^{21}(d+10e)+\frac{5}{19}x^{19}(2d+9e)+\frac{15}{17}x^{17}(3d+8e)+2x^{15}(4d+7e)+\frac{42}{13}x^{13}(5d+6e)+\frac{42}{11}x^{11}(6d+5e)+\frac{10}{3}x^9(7d+4e)+\frac{15}{7}x^7(8d+3e)+x^5(9d+2e)+\frac{1}{3}x^3(10d+e)+dx+\frac{ex^{23}}{23}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] d*x + ((10*d + e)*x^3)/3 + (9*d + 2*e)*x^5 + (15*(8*d + 3*e)*x^7)/7 + (10*(
7*d + 4*e)*x^9)/3 + (42*(6*d + 5*e)*x^11)/11 + (42*(5*d + 6*e)*x^13)/13 + 2
*(4*d + 7*e)*x^15 + (15*(3*d + 8*e)*x^17)/17 + (5*(2*d + 9*e)*x^19)/19 + ((
d + 10*e)*x^21)/21 + (e*x^23)/23
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d+ex^2\right)\left(1+2x^2+x^4\right)^5 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^2)*(1 + 2*x^2 + x^4)^5, x]
```

fricas [A] time = 0.56, size = 130, normalized size = 0.91

$$\frac{1}{23}x^{23}e+\frac{10}{21}x^{21}e+\frac{1}{21}x^{21}d+\frac{45}{19}x^{19}e+\frac{10}{19}x^{19}d+\frac{120}{17}x^{17}e+\frac{45}{17}x^{17}d+14x^{15}e+8x^{15}d+\frac{252}{13}x^{13}e+\frac{210}{13}x^{13}d+\frac{210}{11}x^{11}e+\frac{252}{11}x^{11}d+\frac{40}{3}x^{9}e+\frac{70}{3}x^{9}d+\frac{45}{7}x^{7}e+\frac{120}{7}x^{7}d+2x^{5}e+9x^{5}d+\frac{1}{3}x^{3}e+\frac{10}{3}x^{3}d+xd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/23*x^23*e + 10/21*x^21*e + 1/21*x^21*d + 45/19*x^19*e + 10/19*x^19*d + 12
0/17*x^17*e + 45/17*x^17*d + 14*x^15*e + 8*x^15*d + 252/13*x^13*e + 210/13*
x^13*d + 210/11*x^11*e + 252/11*x^11*d + 40/3*x^9*e + 70/3*x^9*d + 45/7*x^7
*e + 120/7*x^7*d + 2*x^5*e + 9*x^5*d + 1/3*x^3*e + 10/3*x^3*d + x*d
```

giac [A] time = 0.36, size = 141, normalized size = 0.99

$$\frac{1}{23}x^{23}e+\frac{1}{21}dx^{21}+\frac{10}{21}x^{21}e+\frac{10}{19}dx^{19}+\frac{45}{19}x^{19}e+\frac{45}{17}dx^{17}+\frac{120}{17}x^{17}e+8dx^{15}+14x^{15}e+\frac{210}{13}dx^{13}+\frac{252}{13}x^{13}e+\frac{252}{11}dx^{11}+\frac{210}{11}x^{11}e+\frac{70}{3}dx^{9}+\frac{40}{3}x^{9}e+\frac{120}{7}dx^{7}+\frac{45}{7}x^{7}e+9dx^{5}+2x^{5}e+\frac{10}{3}dx^{3}+\frac{1}{3}x^{3}e+dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/23*x^23*e + 1/21*d*x^21 + 10/21*x^21*e + 10/19*d*x^19 + 45/19*x^19*e + 45
/17*d*x^17 + 120/17*x^17*e + 8*d*x^15 + 14*x^15*e + 210/13*d*x^13 + 252/13*
x^13*e + 252/11*d*x^11 + 210/11*x^11*e + 70/3*d*x^9 + 40/3*x^9*e + 120/7*d*
x^7 + 45/7*x^7*e + 9*d*x^5 + 2*x^5*e + 10/3*d*x^3 + 1/3*x^3*e + d*x
```

maple [A] time = 0.00, size = 127, normalized size = 0.89

$$\frac{e\,x^{23}}{23}+\frac{(d+10e)\,x^{21}}{21}+\frac{(10d+45e)\,x^{19}}{19}+\frac{(45d+120e)\,x^{17}}{17}+\frac{(120d+210e)\,x^{15}}{15}+\frac{(210d+252e)\,x^{13}}{13}+\frac{(252d+210e)\,x^{11}}{11}+\frac{(210d+120e)\,x^{9}}{9}+\frac{(120d+45e)\,x^{7}}{7}+\frac{(45d+10e)\,x^{5}}{5}+\frac{(10d+e)\,x^{3}}{3}+dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/23*e*x^23+1/21*(d+10*e)*x^21+1/19*(10*d+45*e)*x^19+1/17*(45*d+120*e)*x^17
+1/15*(120*d+210*e)*x^15+1/13*(210*d+252*e)*x^13+1/11*(252*d+210*e)*x^11+1/
9*(210*d+120*e)*x^9+1/7*(120*d+45*e)*x^7+1/5*(45*d+10*e)*x^5+1/3*(10*d+e)*x
^3+d*x
```

maxima [A] time = 0.58, size = 125, normalized size = 0.87

$$\frac{1}{23}ex^{23}+\frac{1}{21}(d+10e)x^{21}+\frac{5}{19}(2d+9e)x^{19}+\frac{15}{17}(3d+8e)x^{17}+2(4d+7e)x^{15}+\frac{42}{13}(5d+6e)x^{13}+\frac{42}{11}(6d+5e)x^{11}+\frac{10}{3}(7d+4e)x^{9}+\frac{15}{7}(8d+3e)x^{7}+(9d+2e)x^{5}+\frac{1}{3}(10d+e)x^{3}+dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/23*e*x^23 + 1/21*(d + 10*e)*x^21 + 5/19*(2*d + 9*e)*x^19 + 15/17*(3*d + 8
*e)*x^17 + 2*(4*d + 7*e)*x^15 + 42/13*(5*d + 6*e)*x^13 + 42/11*(6*d + 5*e)*
x^11 + 10/3*(7*d + 4*e)*x^9 + 15/7*(8*d + 3*e)*x^7 + (9*d + 2*e)*x^5 + 1/3*
(10*d + e)*x^3 + d*x
```

mupad [B] time = 0.08, size = 120, normalized size = 0.84

$$\frac{e\,x^{23}}{23}+\left(\frac{d}{21}+\frac{10\,e}{21}\right)x^{21}+\left(\frac{10\,d}{19}+\frac{45\,e}{19}\right)x^{19}+\left(\frac{45\,d}{17}+\frac{120\,e}{17}\right)x^{17}+(8\,d+14\,e)\,x^{15}+\left(\frac{210\,d}{13}+\frac{252\,e}{13}\right)x^{13}+\left(\frac{252\,d}{11}+\frac{210\,e}{11}\right)x^{11}+\left(\frac{70\,d}{3}+\frac{40\,e}{3}\right)x^{9}+\left(\frac{120\,d}{7}+\frac{45\,e}{7}\right)x^{7}+(9\,d+2\,e)\,x^{5}+\left(\frac{10\,d}{3}+\frac{e}{3}\right)x^{3}+d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^5*(9*d + 2*e) + x^3*((10*d)/3 + e/3) + x^15*(8*d + 14*e) + x^21*(d/21 + (
10*e)/21) + x^19*((10*d)/19 + (45*e)/19) + x^9*((70*d)/3 + (40*e)/3) + x^7*
((120*d)/7 + (45*e)/7) + x^17*((45*d)/17 + (120*e)/17) + x^11*((252*d)/11 +
 (210*e)/11) + x^13*((210*d)/13 + (252*e)/13) + d*x + (e*x^23)/23
```

sympy [A] time = 0.10, size = 134, normalized size = 0.94

$$dx + \frac{ex^{23}}{23} + x^{21}\left(\frac{d}{21} + \frac{10e}{21}\right) + x^{19}\left(\frac{10d}{19} + \frac{45e}{19}\right) + x^{17}\left(\frac{45d}{17} + \frac{120e}{17}\right) + x^{15}(8d + 14e) + x^{13}\left(\frac{210d}{13} + \frac{252e}{13}\right) + x^{11}\left(\frac{252d}{11} + \frac{210e}{11}\right) + x^{9}\left(\frac{70d}{3} + \frac{40e}{3}\right) + x^{7}\left(\frac{120d}{7} + \frac{45e}{7}\right) + x^{5}(9d + 2e) + x^{3}\left(\frac{10d}{3} + \frac{e}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x + e*x**23/23 + x**21*(d/21 + 10*e/21) + x**19*(10*d/19 + 45*e/19) + x**
17*(45*d/17 + 120*e/17) + x**15*(8*d + 14*e) + x**13*(210*d/13 + 252*e/13)
+ x**11*(252*d/11 + 210*e/11) + x**9*(70*d/3 + 40*e/3) + x**7*(120*d/7 + 45
*e/7) + x**5*(9*d + 2*e) + x**3*(10*d/3 + e/3)
```

3.42 $\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x} \, dx$

Optimal. Leaf size=93

$$\frac{dx^{20}}{20}+\frac{5dx^{18}}{9}+\frac{45dx^{16}}{16}+\frac{60dx^{14}}{7}+\frac{35dx^{12}}{2}+\frac{126dx^{10}}{5}+\frac{105dx^8}{4}+20dx^6+\frac{45dx^4}{4}+5dx^2+d\log(x)+\frac{1}{22}e\left(x^2+1\right)^{11}$$

Rubi [A] time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {28, 446, 80, 43}

$$\frac{dx^{20}}{20}+\frac{5dx^{18}}{9}+\frac{45dx^{16}}{16}+\frac{60dx^{14}}{7}+\frac{35dx^{12}}{2}+\frac{126dx^{10}}{5}+\frac{105dx^8}{4}+20dx^6+\frac{45dx^4}{4}+5dx^2+d\log(x)+\frac{1}{22}e\left(x^2+1\right)^{11}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] 5*d*x^2 + (45*d*x^4)/4 + 20*d*x^6 + (105*d*x^8)/4 + (126*d*x^10)/5 + (35*d*
x^12)/2 + (60*d*x^14)/7 + (45*d*x^16)/16 + (5*d*x^18)/9 + (d*x^20)/20 + (e*
(1 + x^2)^11)/22 + d*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5}{x}\,dx &= \int \frac{\left(1+x^2\right)^{10}\left(d+ex^2\right)}{x}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{(1+x)^{10}(d+ex)}{x}\,dx, x, x^2\right) \\
&= \frac{1}{22}e\left(1+x^2\right)^{11} + \frac{1}{2}d\,\text{Subst}\left(\int \frac{(1+x)^{10}}{x}\,dx, x, x^2\right) \\
&= \frac{1}{22}e\left(1+x^2\right)^{11} + \frac{1}{2}d\,\text{Subst}\left(\int \left(10+\frac{1}{x}+45x+120x^2+210x^3+252x^4+2\right.\right. \\
&= 5dx^2 + \frac{45dx^4}{4} + 20dx^6 + \frac{105dx^8}{4} + \frac{126dx^{10}}{5} + \frac{35dx^{12}}{2} + \frac{60dx^{14}}{7} + \frac{45dx^{16}}{16}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 149, normalized size = 1.60

$$\frac{1}{20}x^{20}(d+10e)+\frac{5}{18}x^{18}(2d+9e)+\frac{15}{16}x^{16}(3d+8e)+\frac{15}{7}x^{14}(4d+7e)+\frac{7}{2}x^{12}(5d+6e)+\frac{21}{5}x^{10}(6d+5e)+\frac{15}{4}x^8(7d+4e)+\frac{5}{2}x^6(8d+3e)+\frac{5}{4}x^4(9d+2e)+\frac{1}{2}x^2(10d+e)+d\log(x)+\frac{ex^{22}}{22}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] ((10*d + e)*x^2)/2 + (5*(9*d + 2*e)*x^4)/4 + (5*(8*d + 3*e)*x^6)/2 + (15*(7
*d + 4*e)*x^8)/4 + (21*(6*d + 5*e)*x^10)/5 + (7*(5*d + 6*e)*x^12)/2 + (15*(
4*d + 7*e)*x^14)/7 + (15*(3*d + 8*e)*x^16)/16 + (5*(2*d + 9*e)*x^18)/18 + (
(d + 10*e)*x^20)/20 + (e*x^22)/22 + d*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] IntegrateAlgebraic[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x, x]
```

fricas [A] time = 0.76, size = 127, normalized size = 1.37

$$\frac{1}{22}ex^{22}+\frac{1}{20}(d+10e)x^{20}+\frac{5}{18}(2d+9e)x^{18}+\frac{15}{16}(3d+8e)x^{16}+\frac{15}{7}(4d+7e)x^{14}+\frac{7}{2}(5d+6e)x^{12}+\frac{21}{5}(6d+5e)x^{10}+\frac{15}{4}(7d+4e)x^8+\frac{5}{2}(8d+3e)x^6+\frac{5}{4}(9d+2e)x^4+\frac{1}{2}(10d+e)x^2+d\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x,x, algorithm="fricas")
```

```
[Out] 1/22*e*x^22 + 1/20*(d + 10*e)*x^20 + 5/18*(2*d + 9*e)*x^18 + 15/16*(3*d + 8
*e)*x^16 + 15/7*(4*d + 7*e)*x^14 + 7/2*(5*d + 6*e)*x^12 + 21/5*(6*d + 5*e)*
x^10 + 15/4*(7*d + 4*e)*x^8 + 5/2*(8*d + 3*e)*x^6 + 5/4*(9*d + 2*e)*x^4 + 1
/2*(10*d + e)*x^2 + d*log(x)
```

giac [A] time = 0.27, size = 145, normalized size = 1.56

$$\frac{1}{22}x^{22}e+\frac{1}{20}dx^{20}+\frac{1}{2}x^{20}e+\frac{5}{9}dx^{18}+\frac{5}{2}x^{18}e+\frac{45}{16}dx^{16}+\frac{15}{2}x^{16}e+\frac{60}{7}dx^{14}+15x^{14}e+\frac{35}{2}dx^{12}+21x^{12}e+\frac{126}{5}dx^{10}+21x^{10}e+\frac{105}{4}dx^8+15x^8e+20dx^6+\frac{15}{2}x^6e+\frac{45}{4}dx^4+\frac{5}{2}x^4e+5dx^2+\frac{1}{2}x^2e+\frac{1}{2}d\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x,x, algorithm="giac")
```

```
[Out] 1/22*x^22*e + 1/20*d*x^20 + 1/2*x^20*e + 5/9*d*x^18 + 5/2*x^18*e + 45/16*d*
x^16 + 15/2*x^16*e + 60/7*d*x^14 + 15*x^14*e + 35/2*d*x^12 + 21*x^12*e + 12
```

```
6/5*d*x^10 + 21*x^10*e + 105/4*d*x^8 + 15*x^8*e + 20*d*x^6 + 15/2*x^6*e + 4
5/4*d*x^4 + 5/2*x^4*e + 5*d*x^2 + 1/2*x^2*e + 1/2*d*log(x^2)
```

maple [A] time = 0.00, size = 132, normalized size = 1.42

$$\frac{e\,x^{22}}{22}+\frac{d\,x^{20}}{20}+\frac{e\,x^{20}}{2}+\frac{5d\,x^{18}}{9}+\frac{5e\,x^{18}}{2}+\frac{45d\,x^{16}}{16}+\frac{15e\,x^{16}}{2}+\frac{60d\,x^{14}}{7}+15e\,x^{14}+\frac{35d\,x^{12}}{2}+21e\,x^{12}+\frac{126d\,x^{10}}{5}+21e\,x^{10}+\frac{105d\,x^{8}}{4}+15e\,x^{8}+20d\,x^{6}+\frac{15e\,x^{6}}{2}+\frac{45d\,x^{4}}{4}+\frac{5e\,x^{4}}{2}+5d\,x^{2}+\frac{e\,x^{2}}{2}+d\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(x^4+2*x^2+1)^5/x,x)
```

```
[Out] 1/22*e*x^22+1/20*d*x^20+1/2*e*x^20+5/9*d*x^18+5/2*e*x^18+45/16*d*x^16+15/2*
x^16*e+60/7*d*x^14+15*e*x^14+35/2*d*x^12+21*e*x^12+126/5*d*x^10+21*e*x^10+1
05/4*d*x^8+15*e*x^8+20*d*x^6+15/2*e*x^6+45/4*d*x^4+5/2*e*x^4+5*d*x^2+1/2*e*
x^2+d*ln(x)
```

maxima [A] time = 0.48, size = 130, normalized size = 1.40

$$\frac{1}{22}ex^{22}+\frac{1}{20}(d+10e)x^{20}+\frac{5}{18}(2d+9e)x^{18}+\frac{15}{16}(3d+8e)x^{16}+\frac{15}{7}(4d+7e)x^{14}+\frac{7}{2}(5d+6e)x^{12}+\frac{21}{5}(6d+5e)x^{10}+\frac{15}{4}(7d+4e)x^{8}+\frac{5}{2}(8d+3e)x^{6}+\frac{5}{4}(9d+2e)x^{4}+\frac{1}{2}(10d+e)x^{2}+\frac{1}{2}d\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x,x, algorithm="maxima")
```

```
[Out] 1/22*e*x^22 + 1/20*(d + 10*e)*x^20 + 5/18*(2*d + 9*e)*x^18 + 15/16*(3*d + 8
*e)*x^16 + 15/7*(4*d + 7*e)*x^14 + 7/2*(5*d + 6*e)*x^12 + 21/5*(6*d + 5*e)*
x^10 + 15/4*(7*d + 4*e)*x^8 + 5/2*(8*d + 3*e)*x^6 + 5/4*(9*d + 2*e)*x^4 + 1
/2*(10*d + e)*x^2 + 1/2*d*log(x^2)
```

mupad [B] time = 0.13, size = 121, normalized size = 1.30

$$x^2\left(5d+\frac{e}{2}\right)+x^{18}\left(\frac{5d}{9}+\frac{5e}{2}\right)+x^6\left(20d+\frac{15e}{2}\right)+x^{20}\left(\frac{d}{20}+\frac{e}{2}\right)+x^4\left(\frac{45d}{4}+\frac{5e}{2}\right)+x^{12}\left(\frac{35d}{2}+21e\right)+x^{16}\left(\frac{45d}{16}+\frac{15e}{2}\right)+x^{14}\left(\frac{60d}{7}+15e\right)+x^8\left(\frac{105d}{4}+15e\right)+x^{10}\left(\frac{126d}{5}+21e\right)+\frac{e\,x^{22}}{22}+d\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(2*x^2 + x^4 + 1)^5)/x,x)
```

```
[Out] x^2*(5*d + e/2) + x^18*((5*d)/9 + (5*e)/2) + x^6*(20*d + (15*e)/2) + x^20*(
d/20 + e/2) + x^4*((45*d)/4 + (5*e)/2) + x^12*((35*d)/2 + 21*e) + x^16*((45
*d)/16 + (15*e)/2) + x^14*((60*d)/7 + 15*e) + x^8*((105*d)/4 + 15*e) + x^10
*((126*d)/5 + 21*e) + (e*x^22)/22 + d*log(x)
```

sympy [A] time = 0.33, size = 131, normalized size = 1.41

$$d\log(x)+\frac{ex^{22}}{22}+x^{20}\left(\frac{d}{20}+\frac{e}{2}\right)+x^{18}\left(\frac{5d}{9}+\frac{5e}{2}\right)+x^{16}\left(\frac{45d}{16}+\frac{15e}{2}\right)+x^{14}\left(\frac{60d}{7}+15e\right)+x^{12}\left(\frac{35d}{2}+21e\right)+x^{10}\left(\frac{126d}{5}+21e\right)+x^{8}\left(\frac{105d}{4}+15e\right)+x^{6}\left(20d+\frac{15e}{2}\right)+x^{4}\left(\frac{45d}{4}+\frac{5e}{2}\right)+x^{2}\left(5d+\frac{e}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x,x)
```

```
[Out] d*log(x) + e*x**22/22 + x**20*(d/20 + e/2) + x**18*(5*d/9 + 5*e/2) + x**16*
(45*d/16 + 15*e/2) + x**14*(60*d/7 + 15*e) + x**12*(35*d/2 + 21*e) + x**10*
(126*d/5 + 21*e) + x**8*(105*d/4 + 15*e) + x**6*(20*d + 15*e/2) + x**4*(45*
d/4 + 5*e/2) + x**2*(5*d + e/2)
```

3.43 $\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2} dx$

Optimal. Leaf size=141

$$\frac{1}{19}x^{19}(d+10e)+\frac{5}{17}x^{17}(2d+9e)+x^{15}(3d+8e)+\frac{30}{13}x^{13}(4d+7e)+\frac{42}{11}x^{11}(5d+6e)+\frac{14}{3}x^9(6d+5e)+\frac{30}{7}x^7(7d+4e)+3x^5($$

Rubi [A] time = 0.08, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {28, 448}

$$\frac{1}{19}x^{19}(d+10e)+\frac{5}{17}x^{17}(2d+9e)+x^{15}(3d+8e)+\frac{30}{13}x^{13}(4d+7e)+\frac{42}{11}x^{11}(5d+6e)+\frac{14}{3}x^9(6d+5e)+\frac{30}{7}x^7(7d+4e)+3x^5(8d+3e)+\frac{5}{3}x^3(9d+2e)+x(10d+e)-\frac{d}{x}+\frac{ex^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -(d/x) + (10*d + e)*x + (5*(9*d + 2*e)*x^3)/3 + 3*(8*d + 3*e)*x^5 + (30*(7*
d + 4*e)*x^7)/7 + (14*(6*d + 5*e)*x^9)/3 + (42*(5*d + 6*e)*x^11)/11 + (30*(
4*d + 7*e)*x^13)/13 + (3*d + 8*e)*x^15 + (5*(2*d + 9*e)*x^17)/17 + ((d + 10
*e)*x^19)/19 + (e*x^21)/21
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2} dx &= \int \frac{(1+x^2)^{10}(d+ex^2)}{x^2} dx \\
&= \int \left(10d\left(1+\frac{e}{10d}\right)+\frac{d}{x^2}+5(9d+2e)x^2+15(8d+3e)x^4+30(7d+4e)x^6+ \right. \\
&= -\frac{d}{x}+(10d+e)x+\frac{5}{3}(9d+2e)x^3+3(8d+3e)x^5+\frac{30}{7}(7d+4e)x^7+\frac{14}{3}(6d+
\end{aligned}$$

Mathematica [A] time = 0.03, size = 141, normalized size = 1.00

$$\frac{1}{19}x^{19}(d+10e)+\frac{5}{17}x^{17}(2d+9e)+x^{15}(3d+8e)+\frac{30}{13}x^{13}(4d+7e)+\frac{42}{11}x^{11}(5d+6e)+\frac{14}{3}x^9(6d+5e)+\frac{30}{7}x^7(7d+4e)+3x^5(8d+3e)+\frac{5}{3}x^3(9d+2e)+x(10d+e)-\frac{d}{x}+\frac{ex^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -(d/x) + (10*d + e)*x + (5*(9*d + 2*e)*x^3)/3 + 3*(8*d + 3*e)*x^5 + (30*(7*
d + 4*e)*x^7)/7 + (14*(6*d + 5*e)*x^9)/3 + (42*(5*d + 6*e)*x^11)/11 + (30*(
4*d + 7*e)*x^13)/13 + (3*d + 8*e)*x^15 + (5*(2*d + 9*e)*x^17)/17 + ((d + 10
*e)*x^19)/19 + (e*x^21)/21
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
[Out] IntegrateAlgebraic[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^2, x]
```

fricas [A] time = 0.88, size = 131, normalized size = 0.93

$$\frac{46189\,ex^{22}+51051\,(d+10\,e)x^{20}+285285\,(2\,d+9\,e)x^{18}+969969\,(3\,d+8\,e)x^{16}+2238390\,(4\,d+7\,e)x^{14}+3703518\,(5\,d+6\,e)x^{12}+4526522\,(6\,d+5\,e)x^{10}+4157010\,(7\,d+4\,e)x^{8}+2909907\,(8\,d+3\,e)x^{6}+1616615\,(9\,d+2\,e)x^{4}+969969\,(10\,d+e)x^{2}-969969\,d}{969969\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x, algorithm="fricas")
[Out] 1/969969*(46189*e*x^22 + 51051*(d + 10*e)*x^20 + 285285*(2*d + 9*e)*x^18 +
969969*(3*d + 8*e)*x^16 + 2238390*(4*d + 7*e)*x^14 + 3703518*(5*d + 6*e)*x^
12 + 4526522*(6*d + 5*e)*x^10 + 4157010*(7*d + 4*e)*x^8 + 2909907*(8*d + 3*
e)*x^6 + 1616615*(9*d + 2*e)*x^4 + 969969*(10*d + e)*x^2 - 969969*d)/x
```

giac [A] time = 0.36, size = 139, normalized size = 0.99

$$\frac{1}{21}\,x^{21}e+\frac{1}{19}\,dx^{19}+\frac{10}{19}\,x^{19}e+\frac{10}{17}\,dx^{17}+\frac{45}{17}\,x^{17}e+3\,dx^{15}+8\,x^{15}e+\frac{120}{13}\,dx^{13}+\frac{210}{13}\,x^{13}e+\frac{210}{11}\,dx^{11}+\frac{252}{11}\,x^{11}e+28\,dx^{9}+\frac{70}{3}\,x^{9}e+30\,dx^{7}+\frac{120}{7}\,x^{7}e+24\,dx^{5}+9\,x^{5}e+15\,dx^{3}+\frac{10}{3}\,x^{3}e+10\,dx+xe-\frac{d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x, algorithm="giac")
[Out] 1/21*x^21*e + 1/19*d*x^19 + 10/19*x^19*e + 10/17*d*x^17 + 45/17*x^17*e + 3*
d*x^15 + 8*x^15*e + 120/13*d*x^13 + 210/13*x^13*e + 210/11*d*x^11 + 252/11*
x^11*e + 28*d*x^9 + 70/3*x^9*e + 30*d*x^7 + 120/7*x^7*e + 24*d*x^5 + 9*x^5*
e + 15*d*x^3 + 10/3*x^3*e + 10*d*x + x*e - d/x
```

maple [A] time = 0.00, size = 129, normalized size = 0.91

$$\frac{e\,x^{21}}{21}+\frac{d\,x^{19}}{19}+\frac{10e\,x^{19}}{19}+\frac{10d\,x^{17}}{17}+\frac{45e\,x^{17}}{17}+3d\,x^{15}+8e\,x^{15}+\frac{120d\,x^{13}}{13}+\frac{210e\,x^{13}}{13}+\frac{210d\,x^{11}}{11}+\frac{252e\,x^{11}}{11}+28d\,x^{9}+\frac{70e\,x^{9}}{3}+30d\,x^{7}+\frac{120e\,x^{7}}{7}+24d\,x^{5}+9e\,x^{5}+15d\,x^{3}+\frac{10e\,x^{3}}{3}+10dx+ex-\frac{d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x)
[Out] 1/21*e*x^21+1/19*x^19*d+10/19*x^19*e+10/17*x^17*d+45/17*x^17*e+3*x^15*d+8*x
^15*e+120/13*x^13*d+210/13*x^13*e+210/11*x^11*d+252/11*x^11*e+28*x^9*d+70/3
*x^9*e+30*x^7*d+120/7*x^7*e+24*d*x^5+9*x^5*e+15*d*x^3+10/3*x^3*e+10*d*x+e*x
-d/x
```

maxima [A] time = 0.50, size = 125, normalized size = 0.89

$$\frac{1}{21}\,ex^{21}+\frac{1}{19}\,(d+10\,e)x^{19}+\frac{5}{17}\,(2\,d+9\,e)x^{17}+(3\,d+8\,e)x^{15}+\frac{30}{13}\,(4\,d+7\,e)x^{13}+\frac{42}{11}\,(5\,d+6\,e)x^{11}+\frac{14}{3}\,(6\,d+5\,e)x^{9}+\frac{30}{7}\,(7\,d+4\,e)x^{7}+3\,(8\,d+3\,e)x^{5}+\frac{5}{3}\,(9\,d+2\,e)x^{3}+(10\,d+e)x-\frac{d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x, algorithm="maxima")
[Out] 1/21*e*x^21 + 1/19*(d + 10*e)*x^19 + 5/17*(2*d + 9*e)*x^17 + (3*d + 8*e)*x^
15 + 30/13*(4*d + 7*e)*x^13 + 42/11*(5*d + 6*e)*x^11 + 14/3*(6*d + 5*e)*x^9
 + 30/7*(7*d + 4*e)*x^7 + 3*(8*d + 3*e)*x^5 + 5/3*(9*d + 2*e)*x^3 + (10*d +
 e)*x - d/x
```

mupad [B] time = 0.08, size = 119, normalized size = 0.84

$$x^{15}\,(3\,d+8\,e)+x^3\,\left(15\,d+\frac{10\,e}{3}\right)+x^5\,(24\,d+9\,e)+x^{19}\,\left(\frac{d}{19}+\frac{10\,e}{19}\right)+x^{17}\,\left(\frac{10\,d}{17}+\frac{45\,e}{17}\right)+x^9\,\left(28\,d+\frac{70\,e}{3}\right)+x^7\,\left(30\,d+\frac{120\,e}{7}\right)+x^{13}\,\left(\frac{120\,d}{13}+\frac{210\,e}{13}\right)+x^{11}\,\left(\frac{210\,d}{11}+\frac{252\,e}{11}\right)+x\,(10\,d+e)-\frac{d}{x}+\frac{e\,x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(2*x^2 + x^4 + 1)^5)/x^2,x)
```

```
[Out] x^15*(3*d + 8*e) + x^3*(15*d + (10*e)/3) + x^5*(24*d + 9*e) + x^19*(d/19 +
(10*e)/19) + x^17*((10*d)/17 + (45*e)/17) + x^9*(28*d + (70*e)/3) + x^7*(30
*d + (120*e)/7) + x^13*((120*d)/13 + (210*e)/13) + x^11*((210*d)/11 + (252*
e)/11) + x*(10*d + e) - d/x + (e*x^21)/21
```

sympy [A] time = 0.32, size = 124, normalized size = 0.88

$$-\frac{d}{x}+\frac{ex^{21}}{21}+x^{19}\left(\frac{d}{19}+\frac{10e}{19}\right)+x^{17}\left(\frac{10d}{17}+\frac{45e}{17}\right)+x^{15}\,(3d+8e)+x^{13}\left(\frac{120d}{13}+\frac{210e}{13}\right)+x^{11}\left(\frac{210d}{11}+\frac{252e}{11}\right)+x^9\left(28d+\frac{70e}{3}\right)+x^7\left(30d+\frac{120e}{7}\right)+x^5\,(24d+9e)+x^3\left(15d+\frac{10e}{3}\right)+x\,(10d+e)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x**2,x)
```

```
[Out] -d/x + e*x**21/21 + x**19*(d/19 + 10*e/19) + x**17*(10*d/17 + 45*e/17) + x*
*15*(3*d + 8*e) + x**13*(120*d/13 + 210*e/13) + x**11*(210*d/11 + 252*e/11)
 + x**9*(28*d + 70*e/3) + x**7*(30*d + 120*e/7) + x**5*(24*d + 9*e) + x**3*
(15*d + 10*e/3) + x*(10*d + e)
```

3.44 $\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3}\,dx$

Optimal. Leaf size=147

$$\frac{1}{18}x^{18}(d+10e)+\frac{5}{16}x^{16}(2d+9e)+\frac{15}{14}x^{14}(3d+8e)+\frac{5}{2}x^{12}(4d+7e)+\frac{21}{5}x^{10}(5d+6e)+\frac{21}{4}x^{8}(6d+5e)+5x^{6}(7d+4e)+\frac{15}{4}x^{4}(8$$

Rubi [A] time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {28, 446, 76}

$$\frac{1}{18}x^{18}(d+10e)+\frac{5}{16}x^{16}(2d+9e)+\frac{15}{14}x^{14}(3d+8e)+\frac{5}{2}x^{12}(4d+7e)+\frac{21}{5}x^{10}(5d+6e)+\frac{21}{4}x^{8}(6d+5e)+5x^{6}(7d+4e)+\frac{15}{4}x^{4}(8d+3e)+\frac{5}{2}x^{2}(9d+2e)+(10d+e)\log(x)-\frac{d}{2x^2}+\frac{ex^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] -d/(2*x^2) + (5*(9*d + 2*e)*x^2)/2 + (15*(8*d + 3*e)*x^4)/4 + 5*(7*d + 4*e)
*x^6 + (21*(6*d + 5*e)*x^8)/4 + (21*(5*d + 6*e)*x^10)/5 + (5*(4*d + 7*e)*x^
12)/2 + (15*(3*d + 8*e)*x^14)/14 + (5*(2*d + 9*e)*x^16)/16 + ((d + 10*e)*x^
18)/18 + (e*x^20)/20 + (10*d + e)*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3}\,dx &= \int \frac{(1+x^2)^{10}(d+ex^2)}{x^3}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(1+x)^{10}(d+ex)}{x^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(5(9d+2e)+\frac{d}{x^2}+\frac{10d+e}{x}+15(8d+3e)x+30(7d+4e)x^2+42(\right.\right.\\
&= -\frac{d}{2x^2}+\frac{5}{2}(9d+2e)x^2+\frac{15}{4}(8d+3e)x^4+5(7d+4e)x^6+\frac{21}{4}(6d+5e)x^8+\frac{21}{5}(5
\end{aligned}$$

Mathematica [A] time = 0.03, size = 147, normalized size = 1.00

$$\frac{1}{18}x^{18}(d+10e)+\frac{5}{16}x^{16}(2d+9e)+\frac{15}{14}x^{14}(3d+8e)+\frac{5}{2}x^{12}(4d+7e)+\frac{21}{5}x^{10}(5d+6e)+\frac{21}{4}x^{8}(6d+5e)+5x^{6}(7d+4e)+\frac{15}{4}x^{4}(8d+3e)+\frac{5}{2}x^{2}(9d+2e)+(10d+e)\log(x)-\frac{d}{2x^2}+\frac{ex^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] -1/2*d/x^2 + (5*(9*d + 2*e)*x^2)/2 + (15*(8*d + 3*e)*x^4)/4 + 5*(7*d + 4*e)
*x^6 + (21*(6*d + 5*e)*x^8)/4 + (21*(5*d + 6*e)*x^10)/5 + (5*(4*d + 7*e)*x^
12)/2 + (15*(3*d + 8*e)*x^14)/14 + (5*(2*d + 9*e)*x^16)/16 + ((d + 10*e)*x^
18)/18 + (e*x^20)/20 + (10*d + e)*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] IntegrateAlgebraic[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^3, x]
```

fricas [A] time = 0.78, size = 133, normalized size = 0.90

$$\frac{252\,ex^{22}+280\,(d+10\,e)x^{20}+1575\,(2\,d+9\,e)x^{18}+5400\,(3\,d+8\,e)x^{16}+12600\,(4\,d+7\,e)x^{14}+21168\,(5\,d+6\,e)x^{12}+26460\,(6\,d+5\,e)x^{10}+25200\,(7\,d+4\,e)x^{8}+18900\,(8\,d+3\,e)x^{6}+12600\,(9\,d+2\,e)x^{4}+5040\,(10\,d+e)x^{2}\log(x)-2520\,d}{5040\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/5040*(252*e*x^22 + 280*(d + 10*e)*x^20 + 1575*(2*d + 9*e)*x^18 + 5400*(3*
d + 8*e)*x^16 + 12600*(4*d + 7*e)*x^14 + 21168*(5*d + 6*e)*x^12 + 26460*(6*
d + 5*e)*x^10 + 25200*(7*d + 4*e)*x^8 + 18900*(8*d + 3*e)*x^6 + 12600*(9*d
+ 2*e)*x^4 + 5040*(10*d + e)*x^2*log(x) - 2520*d)/x^2
```

giac [A] time = 0.26, size = 156, normalized size = 1.06

$$\frac{1}{20}\,x^{20}e+\frac{1}{18}\,dx^{18}+\frac{5}{9}\,x^{18}e+\frac{5}{8}\,dx^{16}+\frac{45}{16}\,x^{16}e+\frac{45}{14}\,dx^{14}+\frac{60}{7}\,x^{14}e+10\,dx^{12}+\frac{35}{2}\,x^{12}e+21\,dx^{10}+\frac{126}{5}\,x^{10}e+\frac{63}{2}\,dx^{8}+\frac{105}{4}\,x^{8}e+35\,dx^{6}+20\,x^{6}e+30\,dx^{4}+\frac{45}{4}\,x^{4}e+\frac{45}{2}\,dx^{2}+5\,x^{2}e+\frac{1}{2}\,(10\,d+e)\log\left(x^2\right)-\frac{10\,dx^2+x^2e+d}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/20*x^20*e + 1/18*d*x^18 + 5/9*x^18*e + 5/8*d*x^16 + 45/16*x^16*e + 45/14*
d*x^14 + 60/7*x^14*e + 10*d*x^12 + 35/2*x^12*e + 21*d*x^10 + 126/5*x^10*e +
 63/2*d*x^8 + 105/4*x^8*e + 35*d*x^6 + 20*x^6*e + 30*d*x^4 + 45/4*x^4*e + 4
5/2*d*x^2 + 5*x^2*e + 1/2*(10*d + e)*log(x^2) - 1/2*(10*d*x^2 + x^2*e + d)/
x^2
```

maple [A] time = 0.01, size = 131, normalized size = 0.89

$$\frac{e\,x^{20}}{20}+\frac{d\,x^{18}}{18}+\frac{5e\,x^{18}}{9}+\frac{5d\,x^{16}}{8}+\frac{45e\,x^{16}}{16}+\frac{45d\,x^{14}}{14}+\frac{60e\,x^{14}}{7}+10d\,x^{12}+\frac{35e\,x^{12}}{2}+21d\,x^{10}+\frac{126e\,x^{10}}{5}+\frac{63d\,x^{8}}{2}+\frac{105e\,x^{8}}{4}+35d\,x^{6}+20e\,x^{6}+30d\,x^{4}+\frac{45e\,x^{4}}{4}+\frac{45d\,x^{2}}{2}+5e\,x^{2}+10d\ln(x)+e\ln(x)-\frac{d}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x)
```

```
[Out] 1/20*e*x^20+1/18*d*x^18+5/9*e*x^18+5/8*d*x^16+45/16*e*x^16+45/14*d*x^14+60/
7*e*x^14+10*d*x^12+35/2*e*x^12+21*d*x^10+126/5*e*x^10+63/2*d*x^8+105/4*e*x^
8+35*d*x^6+20*e*x^6+30*d*x^4+45/4*e*x^4+45/2*d*x^2+5*e*x^2+10*d*ln(x)+ln(x)
*e-1/2*d/x^2
```

maxima [A] time = 0.70, size = 130, normalized size = 0.88

$$\frac{1}{20}ex^{20}+\frac{1}{18}(d+10e)x^{18}+\frac{5}{16}(2d+9e)x^{16}+\frac{15}{14}(3d+8e)x^{14}+\frac{5}{2}(4d+7e)x^{12}+\frac{21}{5}(5d+6e)x^{10}+\frac{21}{4}(6d+5e)x^{8}+5(7d+4e)x^{6}+\frac{15}{4}(8d+3e)x^{4}+\frac{5}{2}(9d+2e)x^{2}+\frac{1}{2}(10d+e)\log\left(x^{2}\right)-\frac{d}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/20*e*x^20 + 1/18*(d + 10*e)*x^18 + 5/16*(2*d + 9*e)*x^16 + 15/14*(3*d + 8
*e)*x^14 + 5/2*(4*d + 7*e)*x^12 + 21/5*(5*d + 6*e)*x^10 + 21/4*(6*d + 5*e)*
x^8 + 5*(7*d + 4*e)*x^6 + 15/4*(8*d + 3*e)*x^4 + 5/2*(9*d + 2*e)*x^2 + 1/2*
(10*d + e)*log(x^2) - 1/2*d/x^2
```

mupad [B] time = 0.08, size = 120, normalized size = 0.82

$$x^{18}\left(\frac{d}{18}+\frac{5e}{9}\right)+x^{2}\left(\frac{45d}{2}+5e\right)+x^{12}\left(10d+\frac{35e}{2}\right)+x^{6}\,(35d+20e)+x^{4}\left(30d+\frac{45e}{4}\right)+x^{16}\left(\frac{5d}{8}+\frac{45e}{16}\right)+x^{14}\left(\frac{45d}{14}+\frac{60e}{7}\right)+x^{10}\left(21d+\frac{126e}{5}\right)+x^{8}\left(\frac{63d}{2}+\frac{105e}{4}\right)-\frac{d}{2x^{2}}+\frac{e\,x^{20}}{20}+\ln(x)\,(10d+e)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(2*x^2 + x^4 + 1)^5)/x^3,x)
```

```
[Out] x^18*(d/18 + (5*e)/9) + x^2*((45*d)/2 + 5*e) + x^12*(10*d + (35*e)/2) + x^6
*(35*d + 20*e) + x^4*(30*d + (45*e)/4) + x^16*((5*d)/8 + (45*e)/16) + x^14*
((45*d)/14 + (60*e)/7) + x^10*(21*d + (126*e)/5) + x^8*((63*d)/2 + (105*e)/
4) - d/(2*x^2) + (e*x^20)/20 + log(x)*(10*d + e)
```

sympy [A] time = 0.39, size = 131, normalized size = 0.89

$$-\frac{d}{2x^{2}}+\frac{ex^{20}}{20}+x^{18}\left(\frac{d}{18}+\frac{5e}{9}\right)+x^{16}\left(\frac{5d}{8}+\frac{45e}{16}\right)+x^{14}\left(\frac{45d}{14}+\frac{60e}{7}\right)+x^{12}\left(10d+\frac{35e}{2}\right)+x^{10}\left(21d+\frac{126e}{5}\right)+x^{8}\left(\frac{63d}{2}+\frac{105e}{4}\right)+x^{6}\,(35d+20e)+x^{4}\left(30d+\frac{45e}{4}\right)+x^{2}\left(\frac{45d}{2}+5e\right)+(10d+e)\log{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x**3,x)
```

```
[Out] -d/(2*x**2) + e*x**20/20 + x**18*(d/18 + 5*e/9) + x**16*(5*d/8 + 45*e/16) +
 x**14*(45*d/14 + 60*e/7) + x**12*(10*d + 35*e/2) + x**10*(21*d + 126*e/5)
+ x**8*(63*d/2 + 105*e/4) + x**6*(35*d + 20*e) + x**4*(30*d + 45*e/4) + x**
2*(45*d/2 + 5*e) + (10*d + e)*log(x)
```

3.45 $\int (fx)^m \left(1+x^2\right)\left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=203

$$\frac{(fx)^{m+23}}{f^{23}(m+23)}+\frac{11(fx)^{m+21}}{f^{21}(m+21)}+\frac{55(fx)^{m+19}}{f^{19}(m+19)}+\frac{165(fx)^{m+17}}{f^{17}(m+17)}+\frac{330(fx)^{m+15}}{f^{15}(m+15)}+\frac{462(fx)^{m+13}}{f^{13}(m+13)}+\frac{462(fx)^{m+11}}{f^{11}(m+11)}+\frac{330(fx)}{f^9(m+}$$

Rubi [A] time = 0.07, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {28, 270}

$$\frac{11(fx)^{m+3}}{f^3(m+3)}+\frac{55(fx)^{m+5}}{f^5(m+5)}+\frac{165(fx)^{m+7}}{f^7(m+7)}+\frac{330(fx)^{m+9}}{f^9(m+9)}+\frac{462(fx)^{m+11}}{f^{11}(m+11)}+\frac{462(fx)^{m+13}}{f^{13}(m+13)}+\frac{330(fx)^{m+15}}{f^{15}(m+15)}+\frac{165(fx)^{m+17}}{f^{17}(m+17)}+\frac{55(fx)^{m+19}}{f^{19}(m+19)}+\frac{11(fx)^{m+21}}{f^{21}(m+21)}+\frac{(fx)^{m+23}}{f^{23}(m+23)}+\frac{(fx)^{m+1}}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (f*x)^(1 + m)/(f*(1 + m)) + (11*(f*x)^(3 + m))/(f^3*(3 + m)) + (55*(f*x)^(5
 + m))/(f^5*(5 + m)) + (165*(f*x)^(7 + m))/(f^7*(7 + m)) + (330*(f*x)^(9 +
m))/(f^9*(9 + m)) + (462*(f*x)^(11 + m))/(f^11*(11 + m)) + (462*(f*x)^(13 +
 m))/(f^13*(13 + m)) + (330*(f*x)^(15 + m))/(f^15*(15 + m)) + (165*(f*x)^(1
7 + m))/(f^17*(17 + m)) + (55*(f*x)^(19 + m))/(f^19*(19 + m)) + (11*(f*x)^(
21 + m))/(f^21*(21 + m)) + (f*x)^(23 + m)/(f^23*(23 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (fx)^m \left(1+x^2\right)\left(1+2x^2+x^4\right)^5 dx &= \int (fx)^m \left(1+x^2\right)^{11} dx\\ &= \int \left((fx)^m+\frac{11(fx)^{2+m}}{f^2}+\frac{55(fx)^{4+m}}{f^4}+\frac{165(fx)^{6+m}}{f^6}+\frac{330(fx)^{8+m}}{f^8}+\cdots\right.\\ &= \frac{(fx)^{1+m}}{f(1+m)}+\frac{11(fx)^{3+m}}{f^3(3+m)}+\frac{55(fx)^{5+m}}{f^5(5+m)}+\frac{165(fx)^{7+m}}{f^7(7+m)}+\frac{330(fx)^{9+m}}{f^9(9+m)}+\cdots\end{aligned}$$

Mathematica [A] time = 0.04, size = 122, normalized size = 0.60

$$x\left(\frac{x^{22}}{m+23}+\frac{11x^{20}}{m+21}+\frac{55x^{18}}{m+19}+\frac{165x^{16}}{m+17}+\frac{330x^{14}}{m+15}+\frac{462x^{12}}{m+13}+\frac{462x^{10}}{m+11}+\frac{330x^8}{m+9}+\frac{165x^6}{m+7}+\frac{55x^4}{m+5}+\frac{11x^2}{m+3}+\frac{1}{m+1}\right)(fx)^m$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x*(f*x)^m*((1 + m)^(-1) + (11*x^2)/(3 + m) + (55*x^4)/(5 + m) + (165*x^6)/(
7 + m) + (330*x^8)/(9 + m) + (462*x^10)/(11 + m) + (462*x^12)/(13 + m) + (3
```

```
30*x^14)/(15 + m) + (165*x^16)/(17 + m) + (55*x^18)/(19 + m) + (11*x^20)/(2
1 + m) + x^22/(23 + m))
```

IntegrateAlgebraic [F] time = 0.56, size = 0, normalized size = 0.00

$$\int (fx)^m \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f*x)^m*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] Defer[IntegrateAlgebraic][(f*x)^m*(1 + x^2)*(1 + 2*x^2 + x^4)^5, x]
```

fricas [B] time = 0.82, size = 759, normalized size = 3.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] ((m^11 + 121*m^10 + 6435*m^9 + 197835*m^8 + 3889578*m^7 + 51069018*m^6 + 45
3714470*m^5 + 2702025590*m^4 + 10431670821*m^3 + 24372200061*m^2 + 29985521
895*m + 13749310575)*x^23 + 11*(m^11 + 123*m^10 + 6635*m^9 + 206505*m^8 + 4
103178*m^7 + 54362574*m^6 + 486687830*m^5 + 2917013970*m^4 + 11320966021*m^
3 + 26560342503*m^2 + 32778930735*m + 15058768725)*x^21 + 55*(m^11 + 125*m^
10 + 6843*m^9 + 215823*m^8 + 4339146*m^7 + 58085538*m^6 + 524676662*m^5 + 3
168601822*m^4 + 12374824773*m^3 + 29178958257*m^2 + 36145916415*m + 1664390
2275)*x^19 + 165*(m^11 + 127*m^10 + 7059*m^9 + 225837*m^8 + 4600554*m^7 + 6
2319894*m^6 + 568863686*m^5 + 3466775738*m^4 + 13643071845*m^3 + 3236840757
9*m^2 + 40283194455*m + 18602008425)*x^17 + 330*(m^11 + 129*m^10 + 7283*m^9
 + 236595*m^8 + 4890858*m^7 + 67166442*m^6 + 620805254*m^5 + 3825379590*m^4
 + 15197565541*m^3 + 36337145829*m^2 + 45488935863*m + 21082276215)*x^15 +
462*(m^11 + 131*m^10 + 7515*m^9 + 248145*m^8 + 5213898*m^7 + 72748638*m^6 +
 682569590*m^5 + 4264053730*m^4 + 17145560901*m^3 + 41408337231*m^2 + 52237
739295*m + 24325703325)*x^13 + 462*(m^11 + 133*m^10 + 7755*m^9 + 260535*m^8
 + 5573898*m^7 + 79216434*m^6 + 756921110*m^5 + 4811326190*m^4 + 1965367130
1*m^3 + 48110244633*m^2 + 61333432335*m + 28748558475)*x^11 + 330*(m^11 + 1
35*m^10 + 8003*m^9 + 273813*m^8 + 5975466*m^7 + 86750118*m^6 + 847550822*m^
5 + 5509501002*m^4 + 22992750373*m^3 + 57365875587*m^2 + 74253243015*m + 35
137127025)*x^9 + 165*(m^11 + 137*m^10 + 8259*m^9 + 288027*m^8 + 6423594*m^7
 + 95564154*m^6 + 959352806*m^5 + 6421988758*m^4 + 27624338085*m^3 + 709302
62349*m^2 + 94034286855*m + 45176306175)*x^7 + 55*(m^11 + 139*m^10 + 8523*m
^9 + 303225*m^8 + 6923658*m^7 + 105911022*m^6 + 1098746774*m^5 + 7643724530
*m^4 + 34359636741*m^3 + 92502445239*m^2 + 128033897103*m + 63246828645)*x^
5 + 11*(m^11 + 141*m^10 + 8795*m^9 + 319455*m^8 + 7481418*m^7 + 118085058*m
^6 + 1274046710*m^5 + 9315318270*m^4 + 44632304581*m^3 + 130403715201*m^2 +
 199334977695*m + 105411381075)*x^3 + (m^11 + 143*m^10 + 9075*m^9 + 336765*
m^8 + 8103018*m^7 + 132426294*m^6 + 1495875590*m^5 + 11641582810*m^4 + 6093
6676581*m^3 + 203363952363*m^2 + 387182170935*m + 316234143225)*x)*(f*x)^m/
(m^12 + 144*m^11 + 9218*m^10 + 345840*m^9 + 8439783*m^8 + 140529312*m^7 + 1
628301884*m^6 + 13137458400*m^5 + 72578259391*m^4 + 264300628944*m^3 + 5905
46123298*m^2 + 703416314160*m + 316234143225)
```

giac [B] time = 0.62, size = 1848, normalized size = 9.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] ((f*x)^m*m^11*x^23 + 121*(f*x)^m*m^10*x^23 + 11*(f*x)^m*m^11*x^21 + 6435*(f
*x)^m*m^9*x^23 + 1353*(f*x)^m*m^10*x^21 + 197835*(f*x)^m*m^8*x^23 + 55*(f*x
```

```
)^m*m^11*x^19 + 72985*(f*x)^m*m^9*x^21 + 3889578*(f*x)^m*m^7*x^23 + 6875*(f
*x)^m*m^10*x^19 + 2271555*(f*x)^m*m^8*x^21 + 51069018*(f*x)^m*m^6*x^23 + 16
5*(f*x)^m*m^11*x^17 + 376365*(f*x)^m*m^9*x^19 + 45134958*(f*x)^m*m^7*x^21 +
 453714470*(f*x)^m*m^5*x^23 + 20955*(f*x)^m*m^10*x^17 + 11870265*(f*x)^m*m^
8*x^19 + 597988314*(f*x)^m*m^6*x^21 + 2702025590*(f*x)^m*m^4*x^23 + 330*(f*
x)^m*m^11*x^15 + 1164735*(f*x)^m*m^9*x^17 + 238653030*(f*x)^m*m^7*x^19 + 53
53566130*(f*x)^m*m^5*x^21 + 10431670821*(f*x)^m*m^3*x^23 + 42570*(f*x)^m*m^
10*x^15 + 37263105*(f*x)^m*m^8*x^17 + 3194704590*(f*x)^m*m^6*x^19 + 3208715
3670*(f*x)^m*m^4*x^21 + 24372200061*(f*x)^m*m^2*x^23 + 462*(f*x)^m*m^11*x^1
3 + 2403390*(f*x)^m*m^9*x^15 + 759091410*(f*x)^m*m^7*x^17 + 28857216410*(f*
x)^m*m^5*x^19 + 124530626231*(f*x)^m*m^3*x^21 + 29985521895*(f*x)^m*m*x^23
+ 60522*(f*x)^m*m^10*x^13 + 78076350*(f*x)^m*m^8*x^15 + 10282782510*(f*x)^m
*m^6*x^17 + 174273100210*(f*x)^m*m^4*x^19 + 292163767533*(f*x)^m*m^2*x^21 +
 13749310575*(f*x)^m*x^23 + 462*(f*x)^m*m^11*x^11 + 3471930*(f*x)^m*m^9*x^1
3 + 1613983140*(f*x)^m*m^7*x^15 + 93862508190*(f*x)^m*m^5*x^17 + 6806153625
15*(f*x)^m*m^3*x^19 + 360568238085*(f*x)^m*m*x^21 + 61446*(f*x)^m*m^10*x^11
 + 114642990*(f*x)^m*m^8*x^13 + 22164925860*(f*x)^m*m^6*x^15 + 572017996770
*(f*x)^m*m^4*x^17 + 1604842704135*(f*x)^m*m^2*x^19 + 165646455975*(f*x)^m*x
^21 + 330*(f*x)^m*m^11*x^9 + 3582810*(f*x)^m*m^9*x^11 + 2408820876*(f*x)^m*
m^7*x^13 + 204865733820*(f*x)^m*m^5*x^15 + 2251106854425*(f*x)^m*m^3*x^17 +
 1988025402825*(f*x)^m*m*x^19 + 44550*(f*x)^m*m^10*x^9 + 120367170*(f*x)^m*
m^8*x^11 + 33609870756*(f*x)^m*m^6*x^13 + 1262375264700*(f*x)^m*m^4*x^15 +
5340787250535*(f*x)^m*m^2*x^17 + 915414625125*(f*x)^m*x^19 + 165*(f*x)^m*m^
11*x^7 + 2640990*(f*x)^m*m^9*x^9 + 2575140876*(f*x)^m*m^7*x^11 + 3153471505
80*(f*x)^m*m^5*x^13 + 5015196628530*(f*x)^m*m^3*x^15 + 6646727085075*(f*x)^
m*m*x^17 + 22605*(f*x)^m*m^10*x^7 + 90358290*(f*x)^m*m^8*x^9 + 36597992508*
(f*x)^m*m^6*x^11 + 1969992823260*(f*x)^m*m^4*x^13 + 11991258123570*(f*x)^m*
m^2*x^15 + 3069331390125*(f*x)^m*x^17 + 55*(f*x)^m*m^11*x^5 + 1362735*(f*x)
^m*m^9*x^7 + 1971903780*(f*x)^m*m^7*x^9 + 349697552820*(f*x)^m*m^5*x^11 + 7
921249136262*(f*x)^m*m^3*x^13 + 15011348834790*(f*x)^m*m*x^15 + 7645*(f*x)^
m*m^10*x^5 + 47524455*(f*x)^m*m^8*x^7 + 28627538940*(f*x)^m*m^6*x^9 + 22228
32699780*(f*x)^m*m^4*x^11 + 19130651800722*(f*x)^m*m^2*x^13 + 6957151150950
*(f*x)^m*x^15 + 11*(f*x)^m*m^11*x^3 + 468765*(f*x)^m*m^9*x^5 + 1059893010*(
f*x)^m*m^7*x^7 + 279691771260*(f*x)^m*m^5*x^9 + 9079996141062*(f*x)^m*m^3*x
^11 + 24133835554290*(f*x)^m*m*x^13 + 1551*(f*x)^m*m^10*x^3 + 16677375*(f*x
)^m*m^8*x^5 + 15768085410*(f*x)^m*m^6*x^7 + 1818135330660*(f*x)^m*m^4*x^9 +
 22226933020446*(f*x)^m*m^2*x^11 + 11238474936150*(f*x)^m*x^13 + (f*x)^m*m^
11*x + 96745*(f*x)^m*m^9*x^3 + 380801190*(f*x)^m*m^7*x^5 + 158293212990*(f*
x)^m*m^5*x^7 + 7587607623090*(f*x)^m*m^3*x^9 + 28336045738770*(f*x)^m*m*x^1
1 + 143*(f*x)^m*m^10*x + 3514005*(f*x)^m*m^8*x^3 + 5825106210*(f*x)^m*m^6*x
^5 + 1059628145070*(f*x)^m*m^4*x^7 + 18930738943710*(f*x)^m*m^2*x^9 + 13281
834015450*(f*x)^m*x^11 + 9075*(f*x)^m*m^9*x + 82295598*(f*x)^m*m^7*x^3 + 60
431072570*(f*x)^m*m^5*x^5 + 4558015784025*(f*x)^m*m^3*x^7 + 24503570194950*
(f*x)^m*m*x^9 + 336765*(f*x)^m*m^8*x + 1298935638*(f*x)^m*m^6*x^3 + 4204048
49150*(f*x)^m*m^4*x^5 + 11703493287585*(f*x)^m*m^2*x^7 + 11595251918250*(f*
x)^m*x^9 + 8103018*(f*x)^m*m^7*x + 14014513810*(f*x)^m*m^5*x^3 + 1889780020
755*(f*x)^m*m^3*x^5 + 15515657331075*(f*x)^m*m*x^7 + 132426294*(f*x)^m*m^6*
x + 102468500970*(f*x)^m*m^4*x^3 + 5087634488145*(f*x)^m*m^2*x^5 + 74540905
18875*(f*x)^m*x^7 + 1495875590*(f*x)^m*m^5*x + 490955350391*(f*x)^m*m^3*x^3
 + 7041864340665*(f*x)^m*m*x^5 + 11641582810*(f*x)^m*m^4*x + 1434440867211*
(f*x)^m*m^2*x^3 + 3478575575475*(f*x)^m*x^5 + 60936676581*(f*x)^m*m^3*x + 2
192684754645*(f*x)^m*m*x^3 + 203363952363*(f*x)^m*m^2*x + 1159525191825*(f*
x)^m*x^3 + 387182170935*(f*x)^m*m*x + 316234143225*(f*x)^m*x)/(m^12 + 144*m
^11 + 9218*m^10 + 345840*m^9 + 8439783*m^8 + 140529312*m^7 + 1628301884*m^6
 + 13137458400*m^5 + 72578259391*m^4 + 264300628944*m^3 + 590546123298*m^2
+ 703416314160*m + 316234143225)
```

maple [B] time = 0.01, size = 1121, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] (f*x)^m*(m^11*x^22+121*m^10*x^22+11*m^11*x^20+6435*m^9*x^22+1353*m^10*x^20+
197835*m^8*x^22+55*m^11*x^18+72985*m^9*x^20+3889578*m^7*x^22+6875*m^10*x^18
+2271555*m^8*x^20+51069018*m^6*x^22+165*m^11*x^16+376365*m^9*x^18+45134958*
m^7*x^20+453714470*m^5*x^22+20955*m^10*x^16+11870265*m^8*x^18+597988314*m^6
*x^20+2702025590*m^4*x^22+330*m^11*x^14+1164735*m^9*x^16+238653030*m^7*x^18
+5353566130*m^5*x^20+10431670821*m^3*x^22+42570*m^10*x^14+37263105*m^8*x^16
+3194704590*m^6*x^18+32087153670*m^4*x^20+24372200061*m^2*x^22+462*m^11*x^1
2+2403390*m^9*x^14+759091410*m^7*x^16+28857216410*m^5*x^18+124530626231*m^3
*x^20+29985521895*m*x^22+60522*m^10*x^12+78076350*m^8*x^14+10282782510*m^6*
x^16+174273100210*m^4*x^18+292163767533*m^2*x^20+13749310575*x^22+462*m^11*
x^10+3471930*m^9*x^12+1613983140*m^7*x^14+93862508190*m^5*x^16+680615362515
*m^3*x^18+360568238085*m*x^20+61446*m^10*x^10+114642990*m^8*x^12+2216492586
0*m^6*x^14+572017996770*m^4*x^16+1604842704135*m^2*x^18+165646455975*x^20+3
30*m^11*x^8+3582810*m^9*x^10+2408820876*m^7*x^12+204865733820*m^5*x^14+2251
106854425*m^3*x^16+1988025402825*m*x^18+44550*m^10*x^8+120367170*m^8*x^10+3
3609870756*m^6*x^12+1262375264700*m^4*x^14+5340787250535*m^2*x^16+915414625
125*x^18+165*m^11*x^6+2640990*m^9*x^8+2575140876*m^7*x^10+315347150580*m^5*
x^12+5015196628530*m^3*x^14+6646727085075*m*x^16+22605*m^10*x^6+90358290*m^
8*x^8+36597992508*m^6*x^10+1969992823260*m^4*x^12+11991258123570*m^2*x^14+3
069331390125*x^16+55*m^11*x^4+1362735*m^9*x^6+1971903780*m^7*x^8+3496975528
20*m^5*x^10+7921249136262*m^3*x^12+15011348834790*m*x^14+7645*m^10*x^4+4752
4455*m^8*x^6+28627538940*m^6*x^8+2222832699780*m^4*x^10+19130651800722*m^2*
x^12+6957151150950*x^14+11*m^11*x^2+468765*m^9*x^4+1059893010*m^7*x^6+27969
1771260*m^5*x^8+9079996141062*m^3*x^10+24133835554290*m*x^12+1551*m^10*x^2+
16677375*m^8*x^4+15768085410*m^6*x^6+1818135330660*m^4*x^8+22226933020446*m
^2*x^10+11238474936150*x^12+m^11+96745*m^9*x^2+380801190*m^7*x^4+1582932129
90*m^5*x^6+7587607623090*m^3*x^8+28336045738770*m*x^10+143*m^10+3514005*m^8
*x^2+5825106210*m^6*x^4+1059628145070*m^4*x^6+18930738943710*m^2*x^8+132818
34015450*x^10+9075*m^9+82295598*m^7*x^2+60431072570*m^5*x^4+4558015784025*m
^3*x^6+24503570194950*m*x^8+336765*m^8+1298935638*m^6*x^2+420404849150*m^4*
x^4+11703493287585*m^2*x^6+11595251918250*x^8+8103018*m^7+14014513810*m^5*x
^2+1889780020755*m^3*x^4+15515657331075*m*x^6+132426294*m^6+102468500970*m^
4*x^2+5087634488145*m^2*x^4+7454090518875*x^6+1495875590*m^5+490955350391*m
^3*x^2+7041864340665*m*x^4+11641582810*m^4+1434440867211*m^2*x^2+3478575575
475*x^4+60936676581*m^3+2192684754645*m*x^2+203363952363*m^2+1159525191825*
x^2+387182170935*m+316234143225)*x/(m+1)/(m+3)/(m+5)/(m+7)/(m+9)/(m+11)/(m+
13)/(m+15)/(m+17)/(m+19)/(m+21)/(m+23)
```

maxima [A] time = 0.97, size = 192, normalized size = 0.95

$$\frac{f^m x^{23} x^m}{m+23} + \frac{11\, f^m x^{21} x^m}{m+21} + \frac{55\, f^m x^{19} x^m}{m+19} + \frac{165\, f^m x^{17} x^m}{m+17} + \frac{330\, f^m x^{15} x^m}{m+15} + \frac{462\, f^m x^{13} x^m}{m+13} + \frac{462\, f^m x^{11} x^m}{m+11} + \frac{330\, f^m x^9 x^m}{m+9} + \frac{165\, f^m x^7 x^m}{m+7} + \frac{55\, f^m x^5 x^m}{m+5} + \frac{11\, f^m x^3 x^m}{m+3} + \frac{(fx)^{m+1}}{f(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] f^m*x^23*x^m/(m + 23) + 11*f^m*x^21*x^m/(m + 21) + 55*f^m*x^19*x^m/(m + 19)
 + 165*f^m*x^17*x^m/(m + 17) + 330*f^m*x^15*x^m/(m + 15) + 462*f^m*x^13*x^m
/(m + 13) + 462*f^m*x^11*x^m/(m + 11) + 330*f^m*x^9*x^m/(m + 9) + 165*f^m*x
^7*x^m/(m + 7) + 55*f^m*x^5*x^m/(m + 5) + 11*f^m*x^3*x^m/(m + 3) + (f*x)^(m
 + 1)/(f*(m + 1))
```

mupad [B] time = 1.25, size = 1483, normalized size = 7.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)*(f*x)^m*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] (x^3*(f*x)^m*(2192684754645*m + 1434440867211*m^2 + 490955350391*m^3 + 1024
68500970*m^4 + 14014513810*m^5 + 1298935638*m^6 + 82295598*m^7 + 3514005*m^
8 + 96745*m^9 + 1551*m^10 + 11*m^11 + 1159525191825))/(703416314160*m + 590
546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628
301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^1
1 + m^12 + 316234143225) + (x^19*(f*x)^m*(1988025402825*m + 1604842704135*m
^2 + 680615362515*m^3 + 174273100210*m^4 + 28857216410*m^5 + 3194704590*m^6
 + 238653030*m^7 + 11870265*m^8 + 376365*m^9 + 6875*m^10 + 55*m^11 + 915414
625125))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 7257825939
1*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 34
5840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^11*(f*x)^m*(283
36045738770*m + 22226933020446*m^2 + 9079996141062*m^3 + 2222832699780*m^4
+ 349697552820*m^5 + 36597992508*m^6 + 2575140876*m^7 + 120367170*m^8 + 358
2810*m^9 + 61446*m^10 + 462*m^11 + 13281834015450))/(703416314160*m + 59054
6123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 162830
1884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11
+ m^12 + 316234143225) + (x^21*(f*x)^m*(360568238085*m + 292163767533*m^2 +
 124530626231*m^3 + 32087153670*m^4 + 5353566130*m^5 + 597988314*m^6 + 4513
4958*m^7 + 2271555*m^8 + 72985*m^9 + 1353*m^10 + 11*m^11 + 165646455975))/(
703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13
137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 +
 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^5*(f*x)^m*(7041864340665*
m + 5087634488145*m^2 + 1889780020755*m^3 + 420404849150*m^4 + 60431072570*
m^5 + 5825106210*m^6 + 380801190*m^7 + 16677375*m^8 + 468765*m^9 + 7645*m^1
0 + 55*m^11 + 3478575575475))/(703416314160*m + 590546123298*m^2 + 26430062
8944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m
^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225)
 + (x^17*(f*x)^m*(6646727085075*m + 5340787250535*m^2 + 2251106854425*m^3 +
 572017996770*m^4 + 93862508190*m^5 + 10282782510*m^6 + 759091410*m^7 + 372
63105*m^8 + 1164735*m^9 + 20955*m^10 + 165*m^11 + 3069331390125))/(70341631
4160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 1313745840
0*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^
10 + 144*m^11 + m^12 + 316234143225) + (x*(f*x)^m*(387182170935*m + 2033639
52363*m^2 + 60936676581*m^3 + 11641582810*m^4 + 1495875590*m^5 + 132426294*
m^6 + 8103018*m^7 + 336765*m^8 + 9075*m^9 + 143*m^10 + m^11 + 316234143225)
)/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 +
 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^
9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^23*(f*x)^m*(2998552189
5*m + 24372200061*m^2 + 10431670821*m^3 + 2702025590*m^4 + 453714470*m^5 +
51069018*m^6 + 3889578*m^7 + 197835*m^8 + 6435*m^9 + 121*m^10 + m^11 + 1374
9310575))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 725782593
91*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 3
45840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^7*(f*x)^m*(155
15657331075*m + 11703493287585*m^2 + 4558015784025*m^3 + 1059628145070*m^4
+ 158293212990*m^5 + 15768085410*m^6 + 1059893010*m^7 + 47524455*m^8 + 1362
735*m^9 + 22605*m^10 + 165*m^11 + 7454090518875))/(703416314160*m + 5905461
23298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 16283018
84*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 +
m^12 + 316234143225) + (x^15*(f*x)^m*(15011348834790*m + 11991258123570*m^2
 + 5015196628530*m^3 + 1262375264700*m^4 + 204865733820*m^5 + 22164925860*m
^6 + 1613983140*m^7 + 78076350*m^8 + 2403390*m^9 + 42570*m^10 + 330*m^11 +
6957151150950))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 725
78259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m
^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^9*(f*x)^
m*(24503570194950*m + 18930738943710*m^2 + 7587607623090*m^3 + 181813533066
0*m^4 + 279691771260*m^5 + 28627538940*m^6 + 1971903780*m^7 + 90358290*m^8
+ 2640990*m^9 + 44550*m^10 + 330*m^11 + 11595251918250))/(703416314160*m +
590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1
628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*
```

```
m^11 + m^12 + 316234143225) + (x^13*(f*x)^m*(24133835554290*m + 19130651800
722*m^2 + 7921249136262*m^3 + 1969992823260*m^4 + 315347150580*m^5 + 336098
70756*m^6 + 2408820876*m^7 + 114642990*m^8 + 3471930*m^9 + 60522*m^10 + 462
*m^11 + 11238474936150))/(703416314160*m + 590546123298*m^2 + 264300628944*
m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 +
8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] Timed out
```

3.46 $\int x^5 (1+x^2) (1+2x^2+x^4)^5 \, dx$

Optimal. Leaf size=34

$$\frac{1}{28}\left(x^2+1\right)^{14}-\frac{1}{13}\left(x^2+1\right)^{13}+\frac{1}{24}\left(x^2+1\right)^{12}$$

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 266, 43}

$$\frac{1}{28}\left(x^2+1\right)^{14}-\frac{1}{13}\left(x^2+1\right)^{13}+\frac{1}{24}\left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (1 + x^2)^12/24 - (1 + x^2)^13/13 + (1 + x^2)^14/28
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx &= \int x^5 \left(1+x^2\right)^{11} \, dx\\
&= \frac{1}{2}\text{Subst}\left(\int x^2(1+x)^{11} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left((1+x)^{11}-2(1+x)^{12}+(1+x)^{13}\right) \, dx, x, x^2\right)\\
&= \frac{1}{24}\left(1+x^2\right)^{12}-\frac{1}{13}\left(1+x^2\right)^{13}+\frac{1}{28}\left(1+x^2\right)^{14}
\end{aligned}$$

Mathematica [B] time = 0.00, size = 85, normalized size = 2.50

$$\frac{x^{28}}{28}+\frac{11x^{26}}{26}+\frac{55x^{24}}{24}+\frac{15x^{22}}{2}+\frac{33x^{20}}{2}+\frac{77x^{18}}{3}+\frac{231x^{16}}{8}+\frac{165x^{14}}{7}+\frac{55x^{12}}{4}+\frac{11x^{10}}{2}+\frac{11x^8}{8}+\frac{x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^6/6 + (11*x^8)/8 + (11*x^10)/2 + (55*x^12)/4 + (165*x^14)/7 + (231*x^16)/
8 + (77*x^18)/3 + (33*x^20)/2 + (15*x^22)/2 + (55*x^24)/24 + (11*x^26)/26 +
 x^28/28
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] IntegrateAlgebraic[x^5*(1 + x^2)*(1 + 2*x^2 + x^4)^5, x]
```

fricas [B] time = 0.56, size = 61, normalized size = 1.79

$$\frac{1}{28}x^{28}+\frac{11}{26}x^{26}+\frac{55}{24}x^{24}+\frac{15}{2}x^{22}+\frac{33}{2}x^{20}+\frac{77}{3}x^{18}+\frac{231}{8}x^{16}+\frac{165}{7}x^{14}+\frac{55}{4}x^{12}+\frac{11}{2}x^{10}+\frac{11}{8}x^{8}+\frac{1}{6}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/3*x^18 + 2
31/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x^8 + 1/6*x^6
```

giac [B] time = 0.34, size = 61, normalized size = 1.79

$$\frac{1}{28}\,x^{28}+\frac{11}{26}\,x^{26}+\frac{55}{24}\,x^{24}+\frac{15}{2}\,x^{22}+\frac{33}{2}\,x^{20}+\frac{77}{3}\,x^{18}+\frac{231}{8}\,x^{16}+\frac{165}{7}\,x^{14}+\frac{55}{4}\,x^{12}+\frac{11}{2}\,x^{10}+\frac{11}{8}\,x^{8}+\frac{1}{6}\,x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/3*x^18 + 2
31/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x^8 + 1/6*x^6
```

maple [B] time = 0.00, size = 62, normalized size = 1.82

$$\frac{1}{28}x^{28}+\frac{11}{26}x^{26}+\frac{55}{24}x^{24}+\frac{15}{2}x^{22}+\frac{33}{2}x^{20}+\frac{77}{3}x^{18}+\frac{231}{8}x^{16}+\frac{165}{7}x^{14}+\frac{55}{4}x^{12}+\frac{11}{2}x^{10}+\frac{11}{8}x^{8}+\frac{1}{6}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/28*x^28+11/26*x^26+55/24*x^24+15/2*x^22+33/2*x^20+77/3*x^18+231/8*x^16+16
5/7*x^14+55/4*x^12+11/2*x^10+11/8*x^8+1/6*x^6
```

maxima [B] time = 0.64, size = 61, normalized size = 1.79

$$\frac{1}{28}\,x^{28}+\frac{11}{26}\,x^{26}+\frac{55}{24}\,x^{24}+\frac{15}{2}\,x^{22}+\frac{33}{2}\,x^{20}+\frac{77}{3}\,x^{18}+\frac{231}{8}\,x^{16}+\frac{165}{7}\,x^{14}+\frac{55}{4}\,x^{12}+\frac{11}{2}\,x^{10}+\frac{11}{8}\,x^{8}+\frac{1}{6}\,x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/3*x^18 + 2
31/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x^8 + 1/6*x^6
```

mupad [B] time = 0.06, size = 61, normalized size = 1.79

$$\frac{x^{28}}{28}+\frac{11\,x^{26}}{26}+\frac{55\,x^{24}}{24}+\frac{15\,x^{22}}{2}+\frac{33\,x^{20}}{2}+\frac{77\,x^{18}}{3}+\frac{231\,x^{16}}{8}+\frac{165\,x^{14}}{7}+\frac{55\,x^{12}}{4}+\frac{11\,x^{10}}{2}+\frac{11\,x^{8}}{8}+\frac{x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^6/6 + (11*x^8)/8 + (11*x^10)/2 + (55*x^12)/4 + (165*x^14)/7 + (231*x^16)/
8 + (77*x^18)/3 + (33*x^20)/2 + (15*x^22)/2 + (55*x^24)/24 + (11*x^26)/26 +
 x^28/28
```

sympy [B] time = 0.07, size = 76, normalized size = 2.24

$$\frac{x^{28}}{28}+\frac{11x^{26}}{26}+\frac{55x^{24}}{24}+\frac{15x^{22}}{2}+\frac{33x^{20}}{2}+\frac{77x^{18}}{3}+\frac{231x^{16}}{8}+\frac{165x^{14}}{7}+\frac{55x^{12}}{4}+\frac{11x^{10}}{2}+\frac{11x^{8}}{8}+\frac{x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**28/28 + 11*x**26/26 + 55*x**24/24 + 15*x**22/2 + 33*x**20/2 + 77*x**18/3
 + 231*x**16/8 + 165*x**14/7 + 55*x**12/4 + 11*x**10/2 + 11*x**8/8 + x**6/6
```

3.47 $\int x^4\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx$

Optimal. Leaf size=83

$$\frac{x^{27}}{27}+\frac{11x^{25}}{25}+\frac{55x^{23}}{23}+\frac{55x^{21}}{7}+\frac{330x^{19}}{19}+\frac{462x^{17}}{17}+\frac{154x^{15}}{5}+\frac{330x^{13}}{13}+15x^{11}+\frac{55x^9}{9}+\frac{11x^7}{7}+\frac{x^5}{5}$$

Rubi [A] time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {28, 270}

$$\frac{x^{27}}{27}+\frac{11x^{25}}{25}+\frac{55x^{23}}{23}+\frac{55x^{21}}{7}+\frac{330x^{19}}{19}+\frac{462x^{17}}{17}+\frac{154x^{15}}{5}+\frac{330x^{13}}{13}+15x^{11}+\frac{55x^9}{9}+\frac{11x^7}{7}+\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*x^15)/5 +
(462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)/23 + (11*x^25)/25 +
 x^27/27
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx &= \int x^4\left(1+x^2\right)^{11}\,dx\\ &= \int \left(x^4+11x^6+55x^8+165x^{10}+330x^{12}+462x^{14}+462x^{16}+330x^{18}+165x\right.\\ &= \frac{x^5}{5}+\frac{11x^7}{7}+\frac{55x^9}{9}+15x^{11}+\frac{330x^{13}}{13}+\frac{154x^{15}}{5}+\frac{462x^{17}}{17}+\frac{330x^{19}}{19}+\frac{55x^{21}}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 83, normalized size = 1.00

$$\frac{x^{27}}{27}+\frac{11x^{25}}{25}+\frac{55x^{23}}{23}+\frac{55x^{21}}{7}+\frac{330x^{19}}{19}+\frac{462x^{17}}{17}+\frac{154x^{15}}{5}+\frac{330x^{13}}{13}+15x^{11}+\frac{55x^9}{9}+\frac{11x^7}{7}+\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*x^15)/5 +
(462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)/23 + (11*x^25)/25 +
 x^27/27
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] IntegrateAlgebraic[x^4*(1 + x^2)*(1 + 2*x^2 + x^4)^5, x]
```

fricas [A] time = 0.76, size = 61, normalized size = 0.73

$$\frac{1}{27}x^{27} + \frac{11}{25}x^{25} + \frac{55}{23}x^{23} + \frac{55}{7}x^{21} + \frac{330}{19}x^{19} + \frac{462}{17}x^{17} + \frac{154}{5}x^{15} + \frac{330}{13}x^{13} + 15x^{11} + \frac{55}{9}x^{9} + \frac{11}{7}x^{7} + \frac{1}{5}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 462/17*x^17
 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7*x^7 + 1/5*x^5
```

giac [A] time = 0.41, size = 61, normalized size = 0.73

$$\frac{1}{27}x^{27} + \frac{11}{25}x^{25} + \frac{55}{23}x^{23} + \frac{55}{7}x^{21} + \frac{330}{19}x^{19} + \frac{462}{17}x^{17} + \frac{154}{5}x^{15} + \frac{330}{13}x^{13} + 15x^{11} + \frac{55}{9}x^{9} + \frac{11}{7}x^{7} + \frac{1}{5}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 462/17*x^17
 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7*x^7 + 1/5*x^5
```

maple [A] time = 0.00, size = 62, normalized size = 0.75

$$\frac{1}{27}x^{27} + \frac{11}{25}x^{25} + \frac{55}{23}x^{23} + \frac{55}{7}x^{21} + \frac{330}{19}x^{19} + \frac{462}{17}x^{17} + \frac{154}{5}x^{15} + \frac{330}{13}x^{13} + 15x^{11} + \frac{55}{9}x^{9} + \frac{11}{7}x^{7} + \frac{1}{5}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/5*x^5+11/7*x^7+55/9*x^9+15*x^11+330/13*x^13+154/5*x^15+462/17*x^17+330/19
*x^19+55/7*x^21+55/23*x^23+11/25*x^25+1/27*x^27
```

maxima [A] time = 0.76, size = 61, normalized size = 0.73

$$\frac{1}{27}x^{27} + \frac{11}{25}x^{25} + \frac{55}{23}x^{23} + \frac{55}{7}x^{21} + \frac{330}{19}x^{19} + \frac{462}{17}x^{17} + \frac{154}{5}x^{15} + \frac{330}{13}x^{13} + 15x^{11} + \frac{55}{9}x^{9} + \frac{11}{7}x^{7} + \frac{1}{5}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 462/17*x^17
 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7*x^7 + 1/5*x^5
```

mupad [B] time = 0.06, size = 61, normalized size = 0.73

$$\frac{x^{27}}{27} + \frac{11\,x^{25}}{25} + \frac{55\,x^{23}}{23} + \frac{55\,x^{21}}{7} + \frac{330\,x^{19}}{19} + \frac{462\,x^{17}}{17} + \frac{154\,x^{15}}{5} + \frac{330\,x^{13}}{13} + 15\,x^{11} + \frac{55\,x^{9}}{9} + \frac{11\,x^{7}}{7} + \frac{x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*x^15)/5 +
(462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)/23 + (11*x^25)/25 +
 x^27/27
```

sympy [A] time = 0.07, size = 75, normalized size = 0.90

$$\frac{x^{27}}{27}+\frac{11x^{25}}{25}+\frac{55x^{23}}{23}+\frac{55x^{21}}{7}+\frac{330x^{19}}{19}+\frac{462x^{17}}{17}+\frac{154x^{15}}{5}+\frac{330x^{13}}{13}+15x^{11}+\frac{55x^{9}}{9}+\frac{11x^{7}}{7}+\frac{x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**27/27 + 11*x**25/25 + 55*x**23/23 + 55*x**21/7 + 330*x**19/19 + 462*x**1
7/17 + 154*x**15/5 + 330*x**13/13 + 15*x**11 + 55*x**9/9 + 11*x**7/7 + x**5
/5
```

3.48 $\int x^3 \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=23

$$\frac{1}{26}\left(x^2+1\right)^{13}-\frac{1}{24}\left(x^2+1\right)^{12}$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 266, 43}

$$\frac{1}{26}\left(x^2+1\right)^{13}-\frac{1}{24}\left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] -(1 + x^2)^12/24 + (1 + x^2)^13/26
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx &= \int x^3 \left(1+x^2\right)^{11} \, dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int x(1+x)^{11} \, dx, x, x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-(1+x)^{11}+(1+x)^{12}\right) \, dx, x, x^2\right) \\
&= -\frac{1}{24}\left(1+x^2\right)^{12}+\frac{1}{26}\left(1+x^2\right)^{13}
\end{aligned}$$

Mathematica [B] time = 0.00, size = 83, normalized size = 3.61

$$\frac{x^{26}}{26}+\frac{11x^{24}}{24}+\frac{5x^{22}}{2}+\frac{33x^{20}}{4}+\frac{55x^{18}}{3}+\frac{231x^{16}}{8}+33x^{14}+\frac{55x^{12}}{2}+\frac{33x^{10}}{2}+\frac{55x^8}{8}+\frac{11x^6}{6}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^4/4 + (11*x^6)/6 + (55*x^8)/8 + (33*x^10)/2 + (55*x^12)/2 + 33*x^14 + (23
1*x^16)/8 + (55*x^18)/3 + (33*x^20)/4 + (5*x^22)/2 + (11*x^24)/24 + x^26/26
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] IntegrateAlgebraic[x^3*(1 + x^2)*(1 + 2*x^2 + x^4)^5, x]
```

fricas [B] time = 0.86, size = 61, normalized size = 2.65

$$\frac{1}{26}x^{26} + \frac{11}{24}x^{24} + \frac{5}{2}x^{22} + \frac{33}{4}x^{20} + \frac{55}{3}x^{18} + \frac{231}{8}x^{16} + 33x^{14} + \frac{55}{2}x^{12} + \frac{33}{2}x^{10} + \frac{55}{8}x^{8} + \frac{11}{6}x^{6} + \frac{1}{4}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8*x^16 + 33
*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1/4*x^4
```

giac [B] time = 0.36, size = 61, normalized size = 2.65

$$\frac{1}{26}\,x^{26} + \frac{11}{24}\,x^{24} + \frac{5}{2}\,x^{22} + \frac{33}{4}\,x^{20} + \frac{55}{3}\,x^{18} + \frac{231}{8}\,x^{16} + 33\,x^{14} + \frac{55}{2}\,x^{12} + \frac{33}{2}\,x^{10} + \frac{55}{8}\,x^{8} + \frac{11}{6}\,x^{6} + \frac{1}{4}\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8*x^16 + 33
*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1/4*x^4
```

maple [B] time = 0.00, size = 62, normalized size = 2.70

$$\frac{1}{26}x^{26} + \frac{11}{24}x^{24} + \frac{5}{2}x^{22} + \frac{33}{4}x^{20} + \frac{55}{3}x^{18} + \frac{231}{8}x^{16} + 33x^{14} + \frac{55}{2}x^{12} + \frac{33}{2}x^{10} + \frac{55}{8}x^{8} + \frac{11}{6}x^{6} + \frac{1}{4}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/26*x^26+11/24*x^24+5/2*x^22+33/4*x^20+55/3*x^18+231/8*x^16+33*x^14+55/2*x
^12+33/2*x^10+55/8*x^8+11/6*x^6+1/4*x^4
```

maxima [B] time = 0.56, size = 61, normalized size = 2.65

$$\frac{1}{26}\,x^{26} + \frac{11}{24}\,x^{24} + \frac{5}{2}\,x^{22} + \frac{33}{4}\,x^{20} + \frac{55}{3}\,x^{18} + \frac{231}{8}\,x^{16} + 33\,x^{14} + \frac{55}{2}\,x^{12} + \frac{33}{2}\,x^{10} + \frac{55}{8}\,x^{8} + \frac{11}{6}\,x^{6} + \frac{1}{4}\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8*x^16 + 33
*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1/4*x^4
```

mupad [B] time = 0.06, size = 61, normalized size = 2.65

$$\frac{x^{26}}{26} + \frac{11\,x^{24}}{24} + \frac{5\,x^{22}}{2} + \frac{33\,x^{20}}{4} + \frac{55\,x^{18}}{3} + \frac{231\,x^{16}}{8} + 33\,x^{14} + \frac{55\,x^{12}}{2} + \frac{33\,x^{10}}{2} + \frac{55\,x^{8}}{8} + \frac{11\,x^{6}}{6} + \frac{x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^4/4 + (11*x^6)/6 + (55*x^8)/8 + (33*x^10)/2 + (55*x^12)/2 + 33*x^14 + (23
1*x^16)/8 + (55*x^18)/3 + (33*x^20)/4 + (5*x^22)/2 + (11*x^24)/24 + x^26/26
```

sympy [B] time = 0.07, size = 75, normalized size = 3.26

$$\frac{x^{26}}{26} + \frac{11x^{24}}{24} + \frac{5x^{22}}{2} + \frac{33x^{20}}{4} + \frac{55x^{18}}{3} + \frac{231x^{16}}{8} + 33x^{14} + \frac{55x^{12}}{2} + \frac{33x^{10}}{2} + \frac{55x^{8}}{8} + \frac{11x^{6}}{6} + \frac{x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**26/26 + 11*x**24/24 + 5*x**22/2 + 33*x**20/4 + 55*x**18/3 + 231*x**16/8
+ 33*x**14 + 55*x**12/2 + 33*x**10/2 + 55*x**8/8 + 11*x**6/6 + x**4/4
```

3.49 $\int x^2 \left(1+x^2\right) \left(1+2 x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=83

$$\frac{x^{25}}{25}+\frac{11 x^{23}}{23}+\frac{55 x^{21}}{21}+\frac{165 x^{19}}{19}+\frac{330 x^{17}}{17}+\frac{154 x^{15}}{5}+\frac{462 x^{13}}{13}+30 x^{11}+\frac{55 x^9}{3}+\frac{55 x^7}{7}+\frac{11 x^5}{5}+\frac{x^3}{3}$$

Rubi [A] time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {28, 270}

$$\frac{x^{25}}{25}+\frac{11 x^{23}}{23}+\frac{55 x^{21}}{21}+\frac{165 x^{19}}{19}+\frac{330 x^{17}}{17}+\frac{154 x^{15}}{5}+\frac{462 x^{13}}{13}+30 x^{11}+\frac{55 x^9}{3}+\frac{55 x^7}{7}+\frac{11 x^5}{5}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^13)/13 + (1
54*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/21 + (11*x^23)/23 +
x^25/25
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int x^2 \left(1+x^2\right) \left(1+2 x^2+x^4\right)^5 \, dx &= \int x^2 \left(1+x^2\right)^{11} \, dx\\ &= \int \left(x^2+11 x^4+55 x^6+165 x^8+330 x^{10}+462 x^{12}+462 x^{14}+330 x^{16}+165 x^1\right.\\ &= \frac{x^3}{3}+\frac{11 x^5}{5}+\frac{55 x^7}{7}+\frac{55 x^9}{3}+30 x^{11}+\frac{462 x^{13}}{13}+\frac{154 x^{15}}{5}+\frac{330 x^{17}}{17}+\frac{165 x^{19}}{19}+ \end{aligned}$$

Mathematica [A] time = 0.00, size = 83, normalized size = 1.00

$$\frac{x^{25}}{25}+\frac{11 x^{23}}{23}+\frac{55 x^{21}}{21}+\frac{165 x^{19}}{19}+\frac{330 x^{17}}{17}+\frac{154 x^{15}}{5}+\frac{462 x^{13}}{13}+30 x^{11}+\frac{55 x^9}{3}+\frac{55 x^7}{7}+\frac{11 x^5}{5}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^13)/13 + (1
54*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/21 + (11*x^23)/23 +
x^25/25
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(1+x^2\right) \left(1+2 x^2+x^4\right)^5 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] IntegrateAlgebraic[x^2*(1 + x^2)*(1 + 2*x^2 + x^4)^5, x]
```

fricas [A] time = 0.57, size = 61, normalized size = 0.73

$$\frac{1}{25}x^{25} + \frac{11}{23}x^{23} + \frac{55}{21}x^{21} + \frac{165}{19}x^{19} + \frac{330}{17}x^{17} + \frac{154}{5}x^{15} + \frac{462}{13}x^{13} + 30x^{11} + \frac{55}{3}x^9 + \frac{55}{7}x^7 + \frac{11}{5}x^5 + \frac{1}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 + 154/5*x^1
5 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*x^5 + 1/3*x^3
```

giac [A] time = 0.30, size = 61, normalized size = 0.73

$$\frac{1}{25}x^{25} + \frac{11}{23}x^{23} + \frac{55}{21}x^{21} + \frac{165}{19}x^{19} + \frac{330}{17}x^{17} + \frac{154}{5}x^{15} + \frac{462}{13}x^{13} + 30x^{11} + \frac{55}{3}x^9 + \frac{55}{7}x^7 + \frac{11}{5}x^5 + \frac{1}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 + 154/5*x^1
5 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*x^5 + 1/3*x^3
```

maple [A] time = 0.00, size = 62, normalized size = 0.75

$$\frac{1}{25}x^{25} + \frac{11}{23}x^{23} + \frac{55}{21}x^{21} + \frac{165}{19}x^{19} + \frac{330}{17}x^{17} + \frac{154}{5}x^{15} + \frac{462}{13}x^{13} + 30x^{11} + \frac{55}{3}x^9 + \frac{55}{7}x^7 + \frac{11}{5}x^5 + \frac{1}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/3*x^3+11/5*x^5+55/7*x^7+55/3*x^9+30*x^11+462/13*x^13+154/5*x^15+330/17*x^
17+165/19*x^19+55/21*x^21+11/23*x^23+1/25*x^25
```

maxima [A] time = 0.88, size = 61, normalized size = 0.73

$$\frac{1}{25}x^{25} + \frac{11}{23}x^{23} + \frac{55}{21}x^{21} + \frac{165}{19}x^{19} + \frac{330}{17}x^{17} + \frac{154}{5}x^{15} + \frac{462}{13}x^{13} + 30x^{11} + \frac{55}{3}x^9 + \frac{55}{7}x^7 + \frac{11}{5}x^5 + \frac{1}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 + 154/5*x^1
5 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*x^5 + 1/3*x^3
```

mupad [B] time = 0.06, size = 61, normalized size = 0.73

$$\frac{x^{25}}{25} + \frac{11\,x^{23}}{23} + \frac{55\,x^{21}}{21} + \frac{165\,x^{19}}{19} + \frac{330\,x^{17}}{17} + \frac{154\,x^{15}}{5} + \frac{462\,x^{13}}{13} + 30\,x^{11} + \frac{55\,x^9}{3} + \frac{55\,x^7}{7} + \frac{11\,x^5}{5} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^13)/13 + (1
54*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/21 + (11*x^23)/23 +
x^25/25
```

sympy [A] time = 0.07, size = 75, normalized size = 0.90

$$\frac{x^{25}}{25} + \frac{11x^{23}}{23} + \frac{55x^{21}}{21} + \frac{165x^{19}}{19} + \frac{330x^{17}}{17} + \frac{154x^{15}}{5} + \frac{462x^{13}}{13} + 30x^{11} + \frac{55x^9}{3} + \frac{55x^7}{7} + \frac{11x^5}{5} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**25/25 + 11*x**23/23 + 55*x**21/21 + 165*x**19/19 + 330*x**17/17 + 154*x*
*15/5 + 462*x**13/13 + 30*x**11 + 55*x**9/3 + 55*x**7/7 + 11*x**5/5 + x**3/
3
```

3.50 $\int x\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx$

Optimal. Leaf size=11

$$\frac{1}{24}\left(x^2+1\right)^{12}$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {28, 261}

$$\frac{1}{24}\left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (1 + x^2)^12/24
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx &= \int x\left(1+x^2\right)^{11}\,dx\\ &= \frac{1}{24}\left(1+x^2\right)^{12}\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{1}{24}\left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (1 + x^2)^12/24
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] IntegrateAlgebraic[x*(1 + x^2)*(1 + 2*x^2 + x^4)^5, x]
```

fricas [B] time = 0.51, size = 61, normalized size = 5.55

$$\frac{1}{24}x^{24} + \frac{1}{2}x^{22} + \frac{11}{4}x^{20} + \frac{55}{6}x^{18} + \frac{165}{8}x^{16} + 33x^{14} + \frac{77}{2}x^{12} + 33x^{10} + \frac{165}{8}x^{8} + \frac{55}{6}x^{6} + \frac{11}{4}x^{4} + \frac{1}{2}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/24*x^24 + 1/2*x^22 + 11/4*x^20 + 55/6*x^18 + 165/8*x^16 + 33*x^14 + 77/2*
x^12 + 33*x^10 + 165/8*x^8 + 55/6*x^6 + 11/4*x^4 + 1/2*x^2
```

giac [B] time = 0.31, size = 76, normalized size = 6.91

$$\frac{1}{24}\left(x^{4} + 2x^{2}\right)^{6} + \frac{1}{4}\left(x^{4} + 2x^{2}\right)^{5} + \frac{5}{8}\left(x^{4} + 2x^{2}\right)^{4} + \frac{1}{4}x^{4} + \frac{5}{6}\left(x^{4} + 2x^{2}\right)^{3} + \frac{5}{8}\left(x^{4} + 2x^{2}\right)^{2} + \frac{1}{2}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/24*(x^4 + 2*x^2)^6 + 1/4*(x^4 + 2*x^2)^5 + 5/8*(x^4 + 2*x^2)^4 + 1/4*x^4
+ 5/6*(x^4 + 2*x^2)^3 + 5/8*(x^4 + 2*x^2)^2 + 1/2*x^2
```

maple [B] time = 0.00, size = 62, normalized size = 5.64

$$\frac{1}{24}x^{24} + \frac{1}{2}x^{22} + \frac{11}{4}x^{20} + \frac{55}{6}x^{18} + \frac{165}{8}x^{16} + 33x^{14} + \frac{77}{2}x^{12} + 33x^{10} + \frac{165}{8}x^{8} + \frac{55}{6}x^{6} + \frac{11}{4}x^{4} + \frac{1}{2}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/24*x^24+1/2*x^22+11/4*x^20+55/6*x^18+165/8*x^16+33*x^14+77/2*x^12+33*x^10
+165/8*x^8+55/6*x^6+11/4*x^4+1/2*x^2
```

maxima [B] time = 0.78, size = 61, normalized size = 5.55

$$\frac{1}{24}x^{24} + \frac{1}{2}x^{22} + \frac{11}{4}x^{20} + \frac{55}{6}x^{18} + \frac{165}{8}x^{16} + 33x^{14} + \frac{77}{2}x^{12} + 33x^{10} + \frac{165}{8}x^{8} + \frac{55}{6}x^{6} + \frac{11}{4}x^{4} + \frac{1}{2}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/24*x^24 + 1/2*x^22 + 11/4*x^20 + 55/6*x^18 + 165/8*x^16 + 33*x^14 + 77/2*
x^12 + 33*x^10 + 165/8*x^8 + 55/6*x^6 + 11/4*x^4 + 1/2*x^2
```

mupad [B] time = 0.06, size = 61, normalized size = 5.55

$$\frac{x^{24}}{24} + \frac{x^{22}}{2} + \frac{11\,x^{20}}{4} + \frac{55\,x^{18}}{6} + \frac{165\,x^{16}}{8} + 33\,x^{14} + \frac{77\,x^{12}}{2} + 33\,x^{10} + \frac{165\,x^{8}}{8} + \frac{55\,x^{6}}{6} + \frac{11\,x^{4}}{4} + \frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^2/2 + (11*x^4)/4 + (55*x^6)/6 + (165*x^8)/8 + 33*x^10 + (77*x^12)/2 + 33*
x^14 + (165*x^16)/8 + (55*x^18)/6 + (11*x^20)/4 + x^22/2 + x^24/24
```

sympy [B] time = 0.07, size = 71, normalized size = 6.45

$$\frac{x^{24}}{24} + \frac{x^{22}}{2} + \frac{11x^{20}}{4} + \frac{55x^{18}}{6} + \frac{165x^{16}}{8} + 33x^{14} + \frac{77x^{12}}{2} + 33x^{10} + \frac{165x^{8}}{8} + \frac{55x^{6}}{6} + \frac{11x^{4}}{4} + \frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**24/24 + x**22/2 + 11*x**20/4 + 55*x**18/6 + 165*x**16/8 + 33*x**14 + 77*
x**12/2 + 33*x**10 + 165*x**8/8 + 55*x**6/6 + 11*x**4/4 + x**2/2
```

3.51 $\int \left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx$

Optimal. Leaf size=73

$$\frac{x^{23}}{23}+\frac{11x^{21}}{21}+\frac{55x^{19}}{19}+\frac{165x^{17}}{17}+22x^{15}+\frac{462x^{13}}{13}+42x^{11}+\frac{110x^9}{3}+\frac{165x^7}{7}+11x^5+\frac{11x^3}{3}+x$$

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {28, 194}

$$\frac{x^{23}}{23}+\frac{11x^{21}}{21}+\frac{55x^{19}}{19}+\frac{165x^{17}}{17}+22x^{15}+\frac{462x^{13}}{13}+42x^{11}+\frac{110x^9}{3}+\frac{165x^7}{7}+11x^5+\frac{11x^3}{3}+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^11 + (462*x^13)/
13 + 22*x^15 + (165*x^17)/17 + (55*x^19)/19 + (11*x^21)/21 + x^23/23
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx &= \int \left(1+x^2\right)^{11}\,dx\\ &= \int \left(1+11x^2+55x^4+165x^6+330x^8+462x^{10}+462x^{12}+330x^{14}+165x^{16}+5\right.\\ &= x+\frac{11x^3}{3}+11x^5+\frac{165x^7}{7}+\frac{110x^9}{3}+42x^{11}+\frac{462x^{13}}{13}+22x^{15}+\frac{165x^{17}}{17}+\frac{55x^{19}}{19}\end{aligned}$$

Mathematica [A] time = 0.00, size = 73, normalized size = 1.00

$$\frac{x^{23}}{23}+\frac{11x^{21}}{21}+\frac{55x^{19}}{19}+\frac{165x^{17}}{17}+22x^{15}+\frac{462x^{13}}{13}+42x^{11}+\frac{110x^9}{3}+\frac{165x^7}{7}+11x^5+\frac{11x^3}{3}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^11 + (462*x^13)/
13 + 22*x^15 + (165*x^17)/17 + (55*x^19)/19 + (11*x^21)/21 + x^23/23
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] IntegrateAlgebraic[(1 + x^2)*(1 + 2*x^2 + x^4)^5, x]
```

fricas [A] time = 1.10, size = 57, normalized size = 0.78

$$\frac{1}{23}x^{23} + \frac{11}{21}x^{21} + \frac{55}{19}x^{19} + \frac{165}{17}x^{17} + 22x^{15} + \frac{462}{13}x^{13} + 42x^{11} + \frac{110}{3}x^{9} + \frac{165}{7}x^{7} + 11x^{5} + \frac{11}{3}x^{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/23*x^23 + 11/21*x^21 + 55/19*x^19 + 165/17*x^17 + 22*x^15 + 462/13*x^13 +
 42*x^11 + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 + x
```

giac [A] time = 0.27, size = 57, normalized size = 0.78

$$\frac{1}{23}\,x^{23} + \frac{11}{21}\,x^{21} + \frac{55}{19}\,x^{19} + \frac{165}{17}\,x^{17} + 22\,x^{15} + \frac{462}{13}\,x^{13} + 42\,x^{11} + \frac{110}{3}\,x^{9} + \frac{165}{7}\,x^{7} + 11\,x^{5} + \frac{11}{3}\,x^{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/23*x^23 + 11/21*x^21 + 55/19*x^19 + 165/17*x^17 + 22*x^15 + 462/13*x^13 +
 42*x^11 + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 + x
```

maple [A] time = 0.00, size = 58, normalized size = 0.79

$$\frac{1}{23}x^{23} + \frac{11}{21}x^{21} + \frac{55}{19}x^{19} + \frac{165}{17}x^{17} + 22x^{15} + \frac{462}{13}x^{13} + 42x^{11} + \frac{110}{3}x^{9} + \frac{165}{7}x^{7} + 11x^{5} + \frac{11}{3}x^{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] x+11/3*x^3+11*x^5+165/7*x^7+110/3*x^9+42*x^11+462/13*x^13+22*x^15+165/17*x^
17+55/19*x^19+11/21*x^21+1/23*x^23
```

maxima [A] time = 0.98, size = 57, normalized size = 0.78

$$\frac{1}{23}\,x^{23} + \frac{11}{21}\,x^{21} + \frac{55}{19}\,x^{19} + \frac{165}{17}\,x^{17} + 22\,x^{15} + \frac{462}{13}\,x^{13} + 42\,x^{11} + \frac{110}{3}\,x^{9} + \frac{165}{7}\,x^{7} + 11\,x^{5} + \frac{11}{3}\,x^{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/23*x^23 + 11/21*x^21 + 55/19*x^19 + 165/17*x^17 + 22*x^15 + 462/13*x^13 +
 42*x^11 + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 + x
```

mupad [B] time = 0.06, size = 57, normalized size = 0.78

$$\frac{x^{23}}{23} + \frac{11\,x^{21}}{21} + \frac{55\,x^{19}}{19} + \frac{165\,x^{17}}{17} + 22\,x^{15} + \frac{462\,x^{13}}{13} + 42\,x^{11} + \frac{110\,x^{9}}{3} + \frac{165\,x^{7}}{7} + 11\,x^{5} + \frac{11\,x^{3}}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^11 + (462*x^13)/
13 + 22*x^15 + (165*x^17)/17 + (55*x^19)/19 + (11*x^21)/21 + x^23/23
```

sympy [A] time = 0.07, size = 68, normalized size = 0.93

$$\frac{x^{23}}{23} + \frac{11x^{21}}{21} + \frac{55x^{19}}{19} + \frac{165x^{17}}{17} + 22x^{15} + \frac{462x^{13}}{13} + 42x^{11} + \frac{110x^{9}}{3} + \frac{165x^{7}}{7} + 11x^{5} + \frac{11x^{3}}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**23/23 + 11*x**21/21 + 55*x**19/19 + 165*x**17/17 + 22*x**15 + 462*x**13/
13 + 42*x**11 + 110*x**9/3 + 165*x**7/7 + 11*x**5 + 11*x**3/3 + x
```

3.52 $\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x} \, dx$

Optimal. Leaf size=80

$$\frac{x^{22}}{22}+\frac{11x^{20}}{20}+\frac{55x^{18}}{18}+\frac{165x^{16}}{16}+\frac{165x^{14}}{7}+\frac{77x^{12}}{2}+\frac{231x^{10}}{5}+\frac{165x^8}{4}+\frac{55x^6}{2}+\frac{55x^4}{4}+\frac{11x^2}{2}+\log(x)$$

Rubi [A] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 266, 43}

$$\frac{x^{22}}{22}+\frac{11x^{20}}{20}+\frac{55x^{18}}{18}+\frac{165x^{16}}{16}+\frac{165x^{14}}{7}+\frac{77x^{12}}{2}+\frac{231x^{10}}{5}+\frac{165x^8}{4}+\frac{55x^6}{2}+\frac{55x^4}{4}+\frac{11x^2}{2}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] (11*x^2)/2 + (55*x^4)/4 + (55*x^6)/2 + (165*x^8)/4 + (231*x^10)/5 + (77*x^1
2)/2 + (165*x^14)/7 + (165*x^16)/16 + (55*x^18)/18 + (11*x^20)/20 + x^22/22
 + Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x} \, dx &= \int \frac{(1+x^2)^{11}}{x} \, dx \\ &= \frac{1}{2}\text{Subst}\left(\int \frac{(1+x)^{11}}{x} \, dx, x, x^2\right) \\ &= \frac{1}{2}\text{Subst}\left(\int \left(11+\frac{1}{x}+55x+165x^2+330x^3+462x^4+462x^5+330x^6+165\right.\right. \\ &= \frac{11x^2}{2}+\frac{55x^4}{4}+\frac{55x^6}{2}+\frac{165x^8}{4}+\frac{231x^{10}}{5}+\frac{77x^{12}}{2}+\frac{165x^{14}}{7}+\frac{165x^{16}}{16}+\frac{55x^1}{18}\end{aligned}$$

Mathematica [A] time = 0.00, size = 80, normalized size = 1.00

$$\frac{x^{22}}{22}+\frac{11x^{20}}{20}+\frac{55x^{18}}{18}+\frac{165x^{16}}{16}+\frac{165x^{14}}{7}+\frac{77x^{12}}{2}+\frac{231x^{10}}{5}+\frac{165x^8}{4}+\frac{55x^6}{2}+\frac{55x^4}{4}+\frac{11x^2}{2}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] (11*x^2)/2 + (55*x^4)/4 + (55*x^6)/2 + (165*x^8)/4 + (231*x^10)/5 + (77*x^1
2)/2 + (165*x^14)/7 + (165*x^16)/16 + (55*x^18)/18 + (11*x^20)/20 + x^22/22
 + Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] IntegrateAlgebraic[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x, x]
```

fricas [A] time = 0.85, size = 58, normalized size = 0.72

$$\frac{1}{22}x^{22}+\frac{11}{20}x^{20}+\frac{55}{18}x^{18}+\frac{165}{16}x^{16}+\frac{165}{7}x^{14}+\frac{77}{2}x^{12}+\frac{231}{5}x^{10}+\frac{165}{4}x^{8}+\frac{55}{2}x^{6}+\frac{55}{4}x^{4}+\frac{11}{2}x^{2}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x,x, algorithm="fricas")
```

```
[Out] 1/22*x^22 + 11/20*x^20 + 55/18*x^18 + 165/16*x^16 + 165/7*x^14 + 77/2*x^12
+ 231/5*x^10 + 165/4*x^8 + 55/2*x^6 + 55/4*x^4 + 11/2*x^2 + log(x)
```

giac [A] time = 0.23, size = 62, normalized size = 0.78

$$\frac{1}{22}x^{22}+\frac{11}{20}x^{20}+\frac{55}{18}x^{18}+\frac{165}{16}x^{16}+\frac{165}{7}x^{14}+\frac{77}{2}x^{12}+\frac{231}{5}x^{10}+\frac{165}{4}x^{8}+\frac{55}{2}x^{6}+\frac{55}{4}x^{4}+\frac{11}{2}x^{2}+\frac{1}{2}\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x,x, algorithm="giac")
```

```
[Out] 1/22*x^22 + 11/20*x^20 + 55/18*x^18 + 165/16*x^16 + 165/7*x^14 + 77/2*x^12
+ 231/5*x^10 + 165/4*x^8 + 55/2*x^6 + 55/4*x^4 + 11/2*x^2 + 1/2*log(x^2)
```

maple [A] time = 0.00, size = 59, normalized size = 0.74

$$\frac{x^{22}}{22}+\frac{11x^{20}}{20}+\frac{55x^{18}}{18}+\frac{165x^{16}}{16}+\frac{165x^{14}}{7}+\frac{77x^{12}}{2}+\frac{231x^{10}}{5}+\frac{165x^{8}}{4}+\frac{55x^{6}}{2}+\frac{55x^{4}}{4}+\frac{11x^{2}}{2}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^4+2*x^2+1)^5/x,x)
```

```
[Out] 11/2*x^2+55/4*x^4+55/2*x^6+165/4*x^8+231/5*x^10+77/2*x^12+165/7*x^14+165/16
*x^16+55/18*x^18+11/20*x^20+1/22*x^22+ln(x)
```

maxima [A] time = 0.82, size = 62, normalized size = 0.78

$$\frac{1}{22}x^{22}+\frac{11}{20}x^{20}+\frac{55}{18}x^{18}+\frac{165}{16}x^{16}+\frac{165}{7}x^{14}+\frac{77}{2}x^{12}+\frac{231}{5}x^{10}+\frac{165}{4}x^{8}+\frac{55}{2}x^{6}+\frac{55}{4}x^{4}+\frac{11}{2}x^{2}+\frac{1}{2}\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x,x, algorithm="maxima")
```

```
[Out] 1/22*x^22 + 11/20*x^20 + 55/18*x^18 + 165/16*x^16 + 165/7*x^14 + 77/2*x^12
+ 231/5*x^10 + 165/4*x^8 + 55/2*x^6 + 55/4*x^4 + 11/2*x^2 + 1/2*log(x^2)
```

mupad [B] time = 0.06, size = 58, normalized size = 0.72

$$\ln(x) + \frac{11\,x^2}{2} + \frac{55\,x^4}{4} + \frac{55\,x^6}{2} + \frac{165\,x^8}{4} + \frac{231\,x^{10}}{5} + \frac{77\,x^{12}}{2} + \frac{165\,x^{14}}{7} + \frac{165\,x^{16}}{16} + \frac{55\,x^{18}}{18} + \frac{11\,x^{20}}{20} + \frac{x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)*(2*x^2 + x^4 + 1)^5)/x,x)
```

```
[Out] log(x) + (11*x^2)/2 + (55*x^4)/4 + (55*x^6)/2 + (165*x^8)/4 + (231*x^10)/5
+ (77*x^12)/2 + (165*x^14)/7 + (165*x^16)/16 + (55*x^18)/18 + (11*x^20)/20
+ x^22/22
```

sympy [A] time = 0.11, size = 75, normalized size = 0.94

$$\frac{x^{22}}{22} + \frac{11x^{20}}{20} + \frac{55x^{18}}{18} + \frac{165x^{16}}{16} + \frac{165x^{14}}{7} + \frac{77x^{12}}{2} + \frac{231x^{10}}{5} + \frac{165x^8}{4} + \frac{55x^6}{2} + \frac{55x^4}{4} + \frac{11x^2}{2} + \log{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**4+2*x**2+1)**5/x,x)
```

```
[Out] x**22/22 + 11*x**20/20 + 55*x**18/18 + 165*x**16/16 + 165*x**14/7 + 77*x**1
2/2 + 231*x**10/5 + 165*x**8/4 + 55*x**6/2 + 55*x**4/4 + 11*x**2/2 + log(x)
```

3.53 $\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2}\,dx$

Optimal. Leaf size=73

$$\frac{x^{21}}{21}+\frac{11x^{19}}{19}+\frac{55x^{17}}{17}+11x^{15}+\frac{330x^{13}}{13}+42x^{11}+\frac{154x^9}{3}+\frac{330x^7}{7}+33x^5+\frac{55x^3}{3}+11x-\frac{1}{x}$$

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {28, 270}

$$\frac{x^{21}}{21}+\frac{11x^{19}}{19}+\frac{55x^{17}}{17}+11x^{15}+\frac{330x^{13}}{13}+42x^{11}+\frac{154x^9}{3}+\frac{330x^7}{7}+33x^5+\frac{55x^3}{3}+11x-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -x^(-1) + 11*x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3 + 42*x^11
+ (330*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19 + x^21/21
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2}\,dx &= \int \frac{(1+x^2)^{11}}{x^2}\,dx\\ &= \int \left(11+\frac{1}{x^2}+55x^2+165x^4+330x^6+462x^8+462x^{10}+330x^{12}+165x^{14}+55\right.\\ &= -\frac{1}{x}+11x+\frac{55x^3}{3}+33x^5+\frac{330x^7}{7}+\frac{154x^9}{3}+42x^{11}+\frac{330x^{13}}{13}+11x^{15}+\frac{55x^{17}}{17}\end{aligned}$$

Mathematica [A] time = 0.00, size = 73, normalized size = 1.00

$$\frac{x^{21}}{21}+\frac{11x^{19}}{19}+\frac{55x^{17}}{17}+11x^{15}+\frac{330x^{13}}{13}+42x^{11}+\frac{154x^9}{3}+\frac{330x^7}{7}+33x^5+\frac{55x^3}{3}+11x-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -x^(-1) + 11*x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3 + 42*x^11
+ (330*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19 + x^21/21
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] IntegrateAlgebraic[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^2, x]
```

fricas [A] time = 0.79, size = 62, normalized size = 0.85

$$\frac{4199\,x^{22}+51051\,x^{20}+285285\,x^{18}+969969\,x^{16}+2238390\,x^{14}+3703518\,x^{12}+4526522\,x^{10}+4157010\,x^{8}+2909907\,x^{6}+1616615\,x^{4}+969969\,x^{2}-88179}{88179\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/88179*(4199*x^22 + 51051*x^20 + 285285*x^18 + 969969*x^16 + 2238390*x^14
+ 3703518*x^12 + 4526522*x^10 + 4157010*x^8 + 2909907*x^6 + 1616615*x^4 + 9
69969*x^2 - 88179)/x
```

giac [A] time = 0.30, size = 59, normalized size = 0.81

$$\frac{1}{21}\,x^{21}+\frac{11}{19}\,x^{19}+\frac{55}{17}\,x^{17}+11\,x^{15}+\frac{330}{13}\,x^{13}+42\,x^{11}+\frac{154}{3}\,x^{9}+\frac{330}{7}\,x^{7}+33\,x^{5}+\frac{55}{3}\,x^{3}+11\,x-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/21*x^21 + 11/19*x^19 + 55/17*x^17 + 11*x^15 + 330/13*x^13 + 42*x^11 + 154
/3*x^9 + 330/7*x^7 + 33*x^5 + 55/3*x^3 + 11*x - 1/x
```

maple [A] time = 0.00, size = 60, normalized size = 0.82

$$\frac{x^{21}}{21}+\frac{11x^{19}}{19}+\frac{55x^{17}}{17}+11x^{15}+\frac{330x^{13}}{13}+42x^{11}+\frac{154x^{9}}{3}+\frac{330x^{7}}{7}+33x^{5}+\frac{55x^{3}}{3}+11x-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^4+2*x^2+1)^5/x^2,x)
```

```
[Out] -1/x+11*x+55/3*x^3+33*x^5+330/7*x^7+154/3*x^9+42*x^11+330/13*x^13+11*x^15+5
5/17*x^17+11/19*x^19+1/21*x^21
```

maxima [A] time = 0.83, size = 59, normalized size = 0.81

$$\frac{1}{21}\,x^{21}+\frac{11}{19}\,x^{19}+\frac{55}{17}\,x^{17}+11\,x^{15}+\frac{330}{13}\,x^{13}+42\,x^{11}+\frac{154}{3}\,x^{9}+\frac{330}{7}\,x^{7}+33\,x^{5}+\frac{55}{3}\,x^{3}+11\,x-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/21*x^21 + 11/19*x^19 + 55/17*x^17 + 11*x^15 + 330/13*x^13 + 42*x^11 + 154
/3*x^9 + 330/7*x^7 + 33*x^5 + 55/3*x^3 + 11*x - 1/x
```

mupad [B] time = 0.06, size = 59, normalized size = 0.81

$$11\,x-\frac{1}{x}+\frac{55\,x^{3}}{3}+33\,x^{5}+\frac{330\,x^{7}}{7}+\frac{154\,x^{9}}{3}+42\,x^{11}+\frac{330\,x^{13}}{13}+11\,x^{15}+\frac{55\,x^{17}}{17}+\frac{11\,x^{19}}{19}+\frac{x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)*(2*x^2 + x^4 + 1)^5)/x^2,x)
```

```
[Out] 11*x - 1/x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3 + 42*x^11 + (3
30*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19 + x^21/21
```

sympy [A] time = 0.10, size = 66, normalized size = 0.90

$$\frac{x^{21}}{21}+\frac{11x^{19}}{19}+\frac{55x^{17}}{17}+11x^{15}+\frac{330x^{13}}{13}+42x^{11}+\frac{154x^{9}}{3}+\frac{330x^{7}}{7}+33x^{5}+\frac{55x^{3}}{3}+11x-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**4+2*x**2+1)**5/x**2,x)
```

```
[Out] x**21/21 + 11*x**19/19 + 55*x**17/17 + 11*x**15 + 330*x**13/13 + 42*x**11 +
 154*x**9/3 + 330*x**7/7 + 33*x**5 + 55*x**3/3 + 11*x - 1/x
```

3.54 $\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3} dx$

Optimal. Leaf size=80

$$\frac{x^{20}}{20}+\frac{11x^{18}}{18}+\frac{55x^{16}}{16}+\frac{165x^{14}}{14}+\frac{55x^{12}}{2}+\frac{231x^{10}}{5}+\frac{231x^8}{4}+55x^6+\frac{165x^4}{4}+\frac{55x^2}{2}-\frac{1}{2x^2}+11\log(x)$$

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 266, 43}

$$\frac{x^{20}}{20}+\frac{11x^{18}}{18}+\frac{55x^{16}}{16}+\frac{165x^{14}}{14}+\frac{55x^{12}}{2}+\frac{231x^{10}}{5}+\frac{231x^8}{4}+55x^6+\frac{165x^4}{4}+\frac{55x^2}{2}-\frac{1}{2x^2}+11\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] -1/(2*x^2) + (55*x^2)/2 + (165*x^4)/4 + 55*x^6 + (231*x^8)/4 + (231*x^10)/5
 + (55*x^12)/2 + (165*x^14)/14 + (55*x^16)/16 + (11*x^18)/18 + x^20/20 + 11
*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3} dx &= \int \frac{(1+x^2)^{11}}{x^3} dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(1+x)^{11}}{x^2} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(55+\frac{1}{x^2}+\frac{11}{x}+165x+330x^2+462x^3+462x^4+330x^5+165\right.\right. \\
&= -\frac{1}{2x^2}+\frac{55x^2}{2}+\frac{165x^4}{4}+55x^6+\frac{231x^8}{4}+\frac{231x^{10}}{5}+\frac{55x^{12}}{2}+\frac{165x^{14}}{14}+\frac{55x^{16}}{16}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 80, normalized size = 1.00

$$\frac{x^{20}}{20}+\frac{11x^{18}}{18}+\frac{55x^{16}}{16}+\frac{165x^{14}}{14}+\frac{55x^{12}}{2}+\frac{231x^{10}}{5}+\frac{231x^8}{4}+55x^6+\frac{165x^4}{4}+\frac{55x^2}{2}-\frac{1}{2x^2}+11\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] -1/2*1/x^2 + (55*x^2)/2 + (165*x^4)/4 + 55*x^6 + (231*x^8)/4 + (231*x^10)/5
 + (55*x^12)/2 + (165*x^14)/14 + (55*x^16)/16 + (11*x^18)/18 + x^20/20 + 11
*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] IntegrateAlgebraic[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^3, x]
```

fricas [A] time = 0.50, size = 64, normalized size = 0.80

$$\frac{252\,x^{22}+3080\,x^{20}+17325\,x^{18}+59400\,x^{16}+138600\,x^{14}+232848\,x^{12}+291060\,x^{10}+277200\,x^{8}+207900\,x^{6}+138600\,x^{4}+55440\,x^{2}\log(x)-2520}{5040\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/5040*(252*x^22 + 3080*x^20 + 17325*x^18 + 59400*x^16 + 138600*x^14 + 2328
48*x^12 + 291060*x^10 + 277200*x^8 + 207900*x^6 + 138600*x^4 + 55440*x^2*lo
g(x) - 2520)/x^2
```

giac [A] time = 0.36, size = 69, normalized size = 0.86

$$\frac{1}{20}\,x^{20}+\frac{11}{18}\,x^{18}+\frac{55}{16}\,x^{16}+\frac{165}{14}\,x^{14}+\frac{55}{2}\,x^{12}+\frac{231}{5}\,x^{10}+\frac{231}{4}\,x^{8}+55\,x^{6}+\frac{165}{4}\,x^{4}+\frac{55}{2}\,x^{2}-\frac{11\,x^{2}+1}{2\,x^{2}}+\frac{11}{2}\,\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/20*x^20 + 11/18*x^18 + 55/16*x^16 + 165/14*x^14 + 55/2*x^12 + 231/5*x^10
+ 231/4*x^8 + 55*x^6 + 165/4*x^4 + 55/2*x^2 - 1/2*(11*x^2 + 1)/x^2 + 11/2*l
og(x^2)
```

maple [A] time = 0.00, size = 61, normalized size = 0.76

$$\frac{x^{20}}{20}+\frac{11x^{18}}{18}+\frac{55x^{16}}{16}+\frac{165x^{14}}{14}+\frac{55x^{12}}{2}+\frac{231x^{10}}{5}+\frac{231x^{8}}{4}+55x^{6}+\frac{165x^{4}}{4}+\frac{55x^{2}}{2}+11\ln(x)-\frac{1}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^4+2*x^2+1)^5/x^3,x)
```

```
[Out] -1/2/x^2+55/2*x^2+165/4*x^4+55*x^6+231/4*x^8+231/5*x^10+55/2*x^12+165/14*x^
14+55/16*x^16+11/18*x^18+1/20*x^20+11*ln(x)
```

maxima [A] time = 0.74, size = 62, normalized size = 0.78

$$\frac{1}{20}\,x^{20}+\frac{11}{18}\,x^{18}+\frac{55}{16}\,x^{16}+\frac{165}{14}\,x^{14}+\frac{55}{2}\,x^{12}+\frac{231}{5}\,x^{10}+\frac{231}{4}\,x^{8}+55\,x^{6}+\frac{165}{4}\,x^{4}+\frac{55}{2}\,x^{2}-\frac{1}{2\,x^{2}}+\frac{11}{2}\,\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/20*x^20 + 11/18*x^18 + 55/16*x^16 + 165/14*x^14 + 55/2*x^12 + 231/5*x^10
+ 231/4*x^8 + 55*x^6 + 165/4*x^4 + 55/2*x^2 - 1/2/x^2 + 11/2*log(x^2)
```

mupad [B] time = 0.06, size = 60, normalized size = 0.75

$$11\ \ln(x) - \frac{1}{2\,x^2} + \frac{55\,x^2}{2} + \frac{165\,x^4}{4} + 55\,x^6 + \frac{231\,x^8}{4} + \frac{231\,x^{10}}{5} + \frac{55\,x^{12}}{2} + \frac{165\,x^{14}}{14} + \frac{55\,x^{16}}{16} + \frac{11\,x^{18}}{18} + \frac{x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)*(2*x^2 + x^4 + 1)^5)/x^3,x)
```

```
[Out] 11*log(x) - 1/(2*x^2) + (55*x^2)/2 + (165*x^4)/4 + 55*x^6 + (231*x^8)/4 + (
231*x^10)/5 + (55*x^12)/2 + (165*x^14)/14 + (55*x^16)/16 + (11*x^18)/18 + x
^20/20
```

sympy [A] time = 0.11, size = 75, normalized size = 0.94

$$\frac{x^{20}}{20} + \frac{11x^{18}}{18} + \frac{55x^{16}}{16} + \frac{165x^{14}}{14} + \frac{55x^{12}}{2} + \frac{231x^{10}}{5} + \frac{231x^8}{4} + 55x^6 + \frac{165x^4}{4} + \frac{55x^2}{2} + 11\log(x) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**4+2*x**2+1)**5/x**3,x)
```

```
[Out] x**20/20 + 11*x**18/18 + 55*x**16/16 + 165*x**14/14 + 55*x**12/2 + 231*x**1
0/5 + 231*x**8/4 + 55*x**6 + 165*x**4/4 + 55*x**2/2 + 11*log(x) - 1/(2*x**2
)
```

3.55 $\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=145

$$\frac{x\left(a+bx^2\right)(bd-ae)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{ex^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{a}\left(a+bx^2\right)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.09, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1250, 459, 321, 205}

$$\frac{x\left(a+bx^2\right)(bd-ae)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{a}\left(a+bx^2\right)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{ex^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((b*d - a*e)*x*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (e*x^3*
(a + b*x^2))/(3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (Sqrt[a]*(b*d - a*e)*(
a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\left(d+ex^2\right)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{x^2\left(d+ex^2\right)}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{ex^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(\left(-3b^2d+3abe\right)\left(ab+b^2x^2\right)\right)\int \frac{x^2}{ab+b^2x^2}\,dx}{3b^2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{(bd-ae)x\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{ex^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(a\left(-3b^2d+3abe\right)\left(ab+b^2x\right.\right.}{3b^3\sqrt{a^2+2abx^2+}}$$
$$= \frac{(bd-ae)x\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{ex^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt{a}\,(bd-ae)\left(a+bx^2\right)\tan^{-1}}{b^{5/2}\sqrt{a^2+2abx^2+b^2x}}$$

Mathematica [A] time = 0.08, size = 80, normalized size = 0.55

$$\frac{\left(a+bx^2\right)\left(\sqrt{b}\,x\left(-3ae+3bd+bex^2\right)+3\sqrt{a}\,(ae-bd)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right)}{3b^{5/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(Sqrt[b]*x*(3*b*d - 3*a*e + b*e*x^2) + 3*Sqrt[a]*(-(b*d) + a*e
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(3*b^(5/2)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 9.87, size = 81, normalized size = 0.56

$$\frac{\left(a+bx^2\right)\left(\frac{\left(a^{3/2}e-\sqrt{a}\,bd\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x\left(-3ae+3bd+bex^2\right)}{3b^2}\right)}{\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*((x*(3*b*d - 3*a*e + b*e*x^2))/(3*b^2) + ((-(Sqrt[a]*b*d) + a^
(3/2)*e)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2)))/Sqrt[(a + b*x^2)^2]
```

fricas [A] time = 0.77, size = 129, normalized size = 0.89

$$\left[\frac{2\,bex^3-3\,(bd-ae)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+6\,(bd-ae)x}{6\,b^2},\ \frac{bex^3-3\,(bd-ae)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+3\,(bd-ae)x}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b*e*x^3 - 3*(b*d - a*e)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) -
a)/(b*x^2 + a)) + 6*(b*d - a*e)*x)/b^2, 1/3*(b*e*x^3 - 3*(b*d - a*e)*sqrt(a
/b)*arctan(b*x*sqrt(a/b)/a) + 3*(b*d - a*e)*x)/b^2]
```

giac [A] time = 0.43, size = 101, normalized size = 0.70

$$-\frac{\left(abd\mathrm{sgn}\left(bx^2+a\right)-a^2e\mathrm{sgn}\left(bx^2+a\right)\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{b^2x^3e\mathrm{sgn}\left(bx^2+a\right)+3\,b^2dx\mathrm{sgn}\left(bx^2+a\right)-3\,abxe\mathrm{sgn}\left(bx^2+a\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(a*b*d*sgn(b*x^2 + a) - a^2*e*sgn(b*x^2 + a))*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*b^2) + 1/3*(b^2*x^3*e*sgn(b*x^2 + a) + 3*b^2*d*x*sgn(b*x^2 + a) - 3*a*
b*x*e*sgn(b*x^2 + a))/b^3
```

maple [A] time = 0.04, size = 90, normalized size = 0.62

$$\frac{\left(b\,x^{2}+a\right)\left(\sqrt{ab}\;be\,x^{3}+3a^{2}e\arctan\left(\frac{bx}{\sqrt{ab}}\right)-3abd\arctan\left(\frac{bx}{\sqrt{ab}}\right)-3\sqrt{ab}\;aex+3\sqrt{ab}\;bdx\right)}{3\sqrt{\left(b\,x^{2}+a\right)^{2}}\;\sqrt{ab}\;b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)*((a*b)^(1/2)*x^3*b*e+3*arctan(b*x/(a*b)^(1/2))*a^2*e-3*arctan
(b*x/(a*b)^(1/2))*a*b*d-3*(a*b)^(1/2)*x*a*e+3*(a*b)^(1/2)*x*b*d)/((b*x^2+a)
^2)^(1/2)/b^2/(a*b)^(1/2)
```

maxima [A] time = 1.52, size = 54, normalized size = 0.37

$$-\frac{\left(abd-a^{2}e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{2}}+\frac{bex^{3}+3\,(bd-ae)x}{3\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(a*b*d - a^2*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b*e*x^3 + 3*(
b*d - a*e)*x)/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2}\,\left(e\,x^{2}+d\right)}{\sqrt{\left(b\,x^{2}+a\right)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^2))/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((x^2*(d + e*x^2))/((a + b*x^2)^2)^(1/2), x)
```

sympy [A] time = 0.36, size = 90, normalized size = 0.62

$$x\left(-\frac{ae}{b^{2}}+\frac{d}{b}\right)-\frac{\sqrt{-\frac{a}{b^{5}}}\;(ae-bd)\log\left(-b^{2}\sqrt{-\frac{a}{b^{5}}}\;+x\right)}{2}+\frac{\sqrt{-\frac{a}{b^{5}}}\;(ae-bd)\log\left(b^{2}\sqrt{-\frac{a}{b^{5}}}\;+x\right)}{2}+\frac{ex^{3}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] x*(-a*e/b**2 + d/b) - sqrt(-a/b**5)*(a*e - b*d)*log(-b**2*sqrt(-a/b**5) + x
)/2 + sqrt(-a/b**5)*(a*e - b*d)*log(b**2*sqrt(-a/b**5) + x)/2 + e*x**3/(3*b
)
```

3.56 $\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}} dx$

Optimal. Leaf size=83

$$\frac{(a+bx^2)(bd-ae)\log(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}$$

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1247, 640, 608, 31}

$$\frac{(a+bx^2)(bd-ae)\log(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (e*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*b^2) + ((b*d - a*e)*(a + b*x^2)*Log[
a + b*x^2])/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(d+ex^2\right)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{d+ex}{\sqrt{a^2+2abx+b^2x^2}}\,dx,x,x^2\right)\\
&= \frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}+\frac{(bd-ae)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx,x,x^2\right)}{2b}\\
&= \frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}+\frac{\left((bd-ae)\left(ab+b^2x^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{ab+b^2x}\,dx,x,x^2\right)}{2b\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}+\frac{(bd-ae)\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.61

$$\frac{\left(a+bx^2\right)\left((bd-ae)\log\left(a+bx^2\right)+bex^2\right)}{2b^2\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(b*e*x^2 + (b*d - a*e)*Log[a + b*x^2]))/(2*b^2*Sqrt[(a + b*x^2
)^2])
```

IntegrateAlgebraic [B] time = 0.46, size = 208, normalized size = 2.51

$$\frac{\left(a\sqrt{b^2}\,e+abe+b^2(-d)-\sqrt{b^2}\,bd\right)\log\left(\sqrt{a^2+2abx^2+b^2x^4}-a-\sqrt{b^2}\,x^2\right)}{4\left(b^2\right)^{3/2}}+\frac{\left(-a\sqrt{b^2}\,e+abe+b^2(-d)+\sqrt{b^2}\,bd\right)\log\left(\sqrt{a^2+2abx^2+b^2x^4}+a-\sqrt{b^2}\,x^2\right)}{4\left(b^2\right)^{3/2}}+\frac{e\sqrt{a^2+2abx^2+b^2x^4}}{4b^2}-\frac{ex^2}{4\sqrt{b^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -1/4*(e*x^2)/Sqrt[b^2] + (e*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*b^2) + ((-(
b^2*d) - b*Sqrt[b^2]*d + a*b*e + a*Sqrt[b^2]*e)*Log[-a - Sqrt[b^2]*x^2 + Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(4*(b^2)^(3/2)) + ((-(b^2*d) + b*Sqrt[b^2]*
d + a*b*e - a*Sqrt[b^2]*e)*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4]])/(4*(b^2)^(3/2))
```

fricas [A] time = 0.83, size = 29, normalized size = 0.35

$$\frac{bex^2+(bd-ae)\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*e*x^2 + (b*d - a*e)*log(b*x^2 + a))/b^2
```

giac [A] time = 0.38, size = 42, normalized size = 0.51

$$\frac{1}{2}\left(\frac{x^2e}{b}+\frac{(bd-ae)\log\left(\left|bx^2+a\right|\right)}{b^2}\right)\operatorname{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(x^2*e/b + (b*d - a*e)*log(abs(b*x^2 + a))/b^2)*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 55, normalized size = 0.66

$$-\frac{\left(b\,x^{2}+a\right)\left(-be\,x^{2}+ae\ln\left(b\,x^{2}+a\right)-bd\ln\left(b\,x^{2}+a\right)\right)}{2\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/2*(b*x^2+a)*(-x^2*e*b+ln(b*x^2+a)*a*e-ln(b*x^2+a)*b*d)/((b*x^2+a)^2)^(1/
2)/b^2
```

maxima [A] time = 0.78, size = 31, normalized size = 0.37

$$\frac{ex^{2}}{2\,b}+\frac{(bd-ae)\log\left(bx^{2}+a\right)}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*e*x^2/b + 1/2*(b*d - a*e)*log(b*x^2 + a)/b^2
```

mupad [B] time = 0.88, size = 103, normalized size = 1.24

$$\frac{e\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{2\,b^{2}}-\frac{a\,b\,e\,\ln\left(a\,b+\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\sqrt{b^{2}}+b^{2}\,x^{2}\right)}{2\,{\left(b^{2}\right)}^{3/2}}+\frac{b^{2}\,d\,\ln\left(b^{2}\,x^{2}+a\,b\right)\,\mathrm{sign}\left(2\,b^{2}\,x^{2}+2\,a\,b\right)}{2\,{\left(b^{2}\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^2))/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] (e*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(2*b^2) - (a*b*e*log(a*b + ((a + b*x^
2)^2)^(1/2)*(b^2)^(1/2) + b^2*x^2))/(2*(b^2)^(3/2)) + (b^2*d*log(a*b + b^2*
x^2)*sign(2*a*b + 2*b^2*x^2))/(2*(b^2)^(3/2))
```

sympy [A] time = 0.28, size = 27, normalized size = 0.33

$$\frac{ex^{2}}{2b}-\frac{(ae-bd)\log\left(a+bx^{2}\right)}{2b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] e*x**2/(2*b) - (a*e - b*d)*log(a + b*x**2)/(2*b**2)
```

3.57 $\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}} \, dx$

Optimal. Leaf size=97

$$\frac{ex\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1148, 388, 205}

$$\frac{\left(a+bx^2\right)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{ex\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (e*x*(a + b*x^2))/(b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d - a*e)*(a + b
*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1148

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_),
x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^
2)^(2*FracPart[p])), Int[(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ
[{a, b, c, d, e, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}} \, dx &= \frac{\left(ab+b^2x^2\right)\int \frac{d+ex^2}{ab+b^2x^2} \, dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{ex\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(\left(-b^2d+abe\right)\left(ab+b^2x^2\right)\right)\int \frac{1}{ab+b^2x^2} \, dx}{b^2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{ex\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(bd-ae)\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 69, normalized size = 0.71

$$-\frac{\left(a+bx^2\right)\left((ae-bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\sqrt{a}\sqrt{b}ex\right)}{\sqrt{a}b^{3/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -(((a + b*x^2)*(-(Sqrt[a]*Sqrt[b]*e*x) + (-(b*d) + a*e)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]]))/(Sqrt[a]*b^(3/2)*Sqrt[(a + b*x^2)^2]))
```

IntegrateAlgebraic [A] time = 8.95, size = 60, normalized size = 0.62

$$\frac{\left(a+bx^2\right)\left(\frac{(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{ex}{b}\right)}{\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x^2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*((e*x)/b + ((b*d - a*e)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*
b^(3/2))))/Sqrt[(a + b*x^2)^2]
```

fricas [A] time = 0.59, size = 98, normalized size = 1.01

$$\left[\frac{2\,abex+\sqrt{-ab}\,(bd-ae)\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab^2},\frac{abex+\sqrt{ab}\,(bd-ae)\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*e*x + sqrt(-a*b)*(b*d - a*e)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a*b^2), (a*b*e*x + sqrt(a*b)*(b*d - a*e)*arctan(sqrt(a*b)*x/a
))/(a*b^2)]
```

giac [A] time = 0.26, size = 59, normalized size = 0.61

$$\frac{x e \mathrm{sgn}\left(bx^2+a\right)}{b}+\frac{\left(bd\mathrm{sgn}\left(bx^2+a\right)-ae\mathrm{sgn}\left(bx^2+a\right)\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] x*e*sgn(b*x^2 + a)/b + (b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 + a))*arctan(b*x
/sqrt(a*b))/(sqrt(a*b)*b)
```

maple [A] time = 0.01, size = 62, normalized size = 0.64

$$\frac{\left(b\,x^2+a\right)\left(-ae\arctan\left(\frac{bx}{\sqrt{ab}}\right)+bd\arctan\left(\frac{bx}{\sqrt{ab}}\right)+\sqrt{ab}\;ex\right)}{\sqrt{\left(b\,x^2+a\right)^2}\;\sqrt{ab}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] (b*x^2+a)*(e*x*(a*b)^(1/2)-arctan(1/(a*b)^(1/2)*b*x)*a*e+arctan(1/(a*b)^(1/
2)*b*x)*b*d)/((b*x^2+a)^2)^(1/2)/b/(a*b)^(1/2)
```

maxima [A] time = 1.51, size = 33, normalized size = 0.34

$$\frac{ex}{b} + \frac{(bd - ae)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] e*x/b + (b*d - a*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2 + d}{\sqrt{\left(b\,x^2 + a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((d + e*x^2)/((a + b*x^2)^2)^(1/2), x)
```

sympy [A] time = 0.32, size = 82, normalized size = 0.85

$$\frac{\sqrt{-\frac{1}{ab^3}}\,(ae - bd)\log\left(-ab\sqrt{-\frac{1}{ab^3}} + x\right)}{2} - \frac{\sqrt{-\frac{1}{ab^3}}\,(ae - bd)\log\left(ab\sqrt{-\frac{1}{ab^3}} + x\right)}{2} + \frac{ex}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] sqrt(-1/(a*b**3))*(a*e - b*d)*log(-a*b*sqrt(-1/(a*b**3)) + x)/2 - sqrt(-1/(
a*b**3))*(a*e - b*d)*log(a*b*sqrt(-1/(a*b**3)) + x)/2 + e*x/b
```

3.58 $\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{d\log(x)\left(a+bx^2\right)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)(bd-ae)\log\left(a+bx^2\right)}{2ab\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1250, 446, 72}

$$\frac{d\log(x)\left(a+bx^2\right)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)(bd-ae)\log\left(a+bx^2\right)}{2ab\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (d*(a + b*x^2)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)*(
a + b*x^2)*Log[a + b*x^2])/(2*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{\left(ab+b^2x^2\right)\int \frac{d+ex^2}{x\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{\left(ab+b^2x^2\right)\operatorname{Subst}\left(\int \frac{d+ex}{x\left(ab+b^2x\right)}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{\left(ab+b^2x^2\right)\operatorname{Subst}\left(\int \left(\frac{d}{abx}+\frac{-bd+ae}{ab(a+bx)}\right)dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{d\left(a+bx^2\right)\log(x)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(bd-ae)\left(a+bx^2\right)\log\left(a+bx^2\right)}{2ab\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.59

$$\frac{\left(a+bx^2\right)\left((ae-bd)\log\left(a+bx^2\right)+2bd\log(x)\right)}{2ab\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*(2*b*d*Log[x] + (-(b*d) + a*e)*Log[a + b*x^2]))/(2*a*b*Sqrt[(a
 + b*x^2)^2])
```

IntegrateAlgebraic [B] time = 0.44, size = 198, normalized size = 2.15

$$\frac{\left(a\sqrt{b^2}\,e-abe-2b\sqrt{b^2}\,d\right)\log\left(\sqrt{a^2+2abx^2+b^2x^4}+a-\sqrt{b^2}\,x^2\right)}{4ab\sqrt{b^2}}+\frac{(2bd-ae)\log\left(-ab\sqrt{a^2+2abx^2+b^2x^4}+a^2b+ab\sqrt{b^2}\,x^2\right)}{4ab}-\frac{e\log\left(\sqrt{a^2+2abx^2+b^2x^4}-a-\sqrt{b^2}\,x^2\right)}{4\sqrt{b^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x^2)/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -1/4*(e*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/Sqrt[b^2
] + ((-2*b*Sqrt[b^2]*d - a*b*e + a*Sqrt[b^2]*e)*Log[a - Sqrt[b^2]*x^2 + Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4]])/(4*a*b*Sqrt[b^2]) + ((2*b*d - a*e)*Log[a^2*b
 + a*b*Sqrt[b^2]*x^2 - a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(4*a*b)
```

fricas [A] time = 0.78, size = 33, normalized size = 0.36

$$\frac{2\,bd\log(x)-(bd-ae)\log\left(bx^2+a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*log(x) - (b*d - a*e)*log(b*x^2 + a))/(a*b)
```

giac [A] time = 0.38, size = 61, normalized size = 0.66

$$\frac{d\log\left(x^2\right)\operatorname{sgn}\left(bx^2+a\right)}{2\,a}-\frac{\left(bd\operatorname{sgn}\left(bx^2+a\right)-ae\operatorname{sgn}\left(bx^2+a\right)\right)\log\left(\left|bx^2+a\right|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*d*log(x^2)*sgn(b*x^2 + a)/a - 1/2*(b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 +
 a))*log(abs(b*x^2 + a))/(a*b)
```

maple [A] time = 0.01, size = 57, normalized size = 0.62

$$\frac{\left(b\,x^2+a\right)\left(ae\ln\left(b\,x^2+a\right)+2bd\ln(x)-bd\ln\left(b\,x^2+a\right)\right)}{2\sqrt{\left(b\,x^2+a\right)^2}\;ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/2*(b*x^2+a)*(2*d*ln(x)*b+a*e*ln(b*x^2+a)-b*d*ln(b*x^2+a))/((b*x^2+a)^2)^(
1/2)/a/b
```

maxima [A] time = 0.73, size = 35, normalized size = 0.38

$$\frac{d\log\left(x^2\right)}{2\,a} - \frac{(bd - ae)\log\left(bx^2 + a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*d*log(x^2)/a - 1/2*(b*d - a*e)*log(b*x^2 + a)/(a*b)
```

mupad [B] time = 0.77, size = 83, normalized size = 0.90

$$\frac{e\,\ln\left(b^2\,x^2 + a\,b\right)\,\mathrm{sign}\left(2\,b^2\,x^2 + 2\,a\,b\right)}{2\,\sqrt{b^2}} - \frac{d\,\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a^2}} - \frac{d\,\ln\left(\sqrt{\left(b\,x^2 + a\right)^2}\,\sqrt{a^2} + a^2 + a\,b\,x^2\right)}{2\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(x*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] (e*log(a*b + b^2*x^2)*sign(2*a*b + 2*b^2*x^2))/(2*(b^2)^(1/2)) - (d*log(1/x
^2))/(2*(a^2)^(1/2)) - (d*log(((a + b*x^2)^2)^(1/2)*(a^2)^(1/2) + a^2 + a*b
*x^2))/(2*(a^2)^(1/2))
```

sympy [A] time = 0.71, size = 26, normalized size = 0.28

$$\frac{d\log(x)}{a} + \frac{(ae - bd)\log\left(\frac{a}{b} + x^2\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] d*log(x)/a + (a*e - b*d)*log(a/b + x**2)/(2*a*b)
```

3.59 $\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=101

$$-\frac{d\left(a+bx^2\right)}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\,\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1250, 453, 205}

$$-\frac{\left(a+bx^2\right)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\,\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)}{ax\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -((d*(a + b*x^2))/(a*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) - ((b*d - a*e)*(a
+ b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{\left(ab+b^2x^2\right)\int \frac{d+ex^2}{x^2\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\ &= -\frac{d\left(a+bx^2\right)}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(\left(b^2d-abe\right)\left(ab+b^2x^2\right)\right)\int \frac{1}{ab+b^2x^2}\,dx}{ab\sqrt{a^2+2abx^2+b^2x^4}}\\ &= -\frac{d\left(a+bx^2\right)}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(bd-ae)\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\,\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.71

$$\frac{(a+bx^2)\left(\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(aex-bdx)-\sqrt{a}\sqrt{b}\,d\right)}{a^{3/2}\sqrt{b}\,x\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*(-(Sqrt[a]*Sqrt[b]*d) + (-(b*d*x) + a*e*x)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]]))/(a^(3/2)*Sqrt[b]*x*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 6.38, size = 63, normalized size = 0.62

$$\frac{(a+bx^2)\left(\frac{(ae-bd)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{d}{ax}\right)}{\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x^2)/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*(-(d/(a*x)) + ((-(b*d) + a*e)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^
(3/2)*Sqrt[b])))/Sqrt[(a + b*x^2)^2]
```

fricas [A] time = 0.70, size = 105, normalized size = 1.04

$$\left[\frac{\sqrt{-ab}\,(bd-ae)x\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)-2\,abd}{2\,a^2bx},-\frac{\sqrt{ab}\,(bd-ae)x\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+abd}{a^2bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a*b)*(b*d - a*e)*x*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) - 2*a*b*d)/(a^2*b*x), -(sqrt(a*b)*(b*d - a*e)*x*arctan(sqrt(a*b)*x/a) + a
*b*d)/(a^2*b*x)]
```

giac [A] time = 0.34, size = 62, normalized size = 0.61

$$-\frac{\left(bd\mathrm{sgn}\left(bx^2+a\right)-ae\mathrm{sgn}\left(bx^2+a\right)\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{d\mathrm{sgn}\left(bx^2+a\right)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 + a))*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a) - d*sgn(b*x^2 + a)/(a*x)
```

maple [A] time = 0.01, size = 67, normalized size = 0.66

$$-\frac{(b\,x^2+a)\left(-aex\arctan\left(\frac{bx}{\sqrt{ab}}\right)+bdx\arctan\left(\frac{bx}{\sqrt{ab}}\right)+\sqrt{ab}\,d\right)}{\sqrt{(b\,x^2+a)^2}\,\sqrt{ab}\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -(b*x^2+a)*(-arctan(1/(a*b)^(1/2)*b*x)*x*a*e+arctan(1/(a*b)^(1/2)*b*x)*x*b*
d+d*(a*b)^(1/2))/((b*x^2+a)^2)^(1/2)/a/x/(a*b)^(1/2)
```

maxima [A] time = 1.22, size = 37, normalized size = 0.37

$$-\frac{(bd-ae)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{d}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(b*d - a*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - d/(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2+d}{x^2\,\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(x^2*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] int((d + e*x^2)/(x^2*((a + b*x^2)^2)^(1/2)), x)
```

sympy [A] time = 0.37, size = 82, normalized size = 0.81

$$-\frac{\sqrt{-\frac{1}{a^3b}}\,(ae-bd)\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^3b}}\,(ae-bd)\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}-\frac{d}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x**2/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -sqrt(-1/(a**3*b))*(a*e - b*d)*log(-a**2*sqrt(-1/(a**3*b)) + x)/2 + sqrt(-1
/(a**3*b))*(a*e - b*d)*log(a**2*sqrt(-1/(a**3*b)) + x)/2 - d/(a*x)
```

3.60 $\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=137

$$-\frac{\log(x)\left(a+bx^2\right)(bd-ae)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(bd-ae)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.10, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1250, 446, 77}

$$-\frac{\log(x)\left(a+bx^2\right)(bd-ae)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(bd-ae)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -(d*(a + b*x^2))/(2*a*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)*(
a + b*x^2)*Log[x])/(a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d - a*e)*(a
+ b*x^2)*Log[a + b*x^2])/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{\left(ab+b^2x^2\right)\int \frac{d+ex^2}{x^3\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}} \\
&= \frac{\left(ab+b^2x^2\right)\operatorname{Subst}\left(\int \frac{d+ex}{x^2\left(ab+b^2x\right)}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}} \\
&= \frac{\left(ab+b^2x^2\right)\operatorname{Subst}\left(\int \left(\frac{d}{abx^2}+\frac{-bd+ae}{a^2bx}+\frac{bd-ae}{a^2(a+bx)}\right)dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}} \\
&= -\frac{d\left(a+bx^2\right)}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)\left(a+bx^2\right)\log(x)}{a^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)\left(a+bx^2\right)\log}{2a^2\sqrt{a^2+2abx^2+}}
\end{aligned}
$$

Mathematica [**A**] time = 0.03, size = 70, normalized size = 0.51

$$
\frac{\left(a+bx^2\right)\left(2x^2\log(x)(ae-bd)+x^2(bd-ae)\log\left(a+bx^2\right)-ad\right)}{2a^2x^2\sqrt{\left(a+bx^2\right)^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*(-(a*d) + 2*(-(b*d) + a*e)*x^2*Log[x] + (b*d - a*e)*x^2*Log[a
+ b*x^2]))/(2*a^2*x^2*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [**B**] time = 1.17, size = 713, normalized size = 5.20

$$
\frac{\frac{\sqrt{b^2}ex^2\log\left(\sqrt{a^2+2abx^2+b^2x^4}+a-\sqrt{b^2}x^2\right)}{2a}-\frac{e\sqrt{a^2+2abx^2+b^2x^4}\log\left(\sqrt{a^2+2abx^2+b^2x^4}+a-\sqrt{b^2}x^2\right)}{2a}-\frac{\sqrt{b^2}ex^2\log\left(a^3+a^2\sqrt{b^2}x^2-a^2\sqrt{a^2+2abx^2+b^2x^4}\right)}{2a}+\frac{e\sqrt{a^2+2abx^2+b^2x^4}\log\left(a^3+a^2\sqrt{b^2}x^2-a^2\sqrt{a^2+2abx^2+b^2x^4}\right)}{2a}-\frac{bd}{a}}{\sqrt{a^2+2abx^2+b^2x^4}-\sqrt{b^2}x^2}+\frac{\left(\sqrt{a^2+2abx^2+b^2x^4}-\sqrt{b^2}x^2\right)^2\left(\frac{bd\log\left(\sqrt{a^2+2abx^2+b^2x^4}+a-\sqrt{b^2}x^2\right)}{2a^2}-\frac{bd\log\left(a^3+a^2\sqrt{b^2}x^2-a^2\sqrt{a^2+2abx^2+b^2x^4}\right)}{2a^2}\right)}{-2\sqrt{b^2}x^2\sqrt{a^2+2abx^2+b^2x^4}+a^2+2abx^2+2b^2x^4}+\frac{-\sqrt{b^2}d\sqrt{a^2+2abx^2+b^2x^4}\left(a^2+4abx^2+4b^2x^4\right)-d\left(-a^3b-5a^2b^2x^2-8ab^3x^4-4b^4x^6\right)}{\sqrt{a^2+2abx^2+b^2x^4}\left(-2a^3bx^2-12a^2b^2x^4-24ab^3x^6-16b^4x^8\right)+\sqrt{b^2}\left(2a^4x^2+14a^3bx^4+36a^2b^2x^6+40ab^3x^8+16b^4x^{10}\right)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x^2)/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-(Sqrt[b^2]*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(a^2 + 4*a*b*x^2 + 4*b^2*x^4
)) - d*(-(a^3*b) - 5*a^2*b^2*x^2 - 8*a*b^3*x^4 - 4*b^4*x^6))/(Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4]*(-2*a^3*b*x^2 - 12*a^2*b^2*x^4 - 24*a*b^3*x^6 - 16*b^4*x
^8) + Sqrt[b^2]*(2*a^4*x^2 + 14*a^3*b*x^4 + 36*a^2*b^2*x^6 + 40*a*b^3*x^8 +
 16*b^4*x^10)) + ((-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^2*((
b*d*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(2*a^2) - (b*
d*Log[a^3 + a^2*Sqrt[b^2]*x^2 - a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(2*a^
2)))/(a^2 + 2*a*b*x^2 + 2*b^2*x^4 - 2*Sqrt[b^2]*x^2*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4]) + (-((b*d)/a) + (Sqrt[b^2]*e*x^2*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4]])/(2*a) - (e*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[a
- Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(2*a) - (Sqrt[b^2]*e*x^
2*Log[a^3 + a^2*Sqrt[b^2]*x^2 - a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(2*a)
 + (e*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[a^3 + a^2*Sqrt[b^2]*x^2 - a^2*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4]])/(2*a))/(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])
```

fricas [**A**] time = 0.71, size = 48, normalized size = 0.35

$$
\frac{(bd-ae)x^2\log\left(bx^2+a\right)-2\,(bd-ae)x^2\log(x)-ad}{2\,a^2x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

[Out] 1/2*((b*d - a*e)*x^2*log(b*x^2 + a) - 2*(b*d - a*e)*x^2*log(x) - a*d)/(a^2*x^2)

giac [A] time = 0.39, size = 131, normalized size = 0.96

$$-\frac{\left(bd\mathrm{sgn}\left(bx^2+a\right)-ae\mathrm{sgn}\left(bx^2+a\right)\right)\log\left(x^2\right)}{2\,a^2}+\frac{\left(b^2d\mathrm{sgn}\left(bx^2+a\right)-abe\mathrm{sgn}\left(bx^2+a\right)\right)\log\left(\left|bx^2+a\right|\right)}{2\,a^2b}+\frac{bdx^2\mathrm{sgn}\left(bx^2+a\right)-ax^2e\mathrm{sgn}\left(bx^2+a\right)-ad\mathrm{sgn}\left(bx^2+a\right)}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="giac")

[Out] -1/2*(b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 + a))*log(x^2)/a^2 + 1/2*(b^2*d*sgn(b*x^2 + a) - a*b*e*sgn(b*x^2 + a))*log(abs(b*x^2 + a))/(a^2*b) + 1/2*(b*d*x^2*sgn(b*x^2 + a) - a*x^2*e*sgn(b*x^2 + a) - a*d*sgn(b*x^2 + a))/(a^2*x^2)

maple [A] time = 0.01, size = 79, normalized size = 0.58

$$\frac{\left(b\,x^2+a\right)\left(2ae\,x^2\ln(x)-ae\,x^2\ln\left(b\,x^2+a\right)-2bd\,x^2\ln(x)+bd\,x^2\ln\left(b\,x^2+a\right)-ad\right)}{2\sqrt{\left(b\,x^2+a\right)^2}\;a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x)

[Out] 1/2*(b*x^2+a)*(2*ln(x)*x^2*a*e-2*ln(x)*x^2*b*d-ln(b*x^2+a)*x^2*a*e+ln(b*x^2+a)*x^2*b*d-a*d)/((b*x^2+a)^2)^(1/2)/x^2/a^2

maxima [A] time = 0.78, size = 48, normalized size = 0.35

$$\frac{(bd-ae)\log\left(bx^2+a\right)}{2\,a^2}-\frac{(bd-ae)\log\left(x^2\right)}{2\,a^2}-\frac{d}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")

[Out] 1/2*(b*d - a*e)*log(b*x^2 + a)/a^2 - 1/2*(b*d - a*e)*log(x^2)/a^2 - 1/2*d/(a*x^2)

mupad [B] time = 0.81, size = 125, normalized size = 0.91

$$\frac{a\,b\,d\,\mathrm{atanh}\left(\frac{a^2+b\,a\,x^2}{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}\right)}{2\,{\left(a^2\right)}^{3/2}}-\frac{e\,\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a^2}}-\frac{d\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,a^2\,x^2}-\frac{e\,\ln\left(\sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{a^2}+a^2+a\,b\,x^2\right)}{2\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2)/(x^3*((a + b*x^2)^2)^(1/2)),x)

[Out] (a*b*d*atanh((a^2 + a*b*x^2)/((a^2)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))))/(2*(a^2)^(3/2)) - (e*log(1/x^2))/(2*(a^2)^(1/2)) - (d*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(2*a^2*x^2) - (e*log(((a + b*x^2)^2)^(1/2)*(a^2)^(1/2) + a^2 + a*b*x^2))/(2*(a^2)^(1/2))

sympy [A] time = 0.73, size = 41, normalized size = 0.30

$$-\frac{d}{2ax^2}+\frac{(ae-bd)\log(x)}{a^2}-\frac{(ae-bd)\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)/x**3/((b*x**2+a)**2)**(1/2),x)

[Out] -d/(2*a*x**2) + (a*e - b*d)*log(x)/a**2 - (a*e - b*d)*log(a/b + x**2)/(2*a**2)

3.61 $\int \frac{x^2\left(d+ex^2\right)}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=153

$$\frac{x(bd-5ae)}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x(bd-ae)}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(3ae+bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.12, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1250, 455, 385, 205}

$$\frac{x(bd-5ae)}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x(bd-ae)}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(3ae+bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((b*d - 5*a*e)*x)/(8*a*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)*
x)/(4*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d + 3*a*e)*(a
+ b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\left(d+ex^2\right)}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{x^2\left(d+ex^2\right)}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{(bd-ae)x}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(ab+b^2x^2\right)\int \frac{-b(bd-ae)-4b^2ex^2}{\left(ab+b^2x^2\right)^2}\,dx}{4b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(bd-5ae)x}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left((bd+3ae)\right(}{8ab^2\sqrt{}}$$

$$= \frac{(bd-5ae)x}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd+3ae)\left(a\right.}{8a^{3/2}b^{5/2}\sqrt{}}$$

Mathematica [A] time = 0.06, size = 108, normalized size = 0.71

$$\frac{\left(a+bx^2\right)^2(3ae+bd)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\sqrt{a}\,\sqrt{b}\,x\left(3a^2e+ab\left(d+5ex^2\right)-b^2dx^2\right)}{8a^{3/2}b^{5/2}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-(Sqrt[a]*Sqrt[b]*x*(3*a^2*e - b^2*d*x^2 + a*b*(d + 5*e*x^2))) + (b*d + 3*
a*e)*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2)*(a + b*x
^2)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 15.92, size = 105, normalized size = 0.69

$$\frac{\left(a+bx^2\right)\left(\frac{(3ae+bd)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}-\frac{x\left(3a^2e+abd+5abex^2-b^2dx^2\right)}{8ab^2\left(a+bx^2\right)^2}\right)}{\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((a + b*x^2)*(-1/8*(x*(a*b*d + 3*a^2*e - b^2*d*x^2 + 5*a*b*e*x^2))/(a*b^2*(
a + b*x^2)^2) + ((b*d + 3*a*e)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5
/2))))/Sqrt[(a + b*x^2)^2]
```

fricas [A] time = 0.93, size = 300, normalized size = 1.96

$$\left[\frac{2\left(ab^3d-5a^2b^2e\right)x^3-\left(\left(b^3d+3ab^2e\right)x^4+a^2bd+3a^3e+2\left(ab^2d+3a^2be\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)-2\left(a^2b^2d+3a^3be\right)x}{16\left(a^2b^5x^4+2a^3b^4x^2+a^4b^3\right)}, \frac{\left(ab^3d-5a^2b^2e\right)x^3+\left(\left(b^3d+3ab^2e\right)x^4+a^2bd+3a^3e+2\left(ab^2d+3a^2be\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-\left(a^2b^2d+3a^3be\right)x}{8\left(a^2b^5x^4+2a^3b^4x^2+a^4b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/16*(2*(a*b^3*d - 5*a^2*b^2*e)*x^3 - ((b^3*d + 3*a*b^2*e)*x^4 + a^2*b*d +
 3*a^3*e + 2*(a*b^2*d + 3*a^2*b*e)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b
)*x - a)/(b*x^2 + a)) - 2*(a^2*b^2*d + 3*a^3*b*e)*x)/(a^2*b^5*x^4 + 2*a^3*b
```

```
^4*x^2 + a^4*b^3), 1/8*((a*b^3*d - 5*a^2*b^2*e)*x^3 + ((b^3*d + 3*a*b^2*e)*
x^4 + a^2*b*d + 3*a^3*e + 2*(a*b^2*d + 3*a^2*b*e)*x^2)*sqrt(a*b)*arctan(sqr
t(a*b)*x/a) - (a^2*b^2*d + 3*a^3*b*e)*x)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4
*b^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 188, normalized size = 1.23

$$-\frac{\left(-3a\,b^2e\,x^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)-b^3d\,x^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)-6a^2be\,x^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)-2a\,b^2d\,x^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)+5\sqrt{ab}\,abe\,x^3-\sqrt{ab}\,b^2d\,x^3-3a^3e\arctan\left(\frac{bx}{\sqrt{ab}}\right)-a^2bd\arctan\left(\frac{bx}{\sqrt{ab}}\right)+3\sqrt{ab}\,a^2ex+\sqrt{ab}\,abdx\right)\left(b\,x^2+a\right)}{8\sqrt{ab}\,\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}a\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/8*(-3*arctan(1/(a*b)^(1/2)*b*x)*x^4*a*b^2*e-arctan(1/(a*b)^(1/2)*b*x)*x^
4*b^3*d+5*(a*b)^(1/2)*x^3*a*b*e-(a*b)^(1/2)*x^3*b^2*d-6*arctan(1/(a*b)^(1/2
)*b*x)*x^2*a^2*b*e-2*arctan(1/(a*b)^(1/2)*b*x)*x^2*a*b^2*d+3*(a*b)^(1/2)*x*
a^2*e+(a*b)^(1/2)*x*a*b*d-3*arctan(1/(a*b)^(1/2)*b*x)*a^3*e-arctan(1/(a*b)^
(1/2)*b*x)*a^2*b*d)*(b*x^2+a)/(a*b)^(1/2)/a/b^2/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 1.49, size = 125, normalized size = 0.82

$$-\frac{1}{8}\,e\left(\frac{5\,bx^3+3\,ax}{b^4x^4+2\,ab^3x^2+a^2b^2}-\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}\right)+\frac{1}{8}\,d\left(\frac{bx^3-ax}{ab^3x^4+2\,a^2b^2x^2+a^3b}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/8*e*((5*b*x^3 + 3*a*x)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2) - 3*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*b^2)) + 1/8*d*((b*x^3 - a*x)/(a*b^3*x^4 + 2*a^2*b^2*x
^2 + a^3*b) + arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(e\,x^2+d\right)}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((x^2*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(d + e x^{2}\right)}{\left(\left(a + b x^{2}\right)^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**2*(d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

3.62 $\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{bd-ae}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1247, 640, 607}

$$-\frac{bd-ae}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -e/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*d - a*e)/(4*b^2*(a + b*x^2)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(d+ex^2\right)}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{d+ex}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= -\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(bd-ae)\operatorname{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx,x,x^2\right)}{2b}\\
&= -\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{bd-ae}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.58

$$\frac{-ae-b\left(d+2ex^2\right)}{4b^2\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-(a*e) - b*(d + 2*e*x^2))/(4*b^2*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [B] time = 0.85, size = 192, normalized size = 2.49

$$\frac{-a^3be+\sqrt{b^2}\sqrt{a^2+2abx^2+b^2x^4}\left(-a^2e+abd+abex^2-b^2dx^2-2b^2ex^4\right)+a^2b^2d+ab^3ex^4+b^4dx^4+2b^4ex^6}{2x^4\left(-2ab^5-2b^6x^2\right)\sqrt{a^2+2abx^2+b^2x^4}+2\sqrt{b^2}\,x^4\left(2a^2b^4+4ab^5x^2+2b^6x^4\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^2*b^2*d - a^3*b*e + b^4*d*x^4 + a*b^3*e*x^4 + 2*b^4*e*x^6 + Sqrt[b^2]*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]*(a*b*d - a^2*e - b^2*d*x^2 + a*b*e*x^2 - 2*b^
2*e*x^4))/(2*x^4*(-2*a*b^5 - 2*b^6*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4] + 2
*Sqrt[b^2]*x^4*(2*a^2*b^4 + 4*a*b^5*x^2 + 2*b^6*x^4))
```

fricas [A] time = 0.85, size = 42, normalized size = 0.55

$$-\frac{2\,bex^2+bd+ae}{4\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*e*x^2 + b*d + a*e)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

giac [A] time = 0.51, size = 40, normalized size = 0.52

$$-\frac{2\,bx^2e+bd+ae}{4\left(bx^2+a\right)^2b^2\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(2*b*x^2*e + b*d + a*e)/((b*x^2 + a)^2*b^2*sgn(b*x^2 + a))
```

maple [A] time = 0.01, size = 38, normalized size = 0.49

$$-\frac{\left(b\,x^2+a\right)\left(2be\,x^2+ae+bd\right)}{4\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/4*(b*x^2+a)*(2*b*e*x^2+a*e+b*d)/b^2/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 0.64, size = 65, normalized size = 0.84

$$-\frac{\left(2\,bx^2+a\right)e}{4\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)}-\frac{d}{4\left(b^3x^4+2\,ab^2x^2+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*x^2 + a)*e/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2) - 1/4*d/(b^3*x^4 + 2
*a*b^2*x^2 + a^2*b)
```

mupad [B] time = 0.18, size = 48, normalized size = 0.62

$$-\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(2\,b\,e\,x^2+a\,e+b\,d\right)}{4\,b^2\,{\left(b\,x^2+a\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] -((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(a*e + b*d + 2*b*e*x^2))/(4*b^2*(a + b*
x^2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(d+ex^2\right)}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x*(d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

3.63 $\int \frac{d+ex^2}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}} dx$

Optimal. Leaf size=156

$$\frac{x(ae+3bd)}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x(bd-ae)}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(ae+3bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.09, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1148, 385, 199, 205}

$$\frac{x(ae+3bd)}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x(bd-ae)}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(ae+3bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((3*b*d + a*e)*x)/(8*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d - a*e)*
x)/(4*a*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((3*b*d + a*e)*(a
+ b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1148

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_),
x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^
2)^(2*FracPart[p])), Int[(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ
[{a, b, c, d, e, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{d+ex^2}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{d+ex^2}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(bd-ae)x}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left((3bd+ae)\left(ab+b^2x^2\right)\right)\int \frac{1}{\left(ab+b^2x^2\right)^2}\,dx}{4a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(3bd+ae)x}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)x}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left((3bd+ae)\left(ab\right.\right.}{8a^2b\sqrt{a^2}}$$

$$= \frac{(3bd+ae)x}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)x}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(3bd+ae)\left(a+\right.}{8a^{5/2}b^{3/2}\sqrt{a^2}}$$

Mathematica [A] time = 0.05, size = 108, normalized size = 0.69

$$\frac{\sqrt{a}\sqrt{b}\,x\left(a^2(-e)+ab\left(5d+ex^2\right)+3b^2dx^2\right)+\left(a+bx^2\right)^2(ae+3bd)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-(a^2*e) + 3*b^2*d*x^2 + a*b*(5*d + e*x^2)) + (3*b*d +
a*e)*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2)*(a + b*x
^2)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 15.78, size = 105, normalized size = 0.67

$$\frac{\left(a+bx^2\right)\left(\frac{(ae+3bd)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}-\frac{x\left(a^2e-5abd-abex^2-3b^2dx^2\right)}{8a^2b\left(a+bx^2\right)^2}\right)}{\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x^2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((a + b*x^2)*(-1/8*(x*(-5*a*b*d + a^2*e - 3*b^2*d*x^2 - a*b*e*x^2))/(a^2*b*
(a + b*x^2)^2) + ((3*b*d + a*e)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(
3/2))))/Sqrt[(a + b*x^2)^2]
```

fricas [A] time = 0.78, size = 301, normalized size = 1.93

$$\left[\frac{2\left(3\,ab^3d+a^2b^2e\right)x^3-\left(\left(3\,b^3d+ab^2e\right)x^4+3\,a^2bd+a^3e+2\left(3\,ab^2d+a^2be\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\left(5\,a^2b^2d-a^3be\right)x}{16\left(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2\right)},\ \frac{\left(3\,ab^3d+a^2b^2e\right)x^3+\left(\left(3\,b^3d+ab^2e\right)x^4+3\,a^2bd+a^3e+2\left(3\,ab^2d+a^2be\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(5\,a^2b^2d-a^3be\right)x}{8\left(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*a*b^3*d + a^2*b^2*e)*x^3 - ((3*b^3*d + a*b^2*e)*x^4 + 3*a^2*b*d
 + a^3*e + 2*(3*a*b^2*d + a^2*b*e)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b
)*x - a)/(b*x^2 + a)) + 2*(5*a^2*b^2*d - a^3*b*e)*x)/(a^3*b^4*x^4 + 2*a^4*b
^3*x^2 + a^5*b^2), 1/8*((3*a*b^3*d + a^2*b^2*e)*x^3 + ((3*b^3*d + a*b^2*e)*
```

```
x^4 + 3*a^2*b*d + a^3*e + 2*(3*a*b^2*d + a^2*b*e)*x^2)*sqrt(a*b)*arctan(sqr
t(a*b)*x/a) + (5*a^2*b^2*d - a^3*b*e)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 + a^5
*b^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 186, normalized size = 1.19

$$\frac{\left(a b^2 e x^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)+3b^3 d x^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)+2a^2 be x^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)+6a b^2 d x^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)+\sqrt{ab}\, abe x^3+3\sqrt{ab}\, b^2 d x^3+a^3 e \arctan\left(\frac{bx}{\sqrt{ab}}\right)+3a^2 bd \arctan\left(\frac{bx}{\sqrt{ab}}\right)-\sqrt{ab}\, a^2 ex+5\sqrt{ab}\, abdx\right)\left(b x^2+a\right)}{8\sqrt{ab}\,\left(\left(b x^2+a\right)^2\right)^{\frac{3}{2}} a^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/8*(a*b^2*e*x^4*arctan(1/(a*b)^(1/2)*b*x)+3*b^3*d*x^4*arctan(1/(a*b)^(1/2)
*b*x)+(a*b)^(1/2)*a*b*e*x^3+3*(a*b)^(1/2)*b^2*d*x^3+2*a^2*b*e*x^2*arctan(1/
(a*b)^(1/2)*b*x)+6*a*b^2*d*x^2*arctan(1/(a*b)^(1/2)*b*x)-(a*b)^(1/2)*a^2*e*
x+5*(a*b)^(1/2)*a*b*d*x+a^3*e*arctan(1/(a*b)^(1/2)*b*x)+3*a^2*b*d*arctan(1/
(a*b)^(1/2)*b*x))*(b*x^2+a)/(a*b)^(1/2)/b/a^2/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 1.67, size = 124, normalized size = 0.79

$$\frac{1}{8}\, d\left(\frac{3\, bx^3+5\, ax}{a^2b^2x^4+2\, a^3bx^2+a^4}+\frac{3\, \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, a^2}\right)+\frac{1}{8}\, e\left(\frac{bx^3-ax}{ab^3x^4+2\, a^2b^2x^2+a^3b}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*d*((3*b*x^3 + 5*a*x)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 3*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*a^2)) + 1/8*e*((b*x^3 - a*x)/(a*b^3*x^4 + 2*a^2*b^2*x^
2 + a^3*b) + arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2+d}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d + e*x^2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

3.64 $\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=161

$$\frac{bd-ae}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d\log(x)\left(a+bx^2\right)}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1250, 446, 77}

$$\frac{bd-ae}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d\log(x)\left(a+bx^2\right)}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] d/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b*d - a*e)/(4*a*b*(a + b*x^2)*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*(a + b*x^2)*Log[x])/(a^3*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) - (d*(a + b*x^2)*Log[a + b*x^2])/(2*a^3*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{d+ex^2}{x\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\mathrm{Subst}\left(\int \frac{d+ex}{x\left(ab+b^2x\right)^3}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\mathrm{Subst}\left(\int \left(\frac{d}{a^3b^3x}+\frac{-bd+ae}{ab^3(a+bx)^3}-\frac{d}{a^2b^2(a+bx)^2}-\frac{d}{a^3b^2(a+bx)}\right)dx,x,x\right.}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{d}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{bd-ae}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d\left(a+bx\right.}{a^3\sqrt{a^2+2a}}$$

Mathematica [A] time = 0.04, size = 92, normalized size = 0.57

$$\frac{a\left(a^2(-e)+3abd+2b^2dx^2\right)+4bd\log(x)\left(a+bx^2\right)^2-2bd\left(a+bx^2\right)^2\log\left(a+bx^2\right)}{4a^3b\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (a*(3*a*b*d - a^2*e + 2*b^2*d*x^2) + 4*b*d*(a + b*x^2)^2*Log[x] - 2*b*d*(a
+ b*x^2)^2*Log[a + b*x^2])/(4*a^3*b*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 0.89, size = 258, normalized size = 1.60

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b^2}x^2}{a}-\frac{\sqrt{a^2+2abx^2+b^2x^4}}{a}\right)}{a^3}+\frac{a^4be-a^3b^2d+a^2b^3ex^4+\sqrt{b^2}\sqrt{a^2+2abx^2+b^2x^4}\left(a^3e-a^2bd-a^2bex^2+ab^2dx^2+2b^3dx^4\right)-3ab^4dx^4-2b^5dx^6}{2a^2b\sqrt{b^2}x^4\left(2a^2b^2+4ab^3x^2+2b^4x^4\right)+2a^2bx^4\left(-2ab^3-2b^4x^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x^2)/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (-(a^3*b^2*d) + a^4*b*e - 3*a*b^4*d*x^4 + a^2*b^3*e*x^4 - 2*b^5*d*x^6 + Sqr
t[b^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-(a^2*b*d) + a^3*e + a*b^2*d*x^2 -
a^2*b*e*x^2 + 2*b^3*d*x^4))/(2*a^2*b*x^4*(-2*a*b^3 - 2*b^4*x^2)*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4] + 2*a^2*b*Sqrt[b^2]*x^4*(2*a^2*b^2 + 4*a*b^3*x^2 + 2*b
^4*x^4)) + (d*ArcTanh[(Sqrt[b^2]*x^2)/a - Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/a
])/a^3
```

fricas [A] time = 0.65, size = 119, normalized size = 0.74

$$\frac{2\,ab^2dx^2+3\,a^2bd-a^3e-2\left(b^3dx^4+2\,ab^2dx^2+a^2bd\right)\log\left(bx^2+a\right)+4\left(b^3dx^4+2\,ab^2dx^2+a^2bd\right)\log(x)}{4\left(a^3b^3x^4+2\,a^4b^2x^2+a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b^2*d*x^2 + 3*a^2*b*d - a^3*e - 2*(b^3*d*x^4 + 2*a*b^2*d*x^2 + a^2
*b*d)*log(b*x^2 + a) + 4*(b^3*d*x^4 + 2*a*b^2*d*x^2 + a^2*b*d)*log(x))/(a^3
*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)
```

giac [A] time = 0.53, size = 96, normalized size = 0.60

$$-\frac{d \log \left(\left|bx^2 + a\right|\right)}{2 a^3 \mathrm{sgn}\left(bx^2 + a\right)} + \frac{d \log \left(|x|\right)}{a^3 \mathrm{sgn}\left(bx^2 + a\right)} + \frac{2 ab^2 dx^2 + 3 a^2 bd - a^3 e}{4 \left(bx^2 + a\right)^2 a^3 b \mathrm{sgn}\left(bx^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*d*log(abs(b*x^2 + a))/(a^3*sgn(b*x^2 + a)) + d*log(abs(x))/(a^3*sgn(b*
x^2 + a)) + 1/4*(2*a*b^2*d*x^2 + 3*a^2*b*d - a^3*e)/((b*x^2 + a)^2*a^3*b*sg
n(b*x^2 + a))
```

maple [A] time = 0.02, size = 133, normalized size = 0.83

$$\frac{\left(4b^3 d x^4 \ln(x) - 2b^3 d x^4 \ln\left(b x^2 + a\right) + 8a b^2 d x^2 \ln(x) - 4a b^2 d x^2 \ln\left(b x^2 + a\right) + 2a b^2 d x^2 + 4a^2 bd \ln(x) - 2a^2 bd \ln\left(b x^2 + a\right) - a^3 e + 3a^2 bd\right)\left(b x^2 + a\right)}{4\left(\left(b x^2 + a\right)^2\right)^{\frac{3}{2}} a^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/4*(4*ln(x)*x^4*b^3*d-2*ln(b*x^2+a)*x^4*b^3*d+8*ln(x)*x^2*a*b^2*d-4*ln(b*x
^2+a)*x^2*a*b^2*d+2*b^2*d*x^2*a+4*ln(x)*a^2*b*d-2*ln(b*x^2+a)*a^2*b*d-a^3*e
+3*a^2*b*d)*(b*x^2+a)/b/a^3/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 0.89, size = 88, normalized size = 0.55

$$\frac{1}{4} d \left(\frac{2 bx^2 + 3 a}{a^2 b^2 x^4 + 2 a^3 bx^2 + a^4} - \frac{2 \log\left(bx^2 + a\right)}{a^3} + \frac{4 \log(x)}{a^3}\right) - \frac{e}{4 \left(b^3 x^4 + 2 ab^2 x^2 + a^2 b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*d*((2*b*x^2 + 3*a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) - 2*log(b*x^2 + a)
/a^3 + 4*log(x)/a^3) - 1/4*e/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e x^2 + d}{x \left(a^2 + 2 a b x^2 + b^2 x^4\right)^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int((d + e*x^2)/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{x \left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/(x*((a + b*x**2)**2)**(3/2)), x)
```

3.65 $\int \frac{d+ex^2}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=190

$$-\frac{x(bd-ae)}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3\left(a+bx^2\right)(5bd-ae)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x(7bd-3ae)}{8a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a\right.}{a^3x\sqrt{a^2+\cdots}}$$

Rubi [A] time = 0.19, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1250, 456, 453, 205}

$$-\frac{x(7bd-3ae)}{8a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x(bd-ae)}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3\left(a+bx^2\right)(5bd-ae)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)}{a^3x\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] -((7*b*d - 3*a*e)*x)/(8*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)
*x)/(4*a^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(a + b*x^2))/(
a^3*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*(5*b*d - a*e)*(a + b*x^2)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{d+ex^2}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{d+ex^2}{x^2\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{(bd-ae)x}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{-\frac{4d}{ab}+\frac{3(bd-ae)x^2}{a^2b}}{x^2\left(ab+b^2x^2\right)^2}\,dx}{4\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{(7bd-3ae)x}{8a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(b^2\left(ab+b^2\right.\right.}{8\sqrt{a^2}}$$

$$= -\frac{(7bd-3ae)x}{8a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\left(a+\right.}{a^3x\sqrt{a^2+2a}}$$

$$= -\frac{(7bd-3ae)x}{8a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\left(a+\right.}{a^3x\sqrt{a^2+2a}}$$

Mathematica [A] time = 0.07, size = 124, normalized size = 0.65

$$\frac{\sqrt{a}\sqrt{b}\left(a^2\left(5ex^2-8d\right)+ab\left(3ex^4-25dx^2\right)-15b^2dx^4\right)+3x\left(a+bx^2\right)^2(ae-5bd)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}\,x\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*(-15*b^2*d*x^4 + a^2*(-8*d + 5*e*x^2) + a*b*(-25*d*x^2 + 3
*e*x^4)) + 3*(-5*b*d + a*e)*x*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8
*a^(7/2)*Sqrt[b]*x*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 19.38, size = 117, normalized size = 0.62

$$\frac{\left(a+bx^2\right)\left(\frac{3(ae-5bd)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}+\frac{-8a^2d+5a^2ex^2-25abdx^2+3abex^4-15b^2dx^4}{8a^3x\left(a+bx^2\right)^2}\right)}{\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x^2)/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] ((a + b*x^2)*((-8*a^2*d - 25*a*b*d*x^2 + 5*a^2*e*x^2 - 15*b^2*d*x^4 + 3*a*b
*e*x^4)/(8*a^3*x*(a + b*x^2)^2) + (3*(-5*b*d + a*e)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(8*a^(7/2)*Sqrt[b])))/Sqrt[(a + b*x^2)^2]
```

fricas [A] time = 0.96, size = 334, normalized size = 1.76

$$\left[-\frac{16a^3bd+6\left(5ab^3d-a^2b^2e\right)x^4+10\left(5a^2b^2d-a^3be\right)x^2-3\left(\left(5b^3d-ab^2e\right)x^5+2\left(5ab^2d-a^2be\right)x^3+\left(5a^2bd-a^3e\right)x\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{16\left(a^4b^3x^5+2a^5b^2x^3+a^6bx\right)}, -\frac{8a^3bd+3\left(5ab^3d-a^2b^2e\right)x^4+5\left(5a^2b^2d-a^3be\right)x^2+3\left(\left(5b^3d-ab^2e\right)x^5+2\left(5ab^2d-a^2be\right)x^3+\left(5a^2bd-a^3e\right)x\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{8\left(a^4b^3x^5+2a^5b^2x^3+a^6bx\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [-1/16*(16*a^3*b*d + 6*(5*a*b^3*d - a^2*b^2*e)*x^4 + 10*(5*a^2*b^2*d - a^3*
b*e)*x^2 - 3*((5*b^3*d - a*b^2*e)*x^5 + 2*(5*a*b^2*d - a^2*b*e)*x^3 + (5*a^
2*b*d - a^3*e)*x)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))
/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x), -1/8*(8*a^3*b*d + 3*(5*a*b^3*d -
a^2*b^2*e)*x^4 + 5*(5*a^2*b^2*d - a^3*b*e)*x^2 + 3*((5*b^3*d - a*b^2*e)*x^5
 + 2*(5*a*b^2*d - a^2*b*e)*x^3 + (5*a^2*b*d - a^3*e)*x)*sqrt(a*b)*arctan(sq
rt(a*b)*x/a))/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 206, normalized size = 1.08

$$\frac{\left(3 a b^2 e x^5 \arctan\left(\frac{b x}{\sqrt{a b}}\right)-15 b^3 d x^5 \arctan\left(\frac{b x}{\sqrt{a b}}\right)+6 a^2 b e x^3 \arctan\left(\frac{b x}{\sqrt{a b}}\right)-30 a b^2 d x^3 \arctan\left(\frac{b x}{\sqrt{a b}}\right)+3 \sqrt{a b}\, a b e x^4-15 \sqrt{a b}\, b^2 d x^4+3 a^3 e x \arctan\left(\frac{b x}{\sqrt{a b}}\right)-15 a^2 b d x \arctan\left(\frac{b x}{\sqrt{a b}}\right)+5 \sqrt{a b}\, a^2 e x^2-25 \sqrt{a b}\, a b d x^2-8 \sqrt{a b}\, a^2 d\right)\left(b x^2+a\right)}{8 \sqrt{a b}\, \left(\left(b x^2+a\right)^2\right)^{\frac{3}{2}} a^3 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/8*(3*arctan(1/(a*b)^(1/2)*b*x)*x^5*a*b^2*e-15*arctan(1/(a*b)^(1/2)*b*x)*x
^5*b^3*d+3*(a*b)^(1/2)*x^4*a*b*e-15*(a*b)^(1/2)*x^4*b^2*d+6*arctan(1/(a*b)^
(1/2)*b*x)*x^3*a^2*b*e-30*arctan(1/(a*b)^(1/2)*b*x)*x^3*a*b^2*d+5*(a*b)^(1/
2)*x^2*a^2*e-25*(a*b)^(1/2)*x^2*a*b*d+3*arctan(1/(a*b)^(1/2)*b*x)*x*a^3*e-1
5*arctan(1/(a*b)^(1/2)*b*x)*x*a^2*b*d-8*(a*b)^(1/2)*a^2*d)*(b*x^2+a)/(a*b)^
(1/2)/x/a^3/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 1.40, size = 134, normalized size = 0.71

$$-\frac{1}{8} d\left(\frac{15 b^2 x^4+25 a b x^2+8 a^2}{a^3 b^2 x^5+2 a^4 b x^3+a^5 x}+\frac{15 b \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, a^3}\right)+\frac{1}{8} e\left(\frac{3 b x^3+5 a x}{a^2 b^2 x^4+2 a^3 b x^2+a^4}+\frac{3 \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/8*d*((15*b^2*x^4 + 25*a*b*x^2 + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*
x) + 15*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)) + 1/8*e*((3*b*x^3 + 5*a*x)
/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2
))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2+d}{x^2\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int((d + e*x^2)/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{x^2 \left(\left(a + bx^2 \right)^2 \right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
[Out] Integral((d + e*x**2)/(x**2*((a + b*x**2)**2)**(3/2)), x)
```

3.66 $\int \frac{d+ex^2}{x^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=223

$$-\frac{bd-ae}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\log(x)\left(a+bx^2\right)(3bd-ae)}{a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(3bd-ae)\log\left(a+bx^2\right)}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{2a^3\sqrt{a^2}}$$

Rubi [A] time = 0.18, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1250, 446, 77}

$$-\frac{bd-ae}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{2bd-ae}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\log(x)\left(a+bx^2\right)(3bd-ae)}{a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(3bd-ae)\log\left(a+bx^2\right)}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)}{2a^3x^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] -(2*b*d - a*e)/(2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*d - a*e)/(4*a^2
*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(a + b*x^2))/(2*a^3*x^2*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((3*b*d - a*e)*(a + b*x^2)*Log[x])/(a^4*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((3*b*d - a*e)*(a + b*x^2)*Log[a + b*x^2
])/(2*a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{d+ex^2}{x^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{d+ex^2}{x^3\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{d+ex}{x^2\left(ab+b^2x\right)^3}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \left(\frac{d}{a^3b^3x^2}+\frac{-3bd+ae}{a^4b^3x}+\frac{bd-ae}{a^2b^2(a+bx)^3}+\frac{2bd-ae}{a^3b^2(a+bx)^2}+\frac{3bd-ae}{a^4b^2(a+b}\right.\right.}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2bd-ae}{2a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{bd-ae}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\left(a\right.}{2a^3x^2\sqrt{a^2+}}$$

Mathematica [A] time = 0.07, size = 130, normalized size = 0.58

$$\frac{a\left(a^2\left(3ex^2-2d\right)+ab\left(2ex^4-9dx^2\right)-6b^2dx^4\right)+4x^2\log(x)\left(a+bx^2\right)^2(ae-3bd)+2x^2\left(a+bx^2\right)^2(3bd-ae)\log\left(a+bx^2\right)}{4a^4x^2\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (a*(-6*b^2*d*x^4 + a^2*(-2*d + 3*e*x^2) + a*b*(-9*d*x^2 + 2*e*x^4)) + 4*(-3
*b*d + a*e)*x^2*(a + b*x^2)^2*Log[x] + 2*(3*b*d - a*e)*x^2*(a + b*x^2)^2*Lo
g[a + b*x^2])/(4*a^4*x^2*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [B] time = 6.04, size = 1473, normalized size = 6.61

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x^2)/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (-(a^15*b^2*d) + a^16*b*e + 2*a^14*b^3*d*x^2 + 49*a^13*b^4*d*x^4 - 3*a^14*b
^3*e*x^4 + 552*a^12*b^5*d*x^6 - 80*a^13*b^4*e*x^6 + 3784*a^11*b^6*d*x^8 - 9
88*a^12*b^5*e*x^8 + 17600*a^10*b^7*d*x^10 - 7488*a^11*b^6*e*x^10 + 58608*a^
9*b^8*d*x^12 - 38896*a^10*b^7*e*x^12 + 143616*a^8*b^9*d*x^14 - 146432*a^9*b
^8*e*x^14 + 261888*a^7*b^10*d*x^16 - 411840*a^8*b^9*e*x^16 + 354816*a^6*b^1
1*d*x^18 - 878592*a^7*b^10*e*x^18 + 352000*a^5*b^12*d*x^20 - 1427712*a^6*b^
11*e*x^20 + 247808*a^4*b^13*d*x^22 - 1757184*a^5*b^12*e*x^22 + 116736*a^3*b
^14*d*x^24 - 1610752*a^4*b^13*e*x^24 + 32768*a^2*b^15*d*x^26 - 1064960*a^3*
b^14*e*x^26 + 4096*a*b^16*d*x^28 - 479232*a^2*b^15*e*x^28 - 131072*a*b^16*e
*x^30 - 16384*b^17*e*x^32 + Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-(a^
14*b*d) + a^15*e - a^13*b^2*d*x^2 - a^14*b*e*x^2 - 48*a^12*b^3*d*x^4 + 4*a^
13*b^2*e*x^4 - 504*a^11*b^4*d*x^6 + 76*a^12*b^3*e*x^6 - 3280*a^10*b^5*d*x^8
 + 912*a^11*b^4*e*x^8 - 14320*a^9*b^6*d*x^10 + 6576*a^10*b^5*e*x^10 - 44288
*a^8*b^7*d*x^12 + 32320*a^9*b^6*e*x^12 - 99328*a^7*b^8*d*x^14 + 114112*a^8*
b^7*e*x^14 - 162560*a^6*b^9*d*x^16 + 297728*a^7*b^8*e*x^16 - 192256*a^5*b^1
0*d*x^18 + 580864*a^6*b^9*e*x^18 - 159744*a^4*b^11*d*x^20 + 846848*a^5*b^10
*e*x^20 - 88064*a^3*b^12*d*x^22 + 910336*a^4*b^11*e*x^22 - 28672*a^2*b^13*d
*x^24 + 700416*a^3*b^12*e*x^24 - 4096*a*b^14*d*x^26 + 364544*a^2*b^13*e*x^2
6 + 114688*a*b^14*e*x^28 + 16384*b^15*e*x^30))/(2*a^2*b*x^4*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4]*(-2*a^14*b^2 - 54*a^13*b^3*x^2 - 676*a^12*b^4*x^4 - 5200*a
^11*b^5*x^6 - 27456*a^10*b^6*x^8 - 105248*a^9*b^7*x^10 - 302016*a^8*b^8*x^1
```

```
2 - 658944*a^7*b^9*x^14 - 1098240*a^6*b^10*x^16 - 1391104*a^5*b^11*x^18 - 1
317888*a^4*b^12*x^20 - 905216*a^3*b^13*x^22 - 425984*a^2*b^14*x^24 - 122880
*a*b^15*x^26 - 16384*b^16*x^28) + 2*a^2*b*Sqrt[b^2]*x^4*(2*a^15*b + 56*a^14
*b^2*x^2 + 730*a^13*b^3*x^4 + 5876*a^12*b^4*x^6 + 32656*a^11*b^5*x^8 + 1327
04*a^10*b^6*x^10 + 407264*a^9*b^7*x^12 + 960960*a^8*b^8*x^14 + 1757184*a^7*
b^9*x^16 + 2489344*a^6*b^10*x^18 + 2708992*a^5*b^11*x^20 + 2223104*a^4*b^12
*x^22 + 1331200*a^3*b^13*x^24 + 548864*a^2*b^14*x^26 + 139264*a*b^15*x^28 +
 16384*b^16*x^30)) + ((-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^
3*((-3*b*d)/a^3 + (Sqrt[b^2]*e*x^2*ArcTanh[(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/a])/a^3 - (e*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(
-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/a])/a^3))/(a^4 + 4*a^3*
b*x^2 + 12*a^2*b^2*x^4 + 16*a*b^3*x^6 + 8*b^4*x^8 - 4*a^2*Sqrt[b^2]*x^2*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4] - 8*a*b*Sqrt[b^2]*x^4*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4] - 8*(b^2)^(3/2)*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*b*d*ArcTa
nh[(Sqrt[b^2]*x^2)/a - Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/a])/a^4
```

fricas [A] time = 0.67, size = 205, normalized size = 0.92

$$-\frac{2\left(3\,ab^2d-a^2be\right)x^4+2\,a^3d+3\left(3\,a^2bd-a^3e\right)x^2-2\left(\left(3\,b^3d-ab^2e\right)x^6+2\left(3\,ab^2d-a^2be\right)x^4+\left(3\,a^2bd-a^3e\right)x^2\right)\log\left(bx^2+a\right)+4\left(\left(3\,b^3d-ab^2e\right)x^6+2\left(3\,ab^2d-a^2be\right)x^4+\left(3\,a^2bd-a^3e\right)x^2\right)\log(x)}{4\left(a^4b^2x^6+2\,a^5bx^4+a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/4*(2*(3*a*b^2*d - a^2*b*e)*x^4 + 2*a^3*d + 3*(3*a^2*b*d - a^3*e)*x^2 - 2
*((3*b^3*d - a*b^2*e)*x^6 + 2*(3*a*b^2*d - a^2*b*e)*x^4 + (3*a^2*b*d - a^3*
e)*x^2)*log(b*x^2 + a) + 4*((3*b^3*d - a*b^2*e)*x^6 + 2*(3*a*b^2*d - a^2*b*
e)*x^4 + (3*a^2*b*d - a^3*e)*x^2)*log(x))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*
x^2)
```

giac [A] time = 0.40, size = 144, normalized size = 0.65

$$-\frac{(3\,bd-ae)\log\left(|x|\right)}{a^4\mathrm{sgn}\left(bx^2+a\right)}+\frac{\left(3\,b^2d-abe\right)\log\left(\left|bx^2+a\right|\right)}{2\,a^4b\mathrm{sgn}\left(bx^2+a\right)}-\frac{2\left(3\,ab^2d-a^2be\right)x^4+2\,a^3d+3\left(3\,a^2bd-a^3e\right)x^2}{4\left(bx^2+a\right)^2a^4x^2\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(3*b*d - a*e)*log(abs(x))/(a^4*sgn(b*x^2 + a)) + 1/2*(3*b^2*d - a*b*e)*log
(abs(b*x^2 + a))/(a^4*b*sgn(b*x^2 + a)) - 1/4*(2*(3*a*b^2*d - a^2*b*e)*x^4
+ 2*a^3*d + 3*(3*a^2*b*d - a^3*e)*x^2)/((b*x^2 + a)^2*a^4*x^2*sgn(b*x^2 + a
))
```

maple [A] time = 0.02, size = 249, normalized size = 1.12

$$\frac{\left(4a\,b^2e\,x^6\ln(x)-2a\,b^2e\,x^6\ln\left(b\,x^2+a\right)-12b^3d\,x^6\ln(x)+6b^3d\,x^6\ln\left(b\,x^2+a\right)+8a^2be\,x^4\ln(x)-4a^2be\,x^4\ln\left(b\,x^2+a\right)-24a\,b^2d\,x^4\ln(x)+12a\,b^2d\,x^4\ln\left(b\,x^2+a\right)+2a^2be\,x^4-6a\,b^2d\,x^4+4a^3e\,x^2\ln(x)-2a^3e\,x^2\ln\left(b\,x^2+a\right)-12a^2bd\,x^2\ln(x)+6a^2bd\,x^2\ln\left(b\,x^2+a\right)+3a^3e\,x^2-9a^2bd\,x^2-2a^3d\right)\left(b\,x^2+a\right)}{4\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/4*(4*ln(x)*x^6*a*b^2*e-12*ln(x)*x^6*b^3*d-2*ln(b*x^2+a)*x^6*a*b^2*e+6*ln(
b*x^2+a)*x^6*b^3*d+8*ln(x)*x^4*a^2*b*e-24*ln(x)*x^4*a*b^2*d-4*ln(b*x^2+a)*x
^4*a^2*b*e+12*ln(b*x^2+a)*x^4*a*b^2*d+2*x^4*a^2*b*e-6*x^4*a*b^2*d+4*ln(x)*x
^2*a^3*e-12*ln(x)*x^2*a^2*b*d-2*ln(b*x^2+a)*x^2*a^3*e+6*ln(b*x^2+a)*x^2*a^2
*b*d+3*x^2*a^3*e-9*x^2*a^2*b*d-2*a^3*d)*(b*x^2+a)/x^2/a^4/((b*x^2+a)^2)^(3/
2)
```

maxima [A] time = 0.84, size = 138, normalized size = 0.62

$$-\frac{1}{4}\,d\left(\frac{6\,b^2x^4 + 9\,abx^2 + 2\,a^2}{a^3b^2x^6 + 2\,a^4bx^4 + a^5x^2} - \frac{6\,b\log\left(bx^2 + a\right)}{a^4} + \frac{12\,b\log(x)}{a^4}\right) + \frac{1}{4}\,e\left(\frac{2\,bx^2 + 3\,a}{a^2b^2x^4 + 2\,a^3bx^2 + a^4} - \frac{2\,\log\left(bx^2 + a\right)}{a^3} + \frac{4\,\log(x)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/4*d*((6*b^2*x^4 + 9*a*b*x^2 + 2*a^2)/(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^
2) - 6*b*log(b*x^2 + a)/a^4 + 12*b*log(x)/a^4) + 1/4*e*((2*b*x^2 + 3*a)/(a^
2*b^2*x^4 + 2*a^3*b*x^2 + a^4) - 2*log(b*x^2 + a)/a^3 + 4*log(x)/a^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2 + d}{x^3\,\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int((d + e*x^2)/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{x^3\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x**3/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/(x**3*((a + b*x**2)**2)**(3/2)), x)
```

3.67 $\int (fx)^m \left(d+ex^2\right) \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx$

Optimal. Leaf size=400

$$\frac{10a^2b^2\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+7}(ae+bd)}{f^7(m+7)\left(a+bx^2\right)}+\frac{b^5e\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+13}}{f^{13}(m+13)\left(a+bx^2\right)}+\frac{b^4\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m-}}{f^{11}(m+11)\left(a+b\right.}$$

Rubi [A] time = 0.24, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1250, 448}

$$\frac{a^4\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+3}(ae+5bd)}{f^3(m+3)\left(a+bx^2\right)}+\frac{5a^3b\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+5}(ae+2bd)}{f^5(m+5)\left(a+bx^2\right)}+\frac{10a^2b^2\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+7}(ae+bd)}{f^7(m+7)\left(a+bx^2\right)}+\frac{5ab^3\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+9}(2ae+bd)}{f^9(m+9)\left(a+bx^2\right)}+\frac{b^4\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+11}(5ae+bd)}{f^{11}(m+11)\left(a+bx^2\right)}+\frac{a^5d\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+1}}{f(m+1)\left(a+bx^2\right)}+\frac{b^5e\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+13}}{f^{13}(m+13)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*(a + b*x^2
)) + (a^4*(5*b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^3
*(3 + m)*(a + b*x^2)) + (5*a^3*b*(2*b*d + a*e)*(f*x)^(5 + m)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(f^5*(5 + m)*(a + b*x^2)) + (10*a^2*b^2*(b*d + a*e)*(f*x
)^(7 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^7*(7 + m)*(a + b*x^2)) + (5*a
*b^3*(b*d + 2*a*e)*(f*x)^(9 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^9*(9 +
 m)*(a + b*x^2)) + (b^4*(b*d + 5*a*e)*(f*x)^(11 + m)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(f^11*(11 + m)*(a + b*x^2)) + (b^5*e*(f*x)^(13 + m)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(f^13*(13 + m)*(a + b*x^2))
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int (fx)^m \left(d+ex^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int (fx)^m\left(ab+b^2x^2\right)^5\left(d+ex^2\right)\,dx}{b^4\left(ab+b^2x^2\right)}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int \left(a^5b^5d(fx)^m+\frac{a^4b^5(5bd+ae)(fx)^{2+m}}{f^2}+\frac{5a^3b^6}{\ }\right.}{\ }\\
&= \frac{a^5d(fx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)\left(a+bx^2\right)}+\frac{a^4(5bd+ae)(fx)^{3+m}\sqrt{a^2+}}{f^3(3+m)\left(a+b\right.}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 160, normalized size = 0.40

$$\frac{x\sqrt{\left(a+bx^2\right)^2}\,(fx)^m\left(\frac{a^5d}{m+1}+\frac{a^4x^2(ae+5bd)}{m+3}+\frac{5a^3bx^4(ae+2bd)}{m+5}+\frac{10a^2b^2x^6(ae+bd)}{m+7}+\frac{b^4x^{10}(5ae+bd)}{m+11}+\frac{5ab^3x^8(2ae+bd)}{m+9}+\frac{b^5ex^{12}}{m+13}\right)}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x*(f*x)^m*Sqrt[(a + b*x^2)^2]*((a^5*d)/(1 + m) + (a^4*(5*b*d + a*e)*x^2)/(
3 + m) + (5*a^3*b*(2*b*d + a*e)*x^4)/(5 + m) + (10*a^2*b^2*(b*d + a*e)*x^6)
/(7 + m) + (5*a*b^3*(b*d + 2*a*e)*x^8)/(9 + m) + (b^4*(b*d + 5*a*e)*x^10)/(
11 + m) + (b^5*e*x^12)/(13 + m)))/(a + b*x^2)
```

IntegrateAlgebraic [F] time = 5.12, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d + ex^2\right) \left(a^2 + 2abx^2 + b^2x^4\right)^{5/2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(
5/2), x]
```

fricas [B] time = 0.86, size = 853, normalized size = 2.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fri
cas")
```

```
[Out] ((b^5*e*m^6 + 36*b^5*e*m^5 + 505*b^5*e*m^4 + 3480*b^5*e*m^3 + 12139*b^5*e*m
^2 + 19524*b^5*e*m + 10395*b^5*e)*x^13 + ((b^5*d + 5*a*b^4*e)*m^6 + 12285*b
^5*d + 61425*a*b^4*e + 38*(b^5*d + 5*a*b^4*e)*m^5 + 555*(b^5*d + 5*a*b^4*e)
*m^4 + 3940*(b^5*d + 5*a*b^4*e)*m^3 + 14039*(b^5*d + 5*a*b^4*e)*m^2 + 22902
*(b^5*d + 5*a*b^4*e)*m)*x^11 + 5*((a*b^4*d + 2*a^2*b^3*e)*m^6 + 15015*a*b^4
*d + 30030*a^2*b^3*e + 40*(a*b^4*d + 2*a^2*b^3*e)*m^5 + 613*(a*b^4*d + 2*a^
2*b^3*e)*m^4 + 4528*(a*b^4*d + 2*a^2*b^3*e)*m^3 + 16627*(a*b^4*d + 2*a^2*b^
3*e)*m^2 + 27688*(a*b^4*d + 2*a^2*b^3*e)*m)*x^9 + 10*((a^2*b^3*d + a^3*b^2*
e)*m^6 + 19305*a^2*b^3*d + 19305*a^3*b^2*e + 42*(a^2*b^3*d + a^3*b^2*e)*m^5
 + 679*(a^2*b^3*d + a^3*b^2*e)*m^4 + 5292*(a^2*b^3*d + a^3*b^2*e)*m^3 + 203
35*(a^2*b^3*d + a^3*b^2*e)*m^2 + 34986*(a^2*b^3*d + a^3*b^2*e)*m)*x^7 + 5*(
(2*a^3*b^2*d + a^4*b*e)*m^6 + 54054*a^3*b^2*d + 27027*a^4*b*e + 44*(2*a^3*b
^2*d + a^4*b*e)*m^5 + 753*(2*a^3*b^2*d + a^4*b*e)*m^4 + 6280*(2*a^3*b^2*d +
 a^4*b*e)*m^3 + 25979*(2*a^3*b^2*d + a^4*b*e)*m^2 + 47436*(2*a^3*b^2*d + a^
4*b*e)*m)*x^5 + ((5*a^4*b*d + a^5*e)*m^6 + 225225*a^4*b*d + 45045*a^5*e + 4
6*(5*a^4*b*d + a^5*e)*m^5 + 835*(5*a^4*b*d + a^5*e)*m^4 + 7540*(5*a^4*b*d +
 a^5*e)*m^3 + 34759*(5*a^4*b*d + a^5*e)*m^2 + 73054*(5*a^4*b*d + a^5*e)*m)*
x^3 + (a^5*d*m^6 + 48*a^5*d*m^5 + 925*a^5*d*m^4 + 9120*a^5*d*m^3 + 48259*a^
5*d*m^2 + 129072*a^5*d*m + 135135*a^5*d)*x)*(f*x)^m/(m^7 + 49*m^6 + 973*m^5
 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135)
```

giac [B] time = 0.69, size = 2213, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="gia
c")
```

```
[Out] ((f*x)^m*b^5*m^6*x^13*e*sgn(b*x^2 + a) + 36*(f*x)^m*b^5*m^5*x^13*e*sgn(b*x^
2 + a) + (f*x)^m*b^5*d*m^6*x^11*sgn(b*x^2 + a) + 5*(f*x)^m*a*b^4*m^6*x^11*e
*sgn(b*x^2 + a) + 505*(f*x)^m*b^5*m^4*x^13*e*sgn(b*x^2 + a) + 38*(f*x)^m*b^
5*d*m^5*x^11*sgn(b*x^2 + a) + 190*(f*x)^m*a*b^4*m^5*x^11*e*sgn(b*x^2 + a) +
```

```
3480*(f*x)^m*b^5*m^3*x^13*e*sgn(b*x^2 + a) + 5*(f*x)^m*a*b^4*d*m^6*x^9*sgn
(b*x^2 + a) + 555*(f*x)^m*b^5*d*m^4*x^11*sgn(b*x^2 + a) + 10*(f*x)^m*a^2*b^
3*m^6*x^9*e*sgn(b*x^2 + a) + 2775*(f*x)^m*a*b^4*m^4*x^11*e*sgn(b*x^2 + a) +
 12139*(f*x)^m*b^5*m^2*x^13*e*sgn(b*x^2 + a) + 200*(f*x)^m*a*b^4*d*m^5*x^9*
sgn(b*x^2 + a) + 3940*(f*x)^m*b^5*d*m^3*x^11*sgn(b*x^2 + a) + 400*(f*x)^m*a
^2*b^3*m^5*x^9*e*sgn(b*x^2 + a) + 19700*(f*x)^m*a*b^4*m^3*x^11*e*sgn(b*x^2
+ a) + 19524*(f*x)^m*b^5*m*x^13*e*sgn(b*x^2 + a) + 10*(f*x)^m*a^2*b^3*d*m^6
*x^7*sgn(b*x^2 + a) + 3065*(f*x)^m*a*b^4*d*m^4*x^9*sgn(b*x^2 + a) + 14039*(
f*x)^m*b^5*d*m^2*x^11*sgn(b*x^2 + a) + 10*(f*x)^m*a^3*b^2*m^6*x^7*e*sgn(b*x
^2 + a) + 6130*(f*x)^m*a^2*b^3*m^4*x^9*e*sgn(b*x^2 + a) + 70195*(f*x)^m*a*b
^4*m^2*x^11*e*sgn(b*x^2 + a) + 10395*(f*x)^m*b^5*x^13*e*sgn(b*x^2 + a) + 42
0*(f*x)^m*a^2*b^3*d*m^5*x^7*sgn(b*x^2 + a) + 22640*(f*x)^m*a*b^4*d*m^3*x^9*
sgn(b*x^2 + a) + 22902*(f*x)^m*b^5*d*m*x^11*sgn(b*x^2 + a) + 420*(f*x)^m*a^
3*b^2*m^5*x^7*e*sgn(b*x^2 + a) + 45280*(f*x)^m*a^2*b^3*m^3*x^9*e*sgn(b*x^2
+ a) + 114510*(f*x)^m*a*b^4*m*x^11*e*sgn(b*x^2 + a) + 10*(f*x)^m*a^3*b^2*d*
m^6*x^5*sgn(b*x^2 + a) + 6790*(f*x)^m*a^2*b^3*d*m^4*x^7*sgn(b*x^2 + a) + 83
135*(f*x)^m*a*b^4*d*m^2*x^9*sgn(b*x^2 + a) + 12285*(f*x)^m*b^5*d*x^11*sgn(b
*x^2 + a) + 5*(f*x)^m*a^4*b*m^6*x^5*e*sgn(b*x^2 + a) + 6790*(f*x)^m*a^3*b^2
*m^4*x^7*e*sgn(b*x^2 + a) + 166270*(f*x)^m*a^2*b^3*m^2*x^9*e*sgn(b*x^2 + a)
 + 61425*(f*x)^m*a*b^4*x^11*e*sgn(b*x^2 + a) + 440*(f*x)^m*a^3*b^2*d*m^5*x^
5*sgn(b*x^2 + a) + 52920*(f*x)^m*a^2*b^3*d*m^3*x^7*sgn(b*x^2 + a) + 138440*
(f*x)^m*a*b^4*d*m*x^9*sgn(b*x^2 + a) + 220*(f*x)^m*a^4*b*m^5*x^5*e*sgn(b*x^
2 + a) + 52920*(f*x)^m*a^3*b^2*m^3*x^7*e*sgn(b*x^2 + a) + 276880*(f*x)^m*a^
2*b^3*m*x^9*e*sgn(b*x^2 + a) + 5*(f*x)^m*a^4*b*d*m^6*x^3*sgn(b*x^2 + a) + 7
530*(f*x)^m*a^3*b^2*d*m^4*x^5*sgn(b*x^2 + a) + 203350*(f*x)^m*a^2*b^3*d*m^2
*x^7*sgn(b*x^2 + a) + 75075*(f*x)^m*a*b^4*d*x^9*sgn(b*x^2 + a) + (f*x)^m*a^
5*m^6*x^3*e*sgn(b*x^2 + a) + 3765*(f*x)^m*a^4*b*m^4*x^5*e*sgn(b*x^2 + a) +
203350*(f*x)^m*a^3*b^2*m^2*x^7*e*sgn(b*x^2 + a) + 150150*(f*x)^m*a^2*b^3*x^
9*e*sgn(b*x^2 + a) + 230*(f*x)^m*a^4*b*d*m^5*x^3*sgn(b*x^2 + a) + 62800*(f*
x)^m*a^3*b^2*d*m^3*x^5*sgn(b*x^2 + a) + 349860*(f*x)^m*a^2*b^3*d*m*x^7*sgn(
b*x^2 + a) + 46*(f*x)^m*a^5*m^5*x^3*e*sgn(b*x^2 + a) + 31400*(f*x)^m*a^4*b*
m^3*x^5*e*sgn(b*x^2 + a) + 349860*(f*x)^m*a^3*b^2*m*x^7*e*sgn(b*x^2 + a) +
(f*x)^m*a^5*d*m^6*x*sgn(b*x^2 + a) + 4175*(f*x)^m*a^4*b*d*m^4*x^3*sgn(b*x^2
 + a) + 259790*(f*x)^m*a^3*b^2*d*m^2*x^5*sgn(b*x^2 + a) + 193050*(f*x)^m*a^
2*b^3*d*x^7*sgn(b*x^2 + a) + 835*(f*x)^m*a^5*m^4*x^3*e*sgn(b*x^2 + a) + 129
895*(f*x)^m*a^4*b*m^2*x^5*e*sgn(b*x^2 + a) + 193050*(f*x)^m*a^3*b^2*x^7*e*s
gn(b*x^2 + a) + 48*(f*x)^m*a^5*d*m^5*x*sgn(b*x^2 + a) + 37700*(f*x)^m*a^4*b
*d*m^3*x^3*sgn(b*x^2 + a) + 474360*(f*x)^m*a^3*b^2*d*m*x^5*sgn(b*x^2 + a) +
 7540*(f*x)^m*a^5*m^3*x^3*e*sgn(b*x^2 + a) + 237180*(f*x)^m*a^4*b*m*x^5*e*s
gn(b*x^2 + a) + 925*(f*x)^m*a^5*d*m^4*x*sgn(b*x^2 + a) + 173795*(f*x)^m*a^4
*b*d*m^2*x^3*sgn(b*x^2 + a) + 270270*(f*x)^m*a^3*b^2*d*x^5*sgn(b*x^2 + a) +
 34759*(f*x)^m*a^5*m^2*x^3*e*sgn(b*x^2 + a) + 135135*(f*x)^m*a^4*b*x^5*e*sg
n(b*x^2 + a) + 9120*(f*x)^m*a^5*d*m^3*x*sgn(b*x^2 + a) + 365270*(f*x)^m*a^4
*b*d*m*x^3*sgn(b*x^2 + a) + 73054*(f*x)^m*a^5*m*x^3*e*sgn(b*x^2 + a) + 4825
9*(f*x)^m*a^5*d*m^2*x*sgn(b*x^2 + a) + 225225*(f*x)^m*a^4*b*d*x^3*sgn(b*x^2
 + a) + 45045*(f*x)^m*a^5*x^3*e*sgn(b*x^2 + a) + 129072*(f*x)^m*a^5*d*m*x*s
gn(b*x^2 + a) + 135135*(f*x)^m*a^5*d*x*sgn(b*x^2 + a))/(m^7 + 49*m^6 + 973*
m^5 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135)
```

maple [B] time = 0.01, size = 1099, normalized size = 2.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] x*(b^5*e*m^6*x^12+36*b^5*e*m^5*x^12+5*a*b^4*e*m^6*x^10+b^5*d*m^6*x^10+505*b
^5*e*m^4*x^12+190*a*b^4*e*m^5*x^10+38*b^5*d*m^5*x^10+3480*b^5*e*m^3*x^12+10
*a^2*b^3*e*m^6*x^8+5*a*b^4*d*m^6*x^8+2775*a*b^4*e*m^4*x^10+555*b^5*d*m^4*x^
10+12139*b^5*e*m^2*x^12+400*a^2*b^3*e*m^5*x^8+200*a*b^4*d*m^5*x^8+19700*a*b
```

```
^4*e*m^3*x^10+3940*b^5*d*m^3*x^10+19524*b^5*e*m*x^12+10*a^3*b^2*e*m^6*x^6+1
0*a^2*b^3*d*m^6*x^6+6130*a^2*b^3*e*m^4*x^8+3065*a*b^4*d*m^4*x^8+70195*a*b^4
*e*m^2*x^10+14039*b^5*d*m^2*x^10+10395*b^5*e*x^12+420*a^3*b^2*e*m^5*x^6+420
*a^2*b^3*d*m^5*x^6+45280*a^2*b^3*e*m^3*x^8+22640*a*b^4*d*m^3*x^8+114510*a*b
^4*e*m*x^10+22902*b^5*d*m*x^10+5*a^4*b*e*m^6*x^4+10*a^3*b^2*d*m^6*x^4+6790*
a^3*b^2*e*m^4*x^6+6790*a^2*b^3*d*m^4*x^6+166270*a^2*b^3*e*m^2*x^8+83135*a*b
^4*d*m^2*x^8+61425*a*b^4*e*x^10+12285*b^5*d*x^10+220*a^4*b*e*m^5*x^4+440*a^
3*b^2*d*m^5*x^4+52920*a^3*b^2*e*m^3*x^6+52920*a^2*b^3*d*m^3*x^6+276880*a^2*
b^3*e*m*x^8+138440*a*b^4*d*m*x^8+a^5*e*m^6*x^2+5*a^4*b*d*m^6*x^2+3765*a^4*b
*e*m^4*x^4+7530*a^3*b^2*d*m^4*x^4+203350*a^3*b^2*e*m^2*x^6+203350*a^2*b^3*d
*m^2*x^6+150150*a^2*b^3*e*x^8+75075*a*b^4*d*x^8+46*a^5*e*m^5*x^2+230*a^4*b*
d*m^5*x^2+31400*a^4*b*e*m^3*x^4+62800*a^3*b^2*d*m^3*x^4+349860*a^3*b^2*e*m*
x^6+349860*a^2*b^3*d*m*x^6+a^5*d*m^6+835*a^5*e*m^4*x^2+4175*a^4*b*d*m^4*x^2
+129895*a^4*b*e*m^2*x^4+259790*a^3*b^2*d*m^2*x^4+193050*a^3*b^2*e*x^6+19305
0*a^2*b^3*d*x^6+48*a^5*d*m^5+7540*a^5*e*m^3*x^2+37700*a^4*b*d*m^3*x^2+23718
0*a^4*b*e*m*x^4+474360*a^3*b^2*d*m*x^4+925*a^5*d*m^4+34759*a^5*e*m^2*x^2+17
3795*a^4*b*d*m^2*x^2+135135*a^4*b*e*x^4+270270*a^3*b^2*d*x^4+9120*a^5*d*m^3
+73054*a^5*e*m*x^2+365270*a^4*b*d*m*x^2+48259*a^5*d*m^2+45045*a^5*e*x^2+225
225*a^4*b*d*x^2+129072*a^5*d*m+135135*a^5*d)*(f*x)^m*((b*x^2+a)^2)^(5/2)/(m
+13)/(m+11)/(m+9)/(m+7)/(m+5)/(m+3)/(m+1)/(b*x^2+a)^5
```

maxima [A] time = 0.82, size = 491, normalized size = 1.23

$$\frac{\left(\left(m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945\right)b^5f^mx^{11}+5\left(m^5+27\,m^4+262\,m^3+1122\,m^2+2041\,m+1155\right)ab^4f^mx^9+10\left(m^5+29\,m^4+302\,m^3+1366\,m^2+2577\,m+1485\right)a^2b^3f^mx^7+10\left(m^5+31\,m^4+350\,m^3+1730\,m^2+3489\,m+2079\right)a^3b^2f^mx^5+5\left(m^5+33\,m^4+406\,m^3+2262\,m^2+5353\,m+3465\right)a^4bf^mx^3+\left(m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395\right)a^5f^mx\right)dx^m}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{\left(\left(m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395\right)b^5f^mx^{13}+5\left(m^5+37\,m^4+518\,m^3+3422\,m^2+10617\,m+12285\right)ab^4f^mx^{11}+10\left(m^5+39\,m^4+574\,m^3+3954\,m^2+12673\,m+15015\right)a^2b^3f^mx^9+10\left(m^5+41\,m^4+638\,m^3+4654\,m^2+15681\,m+19305\right)a^3b^2f^mx^7+5\left(m^5+43\,m^4+710\,m^3+5570\,m^2+20409\,m+27027\right)a^4bf^mx^5+\left(m^5+45\,m^4+790\,m^3+6750\,m^2+28009\,m+45045\right)a^5f^mx^3\right)ex^m}{m^6+48\,m^5+925\,m^4+9120\,m^3+48259\,m^2+129072\,m+135135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="max
ima")
```

```
[Out] ((m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)*b^5*f^m*x^11 + 5*(m^5 +
27*m^4 + 262*m^3 + 1122*m^2 + 2041*m + 1155)*a*b^4*f^m*x^9 + 10*(m^5 + 29*m
^4 + 302*m^3 + 1366*m^2 + 2577*m + 1485)*a^2*b^3*f^m*x^7 + 10*(m^5 + 31*m^4
 + 350*m^3 + 1730*m^2 + 3489*m + 2079)*a^3*b^2*f^m*x^5 + 5*(m^5 + 33*m^4 +
406*m^3 + 2262*m^2 + 5353*m + 3465)*a^4*b*f^m*x^3 + (m^5 + 35*m^4 + 470*m^3
 + 3010*m^2 + 9129*m + 10395)*a^5*f^m*x)*d*x^m/(m^6 + 36*m^5 + 505*m^4 + 34
80*m^3 + 12139*m^2 + 19524*m + 10395) + ((m^5 + 35*m^4 + 470*m^3 + 3010*m^2
 + 9129*m + 10395)*b^5*f^m*x^13 + 5*(m^5 + 37*m^4 + 518*m^3 + 3422*m^2 + 10
617*m + 12285)*a*b^4*f^m*x^11 + 10*(m^5 + 39*m^4 + 574*m^3 + 3954*m^2 + 126
73*m + 15015)*a^2*b^3*f^m*x^9 + 10*(m^5 + 41*m^4 + 638*m^3 + 4654*m^2 + 156
81*m + 19305)*a^3*b^2*f^m*x^7 + 5*(m^5 + 43*m^4 + 710*m^3 + 5570*m^2 + 2040
9*m + 27027)*a^4*b*f^m*x^5 + (m^5 + 45*m^4 + 790*m^3 + 6750*m^2 + 28009*m +
 45045)*a^5*f^m*x^3)*e*x^m/(m^6 + 48*m^5 + 925*m^4 + 9120*m^3 + 48259*m^2 +
 129072*m + 135135)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(f\,x\right)}^m\,\left(e\,x^2+d\right)\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(f x\right)^{m} \left(d + e x^{2}\right) \left(\left(a + b x^{2}\right)^{2}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral((f*x)**m*(d + e*x**2)*((a + b*x**2)**2)**(5/2), x)
```

3.68 $\int (fx)^m \left(d+ex^2\right) \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx$

Optimal. Leaf size=276

$$\frac{b^2\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+7}(3ae+bd)}{f^7(m+7)\left(a+bx^2\right)}+\frac{3ab\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+5}(ae+bd)}{f^5(m+5)\left(a+bx^2\right)}+\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}\,(fx)}{f^3(m+3)\left(a+b\right.}$$

Rubi [A] time = 0.15, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1250, 448}

$$\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+3}(ae+3bd)}{f^3(m+3)\left(a+bx^2\right)}+\frac{3ab\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+5}(ae+bd)}{f^5(m+5)\left(a+bx^2\right)}+\frac{b^2\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+7}(3ae+bd)}{f^7(m+7)\left(a+bx^2\right)}+\frac{a^3d\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+1}}{f(m+1)\left(a+bx^2\right)}+\frac{b^3e\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+9}}{f^9(m+9)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*(a + b*x^2
)) + (a^2*(3*b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^3
*(3 + m)*(a + b*x^2)) + (3*a*b*(b*d + a*e)*(f*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(f^5*(5 + m)*(a + b*x^2)) + (b^2*(b*d + 3*a*e)*(f*x)^(7 + m)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^7*(7 + m)*(a + b*x^2)) + (b^3*e*(f*x)^
(9 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^9*(9 + m)*(a + b*x^2))
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (fx)^m \left(ab+b^2x^2\right)^3 \left(d+ex^2\right) dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a^3b^3d(fx)^m+\frac{a^2b^3(3bd+ae)(fx)^{2+m}}{f^2}+\frac{3ab^4(bd+}{f}\right.}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{a^3d(fx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)\left(a+bx^2\right)}+\frac{a^2(3bd+ae)(fx)^{3+m}\sqrt{a^2+2al}}{f^3(3+m)\left(a+bx^2\right)}$$

Mathematica [A] time = 0.11, size = 112, normalized size = 0.41

$$\frac{x\left(\left(a+bx^2\right)^2\right)^{3/2}(fx)^m\left(\frac{a^3d}{m+1}+\frac{a^2x^2(ae+3bd)}{m+3}+\frac{b^2x^6(3ae+bd)}{m+7}+\frac{3abx^4(ae+bd)}{m+5}+\frac{b^3ex^8}{m+9}\right)}{\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x*(f*x)^m*((a + b*x^2)^2)^(3/2)*((a^3*d)/(1 + m) + (a^2*(3*b*d + a*e)*x^2)
/(3 + m) + (3*a*b*(b*d + a*e)*x^4)/(5 + m) + (b^2*(b*d + 3*a*e)*x^6)/(7 + m
) + (b^3*e*x^8)/(9 + m)))/(a + b*x^2)^3
```

IntegrateAlgebraic [F] time = 3.14, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d + ex^2\right) \left(a^2 + 2abx^2 + b^2x^4\right)^{3/2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(
3/2), x]
```

fricas [A] time = 0.69, size = 381, normalized size = 1.38

$$\frac{\left(\left(b^3em^4 + 16\,b^3em^3 + 86\,b^3em^2 + 176\,b^3em + 105\,b^3e\right)x^9 + \left(\left(b^3d + 3\,ab^2e\right)m^4 + 135\,b^3d + 405\,ab^2e + 18\left(b^3d + 3\,ab^2e\right)m^3 + 104\left(b^3d + 3\,ab^2e\right)m^2 + 222\left(b^3d + 3\,ab^2e\right)m\right)x^7 + 3\left(\left(ab^2d + a^2be\right)m^4 + 189\,ab^2d + 189\,a^2be + 20\left(ab^2d + a^2be\right)m^3 + 130\left(ab^2d + a^2be\right)m^2 + 300\left(ab^2d + a^2be\right)m\right)x^5 + \left(\left(3\,a^2bd + a^3e\right)m^4 + 945\,a^2bd + 315\,a^3e + 22\left(3\,a^2bd + a^3e\right)m^3 + 164\left(3\,a^2bd + a^3e\right)m^2 + 458\left(3\,a^2bd + a^3e\right)m\right)x^3 + \left(a^3dm^4 + 24\,a^3dm^3 + 206\,a^3dm^2 + 744\,a^3dm + 945\,a^3d\right)x\right)\left(fx\right)^m}{m^5 + 25\,m^4 + 230\,m^3 + 950\,m^2 + 1689\,m + 945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fri
cas")
```

```
[Out] ((b^3*e*m^4 + 16*b^3*e*m^3 + 86*b^3*e*m^2 + 176*b^3*e*m + 105*b^3*e)*x^9 +
((b^3*d + 3*a*b^2*e)*m^4 + 135*b^3*d + 405*a*b^2*e + 18*(b^3*d + 3*a*b^2*e)
*m^3 + 104*(b^3*d + 3*a*b^2*e)*m^2 + 222*(b^3*d + 3*a*b^2*e)*m)*x^7 + 3*((a
*b^2*d + a^2*b*e)*m^4 + 189*a*b^2*d + 189*a^2*b*e + 20*(a*b^2*d + a^2*b*e)*
m^3 + 130*(a*b^2*d + a^2*b*e)*m^2 + 300*(a*b^2*d + a^2*b*e)*m)*x^5 + ((3*a^
2*b*d + a^3*e)*m^4 + 945*a^2*b*d + 315*a^3*e + 22*(3*a^2*b*d + a^3*e)*m^3 +
 164*(3*a^2*b*d + a^3*e)*m^2 + 458*(3*a^2*b*d + a^3*e)*m)*x^3 + (a^3*d*m^4
+ 24*a^3*d*m^3 + 206*a^3*d*m^2 + 744*a^3*d*m + 945*a^3*d)*x)*(f*x)^m/(m^5 +
 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

giac [B] time = 0.52, size = 1013, normalized size = 3.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="gia
c")
```

```
[Out] ((f*x)^m*b^3*m^4*x^9*e*sgn(b*x^2 + a) + 16*(f*x)^m*b^3*m^3*x^9*e*sgn(b*x^2
+ a) + (f*x)^m*b^3*d*m^4*x^7*sgn(b*x^2 + a) + 3*(f*x)^m*a*b^2*m^4*x^7*e*sgn
(b*x^2 + a) + 86*(f*x)^m*b^3*m^2*x^9*e*sgn(b*x^2 + a) + 18*(f*x)^m*b^3*d*m^
3*x^7*sgn(b*x^2 + a) + 54*(f*x)^m*a*b^2*m^3*x^7*e*sgn(b*x^2 + a) + 176*(f*x
)^m*b^3*m*x^9*e*sgn(b*x^2 + a) + 3*(f*x)^m*a*b^2*d*m^4*x^5*sgn(b*x^2 + a) +
 104*(f*x)^m*b^3*d*m^2*x^7*sgn(b*x^2 + a) + 3*(f*x)^m*a^2*b*m^4*x^5*e*sgn(b
*x^2 + a) + 312*(f*x)^m*a*b^2*m^2*x^7*e*sgn(b*x^2 + a) + 105*(f*x)^m*b^3*x^
9*e*sgn(b*x^2 + a) + 60*(f*x)^m*a*b^2*d*m^3*x^5*sgn(b*x^2 + a) + 222*(f*x)^
m*b^3*d*m*x^7*sgn(b*x^2 + a) + 60*(f*x)^m*a^2*b*m^3*x^5*e*sgn(b*x^2 + a) +
666*(f*x)^m*a*b^2*m*x^7*e*sgn(b*x^2 + a) + 3*(f*x)^m*a^2*b*d*m^4*x^3*sgn(b*
x^2 + a) + 390*(f*x)^m*a*b^2*d*m^2*x^5*sgn(b*x^2 + a) + 135*(f*x)^m*b^3*d*x
^7*sgn(b*x^2 + a) + (f*x)^m*a^3*m^4*x^3*e*sgn(b*x^2 + a) + 390*(f*x)^m*a^2*
b*m^2*x^5*e*sgn(b*x^2 + a) + 405*(f*x)^m*a*b^2*x^7*e*sgn(b*x^2 + a) + 66*(f
*x)^m*a^2*b*d*m^3*x^3*sgn(b*x^2 + a) + 900*(f*x)^m*a*b^2*d*m*x^5*sgn(b*x^2
+ a) + 22*(f*x)^m*a^3*m^3*x^3*e*sgn(b*x^2 + a) + 900*(f*x)^m*a^2*b*m*x^5*e*
sgn(b*x^2 + a) + (f*x)^m*a^3*d*m^4*x*sgn(b*x^2 + a) + 492*(f*x)^m*a^2*b*d*m
```

```
^2*x^3*sgn(b*x^2 + a) + 567*(f*x)^m*a*b^2*d*x^5*sgn(b*x^2 + a) + 164*(f*x)^
m*a^3*m^2*x^3*e*sgn(b*x^2 + a) + 567*(f*x)^m*a^2*b*x^5*e*sgn(b*x^2 + a) + 2
4*(f*x)^m*a^3*d*m^3*x*sgn(b*x^2 + a) + 1374*(f*x)^m*a^2*b*d*m*x^3*sgn(b*x^2
 + a) + 458*(f*x)^m*a^3*m*x^3*e*sgn(b*x^2 + a) + 206*(f*x)^m*a^3*d*m^2*x*sg
n(b*x^2 + a) + 945*(f*x)^m*a^2*b*d*x^3*sgn(b*x^2 + a) + 315*(f*x)^m*a^3*x^3
*e*sgn(b*x^2 + a) + 744*(f*x)^m*a^3*d*m*x*sgn(b*x^2 + a) + 945*(f*x)^m*a^3*
d*x*sgn(b*x^2 + a))/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

maple [B] time = 0.01, size = 495, normalized size = 1.79

$$\frac{\left(b^{3} e m^{4} x^{8}+16 b^{3} e m^{3} x^{8}+3 a b^{2} e m^{4} x^{6}+b^{3} d m^{4} x^{6}+86 b^{3} e m^{2} x^{8}+54 a b^{2} e m^{3} x^{6}+18 b^{3} d m^{3} x^{6}+176 b^{3} e m x^{8}+3 a^{2} b e m^{4} x^{4}+3 a b^{2} d m^{4} x^{4}+312 a b^{2} e m^{2} x^{6}+104 b^{3} d m^{2} x^{6}+105 b^{3} e x^{8}+60 a^{2} b e m^{3} x^{4}+60 a b^{2} d m^{3} x^{4}+666 a b^{2} e m x^{6}+222 b^{3} d m x^{6}+a^{3} e m^{4} x^{2}+3 a^{2} b d m^{4} x^{2}+390 a^{2} b e m^{2} x^{4}+390 a b^{2} d m^{2} x^{4}+405 a b^{2} e x^{6}+135 b^{3} d x^{6}+22 a^{3} e m^{3} x^{2}+66 a^{2} b d m^{3} x^{2}+900 a^{2} b e m x^{4}+900 a b^{2} d m x^{4}+a^{3} d m^{4}+164 a^{3} e m^{2} x^{2}+492 a^{2} b d m^{2} x^{2}+567 a^{2} b e x^{4}+567 a b^{2} d x^{4}+24 a^{3} d m^{3}+458 a^{3} e m x^{2}+1374 a^{2} b d m x^{2}+206 a^{3} d m^{2}+315 a^{3} e x^{2}+945 a^{2} b d x^{2}+744 a^{3} d m+945 a^{3} d\right) \left(\left(b x^{2}+a\right)^{2}\right)^{\frac{3}{2}} x \left(f x\right)^{m}}{\left(m+9\right) \left(m+7\right) \left(m+5\right) \left(m+3\right) \left(m+1\right) \left(b x^{2}+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] x*(b^3*e*m^4*x^8+16*b^3*e*m^3*x^8+3*a*b^2*e*m^4*x^6+b^3*d*m^4*x^6+86*b^3*e*
m^2*x^8+54*a*b^2*e*m^3*x^6+18*b^3*d*m^3*x^6+176*b^3*e*m*x^8+3*a^2*b*e*m^4*x
^4+3*a*b^2*d*m^4*x^4+312*a*b^2*e*m^2*x^6+104*b^3*d*m^2*x^6+105*b^3*e*x^8+60
*a^2*b*e*m^3*x^4+60*a*b^2*d*m^3*x^4+666*a*b^2*e*m*x^6+222*b^3*d*m*x^6+a^3*e
*m^4*x^2+3*a^2*b*d*m^4*x^2+390*a^2*b*e*m^2*x^4+390*a*b^2*d*m^2*x^4+405*a*b^
2*e*x^6+135*b^3*d*x^6+22*a^3*e*m^3*x^2+66*a^2*b*d*m^3*x^2+900*a^2*b*e*m*x^4
+900*a*b^2*d*m*x^4+a^3*d*m^4+164*a^3*e*m^2*x^2+492*a^2*b*d*m^2*x^2+567*a^2*
b*e*x^4+567*a*b^2*d*x^4+24*a^3*d*m^3+458*a^3*e*m*x^2+1374*a^2*b*d*m*x^2+206
*a^3*d*m^2+315*a^3*e*x^2+945*a^2*b*d*x^2+744*a^3*d*m+945*a^3*d)*(f*x)^m*((b
*x^2+a)^2)^(3/2)/(m+9)/(m+7)/(m+5)/(m+3)/(m+1)/(b*x^2+a)^3
```

maxima [A] time = 0.96, size = 243, normalized size = 0.88

$$\frac{{\left({\left(m^{3}+9 m^{2}+23 m+15\right)} b^{3} f^{m} x^{7}+3 {\left(m^{3}+11 m^{2}+31 m+21\right)} a b^{2} f^{m} x^{5}+3 {\left(m^{3}+13 m^{2}+47 m+35\right)} a^{2} b f^{m} x^{3}+{\left(m^{3}+15 m^{2}+71 m+105\right)} a^{3} f^{m} x\right)} d x^{m}}{m^{4}+16 m^{3}+86 m^{2}+176 m+105}+\frac{{\left({\left(m^{3}+15 m^{2}+71 m+105\right)} b^{3} f^{m} x^{9}+3 {\left(m^{3}+17 m^{2}+87 m+135\right)} a b^{2} f^{m} x^{7}+3 {\left(m^{3}+19 m^{2}+111 m+189\right)} a^{2} b f^{m} x^{5}+{\left(m^{3}+21 m^{2}+143 m+315\right)} a^{3} f^{m} x^{3}\right)} e x^{m}}{m^{4}+24 m^{3}+206 m^{2}+744 m+945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="max
ima")
```

```
[Out] ((m^3 + 9*m^2 + 23*m + 15)*b^3*f^m*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*a*b^2
*f^m*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*a^2*b*f^m*x^3 + (m^3 + 15*m^2 + 71*
m + 105)*a^3*f^m*x)*d*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105) + ((m^3 + 1
5*m^2 + 71*m + 105)*b^3*f^m*x^9 + 3*(m^3 + 17*m^2 + 87*m + 135)*a*b^2*f^m*x
^7 + 3*(m^3 + 19*m^2 + 111*m + 189)*a^2*b*f^m*x^5 + (m^3 + 21*m^2 + 143*m +
 315)*a^3*f^m*x^3)*e*x^m/(m^4 + 24*m^3 + 206*m^2 + 744*m + 945)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(f\,x\right)}^m\,\left(e\,x^2+d\right)\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(f x\right)^{m} \left(d + e x^{2}\right) \left(\left(a + b x^{2}\right)^{2}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)*((a + b*x**2)**2)**(3/2), x)
```

3.69 $\int (fx)^m \left(d+ex^2\right) \sqrt{a^2+2abx^2+b^2x^4}\, dx$

Optimal. Leaf size=153

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+3}(ae+bd)}{f^3(m+3)\left(a+bx^2\right)}+\frac{ad\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+1}}{f(m+1)\left(a+bx^2\right)}+\frac{be\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+5}}{f^5(m+5)\left(a+bx^2\right)}$$

Rubi [A] time = 0.08, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1250, 448}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+3}(ae+bd)}{f^3(m+3)\left(a+bx^2\right)}+\frac{ad\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+1}}{f(m+1)\left(a+bx^2\right)}+\frac{be\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+5}}{f^5(m+5)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*(a + b*x^2))
 + ((b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^3*(3 + m)
*(a + b*x^2)) + (b*e*(f*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^5*(5
 + m)*(a + b*x^2))
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)\sqrt{a^2+2abx^2+b^2x^4}\, dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (fx)^m \left(ab+b^2x^2\right)\left(d+ex^2\right) dx}{ab+b^2x^2}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(abd(fx)^m+\frac{b(bd+ae)(fx)^{2+m}}{f^2}+\frac{b^2e(fx)^{4+m}}{f^4}\right) d\ldots}{ab+b^2x^2}$$

$$= \frac{ad(fx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)\left(a+bx^2\right)}+\frac{(bd+ae)(fx)^{3+m}\sqrt{a^2+2abx^2\ldots}}{f^3(3+m)\left(a+bx^2\right)}$$

Mathematica [A] time = 0.06, size = 86, normalized size = 0.56

$$\frac{x\sqrt{\left(a+bx^2\right)^2}\,(fx)^m\left(a(m+5)\left(d(m+3)+e(m+1)x^2\right)+b(m+1)x^2\left(d(m+5)+e(m+3)x^2\right)\right)}{(m+1)(m+3)(m+5)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (x*(f*x)^m*Sqrt[(a + b*x^2)^2]*(a*(5 + m)*(d*(3 + m) + e*(1 + m)*x^2) + b*(
1 + m)*x^2*(d*(5 + m) + e*(3 + m)*x^2)))/((1 + m)*(3 + m)*(5 + m)*(a + b*x^
2))
```

IntegrateAlgebraic [F] time = 1.60, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d + ex^2\right) \sqrt{a^2 + 2abx^2 + b^2x^4}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f*x)^m*(d + e*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] Defer[IntegrateAlgebraic][(f*x)^m*(d + e*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4], x]
```

fricas [A] time = 0.73, size = 94, normalized size = 0.61

$$\frac{\left(\left(bem^2 + 4\,bem + 3\,be\right)x^5 + \left((bd + ae)m^2 + 5\,bd + 5\,ae + 6\,(bd + ae)m\right)x^3 + \left(adm^2 + 8\,adm + 15\,ad\right)x\right)\left(fx\right)^m}{m^3 + 9\,m^2 + 23\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="fri
cas")
```

```
[Out] ((b*e*m^2 + 4*b*e*m + 3*b*e)*x^5 + ((b*d + a*e)*m^2 + 5*b*d + 5*a*e + 6*(b*
d + a*e)*m)*x^3 + (a*d*m^2 + 8*a*d*m + 15*a*d)*x)*(f*x)^m/(m^3 + 9*m^2 + 23
*m + 15)
```

giac [B] time = 0.32, size = 269, normalized size = 1.76

$$\frac{\left(fx\right)^m bm^2x^5e\mathrm{sgn}\left(bx^2 + a\right) + 4\,\left(fx\right)^m bmx^5e\mathrm{sgn}\left(bx^2 + a\right) + \left(fx\right)^m bdm^2x^3\mathrm{sgn}\left(bx^2 + a\right) + \left(fx\right)^m am^2x^3e\mathrm{sgn}\left(bx^2 + a\right) + 3\,\left(fx\right)^m bx^5e\mathrm{sgn}\left(bx^2 + a\right) + 6\,\left(fx\right)^m bdmx^3\mathrm{sgn}\left(bx^2 + a\right) + 6\,\left(fx\right)^m amx^3e\mathrm{sgn}\left(bx^2 + a\right) + \left(fx\right)^m adm^2x\mathrm{sgn}\left(bx^2 + a\right) + 5\,\left(fx\right)^m bdx^3\mathrm{sgn}\left(bx^2 + a\right) + 5\,\left(fx\right)^m ax^3e\mathrm{sgn}\left(bx^2 + a\right) + 8\,\left(fx\right)^m admx\mathrm{sgn}\left(bx^2 + a\right) + 15\,\left(fx\right)^m adx\mathrm{sgn}\left(bx^2 + a\right)}{m^3 + 9\,m^2 + 23\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="gia
c")
```

```
[Out] ((f*x)^m*b*m^2*x^5*e*sgn(b*x^2 + a) + 4*(f*x)^m*b*m*x^5*e*sgn(b*x^2 + a) +
(f*x)^m*b*d*m^2*x^3*sgn(b*x^2 + a) + (f*x)^m*a*m^2*x^3*e*sgn(b*x^2 + a) + 3
*(f*x)^m*b*x^5*e*sgn(b*x^2 + a) + 6*(f*x)^m*b*d*m*x^3*sgn(b*x^2 + a) + 6*(f
*x)^m*a*m*x^3*e*sgn(b*x^2 + a) + (f*x)^m*a*d*m^2*x*sgn(b*x^2 + a) + 5*(f*x)
^m*b*d*x^3*sgn(b*x^2 + a) + 5*(f*x)^m*a*x^3*e*sgn(b*x^2 + a) + 8*(f*x)^m*a*
d*m*x*sgn(b*x^2 + a) + 15*(f*x)^m*a*d*x*sgn(b*x^2 + a))/(m^3 + 9*m^2 + 23*m
 + 15)
```

maple [A] time = 0.01, size = 131, normalized size = 0.86

$$\frac{\left(be\,m^2x^4 + 4bem\,x^4 + ae\,m^2x^2 + bd\,m^2x^2 + 3be\,x^4 + 6aem\,x^2 + 6bdm\,x^2 + ad\,m^2 + 5ae\,x^2 + 5bd\,x^2 + 8adm + 15ad\right)\sqrt{\left(b\,x^2 + a\right)^2}\;x\left(fx\right)^m}{(m + 5)\,(m + 3)\,(m + 1)\left(b\,x^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
```

```
[Out] x*(b*e*m^2*x^4+4*b*e*m*x^4+a*e*m^2*x^2+b*d*m^2*x^2+3*b*e*x^4+6*a*e*m*x^2+6*
b*d*m*x^2+a*d*m^2+5*a*e*x^2+5*b*d*x^2+8*a*d*m+15*a*d)*(f*x)^m*((b*x^2+a)^2)
^(1/2)/(m+5)/(m+3)/(m+1)/(b*x^2+a)
```

maxima [A] time = 0.81, size = 75, normalized size = 0.49

$$\frac{\left(bf^m(m + 1)x^3 + af^m(m + 3)x\right)dx^m}{m^2 + 4\,m + 3} + \frac{\left(bf^m(m + 3)x^5 + af^m(m + 5)x^3\right)ex^m}{m^2 + 8\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="max
ima")
```

```
[Out] (b*f^m*(m + 1)*x^3 + a*f^m*(m + 3)*x)*d*x^m/(m^2 + 4*m + 3) + (b*f^m*(m + 3
)*x^5 + a*f^m*(m + 5)*x^3)*e*x^m/(m^2 + 8*m + 15)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(f\,x\right)^m\,\left(e\,x^2+d\right)\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2),x)
```

```
[Out] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{m} \left(d + ex^{2}\right) \sqrt{\left(a + bx^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)*sqrt((a + b*x**2)**2), x)
```

3.70 $\int x\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$

Optimal. Leaf size=34

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{p+1}}{4b(p+1)}$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1247, 629}

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{p+1}}{4b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (a^2 + 2*a*b*x^2 + b^2*x^4)^(1 + p)/(4*b*(1 + p))
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx &= \frac{1}{2}\text{Subst}\left(\int (a+bx)\left(a^2+2abx+b^2x^2\right)^p\,dx,x,x^2\right)\\ &= \frac{\left(a^2+2abx^2+b^2x^4\right)^{1+p}}{4b(1+p)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.74

$$\frac{\left(\left(a+bx^2\right)^2\right)^{p+1}}{4b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)^2)^(1 + p)/(4*b*(1 + p))
```

IntegrateAlgebraic [F] time = 0.13, size = 0, normalized size = 0.00

$$\int x\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
[Out] Defer[IntegrateAlgebraic][x*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p, x]
```

fricas [A] time = 0.71, size = 47, normalized size = 1.38

$$\frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)\left(b^2x^4 + 2\,abx^2 + a^2\right)^p}{4\left(bp + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
[Out] 1/4*(b^2*x^4 + 2*a*b*x^2 + a^2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(b*p + b)
```

giac [A] time = 0.29, size = 32, normalized size = 0.94

$$\frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{p+1}}{4\,b\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
[Out] 1/4*(b^2*x^4 + 2*a*b*x^2 + a^2)^(p + 1)/(b*(p + 1))
```

maple [A] time = 0.00, size = 40, normalized size = 1.18

$$\frac{\left(b\,x^2 + a\right)^2 \left(b^2x^4 + 2ab\,x^2 + a^2\right)^p}{4\left(p+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
[Out] 1/4*(b*x^2+a)^2/b/(1+p)*(b^2*x^4+2*a*b*x^2+a^2)^p
```

maxima [B] time = 0.72, size = 86, normalized size = 2.53

$$\frac{\left(bx^2 + a\right)\left(bx^2 + a\right)^{2p} a}{2\,b\left(2\,p+1\right)} + \frac{\left(b^2\left(2\,p+1\right)x^4 + 2\,abpx^2 - a^2\right)\left(bx^2 + a\right)^{2p}}{4\left(2\,p^2 + 3\,p + 1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
[Out] 1/2*(b*x^2 + a)*(b*x^2 + a)^(2*p)*a/(b*(2*p + 1)) + 1/4*(b^2*(2*p + 1)*x^4
+ 2*a*b*p*x^2 - a^2)*(b*x^2 + a)^(2*p)/((2*p^2 + 3*p + 1)*b)
```

mupad [B] time = 0.14, size = 59, normalized size = 1.74

$$\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^p \left(\frac{a^2}{4\,b\,\left(p+1\right)} + \frac{a\,x^2}{2\,\left(p+1\right)} + \frac{b\,x^4}{4\,\left(p+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^p*(a^2/(4*b*(p + 1)) + (a*x^2)/(2*(p + 1)) + (b
*x^4)/(4*(p + 1)))
```

sympy [A] time = 9.60, size = 165, normalized size = 4.85

$$\begin{cases} \frac{x^2}{2a} & \text{for } b = 0 \wedge p = -1 \\ \frac{ax^2(a^2)^p}{2} & \text{for } b = 0 \\ \frac{\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b} + \frac{\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b} & \text{for } p = -1 \\ \frac{a^2(a^2+2abx^2+b^2x^4)^p}{4bp+4b} + \frac{2abx^2(a^2+2abx^2+b^2x^4)^p}{4bp+4b} + \frac{b^2x^4(a^2+2abx^2+b^2x^4)^p}{4bp+4b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((x**2/(2*a), Eq(b, 0) & Eq(p, -1)), (a*x**2*(a**2)**p/2, Eq(b, 0)
), (log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b) + log(I*sqrt(a)*sqrt(1/b) + x)/(2*b
), Eq(p, -1)), (a**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(4*b*p + 4*b) + 2*a
*b*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(4*b*p + 4*b) + b**2*x**4*(a**2
+ 2*a*b*x**2 + b**2*x**4)**p/(4*b*p + 4*b), True))
```

3.71 $\int x^3 \left(a+bx^2\right) \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=86

$$\frac{\left(a+bx^2\right)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(2p+3)}-\frac{a\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(p+1)}$$

Rubi [A] time = 0.09, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1249, 770, 21, 43}

$$\frac{\left(a+bx^2\right)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(2p+3)}-\frac{a\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] -(a*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^2*(1 + p)) + ((a + b*
x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^2*(3 + 2*p))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 770

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPa
rt[p]*(b/2 + c*x)^(2*FracPart[p])), Int[(d + e*x)^m*(f + g*x)*(b/2 + c*x)^(
2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rule 1249

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[b
^2 - 4*a*c, 0] &&  !IntegerQ[p] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int x^3\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx = \frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)\left(a^2+2abx+b^2x^2\right)^p\,dx,x,x^2\right)$$
$$= \frac{1}{2}\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\operatorname{Subst}\left(\int x(a+bx)\left(ab+b^2\right.\right.$$
$$= \frac{\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\operatorname{Subst}\left(\int x\left(ab+b^2x\right)^{1+2p}\,dx,\right.}{2b}$$
$$= \frac{\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\operatorname{Subst}\left(\int\left(-\frac{a\left(ab+b^2x\right)^{1+2p}}{b}+\frac{(ab-}{}\right.\right.}{2b}$$
$$= -\frac{a\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(1+p)}+\frac{\left(a+bx^2\right)^3\left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(3+2p)}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.52

$$\frac{\left(\left(a+bx^2\right)^2\right)^{p+1}\left(2b(p+1)x^2-a\right)}{4b^2(p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (((a + b*x^2)^2)^(1 + p)*(-a + 2*b*(1 + p)*x^2))/(4*b^2*(1 + p)*(3 + 2*p))
```

IntegrateAlgebraic [F] time = 0.29, size = 0, normalized size = 0.00

$$\int x^3\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p, x]
```

fricas [A] time = 0.66, size = 92, normalized size = 1.07

$$\frac{\left(2\left(b^3p+b^3\right)x^6+2\,a^2bpx^2+\left(4\,ab^2p+3\,ab^2\right)x^4-a^3\right)\left(b^2x^4+2\,abx^2+a^2\right)^p}{4\left(2\,b^2p^2+5\,b^2p+3\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^3*p + b^3)*x^6 + 2*a^2*b*p*x^2 + (4*a*b^2*p + 3*a*b^2)*x^4 - a^3)
*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(2*b^2*p^2 + 5*b^2*p + 3*b^2)
```

giac [B] time = 0.43, size = 196, normalized size = 2.28

$$\frac{2\left(b^2x^4+2\,abx^2+a^2\right)^pb^3px^6+2\left(b^2x^4+2\,abx^2+a^2\right)^pb^3x^6+4\left(b^2x^4+2\,abx^2+a^2\right)^pab^2px^4+3\left(b^2x^4+2\,abx^2+a^2\right)^pab^2x^4+2\left(b^2x^4+2\,abx^2+a^2\right)^pa^2bpx^2-\left(b^2x^4+2\,abx^2+a^2\right)^pa^3}{4\left(2\,b^2p^2+5\,b^2p+3\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/4*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^3*p*x^6 + 2*(b^2*x^4 + 2*a*b*x^2 + a
^2)^p*b^3*x^6 + 4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^2*p*x^4 + 3*(b^2*x^4 +
```

```
2*a*b*x^2 + a^2)^p*a*b^2*x^4 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b*p*x^2
- (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^3)/(2*b^2*p^2 + 5*b^2*p + 3*b^2)
```

maple [A] time = 0.01, size = 62, normalized size = 0.72

$$-\frac{\left(-2x^2pb-2b\,x^2+a\right)\left(b\,x^2+a\right)^2\left(b^2x^4+2ab\,x^2+a^2\right)^p}{4\left(2p^2+5p+3\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] -1/4*(b^2*x^4+2*a*b*x^2+a^2)^p*(-2*b*p*x^2-2*b*x^2+a)*(b*x^2+a)^2/b^2/(2*p^
2+5*p+3)
```

maxima [A] time = 0.74, size = 135, normalized size = 1.57

$$\frac{\left(b^2\left(2\,p+1\right)x^4+2\,abpx^2-a^2\right)\left(bx^2+a\right)^{2\,p}a}{4\left(2\,p^2+3\,p+1\right)b^2}+\frac{\left(\left(2\,p^2+3\,p+1\right)b^3x^6+\left(2\,p^2+p\right)ab^2x^4-2\,a^2bpx^2+a^3\right)\left(bx^2+a\right)^{2\,p}}{2\left(4\,p^3+12\,p^2+11\,p+3\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*(2*p + 1)*x^4 + 2*a*b*p*x^2 - a^2)*(b*x^2 + a)^(2*p)*a/((2*p^2 + 3
*p + 1)*b^2) + 1/2*((2*p^2 + 3*p + 1)*b^3*x^6 + (2*p^2 + p)*a*b^2*x^4 - 2*a
^2*b*p*x^2 + a^3)*(b*x^2 + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^2)
```

mupad [B] time = 0.17, size = 108, normalized size = 1.26

$${\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^p\,\left(\frac{b\,x^6\,\left(p+1\right)}{2\,\left(2\,p^2+5\,p+3\right)}-\frac{a^3}{4\,b^2\,\left(2\,p^2+5\,p+3\right)}+\frac{a\,x^4\,\left(4\,p+3\right)}{4\,\left(2\,p^2+5\,p+3\right)}+\frac{a^2\,p\,x^2}{2\,b\,\left(2\,p^2+5\,p+3\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^p*((b*x^6*(p + 1))/(2*(5*p + 2*p^2 + 3)) - a^3/
(4*b^2*(5*p + 2*p^2 + 3)) + (a*x^4*(4*p + 3))/(4*(5*p + 2*p^2 + 3)) + (a^2*
p*x^2)/(2*b*(5*p + 2*p^2 + 3)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{ax^4\left(a^2\right)^p}{4} & \text{for}\: b = 0 \\ \int \frac{x^3\left(a+bx^2\right)}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\, dx & \text{for}\: p = -\frac{3}{2} \\ -\frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^2}-\frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^2}+\frac{x^2}{2b} & \text{for}\: p = -1 \\ -\frac{a^3\left(a^2+2abx^2+b^2x^4\right)^p}{8b^2p^2+20b^2p+12b^2}+\frac{2a^2bpx^2\left(a^2+2abx^2+b^2x^4\right)^p}{8b^2p^2+20b^2p+12b^2}+\frac{4ab^2px^4\left(a^2+2abx^2+b^2x^4\right)^p}{8b^2p^2+20b^2p+12b^2}+\frac{3ab^2x^4\left(a^2+2abx^2+b^2x^4\right)^p}{8b^2p^2+20b^2p+12b^2}+\frac{2b^3px^6\left(a^2+2abx^2+b^2x^4\right)^p}{8b^2p^2+20b^2p+12b^2}+\frac{2b^3x^6\left(a^2+2abx^2+b^2x^4\right)^p}{8b^2p^2+20b^2p+12b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((a*x**4*(a**2)**p/4, Eq(b, 0)), (Integral(x**3*(a + b*x**2)/((a +
 b*x**2)**2)**(3/2), x), Eq(p, -3/2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2
*b**2) - a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)),
(-a**3*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**
2) + 2*a**2*b*p*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 20*b
**2*p + 12*b**2) + 4*a*b**2*p*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b*
*2*p**2 + 20*b**2*p + 12*b**2) + 3*a*b**2*x**4*(a**2 + 2*a*b*x**2 + b**2*x*
*4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**2) + 2*b**3*p*x**6*(a**2 + 2*a*b*x*
*2 + b**2*x**4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**2) + 2*b**3*x**6*(a**2
+ 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**2), True))
```

3.72 $\int x^5 \left(a+bx^2\right) \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=128

$$\frac{\left(a+bx^2\right)^4 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(p+2)}-\frac{a\left(a+bx^2\right)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{b^3(2p+3)}+\frac{a^2\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(p+1)}$$

Rubi [A] time = 0.13, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1249, 770, 21, 43}

$$\frac{\left(a+bx^2\right)^4 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(p+2)}-\frac{a\left(a+bx^2\right)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{b^3(2p+3)}+\frac{a^2\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (a^2*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^3*(1 + p)) - (a*(a +
 b*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(b^3*(3 + 2*p)) + ((a + b*x^2)^4*(
a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^3*(2 + p))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 770

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPa
rt[p]*(b/2 + c*x)^(2*FracPart[p])), Int[(d + e*x)^m*(f + g*x)*(b/2 + c*x)^(
2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rule 1249

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[b
^2 - 4*a*c, 0] &&  !IntegerQ[p] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^5\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2(a+bx)\left(a^2+2abx+b^2x^2\right)^p\,dx,x,x^2\right)\\
&= \frac{1}{2}\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\operatorname{Subst}\left(\int x^2(a+bx)\left(ab\right.\right.\\
&= \frac{\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\operatorname{Subst}\left(\int x^2\left(ab+b^2x\right)^{1+2p}\right.}{2b}\\
&= \frac{\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\operatorname{Subst}\left(\int\left(\frac{a^2\left(ab+b^2x\right)^{1+2p}}{b^2}-\right.\right.}{2b}\\
&= \frac{a^2\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(1+p)}-\frac{a\left(a+bx^2\right)^3\left(a^2+2abx^2+b^2\right.}{b^3(3+2p)}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 68, normalized size = 0.53

$$
\frac{\left(\left(a+bx^2\right)^2\right)^{p+1}\left(a^2-2ab(p+1)x^2+b^2\left(2p^2+5p+3\right)x^4\right)}{4b^3(p+1)(p+2)(2p+3)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (((a + b*x^2)^2)^(1 + p)*(a^2 - 2*a*b*(1 + p)*x^2 + b^2*(3 + 5*p + 2*p^2)*x
^4))/(4*b^3*(1 + p)*(2 + p)*(3 + 2*p))
```

IntegrateAlgebraic [F] time = 0.42, size = 0, normalized size = 0.00

$$
\int x^5\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^5*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p, x]
```

fricas [A] time = 0.77, size = 140, normalized size = 1.09

$$
\frac{\left(\left(2\,b^4p^2+5\,b^4p+3\,b^4\right)x^8-2\,a^3bpx^2+4\left(ab^3p^2+2\,ab^3p+ab^3\right)x^6+\left(2\,a^2b^2p^2+a^2b^2p\right)x^4+a^4\right)\left(b^2x^4+2\,abx^2+a^2\right)^p}{4\left(2\,b^3p^3+9\,b^3p^2+13\,b^3p+6\,b^3\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/4*((2*b^4*p^2 + 5*b^4*p + 3*b^4)*x^8 - 2*a^3*b*p*x^2 + 4*(a*b^3*p^2 + 2*a
*b^3*p + a*b^3)*x^6 + (2*a^2*b^2*p^2 + a^2*b^2*p)*x^4 + a^4)*(b^2*x^4 + 2*a
*b*x^2 + a^2)^p/(2*b^3*p^3 + 9*b^3*p^2 + 13*b^3*p + 6*b^3)
```

giac [B] time = 0.38, size = 331, normalized size = 2.59

$$
\frac{2\left(b^2x^4+2\,abx^2+a^2\right)^pb^4p^2x^8+5\left(b^2x^4+2\,abx^2+a^2\right)^pb^4px^8+4\left(b^2x^4+2\,abx^2+a^2\right)^pab^3p^2x^6+3\left(b^2x^4+2\,abx^2+a^2\right)^pb^4x^8+8\left(b^2x^4+2\,abx^2+a^2\right)^pab^3px^6+2\left(b^2x^4+2\,abx^2+a^2\right)^pa^2b^2p^2x^4+4\left(b^2x^4+2\,abx^2+a^2\right)^pab^3x^6+\left(b^2x^4+2\,abx^2+a^2\right)^pa^2b^2px^4-2\left(b^2x^4+2\,abx^2+a^2\right)^pa^3bpx^2+\left(b^2x^4+2\,abx^2+a^2\right)^pa^4}{4\left(2\,b^3p^3+9\,b^3p^2+13\,b^3p+6\,b^3\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/4*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*p^2*x^8 + 5*(b^2*x^4 + 2*a*b*x^2 +
 a^2)^p*b^4*p*x^8 + 4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^3*p^2*x^6 + 3*(b^2*
x^4 + 2*a*b*x^2 + a^2)^p*b^4*x^8 + 8*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^3*p*
x^6 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p^2*x^4 + 4*(b^2*x^4 + 2*a*b*
x^2 + a^2)^p*a*b^3*x^6 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p*x^4 - 2*(b
^2*x^4 + 2*a*b*x^2 + a^2)^p*a^3*b*p*x^2 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^4
)/(2*b^3*p^3 + 9*b^3*p^2 + 13*b^3*p + 6*b^3)
```

maple [A] time = 0.01, size = 99, normalized size = 0.77

$$\frac{\left(b\,x^{2}+a\right)^{2}\left(2b^{2}p^{2}x^{4}+5b^{2}p\,x^{4}+3b^{2}x^{4}-2abp\,x^{2}-2ab\,x^{2}+a^{2}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{p}}{4\left(2p^{3}+9p^{2}+13p+6\right)b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] 1/4*(b*x^2+a)^2*(2*b^2*p^2*x^4+5*b^2*p*x^4+3*b^2*x^4-2*a*b*p*x^2-2*a*b*x^2+
a^2)*(b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(2*p^3+9*p^2+13*p+6)
```

maxima [A] time = 0.70, size = 196, normalized size = 1.53

$$\frac{\left(\left(2\,p^{2}+3\,p+1\right)b^{3}x^{6}+\left(2\,p^{2}+p\right)ab^{2}x^{4}-2\,a^{2}bpx^{2}+a^{3}\right)\left(bx^{2}+a\right)^{2\,p}a}{2\left(4\,p^{3}+12\,p^{2}+11\,p+3\right)b^{3}}+\frac{\left(\left(4\,p^{3}+12\,p^{2}+11\,p+3\right)b^{4}x^{8}+2\left(2\,p^{3}+3\,p^{2}+p\right)ab^{3}x^{6}-3\left(2\,p^{2}+p\right)a^{2}b^{2}x^{4}+6\,a^{3}bpx^{2}-3\,a^{4}\right)\left(bx^{2}+a\right)^{2\,p}}{4\left(4\,p^{4}+20\,p^{3}+35\,p^{2}+25\,p+6\right)b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((2*p^2 + 3*p + 1)*b^3*x^6 + (2*p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + a^
3)*(b*x^2 + a)^(2*p)*a/((4*p^3 + 12*p^2 + 11*p + 3)*b^3) + 1/4*((4*p^3 + 12
*p^2 + 11*p + 3)*b^4*x^8 + 2*(2*p^3 + 3*p^2 + p)*a*b^3*x^6 - 3*(2*p^2 + p)*
a^2*b^2*x^4 + 6*a^3*b*p*x^2 - 3*a^4)*(b*x^2 + a)^(2*p)/((4*p^4 + 20*p^3 + 3
5*p^2 + 25*p + 6)*b^3)
```

mupad [B] time = 0.20, size = 169, normalized size = 1.32

$$\left(a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}\right)^{p}\left(\frac{a^{4}}{4\,b^{3}\,\left(2\,p^{3}+9\,p^{2}+13\,p+6\right)}+\frac{a\,x^{6}\,{\left(p+1\right)}^{2}}{2\,p^{3}+9\,p^{2}+13\,p+6}+\frac{b\,x^{8}\,\left(2\,p^{2}+5\,p+3\right)}{4\,\left(2\,p^{3}+9\,p^{2}+13\,p+6\right)}-\frac{a^{3}\,p\,x^{2}}{2\,b^{2}\,\left(2\,p^{3}+9\,p^{2}+13\,p+6\right)}+\frac{a^{2}\,p\,x^{4}\,\left(2\,p+1\right)}{4\,b\,\left(2\,p^{3}+9\,p^{2}+13\,p+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^p*(a^4/(4*b^3*(13*p + 9*p^2 + 2*p^3 + 6)) + (a*
x^6*(p + 1)^2)/(13*p + 9*p^2 + 2*p^3 + 6) + (b*x^8*(5*p + 2*p^2 + 3))/(4*(1
3*p + 9*p^2 + 2*p^3 + 6)) - (a^3*p*x^2)/(2*b^2*(13*p + 9*p^2 + 2*p^3 + 6))
+ (a^2*p*x^4*(2*p + 1))/(4*b*(13*p + 9*p^2 + 2*p^3 + 6)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{ax^{6}\left(a^{2}\right)^{p}}{6} & \text{for}\: b = 0 \\ \frac{2a^{2}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{2a^{2}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{3a^{2}}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{4abx^{2}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{4abx^{2}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{4abx^{2}}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{2b^{2}x^{4}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{2b^{2}x^{4}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}} & \text{for}\: p = -2 \\ \int \frac{x^{5}\left(a+bx^{2}\right)}{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}\, dx & \text{for}\: p = -\frac{3}{2} \\ \frac{a^{2}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^{3}}+\frac{a^{2}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^{3}}-\frac{ax^{2}}{2b^{2}}+\frac{x^{4}}{4b} & \text{for}\: p = -1 \\ \frac{a^{4}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{p}}{8b^{3}p^{3}+36b^{3}p^{2}+52b^{3}p+24b^{3}}-\frac{2a^{3}bpx^{2}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{p}}{8b^{3}p^{3}+36b^{3}p^{2}+52b^{3}p+24b^{3}}+\frac{2a^{2}b^{2}p^{2}x^{4}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{p}}{8b^{3}p^{3}+36b^{3}p^{2}+52b^{3}p+24b^{3}}+\frac{a^{2}b^{2}px^{4}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{p}}{8b^{3}p^{3}+36b^{3}p^{2}+52b^{3}p+24b^{3}}+\frac{4ab^{3}p^{2}x^{6}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{p}}{8b^{3}p^{3}+36b^{3}p^{2}+52b^{3}p+24b^{3}}+\frac{8ab^{3}px^{6}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{p}}{8b^{3}p^{3}+36b^{3}p^{2}+52b^{3}p+24b^{3}}+\frac{4ab^{3}x^{6}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{p}}{8b^{3}p^{3}+36b^{3}p^{2}+52b^{3}p+24b^{3}}+\frac{2b^{4}p^{2}x^{8}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{p}}{8b^{3}p^{3}+36b^{3}p^{2}+52b^{3}p+24b^{3}}+\frac{5b^{4}px^{8}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{p}}{8b^{3}p^{3}+36b^{3}p^{2}+52b^{3}p+24b^{3}}+\frac{3b^{4}x^{8}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{p}}{8b^{3}p^{3}+36b^{3}p^{2}+52b^{3}p+24b^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((a*x**6*(a**2)**p/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)*sqrt(1/b)
+ x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*sqrt(a)*sqr
```

```
t(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b*
*3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x
)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*sqrt(a)*sq
rt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a*
*2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sqrt(a)*sqrt(1/
b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(I*sqr
t(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -2))
, (Integral(x**5*(a + b*x**2)/((a + b*x**2)**2)**(3/2), x), Eq(p, -3/2)), (
a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) + a**2*log(I*sqrt(a)*sqrt(1/b)
+ x)/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (a**4*(a**2 + 2*a
*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b**3)
- 2*a**3*b*p*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3
*p**2 + 52*b**3*p + 24*b**3) + 2*a**2*b**2*p**2*x**4*(a**2 + 2*a*b*x**2 + b
**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b**3) + a**2*b**2
*p*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52
*b**3*p + 24*b**3) + 4*a*b**3*p**2*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/
(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b**3) + 8*a*b**3*p*x**6*(a**2
+ 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b
**3) + 4*a*b**3*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b
**3*p**2 + 52*b**3*p + 24*b**3) + 2*b**4*p**2*x**8*(a**2 + 2*a*b*x**2 + b**
2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b**3) + 5*b**4*p*x*
*8*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3
*p + 24*b**3) + 3*b**4*x**8*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3
 + 36*b**3*p**2 + 52*b**3*p + 24*b**3), True))
```

3.73 $\int x^3 \left(A+Bx^2\right) \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=166

$$\frac{1}{4}a^3Ax^4+\frac{1}{6}a^2x^6(aB+3Ab)+\frac{3}{14}cx^{14}\left(aBc+Abc+b^2B\right)+\frac{3}{8}ax^8\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{12}x^{12}\left(3aAc^2+6abBc+3Ab\right.$$

Rubi [A] time = 0.39, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1251, 765}

$$\frac{1}{6}a^2x^6(aB+3Ab)+\frac{1}{4}a^3Ax^4+\frac{1}{12}x^{12}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{3}{14}cx^{14}\left(aBc+Abc+b^2B\right)+\frac{1}{10}x^{10}\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{3}{8}ax^8\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{16}c^2x^{16}(Ac+3bB)+\frac{1}{18}Bc^3x^{18}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^4)/4 + (a^2*(3*A*b + a*B)*x^6)/6 + (3*a*(a*b*B + A*(b^2 + a*c))*x^
8)/8 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^10)/10 + ((b^3*B + 3*A*b^
2*c + 6*a*b*B*c + 3*a*A*c^2)*x^12)/12 + (3*c*(b^2*B + A*b*c + a*B*c)*x^14)/
14 + (c^2*(3*b*B + A*c)*x^16)/16 + (B*c^3*x^18)/18
```

Rule 765

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + b*x + c
*x^2)^p, x], x] /; FreeQ[{a, b, c, e, f, g, m}, x] && IntegerQ[p] && (GtQ[p
, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(A+Bx^2\right) \left(a+bx^2+cx^4\right)^3 \, dx &= \frac{1}{2}\text{Subst}\left(\int x(A+Bx)\left(a+bx+cx^2\right)^3 \, dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(a^3Ax+a^2(3Ab+aB)x^2+3a\left(abB+A\left(b^2+ac\right)\right)x^3+\left(3aB\right.\right.\right.\\ &= \frac{1}{4}a^3Ax^4+\frac{1}{6}a^2(3Ab+aB)x^6+\frac{3}{8}a\left(abB+A\left(b^2+ac\right)\right)x^8+\frac{1}{10}\left(3aB\left(b^2+\right.\right.\end{aligned}$$

Mathematica [A] time = 0.05, size = 166, normalized size = 1.00

$$\frac{1}{4}a^3Ax^4+\frac{1}{6}a^2x^6(aB+3Ab)+\frac{3}{14}cx^{14}\left(aBc+Abc+b^2B\right)+\frac{3}{8}ax^8\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{12}x^{12}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{10}x^{10}\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{16}c^2x^{16}(Ac+3bB)+\frac{1}{18}Bc^3x^{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^4)/4 + (a^2*(3*A*b + a*B)*x^6)/6 + (3*a*(a*b*B + A*(b^2 + a*c))*x^
8)/8 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^10)/10 + ((b^3*B + 3*A*b^
2*c + 6*a*b*B*c + 3*a*A*c^2)*x^12)/12 + (3*c*(b^2*B + A*b*c + a*B*c)*x^14)/
14 + (c^2*(3*b*B + A*c)*x^16)/16 + (B*c^3*x^18)/18
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3, x]
```

fricas [A] time = 0.71, size = 193, normalized size = 1.16

$$\frac{1}{18}x^{18}c^3B+\frac{3}{16}x^{16}c^2bB+\frac{1}{16}x^{16}c^3A+\frac{3}{14}x^{14}cb^2B+\frac{3}{14}x^{14}c^2aB+\frac{3}{14}x^{14}c^2bA+\frac{1}{12}x^{12}b^3B+\frac{1}{2}x^{12}cbaB+\frac{1}{4}x^{12}cb^2A+\frac{1}{4}x^{12}c^2aA+\frac{3}{10}x^{10}b^2aB+\frac{3}{10}x^{10}ca^2B+\frac{1}{10}x^{10}b^3A+\frac{3}{5}x^{10}cbaA+\frac{3}{8}x^8ba^2B+\frac{3}{8}x^8b^2aA+\frac{3}{8}x^8ca^2A+\frac{1}{6}x^6a^3B+\frac{1}{2}x^6ba^2A+\frac{1}{4}x^4a^3A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/18*x^18*c^3*B + 3/16*x^16*c^2*b*B + 1/16*x^16*c^3*A + 3/14*x^14*c*b^2*B +
 3/14*x^14*c^2*a*B + 3/14*x^14*c^2*b*A + 1/12*x^12*b^3*B + 1/2*x^12*c*b*a*B
 + 1/4*x^12*c*b^2*A + 1/4*x^12*c^2*a*A + 3/10*x^10*b^2*a*B + 3/10*x^10*c*a^
2*B + 1/10*x^10*b^3*A + 3/5*x^10*c*b*a*A + 3/8*x^8*b*a^2*B + 3/8*x^8*b^2*a*
A + 3/8*x^8*c*a^2*A + 1/6*x^6*a^3*B + 1/2*x^6*b*a^2*A + 1/4*x^4*a^3*A
```

giac [A] time = 0.34, size = 193, normalized size = 1.16

$$\frac{1}{18}Bc^3x^{18}+\frac{3}{16}Bbc^2x^{16}+\frac{1}{16}Ac^3x^{16}+\frac{3}{14}Bb^2cx^{14}+\frac{3}{14}Bac^2x^{14}+\frac{3}{14}Abc^2x^{14}+\frac{1}{12}Bb^3x^{12}+\frac{1}{2}Babcx^{12}+\frac{1}{4}Ab^2cx^{12}+\frac{1}{4}Aac^2x^{12}+\frac{3}{10}Bab^2x^{10}+\frac{1}{10}Ab^3x^{10}+\frac{3}{10}Ba^2cx^{10}+\frac{3}{5}Aabcx^{10}+\frac{3}{8}Ba^2bx^8+\frac{3}{8}Aab^2x^8+\frac{3}{8}Aa^2cx^8+\frac{1}{6}Ba^3x^6+\frac{1}{2}Aa^2bx^6+\frac{1}{4}Aa^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/18*B*c^3*x^18 + 3/16*B*b*c^2*x^16 + 1/16*A*c^3*x^16 + 3/14*B*b^2*c*x^14 +
 3/14*B*a*c^2*x^14 + 3/14*A*b*c^2*x^14 + 1/12*B*b^3*x^12 + 1/2*B*a*b*c*x^12
 + 1/4*A*b^2*c*x^12 + 1/4*A*a*c^2*x^12 + 3/10*B*a*b^2*x^10 + 1/10*A*b^3*x^1
0 + 3/10*B*a^2*c*x^10 + 3/5*A*a*b*c*x^10 + 3/8*B*a^2*b*x^8 + 3/8*A*a*b^2*x^
8 + 3/8*A*a^2*c*x^8 + 1/6*B*a^3*x^6 + 1/2*A*a^2*b*x^6 + 1/4*A*a^3*x^4
```

maple [A] time = 0.00, size = 226, normalized size = 1.36

$$\frac{Bc^3x^{18}}{18}+\frac{\left(Ac^3+3Bbc^2\right)x^{16}}{16}+\frac{\left(3Abc^2+\left(ac^2+2b^2c+\left(2ac+b^2\right)c\right)B\right)x^{14}}{14}+\frac{\left(\left(ac^2+2b^2c+\left(2ac+b^2\right)c\right)A+\left(4abc+\left(2ac+b^2\right)b\right)B\right)x^{12}}{12}+\frac{\left(\left(4abc+\left(2ac+b^2\right)b\right)A+\left(a^2c+2ab^2+\left(2ac+b^2\right)a\right)B\right)x^{10}}{10}+\frac{Aa^3x^4}{4}+\frac{\left(3Ba^2b+\left(a^2c+2ab^2+\left(2ac+b^2\right)a\right)A\right)x^8}{8}+\frac{\left(3Aa^2b+Ba^3\right)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/18*B*c^3*x^18+1/16*(A*c^3+3*B*b*c^2)*x^16+1/14*(3*A*b*c^2+B*(a*c^2+2*b^2*
c+c*(2*a*c+b^2)))*x^14+1/12*(A*(a*c^2+2*b^2*c+c*(2*a*c+b^2))+B*(4*a*b*c+b*(
2*a*c+b^2)))*x^12+1/10*(A*(4*a*b*c+b*(2*a*c+b^2))+B*(a*(2*a*c+b^2)+2*b^2*a+
c*a^2))*x^10+1/8*(A*(a*(2*a*c+b^2)+2*b^2*a+c*a^2)+3*B*a^2*b)*x^8+1/6*(3*A*a
^2*b+B*a^3)*x^6+1/4*a^3*A*x^4
```

maxima [A] time = 0.60, size = 166, normalized size = 1.00

$$\frac{1}{18}Bc^3x^{18}+\frac{1}{16}\left(3Bbc^2+Ac^3\right)x^{16}+\frac{3}{14}\left(Bb^2c+(Ba+Ab)c^2\right)x^{14}+\frac{1}{12}\left(Bb^3+3Aac^2+3\left(2Bab+Ab^2\right)c\right)x^{12}+\frac{1}{10}\left(3Bab^2+Ab^3+3\left(Ba^2+2Aab\right)c\right)x^{10}+\frac{3}{8}\left(Ba^2b+Aab^2+Aa^2c\right)x^8+\frac{1}{4}Aa^3x^4+\frac{1}{6}\left(Ba^3+3Aa^2b\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*B*c^3*x^18 + 1/16*(3*B*b*c^2 + A*c^3)*x^16 + 3/14*(B*b^2*c + (B*a + A*
b)*c^2)*x^14 + 1/12*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^12 + 1/10
```

```
*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^10 + 3/8*(B*a^2*b + A*a*b^2
+ A*a^2*c)*x^8 + 1/4*A*a^3*x^4 + 1/6*(B*a^3 + 3*A*a^2*b)*x^6
```

mupad [B] time = 0.08, size = 169, normalized size = 1.02

$$x^{10}\left(\frac{3\,B\,c\,a^2}{10}+\frac{3\,B\,a\,b^2}{10}+\frac{3\,A\,c\,a\,b}{5}+\frac{A\,b^3}{10}\right)+x^{12}\left(\frac{B\,b^3}{12}+\frac{A\,b^2\,c}{4}+\frac{B\,a\,b\,c}{2}+\frac{A\,a\,c^2}{4}\right)+x^6\left(\frac{B\,a^3}{6}+\frac{A\,b\,a^2}{2}\right)+x^{16}\left(\frac{A\,c^3}{16}+\frac{3\,B\,b\,c^2}{16}\right)+x^8\left(\frac{3\,B\,a^2\,b}{8}+\frac{3\,A\,c\,a^2}{8}+\frac{3\,A\,a\,b^2}{8}\right)+x^{14}\left(\frac{3\,B\,b^2\,c}{14}+\frac{3\,A\,b\,c^2}{14}+\frac{3\,B\,a\,c^2}{14}\right)+\frac{A\,a^3\,x^4}{4}+\frac{B\,c^3\,x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^10*((A*b^3)/10 + (3*B*a*b^2)/10 + (3*B*a^2*c)/10 + (3*A*a*b*c)/5) + x^12*
((B*b^3)/12 + (A*a*c^2)/4 + (A*b^2*c)/4 + (B*a*b*c)/2) + x^6*((B*a^3)/6 + (
A*a^2*b)/2) + x^16*((A*c^3)/16 + (3*B*b*c^2)/16) + x^8*((3*A*a*b^2)/8 + (3*
A*a^2*c)/8 + (3*B*a^2*b)/8) + x^14*((3*A*b*c^2)/14 + (3*B*a*c^2)/14 + (3*B*
b^2*c)/14) + (A*a^3*x^4)/4 + (B*c^3*x^18)/18
```

sympy [A] time = 0.10, size = 202, normalized size = 1.22

$$\frac{Aa^3x^4}{4}+\frac{Bc^3x^{18}}{18}+x^{16}\left(\frac{Ac^3}{16}+\frac{3Bbc^2}{16}\right)+x^{14}\left(\frac{3Abc^2}{14}+\frac{3Bac^2}{14}+\frac{3Bb^2c}{14}\right)+x^{12}\left(\frac{Aac^2}{4}+\frac{Ab^2c}{4}+\frac{Babc}{2}+\frac{Bb^3}{12}\right)+x^{10}\left(\frac{3Aabc}{5}+\frac{Ab^3}{10}+\frac{3Ba^2c}{10}+\frac{3Bab^2}{10}\right)+x^8\left(\frac{3Aa^2c}{8}+\frac{3Aab^2}{8}+\frac{3Ba^2b}{8}\right)+x^6\left(\frac{Aa^2b}{2}+\frac{Ba^3}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x**4/4 + B*c**3*x**18/18 + x**16*(A*c**3/16 + 3*B*b*c**2/16) + x**14
*(3*A*b*c**2/14 + 3*B*a*c**2/14 + 3*B*b**2*c/14) + x**12*(A*a*c**2/4 + A*b*
*2*c/4 + B*a*b*c/2 + B*b**3/12) + x**10*(3*A*a*b*c/5 + A*b**3/10 + 3*B*a**2
*c/10 + 3*B*a*b**2/10) + x**8*(3*A*a**2*c/8 + 3*A*a*b**2/8 + 3*B*a**2*b/8)
+ x**6*(A*a**2*b/2 + B*a**3/6)
```

3.74 $\int x^2 \left(A+Bx^2\right) \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=166

$$\frac{1}{3}a^3Ax^3+\frac{1}{5}a^2x^5(aB+3Ab)+\frac{3}{13}cx^{13}\left(aBc+Abc+b^2B\right)+\frac{3}{7}ax^7\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{11}x^{11}\left(3aAc^2+6abBc+3\right.$$

Rubi [A] time = 0.15, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1261}

$$\frac{1}{5}a^2x^5(aB+3Ab)+\frac{1}{3}a^3Ax^3+\frac{1}{11}x^{11}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{3}{13}cx^{13}\left(aBc+Abc+b^2B\right)+\frac{1}{9}x^9\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{3}{7}ax^7\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{15}c^2x^{15}(Ac+3bB)+\frac{1}{17}Bc^3x^{17}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^3)/3 + (a^2*(3*A*b + a*B)*x^5)/5 + (3*a*(a*b*B + A*(b^2 + a*c))*x^
7)/7 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^9)/9 + ((b^3*B + 3*A*b^2*
c + 6*a*b*B*c + 3*a*A*c^2)*x^11)/11 + (3*c*(b^2*B + A*b*c + a*B*c)*x^13)/13
 + (c^2*(3*b*B + A*c)*x^15)/15 + (B*c^3*x^17)/17
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx &= \int \left(a^3Ax^2+a^2(3Ab+aB)x^4+3a\left(abB+A\left(b^2+ac\right)\right)x^6+\left(3aB\left(b^2+\right.\right.\right.\\ &= \frac{1}{3}a^3Ax^3+\frac{1}{5}a^2(3Ab+aB)x^5+\frac{3}{7}a\left(abB+A\left(b^2+ac\right)\right)x^7+\frac{1}{9}\left(3aB\left(b^2\right.\right.\end{aligned}$$

Mathematica [A] time = 0.05, size = 166, normalized size = 1.00

$$\frac{1}{3}a^3Ax^3+\frac{1}{5}a^2x^5(aB+3Ab)+\frac{3}{13}cx^{13}\left(aBc+Abc+b^2B\right)+\frac{3}{7}ax^7\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{11}x^{11}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{9}x^9\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{15}c^2x^{15}(Ac+3bB)+\frac{1}{17}Bc^3x^{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^3)/3 + (a^2*(3*A*b + a*B)*x^5)/5 + (3*a*(a*b*B + A*(b^2 + a*c))*x^
7)/7 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^9)/9 + ((b^3*B + 3*A*b^2*
c + 6*a*b*B*c + 3*a*A*c^2)*x^11)/11 + (3*c*(b^2*B + A*b*c + a*B*c)*x^13)/13
 + (c^2*(3*b*B + A*c)*x^15)/15 + (B*c^3*x^17)/17
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3, x]
```

fricas [A] time = 0.61, size = 193, normalized size = 1.16

$$\frac{1}{17}x^{17}c^3B+\frac{1}{5}x^{15}c^2bB+\frac{1}{15}x^{15}c^3A+\frac{3}{13}x^{13}cb^2B+\frac{3}{13}x^{13}c^2aB+\frac{3}{13}x^{13}c^2bA+\frac{1}{11}x^{11}b^3B+\frac{6}{11}x^{11}cbaB+\frac{3}{11}x^{11}cb^2A+\frac{3}{11}x^{11}c^2aA+\frac{1}{3}x^9b^2aB+\frac{1}{3}x^9ca^2B+\frac{1}{9}x^9b^3A+\frac{2}{3}x^9cbaA+\frac{3}{7}x^7ba^2B+\frac{3}{7}x^7b^2aA+\frac{3}{7}x^7ca^2A+\frac{1}{5}x^5a^3B+\frac{3}{5}x^5ba^2A+\frac{1}{3}x^3a^3A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*c^3*B + 1/5*x^15*c^2*b*B + 1/15*x^15*c^3*A + 3/13*x^13*c*b^2*B +
3/13*x^13*c^2*a*B + 3/13*x^13*c^2*b*A + 1/11*x^11*b^3*B + 6/11*x^11*c*b*a*B
 + 3/11*x^11*c*b^2*A + 3/11*x^11*c^2*a*A + 1/3*x^9*b^2*a*B + 1/3*x^9*c*a^2*
B + 1/9*x^9*b^3*A + 2/3*x^9*c*b*a*A + 3/7*x^7*b*a^2*B + 3/7*x^7*b^2*a*A + 3
/7*x^7*c*a^2*A + 1/5*x^5*a^3*B + 3/5*x^5*b*a^2*A + 1/3*x^3*a^3*A
```

giac [A] time = 0.27, size = 193, normalized size = 1.16

$$\frac{1}{17}Bc^3x^{17}+\frac{1}{5}Bbc^2x^{15}+\frac{1}{15}Ac^3x^{15}+\frac{3}{13}Bb^2cx^{13}+\frac{3}{13}Bac^2x^{13}+\frac{3}{13}Abc^2x^{13}+\frac{1}{11}Bb^3x^{11}+\frac{6}{11}Babcx^{11}+\frac{3}{11}Ab^2cx^{11}+\frac{3}{11}Aac^2x^{11}+\frac{1}{3}Bab^2x^9+\frac{1}{9}Ab^3x^9+\frac{1}{3}Ba^2cx^9+\frac{2}{3}Aabcx^9+\frac{3}{7}Ba^2bx^7+\frac{3}{7}Aab^2x^7+\frac{3}{7}Aa^2cx^7+\frac{1}{5}Ba^3x^5+\frac{3}{5}Aa^2bx^5+\frac{1}{3}Aa^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/17*B*c^3*x^17 + 1/5*B*b*c^2*x^15 + 1/15*A*c^3*x^15 + 3/13*B*b^2*c*x^13 +
3/13*B*a*c^2*x^13 + 3/13*A*b*c^2*x^13 + 1/11*B*b^3*x^11 + 6/11*B*a*b*c*x^11
 + 3/11*A*b^2*c*x^11 + 3/11*A*a*c^2*x^11 + 1/3*B*a*b^2*x^9 + 1/9*A*b^3*x^9
+ 1/3*B*a^2*c*x^9 + 2/3*A*a*b*c*x^9 + 3/7*B*a^2*b*x^7 + 3/7*A*a*b^2*x^7 + 3
/7*A*a^2*c*x^7 + 1/5*B*a^3*x^5 + 3/5*A*a^2*b*x^5 + 1/3*A*a^3*x^3
```

maple [A] time = 0.00, size = 226, normalized size = 1.36

$$\frac{Bc^3x^{17}}{17}+\frac{\left(Ac^3+3Bbc^2\right)x^{15}}{15}+\frac{\left(3Abc^2+\left(ac^2+2b^2c+\left(2ac+b^2\right)c\right)B\right)x^{13}}{13}+\frac{\left(\left(ac^2+2b^2c+\left(2ac+b^2\right)c\right)A+\left(4abc+\left(2ac+b^2\right)b\right)B\right)x^{11}}{11}+\frac{\left(\left(4abc+\left(2ac+b^2\right)b\right)A+\left(a^2c+2ab^2+\left(2ac+b^2\right)a\right)B\right)x^9}{9}+\frac{Aa^3x^3}{3}+\frac{\left(3Ba^2b+\left(a^2c+2ab^2+\left(2ac+b^2\right)a\right)A\right)x^7}{7}+\frac{\left(3Aa^2b+Ba^3\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/17*B*c^3*x^17+1/15*(A*c^3+3*B*b*c^2)*x^15+1/13*(3*A*b*c^2+(a*c^2+2*b^2*c+
(2*a*c+b^2)*c)*B)*x^13+1/11*((a*c^2+2*b^2*c+(2*a*c+b^2)*c)*A+(4*a*b*c+(2*a*
c+b^2)*b)*B)*x^11+1/9*((4*a*b*c+(2*a*c+b^2)*b)*A+(a^2*c+2*a*b^2+(2*a*c+b^2)
*a)*B)*x^9+1/7*(3*B*a^2*b+(a^2*c+2*a*b^2+(2*a*c+b^2)*a)*A)*x^7+1/5*(3*A*a^2
*b+B*a^3)*x^5+1/3*a^3*A*x^3
```

maxima [A] time = 0.73, size = 166, normalized size = 1.00

$$\frac{1}{17}Bc^3x^{17}+\frac{1}{15}\left(3Bbc^2+Ac^3\right)x^{15}+\frac{3}{13}\left(Bb^2c+\left(Ba+Ab\right)c^2\right)x^{13}+\frac{1}{11}\left(Bb^3+3Aac^2+3\left(2Bab+Ab^2\right)c\right)x^{11}+\frac{1}{9}\left(3Bab^2+Ab^3+3\left(Ba^2+2Aab\right)c\right)x^9+\frac{3}{7}\left(Ba^2b+Aab^2+Aa^2c\right)x^7+\frac{1}{3}Aa^3x^3+\frac{1}{5}\left(Ba^3+3Aa^2b\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/17*B*c^3*x^17 + 1/15*(3*B*b*c^2 + A*c^3)*x^15 + 3/13*(B*b^2*c + (B*a + A*
b)*c^2)*x^13 + 1/11*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^11 + 1/9*
(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^9 + 3/7*(B*a^2*b + A*a*b^2 +
A*a^2*c)*x^7 + 1/3*A*a^3*x^3 + 1/5*(B*a^3 + 3*A*a^2*b)*x^5
```

mupad [B] time = 0.10, size = 169, normalized size = 1.02

$$x^9\left(\frac{B\,c\,a^2}{3}+\frac{B\,a\,b^2}{3}+\frac{2\,A\,c\,a\,b}{3}+\frac{A\,b^3}{9}\right)+x^{11}\left(\frac{B\,b^3}{11}+\frac{3\,A\,b^2\,c}{11}+\frac{6\,B\,a\,b\,c}{11}+\frac{3\,A\,a\,c^2}{11}\right)+x^5\left(\frac{B\,a^3}{5}+\frac{3\,A\,b\,a^2}{5}\right)+x^{15}\left(\frac{A\,c^3}{15}+\frac{B\,b\,c^2}{5}\right)+x^7\left(\frac{3\,B\,a^2\,b}{7}+\frac{3\,A\,c\,a^2}{7}+\frac{3\,A\,a\,b^2}{7}\right)+x^{13}\left(\frac{3\,B\,b^2\,c}{13}+\frac{3\,A\,b\,c^2}{13}+\frac{3\,B\,a\,c^2}{13}\right)+\frac{A\,a^3\,x^3}{3}+\frac{B\,c^3\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^9*((A*b^3)/9 + (B*a*b^2)/3 + (B*a^2*c)/3 + (2*A*a*b*c)/3) + x^11*((B*b^3)
/11 + (3*A*a*c^2)/11 + (3*A*b^2*c)/11 + (6*B*a*b*c)/11) + x^5*((B*a^3)/5 +
(3*A*a^2*b)/5) + x^15*((A*c^3)/15 + (B*b*c^2)/5) + x^7*((3*A*a*b^2)/7 + (3*
A*a^2*c)/7 + (3*B*a^2*b)/7) + x^13*((3*A*b*c^2)/13 + (3*B*a*c^2)/13 + (3*B*
b^2*c)/13) + (A*a^3*x^3)/3 + (B*c^3*x^17)/17
```

sympy [A] time = 0.10, size = 204, normalized size = 1.23

$$\frac{Aa^3x^3}{3} + \frac{Bc^3x^{17}}{17} + x^{15}\left(\frac{Ac^3}{15} + \frac{Bbc^2}{5}\right) + x^{13}\left(\frac{3Abc^2}{13} + \frac{3Bac^2}{13} + \frac{3Bb^2c}{13}\right) + x^{11}\left(\frac{3Aac^2}{11} + \frac{3Ab^2c}{11} + \frac{6Babc}{11} + \frac{Bb^3}{11}\right) + x^9\left(\frac{2Aabc}{3} + \frac{Ab^3}{9} + \frac{Ba^2c}{3} + \frac{Bab^2}{3}\right) + x^7\left(\frac{3Aa^2c}{7} + \frac{3Aab^2}{7} + \frac{3Ba^2b}{7}\right) + x^5\left(\frac{3Aa^2b}{5} + \frac{Ba^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x**3/3 + B*c**3*x**17/17 + x**15*(A*c**3/15 + B*b*c**2/5) + x**13*(3
*A*b*c**2/13 + 3*B*a*c**2/13 + 3*B*b**2*c/13) + x**11*(3*A*a*c**2/11 + 3*A*
b**2*c/11 + 6*B*a*b*c/11 + B*b**3/11) + x**9*(2*A*a*b*c/3 + A*b**3/9 + B*a*
*2*c/3 + B*a*b**2/3) + x**7*(3*A*a**2*c/7 + 3*A*a*b**2/7 + 3*B*a**2*b/7) +
x**5*(3*A*a**2*b/5 + B*a**3/5)
```

3.75 $\int x\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=166

$$\frac{1}{2}a^3Ax^2+\frac{1}{4}a^2x^4(aB+3Ab)+\frac{1}{4}cx^{12}\left(aBc+Abc+b^2B\right)+\frac{1}{2}ax^6\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{10}x^{10}\left(3aAc^2+6abBc+3Ab^2\right.$$

Rubi $[\mathbf{A}]$ time = 0.29, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1247, 631}

$$\frac{1}{4}a^2x^4(aB+3Ab)+\frac{1}{2}a^3Ax^2+\frac{1}{10}x^{10}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{4}cx^{12}\left(aBc+Abc+b^2B\right)+\frac{1}{8}x^8\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{2}ax^6\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{14}c^2x^{14}(Ac+3bB)+\frac{1}{16}Bc^3x^{16}$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^2)/2 + (a^2*(3*A*b + a*B)*x^4)/4 + (a*(a*b*B + A*(b^2 + a*c))*x^6)
/2 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^8)/8 + ((b^3*B + 3*A*b^2*c
+ 6*a*b*B*c + 3*a*A*c^2)*x^10)/10 + (c*(b^2*B + A*b*c + a*B*c)*x^12)/4 + (c
^2*(3*b*B + A*c)*x^14)/14 + (B*c^3*x^16)/16
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx&=\frac{1}{2}\text{Subst}\left(\int(A+Bx)\left(a+bx+cx^2\right)^3\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(a^3A+a^2(3Ab+aB)x+3a\left(abB+A\left(b^2+ac\right)\right)x^2+\left(3aB\left(b^2\right.\right.\right.\right.\\&=\frac{1}{2}a^3Ax^2+\frac{1}{4}a^2(3Ab+aB)x^4+\frac{1}{2}a\left(abB+A\left(b^2+ac\right)\right)x^6+\frac{1}{8}\left(3aB\left(b^2+a\right.\right.\end{aligned}$$

Mathematica $[\mathbf{A}]$ time = 0.06, size = 154, normalized size = 0.93

$$\frac{1}{560}x^2\left(280a^3A+140a^2x^2(aB+3Ab)+140cx^{10}\left(aBc+Abc+b^2B\right)+280ax^4\left(A\left(ac+b^2\right)+abB\right)+56x^8\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+70x^6\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+40c^2x^{12}(Ac+3bB)+35Bc^3x^{14}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^2*(280*a^3*A + 140*a^2*(3*A*b + a*B)*x^2 + 280*a*(a*b*B + A*(b^2 + a*c))
*x^4 + 70*(3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6 + 56*(b^3*B + 3*A*b^2
*c + 6*a*b*B*c + 3*a*A*c^2)*x^8 + 140*c*(b^2*B + A*b*c + a*B*c)*x^10 + 40*c
^2*(3*b*B + A*c)*x^12 + 35*B*c^3*x^14))/560
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3, x]
```

fricas [A] time = 0.60, size = 193, normalized size = 1.16

$$\frac{1}{16}x^{16}c^3B+\frac{3}{14}x^{14}c^2bB+\frac{1}{14}x^{14}c^3A+\frac{1}{4}x^{12}cb^2B+\frac{1}{4}x^{12}c^2aB+\frac{1}{4}x^{12}c^2bA+\frac{1}{10}x^{10}b^3B+\frac{3}{5}x^{10}cbaB+\frac{3}{10}x^{10}cb^2A+\frac{3}{10}x^{10}c^2aA+\frac{3}{8}x^8b^2aB+\frac{3}{8}x^8ca^2B+\frac{1}{8}x^8b^3A+\frac{3}{4}x^8cbaA+\frac{1}{2}x^6ba^2B+\frac{1}{2}x^6b^2aA+\frac{1}{2}x^6ca^2A+\frac{1}{4}x^4a^3B+\frac{3}{4}x^4ba^2A+\frac{1}{2}x^2a^3A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*c^3*B + 3/14*x^14*c^2*b*B + 1/14*x^14*c^3*A + 1/4*x^12*c*b^2*B +
1/4*x^12*c^2*a*B + 1/4*x^12*c^2*b*A + 1/10*x^10*b^3*B + 3/5*x^10*c*b*a*B +
3/10*x^10*c*b^2*A + 3/10*x^10*c^2*a*A + 3/8*x^8*b^2*a*B + 3/8*x^8*c*a^2*B +
 1/8*x^8*b^3*A + 3/4*x^8*c*b*a*A + 1/2*x^6*b*a^2*B + 1/2*x^6*b^2*a*A + 1/2*
x^6*c*a^2*A + 1/4*x^4*a^3*B + 3/4*x^4*b*a^2*A + 1/2*x^2*a^3*A
```

giac [A] time = 0.29, size = 193, normalized size = 1.16

$$\frac{1}{16}Bc^3x^{16}+\frac{3}{14}Bbc^2x^{14}+\frac{1}{14}Ac^3x^{14}+\frac{1}{4}Bb^2cx^{12}+\frac{1}{4}Bac^2x^{12}+\frac{1}{4}Abc^2x^{12}+\frac{1}{10}Bb^3x^{10}+\frac{3}{5}Babcx^{10}+\frac{3}{10}Ab^2cx^{10}+\frac{3}{10}Aac^2x^{10}+\frac{3}{8}Bab^2x^8+\frac{1}{8}Ab^3x^8+\frac{3}{8}Ba^2cx^8+\frac{3}{4}Aabcx^8+\frac{1}{2}Ba^2bx^6+\frac{1}{2}Aab^2x^6+\frac{1}{2}Aa^2cx^6+\frac{1}{4}Ba^3x^4+\frac{3}{4}Aa^2bx^4+\frac{1}{2}Aa^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*B*c^3*x^16 + 3/14*B*b*c^2*x^14 + 1/14*A*c^3*x^14 + 1/4*B*b^2*c*x^12 +
1/4*B*a*c^2*x^12 + 1/4*A*b*c^2*x^12 + 1/10*B*b^3*x^10 + 3/5*B*a*b*c*x^10 +
3/10*A*b^2*c*x^10 + 3/10*A*a*c^2*x^10 + 3/8*B*a*b^2*x^8 + 1/8*A*b^3*x^8 + 3
/8*B*a^2*c*x^8 + 3/4*A*a*b*c*x^8 + 1/2*B*a^2*b*x^6 + 1/2*A*a*b^2*x^6 + 1/2*
A*a^2*c*x^6 + 1/4*B*a^3*x^4 + 3/4*A*a^2*b*x^4 + 1/2*A*a^3*x^2
```

maple [A] time = 0.00, size = 226, normalized size = 1.36

$$\frac{Bc^3x^{16}}{16}+\frac{\left(Ac^3+3Bbc^2\right)x^{14}}{14}+\frac{\left(3Abc^2+\left(ac^2+2b^2c+\left(2ac+b^2\right)c\right)B\right)x^{12}}{12}+\frac{\left(\left(ac^2+2b^2c+\left(2ac+b^2\right)c\right)A+\left(4abc+\left(2ac+b^2\right)b\right)B\right)x^{10}}{10}+\frac{\left(\left(4abc+\left(2ac+b^2\right)b\right)A+\left(a^2c+2ab^2+\left(2ac+b^2\right)a\right)B\right)x^8}{8}+\frac{Aa^3x^2}{2}+\frac{\left(3Ba^2b+\left(a^2c+2ab^2+\left(2ac+b^2\right)a\right)A\right)x^6}{6}+\frac{\left(3Aa^2b+Ba^3\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/16*B*c^3*x^16+1/14*(A*c^3+3*B*b*c^2)*x^14+1/12*(3*A*b*c^2+(a*c^2+2*b^2*c+
(2*a*c+b^2)*c)*B)*x^12+1/10*((a*c^2+2*b^2*c+(2*a*c+b^2)*c)*A+(4*a*b*c+(2*a*
c+b^2)*b)*B)*x^10+1/8*((4*a*b*c+(2*a*c+b^2)*b)*A+(a^2*c+2*a*b^2+(2*a*c+b^2)
*a)*B)*x^8+1/6*(3*B*a^2*b+(a^2*c+2*a*b^2+(2*a*c+b^2)*a)*A)*x^6+1/4*(3*A*a^2
*b+B*a^3)*x^4+1/2*a^3*A*x^2
```

maxima [A] time = 0.77, size = 166, normalized size = 1.00

$$\frac{1}{16}Bc^3x^{16}+\frac{1}{14}\left(3Bbc^2+Ac^3\right)x^{14}+\frac{1}{4}\left(Bb^2c+(Ba+Ab)c^2\right)x^{12}+\frac{1}{10}\left(Bb^3+3Aac^2+3\left(2Bab+Ab^2\right)c\right)x^{10}+\frac{1}{8}\left(3Bab^2+Ab^3+3\left(Ba^2+2Aab\right)c\right)x^8+\frac{1}{2}\left(Ba^2b+Aab^2+Aa^2c\right)x^6+\frac{1}{2}Aa^3x^2+\frac{1}{4}\left(Ba^3+3Aa^2b\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*B*c^3*x^16 + 1/14*(3*B*b*c^2 + A*c^3)*x^14 + 1/4*(B*b^2*c + (B*a + A*b
)*c^2)*x^12 + 1/10*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^10 + 1/8*(
```

```
3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^8 + 1/2*(B*a^2*b + A*a*b^2 + A
*a^2*c)*x^6 + 1/2*A*a^3*x^2 + 1/4*(B*a^3 + 3*A*a^2*b)*x^4
```

mupad [B] time = 0.05, size = 169, normalized size = 1.02

$$x^8\left(\frac{3\,B\,c\,a^2}{8}+\frac{3\,B\,a\,b^2}{8}+\frac{3\,A\,c\,a\,b}{4}+\frac{A\,b^3}{8}\right)+x^{10}\left(\frac{B\,b^3}{10}+\frac{3\,A\,b^2\,c}{10}+\frac{3\,B\,a\,b\,c}{5}+\frac{3\,A\,a\,c^2}{10}\right)+x^4\left(\frac{B\,a^3}{4}+\frac{3\,A\,b\,a^2}{4}\right)+x^{14}\left(\frac{A\,c^3}{14}+\frac{3\,B\,b\,c^2}{14}\right)+x^6\left(\frac{B\,a^2\,b}{2}+\frac{A\,c\,a^2}{2}+\frac{A\,a\,b^2}{2}\right)+x^{12}\left(\frac{B\,b^2\,c}{4}+\frac{A\,b\,c^2}{4}+\frac{B\,a\,c^2}{4}\right)+\frac{A\,a^3\,x^2}{2}+\frac{B\,c^3\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^8*((A*b^3)/8 + (3*B*a*b^2)/8 + (3*B*a^2*c)/8 + (3*A*a*b*c)/4) + x^10*((B*
b^3)/10 + (3*A*a*c^2)/10 + (3*A*b^2*c)/10 + (3*B*a*b*c)/5) + x^4*((B*a^3)/4
 + (3*A*a^2*b)/4) + x^14*((A*c^3)/14 + (3*B*b*c^2)/14) + x^6*((A*a*b^2)/2 +
 (A*a^2*c)/2 + (B*a^2*b)/2) + x^12*((A*b*c^2)/4 + (B*a*c^2)/4 + (B*b^2*c)/4
) + (A*a^3*x^2)/2 + (B*c^3*x^16)/16
```

sympy [A] time = 0.10, size = 199, normalized size = 1.20

$$\frac{Aa^3x^2}{2}+\frac{Bc^3x^{16}}{16}+x^{14}\left(\frac{Ac^3}{14}+\frac{3Bbc^2}{14}\right)+x^{12}\left(\frac{Abc^2}{4}+\frac{Bac^2}{4}+\frac{Bb^2c}{4}\right)+x^{10}\left(\frac{3Aac^2}{10}+\frac{3Ab^2c}{10}+\frac{3Babc}{5}+\frac{Bb^3}{10}\right)+x^8\left(\frac{3Aabc}{4}+\frac{Ab^3}{8}+\frac{3Ba^2c}{8}+\frac{3Bab^2}{8}\right)+x^6\left(\frac{Aa^2c}{2}+\frac{Aab^2}{2}+\frac{Ba^2b}{2}\right)+x^4\left(\frac{3Aa^2b}{4}+\frac{Ba^3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x**2/2 + B*c**3*x**16/16 + x**14*(A*c**3/14 + 3*B*b*c**2/14) + x**12
*(A*b*c**2/4 + B*a*c**2/4 + B*b**2*c/4) + x**10*(3*A*a*c**2/10 + 3*A*b**2*c
/10 + 3*B*a*b*c/5 + B*b**3/10) + x**8*(3*A*a*b*c/4 + A*b**3/8 + 3*B*a**2*c/
8 + 3*B*a*b**2/8) + x**6*(A*a**2*c/2 + A*a*b**2/2 + B*a**2*b/2) + x**4*(3*A
*a**2*b/4 + B*a**3/4)
```

3.76 $\int \left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=161

$$a^3Ax+\frac{1}{3}a^2x^3(aB+3Ab)+\frac{3}{11}cx^{11}\left(aBc+Abc+b^2B\right)+\frac{3}{5}ax^5\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{9}x^9\left(3aAc^2+6abBc+3Ab^2c\right.$$

Rubi [A] time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1153}

$$\frac{1}{3}a^2x^3(aB+3Ab)+a^3Ax+\frac{1}{9}x^9\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{3}{11}cx^{11}\left(aBc+Abc+b^2B\right)+\frac{1}{7}x^7\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{3}{5}ax^5\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{13}c^2x^{13}(Ac+3bB)+\frac{1}{15}Bc^3x^{15}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] a^3*A*x + (a^2*(3*A*b + a*B)*x^3)/3 + (3*a*(a*b*B + A*(b^2 + a*c))*x^5)/5 +
 ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^7)/7 + ((b^3*B + 3*A*b^2*c + 6*
a*b*B*c + 3*a*A*c^2)*x^9)/9 + (3*c*(b^2*B + A*b*c + a*B*c)*x^11)/11 + (c^2*
(3*b*B + A*c)*x^13)/13 + (B*c^3*x^15)/15
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx &= \int \left(a^3A+a^2(3Ab+aB)x^2+3a\left(abB+A\left(b^2+ac\right)\right)x^4+\left(3aB\left(b^2+ac\right)\right.\right.\\ &= a^3Ax+\frac{1}{3}a^2(3Ab+aB)x^3+\frac{3}{5}a\left(abB+A\left(b^2+ac\right)\right)x^5+\frac{1}{7}\left(3aB\left(b^2+ac\right.\right.\end{aligned}$$

Mathematica [A] time = 0.05, size = 161, normalized size = 1.00

$$a^3Ax+\frac{1}{3}a^2x^3(aB+3Ab)+\frac{3}{11}cx^{11}\left(aBc+Abc+b^2B\right)+\frac{3}{5}ax^5\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{9}x^9\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{7}x^7\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{13}c^2x^{13}(Ac+3bB)+\frac{1}{15}Bc^3x^{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] a^3*A*x + (a^2*(3*A*b + a*B)*x^3)/3 + (3*a*(a*b*B + A*(b^2 + a*c))*x^5)/5 +
 ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^7)/7 + ((b^3*B + 3*A*b^2*c + 6*
a*b*B*c + 3*a*A*c^2)*x^9)/9 + (3*c*(b^2*B + A*b*c + a*B*c)*x^11)/11 + (c^2*
(3*b*B + A*c)*x^13)/13 + (B*c^3*x^15)/15
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)*(a + b*x^2 + c*x^4)^3, x]
```

fricas [A] time = 0.76, size = 189, normalized size = 1.17

$$\frac{1}{15}x^{15}c^3B+\frac{3}{13}x^{13}c^2bB+\frac{1}{13}x^{13}c^3A+\frac{3}{11}x^{11}cb^2B+\frac{3}{11}x^{11}c^2aB+\frac{3}{11}x^{11}c^2bA+\frac{1}{9}x^9b^3B+\frac{2}{3}x^9cbaB+\frac{1}{3}x^9cb^2A+\frac{1}{3}x^9c^2aA+\frac{3}{7}x^7b^2aB+\frac{3}{7}x^7ca^2B+\frac{1}{7}x^7b^3A+\frac{6}{7}x^7cbaA+\frac{3}{5}x^5ba^2B+\frac{3}{5}x^5b^2aA+\frac{3}{5}x^5ca^2A+\frac{1}{3}x^3a^3B+x^3ba^2A+xa^3A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*c^3*B + 3/13*x^13*c^2*b*B + 1/13*x^13*c^3*A + 3/11*x^11*c*b^2*B +
 3/11*x^11*c^2*a*B + 3/11*x^11*c^2*b*A + 1/9*x^9*b^3*B + 2/3*x^9*c*b*a*B +
1/3*x^9*c*b^2*A + 1/3*x^9*c^2*a*A + 3/7*x^7*b^2*a*B + 3/7*x^7*c*a^2*B + 1/7
*x^7*b^3*A + 6/7*x^7*c*b*a*A + 3/5*x^5*b*a^2*B + 3/5*x^5*b^2*a*A + 3/5*x^5*
c*a^2*A + 1/3*x^3*a^3*B + x^3*b*a^2*A + x*a^3*A
```

giac [A] time = 0.26, size = 189, normalized size = 1.17

$$\frac{1}{15}Bc^3x^{15}+\frac{3}{13}Bbc^2x^{13}+\frac{1}{13}Ac^3x^{13}+\frac{3}{11}Bb^2cx^{11}+\frac{3}{11}Bac^2x^{11}+\frac{3}{11}Abc^2x^{11}+\frac{1}{9}Bb^3x^9+\frac{2}{3}Babcx^9+\frac{1}{3}Ab^2cx^9+\frac{1}{3}Aac^2x^9+\frac{3}{7}Bab^2x^7+\frac{1}{7}Ab^3x^7+\frac{3}{7}Ba^2cx^7+\frac{6}{7}Aabcx^7+\frac{3}{5}Ba^2bx^5+\frac{3}{5}Aab^2x^5+\frac{3}{5}Aa^2cx^5+\frac{1}{3}Ba^3x^3+Aa^2bx^3+Aa^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/15*B*c^3*x^15 + 3/13*B*b*c^2*x^13 + 1/13*A*c^3*x^13 + 3/11*B*b^2*c*x^11 +
 3/11*B*a*c^2*x^11 + 3/11*A*b*c^2*x^11 + 1/9*B*b^3*x^9 + 2/3*B*a*b*c*x^9 +
1/3*A*b^2*c*x^9 + 1/3*A*a*c^2*x^9 + 3/7*B*a*b^2*x^7 + 1/7*A*b^3*x^7 + 3/7*B
*a^2*c*x^7 + 6/7*A*a*b*c*x^7 + 3/5*B*a^2*b*x^5 + 3/5*A*a*b^2*x^5 + 3/5*A*a^
2*c*x^5 + 1/3*B*a^3*x^3 + A*a^2*b*x^3 + A*a^3*x
```

maple [A] time = 0.00, size = 223, normalized size = 1.39

$$\frac{Bc^3x^{15}}{15}+\frac{\left(Ac^3+3Bbc^2\right)x^{13}}{13}+\frac{\left(3Abc^2+\left(ac^2+2b^2c+\left(2ac+b^2\right)c\right)B\right)x^{11}}{11}+\frac{\left(\left(ac^2+2b^2c+\left(2ac+b^2\right)c\right)A+\left(4abc+\left(2ac+b^2\right)b\right)B\right)x^9}{9}+\frac{\left(\left(4abc+\left(2ac+b^2\right)b\right)A+\left(a^2c+2ab^2+\left(2ac+b^2\right)a\right)B\right)x^7}{7}+Aa^3x+\frac{\left(3Ba^2b+\left(a^2c+2ab^2+\left(2ac+b^2\right)a\right)A\right)x^5}{5}+\frac{\left(3Aa^2b+Ba^3\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/15*B*c^3*x^15+1/13*(A*c^3+3*B*b*c^2)*x^13+1/11*(3*A*b*c^2+(a*c^2+2*b^2*c+
(2*a*c+b^2)*c)*B)*x^11+1/9*((a*c^2+2*b^2*c+(2*a*c+b^2)*c)*A+(4*a*b*c+(2*a*c
+b^2)*b)*B)*x^9+1/7*((4*a*b*c+(2*a*c+b^2)*b)*A+(a^2*c+2*a*b^2+(2*a*c+b^2)*a
)*B)*x^7+1/5*(3*B*a^2*b+(a^2*c+2*a*b^2+(2*a*c+b^2)*a)*A)*x^5+1/3*(3*A*a^2*b
+B*a^3)*x^3+a^3*A*x
```

maxima [A] time = 0.73, size = 163, normalized size = 1.01

$$\frac{1}{15}Bc^3x^{15}+\frac{1}{13}\left(3Bbc^2+Ac^3\right)x^{13}+\frac{3}{11}\left(Bb^2c+(Ba+Ab)c^2\right)x^{11}+\frac{1}{9}\left(Bb^3+3Aac^2+3\left(2Bab+Ab^2\right)c\right)x^9+\frac{1}{7}\left(3Bab^2+Ab^3+3\left(Ba^2+2Aab\right)c\right)x^7+\frac{3}{5}\left(Ba^2b+Aab^2+Aa^2c\right)x^5+Aa^3x+\frac{1}{3}\left(Ba^3+3Aa^2b\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/15*B*c^3*x^15 + 1/13*(3*B*b*c^2 + A*c^3)*x^13 + 3/11*(B*b^2*c + (B*a + A*
b)*c^2)*x^11 + 1/9*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^9 + 1/7*(3
*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^7 + 3/5*(B*a^2*b + A*a*b^2 + A*
a^2*c)*x^5 + A*a^3*x + 1/3*(B*a^3 + 3*A*a^2*b)*x^3
```

mupad [B] time = 0.05, size = 165, normalized size = 1.02

$$x^7\left(\frac{3Bca^2}{7}+\frac{3Bab^2}{7}+\frac{6Acab}{7}+\frac{Ab^3}{7}\right)+x^9\left(\frac{Bb^3}{9}+\frac{Ab^2c}{3}+\frac{2Babc}{3}+\frac{Aac^2}{3}\right)+x^3\left(\frac{Ba^3}{3}+Aba^2\right)+x^{13}\left(\frac{Ac^3}{13}+\frac{3Bbc^2}{13}\right)+x^5\left(\frac{3Ba^2b}{5}+\frac{3Aca^2}{5}+\frac{3Aab^2}{5}\right)+x^{11}\left(\frac{3Bb^2c}{11}+\frac{3Abc^2}{11}+\frac{3Bac^2}{11}\right)+\frac{Bc^3x^{15}}{15}+Aa^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^7*((A*b^3)/7 + (3*B*a*b^2)/7 + (3*B*a^2*c)/7 + (6*A*a*b*c)/7) + x^9*((B*b
^3)/9 + (A*a*c^2)/3 + (A*b^2*c)/3 + (2*B*a*b*c)/3) + x^3*((B*a^3)/3 + A*a^2
*b) + x^13*((A*c^3)/13 + (3*B*b*c^2)/13) + x^5*((3*A*a*b^2)/5 + (3*A*a^2*c)
/5 + (3*B*a^2*b)/5) + x^11*((3*A*b*c^2)/11 + (3*B*a*c^2)/11 + (3*B*b^2*c)/1
1) + (B*c^3*x^15)/15 + A*a^3*x
```

sympy [A] time = 0.10, size = 199, normalized size = 1.24

$$Aa^3x + \frac{Bc^3x^{15}}{15} + x^{13}\left(\frac{Ac^3}{13} + \frac{3Bbc^2}{13}\right) + x^{11}\left(\frac{3Abc^2}{11} + \frac{3Bac^2}{11} + \frac{3Bb^2c}{11}\right) + x^9\left(\frac{Aac^2}{3} + \frac{Ab^2c}{3} + \frac{2Babc}{3} + \frac{Bb^3}{9}\right) + x^7\left(\frac{6Aabc}{7} + \frac{Ab^3}{7} + \frac{3Ba^2c}{7} + \frac{3Bab^2}{7}\right) + x^5\left(\frac{3Aa^2c}{5} + \frac{3Aab^2}{5} + \frac{3Ba^2b}{5}\right) + x^3\left(Aa^2b + \frac{Ba^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x + B*c**3*x**15/15 + x**13*(A*c**3/13 + 3*B*b*c**2/13) + x**11*(3*A
*b*c**2/11 + 3*B*a*c**2/11 + 3*B*b**2*c/11) + x**9*(A*a*c**2/3 + A*b**2*c/3
 + 2*B*a*b*c/3 + B*b**3/9) + x**7*(6*A*a*b*c/7 + A*b**3/7 + 3*B*a**2*c/7 +
3*B*a*b**2/7) + x**5*(3*A*a**2*c/5 + 3*A*a*b**2/5 + 3*B*a**2*b/5) + x**3*(A
*a**2*b + B*a**3/3)
```

3.77 $\int \frac{\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3}{x}\,dx$

Optimal. Leaf size=162

$$a^3A\log(x)+\frac{1}{2}a^2x^2(aB+3Ab)+\frac{3}{10}cx^{10}\left(aBc+Abc+b^2B\right)+\frac{3}{4}ax^4\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{8}x^8\left(3aAc^2+6abBc+3A\right.$$

Rubi [A] time = 0.23, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1251, 765}

$$\frac{1}{2}a^2x^2(aB+3Ab)+a^3A\log(x)+\frac{1}{8}x^8\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{3}{10}cx^{10}\left(aBc+Abc+b^2B\right)+\frac{1}{6}x^6\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{3}{4}ax^4\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{12}c^2x^{12}(Ac+3bB)+\frac{1}{14}Bc^3x^{14}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x]
```

```
[Out] (a^2*(3*A*b + a*B)*x^2)/2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^4)/4 + ((3*a*B*(
b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6)/6 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c +
3*a*A*c^2)*x^8)/8 + (3*c*(b^2*B + A*b*c + a*B*c)*x^10)/10 + (c^2*(3*b*B + A
*c)*x^12)/12 + (B*c^3*x^14)/14 + a^3*A*Log[x]
```

Rule 765

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + b*x + c
*x^2)^p, x], x] /; FreeQ[{a, b, c, e, f, g, m}, x] && IntegerQ[p] && (GtQ[p
, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(a^2(3Ab+aB)+\frac{a^3A}{x}+3a\left(abB+A\left(b^2+ac\right)\right)x+\left(3aB\left(b^2+\right.\right.\right.\right.\\ &= \frac{1}{2}a^2(3Ab+aB)x^2+\frac{3}{4}a\left(abB+A\left(b^2+ac\right)\right)x^4+\frac{1}{6}\left(3aB\left(b^2+ac\right)+A\left(b^3+\right.\right.\end{aligned}$$

Mathematica [A] time = 0.06, size = 162, normalized size = 1.00

$$a^3A\log(x)+\frac{1}{2}a^2x^2(aB+3Ab)+\frac{3}{10}cx^{10}\left(aBc+Abc+b^2B\right)+\frac{3}{4}ax^4\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{8}x^8\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{6}x^6\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{12}c^2x^{12}(Ac+3bB)+\frac{1}{14}Bc^3x^{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x]
```

```
[Out] (a^2*(3*A*b + a*B)*x^2)/2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^4)/4 + ((3*a*B*(
b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6)/6 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c +
```

```
3*a*A*c^2)*x^8)/8 + (3*c*(b^2*B + A*b*c + a*B*c)*x^10)/10 + (c^2*(3*b*B + A
*c)*x^12)/12 + (B*c^3*x^14)/14 + a^3*A*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(a + bx^2 + cx^4)^3}{x} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x]
```

```
[Out] IntegrateAlgebraic[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x, x]
```

fricas [A] time = 0.82, size = 164, normalized size = 1.01

$$\frac{1}{14} Bc^3x^{14} + \frac{1}{12}(3Bbc^2 + Ac^3)x^{12} + \frac{3}{10}(Bb^2c + (Ba + Ab)c^2)x^{10} + \frac{1}{8}(Bb^3 + 3Aac^2 + 3(2Bab + Ab^2)c)x^8 + \frac{1}{6}(3Bab^2 + Ab^3 + 3(Ba^2 + 2Aab)c)x^6 + \frac{3}{4}(Ba^2b + Aab^2 + Aa^2c)x^4 + Aa^3\log(x) + \frac{1}{2}(Ba^3 + 3Aa^2b)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/14*B*c^3*x^14 + 1/12*(3*B*b*c^2 + A*c^3)*x^12 + 3/10*(B*b^2*c + (B*a + A*
b)*c^2)*x^10 + 1/8*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^8 + 1/6*(3
*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 3/4*(B*a^2*b + A*a*b^2 + A*
a^2*c)*x^4 + A*a^3*log(x) + 1/2*(B*a^3 + 3*A*a^2*b)*x^2
```

giac [A] time = 0.34, size = 193, normalized size = 1.19

$$\frac{1}{14} Bc^3x^{14} + \frac{1}{4} Bbc^2x^{12} + \frac{1}{12} Ac^3x^{12} + \frac{3}{10} Bb^2cx^{10} + \frac{3}{10} Bac^2x^{10} + \frac{3}{10} Abc^2x^{10} + \frac{1}{8} Bb^3x^8 + \frac{3}{4} Babcx^8 + \frac{3}{8} Ab^2cx^8 + \frac{3}{8} Aac^2x^8 + \frac{1}{2} Bab^2x^6 + \frac{1}{6} Ab^3x^6 + \frac{1}{2} Ba^2cx^6 + Aabcx^6 + \frac{3}{4} Ba^2bx^4 + \frac{3}{4} Aab^2x^4 + \frac{3}{4} Aa^2cx^4 + \frac{1}{2} Ba^3x^2 + \frac{3}{2} Aa^2bx^2 + \frac{1}{2} Aa^3\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/14*B*c^3*x^14 + 1/4*B*b*c^2*x^12 + 1/12*A*c^3*x^12 + 3/10*B*b^2*c*x^10 +
3/10*B*a*c^2*x^10 + 3/10*A*b*c^2*x^10 + 1/8*B*b^3*x^8 + 3/4*B*a*b*c*x^8 + 3
/8*A*b^2*c*x^8 + 3/8*A*a*c^2*x^8 + 1/2*B*a*b^2*x^6 + 1/6*A*b^3*x^6 + 1/2*B*
a^2*c*x^6 + A*a*b*c*x^6 + 3/4*B*a^2*b*x^4 + 3/4*A*a*b^2*x^4 + 3/4*A*a^2*c*x
^4 + 1/2*B*a^3*x^2 + 3/2*A*a^2*b*x^2 + 1/2*A*a^3*log(x^2)
```

maple [A] time = 0.00, size = 191, normalized size = 1.18

$$\frac{Bc^3x^{14}}{14} + \frac{Ac^3x^{12}}{12} + \frac{Bbc^2x^{12}}{4} + \frac{3Abc^2x^{10}}{10} + \frac{3Bac^2x^{10}}{10} + \frac{3Bb^2cx^{10}}{10} + \frac{3Aac^2x^8}{8} + \frac{3Ab^2cx^8}{8} + \frac{3Babcx^8}{4} + \frac{Bb^3x^8}{8} + Aabcx^6 + \frac{Ab^3x^6}{6} + \frac{Ba^2cx^6}{2} + \frac{Bab^2x^6}{2} + \frac{3Aa^2cx^4}{4} + \frac{3Aab^2x^4}{4} + \frac{3Ba^2bx^4}{4} + \frac{3Aa^2bx^2}{2} + \frac{Ba^3x^2}{2} + Aa^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x)
```

```
[Out] 1/14*B*c^3*x^14+1/12*A*x^12*c^3+1/4*B*x^12*b*c^2+3/10*A*x^10*b*c^2+3/10*B*x
^10*a*c^2+3/10*B*x^10*b^2*c+3/8*A*x^8*a*c^2+3/8*A*x^8*b^2*c+3/4*B*x^8*a*b*c
+1/8*B*x^8*b^3+A*x^6*a*b*c+1/6*A*x^6*b^3+1/2*B*x^6*a^2*c+1/2*B*x^6*a*b^2+3/
4*A*x^4*a^2*c+3/4*A*x^4*a*b^2+3/4*B*x^4*a^2*b+3/2*A*x^2*a^2*b+1/2*B*x^2*a^3
+a^3*A*ln(x)
```

maxima [A] time = 0.79, size = 167, normalized size = 1.03

$$\frac{1}{14} Bc^3x^{14} + \frac{1}{12}(3Bbc^2 + Ac^3)x^{12} + \frac{3}{10}(Bb^2c + (Ba + Ab)c^2)x^{10} + \frac{1}{8}(Bb^3 + 3Aac^2 + 3(2Bab + Ab^2)c)x^8 + \frac{1}{6}(3Bab^2 + Ab^3 + 3(Ba^2 + 2Aab)c)x^6 + \frac{3}{4}(Ba^2b + Aab^2 + Aa^2c)x^4 + \frac{1}{2} Aa^3\log(x^2) + \frac{1}{2}(Ba^3 + 3Aa^2b)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/14*B*c^3*x^14 + 1/12*(3*B*b*c^2 + A*c^3)*x^12 + 3/10*(B*b^2*c + (B*a + A*
b)*c^2)*x^10 + 1/8*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^8 + 1/6*(3
*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 3/4*(B*a^2*b + A*a*b^2 + A*
a^2*c)*x^4 + 1/2*A*a^3*log(x^2) + 1/2*(B*a^3 + 3*A*a^2*b)*x^2
```

mupad [B] time = 0.10, size = 166, normalized size = 1.02

$$x^6\left(\frac{B\,c\,a^2}{2}+\frac{B\,a\,b^2}{2}+A\,c\,a\,b+\frac{A\,b^3}{6}\right)+x^8\left(\frac{B\,b^3}{8}+\frac{3\,A\,b^2\,c}{8}+\frac{3\,B\,a\,b\,c}{4}+\frac{3\,A\,a\,c^2}{8}\right)+x^2\left(\frac{B\,a^3}{2}+\frac{3\,A\,b\,a^2}{2}\right)+x^{12}\left(\frac{A\,c^3}{12}+\frac{B\,b\,c^2}{4}\right)+x^4\left(\frac{3\,B\,a^2\,b}{4}+\frac{3\,A\,c\,a^2}{4}+\frac{3\,A\,a\,b^2}{4}\right)+x^{10}\left(\frac{3\,B\,b^2\,c}{10}+\frac{3\,A\,b\,c^2}{10}+\frac{3\,B\,a\,c^2}{10}\right)+\frac{B\,c^3\,x^{14}}{14}+A\,a^3\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x)
```

```
[Out] x^6*((A*b^3)/6 + (B*a*b^2)/2 + (B*a^2*c)/2 + A*a*b*c) + x^8*((B*b^3)/8 + (3
*A*a*c^2)/8 + (3*A*b^2*c)/8 + (3*B*a*b*c)/4) + x^2*((B*a^3)/2 + (3*A*a^2*b)
/2) + x^12*((A*c^3)/12 + (B*b*c^2)/4) + x^4*((3*A*a*b^2)/4 + (3*A*a^2*c)/4
+ (3*B*a^2*b)/4) + x^10*((3*A*b*c^2)/10 + (3*B*a*c^2)/10 + (3*B*b^2*c)/10)
+ (B*c^3*x^14)/14 + A*a^3*log(x)
```

sympy [A] time = 0.31, size = 199, normalized size = 1.23

$$Aa^3\log(x)+\frac{Bc^3x^{14}}{14}+x^{12}\left(\frac{Ac^3}{12}+\frac{Bbc^2}{4}\right)+x^{10}\left(\frac{3Abc^2}{10}+\frac{3Bac^2}{10}+\frac{3Bb^2c}{10}\right)+x^8\left(\frac{3Aac^2}{8}+\frac{3Ab^2c}{8}+\frac{3Babc}{4}+\frac{Bb^3}{8}\right)+x^6\left(Aabc+\frac{Ab^3}{6}+\frac{Ba^2c}{2}+\frac{Bab^2}{2}\right)+x^4\left(\frac{3Aa^2c}{4}+\frac{3Aab^2}{4}+\frac{3Ba^2b}{4}\right)+x^2\left(\frac{3Aa^2b}{2}+\frac{Ba^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x,x)
```

```
[Out] A*a**3*log(x) + B*c**3*x**14/14 + x**12*(A*c**3/12 + B*b*c**2/4) + x**10*(3
*A*b*c**2/10 + 3*B*a*c**2/10 + 3*B*b**2*c/10) + x**8*(3*A*a*c**2/8 + 3*A*b*
*2*c/8 + 3*B*a*b*c/4 + B*b**3/8) + x**6*(A*a*b*c + A*b**3/6 + B*a**2*c/2 +
B*a*b**2/2) + x**4*(3*A*a**2*c/4 + 3*A*a*b**2/4 + 3*B*a**2*b/4) + x**2*(3*A
*a**2*b/2 + B*a**3/2)
```

3.78 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2}\,dx$

Optimal. Leaf size=156

$$-\frac{a^3A}{x}+a^2x(aB+3Ab)+\frac{1}{3}cx^9\left(aBc+Abc+b^2B\right)+ax^3\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{7}x^7\left(3aAc^2+6abBc+3Ab^2c+b^3\right.$$

Rubi [A] time = 0.11, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1261}

$$a^2x(aB+3Ab)-\frac{a^3A}{x}+\frac{1}{7}x^7\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{3}cx^9\left(aBc+Abc+b^2B\right)+\frac{1}{5}x^5\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+ax^3\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{11}c^2x^{11}(Ac+3bB)+\frac{1}{13}Bc^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2,x]
```

```
[Out] -((a^3*A)/x) + a^2*(3*A*b + a*B)*x + a*(a*b*B + A*(b^2 + a*c))*x^3 + ((3*a*
B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^5)/5 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c
 + 3*a*A*c^2)*x^7)/7 + (c*(b^2*B + A*b*c + a*B*c)*x^9)/3 + (c^2*(3*b*B + A*
c)*x^11)/11 + (B*c^3*x^13)/13
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3}{x^2}\,dx=\int \left(a^2(3Ab+aB)+\frac{a^3A}{x^2}+3a\left(abB+A\left(b^2+ac\right)\right)x^2+\left(3aB\left(b^2+ac\right)+\right.\right.$$
$$=-\frac{a^3A}{x}+a^2(3Ab+aB)x+a\left(abB+A\left(b^2+ac\right)\right)x^3+\frac{1}{5}\left(3aB\left(b^2+ac\right)+\right.$$

Mathematica [A] time = 0.08, size = 156, normalized size = 1.00

$$-\frac{a^3A}{x}+a^2x(aB+3Ab)+\frac{1}{3}cx^9\left(aBc+Abc+b^2B\right)+ax^3\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{7}x^7\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{5}x^5\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{11}c^2x^{11}(Ac+3bB)+\frac{1}{13}Bc^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2,x]
```

```
[Out] -((a^3*A)/x) + a^2*(3*A*b + a*B)*x + a*(a*b*B + A*(b^2 + a*c))*x^3 + ((3*a*
B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^5)/5 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c
 + 3*a*A*c^2)*x^7)/7 + (c*(b^2*B + A*b*c + a*B*c)*x^9)/3 + (c^2*(3*b*B + A*
c)*x^11)/11 + (B*c^3*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2,x]
```

```
[Out] IntegrateAlgebraic[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2, x]
```

fricas [A] time = 0.96, size = 168, normalized size = 1.08

$$\frac{1155\,Bc^3x^{14}+1365\,(3\,Bbc^2+Ac^3)x^{12}+5005\,(Bb^2c+(Ba+Ab)c^2)x^{10}+2145\,(Bb^3+3\,Aac^2+3\,(2\,Bab+Ab^2)c)x^8+3003\,(3\,Bab^2+Ab^3+3\,(Ba^2+2\,Aab)c)x^6+15015\,(Ba^2b+Aab^2+Aa^2c)x^4-15015\,Aa^3+15015\,(Ba^3+3\,Aa^2b)x^2}{15015\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/15015*(1155*B*c^3*x^14 + 1365*(3*B*b*c^2 + A*c^3)*x^12 + 5005*(B*b^2*c +
(B*a + A*b)*c^2)*x^10 + 2145*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^
8 + 3003*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 15015*(B*a^2*b +
 A*a*b^2 + A*a^2*c)*x^4 - 15015*A*a^3 + 15015*(B*a^3 + 3*A*a^2*b)*x^2)/x
```

giac [A] time = 0.31, size = 185, normalized size = 1.19

$$\frac{1}{13}\,Bc^3x^{13}+\frac{3}{11}\,Bbc^2x^{11}+\frac{1}{11}\,Ac^3x^{11}+\frac{1}{3}\,Bb^2cx^9+\frac{1}{3}\,Bac^2x^9+\frac{1}{3}\,Abc^2x^9+\frac{1}{7}\,Bb^3x^7+\frac{6}{7}\,Babcx^7+\frac{3}{7}\,Ab^2cx^7+\frac{3}{7}\,Aac^2x^7+\frac{3}{5}\,Bab^2x^5+\frac{1}{5}\,Ab^3x^5+\frac{3}{5}\,Ba^2cx^5+\frac{6}{5}\,Aabcx^5+Ba^2bx^3+Aab^2x^3+Aa^2cx^3+Ba^3x+3\,Aa^2bx-\frac{Aa^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/13*B*c^3*x^13 + 3/11*B*b*c^2*x^11 + 1/11*A*c^3*x^11 + 1/3*B*b^2*c*x^9 + 1
/3*B*a*c^2*x^9 + 1/3*A*b*c^2*x^9 + 1/7*B*b^3*x^7 + 6/7*B*a*b*c*x^7 + 3/7*A*
b^2*c*x^7 + 3/7*A*a*c^2*x^7 + 3/5*B*a*b^2*x^5 + 1/5*A*b^3*x^5 + 3/5*B*a^2*c
*x^5 + 6/5*A*a*b*c*x^5 + B*a^2*b*x^3 + A*a*b^2*x^3 + A*a^2*c*x^3 + B*a^3*x
+ 3*A*a^2*b*x - A*a^3/x
```

maple [A] time = 0.00, size = 186, normalized size = 1.19

$$\frac{Bc^3x^{13}}{13}+\frac{Ac^3x^{11}}{11}+\frac{3Bbc^2x^{11}}{11}+\frac{Abc^2x^9}{3}+\frac{Bac^2x^9}{3}+\frac{Bb^2cx^9}{3}+\frac{3Aac^2x^7}{7}+\frac{3Ab^2cx^7}{7}+\frac{6Babcx^7}{7}+\frac{Bb^3x^7}{7}+\frac{6Aabcx^5}{5}+\frac{Ab^3x^5}{5}+\frac{3Ba^2cx^5}{5}+\frac{3Bab^2x^5}{5}+Aa^2cx^3+Aab^2x^3+Ba^2bx^3+3Aa^2bx+Ba^3x-\frac{Aa^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x)
```

```
[Out] 1/13*B*c^3*x^13+1/11*A*x^11*c^3+3/11*B*x^11*b*c^2+1/3*A*x^9*b*c^2+1/3*B*x^9
*a*c^2+1/3*B*x^9*b^2*c+3/7*A*x^7*a*c^2+3/7*A*x^7*b^2*c+6/7*B*x^7*a*b*c+1/7*
B*x^7*b^3+6/5*A*x^5*a*b*c+1/5*A*x^5*b^3+3/5*B*x^5*a^2*c+3/5*B*x^5*a*b^2+A*x
^3*a^2*c+A*x^3*a*b^2+B*x^3*a^2*b+3*A*a^2*b*x+B*a^3*x-a^3*A/x
```

maxima [A] time = 0.71, size = 162, normalized size = 1.04

$$\frac{1}{13}\,Bc^3x^{13}+\frac{1}{11}\,(3\,Bbc^2+Ac^3)x^{11}+\frac{1}{3}\,(Bb^2c+(Ba+Ab)c^2)x^9+\frac{1}{7}\,(Bb^3+3\,Aac^2+3\,(2\,Bab+Ab^2)c)x^7+\frac{1}{5}\,(3\,Bab^2+Ab^3+3\,(Ba^2+2\,Aab)c)x^5+(Ba^2b+Aab^2+Aa^2c)x^3-\frac{Aa^3}{x}+(Ba^3+3\,Aa^2b)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/13*B*c^3*x^13 + 1/11*(3*B*b*c^2 + A*c^3)*x^11 + 1/3*(B*b^2*c + (B*a + A*b
)*c^2)*x^9 + 1/7*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^7 + 1/5*(3*B
*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^5 + (B*a^2*b + A*a*b^2 + A*a^2*c)
*x^3 - A*a^3/x + (B*a^3 + 3*A*a^2*b)*x
```

mupad [B] time = 0.05, size = 163, normalized size = 1.04

$$x^5\left(\frac{3\,B\,c\,a^2}{5}+\frac{3\,B\,a\,b^2}{5}+\frac{6\,A\,c\,a\,b}{5}+\frac{A\,b^3}{5}\right)+x^7\left(\frac{B\,b^3}{7}+\frac{3\,A\,b^2\,c}{7}+\frac{6\,B\,a\,b\,c}{7}+\frac{3\,A\,a\,c^2}{7}\right)+x\,\left(B\,a^3+3\,A\,b\,a^2\right)+x^{11}\left(\frac{A\,c^3}{11}+\frac{3\,B\,b\,c^2}{11}\right)+x^3\,\left(B\,a^2\,b+A\,c\,a^2+A\,a\,b^2\right)+x^9\left(\frac{B\,b^2\,c}{3}+\frac{A\,b\,c^2}{3}+\frac{B\,a\,c^2}{3}\right)-\frac{A\,a^3}{x}+\frac{B\,c^3\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2,x)
```

```
[Out] x^5*((A*b^3)/5 + (3*B*a*b^2)/5 + (3*B*a^2*c)/5 + (6*A*a*b*c)/5) + x^7*((B*b
^3)/7 + (3*A*a*c^2)/7 + (3*A*b^2*c)/7 + (6*B*a*b*c)/7) + x*(B*a^3 + 3*A*a^2
*b) + x^11*((A*c^3)/11 + (3*B*b*c^2)/11) + x^3*(A*a*b^2 + A*a^2*c + B*a^2*b
) + x^9*((A*b*c^2)/3 + (B*a*c^2)/3 + (B*b^2*c)/3) - (A*a^3)/x + (B*c^3*x^13
)/13
```

sympy [A] time = 0.31, size = 185, normalized size = 1.19

$$-\frac{Aa^3}{x}+\frac{Bc^3x^{13}}{13}+x^{11}\left(\frac{Ac^3}{11}+\frac{3Bbc^2}{11}\right)+x^9\left(\frac{Abc^2}{3}+\frac{Bac^2}{3}+\frac{Bb^2c}{3}\right)+x^7\left(\frac{3Aac^2}{7}+\frac{3Ab^2c}{7}+\frac{6Babc}{7}+\frac{Bb^3}{7}\right)+x^5\left(\frac{6Aabc}{5}+\frac{Ab^3}{5}+\frac{3Ba^2c}{5}+\frac{3Bab^2}{5}\right)+x^3\left(Aa^2c+Aab^2+Ba^2b\right)+x\left(3Aa^2b+Ba^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x**2,x)
```

```
[Out] -A*a**3/x + B*c**3*x**13/13 + x**11*(A*c**3/11 + 3*B*b*c**2/11) + x**9*(A*b
*c**2/3 + B*a*c**2/3 + B*b**2*c/3) + x**7*(3*A*a*c**2/7 + 3*A*b**2*c/7 + 6*
B*a*b*c/7 + B*b**3/7) + x**5*(6*A*a*b*c/5 + A*b**3/5 + 3*B*a**2*c/5 + 3*B*a
*b**2/5) + x**3*(A*a**2*c + A*a*b**2 + B*a**2*b) + x*(3*A*a**2*b + B*a**3)
```

3.79 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^3} dx$

Optimal. Leaf size=162

$$-\frac{a^3A}{2x^2}+a^2\log(x)(aB+3Ab)+\frac{3}{8}cx^8\left(aBc+Abc+b^2B\right)+\frac{3}{2}ax^2\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{6}x^6\left(3aAc^2+6abBc+3Ab^2c\right.$$

Rubi [A] time = 0.23, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1251, 765}

$$a^2\log(x)(aB+3Ab)-\frac{a^3A}{2x^2}+\frac{1}{6}x^6\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{3}{8}cx^8\left(aBc+Abc+b^2B\right)+\frac{1}{4}x^4\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{3}{2}ax^2\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{10}c^2x^{10}(Ac+3bB)+\frac{1}{12}Bc^3x^{12}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x]
```

```
[Out] -(a^3*A)/(2*x^2) + (3*a*(a*b*B + A*(b^2 + a*c))*x^2)/2 + ((3*a*B*(b^2 + a*c
) + A*(b^3 + 6*a*b*c))*x^4)/4 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2
)*x^6)/6 + (3*c*(b^2*B + A*b*c + a*B*c)*x^8)/8 + (c^2*(3*b*B + A*c)*x^10)/1
0 + (B*c^3*x^12)/12 + a^2*(3*A*b + a*B)*Log[x]
```

Rule 765

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + b*x + c
*x^2)^p, x], x] /; FreeQ[{a, b, c, e, f, g, m}, x] && IntegerQ[p] && (GtQ[p
, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(3a\left(abB+A\left(b^2+ac\right)\right)+\frac{a^3A}{x^2}+\frac{a^2(3Ab+aB)}{x}+\left(3aB\left(b^2+a\right.\right.\right.\right.\\
&= -\frac{a^3A}{2x^2}+\frac{3}{2}a\left(abB+A\left(b^2+ac\right)\right)x^2+\frac{1}{4}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^4+
\end{aligned}$$

Mathematica [A] time = 0.07, size = 162, normalized size = 1.00

$$-\frac{a^3A}{2x^2}+a^2\log(x)(aB+3Ab)+\frac{3}{8}cx^8\left(aBc+Abc+b^2B\right)+\frac{3}{2}ax^2\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{6}x^6\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{4}x^4\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{10}c^2x^{10}(Ac+3bB)+\frac{1}{12}Bc^3x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x]
```

```
[Out] -1/2*(a^3*A)/x^2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^2)/2 + ((3*a*B*(b^2 + a*c
) + A*(b^3 + 6*a*b*c))*x^4)/4 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2
```

```
)*x^6)/6 + (3*c*(b^2*B + A*b*c + a*B*c)*x^8)/8 + (c^2*(3*b*B + A*c)*x^10)/1
0 + (B*c^3*x^12)/12 + a^2*(3*A*b + a*B)*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x]
```

```
[Out] IntegrateAlgebraic[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3, x]
```

fricas [A] time = 0.84, size = 170, normalized size = 1.05

$$\frac{10\,Bc^3x^{14}+12\left(3\,Bbc^2+Ac^3\right)x^{12}+45\left(Bb^2c+(Ba+Ab)c^2\right)x^{10}+20\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^8+30\left(3\,Bab^2+Ab^3+3\left(Ba^2+2\,Aab\right)c\right)x^6+180\left(Ba^2b+Aab^2+Aa^2c\right)x^4-60\,Aa^3+120\left(Ba^3+3\,Aa^2b\right)x^2\log(x)}{120\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/120*(10*B*c^3*x^14 + 12*(3*B*b*c^2 + A*c^3)*x^12 + 45*(B*b^2*c + (B*a + A
*b)*c^2)*x^10 + 20*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^8 + 30*(3*
B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 180*(B*a^2*b + A*a*b^2 + A*a
^2*c)*x^4 - 60*A*a^3 + 120*(B*a^3 + 3*A*a^2*b)*x^2*log(x))/x^2
```

giac [A] time = 0.40, size = 212, normalized size = 1.31

$$\frac{1}{12}\,Bc^3x^{12}+\frac{3}{10}\,Bbc^2x^{10}+\frac{1}{10}\,Ac^3x^{10}+\frac{3}{8}\,Bb^2cx^8+\frac{3}{8}\,Bac^2x^8+\frac{3}{8}\,Abc^2x^8+\frac{1}{6}\,Bb^3x^6+Babcx^6+\frac{1}{2}\,Ab^2cx^6+\frac{1}{2}\,Aac^2x^6+\frac{3}{4}\,Bab^2x^4+\frac{1}{4}\,Ab^3x^4+\frac{3}{4}\,Ba^2cx^4+\frac{3}{2}\,Aabcx^4+\frac{3}{2}\,Ba^2bx^2+\frac{3}{2}\,Aab^2x^2+\frac{3}{2}\,Aa^2cx^2+\frac{1}{2}\left(Ba^3+3\,Aa^2b\right)\log\left(x^2\right)-\frac{Ba^3x^2+3\,Aa^2bx^2+Aa^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/12*B*c^3*x^12 + 3/10*B*b*c^2*x^10 + 1/10*A*c^3*x^10 + 3/8*B*b^2*c*x^8 + 3
/8*B*a*c^2*x^8 + 3/8*A*b*c^2*x^8 + 1/6*B*b^3*x^6 + B*a*b*c*x^6 + 1/2*A*b^2*
c*x^6 + 1/2*A*a*c^2*x^6 + 3/4*B*a*b^2*x^4 + 1/4*A*b^3*x^4 + 3/4*B*a^2*c*x^4
 + 3/2*A*a*b*c*x^4 + 3/2*B*a^2*b*x^2 + 3/2*A*a*b^2*x^2 + 3/2*A*a^2*c*x^2 +
1/2*(B*a^3 + 3*A*a^2*b)*log(x^2) - 1/2*(B*a^3*x^2 + 3*A*a^2*b*x^2 + A*a^3)/
x^2
```

maple [A] time = 0.01, size = 190, normalized size = 1.17

$$\frac{B\,c^3x^{12}}{12}+\frac{A\,c^3x^{10}}{10}+\frac{3Bb\,c^2x^{10}}{10}+\frac{3Ab\,c^2x^8}{8}+\frac{3Ba\,c^2x^8}{8}+\frac{3B\,b^2c\,x^8}{8}+\frac{Aa\,c^2x^6}{2}+\frac{A\,b^2c\,x^6}{2}+Babc\,x^6+\frac{B\,b^3x^6}{6}+\frac{3Aabc\,x^4}{2}+\frac{A\,b^3x^4}{4}+\frac{3B\,a^2c\,x^4}{4}+\frac{3Ba\,b^2x^4}{4}+\frac{3A\,a^2c\,x^2}{2}+\frac{3Aa\,b^2x^2}{2}+\frac{3B\,a^2b\,x^2}{2}+3A\,a^2b\ln(x)+B\,a^3\ln(x)-\frac{A\,a^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x)
```

```
[Out] 1/12*B*c^3*x^12+1/10*A*x^10*c^3+3/10*B*x^10*b*c^2+3/8*A*x^8*b*c^2+3/8*B*x^8
*a*c^2+3/8*B*x^8*b^2*c+1/2*A*x^6*a*c^2+1/2*A*x^6*b^2*c+B*x^6*a*b*c+1/6*B*x^
6*b^3+3/2*A*x^4*a*b*c+1/4*A*x^4*b^3+3/4*B*x^4*a^2*c+3/4*B*x^4*a*b^2+3/2*A*x
^2*a^2*c+3/2*A*x^2*a*b^2+3/2*B*x^2*a^2*b+3*A*ln(x)*a^2*b+B*ln(x)*a^3-1/2*a^
3*A/x^2
```

maxima [A] time = 0.79, size = 167, normalized size = 1.03

$$\frac{1}{12}\,Bc^3x^{12}+\frac{1}{10}\left(3\,Bbc^2+Ac^3\right)x^{10}+\frac{3}{8}\left(Bb^2c+(Ba+Ab)c^2\right)x^8+\frac{1}{6}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^6+\frac{1}{4}\left(3\,Bab^2+Ab^3+3\left(Ba^2+2\,Aab\right)c\right)x^4+\frac{3}{2}\left(Ba^2b+Aab^2+Aa^2c\right)x^2-\frac{Aa^3}{2\,x^2}+\frac{1}{2}\left(Ba^3+3\,Aa^2b\right)\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*B*c^3*x^12 + 1/10*(3*B*b*c^2 + A*c^3)*x^10 + 3/8*(B*b^2*c + (B*a + A*b
)*c^2)*x^8 + 1/6*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^6 + 1/4*(3*B
*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^4 + 3/2*(B*a^2*b + A*a*b^2 + A*a^
2*c)*x^2 - 1/2*A*a^3/x^2 + 1/2*(B*a^3 + 3*A*a^2*b)*log(x^2)
```

mupad [B] time = 0.06, size = 166, normalized size = 1.02

$$x^4\left(\frac{3Bca^2}{4}+\frac{3Bab^2}{4}+\frac{3Acab}{2}+\frac{Ab^3}{4}\right)+x^6\left(\frac{Bb^3}{6}+\frac{Ab^2c}{2}+Babc+\frac{Aac^2}{2}\right)+x^{10}\left(\frac{Ac^3}{10}+\frac{3Bbc^2}{10}\right)+\ln(x)\left(Ba^3+3Aba^2\right)+x^2\left(\frac{3Ba^2b}{2}+\frac{3Aca^2}{2}+\frac{3Aab^2}{2}\right)+x^8\left(\frac{3Bb^2c}{8}+\frac{3Abc^2}{8}+\frac{3Bac^2}{8}\right)-\frac{Aa^3}{2x^2}+\frac{Bc^3x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x)
```

```
[Out] x^4*((A*b^3)/4 + (3*B*a*b^2)/4 + (3*B*a^2*c)/4 + (3*A*a*b*c)/2) + x^6*((B*b
^3)/6 + (A*a*c^2)/2 + (A*b^2*c)/2 + B*a*b*c) + x^10*((A*c^3)/10 + (3*B*b*c^
2)/10) + log(x)*(B*a^3 + 3*A*a^2*b) + x^2*((3*A*a*b^2)/2 + (3*A*a^2*c)/2 +
(3*B*a^2*b)/2) + x^8*((3*A*b*c^2)/8 + (3*B*a*c^2)/8 + (3*B*b^2*c)/8) - (A*a
^3)/(2*x^2) + (B*c^3*x^12)/12
```

sympy [A] time = 0.40, size = 197, normalized size = 1.22

$$-\frac{Aa^3}{2x^2}+\frac{Bc^3x^{12}}{12}+a^2(3Ab+Ba)\log(x)+x^{10}\left(\frac{Ac^3}{10}+\frac{3Bbc^2}{10}\right)+x^8\left(\frac{3Abc^2}{8}+\frac{3Bac^2}{8}+\frac{3Bb^2c}{8}\right)+x^6\left(\frac{Aac^2}{2}+\frac{Ab^2c}{2}+Babc+\frac{Bb^3}{6}\right)+x^4\left(\frac{3Aabc}{2}+\frac{Ab^3}{4}+\frac{3Ba^2c}{4}+\frac{3Bab^2}{4}\right)+x^2\left(\frac{3Aa^2c}{2}+\frac{3Aab^2}{2}+\frac{3Ba^2b}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x**3,x)
```

```
[Out] -A*a**3/(2*x**2) + B*c**3*x**12/12 + a**2*(3*A*b + B*a)*log(x) + x**10*(A*c
**3/10 + 3*B*b*c**2/10) + x**8*(3*A*b*c**2/8 + 3*B*a*c**2/8 + 3*B*b**2*c/8)
 + x**6*(A*a*c**2/2 + A*b**2*c/2 + B*a*b*c + B*b**3/6) + x**4*(3*A*a*b*c/2
+ A*b**3/4 + 3*B*a**2*c/4 + 3*B*a*b**2/4) + x**2*(3*A*a**2*c/2 + 3*A*a*b**2
/2 + 3*B*a**2*b/2)
```

3.80 $\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=133

$$\frac{(-aBc - Abc + b^2B)\log(a + bx^2 + cx^4)}{4c^3} + \frac{(2aAc^2 - 3abBc - Ab^2c + b^3B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2 - 4ac}} - \frac{x^2(bB - Ac)}{2c^2} + \frac{Bx^4}{4c}$$

Rubi [A] time = 0.21, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1251, 800, 634, 618, 206, 628}

$$\frac{(-aBc - Abc + b^2B)\log(a + bx^2 + cx^4)}{4c^3} + \frac{(2aAc^2 - 3abBc - Ab^2c + b^3B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2 - 4ac}} - \frac{x^2(bB - Ac)}{2c^2} + \frac{Bx^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*B - A*c)*x^2)/(2*c^2) + (B*x^4)/(4*c) + ((b^3*B - A*b^2*c - 3*a*b*B*c
+ 2*a*A*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*
a*c]) + ((b^2*B - A*b*c - a*B*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
```

```
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\left(A+Bx^2\right)}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(A+Bx)}{a+bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{bB-Ac}{c^2}+\frac{Bx}{c}+\frac{a(bB-Ac)+\left(b^2B-Abc-aBc\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx,x,x^2\right)\\
&= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{\operatorname{Subst}\left(\int \frac{a(bB-Ac)+\left(b^2B-Abc-aBc\right)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2c^2}\\
&= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{\left(b^2B-Abc-aBc\right)\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^3}-\frac{\left(b^3B-Ab^2c-3\right.}{}\\
&= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{\left(b^2B-Abc-aBc\right)\log\left(a+bx^2+cx^4\right)}{4c^3}+\frac{\left(b^3B-Ab^2c-3abBc+\right.}{}\\
&= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{\left(b^3B-Ab^2c-3abBc+2aAc^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{\left(b^2B-Abc-\right.}{}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 126, normalized size = 0.95

$$
\frac{\left(-aBc-Abc+b^2B\right)\log\left(a+bx^2+cx^4\right)+\frac{2\left(-2aAc^2+3abBc+Ab^2c+b^3(-B)\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+2cx^2(Ac-bB)+Bc^2x^4}{4c^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(-(b*B) + A*c)*x^2 + B*c^2*x^4 + (2*(-(b^3*B) + A*b^2*c + 3*a*b*B*c -
2*a*A*c^2)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (
b^2*B - A*b*c - a*B*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^5\left(A+Bx^2\right)}{a+bx^2+cx^4}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4), x]
```

fricas [A] time = 1.59, size = 421, normalized size = 3.17

$$
\left[\frac{\left(Bb^2c^2-4\,Bac^3\right)x^4-2\left(Bb^3c+4\,Aac^3-\left(4\,Bab+Ab^2\right)c^2\right)x^2+\left(Bb^3+2\,Aac^2-\left(3\,Bab+Ab^2\right)c\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\left(Bb^4+4\left(Ba^2+Aab\right)c^2-\left(5\,Bab^2+Ab^3\right)c\right)\log\left(cx^4+bx^2+a\right)}{4\left(b^2c^3-4\,ac^4\right)},\ \frac{\left(Bb^2c^2-4\,Bac^3\right)x^4-2\left(Bb^3c+4\,Aac^3-\left(4\,Bab+Ab^2\right)c^2\right)x^2+2\left(Bb^3+2\,Aac^2-\left(3\,Bab+Ab^2\right)c\right)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+\left(Bb^4+4\left(Ba^2+Aab\right)c^2-\left(5\,Bab^2+Ab^3\right)c\right)\log\left(cx^4+bx^2+a\right)}{4\left(b^2c^3-4\,ac^4\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((B*b^2*c^2 - 4*B*a*c^3)*x^4 - 2*(B*b^3*c + 4*A*a*c^3 - (4*B*a*b + A*b
^2)*c^2)*x^2 + (B*b^3 + 2*A*a*c^2 - (3*B*a*b + A*b^2)*c)*sqrt(b^2 - 4*a*c)*
```

```
log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))
/(c*x^4 + b*x^2 + a)) + (B*b^4 + 4*(B*a^2 + A*a*b)*c^2 - (5*B*a*b^2 + A*b^3
)*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4), 1/4*((B*b^2*c^2 - 4*B*a*c
^3)*x^4 - 2*(B*b^3*c + 4*A*a*c^3 - (4*B*a*b + A*b^2)*c^2)*x^2 + 2*(B*b^3 +
2*A*a*c^2 - (3*B*a*b + A*b^2)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*s
qrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (B*b^4 + 4*(B*a^2 + A*a*b)*c^2 - (5*B*a*
b^2 + A*b^3)*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]
```

giac [A] time = 1.87, size = 126, normalized size = 0.95

$$\frac{Bcx^4 - 2\,Bbx^2 + 2\,Acx^2}{4\,c^2} + \frac{\left(Bb^2 - Bac - Abc\right)\log\left(cx^4 + bx^2 + a\right)}{4\,c^3} - \frac{\left(Bb^3 - 3\,Babc - Ab^2c + 2\,Aac^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(B*c*x^4 - 2*B*b*x^2 + 2*A*c*x^2)/c^2 + 1/4*(B*b^2 - B*a*c - A*b*c)*log
(c*x^4 + b*x^2 + a)/c^3 - 1/2*(B*b^3 - 3*B*a*b*c - A*b^2*c + 2*A*a*c^2)*arc
tan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

maple [B] time = 0.01, size = 261, normalized size = 1.96

$$\frac{B\,x^4}{4c} - \frac{Aa\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c} + \frac{A\,b^2\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;c^2} + \frac{A\,x^2}{2c} + \frac{3Bab\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;c^2} - \frac{B\,b^3\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;c^3} - \frac{Bb\,x^2}{2c^2} - \frac{Ab\ln\left(c\,x^4+b\,x^2+a\right)}{4c^2} - \frac{Ba\ln\left(c\,x^4+b\,x^2+a\right)}{4c^2} + \frac{B\,b^2\ln\left(c\,x^4+b\,x^2+a\right)}{4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*B*x^4/c+1/2/c*A*x^2-1/2/c^2*B*x^2*b-1/4/c^2*ln(c*x^4+b*x^2+a)*A*b-1/4/c
^2*ln(c*x^4+b*x^2+a)*a*B+1/4/c^3*ln(c*x^4+b*x^2+a)*b^2*B-1/c/(4*a*c-b^2)^(1
/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*A+3/2/c^2/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*B+1/2/c^2/(4*a*c-b^2)^(1/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^2-1/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^2+b)/(4*a*c-b^2)^(1/2))*b^3*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.46, size = 1343, normalized size = 10.10

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^2*(A/(2*c) - (B*b)/(2*c^2)) + (B*x^4)/(4*c) - (log(a + b*x^2 + c*x^4)*(2*
B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(2*(16*a*c^4
 - 4*b^2*c^3)) + (atan((2*c^4*(4*a*c - b^2)*(x^2*(((((6*A*b^2*c^4 - 6*B*b^3
*c^3 - 4*A*a*c^5 + 10*B*a*b*c^4)/c^4 - (4*b*c^2*(2*B*b^4 + 8*B*a^2*c^2 - 2*
```

```
A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(B*b^3 + 2*A
*a*c^2 - A*b^2*c - 3*B*a*b*c))/(8*c^3*(4*a*c - b^2)^(1/2)) - (b*(B*b^3 + 2*
A*a*c^2 - A*b^2*c - 3*B*a*b*c)*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b
*c^2 - 10*B*a*b^2*c))/(2*c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a -
 (b*((((6*A*b^2*c^4 - 6*B*b^3*c^3 - 4*A*a*c^5 + 10*B*a*b*c^4)/c^4 - (4*b*c^
2*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(16*a*c
^4 - 4*b^2*c^3))*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*
b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (B^2*b^5 + A^2*b^3*c^2 - 2*A*B*b^4*c -
 A*B*a^2*c^3 - A^2*a*b*c^3 - 3*B^2*a*b^3*c + 2*B^2*a^2*b*c^2 + 4*A*B*a*b^2*
c^2)/c^4 + (b*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c)^2)/(2*c^4*(4*a*c -
b^2))))/(2*a*(4*a*c - b^2)^(1/2))) + ((((8*B*a^2*c^4 + 8*A*a*b*c^4 - 8*B*a*
b^2*c^3)/c^4 - (8*a*c^2*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 -
10*B*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a
*b*c))/(8*c^3*(4*a*c - b^2)^(1/2)) - (a*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*
a*b*c)*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(c
*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a - (b*((((8*B*a^2*c^4 + 8*A*
a*b*c^4 - 8*B*a*b^2*c^3)/c^4 - (8*a*c^2*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c
+ 8*A*a*b*c^2 - 10*B*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(2*B*b^4 + 8*B*a^2*c
^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) -
(B^2*a*b^4 + B^2*a^3*c^2 + A^2*a*b^2*c^2 - 2*B^2*a^2*b^2*c - 2*A*B*a*b^3*c
+ 2*A*B*a^2*b*c^2)/c^4 + (a*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c)^2)/(c
^4*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(B^2*b^6 + 4*A^2*a^2*c^4 +
A^2*b^4*c^2 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2
*c^3 + 10*A*B*a*b^3*c^2 - 12*A*B*a^2*b*c^3))*(B*b^3 + 2*A*a*c^2 - A*b^2*c -
 3*B*a*b*c))/(2*c^3*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 43.89, size = 620, normalized size = 4.66

$$\frac{Bx^4}{4c}+x^2\left(\frac{A}{2c}-\frac{Bb}{2c^2}\right)+\left(-\frac{\sqrt{-4ac+b^2}\left(-2Aac^2+Ab^2c+3Babc-Bb^3\right)}{4c^3\left(4ac-b^2\right)}-\frac{Abc+Bac-Bb^2}{4c^3}\right)\log\left(x^2+\frac{Aabc+2Ba^2c-Bab^2+8ac^3\left(-\frac{\sqrt{-4ac+b^2}\left(-2Aac^2+Ab^2c+3Babc-Bb^3\right)}{4c^3\left(4ac-b^2\right)}-\frac{Abc+Bac-Bb^2}{4c^3}\right)-2b^2c^2\left(-\frac{\sqrt{-4ac+b^2}\left(-2Aac^2+Ab^2c+3Babc-Bb^3\right)}{4c^3\left(4ac-b^2\right)}-\frac{Abc+Bac-Bb^2}{4c^3}\right)}{-2Aac^2+Ab^2c+3Babc-Bb^3}\right)+\left(\frac{\sqrt{-4ac+b^2}\left(-2Aac^2+Ab^2c+3Babc-Bb^3\right)}{4c^3\left(4ac-b^2\right)}-\frac{Abc+Bac-Bb^2}{4c^3}\right)\log\left(x^2+\frac{Aabc+2Ba^2c-Bab^2+8ac^3\left(\frac{\sqrt{-4ac+b^2}\left(-2Aac^2+Ab^2c+3Babc-Bb^3\right)}{4c^3\left(4ac-b^2\right)}-\frac{Abc+Bac-Bb^2}{4c^3}\right)-2b^2c^2\left(\frac{\sqrt{-4ac+b^2}\left(-2Aac^2+Ab^2c+3Babc-Bb^3\right)}{4c^3\left(4ac-b^2\right)}-\frac{Abc+Bac-Bb^2}{4c^3}\right)}{-2Aac^2+Ab^2c+3Babc-Bb^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] B*x**4/(4*c) + x**2*(A/(2*c) - B*b/(2*c**2)) + (-sqrt(-4*a*c + b**2)*(-2*A*
a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3)/(4*c**3*(4*a*c - b**2)) - (A*b*c +
B*a*c - B*b**2)/(4*c**3))*log(x**2 + (A*a*b*c + 2*B*a**2*c - B*a*b**2 + 8*a
*c**3*(-sqrt(-4*a*c + b**2)*(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3)/(
4*c**3*(4*a*c - b**2)) - (A*b*c + B*a*c - B*b**2)/(4*c**3)) - 2*b**2*c**2*(
-sqrt(-4*a*c + b**2)*(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3)/(4*c**3*
(4*a*c - b**2)) - (A*b*c + B*a*c - B*b**2)/(4*c**3)))/(-2*A*a*c**2 + A*b**2
*c + 3*B*a*b*c - B*b**3)) + (sqrt(-4*a*c + b**2)*(-2*A*a*c**2 + A*b**2*c +
3*B*a*b*c - B*b**3)/(4*c**3*(4*a*c - b**2)) - (A*b*c + B*a*c - B*b**2)/(4*c
**3))*log(x**2 + (A*a*b*c + 2*B*a**2*c - B*a*b**2 + 8*a*c**3*(sqrt(-4*a*c +
 b**2)*(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3)/(4*c**3*(4*a*c - b**2)
) - (A*b*c + B*a*c - B*b**2)/(4*c**3)) - 2*b**2*c**2*(sqrt(-4*a*c + b**2)*(
-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3)/(4*c**3*(4*a*c - b**2)) - (A*b
*c + B*a*c - B*b**2)/(4*c**3)))/(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**
3))
```

3.81 $\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=97

$$-\frac{\left(-2aBc-Abc+b^2B\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{(bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{Bx^2}{2c}$$

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1251, 773, 634, 618, 206, 628}

$$-\frac{\left(-2aBc-Abc+b^2B\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{(bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{Bx^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (B*x^2)/(2*c) - ((b^2*B - A*b*c - 2*a*B*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) - ((b*B - A*c)*Log[a + b*x^2 + c*x^4])/
(4*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
```

```
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \left(A+Bx^2\right)}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(A+Bx)}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= \frac{Bx^2}{2c} + \frac{\operatorname{Subst}\left(\int \frac{-aB+(-bB+Ac)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c}\\
&= \frac{Bx^2}{2c} - \frac{(bB-Ac)\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2} + \frac{\left(b^2B-Abc-2aBc\right)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,\right.}{4c^2}\\
&= \frac{Bx^2}{2c} - \frac{(bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^2} - \frac{\left(b^2B-Abc-2aBc\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2\right.}{2c^2}\\
&= \frac{Bx^2}{2c} - \frac{\left(b^2B-Abc-2aBc\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}} - \frac{(bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^2}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 93, normalized size = 0.96

$$\frac{\frac{2\left(-2aBc-Abc+b^2B\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + (Ac-bB)\log\left(a+bx^2+cx^4\right) + 2Bcx^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*B*c*x^2 + (2*(b^2*B - A*b*c - 2*a*B*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 +
4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-(b*B) + A*c)*Log[a + b*x^2 + c*x^4])/(4*c^2
)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(A+Bx^2\right)}{a+bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4), x]
```

fricas [A] time = 0.77, size = 312, normalized size = 3.22

$$\left[\frac{2\left(Bb^2c-4\,Bac^2\right)x^2-\left(Bb^2-(2\,Ba+Ab)c\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(Bb^3+4\,Aac^2-\left(4\,Bab+Ab^2\right)c\right)\log\left(cx^4+bx^2+a\right)}{4\left(b^2c^2-4\,ac^3\right)}, \frac{2\left(Bb^2c-4\,Bac^2\right)x^2-2\left(Bb^2-(2\,Ba+Ab)c\right)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-\left(Bb^3+4\,Aac^2-\left(4\,Bab+Ab^2\right)c\right)\log\left(cx^4+bx^2+a\right)}{4\left(b^2c^2-4\,ac^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(B*b^2*c - 4*B*a*c^2)*x^2 - (B*b^2 - (2*B*a + A*b)*c)*sqrt(b^2 - 4*
a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4 + b*x^2 + a)) - (B*b^3 + 4*A*a*c^2 - (4*B*a*b + A*b^2)*c)*log(
c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1/4*(2*(B*b^2*c - 4*B*a*c^2)*x^2 -
```

```
2*(B*b^2 - (2*B*a + A*b)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(
-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (B*b^3 + 4*A*a*c^2 - (4*B*a*b + A*b^2)*c)*lo
g(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

giac [A] time = 1.81, size = 91, normalized size = 0.94

$$\frac{Bx^2}{2c} - \frac{(Bb - Ac)\log\left(cx^4 + bx^2 + a\right)}{4c^2} + \frac{\left(Bb^2 - 2Bac - Abc\right)\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/c - 1/4*(B*b - A*c)*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(B*b^2 - 2*B
*a*c - A*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*
c^2)
```

maple [A] time = 0.00, size = 175, normalized size = 1.80

$$-\frac{Ab\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\,c} - \frac{Ba\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c} + \frac{Bb^2\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\,c^2} + \frac{Bx^2}{2c} + \frac{A\ln\left(cx^4+bx^2+a\right)}{4c} - \frac{Bb\ln\left(cx^4+bx^2+a\right)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*B*x^2/c+1/4/c*ln(c*x^4+b*x^2+a)*A-1/4/c^2*ln(c*x^4+b*x^2+a)*b*B-1/c/(4*
a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*B-1/2/c/(4*a*c-b^2)^
(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b+1/2/c^2/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.65, size = 979, normalized size = 10.09

$$\frac{Bx^2}{2c} + \frac{\ln\left(cx^4+bx^2+a\right)\left(2Bb^3-2Ab^2c-8Babc+8Aac^2\right)}{2\left(16ac^3-4b^2c^2\right)} - \frac{\operatorname{atan}\left([illegible]\right)\left(-Bb^2+Acb+2Bac\right)}{2c^2\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (B*x^2)/(2*c) + (log(a + b*x^2 + c*x^4)*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c -
8*B*a*b*c))/(2*(16*a*c^3 - 4*b^2*c^2)) - (atan((2*c^2*(4*a*c - b^2)*(((((8*
A*a*c^3 - 8*B*a*b*c^2)/c^2 - (8*a*c^2*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c - 8*
B*a*b*c))/(16*a*c^3 - 4*b^2*c^2))*(A*b*c - B*b^2 + 2*B*a*c))/(8*c^2*(4*a*c
- b^2)^(1/2)) - (a*(A*b*c - B*b^2 + 2*B*a*c)*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2
*c - 8*B*a*b*c))/((4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2)))/a + x^2*((((
(6*A*b*c^3 - 6*B*b^2*c^2 + 4*B*a*c^3)/c^2 - (4*b*c^2*(2*B*b^3 + 8*A*a*c^2 -
 2*A*b^2*c - 8*B*a*b*c))/(16*a*c^3 - 4*b^2*c^2))*(A*b*c - B*b^2 + 2*B*a*c))
```

```
/(8*c^2*(4*a*c - b^2)^(1/2)) - (b*(A*b*c - B*b^2 + 2*B*a*c)*(2*B*b^3 + 8*A*
a*c^2 - 2*A*b^2*c - 8*B*a*b*c))/(2*(4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^
2)))/a + (b*((((6*A*b*c^3 - 6*B*b^2*c^2 + 4*B*a*c^3)/c^2 - (4*b*c^2*(2*B*b^
3 + 8*A*a*c^2 - 2*A*b^2*c - 8*B*a*b*c))/(16*a*c^3 - 4*b^2*c^2))*(2*B*b^3 +
8*A*a*c^2 - 2*A*b^2*c - 8*B*a*b*c))/(2*(16*a*c^3 - 4*b^2*c^2)) - (B^2*b^3 +
 A^2*b*c^2 + A*B*a*c^2 - 2*A*B*b^2*c - B^2*a*b*c)/c^2 + (b*(A*b*c - B*b^2 +
 2*B*a*c)^2)/(2*c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) + (b*((((8*
A*a*c^3 - 8*B*a*b*c^2)/c^2 - (8*a*c^2*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c - 8*
B*a*b*c))/(16*a*c^3 - 4*b^2*c^2))*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c - 8*B*a*
b*c))/(2*(16*a*c^3 - 4*b^2*c^2)) - (A^2*a*c^2 + B^2*a*b^2 - 2*A*B*a*b*c)/c^
2 + (a*(A*b*c - B*b^2 + 2*B*a*c)^2)/(c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2
)^(1/2))))/(B^2*b^4 + A^2*b^2*c^2 + 4*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*B^2*a*b
^2*c + 4*A*B*a*b*c^2))*(A*b*c - B*b^2 + 2*B*a*c))/(2*c^2*(4*a*c - b^2)^(1/2
))
```

sympy [B] time = 10.55, size = 434, normalized size = 4.47

$$\frac{Bx^2}{2c} + \left(-\frac{-Ac + Bb}{4c^2} - \frac{\sqrt{-4ac + b^2}\left(Abc + 2Bac - Bb^2\right)}{4c^2\left(4ac - b^2\right)}\right)\log\left(x^2 + \frac{2Aac - Bab - 8ac^2\left(-\frac{-Ac+Bb}{4c^2} - \frac{\sqrt{-4ac+b^2}\left(Abc+2Bac-Bb^2\right)}{4c^2\left(4ac-b^2\right)}\right) + 2b^2c\left(-\frac{-Ac+Bb}{4c^2} - \frac{\sqrt{-4ac+b^2}\left(Abc+2Bac-Bb^2\right)}{4c^2\left(4ac-b^2\right)}\right)}{Abc + 2Bac - Bb^2}\right) + \left(-\frac{-Ac + Bb}{4c^2} + \frac{\sqrt{-4ac + b^2}\left(Abc + 2Bac - Bb^2\right)}{4c^2\left(4ac - b^2\right)}\right)\log\left(x^2 + \frac{2Aac - Bab - 8ac^2\left(-\frac{-Ac+Bb}{4c^2} + \frac{\sqrt{-4ac+b^2}\left(Abc+2Bac-Bb^2\right)}{4c^2\left(4ac-b^2\right)}\right) + 2b^2c\left(-\frac{-Ac+Bb}{4c^2} + \frac{\sqrt{-4ac+b^2}\left(Abc+2Bac-Bb^2\right)}{4c^2\left(4ac-b^2\right)}\right)}{Abc + 2Bac - Bb^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] B*x**2/(2*c) + (-(-A*c + B*b)/(4*c**2) - sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a
*c - B*b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (2*A*a*c - B*a*b - 8*a*c**
2*(-(-A*c + B*b)/(4*c**2) - sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a*c - B*b**2)/
(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-(-A*c + B*b)/(4*c**2) - sqrt(-4*a*c +
 b**2)*(A*b*c + 2*B*a*c - B*b**2)/(4*c**2*(4*a*c - b**2))))/(A*b*c + 2*B*a*
c - B*b**2)) + (-(-A*c + B*b)/(4*c**2) + sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a
*c - B*b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (2*A*a*c - B*a*b - 8*a*c**
2*(-(-A*c + B*b)/(4*c**2) + sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a*c - B*b**2)/
(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-(-A*c + B*b)/(4*c**2) + sqrt(-4*a*c +
 b**2)*(A*b*c + 2*B*a*c - B*b**2)/(4*c**2*(4*a*c - b**2))))/(A*b*c + 2*B*a*
c - B*b**2))
```

3.82 $\int \frac{x(A+Bx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=71

$$\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c}$$

Rubi [A] time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1247, 634, 618, 206, 628}

$$\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*
a*c]) + (B*Log[a + b*x^2 + c*x^4])/(4*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(A+Bx^2\right)}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= \frac{B\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c} + \frac{(-bB+2Ac)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c}\\
&= \frac{B\log\left(a+bx^2+cx^4\right)}{4c} - \frac{(-bB+2Ac)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c}\\
&= \frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{B\log\left(a+bx^2+cx^4\right)}{4c}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 71, normalized size = 1.00

$$
\frac{B\log\left(a+bx^2+cx^4\right) - \frac{2(bB-2Ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-2*(b*B - 2*A*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*
a*c] + B*Log[a + b*x^2 + c*x^4])/(4*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x\left(A+Bx^2\right)}{a+bx^2+cx^4}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[(x*(A + B*x^2))/(a + b*x^2 + c*x^4), x]
```

fricas [A] time = 0.83, size = 219, normalized size = 3.08

$$
\left[-\frac{(Bb-2\,Ac)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right) - \left(Bb^2-4\,Bac\right)\log\left(cx^4+bx^2+a\right)}{4\left(b^2c-4\,ac^2\right)}, \frac{2\,(Bb-2\,Ac)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right) + \left(Bb^2-4\,Bac\right)\log\left(cx^4+bx^2+a\right)}{4\left(b^2c-4\,ac^2\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((B*b - 2*A*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2
*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (B*b^2 - 4*B
*a*c)*log(c*x^4 + b*x^2 + a))/(b^2*c - 4*a*c^2), 1/4*(2*(B*b - 2*A*c)*sqrt(
-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (B*
b^2 - 4*B*a*c)*log(c*x^4 + b*x^2 + a))/(b^2*c - 4*a*c^2)]
```

giac [A] time = 1.71, size = 67, normalized size = 0.94

$$
\frac{B\log\left(cx^4+bx^2+a\right)}{4\,c} - \frac{(Bb-2\,Ac)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,c}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*B*log(c*x^4 + b*x^2 + a)/c - 1/2*(B*b - 2*A*c)*arctan((2*c*x^2 + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

maple [A] time = 0.00, size = 98, normalized size = 1.38

$$\frac{A\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{Bb\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\,c}+\frac{B\ln\left(cx^4+bx^2+a\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*B*ln(c*x^4+b*x^2+a)/c+1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2
)^(1/2))*A-1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B*b/
c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.50, size = 606, normalized size = 8.54

$$-\frac{\ln\left(cx^4+bx^2+a\right)\left(2Bb^2-8Bac\right)}{2\left(16ac^2-4b^2c\right)}-\frac{\mathrm{atan}\left(\frac{2\left(4ac-b^2\right)\left(x^2\left(\frac{\frac{(2Ac-Bb)\left(6Bbc-4Ac^2+\frac{4bc^2\left(2Bb^2-8Bac\right)}{16ac^2-4b^2c}\right)}{8c\sqrt{4ac-b^2}}+\frac{bc\left(2Bb^2-8Bac\right)(2Ac-Bb)}{2\left(16ac^2-4b^2c\right)\sqrt{4ac-b^2}}}{a}+\frac{b\left(B^2b-ABc-\frac{b(2Ac-Bb)^2}{2\left(4ac-b^2\right)}+\frac{\left(2Bb^2-8Bac\right)\left(6Bbc-4Ac^2+\frac{4bc^2\left(2Bb^2-8Bac\right)}{16ac^2-4b^2c}\right)}{2\left(16ac^2-4b^2c\right)}\right)}{2a\sqrt{4ac-b^2}}\right)+\frac{\frac{\left(8Bac+\frac{8ac^2\left(2Bb^2-8Bac\right)}{16ac^2-4b^2c}\right)(2Ac-Bb)}{8c\sqrt{4ac-b^2}}+\frac{ac\left(2Bb^2-8Bac\right)(2Ac-Bb)}{\left(16ac^2-4b^2c\right)\sqrt{4ac-b^2}}}{a}+\frac{b\left(B^2a+\frac{\left(2Bb^2-8Bac\right)\left(8Bac+\frac{8ac^2\left(2Bb^2-8Bac\right)}{16ac^2-4b^2c}\right)}{2\left(16ac^2-4b^2c\right)}-\frac{a(2Ac-Bb)^2}{4ac-b^2}\right)}{2a\sqrt{4ac-b^2}}\right)}{4A^2c^2-4ABbc+B^2b^2}\right)(2Ac-Bb)}{2c\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] - (log(a + b*x^2 + c*x^4)*(2*B*b^2 - 8*B*a*c))/(2*(16*a*c^2 - 4*b^2*c)) - (
atan((2*(4*a*c - b^2)*(x^2*((((2*A*c - B*b)*(6*B*b*c - 4*A*c^2 + (4*b*c^2*(
2*B*b^2 - 8*B*a*c))/(16*a*c^2 - 4*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) + (b*c
*(2*B*b^2 - 8*B*a*c)*(2*A*c - B*b))/(2*(16*a*c^2 - 4*b^2*c)*(4*a*c - b^2)^(
1/2)))/a + (b*(B^2*b - A*B*c - (b*(2*A*c - B*b)^2)/(2*(4*a*c - b^2)) + ((2*
B*b^2 - 8*B*a*c)*(6*B*b*c - 4*A*c^2 + (4*b*c^2*(2*B*b^2 - 8*B*a*c))/(16*a*c
^2 - 4*b^2*c)))/(2*(16*a*c^2 - 4*b^2*c))))/(2*a*(4*a*c - b^2)^(1/2))) + (((
8*B*a*c + (8*a*c^2*(2*B*b^2 - 8*B*a*c))/(16*a*c^2 - 4*b^2*c))*(2*A*c - B*b)
)/(8*c*(4*a*c - b^2)^(1/2)) + (a*c*(2*B*b^2 - 8*B*a*c)*(2*A*c - B*b))/((16*
a*c^2 - 4*b^2*c)*(4*a*c - b^2)^(1/2)))/a + (b*(B^2*a + ((2*B*b^2 - 8*B*a*c)
*(8*B*a*c + (8*a*c^2*(2*B*b^2 - 8*B*a*c))/(16*a*c^2 - 4*b^2*c)))/(2*(16*a*c
^2 - 4*b^2*c)) - (a*(2*A*c - B*b)^2)/(4*a*c - b^2)))/(2*a*(4*a*c - b^2)^(1/
2))))/(4*A^2*c^2 + B^2*b^2 - 4*A*B*b*c))*(2*A*c - B*b))/(2*c*(4*a*c - b^2)^
(1/2))
```

sympy [B] time = 3.59, size = 287, normalized size = 4.04

$$\left(\frac{B}{4c}-\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{-Ab+2Ba-8ac\left(\frac{B}{4c}-\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}\right)+2b^2\left(\frac{B}{4c}-\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}\right)}{-2Ac+Bb}\right)+\left(\frac{B}{4c}+\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{-Ab+2Ba-8ac\left(\frac{B}{4c}+\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}\right)+2b^2\left(\frac{B}{4c}+\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}\right)}{-2Ac+Bb}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] (B/(4*c) - (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)))*log(x**
2 + (-A*b + 2*B*a - 8*a*c*(B/(4*c) - (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*
c*(4*a*c - b**2))) + 2*b**2*(B/(4*c) - (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(
4*c*(4*a*c - b**2))))/(-2*A*c + B*b)) + (B/(4*c) + (-2*A*c + B*b)*sqrt(-4*a
*c + b**2)/(4*c*(4*a*c - b**2)))*log(x**2 + (-A*b + 2*B*a - 8*a*c*(B/(4*c)
+ (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2))) + 2*b**2*(B/(4*c
) + (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2))))/(-2*A*c + B*b
))
```

3.83 $\int \frac{A+Bx^2}{x(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=78

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}+\frac{A\log(x)}{a}$$

Rubi [A] time = 0.14, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1251, 800, 634, 618, 206, 628}

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}+\frac{A\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*
a*c]) + (A*Log[x])/a - (A*Log[a + b*x^2 + c*x^4])/(4*a)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
```

```
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x\left(a+bx+cx^2\right)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{ax}+\frac{-Ab+aB-Acx}{a\left(a+bx+cx^2\right)}\right)dx,x,x^2\right)\\ &= \frac{A\log(x)}{a}+\frac{\text{Subst}\left(\int \frac{-Ab+aB-Acx}{a+bx+cx^2}\,dx,x,x^2\right)}{2a}\\ &= \frac{A\log(x)}{a}-\frac{A\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a}+\frac{(-Ab+2aB)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4a}\\ &= \frac{A\log(x)}{a}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}-\frac{(-Ab+2aB)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2a}\\ &= \frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}+\frac{A\log(x)}{a}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}\end{aligned}$$

Mathematica [A] time = 0.10, size = 128, normalized size = 1.64

$$\frac{-\left(A\left(\sqrt{b^2-4ac}+b\right)-2aB\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)+\left(A\left(b-\sqrt{b^2-4ac}\right)-2aB\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)+4A\log(x)\sqrt{b^2-4ac}}{4a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (4*A*Sqrt[b^2 - 4*a*c]*Log[x] - (-2*a*B + A*(b + Sqrt[b^2 - 4*a*c]))*Log[b
- Sqrt[b^2 - 4*a*c] + 2*c*x^2] + (-2*a*B + A*(b - Sqrt[b^2 - 4*a*c]))*Log[b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)), x]
```

fricas [A] time = 1.14, size = 249, normalized size = 3.19

$$\left[-\frac{(2Ba-Ab)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac+\left(2cx^2+b\right)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)+\left(Ab^2-4Aac\right)\log\left(cx^4+bx^2+a\right)-4\left(Ab^2-4Aac\right)\log(x)}{4\left(ab^2-4a^2c\right)},-\frac{2(2Ba-Ab)\sqrt{-b^2+4ac}\arctan\left(-\frac{\left(2cx^2+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)+\left(Ab^2-4Aac\right)\log\left(cx^4+bx^2+a\right)-4\left(Ab^2-4Aac\right)\log(x)}{4\left(ab^2-4a^2c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*B*a - A*b)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2
*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (A*b^2 - 4*A
*a*c)*log(c*x^4 + b*x^2 + a) - 4*(A*b^2 - 4*A*a*c)*log(x))/(a*b^2 - 4*a^2*c
```

```
), -1/4*(2*(2*B*a - A*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2
 + 4*a*c)/(b^2 - 4*a*c)) + (A*b^2 - 4*A*a*c)*log(c*x^4 + b*x^2 + a) - 4*(A*
b^2 - 4*A*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

giac [A] time = 1.61, size = 78, normalized size = 1.00

$$-\frac{A\log\left(cx^4+bx^2+a\right)}{4\,a}+\frac{A\log\left(x^2\right)}{2\,a}+\frac{(2\,Ba-Ab)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*A*log(c*x^4 + b*x^2 + a)/a + 1/2*A*log(x^2)/a + 1/2*(2*B*a - A*b)*arct
an((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)
```

maple [A] time = 0.01, size = 105, normalized size = 1.35

$$-\frac{Ab\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\,a}+\frac{B\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{A\ln(x)}{a}-\frac{A\ln\left(cx^4+bx^2+a\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] A*ln(x)/a-1/4*A*ln(c*x^4+b*x^2+a)/a-1/2/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
+b)/(4*a*c-b^2)^(1/2))*A*b+1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^
2)^(1/2))*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.48, size = 2424, normalized size = 31.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (A*log(x))/a - (log((A*B^2*c^2 + ((A + a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^
2)))^(1/2))*(B^2*a*c^2 + ((A + a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))^(1/
2))*(4*A*b^2*c^2 + 2*c^2*x^2*(5*A*b*c - 4*B*b^2 + 10*B*a*c) - 4*B*a*b*c^2 +
 (b*c^2*(A + a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*x
^2 - 10*a*c*x^2))/a))/(4*a) - 4*A*B*b*c^2 - B*c^2*x^2*(5*A*c + B*b)))/(4*a)
 + B^3*c^2*x^2)*(A*B^2*c^2 + ((A - a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))
^(1/2))*(B^2*a*c^2 + ((A - a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))^(1/2))*
(4*A*b^2*c^2 + 2*c^2*x^2*(5*A*b*c - 4*B*b^2 + 10*B*a*c) - 4*B*a*b*c^2 + (b*
c^2*(A - a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*x^2 -
 10*a*c*x^2))/a))/(4*a) - 4*A*B*b*c^2 - B*c^2*x^2*(5*A*c + B*b)))/(4*a) + B
^3*c^2*x^2))*(2*A*b^2 - 8*A*a*c))/(2*(4*a*b^2 - 16*a^2*c)) - (atan((2*(4*a*
c - b^2)^(3/2)*(3*A*b^3 - B*a*b^2 + B*a^2*c - 8*A*a*b*c)*(A*B^2*c^2 + ((2*A
```

```
*b^2 - 8*A*a*c)*(((2*A*b^2 - 8*A*a*c)*(4*A*b^2*c^2 - 4*B*a*b*c^2 + (2*a*b^2
*c^2*(2*A*b^2 - 8*A*a*c))/(4*a*b^2 - 16*a^2*c)))/(2*(4*a*b^2 - 16*a^2*c)) +
 B^2*a*c^2 - 4*A*B*b*c^2))/(2*(4*a*b^2 - 16*a^2*c)) - ((A*b - 2*B*a)*(((A*b
 - 2*B*a)*(4*A*b^2*c^2 - 4*B*a*b*c^2 + (2*a*b^2*c^2*(2*A*b^2 - 8*A*a*c))/(4
*a*b^2 - 16*a^2*c)))/(4*a*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*A*b^2 - 8*A*a*
c)*(A*b - 2*B*a))/(2*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2))))/(4*a*(4*a*
c - b^2)^(1/2)) - (b^2*c^2*(2*A*b^2 - 8*A*a*c)*(A*b - 2*B*a)^2)/(8*a*(4*a*b
^2 - 16*a^2*c)*(4*a*c - b^2))))/(c^2*(A^2*b^2*c^2 + 4*B^2*a^2*c^2 - 4*A*B*a
*b*c^2)*(6*A^2*b^2 - B^2*a^2 - 25*A^2*a*c + A*B*a*b)) - (16*a^3*x^2*(((3*A*
b^3 - B*a*b^2 + B*a^2*c - 8*A*a*b*c)*(((2*A*b^2 - 8*A*a*c)*(B^2*b*c^2 + 5*A
*B*c^3 - ((2*A*b^2 - 8*A*a*c)*(((2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^
3))/(2*(4*a*b^2 - 16*a^2*c)) - 8*B*b^2*c^2 + 10*A*b*c^3 + 20*B*a*c^3))/(2*(
4*a*b^2 - 16*a^2*c))))/(2*(4*a*b^2 - 16*a^2*c)) - B^3*c^2 + ((A*b - 2*B*a)*
(((A*b - 2*B*a)*(((2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^3))/(2*(4*a*b^
2 - 16*a^2*c)) - 8*B*b^2*c^2 + 10*A*b*c^3 + 20*B*a*c^3))/(4*a*(4*a*c - b^2)
^(1/2)) + ((2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^3)*(A*b - 2*B*a))/(8*
a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2))))/(4*a*(4*a*c - b^2)^(1/2)) + (
(2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^3)*(A*b - 2*B*a)^2)/(32*a^2*(4*a
*b^2 - 16*a^2*c)*(4*a*c - b^2))))/(8*a^3*c^2*(6*A^2*b^2 - B^2*a^2 - 25*A^2*
a*c + A*B*a*b)) + ((((12*b^3*c^2 - 40*a*b*c^3)*(A*b - 2*B*a)^3)/(64*a^3*(4*
a*c - b^2)^(3/2)) - ((2*A*b^2 - 8*A*a*c)*(((A*b - 2*B*a)*(((2*A*b^2 - 8*A*a
*c)*(12*b^3*c^2 - 40*a*b*c^3))/(2*(4*a*b^2 - 16*a^2*c)) - 8*B*b^2*c^2 + 10*
A*b*c^3 + 20*B*a*c^3))/(4*a*(4*a*c - b^2)^(1/2)) + ((2*A*b^2 - 8*A*a*c)*(12
*b^3*c^2 - 40*a*b*c^3)*(A*b - 2*B*a))/(8*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^
2)^(1/2))))/(2*(4*a*b^2 - 16*a^2*c)) + ((A*b - 2*B*a)*(B^2*b*c^2 + 5*A*B*c^
3 - ((2*A*b^2 - 8*A*a*c)*(((2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^3))/(
2*(4*a*b^2 - 16*a^2*c)) - 8*B*b^2*c^2 + 10*A*b*c^3 + 20*B*a*c^3))/(2*(4*a*b
^2 - 16*a^2*c))))/(4*a*(4*a*c - b^2)^(1/2)))*(3*A*b^4 + 10*A*a^2*c^2 - B*a*
b^3 - 14*A*a*b^2*c + 3*B*a^2*b*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(6*A^2*b^
2 - B^2*a^2 - 25*A^2*a*c + A*B*a*b)))*(4*a*c - b^2)^(3/2))/(A^2*b^2*c^2 + 4
*B^2*a^2*c^2 - 4*A*B*a*b*c^2) + (2*(4*a*c - b^2)*(((2*A*b^2 - 8*A*a*c)*(((A
*b - 2*B*a)*(4*A*b^2*c^2 - 4*B*a*b*c^2 + (2*a*b^2*c^2*(2*A*b^2 - 8*A*a*c))/
(4*a*b^2 - 16*a^2*c)))/(4*a*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*A*b^2 - 8*A*
a*c)*(A*b - 2*B*a))/(2*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2))))/(2*(4*a*
b^2 - 16*a^2*c)) + ((A*b - 2*B*a)*(((2*A*b^2 - 8*A*a*c)*(4*A*b^2*c^2 - 4*B*
a*b*c^2 + (2*a*b^2*c^2*(2*A*b^2 - 8*A*a*c))/(4*a*b^2 - 16*a^2*c)))/(2*(4*a*
b^2 - 16*a^2*c)) + B^2*a*c^2 - 4*A*B*b*c^2))/(4*a*(4*a*c - b^2)^(1/2)) - (b
^2*c^2*(A*b - 2*B*a)^3)/(16*a^2*(4*a*c - b^2)^(3/2)))*(3*A*b^4 + 10*A*a^2*c
^2 - B*a*b^3 - 14*A*a*b^2*c + 3*B*a^2*b*c))/(c^2*(A^2*b^2*c^2 + 4*B^2*a^2*c
^2 - 4*A*B*a*b*c^2)*(6*A^2*b^2 - B^2*a^2 - 25*A^2*a*c + A*B*a*b)))*(A*b - 2
*B*a))/(2*a*(4*a*c - b^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.84 $\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=112

$$-\frac{\left(-2aAc-abB+Ab^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{2ax^2}$$

Rubi [A] time = 0.25, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1251, 800, 634, 618, 206, 628}

$$-\frac{\left(-2aAc-abB+Ab^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -A/(2*a*x^2) - ((A*b^2 - a*b*B - 2*a*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - ((A*b - a*B)*Log[x])/a^2 + ((A*b - a*B
)*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
```

```
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{ax^2}+\frac{-Ab+aB}{a^2x}+\frac{-abB+A\left(b^2-ac\right)+(Ab-aB)cx}{a^2\left(a+bx+cx^2\right)}\right)dx, x, x^2\right)$$

$$= -\frac{A}{2ax^2}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{\text{Subst}\left(\int \frac{-abB+A\left(b^2-ac\right)+(Ab-aB)cx}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2}$$

$$= -\frac{A}{2ax^2}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2}+\frac{\left(-abB+A\left(b^2\right.\right.}{}$$

$$= -\frac{A}{2ax^2}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\left(-abB+A\left(b^2-2ac\right)\right)}{}$$

$$= -\frac{A}{2ax^2}+\frac{\left(abB-A\left(b^2-2ac\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\log}{4}$$

Mathematica [A] time = 0.22, size = 186, normalized size = 1.66

$$\frac{\frac{\left(A\left(b\sqrt{b^2-4ac}-2ac+b^2\right)-aB\left(\sqrt{b^2-4ac}+b\right)\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(A\left(b\sqrt{b^2-4ac}+2ac-b^2\right)+aB\left(b-\sqrt{b^2-4ac}\right)\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}+4\log(x)(aB-Ab)-\frac{2aA}{x^2}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*A)/x^2 + 4*(-(A*b) + a*B)*Log[x] + ((-(a*B*(b + Sqrt[b^2 - 4*a*c]))
+ A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^
2])/Sqrt[b^2 - 4*a*c] + ((a*B*(b - Sqrt[b^2 - 4*a*c]) + A*(-b^2 + 2*a*c + b
*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c
])/(4*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)), x]
```

fricas [A] time = 1.08, size = 385, normalized size = 3.44

$$\left[\frac{\left(Bab-Ab^2+2\,Aac\right)\sqrt{b^2-4\,ac}\,x^2\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-2\,Aab^2+8\,Aa^2c-\left(Bab^2-Ab^3-4\left(Ba^2-Aab\right)c\right)x^2\log\left(cx^4+bx^2+a\right)+4\left(Bab^2-Ab^3-4\left(Ba^2-Aab\right)c\right)x^2\log(x)}{4\left(a^2b^2-4\,a^3c\right)x^2},\ \frac{2\left(Bab-Ab^2+2\,Aac\right)\sqrt{-b^2+4\,ac}\,x^2\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-2\,Aab^2+8\,Aa^2c-\left(Bab^2-Ab^3-4\left(Ba^2-Aab\right)c\right)x^2\log\left(cx^4+bx^2+a\right)+4\left(Bab^2-Ab^3-4\left(Ba^2-Aab\right)c\right)x^2\log(x)}{4\left(a^2b^2-4\,a^3c\right)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((B*a*b - A*b^2 + 2*A*a*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 + 2*b*
c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a))
 - 2*A*a*b^2 + 8*A*a^2*c - (B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*log(
c*x^4 + b*x^2 + a) + 4*(B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*log(x))/
((a^2*b^2 - 4*a^3*c)*x^2), 1/4*(2*(B*a*b - A*b^2 + 2*A*a*c)*sqrt(-b^2 + 4*a
*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*A*a*b^2
 + 8*A*a^2*c - (B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*log(c*x^4 + b*x^
2 + a) + 4*(B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*log(x))/((a^2*b^2 -
4*a^3*c)*x^2)]
```

giac [A] time = 1.87, size = 124, normalized size = 1.11

$$-\frac{(Ba-Ab)\log\left(cx^4+bx^2+a\right)}{4\,a^2}+\frac{(Ba-Ab)\log\left(x^2\right)}{2\,a^2}-\frac{\left(Bab-Ab^2+2\,Aac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,a^2}-\frac{Bax^2-Abx^2+Aa}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(B*a - A*b)*log(c*x^4 + b*x^2 + a)/a^2 + 1/2*(B*a - A*b)*log(x^2)/a^2
- 1/2*(B*a*b - A*b^2 + 2*A*a*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(s
qrt(-b^2 + 4*a*c)*a^2) - 1/2*(B*a*x^2 - A*b*x^2 + A*a)/(a^2*x^2)
```

maple [A] time = 0.01, size = 191, normalized size = 1.71

$$-\frac{Ac\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,a}+\frac{Ab^2\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\,a^2}-\frac{Bb\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\,a}-\frac{Ab\ln(x)}{a^2}+\frac{Ab\ln\left(cx^4+bx^2+a\right)}{4a^2}+\frac{B\ln(x)}{a}-\frac{B\ln\left(cx^4+bx^2+a\right)}{4a}-\frac{A}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*A/a/x^2-1/a^2*ln(x)*A*b+1/a*ln(x)*B+1/4/a^2*ln(c*x^4+b*x^2+a)*A*b-1/4/
a*ln(c*x^4+b*x^2+a)*B-1/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^
(1/2))*A*c+1/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*
A*b^2-1/2/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.85, size = 3729, normalized size = 33.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] - A/(2*a*x^2) - (log(x)*(A*b - B*a))/a^2 - (log(((A^3*c^5*x^2)/a^3 - (((((4
*b*c^2*(A*a*c - A*b^2 + B*a*b))/a - (2*c^3*x^2*(A*b^2 + 10*A*a*c - 5*B*a*b)
)/a + (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(B*a - A*b + a^2*(-(2*A*a*c - A
*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^(1/2)))/a^2)*(B*a - A*b + a^2*(-(2*A*a
*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(4*a^2) + (A*c^3*(A*a*c
- 4*A*b^2 + 4*B*a*b))/a^2 - (A*c^4*x^2*(6*A*b - 5*B*a))/a^2)*(B*a - A*b + a
```

```
^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(4*a^2) + (A^
2*c^4*(A*b - B*a))/a^3)*((((((2*c^3*x^2*(A*b^2 + 10*A*a*c - 5*B*a*b))/a - (
4*b*c^2*(A*a*c - A*b^2 + B*a*b))/a + (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*
(A*b - B*a + a^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^(1/2)))
/a^2)*(A*b - B*a + a^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^(
1/2)))/(4*a^2) + (A*c^3*(A*a*c - 4*A*b^2 + 4*B*a*b))/a^2 - (A*c^4*x^2*(6*A*
b - 5*B*a))/a^2)*(A*b - B*a + a^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c
 - b^2)))^(1/2)))/(4*a^2) + (A^3*c^5*x^2)/a^3 + (A^2*c^4*(A*b - B*a))/a^3))
*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2))
- (atan((16*a^6*x^2*((((((5*A*B*a^2*c^4 - 6*A^2*a*b*c^4)/a^3 - (((20*A*a^3*
c^4 - 10*B*a^3*b*c^3 + 2*A*a^2*b^2*c^3)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c
^2)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*a^3*(16*a^3*c - 4*a^2
*b^2)))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2
*b^2)))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2
*b^2)) - (A^3*c^5)/a^3 + (((((20*A*a^3*c^4 - 10*B*a^3*b*c^3 + 2*A*a^2*b^2*c
^3)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c
 - 8*A*a*b*c))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(2*A*a*c - A*b^2 + B*a*b))/(
4*a^2*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*a*c - A*
b^2 + B*a*b)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(8*a^5*(4*a*c -
 b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(2*A*a*c - A*b^2 + B*a*b))/(4*a^2*(4*a
*c - b^2)^(1/2)) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*a*c - A*b^2 + B*a*
b)^2*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(32*a^7*(4*a*c - b^2)*(
16*a^3*c - 4*a^2*b^2)))*(3*A*b^4 + A*a^2*c^2 - 3*B*a*b^3 - 9*A*a*b^2*c + 8*
B*a^2*b*c))/(8*a^3*c^2*(25*B^2*a^3*c - 6*A^2*b^4 + A^2*a^2*c^2 - 6*B^2*a^2*
b^2 + 12*A*B*a*b^3 + 24*A^2*a*b^2*c - 49*A*B*a^2*b*c)) + (((((5*A*B*a^2*c^4
 - 6*A^2*a*b*c^4)/a^3 - (((20*A*a^3*c^4 - 10*B*a^3*b*c^3 + 2*A*a^2*b^2*c^3)
/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c -
8*A*a*b*c))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*
c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*A*a*c - A*b^2 + B*a*b))/(4*a
^2*(4*a*c - b^2)^(1/2)) - (((((20*A*a^3*c^4 - 10*B*a^3*b*c^3 + 2*A*a^2*b^2*
c^3)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*
c - 8*A*a*b*c))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(2*A*a*c - A*b^2 + B*a*b))/
(4*a^2*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*a*c - A
*b^2 + B*a*b)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(8*a^5*(4*a*c
 - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*
A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2)) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*
A*a*c - A*b^2 + B*a*b)^3)/(64*a^9*(4*a*c - b^2)^(3/2)))*(6*A*b^5 - 20*B*a^3
*c^2 - 6*B*a*b^4 - 30*A*a*b^3*c + 26*A*a^2*b*c^2 + 28*B*a^2*b^2*c))/(16*a^3
*c^2*(4*a*c - b^2)^(1/2)*(25*B^2*a^3*c - 6*A^2*b^4 + A^2*a^2*c^2 - 6*B^2*a^
2*b^2 + 12*A*B*a*b^3 + 24*A^2*a*b^2*c - 49*A*B*a^2*b*c)))*(4*a*c - b^2)^(3/
2))/(4*A^2*a^2*c^4 + A^2*b^4*c^2 + B^2*a^2*b^2*c^2 - 4*A^2*a*b^2*c^3 - 2*A*
B*a*b^3*c^2 + 4*A*B*a^2*b*c^3) + (a^3*(4*a*c - b^2)*((((((4*A*a^3*b*c^3 - 4
*A*a^2*b^3*c^2 + 4*B*a^3*b^2*c^2)/a^3 + (2*a*b^2*c^2*(2*A*b^3 - 2*B*a*b^2 +
 8*B*a^2*c - 8*A*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*A*a*c - A*b^2 + B*a*b))
/(4*a^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*A*a*c - A*b^2 + B*a*b)*(2*A*b^3
- 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*a*(4*a*c - b^2)^(1/2)*(16*a^3*c -
4*a^2*b^2)))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c -
4*a^2*b^2)) + (((A^2*a^2*c^4 - 4*A^2*a*b^2*c^3 + 4*A*B*a^2*b*c^3)/a^3 + (((
4*A*a^3*b*c^3 - 4*A*a^2*b^3*c^2 + 4*B*a^3*b^2*c^2)/a^3 + (2*a*b^2*c^2*(2*A*
b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*A*b^3
- 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*A*a*c
- A*b^2 + B*a*b))/(4*a^2*(4*a*c - b^2)^(1/2)) - (b^2*c^2*(2*A*a*c - A*b^2 +
 B*a*b)^3)/(16*a^5*(4*a*c - b^2)^(3/2)))*(6*A*b^5 - 20*B*a^3*c^2 - 6*B*a*b^
4 - 30*A*a*b^3*c + 26*A*a^2*b*c^2 + 28*B*a^2*b^2*c))/(c^2*(4*A^2*a^2*c^4 +
A^2*b^4*c^2 + B^2*a^2*b^2*c^2 - 4*A^2*a*b^2*c^3 - 2*A*B*a*b^3*c^2 + 4*A*B*a
^2*b*c^3)*(25*B^2*a^3*c - 6*A^2*b^4 + A^2*a^2*c^2 - 6*B^2*a^2*b^2 + 12*A*B*
a*b^3 + 24*A^2*a*b^2*c - 49*A*B*a^2*b*c)) - (2*a^3*(4*a*c - b^2)^(3/2)*((A^
3*b*c^4 - A^2*B*a*c^4)/a^3 - (((A^2*a^2*c^4 - 4*A^2*a*b^2*c^3 + 4*A*B*a^2*b
*c^3)/a^3 + (((4*A*a^3*b*c^3 - 4*A*a^2*b^3*c^2 + 4*B*a^3*b^2*c^2)/a^3 + (2*
```

```
a*b^2*c^2*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(16*a^3*c - 4*a^2*
b^2))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b
^2)))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b
^2)) + (((((4*A*a^3*b*c^3 - 4*A*a^2*b^3*c^2 + 4*B*a^3*b^2*c^2)/a^3 + (2*a*b
^2*c^2*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(16*a^3*c - 4*a^2*b^2
))*(2*A*a*c - A*b^2 + B*a*b))/(4*a^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*A*a
*c - A*b^2 + B*a*b)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*a*(4*
a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(2*A*a*c - A*b^2 + B*a*b))/(4*a^2
*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*A*a*c - A*b^2 + B*a*b)^2*(2*A*b^3 - 2*B
*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(8*a^3*(4*a*c - b^2)*(16*a^3*c - 4*a^2*b^2
)))*(3*A*b^4 + A*a^2*c^2 - 3*B*a*b^3 - 9*A*a*b^2*c + 8*B*a^2*b*c))/(c^2*(4*
A^2*a^2*c^4 + A^2*b^4*c^2 + B^2*a^2*b^2*c^2 - 4*A^2*a*b^2*c^3 - 2*A*B*a*b^3
*c^2 + 4*A*B*a^2*b*c^3)*(25*B^2*a^3*c - 6*A^2*b^4 + A^2*a^2*c^2 - 6*B^2*a^2
*b^2 + 12*A*B*a*b^3 + 24*A^2*a*b^2*c - 49*A*B*a^2*b*c)))*(2*A*a*c - A*b^2 +
 B*a*b))/(2*a^2*(4*a*c - b^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.85 $\int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=261

$$\frac{\left(-\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\tan^{-1}}{\sqrt{2}\,c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 1.49, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1279, 1166, 205}

$$\frac{\left(-\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x(bB-Ac)}{c^2}+\frac{Bx^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(((b*B - A*c)*x)/c^2) + (B*x^3)/(3*c) + ((b^2*B - A*b*c - a*B*c - (b^3*B -
 A*b^2*c - 3*a*b*B*c + 2*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a
*c]]) + ((b^2*B - A*b*c - a*B*c + (b^3*B - A*b^2*c - 3*a*b*B*c + 2*a*A*c^2)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4 \left(A+Bx^2\right)}{a+bx^2+cx^4}\,dx &= \frac{Bx^3}{3c} - \frac{\int \frac{x^2\left(3aB+3(bB-Ac)x^2\right)}{a+bx^2+cx^4}\,dx}{3c} \\
&= -\frac{(bB-Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{\int \frac{3a(bB-Ac)+3\left(b^2B-Abc-aBc\right)x^2}{a+bx^2+cx^4}\,dx}{3c^2} \\
&= -\frac{(bB-Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{\left(b^2B-Abc-aBc-\frac{b^3B-Ab^2c-3abBc+2aAc^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2} + \\
&= -\frac{(bB-Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{\left(b^2B-Abc-aBc-\frac{b^3B-Ab^2c-3abBc+2aAc^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} +
\end{aligned}
$$

Mathematica [A] time = 0.40, size = 327, normalized size = 1.25

$$
\frac{\left(-Abc\sqrt{b^2-4ac}-2aAc^2+b^2B\sqrt{b^2-4ac}-aBc\sqrt{b^2-4ac}+3abBc+Ab^2c+b^3(-B)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(-Abc\sqrt{b^2-4ac}+2aAc^2+b^2B\sqrt{b^2-4ac}-aBc\sqrt{b^2-4ac}-3abBc-Ab^2c+b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x(Ac-bB)}{c^2} + \frac{Bx^3}{3c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-(b*B) + A*c)*x)/c^2 + (B*x^3)/(3*c) + ((-(b^3*B) + A*b^2*c + 3*a*b*B*c -
 2*a*A*c^2 + b^2*B*Sqrt[b^2 - 4*a*c] - A*b*c*Sqrt[b^2 - 4*a*c] - a*B*c*Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sq
rt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3*B - A*
b^2*c - 3*a*b*B*c + 2*a*A*c^2 + b^2*B*Sqrt[b^2 - 4*a*c] - A*b*c*Sqrt[b^2 -
4*a*c] - a*B*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*
c]])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^4 \left(A+Bx^2\right)}{a+bx^2+cx^4}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4), x]
```

fricas [B] time = 3.52, size = 5140, normalized size = 19.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*B*c*x^3 + 3*sqrt(1/2)*c^2*sqrt(-(B^2*b^5 - (4*A*B*a^2 + 3*A^2*a*b)*c
^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*A*B*b^4)*
c + (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^
3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2
+ 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*
A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2
*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b
^2*c^5 - 4*a*c^6))*log(-2*(B^4*a^2*b^4 - A*B^3*a*b^5 - A^4*a^2*c^4 + (5*A^3
```

```
*B*a^2*b + A^4*a*b^2)*c^3 + (B^4*a^4 + 3*A*B^3*a^3*b - 6*A^2*B^2*a^2*b^2 -
3*A^3*B*a*b^3)*c^2 - (3*B^4*a^3*b^2 - A*B^3*a^2*b^3 - 3*A^2*B^2*a*b^4)*c)*x
 + sqrt(1/2)*(B^3*b^7 - 4*A^3*a^2*c^5 + (4*A*B^2*a^3 + 20*A^2*B*a^2*b + 5*A
^3*a*b^2)*c^4 - (4*B^3*a^3*b + 29*A*B^2*a^2*b^2 + 17*A^2*B*a*b^3 + A^3*b^4)
*c^3 + (13*B^3*a^2*b^3 + 19*A*B^2*a*b^4 + 3*A^2*B*b^5)*c^2 - (7*B^3*a*b^5 +
 3*A*B^2*b^6)*c - (B*b^4*c^5 + 4*(2*B*a^2 + A*a*b)*c^7 - (6*B*a*b^2 + A*b^3
)*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a
*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3
+ A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2
*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2
*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(B^2*b^5 - (4
*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*
B^2*a*b^3 + 2*A*B*b^4)*c + (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6
- 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*
b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 +
 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 +
 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^
2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 3*sqrt(1/2)*c^2*sqrt(-(B^2*b^5
 - (4*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2
- (5*B^2*a*b^3 + 2*A*B*b^4)*c + (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4*a^2
*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3
*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*
b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*
b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c
)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-2*(B^4*a^2*b^4 - A*B^3*
a*b^5 - A^4*a^2*c^4 + (5*A^3*B*a^2*b + A^4*a*b^2)*c^3 + (B^4*a^4 + 3*A*B^3*
a^3*b - 6*A^2*B^2*a^2*b^2 - 3*A^3*B*a*b^3)*c^2 - (3*B^4*a^3*b^2 - A*B^3*a^2
*b^3 - 3*A^2*B^2*a*b^4)*c)*x - sqrt(1/2)*(B^3*b^7 - 4*A^3*a^2*c^5 + (4*A*B^
2*a^3 + 20*A^2*B*a^2*b + 5*A^3*a*b^2)*c^4 - (4*B^3*a^3*b + 29*A*B^2*a^2*b^2
 + 17*A^2*B*a*b^3 + A^3*b^4)*c^3 + (13*B^3*a^2*b^3 + 19*A*B^2*a*b^4 + 3*A^2
*B*b^5)*c^2 - (7*B^3*a*b^5 + 3*A*B^2*b^6)*c - (B*b^4*c^5 + 4*(2*B*a^2 + A*a
*b)*c^7 - (6*B*a*b^2 + A*b^3)*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2
*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B
^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^
2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*
b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a
*c^11)))*sqrt(-(B^2*b^5 - (4*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*
B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*A*B*b^4)*c + (b^2*c^5 - 4*a*c^6)*
sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c
^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b
^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*
b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4
*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) + 3*
sqrt(1/2)*c^2*sqrt(-(B^2*b^5 - (4*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b +
 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*A*B*b^4)*c - (b^2*c^5 - 4*a*
c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b
^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 +
A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A
^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(
3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*
log(-2*(B^4*a^2*b^4 - A*B^3*a*b^5 - A^4*a^2*c^4 + (5*A^3*B*a^2*b + A^4*a*b^
2)*c^3 + (B^4*a^4 + 3*A*B^3*a^3*b - 6*A^2*B^2*a^2*b^2 - 3*A^3*B*a*b^3)*c^2
- (3*B^4*a^3*b^2 - A*B^3*a^2*b^3 - 3*A^2*B^2*a*b^4)*c)*x + sqrt(1/2)*(B^3*b
^7 - 4*A^3*a^2*c^5 + (4*A*B^2*a^3 + 20*A^2*B*a^2*b + 5*A^3*a*b^2)*c^4 - (4*
B^3*a^3*b + 29*A*B^2*a^2*b^2 + 17*A^2*B*a*b^3 + A^3*b^4)*c^3 + (13*B^3*a^2*
b^3 + 19*A*B^2*a*b^4 + 3*A^2*B*b^5)*c^2 - (7*B^3*a*b^5 + 3*A*B^2*b^6)*c + (
B*b^4*c^5 + 4*(2*B*a^2 + A*a*b)*c^7 - (6*B*a*b^2 + A*b^3)*c^6)*sqrt((B^4*b^
8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^
4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*
```

```
(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (
11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A
*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(B^2*b^5 - (4*A*B*a^2 + 3*A^2*a*
b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*A*B*b
^4)*c - (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 +
4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*
b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 +
 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6
*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11))
)/(b^2*c^5 - 4*a*c^6))) - 3*sqrt(1/2)*c^2*sqrt(-(B^2*b^5 - (4*A*B*a^2 + 3*A
^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*
A*B*b^4)*c - (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a
^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2
*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*
b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^
5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c
^11)))/(b^2*c^5 - 4*a*c^6))*log(-2*(B^4*a^2*b^4 - A*B^3*a*b^5 - A^4*a^2*c^4
 + (5*A^3*B*a^2*b + A^4*a*b^2)*c^3 + (B^4*a^4 + 3*A*B^3*a^3*b - 6*A^2*B^2*a
^2*b^2 - 3*A^3*B*a*b^3)*c^2 - (3*B^4*a^3*b^2 - A*B^3*a^2*b^3 - 3*A^2*B^2*a*
b^4)*c)*x - sqrt(1/2)*(B^3*b^7 - 4*A^3*a^2*c^5 + (4*A*B^2*a^3 + 20*A^2*B*a^
2*b + 5*A^3*a*b^2)*c^4 - (4*B^3*a^3*b + 29*A*B^2*a^2*b^2 + 17*A^2*B*a*b^3 +
 A^3*b^4)*c^3 + (13*B^3*a^2*b^3 + 19*A*B^2*a*b^4 + 3*A^2*B*b^5)*c^2 - (7*B^
3*a*b^5 + 3*A*B^2*b^6)*c + (B*b^4*c^5 + 4*(2*B*a^2 + A*a*b)*c^7 - (6*B*a*b^
2 + A*b^3)*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*
b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3
*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*
a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6
)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(B^2
*b^5 - (4*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*
c^2 - (5*B^2*a*b^3 + 2*A*B*b^4)*c - (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4
*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A
*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*
a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*
a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^
7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 6*(B*b - A*c)*x)/c^2
```

giac [B] time = 3.79, size = 4391, normalized size = 16.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*((2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*A*c^2 - (
2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
```

```
c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4
*a*c)*a^2*c^4)*B*c^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3
 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 16*a^2*b^2*c^5
 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^6 + 32*a^3*c^6 - 2*(b^2
- 4*a*c)*a*b^2*c^4 + 8*(b^2 - 4*a*c)*a^2*c^5)*A*abs(c) - 2*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a
*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a
*c)*a^2*b*c^4)*B*abs(c) - (2*b^5*c^5 - 12*a*b^3*c^6 + 16*a^2*b*c^7 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 + 6*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 4*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^5 + 4*(b^2 - 4*a*
c)*a*b*c^6)*A + (2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*
a*b^2*c^5)*B)*arctan(2*sqrt(1/2)*x/sqrt((b*c^3 + sqrt(b^2*c^6 - 4*a*c^7))/c
^4))/((a*b^4*c^4 - 8*a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 +
 a*b^2*c^6 - 4*a^2*c^7)*c^2) + 1/8*((2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^
5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b
^2 - 4*a*c)*a*b*c^4)*A*c^2 - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 3
2*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 +
 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 -
 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*B*c^2 - 2*(sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 2*a*b^4
*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^2*c^5 + 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*c^6 - 32*a^3*c^6 + 2*(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)
```

```
*A*abs(c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4
+ sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4
*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*B*abs(c) - (2*b^5*c^5 - 12*a*b
^3*c^6 + 16*a^2*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^5*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^4*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*
c^5 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 2*(b^2 -
4*a*c)*b^3*c^5 + 4*(b^2 - 4*a*c)*a*b*c^6)*A + (2*b^6*c^4 - 14*a*b^4*c^5 + 2
4*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*
c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 -
 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 -
 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a
*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5)*B)*arctan(2*sqrt(1/2)*x/sqrt((b*c^
3 - sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^4 - 8*a^2*b^2*c^5 - 2*a*b^3*c^
5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^7)*c^2) + 1/3*(B*c^2*x^3
 - 3*B*b*c*x + 3*A*c^2*x)/c^3
```

maple [B] time = 0.05, size = 825, normalized size = 3.16

$$\frac{\sqrt{2}\, A a \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, A a \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, A b^2 \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\, c}-\frac{\sqrt{2}\, A b^2 \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\, c}-\frac{3\sqrt{2}\, B a b \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\, c}-\frac{3\sqrt{2}\, B a b \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\, c}+\frac{\sqrt{2}\, B b^3 \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\, c^2}+\frac{\sqrt{2}\, B b^3 \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\, c^2}+\frac{B x^3}{3c}+\frac{\sqrt{2}\, A b \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\, c}-\frac{\sqrt{2}\, A b \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\, c}+\frac{\sqrt{2}\, B a \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\, c}-\frac{\sqrt{2}\, B a \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\, c}-\frac{\sqrt{2}\, B b^2 \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\, c^2}+\frac{\sqrt{2}\, B b^2 \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\, c^2}+\frac{A x}{c}-\frac{B b x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3*B*x^3/c+1/c*A*x-1/c^2*b*B*x+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b+1/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*A-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*A*b^2+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*B-1/2/c^2*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*b^2*B-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*B+1/2/c^2/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*B-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b+1/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*A-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*A*b^2-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*B+1/2/c^2*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*B-
3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*B+1/2/c^2/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*b^3*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{Bcx^3 - 3\,(Bb - Ac)x}{3\,c^2} - \frac{-\int \frac{Bab-Aac+\left(Bb^2-(Ba+Ab)c\right)x^2}{cx^4+bx^2+a}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*(B*c*x^3 - 3*(B*b - A*c)*x)/c^2 - integrate(-(B*a*b - A*a*c + (B*b^2 -
(B*a + A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/c^2
```

mupad [B] time = 1.59, size = 10177, normalized size = 38.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x*(A/c - (B*b)/c^2) - atan(((((16*A*a^2*c^5 - 4*A*a*b^2*c^4 + 4*B*a*b^3*c^3
 - 16*B*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(B^2*b^7 + A^2*b^5
*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2
+ A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/
2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 -
A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3
- 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(
-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^
2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2
*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*
c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*
B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*
(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*
b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c -
b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^
4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(B^2*b^6 + 2*A^2*a^2*c^4 + A^2*b^4*c^2
- 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a
*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b*c^3))/c^3)*(-(B^2*b^7 + A^2*b^5*
c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 +
 A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2
) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A
^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 -
 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-
(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2
*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((16*A*a^2*c^5 - 4*A*a*b^2*c^4
+ 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(B^
2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B
^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A
^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A
*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2
- 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(
1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(B^2*b^7 + A^
2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3
*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3
)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c
^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2
*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^
3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(
16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(B^2*b^6 + 2*A^2*a^2*c^4
 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*a*
```

b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b*c^3))/c^3)*(-(B^2
*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^
2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c
- b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^
2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*
B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 -
2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1
/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((((16*A*a^2*c^5 -
4*A*a*b^2*c^4 + 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16
*a*b*c^6)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A
*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a
*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a
^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16
*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*
(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c +
25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-
(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 +
12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 -
36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4
*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(B^2*b^6
+ 2*A^2*a^2*c^4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*b^2
*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b*c^
3))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*
B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a
^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*
b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^
3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*
A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a
c - b^2)^3)^(1/2))/(8(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((16
*A*a^2*c^5 - 4*A*a*b^2*c^4 + 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 + (2*x*(4*
b^3*c^5 - 16*a*b*c^6)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)
^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^
(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5
*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2
) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3
)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a
*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^
(1/2))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2
*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^
2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2
*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2
*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) +
16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(
4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2
x(B^2*b^6 + 2*A^2*a^2*c^4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9
*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*
A*B*a^2*b*c^3))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^
(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(
1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*
c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2)
- 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)
^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*
b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(
1/2) + (2*(B^3*a^4*c - B^3*a^3*b^2 + A*B^2*a^2*b^3 + A^2*B*a^3*c^2 + A^3*a^
2*b*c^2 - 2*A^2*B*a^2*b^2*c))/c^3))*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4
*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*

a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i - atan(((((16*A*a^2*c^5 - 4*A*a*b^2*c^4 + 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(B^2*b^6 + 2*A^2*a^2*c^4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b*c^3))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((16*A*a^2*c^5 - 4*A*a*b^2*c^4 + 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(B^2*b^6 + 2*A^2*a^2*c^4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b*c^3))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((((16*A*a^2*c^5 - 4*A*a*b^2*c^4 + 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^

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2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) +
 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^
3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*
c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2
*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^
3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^
3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*
b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(B^2*b
^6 + 2*A^2*a^2*c^4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*
b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b
*c^3))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2
*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^
2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2
*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2
*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) +
16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(
4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + ((
(16*A*a^2*c^5 - 4*A*a*b^2*c^4 + 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 + (2*x*
(4*b^3*c^5 - 16*a*b*c^6)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)
^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^
3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*
b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(
1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2
)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*
B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)
))^(1/2))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2)
- 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) -
 B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*
A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*
B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2)
 + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) +
 (2*x*(B^2*b^6 + 2*A^2*a^2*c^4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c
+ 9*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 -
10*A*B*a^2*b*c^3))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^
3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3
)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b
^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1
/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)
^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B
*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))
)^(1/2) + (2*(B^3*a^4*c - B^3*a^3*b^2 + A*B^2*a^2*b^3 + A^2*B*a^3*c^2 + A^3
*a^2*b*c^2 - 2*A^2*B*a^2*b^2*c))/c^3))*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(
-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-
(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*
c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a
*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*
(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3
)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5
- 8*a*b^2*c^6)))^(1/2)*2i + (B*x^3)/(3*c)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.86 $\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=208

$$-\frac{\left(-\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{Bx}{c}$$

Rubi [A] time = 0.53, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1279, 1166, 205}

$$-\frac{\left(-\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{Bx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (B*x)/c - ((b*B - A*c - (b^2*B - A*b*c - 2*a*B*c)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) - ((b*B - A*c + (b^2*B - A*b*c - 2*a*B*c)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^2 \left(A+Bx^2\right)}{a+bx^2+cx^4}\,dx = \frac{Bx}{c} - \frac{\int \frac{aB+(bB-Ac)x^2}{a+bx^2+cx^4}\,dx}{c}$$
$$= \frac{Bx}{c} - \frac{\left(bB-Ac-\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} - \frac{\left(bB-Ac+\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-\ldots}}}{2c}$$
$$= \frac{Bx}{c} - \frac{\left(bB-Ac-\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(bB-Ac+\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\ldots}{\sqrt{b\ldots}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.16, size = 251, normalized size = 1.21

$$-\frac{\left(-Ac\sqrt{b^2-4ac}+bB\sqrt{b^2-4ac}+2aBc+Abc+b^2(-B)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(-Ac\sqrt{b^2-4ac}+bB\sqrt{b^2-4ac}-2aBc-Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{Bx}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (B*x)/c - ((-(b^2*B) + A*b*c + 2*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - A*c*Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b^2*B - A*b*
c - 2*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - A*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*
c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(A+Bx^2\right)}{a+bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4), x]
```

fricas [B] time = 1.30, size = 2632, normalized size = 12.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*
B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A
^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 +
2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2
 - A*B^3*b^3 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b
^2)*c)*x + sqrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b +
A^2*B*b^2)*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c - (B*b^3*c^3 + 8*A*a*c^5 - 2
*(2*B*a*b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)
*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3
*b^3)*c)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*
B^2*a*b + 2*A*B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A
^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2
*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) -
```

```
sqrt(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^
2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B
*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*
B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2 - A
*B^3*b^3 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b^2)*
c)*x - sqrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*
B*b^2)*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c - (B*b^3*c^3 + 8*A*a*c^5 - 2*(2*
B*a*b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3
 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3
)*c)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*
a*b + 2*A*B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B
^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^
4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqr
t(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c
 - (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*
c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*
b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2 - A*B^3
*b^3 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b^2)*c)*x
 + sqrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*B*b^
2)*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c + (B*b^3*c^3 + 8*A*a*c^5 - 2*(2*B*a*
b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (
B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)
/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b
+ 2*A*B*b^2)*c - (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a
 + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*
b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/
2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c - (
b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3
+ (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)
*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2 - A*B^3*b^3
 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b^2)*c)*x - s
qrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*B*b^2)*c
^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c + (B*b^3*c^3 + 8*A*a*c^5 - 2*(2*B*a*b +
A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*
a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^
2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*
A*B*b^2)*c - (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2
*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2
+ 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + 2*B*x)/c
```

giac [B] time = 3.50, size = 3179, normalized size = 15.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] B*x/c + 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*A*c^2 - (2*b
^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
```

```
*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^
2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2
*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*B*c^2 - 2*(sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*
a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2
*c^4)*B*abs(c) - (2*b^4*c^5 - 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^5)*A + (2*b^5*c^4 - 12*a*b^3*c^
5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^
3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 -
 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c
)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*B)*arctan(2*sqrt(1/2)*x/sqrt((b*c + sq
rt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16
*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*((2*b^4*c^3 - 16
*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c
^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c
)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*A*c^2 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a
^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 +
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(
b^2 - 4*a*c)*a*b*c^3)*B*c^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*
b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 +
2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*B*abs(c) - (2*b^4*c^5
- 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 2*(b^2
 - 4*a*c)*b^2*c^5)*A + (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*
a*b*c^5)*B)*arctan(2*sqrt(1/2)*x/sqrt((b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2))
```

```
/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b
^2*c^5 - 4*a^2*c^6)*c^2)
```

maple [B] time = 0.03, size = 560, normalized size = 2.69

$$\frac{\sqrt{2}\,Ab\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,Ab\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,Ba\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,Ba\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,B\,b^2\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\,c}-\frac{\sqrt{2}\,B\,b^2\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\,c}-\frac{\sqrt{2}\,A\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,Bb\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\,c}-\frac{\sqrt{2}\,Bb\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\,c}+\frac{Bx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] B*x/c-1/2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*A*b+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*a*B-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*B+1/2*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)*A+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b-1/2/c*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*b*B+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*B-1/2/c/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{Bx}{c}+\frac{-\int\frac{(Bb-Ac)x^2+Ba}{cx^4+bx^2+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] B*x/c + integrate(-((B*b - A*c)*x^2 + B*a)/(c*x^4 + b*x^2 + a), x)/c
```

mupad [B] time = 1.26, size = 6366, normalized size = 30.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (B*x)/c - atan(((((16*B*a^2*c^3 - 4*B*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a
*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b
^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3
- 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A
*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c -
b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2
*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 1
2*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8
*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(B^2*b^4 - 2*A^2*a*c^3
 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*B^2*a*b^2*c + 6*A*B*a*b*c^
2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^
2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 -
 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*
B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3
```

```
 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*B*a^2*c^3 - 4*B*a*b^2*c^2)/c + (2*x*(4*b
^3*c^3 - 16*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^
(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4
*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^
2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2
*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^
2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c
 - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)
^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*
a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(B^2*b^4
- 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*B^2*a*b^2*c +
 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(
1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*
A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2
*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*
c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*B*a^2*c^3 - 4*B*a*b^2*c^2)/
c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a
*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*
B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2
) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^
2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5 + A^2*b^3
*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2)
- 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-
(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1
/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (
2*x*(B^2*b^4 - 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*
B^2*a*b^2*c + 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B
*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2)
 + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2
)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*B*a^2*c^3 - 4*B*a
*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*
c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4
*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^
2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*
B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5
 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)
^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c +
B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c -
b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))
^(1/2) + (2*x*(B^2*b^4 - 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*
b^3*c - 4*B^2*a*b^2*c + 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c
^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*
c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2
)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B
*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(A^3*a*c^2
 - B^3*a^2*b + A*B^2*a*b^2 + A*B^2*a^2*c - 2*A^2*B*a*b*c))/c))*(-(B^2*b^5 +
 A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3
)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^
2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^
2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(
1/2)*2i - atan(((((16*B*a^2*c^3 - 4*B*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a
*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b
^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3
- 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A
*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a*c -
b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2
*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 1
```

```
2*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8
*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(B^2*b^4 - 2*A^2*a*c^3
 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*B^2*a*b^2*c + 6*A*B*a*b*c^
2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b^
2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 -
 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A*
B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3
 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*B*a^2*c^3 - 4*B*a*b^2*c^2)/c + (2*x*(4*b
^3*c^3 - 16*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^
(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4
*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^
2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2
*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^
2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c
 - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)
^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*
a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(B^2*b^4
- 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*B^2*a*b^2*c +
 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^(
1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*
A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2
*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*
c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*B*a^2*c^3 - 4*B*a*b^2*c^2)/
c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a
*c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*
B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^(1/2
) + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^
2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5 + A^2*b^3
*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2)
- 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-
(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1
/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (
2*x*(B^2*b^4 - 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*
B^2*a*b^2*c + 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B
*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^(1/2)
 + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2
)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*B*a^2*c^3 - 4*B*a
*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*
c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4
*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^
2)^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*
B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5
 + A^2*b^3*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)
^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c -
B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c -
b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))
^(1/2) + (2*x*(B^2*b^4 - 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*
b^3*c - 4*B^2*a*b^2*c + 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c
^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*
c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2
)^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B
*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(A^3*a*c^2
 - B^3*a^2*b + A*B^2*a*b^2 + A*B^2*a^2*c - 2*A^2*B*a*b*c))/c))*(-(B^2*b^5 +
 A^2*b^3*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3
)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^
2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^
2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(
1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.87 $\int \frac{A+Bx^2}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=172

$$\frac{\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{bB-2Ac}{\sqrt{b^2-4ac}}+B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.20, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1166, 205}

$$\frac{\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{bB-2Ac}{\sqrt{b^2-4ac}}+B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((B - (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((B +
(b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx+\frac{1}{2}\left(B+\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-\ldots}}\\ &= \frac{\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(B+\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 173, normalized size = 1.01

$$\frac{\frac{\left(B\sqrt{b^2-4ac}+2Ac-bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(B\sqrt{b^2-4ac}-2Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (((-(b*B) + 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((b*B - 2*A*c + B*Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqr
t[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{a + bx^2 + cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(a + b*x^2 + c*x^4), x]
```

fricas [B] time = 1.33, size = 1569, normalized size = 9.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*
sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2
*c - 4*a^2*c^2))*log(-2*(B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*x + sqr
t(1/2)*(A*B^2*a*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*c + (4*(2*B*a^3
 - A*a^2*b)*c^2 - (2*B*a^2*b^2 - A*a*b^3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c
+ A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c
 + (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*
c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*sqrt(-(B^2*a*b -
 (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c
+ A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(B^4*a
^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*x - sqrt(1/2)*(A*B^2*a*b^2 + 4*A^3*a*
c^2 - (4*A*B^2*a^2 + A^3*b^2)*c + (4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B*a^2*b^2
 - A*a*b^3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^
3*c^3)))*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*sqrt(
(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c -
4*a^2*c^2))) + 1/2*sqrt(1/2)*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c - (a*b^2*
c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^
3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A
^4*c^2)*x + sqrt(1/2)*(A*B^2*a*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*
c - (4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B*a^2*b^2 - A*a*b^3)*c)*sqrt((B^4*a^2 -
 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(B^2*a*b - (4*A
*B*a - A^2*b)*c - (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4
*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*s
qrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c - (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 -
 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2)
)*log(-2*(B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*x - sqrt(1/2)*(A*B^2*a
*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*c - (4*(2*B*a^3 - A*a^2*b)*c^2
 - (2*B*a^2*b^2 - A*a*b^3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2
*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c - (a*b^2*c - 4
*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3
)))/(a*b^2*c - 4*a^2*c^2)))
```

giac [B] time = 2.42, size = 1400, normalized size = 8.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] 1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)
*b*c^2)*A - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*B)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2
- 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2
+ a*b^2*c^2 - 4*a^2*c^3)*abs(c)) + 1/4*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^
2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*
c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*
c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b
^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*A + 2*(2*a*b^2*c^2 - 8*a^2*c^3 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*B)*arcta
n(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*
a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c))

maple [B] time = 0.02, size = 328, normalized size = 1.91

$$-\frac{\sqrt{2}\, Ac \,\mathrm{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, Ac \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, Bb \,\mathrm{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, Bb \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, B \,\mathrm{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, B \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)/(c*x^4+b*x^2+a),x)

[Out] -c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A-1/2*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B+1
/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B-c/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)*A+1/2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*b*B

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{cx^4 + bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 1.00, size = 4109, normalized size = 23.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] - atan((((-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*
(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c -
 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(x*(8*b^3
*c^2 - 32*a*b*c^3)*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*
c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2
*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)
 - 4*A*b^2*c^2 + 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4
*A*B*b*c^2))*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^
2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b
*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i +
((-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c
 - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a
*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(4*A*b^2*c^2 + x*
(8*b^3*c^2 - 32*a*b*c^3)*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^
2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 -
 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))
^(1/2) - 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c
^2))*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4
*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A
*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i)/(((-(B^2*
a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^
3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/
(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(x*(8*b^3*c^2 - 32*a*b*c^
3)*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a
*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B
*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - 4*A*b^2*c^2 +
 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c^2))*(-(
B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b
^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2
*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - ((-(B^2*a*b^3 + B^2
*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) +
16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*
c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(4*A*b^2*c^2 + x*(8*b^3*c^2 - 32*a*b
*c^3)*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(
4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*
A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - 16*A*a*c^3
) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c^2))*(-(B^2*a*b^3 +
 B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2
) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*
a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + 2*A^2*B*c^2 + 2*B^3*a*c - 2*A*
B^2*b*c))*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c
*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c
- 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*2i - ata
n((((-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*
a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*
B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(x*(8*b^3*c^2
- 32*a*b*c^3)*(-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A
^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*
b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - 4*
A*b^2*c^2 + 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*
```

```
b*c^2))*(-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(
-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c -
4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i + ((-(B
^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c - b^
2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*
c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(4*A*b^2*c^2 + x*(8*b^
3*c^2 - 32*a*b*c^3)*(-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3
*c + A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^
2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2
) - 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c^2))*
(-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c
- b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*
b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i)/(((-(B^2*a*b^3
 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c - b^2)^3)^(1
/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(1
6*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(x*(8*b^3*c^2 - 32*a*b*c^3)*(-
(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c -
b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^
2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - 4*A*b^2*c^2 + 16*A
*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c^2))*(-(B^2*a
*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c - b^2)^3
)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(
8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - ((-(B^2*a*b^3 - B^2*a*(-
(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*
B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 -
 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(4*A*b^2*c^2 + x*(8*b^3*c^2 - 32*a*b*c^3)
*(-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c
 - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a
*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - 16*A*a*c^3) + x
*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c^2))*(-(B^2*a*b^3 - B^2*
a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 1
6*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c
^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + 2*A^2*B*c^2 + 2*B^3*a*c - 2*A*B^2*b
*c))*(-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4
*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A
*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*2i
```

sympy [A] time = 16.96, size = 314, normalized size = 1.83

$$\operatorname{RootSum}\left(t^4\left(256a^3c^3-128a^2b^2c^2+16ab^4c\right)+t^2\left(-16A^2abc^2+4A^2b^3c+64ABa^2c^2-16ABab^2c-16B^2a^2bc+4B^2ab^3\right)+A^4c^2-2A^3Bbc+2A^2B^2ac+A^2B^2b^2-2AB^3ab+B^4a^2,\left(t\mapsto t\log\left(x+\frac{-32t^3Aa^2bc^2+8t^3Aab^3c+64t^3Ba^3c^2-16t^3Ba^2b^2c-4tA^3ac^2+2tA^3b^2c-6tA^2Babc+12tAB^2a^2c-2tB^3a^2b}{-A^4c^2+A^3Bbc-AB^3ab+B^4a^2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**3 - 128*a**2*b**2*c**2 + 16*a*b**4*c) + _t**2*(-
16*A**2*a*b*c**2 + 4*A**2*b**3*c + 64*A*B*a**2*c**2 - 16*A*B*a*b**2*c - 16*
B**2*a**2*b*c + 4*B**2*a*b**3) + A**4*c**2 - 2*A**3*B*b*c + 2*A**2*B**2*a*c
 + A**2*B**2*b**2 - 2*A*B**3*a*b + B**4*a**2, Lambda(_t, _t*log(x + (-32*_t
**3*A*a**2*b*c**2 + 8*_t**3*A*a*b**3*c + 64*_t**3*B*a**3*c**2 - 16*_t**3*B*
a**2*b**2*c - 4*_t*A**3*a*c**2 + 2*_t*A**3*b**2*c - 6*_t*A**2*B*a*b*c + 12*
_t*A*B**2*a**2*c - 2*_t*B**3*a**2*b)/(-A**4*c**2 + A**3*B*b*c - A*B**3*a*b
+ B**4*a**2))))
```

3.88 $\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=189

$$-\frac{\sqrt{c}\left(\frac{Ab-2aB}{\sqrt{b^2-4ac}}+A\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{A}{ax}$$

Rubi [A] time = 0.40, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1281, 1166, 205}

$$-\frac{\sqrt{c}\left(\frac{Ab-2aB}{\sqrt{b^2-4ac}}+A\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{A}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(A/(a*x)) - (Sqrt[c]*(A + (A*b - 2*a*B)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) - (Sqrt[c]*(A - (A*b - 2*a*B)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]
])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx = -\frac{A}{ax} - \frac{\int \frac{Ab-aB+Acx^2}{a+bx^2+cx^4}\,dx}{a}$$
$$= -\frac{A}{ax} - \frac{\left(c\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a} - \frac{\left(c\left(A+\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+c x^2}\,dx}{2a}$$
$$= -\frac{A}{ax} - \frac{\sqrt{c}\left(A+\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.29, size = 206, normalized size = 1.09

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(A\left(\sqrt{b^2-4ac}+b\right)-2aB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(A\left(\sqrt{b^2-4ac}-b\right)+2aB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2A}{x}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*((2*A)/x + (Sqrt[2]*Sqrt[c]*(-2*a*B + A*(b + Sqrt[b^2 - 4*a*c]))*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt
[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*a*B + A*(-b + Sqrt[b^2 - 4*a
*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)), x]
```

fricas [B] time = 1.47, size = 2914, normalized size = 15.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*a*x*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 -
3*A^2*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^
2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*
B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(2*(A
^4*a*c^3 + (A^3*B*a*b - A^4*b^2)*c^2 - (B^4*a^3 - 3*A*B^3*a^2*b + 3*A^2*B^2
*a*b^2 - A^3*B*b^3)*c)*x + sqrt(1/2)*(B^3*a^3*b^2 - 3*A*B^2*a^2*b^3 + 3*A^2
*B*a*b^4 - A^3*b^5 + 4*(A^2*B*a^3 - A^3*a^2*b)*c^2 - (4*B^3*a^4 - 12*A*B^2*
a^3*b + 13*A^2*B*a^2*b^2 - 5*A^3*a*b^3)*c - (B*a^4*b^3 - A*a^3*b^4 - 8*A*a^
5*c^2 - 2*(2*B*a^5*b - 3*A*a^4*b^2)*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^
2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*
A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(B^2*a^2*b - 2*A*B*
```

```
a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((B^4
*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^
2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c))
)/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^
2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A
*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*
(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2
 - 4*a^4*c))*log(2*(A^4*a*c^3 + (A^3*B*a*b - A^4*b^2)*c^2 - (B^4*a^3 - 3*A*
B^3*a^2*b + 3*A^2*B^2*a*b^2 - A^3*B*b^3)*c)*x - sqrt(1/2)*(B^3*a^3*b^2 - 3*
A*B^2*a^2*b^3 + 3*A^2*B*a*b^4 - A^3*b^5 + 4*(A^2*B*a^3 - A^3*a^2*b)*c^2 - (
4*B^3*a^4 - 12*A*B^2*a^3*b + 13*A^2*B*a^2*b^2 - 5*A^3*a*b^3)*c - (B*a^4*b^3
 - A*a^3*b^4 - 8*A*a^5*c^2 - 2*(2*B*a^5*b - 3*A*a^4*b^2)*c)*sqrt((B^4*a^4 -
 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2
- 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))*sqrt
(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c + (a^3*b^2
 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b
^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)
/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) + sqrt(1/2)*a*x*sqrt(-(B^2*a^2
*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c - (a^3*b^2 - 4*a^4*c
)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b
^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2
- 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(2*(A^4*a*c^3 + (A^3*B*a*b - A^4*b^2)*
c^2 - (B^4*a^3 - 3*A*B^3*a^2*b + 3*A^2*B^2*a*b^2 - A^3*B*b^3)*c)*x + sqrt(1
/2)*(B^3*a^3*b^2 - 3*A*B^2*a^2*b^3 + 3*A^2*B*a*b^4 - A^3*b^5 + 4*(A^2*B*a^3
 - A^3*a^2*b)*c^2 - (4*B^3*a^4 - 12*A*B^2*a^3*b + 13*A^2*B*a^2*b^2 - 5*A^3*
a*b^3)*c + (B*a^4*b^3 - A*a^3*b^4 - 8*A*a^5*c^2 - 2*(2*B*a^5*b - 3*A*a^4*b^
2)*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A
^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*
b^2 - 4*a^7*c)))*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*
A^2*a*b)*c - (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*
a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*
a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/
2)*a*x*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c
 - (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 -
4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^
4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(2*(A^4*a*c^3 + (
A^3*B*a*b - A^4*b^2)*c^2 - (B^4*a^3 - 3*A*B^3*a^2*b + 3*A^2*B^2*a*b^2 - A^3
*B*b^3)*c)*x - sqrt(1/2)*(B^3*a^3*b^2 - 3*A*B^2*a^2*b^3 + 3*A^2*B*a*b^4 - A
^3*b^5 + 4*(A^2*B*a^3 - A^3*a^2*b)*c^2 - (4*B^3*a^4 - 12*A*B^2*a^3*b + 13*A
^2*B*a^2*b^2 - 5*A^3*a*b^3)*c + (B*a^4*b^3 - A*a^3*b^4 - 8*A*a^5*c^2 - 2*(2
*B*a^5*b - 3*A*a^4*b^2)*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^
2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b
+ A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*
b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c - (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B
^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A
^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 -
 4*a^4*c))) - 2*A)/(a*x)
```

giac [B] time = 3.57, size = 2805, normalized size = 14.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -A/(a*x) - 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
```

```
^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*A*a^2 + 2*(sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c -
2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*
c)*a^2*b*c^2)*A*abs(a) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4
 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - 2*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*a^2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b
*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 16*a^3*b^2*c^2
 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 32*a^4*c^3 + 2*(b^2
- 4*a*c)*a^2*b^2*c - 8*(b^2 - 4*a*c)*a^3*c^2)*B*abs(a) + (2*a^2*b^4*c^2 - 8
*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2
*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2
*(b^2 - 4*a*c)*a^2*b^2*c^2)*A - 2*(2*a^3*b^3*c^2 - 8*a^4*b*c^3 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3 + 4*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c + 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^3*b*c^2)*B)*
arctan(2*sqrt(1/2)*x/sqrt((a*b + sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3*b^4
 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a
^4*c^3)*abs(a)*abs(c)) + 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*A
*a^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*
c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^3*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c +
 8*(b^2 - 4*a*c)*a^2*b*c^2)*A*abs(a) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - 2*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*a^2*b^4*c + 16*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 -
16*a^3*b^2*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 32*a^4
*c^3 - 2*(b^2 - 4*a*c)*a^2*b^2*c + 8*(b^2 - 4*a*c)*a^3*c^2)*B*abs(a) + (2*a
^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2)*A - 2*(2*a^3*b^3*c^2 - 8*a^4*b*c^
3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*
a^3*b*c^2)*B)*arctan(2*sqrt(1/2)*x/sqrt((a*b - sqrt(a^2*b^2 - 4*a^3*c))/(a*
c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3
```

```
*b^2*c^2 - 4*a^4*c^3)*abs(a)*abs(c))
```

maple [B] time = 0.03, size = 353, normalized size = 1.87

$$\frac{\sqrt{2}\, Abc \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\; a}+\frac{\sqrt{2}\, Abc \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\; a}-\frac{\sqrt{2}\, Bc \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, Bc \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, Ac \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\; a}-\frac{\sqrt{2}\, Ac \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\; a}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] -A/a/x+1/2/a*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A+1/2/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)*A*b-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B-1/2/a*c*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*c*x)*A+1/2/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b-c/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((A*c*x^2 - B*a + A*b)/(c*x^4 + b*x^2 + a), x)/a - A/(a*x)
```

mupad [B] time = 1.35, size = 6335, normalized size = 33.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)),x)
```

```
[Out] - atan(((x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*b
*c^3) + (-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a
^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c
- A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A
*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^
2 - 8*a^4*b^2*c)))^(1/2)*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b^5 + B^2
*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1
/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2
)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3
)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)
 - 16*B*a^6*c^3 + 16*A*a^5*b*c^3 - 4*A*a^4*b^3*c^2 + 4*B*a^5*b^2*c^2))*(-(A
^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*(-(4*a*c
- b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c - A^2*a*c*(-
(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*
a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^
2*c)))^(1/2)*1i + (x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4
*A*B*a^4*b*c^3) + (-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/
2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^
2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b
*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 +
 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*B*a^6*c^3 + x*(32*a^6*b*c^3 - 8*a^5*
b^3*c^2)*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*
a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c
 - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*
```

```
A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c
^2 - 8*a^4*b^2*c)))^(1/2) - 16*A*a^5*b*c^3 + 4*A*a^4*b^3*c^2 - 4*B*a^5*b^2*
c^2))*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2
*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c -
A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A*B
*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2
- 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b
^2*c^3 + 4*A*B*a^4*b*c^3) + (-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b
^2)^3)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*
c^2 - 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12
*A^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*
(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c
^2)*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*(
-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c - A^
2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A*B*a
*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 -
8*a^4*b^2*c)))^(1/2) - 16*B*a^6*c^3 + 16*A*a^5*b*c^3 - 4*A*a^4*b^3*c^2 + 4*
B*a^5*b^2*c^2))*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2)
 + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*
a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c
^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - (x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^
2*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) + (-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*
a*c - b^2)^3)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A
*B*a^3*c^2 - 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b
*c + 12*A^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2
*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*B*a^6*c^3 + x*(32*a
^6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2
)^3)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^
2 - 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A
^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a
^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - 16*A*a^5*b*c^3 + 4*A*a^4*b^3*c
^2 - 4*B*a^5*b^2*c^2))*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3
)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 -
 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*
a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*
b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 2*A^3*a^3*c^4 + 2*A*B^2*a^4*c^3 -
 2*A^2*B*a^3*b*c^3))*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^
(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7
*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^
2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^
4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - atan(((x*(4*A^2*a^4*c^4 - 4*B^2*
a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) + (-(A^2*b^5 + B^2*a^2*b^3 -
 A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*
B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^(1/2)
 - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) +
12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(x*(32*a^
6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b^5 + B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)
^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2
 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^
2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^
3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - 16*B*a^6*c^3 + 16*A*a^5*b*c^3 -
 4*A*a^4*b^3*c^2 + 4*B*a^5*b^2*c^2))*(-(A^2*b^5 + B^2*a^2*b^3 - A^2*b^2*(-(
4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16
*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3
*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b
^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i + (x*(4*A^2*a^4*c^
4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) + (-(A^2*b^5 + B^2
*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1
```

```
/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2
)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3
)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)
*(16*B*a^6*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b^5 + B^2*a^2*b^3
- A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A
*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^(1/2
) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) +
 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - 16*A*a
^5*b*c^3 + 4*A*a^4*b^3*c^2 - 4*B*a^5*b^2*c^2))*(-(A^2*b^5 + B^2*a^2*b^3 - A
^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*
a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) -
 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12
*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*
A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) + (-(A^2
*b^5 + B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c -
b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4
*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*
c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*
c)))^(1/2)*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b^5 + B^2*a^2*b^3 - A^2
*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*
b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4
*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A
*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - 16*B*a^6*c^
3 + 16*A*a^5*b*c^3 - 4*A*a^4*b^3*c^2 + 4*B*a^5*b^2*c^2))*(-(A^2*b^5 + B^2*a
^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1/2
) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^
3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^
(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) -
 (x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) +
 (-(A^2*b^5 + B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4
*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a
*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*
(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a
^4*b^2*c)))^(1/2)*(16*B*a^6*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b
^5 + B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^
2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a
*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c
- b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)
))^(1/2) - 16*A*a^5*b*c^3 + 4*A*a^4*b^3*c^2 - 4*B*a^5*b^2*c^2))*(-(A^2*b^5
+ B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^
3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c
- b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b
^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^
(1/2) + 2*A^3*a^3*c^4 + 2*A*B^2*a^4*c^3 - 2*A^2*B*a^3*b*c^3))*(-(A^2*b^5 +
B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)
^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c -
b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2
)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1
/2)*2i - A/(a*x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.89 $\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=271

$$-\frac{\sqrt{c}\left(aB\left(\sqrt{b^2-4ac}+b\right)-A\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(aB\left(b-\sqrt{b^2-4ac}\right)-A\left(\right.\right.}{\sqrt{2}\,a^2\sqrt{b}}$$

Rubi [A] time = 0.65, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1281, 1166, 205}

$$-\frac{\sqrt{c}\left(aB\left(\sqrt{b^2-4ac}+b\right)-A\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(aB\left(b-\sqrt{b^2-4ac}\right)-A\left(-b\sqrt{b^2-4ac}-2ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\,\sqrt{\sqrt{b^2-4ac}+b}}+\frac{Ab-aB}{a^2x}-\frac{A}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -A/(3*a*x^3) + (A*b - a*B)/(a^2*x) - (Sqrt[c]*(a*B*(b + Sqrt[b^2 - 4*a*c])
- A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) + (Sqrt[c]*(a*B*(b - Sqrt[b^2 - 4*a*c]) - A*(b^2 - 2*a*c - b*Sqrt[b
^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx = -\frac{A}{3ax^3} - \frac{\int \frac{3(Ab-aB)+3Acx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx}{3a}$$

$$= -\frac{A}{3ax^3} + \frac{Ab-aB}{a^2x} + \frac{\int \frac{3\left(Ab^2-abB-aAc\right)+3(Ab-aB)cx^2}{a+bx^2+cx^4}\,dx}{3a^2}$$

$$= -\frac{A}{3ax^3} + \frac{Ab-aB}{a^2x} + \frac{\left(c\left(aB\left(b-\sqrt{b^2-4ac}\right)-A\left(b^2-2ac-b\sqrt{b^2-4ac}\right)\right)\right)\int \frac{}{\frac{b}{2}+\frac{1}{2}\sqrt{b^{\cdots}}}}{2a^2\sqrt{b^2-4ac}}$$

$$= -\frac{A}{3ax^3} + \frac{Ab-aB}{a^2x} - \frac{\sqrt{c}\left(aB\left(b+\sqrt{b^2-4ac}\right)-A\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\cdots\right.}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.32, size = 267, normalized size = 0.99

$$\frac{-\frac{3\sqrt{2}\sqrt{c}\left(aB\left(\sqrt{b^2-4ac}+b\right)-A\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(A\left(b\sqrt{b^2-4ac}+2ac-b^2\right)+aB\left(b-\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{6Ab-6aB}{x}-\frac{2aA}{x^3}}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*A)/x^3 + (6*A*b - 6*a*B)/x - (3*Sqrt[2]*Sqrt[c]*(a*B*(b + Sqrt[b^2 -
 4*a*c]) - A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (3*Sqrt[2]*Sqrt[c]*(a*B*(b - Sqrt[b^2 - 4*a*c]) + A*(-b^2 + 2*a*c +
b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)), x]
```

fricas [B] time = 3.55, size = 5442, normalized size = 20.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(1/2)*a^2*x^3*sqrt(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*
B*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c +
(a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6
 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b +
 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^
3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A
^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))
/(a^5*b^2 - 4*a^6*c))*log(2*(A^4*a^2*c^5 + 3*(A^3*B*a^2*b - A^4*a*b^2)*c^4
```

```
- (B^4*a^4 - 5*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - A^3*B*a*b^3 - A^4*b^4)*c^3
 + (B^4*a^3*b^2 - 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 - A^3*B*b^5)*c^2)*x + s
qrt(1/2)*(B^3*a^3*b^5 - 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 - A^3*b^8 - 4*A^3*a
^4*c^4 + (4*A*B^2*a^5 - 20*A^2*B*a^4*b + 17*A^3*a^3*b^2)*c^3 + (4*B^3*a^5*b
 - 25*A*B^2*a^4*b^2 + 41*A^2*B*a^3*b^3 - 20*A^3*a^2*b^4)*c^2 - (5*B^3*a^4*b
^3 - 18*A*B^2*a^3*b^4 + 21*A^2*B*a^2*b^5 - 8*A^3*a*b^6)*c - (B*a^6*b^4 - A*
a^5*b^5 + 4*(2*B*a^8 - 3*A*a^7*b)*c^2 - (6*B*a^7*b^2 - 7*A*a^6*b^3)*c)*sqrt
((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b
^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B
^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2
*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*
B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))*sqrt(-(B^2*a^2*b^3 - 2*
A*B*a*b^4 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*
a^2*b^2 + 5*A^2*a*b^3)*c + (a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*
a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^
2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b +
 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b
^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)
*c)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 3*sqrt(1/2)*a^2*x^3*sqr
t(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (
3*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c + (a^5*b^2 - 4*a^6*c)*sqrt((B^
4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 +
 A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a
^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4
)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^
2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*
(A^4*a^2*c^5 + 3*(A^3*B*a^2*b - A^4*a*b^2)*c^4 - (B^4*a^4 - 5*A*B^3*a^3*b +
 6*A^2*B^2*a^2*b^2 - A^3*B*a*b^3 - A^4*b^4)*c^3 + (B^4*a^3*b^2 - 3*A*B^3*a^
2*b^3 + 3*A^2*B^2*a*b^4 - A^3*B*b^5)*c^2)*x - sqrt(1/2)*(B^3*a^3*b^5 - 3*A*
B^2*a^2*b^6 + 3*A^2*B*a*b^7 - A^3*b^8 - 4*A^3*a^4*c^4 + (4*A*B^2*a^5 - 20*A
^2*B*a^4*b + 17*A^3*a^3*b^2)*c^3 + (4*B^3*a^5*b - 25*A*B^2*a^4*b^2 + 41*A^2
*B*a^3*b^3 - 20*A^3*a^2*b^4)*c^2 - (5*B^3*a^4*b^3 - 18*A*B^2*a^3*b^4 + 21*A
^2*B*a^2*b^5 - 8*A^3*a*b^6)*c - (B*a^6*b^4 - A*a^5*b^5 + 4*(2*B*a^8 - 3*A*a
^7*b)*c^2 - (6*B*a^7*b^2 - 7*A*a^6*b^3)*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*
b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^
2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*
A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 -
 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/
(a^10*b^2 - 4*a^11*c)))*sqrt(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*B
*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c + (
a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6
 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b +
3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3
*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^
2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))/
(a^5*b^2 - 4*a^6*c))) + 3*sqrt(1/2)*a^2*x^3*sqrt(-(B^2*a^2*b^3 - 2*A*B*a*b^
4 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2
+ 5*A^2*a*b^3)*c - (a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5
+ 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^
5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*
B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A
*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^1
0*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(A^4*a^2*c^5 + 3*(A^3*B*a^2*
b - A^4*a*b^2)*c^4 - (B^4*a^4 - 5*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - A^3*B*a
*b^3 - A^4*b^4)*c^3 + (B^4*a^3*b^2 - 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 - A^
3*B*b^5)*c^2)*x + sqrt(1/2)*(B^3*a^3*b^5 - 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7
- A^3*b^8 - 4*A^3*a^4*c^4 + (4*A*B^2*a^5 - 20*A^2*B*a^4*b + 17*A^3*a^3*b^2)
*c^3 + (4*B^3*a^5*b - 25*A*B^2*a^4*b^2 + 41*A^2*B*a^3*b^3 - 20*A^3*a^2*b^4)
*c^2 - (5*B^3*a^4*b^3 - 18*A*B^2*a^3*b^4 + 21*A^2*B*a^2*b^5 - 8*A^3*a*b^6)*
```

```
c + (B*a^6*b^4 - A*a^5*b^5 + 4*(2*B*a^8 - 3*A*a^7*b)*c^2 - (6*B*a^7*b^2 - 7
*A*a^6*b^3)*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*
A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^
4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a
^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^
2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))*sqrt
(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (3
*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c - (a^5*b^2 - 4*a^6*c)*sqrt((B^4
*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 +
A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^
6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)
*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2
*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 3*sq
rt(1/2)*a^2*x^3*sqrt(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*B*a^3 - 5
*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c - (a^5*b^2
- 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*
B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^
3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b
^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^
3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2
 - 4*a^6*c))*log(2*(A^4*a^2*c^5 + 3*(A^3*B*a^2*b - A^4*a*b^2)*c^4 - (B^4*a^
4 - 5*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - A^3*B*a*b^3 - A^4*b^4)*c^3 + (B^4*a
^3*b^2 - 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 - A^3*B*b^5)*c^2)*x - sqrt(1/2)*
(B^3*a^3*b^5 - 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 - A^3*b^8 - 4*A^3*a^4*c^4 +
(4*A*B^2*a^5 - 20*A^2*B*a^4*b + 17*A^3*a^3*b^2)*c^3 + (4*B^3*a^5*b - 25*A*B
^2*a^4*b^2 + 41*A^2*B*a^3*b^3 - 20*A^3*a^2*b^4)*c^2 - (5*B^3*a^4*b^3 - 18*A
*B^2*a^3*b^4 + 21*A^2*B*a^2*b^5 - 8*A^3*a*b^6)*c + (B*a^6*b^4 - A*a^5*b^5 +
 4*(2*B*a^8 - 3*A*a^7*b)*c^2 - (6*B*a^7*b^2 - 7*A*a^6*b^3)*c)*sqrt((B^4*a^4
*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*
a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 -
8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2
 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5
 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))*sqrt(-(B^2*a^2*b^3 - 2*A*B*a*b^4
 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2 +
 5*A^2*a*b^3)*c - (a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 +
 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5
 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B
^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*
B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10
*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 6*(B*a - A*b)*x^2 - 2*A*a)/(a^2*
x^3)
```

giac [B] time = 3.35, size = 2870, normalized size = 10.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*
c - 2*b^6*c + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 10*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^4*c^2 + 18*a*b^4*c^2 + 2*b^5*c^2 - 16*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^3 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 48*a^2*b^2*
c^3 - 14*a*b^3*c^3 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32
*a^3*c^4 + 24*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^5 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c
```

```
 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 -
 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 3*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*(b^2 - 4*a*c
)*b^4*c - 10*(b^2 - 4*a*c)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4
*a*c)*a^2*c^3 + 6*(b^2 - 4*a*c)*a*b*c^3)*A - (sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 +
16*a^2*b^3*c^2 + 2*a*b^4*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^3 - 32*a^3*b*c^3 - 12*a^2*b^2*c^3 + 16*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*c^3 + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2 - 2
*(b^2 - 4*a*c)*a*b^2*c^2 + 4*(b^2 - 4*a*c)*a^2*c^3)*B)*arctan(2*sqrt(1/2)*x
/sqrt((a^2*b + sqrt(a^4*b^2 - 4*a^5*c))/(a^2*c)))/((a^3*b^4 - 8*a^4*b^2*c -
 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c))
+ 1/4*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
5*c + 2*b^6*c + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 10
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^4*c^2 - 18*a*b^4*c^2 - 2*b^5*c^2 - 16*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^3 - 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 48*a^2*b^
2*c^3 + 14*a*b^3*c^3 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 -
32*a^3*c^4 - 24*a^2*b*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*b^5 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b^3*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4
*c + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2
 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 +
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 3*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a
*c)*b^4*c + 10*(b^2 - 4*a*c)*a*b^2*c^2 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 -
 4*a*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b*c^3)*A - (sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c -
2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2
- 16*a^2*b^3*c^2 - 2*a*b^4*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^3 + 32*a^3*b*c^3 + 12*a^2*b^2*c^3 - 16*a^3*c^4 + sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^2*b*c^2 +
 2*(b^2 - 4*a*c)*a*b^2*c^2 - 4*(b^2 - 4*a*c)*a^2*c^3)*B)*arctan(2*sqrt(1/2)
*x/sqrt((a^2*b - sqrt(a^4*b^2 - 4*a^5*c))/(a^2*c)))/((a^3*b^4 - 8*a^4*b^2*c
 - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c)
) - 1/3*(3*B*a*x^2 - 3*A*b*x^2 + A*a)/(a^2*x^3)
```

maple [B] time = 0.03, size = 611, normalized size = 2.25

$$\frac{\sqrt{2}\, A c^{2} \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{\sqrt{-4ac+b^{2}}\, \sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}\; a}+\frac{\sqrt{2}\, A c^{2} \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{\sqrt{-4ac+b^{2}}\, \sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}\; a}-\frac{\sqrt{2}\, A b^{2} c \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{-4ac+b^{2}}\, \sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}\; a^{2}}-\frac{\sqrt{2}\, A b^{2} c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{-4ac+b^{2}}\, \sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}\; a^{2}}+\frac{\sqrt{2}\, B b c \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{-4ac+b^{2}}\, \sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}\; a}+\frac{\sqrt{2}\, B b c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{-4ac+b^{2}}\, \sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}\; a}-\frac{\sqrt{2}\, A b c \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}\; a^{2}}+\frac{\sqrt{2}\, A b c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}\; a^{2}}+\frac{\sqrt{2}\, B c \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}\; a}-\frac{\sqrt{2}\, B c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}\; a}+\frac{A b}{a^{2} x}-\frac{B}{a x}-\frac{A}{3 a x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/3*A/a/x^3+1/a^2/x*A*b-1/a/x*B-1/2/a^2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b+1/a*c^
2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A-1/2/a^2*c/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)*A*b^2+1/2/a*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B+1/2/a*c/(-4*a*c+
b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B+1/2/a^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b+1/a*c
^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A-1/2/a^2*c/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*c*x)*A*b^2-1/2/a*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B+1/2/a*c/(-4*a*c+b^2)^(1/
2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*b*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-\int \frac{(Ba-Ab)cx^2+Bab-Ab^2+Aac}{cx^4+bx^2+a}\,dx}{a^2} - \frac{3\,(Ba-Ab)x^2+Aa}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(-((B*a - A*b)*c*x^2 + B*a*b - A*b^2 + A*a*c)/(c*x^4 + b*x^2 + a),
 x)/a^2 - 1/3*(3*(B*a - A*b)*x^2 + A*a)/(a^2*x^3)
```

mupad [B] time = 2.19, size = 10101, normalized size = 37.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)),x)
```

```
[Out] - (A/(3*a) - (x^2*(A*b - B*a))/a^2)/x^3 - atan((((-(A^2*b^7 + B^2*a^2*b^5 +
 A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 1
6*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2
*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^
2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) +
 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 1
6*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(16*A*a^10*c^4 + x*(32*a^11*b*c^3 - 8*a^10
*b^3*c^2)*(-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A
*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*
a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*
a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^
3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*
A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B
*a^10*b*c^3 + 4*A*a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) - x*(4*
A^2*a^8*c^5 - 4*B^2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2
*a^8*b^2*c^3 - 4*A*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*
b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 +
 A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2
) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 1
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2*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 -
 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1
/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^
4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*1i - ((-(A^2*b^7 + B^2*a^2*b^5 + A^2*
b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c
^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B
*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*
b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b
^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A
*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7
*c^2 - 8*a^6*b^2*c)))^(1/2)*(16*A*a^10*c^4 - x*(32*a^11*b*c^3 - 8*a^10*b^3*
c^2)*(-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*
b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b
^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b
*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1
/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a
*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c -
 b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B*a^10
*b*c^3 + 4*A*a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) + x*(4*A^2*a
^8*c^5 - 4*B^2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*
b^2*c^3 - 4*A*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 +
 A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 1
6*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2
*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^
2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) +
 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 1
6*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*1i)/(((-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(
-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-
(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*
c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2
 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*
(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^
2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2
 - 8*a^6*b^2*c)))^(1/2)*(16*A*a^10*c^4 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*
(-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 +
 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-
(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3
 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) -
 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*
(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B*a^10*b*c^
3 + 4*A*a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) - x*(4*A^2*a^8*c^
5 - 4*B^2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*b^2*c
^3 - 4*A*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 + A^2*
b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c
^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B
*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*
b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b
^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A
*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7
*c^2 - 8*a^6*b^2*c)))^(1/2) + ((-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c
 - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c -
 b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*
A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a
^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c
 - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c
+ 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*
b^2*c)))^(1/2)*(16*A*a^10*c^4 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(A^2*b
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^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*
a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c -
 b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*
a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*
a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c
 - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2
))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B*a^10*b*c^3 + 4*A*
a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) + x*(4*A^2*a^8*c^5 - 4*B^
2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*b^2*c^3 - 4*A
*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4
*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3
 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 -
B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(
4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b
^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8
*a^6*b^2*c)))^(1/2) + 2*B^3*a^8*c^4 + 2*A^2*B*a^7*c^5 - 2*A^3*a^6*b*c^5 - 4
*A*B^2*a^7*b*c^4 + 2*A^2*B*a^6*b^2*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4
*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*
(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^
4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c
^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*
c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*
a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^
2 - 8*a^6*b^2*c)))^(1/2)*2i - atan((((-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(
4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4
*a*c - b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^
3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 +
 B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-
(4*a*c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*
b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 -
8*a^6*b^2*c)))^(1/2)*(16*A*a^10*c^4 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-
(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 2
5*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4
*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 -
7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 3
6*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-
(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3
)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B*a^10*b*c^3
+ 4*A*a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) - x*(4*A^2*a^8*c^5
- 4*B^2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*b^2*c^3
 - 4*A*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^
4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2
*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a
^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*
c^2 + B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2
*c*(-(4*a*c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B
*a^2*b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c
^2 - 8*a^6*b^2*c)))^(1/2)*1i - ((-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c
 - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c
- b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9
*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*
a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*
c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c
 - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6
*b^2*c)))^(1/2)*(16*A*a^10*c^4 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(A^2*
b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2
*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c
- b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2
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*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B
*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*
c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B*a^10*b*c^3 + 4*A
*a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) + x*(4*A^2*a^8*c^5 - 4*B
^2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*b^2*c^3 - 4*
A*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(
4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4
*a*c - b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^
3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 +
 B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-
(4*a*c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*
b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 -
8*a^6*b^2*c)))^(1/2)*1i)/(((-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^
2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2
)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*
a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c
*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c - b
^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c - 4*
A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*
c)))^(1/2)*(16*A*a^10*c^4 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(A^2*b^7 +
 B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*
b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2
)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*
b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*
b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b
^2)^3)^(1/2) + 16*A*B*a^2*b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(
8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B*a^10*b*c^3 + 4*A*a^8*
b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) - x*(4*A^2*a^8*c^5 - 4*B^2*a^
9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*b^2*c^3 - 4*A*B*a
^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c
 - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c
- b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9
*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*
a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*
c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c
 - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6
*b^2*c)))^(1/2) + ((-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^2)^3)^(1
/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/
2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c
- 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c*(-(4*a*
c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(
1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c - 4*A*B*a^2*
b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/
2)*(16*A*a^10*c^4 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(A^2*b^7 + B^2*a^2
*b^5 - A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2
- A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/
2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c +
12*B^2*a^4*b*c^2 + B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2
+ 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(
1/2) + 16*A*B*a^2*b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b
^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B*a^10*b*c^3 + 4*A*a^8*b^4*c^2
- 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) + x*(4*A^2*a^8*c^5 - 4*B^2*a^9*c^4 +
2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*b^2*c^3 - 4*A*B*a^7*b^3*c
^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^2)^
3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2)^3
)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b
^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c*(-
(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c - b^2)
```

```
^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c - 4*A*B
*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c))
)^(1/2) + 2*B^3*a^8*c^4 + 2*A^2*B*a^7*c^5 - 2*A^3*a^6*b*c^5 - 4*A*B^2*a^7*b
*c^4 + 2*A^2*B*a^6*b^2*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c -
b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b
^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^
2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3
*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c -
 b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c -
4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^
2*c)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.90 $\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=212

$$-\frac{\left(12a^2Bc^2+6aAbc^2-12ab^2Bc-Ab^3c+2b^4B\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(-6aBc-Abc+2b^2B\right)}{2c^2\left(b^2-4ac\right)}-\frac{x^4\left(x^2\left(-2aBc\right.\right.}{2c\left(b^2\right.}$$

Rubi [A] time = 0.38, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1251, 818, 773, 634, 618, 206, 628}

$$-\frac{\left(12a^2Bc^2+6aAbc^2-12ab^2Bc-Ab^3c+2b^4B\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(-6aBc-Abc+2b^2B\right)}{2c^2\left(b^2-4ac\right)}-\frac{x^4\left(x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{(2bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*b^2*B - A*b*c - 6*a*B*c)*x^2)/(2*c^2*(b^2 - 4*a*c)) - (x^4*(a*(b*B - 2*
A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) - ((2*b^4*B - A*b^3*c - 12*a*b^2*B*c + 6*a*A*b*c^2 + 12*a^2*B*c^2)*ArcT
anh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)^(3/2)) - ((2*b*B
 - A*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^3(A+Bx)}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{x^4\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\mathrm{Subst}\left(\int \frac{x\left(2a(bB-2Ac)+\left(2b^2B-Abc-6aBc\right)x\right)}{a+bx+cx^2}\,d\right.}{2c\left(b^2-4ac\right)}$$

$$= \frac{\left(2b^2B-Abc-6aBc\right)x^2}{2c^2\left(b^2-4ac\right)} - \frac{x^4\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\mathrm{Subst}\left(\int \frac{-a(2b}{}\right.}{}$$

$$= \frac{\left(2b^2B-Abc-6aBc\right)x^2}{2c^2\left(b^2-4ac\right)} - \frac{x^4\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{(2bB-Ac)\,\mathrm{Su}}{}$$

$$= \frac{\left(2b^2B-Abc-6aBc\right)x^2}{2c^2\left(b^2-4ac\right)} - \frac{x^4\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{(2bB-Ac)\log}{4}$$

$$= \frac{\left(2b^2B-Abc-6aBc\right)x^2}{2c^2\left(b^2-4ac\right)} - \frac{x^4\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2b^4B-Ab^3c\right.}{}$$

Mathematica [A] time = 0.28, size = 208, normalized size = 0.98

$$\frac{-\frac{2\left(a^2c\left(2c\left(A+Bx^2\right)-3bB\right)+ab\left(-bc\left(A+4Bx^2\right)+3Ac^2x^2+b^2B\right)+b^3x^2(bB-Ac)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2\left(12a^2Bc^2+6aAbc^2-12ab^2Bc-Ab^3c+2b^4B\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+(Ac-2bB)\log\left(a+bx^2+cx^4\right)+2Bcx^2}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*B*c*x^2 - (2*(b^3*(b*B - A*c)*x^2 + a^2*c*(-3*b*B + 2*c*(A + B*x^2)) + a
*b*(b^2*B + 3*A*c^2*x^2 - b*c*(A + 4*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)) - (2*(2*b^4*B - A*b^3*c - 12*a*b^2*B*c + 6*a*A*b*c^2 + 12*a^2*B*c^2
```

```
)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (-2*b*B
+ A*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7 \left(A + Bx^2\right)}{\left(a + bx^2 + cx^4\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
[Out] IntegrateAlgebraic[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2, x]
```

fricas [B] time = 0.78, size = 1323, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
[Out] [-1/4*(2*B*a*b^5 - 16*A*a^3*c^3 - 2*(B*b^4*c^2 - 8*B*a*b^2*c^3 + 16*B*a^2*c
^4)*x^6 - 2*(B*b^5*c - 8*B*a*b^3*c^2 + 16*B*a^2*b*c^3)*x^4 + 12*(2*B*a^3*b
+ A*a^2*b^2)*c^2 + 2*(B*b^6 - 12*(2*B*a^3 + A*a^2*b)*c^3 + (26*B*a^2*b^2 +
7*A*a*b^3)*c^2 - (9*B*a*b^4 + A*b^5)*c)*x^2 + (2*B*a*b^4 + (2*B*b^4*c + 6*(
2*B*a^2 + A*a*b)*c^3 - (12*B*a*b^2 + A*b^3)*c^2)*x^4 + 6*(2*B*a^3 + A*a^2*b
)*c^2 + (2*B*b^5 + 6*(2*B*a^2*b + A*a*b^2)*c^2 - (12*B*a*b^3 + A*b^4)*c)*x^
2 - (12*B*a^2*b^2 + A*a*b^3)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^
2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 2
*(7*B*a^2*b^3 + A*a*b^4)*c + (2*B*a*b^5 - 16*A*a^3*c^3 + (2*B*b^5*c - 16*A*
a^2*c^4 + 8*(4*B*a^2*b + A*a*b^2)*c^3 - (16*B*a*b^3 + A*b^4)*c^2)*x^4 + 8*(
4*B*a^3*b + A*a^2*b^2)*c^2 + (2*B*b^6 - 16*A*a^2*b*c^3 + 8*(4*B*a^2*b^2 + A
*a*b^3)*c^2 - (16*B*a*b^4 + A*b^5)*c)*x^2 - (16*B*a^2*b^3 + A*a*b^4)*c)*log
(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8
*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2),
 -1/4*(2*B*a*b^5 - 16*A*a^3*c^3 - 2*(B*b^4*c^2 - 8*B*a*b^2*c^3 + 16*B*a^2*c
^4)*x^6 - 2*(B*b^5*c - 8*B*a*b^3*c^2 + 16*B*a^2*b*c^3)*x^4 + 12*(2*B*a^3*b
+ A*a^2*b^2)*c^2 + 2*(B*b^6 - 12*(2*B*a^3 + A*a^2*b)*c^3 + (26*B*a^2*b^2 +
7*A*a*b^3)*c^2 - (9*B*a*b^4 + A*b^5)*c)*x^2 + 2*(2*B*a*b^4 + (2*B*b^4*c + 6
*(2*B*a^2 + A*a*b)*c^3 - (12*B*a*b^2 + A*b^3)*c^2)*x^4 + 6*(2*B*a^3 + A*a^2
*b)*c^2 + (2*B*b^5 + 6*(2*B*a^2*b + A*a*b^2)*c^2 - (12*B*a*b^3 + A*b^4)*c)*
x^2 - (12*B*a^2*b^2 + A*a*b^3)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*
sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*(7*B*a^2*b^3 + A*a*b^4)*c + (2*B*a*b^
5 - 16*A*a^3*c^3 + (2*B*b^5*c - 16*A*a^2*c^4 + 8*(4*B*a^2*b + A*a*b^2)*c^3
- (16*B*a*b^3 + A*b^4)*c^2)*x^4 + 8*(4*B*a^3*b + A*a^2*b^2)*c^2 + (2*B*b^6
- 16*A*a^2*b*c^3 + 8*(4*B*a^2*b^2 + A*a*b^3)*c^2 - (16*B*a*b^4 + A*b^5)*c)*
x^2 - (16*B*a^2*b^3 + A*a*b^4)*c)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^
2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^
3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2)]
```

giac [A] time = 1.65, size = 239, normalized size = 1.13

$$\frac{Bx^2}{2\,c^2} + \frac{\left(2\,Bb^4 - 12\,Bab^2c - Ab^3c + 12\,Ba^2c^2 + 6\,Aabc^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\left(b^2c^3 - 4\,ac^4\right)\sqrt{-b^2 + 4\,ac}} + \frac{2\,Bb^3x^4 - 8\,Babcx^4 - Ab^2cx^4 + 4\,Aac^2x^4 + Ab^3x^2 - 4\,Ba^2cx^2 - 2\,Aabcx^2 - 2\,Ba^2b + Aab^2}{4\,\left(cx^4 + bx^2 + a\right)\left(b^2c^2 - 4\,ac^3\right)} - \frac{(2\,Bb - Ac)\log\left(cx^4 + bx^2 + a\right)}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
[Out] 1/2*B*x^2/c^2 + 1/2*(2*B*b^4 - 12*B*a*b^2*c - A*b^3*c + 12*B*a^2*c^2 + 6*A*
a*b*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 - 4*a*c^4)*sqrt
(-b^2 + 4*a*c)) + 1/4*(2*B*b^3*x^4 - 8*B*a*b*c*x^4 - A*b^2*c*x^4 + 4*A*a*c^
```

```
2*x^4 + A*b^3*x^2 - 4*B*a^2*c*x^2 - 2*A*a*b*c*x^2 - 2*B*a^2*b + A*a*b^2)/((
c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) - 1/4*(2*B*b - A*c)*log(c*x^4 + b*x
^2 + a)/c^3
```

maple [B] time = 0.02, size = 689, normalized size = 3.25

$$\frac{3Aab\,x^{2}}{2\left(c\,x^{4}+b\,x^{2}+a\right)\left(4ac-b^{2}\right)c}-\frac{A\,b^{3}x^{2}}{2\left(c\,x^{4}+b\,x^{2}+a\right)\left(4ac-b^{2}\right)c^{2}}+\frac{B\,a^{2}x^{2}}{\left(c\,x^{4}+b\,x^{2}+a\right)\left(4ac-b^{2}\right)c}-\frac{2Ba\,b^{2}x^{2}}{\left(c\,x^{4}+b\,x^{2}+a\right)\left(4ac-b^{2}\right)c^{2}}+\frac{B\,b^{4}x^{2}}{2\left(c\,x^{4}+b\,x^{2}+a\right)\left(4ac-b^{2}\right)c^{3}}-\frac{3Aab\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}c}+\frac{A\,b^{3}\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{2\left(4ac-b^{2}\right)^{\frac{3}{2}}c^{2}}-\frac{6B\,a^{2}\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}c}+\frac{6Ba\,b^{2}\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}c^{2}}-\frac{B\,b^{4}\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}c^{3}}+\frac{A\,a^{2}}{\left(c\,x^{4}+b\,x^{2}+a\right)\left(4ac-b^{2}\right)c}-\frac{Aa\,b^{2}}{2\left(c\,x^{4}+b\,x^{2}+a\right)\left(4ac-b^{2}\right)c^{2}}+\frac{Aa\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{\left(4ac-b^{2}\right)c}-\frac{A\,b^{2}\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{4\left(4ac-b^{2}\right)c^{2}}-\frac{3B\,a^{2}b}{2\left(c\,x^{4}+b\,x^{2}+a\right)\left(4ac-b^{2}\right)c^{2}}+\frac{Ba\,b^{3}}{2\left(c\,x^{4}+b\,x^{2}+a\right)\left(4ac-b^{2}\right)c^{3}}-\frac{2Bab\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{\left(4ac-b^{2}\right)c^{2}}+\frac{B\,b^{3}\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2\left(4ac-b^{2}\right)c^{3}}+\frac{B\,x^{2}}{2c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*B*x^2/c^2+3/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*A*b-1/2/c^2/(c*x^4+b*
x^2+a)/(4*a*c-b^2)*x^2*A*b^3+1/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a^2*B-2/c^
2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*b^2*B+1/2/c^3/(c*x^4+b*x^2+a)/(4*a*c-b^
2)*x^2*b^4*B+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*A-1/2/c^2/(c*x^4+b*x^2+a)*
a/(4*a*c-b^2)*A*b^2-3/2/c^2/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*b*B+1/2/c^3/(c*
x^4+b*x^2+a)*a/(4*a*c-b^2)*b^3*B+1/c/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a*A-1/4/
c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*A*b^2-2/c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)
*a*b*B+1/2/c^3/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^3*B-3/c/(4*a*c-b^2)^(3/2)*ar
ctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*a*b-6/c/(4*a*c-b^2)^(3/2)*arctan((2*c
*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*B+6/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)
/(4*a*c-b^2)^(1/2))*B*a*b^2+1/2/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4
*a*c-b^2)^(1/2))*b^3*A-1/c^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^
2)^(1/2))*b^4*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.83, size = 2282, normalized size = 10.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((a*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c))/(2*c*(4*a*c - b^2)) + (x^2*(
B*b^4 + 2*B*a^2*c^2 - A*b^3*c + 3*A*a*b*c^2 - 4*B*a*b^2*c))/(2*c*(4*a*c - b
^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) + (B*x^2)/(2*c^2) + (log(a + b*x^2 + c*
x^4)*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 -
 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/(2*(256*a^3*c^6 -
 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)) - (atan(((8*a*c^5*(4*a*c - b^
2)^3 - 2*b^2*c^4*(4*a*c - b^2)^3)*(x^2*(((((24*B*a^2*c^5 - 6*A*b^3*c^4 + 12
*B*b^4*c^3 + 28*A*a*b*c^5 - 56*B*a*b^2*c^4)/(4*a*c^5 - b^2*c^4) + ((8*b^3*c
^6 - 32*a*b*c^7)*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c + 24*A
*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/(2*(4
*a*c^5 - b^2*c^4)*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5
)))*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c + 6*A*a*b*c^2 - 12*B*a*b^2*c))/(8*c^3
*(4*a*c - b^2)^(3/2)) + ((8*b^3*c^6 - 32*a*b*c^7)*(2*B*b^4 + 12*B*a^2*c^2 -
 A*b^3*c + 6*A*a*b*c^2 - 12*B*a*b^2*c)*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c
 - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192
*B*a^2*b^3*c^2))/(16*c^3*(4*a*c - b^2)^(3/2)*(4*a*c^5 - b^2*c^4)*(256*a^3*c
```

```
^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))/(a*(4*a*c - b^2)) + (b*(
(4*B^2*b^5 + A^2*b^3*c^2 - 4*A*B*b^4*c - 6*A*B*a^2*c^3 - 5*A^2*a*b*c^3 - 20
*B^2*a*b^3*c + 12*B^2*a^2*b*c^2 + 20*A*B*a*b^2*c^2)/(4*a*c^5 - b^2*c^4) + (
((24*B*a^2*c^5 - 6*A*b^3*c^4 + 12*B*b^4*c^3 + 28*A*a*b*c^5 - 56*B*a*b^2*c^4
)/(4*a*c^5 - b^2*c^4) + ((8*b^3*c^6 - 32*a*b*c^7)*(4*B*b^7 + 128*A*a^3*c^4
- 2*A*b^6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^
2*c^3 + 192*B*a^2*b^3*c^2))/(2*(4*a*c^5 - b^2*c^4)*(256*a^3*c^6 - 4*b^6*c^3
 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c -
 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192*B
*a^2*b^3*c^2))/(2*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5
)) - (((b^3*c^6)/2 - 2*a*b*c^7)*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c + 6*A*a*b
*c^2 - 12*B*a*b^2*c)^2)/(c^6*(4*a*c - b^2)^3*(4*a*c^5 - b^2*c^4))))/(2*a*(4
*a*c - b^2)^(3/2))) + ((((8*A*a*c^4 - 16*B*a*b*c^3)/c^4 - (8*a*c^2*(4*B*b^7
 + 128*A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*
c^3 - 96*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/(256*a^3*c^6 - 4*b^6*c^3 + 48*
a*b^4*c^4 - 192*a^2*b^2*c^5))*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c + 6*A*a*b*c
^2 - 12*B*a*b^2*c))/(8*c^3*(4*a*c - b^2)^(3/2)) - (a*(2*B*b^4 + 12*B*a^2*c^
2 - A*b^3*c + 6*A*a*b*c^2 - 12*B*a*b^2*c)*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^
6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 +
192*B*a^2*b^3*c^2))/(c*(4*a*c - b^2)^(3/2)*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*
b^4*c^4 - 192*a^2*b^2*c^5)))/(a*(4*a*c - b^2)) + (b*((((8*A*a*c^4 - 16*B*a*
b*c^3)/c^4 - (8*a*c^2*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c +
 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/
(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5))*(4*B*b^7 + 128*
A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 9
6*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/(2*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^
4*c^4 - 192*a^2*b^2*c^5)) - (A^2*a*c^2 + 4*B^2*a*b^2 - 4*A*B*a*b*c)/c^4 + (
a*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c + 6*A*a*b*c^2 - 12*B*a*b^2*c)^2)/(c^4*(
4*a*c - b^2)^3)))/(2*a*(4*a*c - b^2)^(3/2))))/(4*B^2*b^8 + A^2*b^6*c^2 + 14
4*B^2*a^4*c^4 - 4*A*B*b^7*c + 36*A^2*a^2*b^2*c^4 + 192*B^2*a^2*b^4*c^2 - 28
8*B^2*a^3*b^2*c^3 - 48*B^2*a*b^6*c - 12*A^2*a*b^4*c^3 - 168*A*B*a^2*b^3*c^3
 + 48*A*B*a*b^5*c^2 + 144*A*B*a^3*b*c^4))*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c
 + 6*A*a*b*c^2 - 12*B*a*b^2*c))/(2*c^3*(4*a*c - b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.91 $\int \frac{x^5\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=147

$$-\frac{x^2\left(x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(4aAc^2-6abBc+b^3B\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Rubi [A] time = 0.17, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1251, 818, 634, 618, 206, 628}

$$\frac{\left(4aAc^2-6abBc+b^3B\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}}-\frac{x^2\left(x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(x^2*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(2*c*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)) + ((b^3*B - 6*a*b*B*c + 4*a*A*c^2)*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^(3/2)) + (B*Log[a + b*x^2 + c*
x^4])/(4*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
```

```
p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^5\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2(A+Bx)}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)$$

$$= -\frac{x^2\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{a(bB-2Ac)+B\left(b^2-4ac\right)x}{a+bx+cx^2}\,dx,x,x^2\right.}{2c\left(b^2-4ac\right)}$$

$$= -\frac{x^2\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{B\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^2} - \frac{\left(b^3B\right.}{}$$

$$= -\frac{x^2\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{B\log\left(a+bx^2+cx^4\right)}{4c^2} + \frac{\left(b^3B-6abB\right.}{}$$

$$= -\frac{x^2\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^3B-6abBc+4aAc^2\right)\tanh^{-1}\left(\frac{b+2}{\sqrt{b^2-}}\right.}{2c^2\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.19, size = 160, normalized size = 1.09

$$\frac{-\frac{2\left(2a^2Bc+a\left(bc\left(A+3Bx^2\right)-2Ac^2x^2+b^2(-B)\right)+b^2x^2(Ac-bB)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2\left(4aAc^2-6abBc+b^3B\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+B\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*(2*a^2*B*c + b^2*(-(b*B) + A*c)*x^2 + a*(-(b^2*B) - 2*A*c^2*x^2 + b*c*
(A + 3*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(b^3*B - 6*a*b*B*
c + 4*a*A*c^2)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/
2) + B*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2, x]
```

fricas [B] time = 1.01, size = 849, normalized size = 5.78

$$\left[\frac{2\,Bab^4 + 8\left(2\,Ba^3 + Aa^2b\right)c^2 + 2\left(Bb^5 - 8\,Aa^2c^3 + 6\left(2\,Ba^2b + Aab^2\right)c^2 - \left(7\,Bab^3 + Ab^4\right)c\right)x^2 - \left(Bab^3 - 6\,Ba^2bc + 4\,Aa^2c^2 + \left(Bb^3c - 6\,Babc^2 + 4\,Aac^3\right)x^4 + \left(Bb^4 - 6\,Bab^2c + 4\,Aabc^2\right)x^2\right)\sqrt{b^2 - 4\,ac}\,\log\left(\frac{2\,c^2x^4 + 2\,bcx^2 + b^2 - 2\,ac - \left(2\,cx^2 + b\right)\sqrt{b^2 - 4\,ac}}{cx^4 + bx^2 + a}\right) - 2\left(6\,Ba^2b^2 + Aab^3\right)c + \left(Bab^4 - 8\,Ba^2b^2c + 16\,Ba^3c^2 + \left(Bb^4c - 8\,Bab^2c^2 + 16\,Ba^2c^3\right)x^4 + \left(Bb^5 - 8\,Bab^3c + 16\,Ba^2bc^2\right)x^2\right)\log\left(cx^4 + bx^2 + a\right)}{4\left(ab^4c^2 - 8\,a^2b^2c^3 + 16\,a^3c^4 + \left(b^4c^3 - 8\,ab^2c^4 + 16\,a^2c^5\right)x^4 + \left(b^5c^2 - 8\,ab^3c^3 + 16\,a^2bc^4\right)x^2\right)}, \frac{2\,Bab^4 + 8\left(2\,Ba^3 + Aa^2b\right)c^2 + 2\left(Bb^5 - 8\,Aa^2c^3 + 6\left(2\,Ba^2b + Aab^2\right)c^2 - \left(7\,Bab^3 + Ab^4\right)c\right)x^2 + 2\left(Bab^3 - 6\,Ba^2bc + 4\,Aa^2c^2 + \left(Bb^3c - 6\,Babc^2 + 4\,Aac^3\right)x^4 + \left(Bb^4 - 6\,Bab^2c + 4\,Aabc^2\right)x^2\right)\sqrt{-b^2 + 4\,ac}\,\arctan\left(-\frac{\left(2\,cx^2 + b\right)\sqrt{-b^2 + 4\,ac}}{b^2 - 4\,ac}\right) - 2\left(6\,Ba^2b^2 + Aab^3\right)c + \left(Bab^4 - 8\,Ba^2b^2c + 16\,Ba^3c^2 + \left(Bb^4c - 8\,Bab^2c^2 + 16\,Ba^2c^3\right)x^4 + \left(Bb^5 - 8\,Bab^3c + 16\,Ba^2bc^2\right)x^2\right)\log\left(cx^4 + bx^2 + a\right)}{4\left(ab^4c^2 - 8\,a^2b^2c^3 + 16\,a^3c^4 + \left(b^4c^3 - 8\,ab^2c^4 + 16\,a^2c^5\right)x^4 + \left(b^5c^2 - 8\,ab^3c^3 + 16\,a^2bc^4\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*B*a*b^4 + 8*(2*B*a^3 + A*a^2*b)*c^2 + 2*(B*b^5 - 8*A*a^2*c^3 + 6*(2
*B*a^2*b + A*a*b^2)*c^2 - (7*B*a*b^3 + A*b^4)*c)*x^2 - (B*a*b^3 - 6*B*a^2*b
*c + 4*A*a^2*c^2 + (B*b^3*c - 6*B*a*b*c^2 + 4*A*a*c^3)*x^4 + (B*b^4 - 6*B*a
*b^2*c + 4*A*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b
^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 2*(6*B
*a^2*b^2 + A*a*b^3)*c + (B*a*b^4 - 8*B*a^2*b^2*c + 16*B*a^3*c^2 + (B*b^4*c
- 8*B*a*b^2*c^2 + 16*B*a^2*c^3)*x^4 + (B*b^5 - 8*B*a*b^3*c + 16*B*a^2*b*c^2
)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b
^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*
c^4)*x^2), 1/4*(2*B*a*b^4 + 8*(2*B*a^3 + A*a^2*b)*c^2 + 2*(B*b^5 - 8*A*a^2*
c^3 + 6*(2*B*a^2*b + A*a*b^2)*c^2 - (7*B*a*b^3 + A*b^4)*c)*x^2 + 2*(B*a*b^3
 - 6*B*a^2*b*c + 4*A*a^2*c^2 + (B*b^3*c - 6*B*a*b*c^2 + 4*A*a*c^3)*x^4 + (B
*b^4 - 6*B*a*b^2*c + 4*A*a*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2
+ b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*(6*B*a^2*b^2 + A*a*b^3)*c + (B*a
*b^4 - 8*B*a^2*b^2*c + 16*B*a^3*c^2 + (B*b^4*c - 8*B*a*b^2*c^2 + 16*B*a^2*c
^3)*x^4 + (B*b^5 - 8*B*a*b^3*c + 16*B*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a
))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^
2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2)]
```

giac [A] time = 1.62, size = 194, normalized size = 1.32

$$-\frac{\left(Bb^3 - 6\,Babc + 4\,Aac^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(b^2c^2 - 4\,ac^3\right)\sqrt{-b^2 + 4\,ac}} + \frac{B\log\left(cx^4 + bx^2 + a\right)}{4\,c^2} - \frac{Bb^2cx^4 - 4\,Bac^2x^4 - Bb^3x^2 + 2\,Babcx^2 + 2\,Ab^2cx^2 - 4\,Aac^2x^2 - Bab^2 + 2\,Aabc}{4\left(cx^4 + bx^2 + a\right)\left(b^2c^2 - 4\,ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*b^3 - 6*B*a*b*c + 4*A*a*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c
))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) + 1/4*B*log(c*x^4 + b*x^2 + a)/
c^2 - 1/4*(B*b^2*c*x^4 - 4*B*a*c^2*x^4 - B*b^3*x^2 + 2*B*a*b*c*x^2 + 2*A*b^
2*c*x^2 - 4*A*a*c^2*x^2 - B*a*b^2 + 2*A*a*b*c)/((c*x^4 + b*x^2 + a)*(b^2*c^
2 - 4*a*c^3))
```

maple [B] time = 0.01, size = 286, normalized size = 1.95

$$\frac{2Aa\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}} - \frac{3Bab\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}c} + \frac{B\,b^3\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(4ac-b^2\right)^{\frac{3}{2}}c^2} + \frac{Ba\ln\left(c\,x^4+b\,x^2+a\right)}{\left(4ac-b^2\right)c} - \frac{B\,b^2\ln\left(c\,x^4+b\,x^2+a\right)}{4\left(4ac-b^2\right)c^2} + \frac{-\frac{\left(2aA\,c^2-A\,b^2c-3abBc+b^3B\right)x^2}{\left(4ac-b^2\right)c^2} + \frac{\left(Abc+2aBc-b^2B\right)a}{\left(4ac-b^2\right)c^2}}{2c\,x^4+2b\,x^2+2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(-1/c^2*(2*A*a*c^2-A*b^2*c-3*B*a*b*c+B*b^3)/(4*a*c-b^2)*x^2+a*(A*b*c+2*
B*a*c-B*b^2)/c^2/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)/c*ln(c*x^4+b*x^
2+a)*a*B-1/4/(4*a*c-b^2)/c^2*ln(c*x^4+b*x^2+a)*b^2*B+2/(4*a*c-b^2)^(3/2)*ar
ctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*A-3/(4*a*c-b^2)^(3/2)/c*arctan((2*c*x
^2+b)/(4*a*c-b^2)^(1/2))*a*b*B+1/2/(4*a*c-b^2)^(3/2)/c^2*arctan((2*c*x^2+b)
/(4*a*c-b^2)^(1/2))*b^3*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.22, size = 1527, normalized size = 10.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((x^2*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c))/(2*c^2*(4*a*c - b^2)) -
(a*(A*b*c - B*b^2 + 2*B*a*c))/(2*c^2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) -
(log(a + b*x^2 + c*x^4)*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*
b^2*c^2))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) -
(atan(((8*a*c^3*(4*a*c - b^2)^3 - 2*b^2*c^2*(4*a*c - b^2)^3)*((((8*B*a + (8
*a*c^2*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/(256*a^
3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))*(B*b^3 + 4*A*a*c^2 - 6
*B*a*b*c))/(8*c^2*(4*a*c - b^2)^(3/2)) + (a*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c)
*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/((4*a*c - b^2
)^(3/2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*
a*c - b^2)) - x^2*(((((6*B*b^3*c^2 + 8*A*a*c^4 - 28*B*a*b*c^3)/(4*a*c^3 - b
^2*c^2) + ((8*b^3*c^4 - 32*a*b*c^5)*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c
 + 96*B*a^2*b^2*c^2))/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*
a*b^4*c^3 - 192*a^2*b^2*c^4)))*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c))/(8*c^2*(4*a
*c - b^2)^(3/2)) + ((8*b^3*c^4 - 32*a*b*c^5)*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c
)*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/(16*c^2*(4*a
*c - b^2)^(3/2)*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3
 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) + (b*((B^2*b^3 + 2*A*B*a*c^2 - 5*B^
2*a*b*c)/(4*a*c^3 - b^2*c^2) + (((6*B*b^3*c^2 + 8*A*a*c^4 - 28*B*a*b*c^3)/(
4*a*c^3 - b^2*c^2) + ((8*b^3*c^4 - 32*a*b*c^5)*(2*B*b^6 - 128*B*a^3*c^3 - 2
4*B*a*b^4*c + 96*B*a^2*b^2*c^2))/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^
6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a
*b^4*c + 96*B*a^2*b^2*c^2))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 19
2*a^2*b^2*c^4)) - (((b^3*c^4)/2 - 2*a*b*c^5)*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c
)^2)/(c^4*(4*a*c - b^2)^3*(4*a*c^3 - b^2*c^2))))/(2*a*(4*a*c - b^2)^(3/2)))
 + (b*(((8*B*a + (8*a*c^2*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^
2*b^2*c^2))/(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))*(2*
B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/(2*(256*a^3*c^5 -
 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) + (B^2*a)/c^2 - (a*(B*b^3 + 4
*A*a*c^2 - 6*B*a*b*c)^2)/(c^2*(4*a*c - b^2)^3)))/(2*a*(4*a*c - b^2)^(3/2)))
)/(B^2*b^6 + 16*A^2*a^2*c^4 + 36*B^2*a^2*b^2*c^2 - 12*B^2*a*b^4*c + 8*A*B*a
*b^3*c^2 - 48*A*B*a^2*b*c^3))*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c))/(2*c^2*(4*a*
c - b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.92 $\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=107

$$-\frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Rubi [A] time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1251, 777, 618, 206}

$$-\frac{x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)/(2*c*(b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)) - ((A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(b^2 - 4*a*c)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(A+Bx)}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$
$$= -\frac{a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{(Ab-2aB)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2\left(b^2-4ac\right)}$$
$$= -\frac{a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{(Ab-2aB)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2\right.}{b^2-4ac}$$
$$= -\frac{a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.08, size = 111, normalized size = 1.04

$$\frac{-2ac\left(A+Bx^2\right)+abB+bx^2(bB-Ac)}{2c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)} - \frac{(Ab-2aB)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a*b*B + b*(b*B - A*c)*x^2 - 2*a*c*(A + B*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b*
x^2 + c*x^4)) - ((A*b - 2*a*B)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-
b^2 + 4*a*c)^(3/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2, x]
```

fricas [B] time = 0.80, size = 538, normalized size = 5.03

$$\left[-\frac{Bab^3+8Aa^2c^2+\left(Bb^4+4\left(2Ba^2+Aab\right)c^2-\left(6Bab^2+Ab^3\right)c\right)x^2-\left((2Ba-Ab)c^2x^4+\left(2Bab-Ab^2\right)cx^2+\left(2Ba^2-Aab\right)c\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac+\left(2cx^2+b\right)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)-2\left(2Ba^2b+Aab^2\right)c}{2\left(ab^4c-8a^2b^2c^2+16a^3c^3+\left(b^4c^2-8ab^2c^3+16a^2c^4\right)x^4+\left(b^5c-8ab^3c^2+16a^2bc^3\right)x^2\right)}, -\frac{Bab^3+8Aa^2c^2+\left(Bb^4+4\left(2Ba^2+Aab\right)c^2-\left(6Bab^2+Ab^3\right)c\right)x^2-2\left((2Ba-Ab)c^2x^4+\left(2Bab-Ab^2\right)cx^2+\left(2Ba^2-Aab\right)c\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{\left(2cx^2+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)-2\left(2Ba^2b+Aab^2\right)c}{2\left(ab^4c-8a^2b^2c^2+16a^3c^3+\left(b^4c^2-8ab^2c^3+16a^2c^4\right)x^4+\left(b^5c-8ab^3c^2+16a^2bc^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(B*a*b^3 + 8*A*a^2*c^2 + (B*b^4 + 4*(2*B*a^2 + A*a*b)*c^2 - (6*B*a*b^
2 + A*b^3)*c)*x^2 - ((2*B*a - A*b)*c^2*x^4 + (2*B*a*b - A*b^2)*c*x^2 + (2*B
*a^2 - A*a*b)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c
 + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 2*(2*B*a^2*b + A
*a*b^2)*c)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 +
 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2), -1/2*(B*a*b^3
 + 8*A*a^2*c^2 + (B*b^4 + 4*(2*B*a^2 + A*a*b)*c^2 - (6*B*a*b^2 + A*b^3)*c)*
x^2 - 2*((2*B*a - A*b)*c^2*x^4 + (2*B*a*b - A*b^2)*c*x^2 + (2*B*a^2 - A*a*b
)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a
*c)) - 2*(2*B*a^2*b + A*a*b^2)*c)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (
```

```
b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c
^3)*x^2)]
```

giac [A] time = 1.72, size = 120, normalized size = 1.12

$$-\frac{(2\,Ba - Ab)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2 - 4\,ac\right)\sqrt{-b^2 + 4\,ac}} - \frac{Bb^2x^2 - 2\,Bacx^2 - Abcx^2 + Bab - 2\,Aac}{2\left(cx^4 + bx^2 + a\right)\left(b^2c - 4\,ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -(2*B*a - A*b)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt
(-b^2 + 4*a*c)) - 1/2*(B*b^2*x^2 - 2*B*a*c*x^2 - A*b*c*x^2 + B*a*b - 2*A*a*
c)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))
```

maple [A] time = 0.01, size = 158, normalized size = 1.48

$$-\frac{Ab\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}} + \frac{2Ba\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}} + \frac{-\frac{\left(Abc+2aBc-b^2B\right)x^2}{\left(4ac-b^2\right)c} - \frac{(2Ac-bB)a}{\left(4ac-b^2\right)c}}{2c\,x^4 + 2b\,x^2 + 2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(-(A*b*c+2*B*a*c-B*b^2)/c/(4*a*c-b^2)*x^2-a*(2*A*c-B*b)/(4*a*c-b^2)/c)/
(c*x^4+b*x^2+a)-1/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A
*b+2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.32, size = 283, normalized size = 2.64

$$-\frac{\frac{x^2\left(-B\,b^2+A\,c\,b+2\,B\,a\,c\right)}{2\,c\left(4\,a\,c-b^2\right)} + \frac{a\,(2\,A\,c-B\,b)}{2\,c\left(4\,a\,c-b^2\right)}}{c\,x^4 + b\,x^2 + a} - \frac{\mathrm{atan}\left(\frac{\left(4\,a\,c-b^2\right)^4\left(x^2\left(\frac{(A\,b-2\,B\,a)\left(A\,b\,c^2-2\,B\,a\,c^2\right)}{a\left(4\,a\,c-b^2\right)^{7/2}} + \frac{\left(2\,b^3\,c^2-8\,a\,b\,c^3\right)(A\,b-2\,B\,a)^2\left(b^3-4\,a\,b\,c\right)}{2\,a\left(4\,a\,c-b^2\right)^{13/2}}\right) - \frac{2\,c^2\,(A\,b-2\,B\,a)^2\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{11/2}}\right)}{2\,A^2\,b^2\,c^2-8\,A\,B\,a\,b\,c^2+8\,B^2\,a^2\,c^2}\right)(A\,b - 2\,B\,a)}{\left(4\,a\,c - b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((x^2*(A*b*c - B*b^2 + 2*B*a*c))/(2*c*(4*a*c - b^2)) + (a*(2*A*c - B*b))/
(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - (atan(((4*a*c - b^2)^4*(x^2*(((A
*b - 2*B*a)*(A*b*c^2 - 2*B*a*c^2))/(a*(4*a*c - b^2)^(7/2)) + ((2*b^3*c^2 -
8*a*b*c^3)*(A*b - 2*B*a)^2*(b^3 - 4*a*b*c))/(2*a*(4*a*c - b^2)^(13/2))) - (
2*c^2*(A*b - 2*B*a)^2*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(11/2)))/(2*A^2*b^2*c^
2 + 8*B^2*a^2*c^2 - 8*A*B*a*b*c^2))*(A*b - 2*B*a))/(4*a*c - b^2)^(3/2)
```

sympy [B] time = 5.38, size = 394, normalized size = 3.68

$$-\frac{\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(-Ab+2Ba\right)\log\left(x^2+\frac{-Ab^2+2Bab-16a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(-Ab+2Ba\right)+8ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(-Ab+2Ba\right)-b^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(-Ab+2Ba\right)}{-2Abc+4Bac}\right)}{2}+\frac{\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(-Ab+2Ba\right)\log\left(x^2+\frac{-Ab^2+2Bab+16a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(-Ab+2Ba\right)-8ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(-Ab+2Ba\right)+b^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(-Ab+2Ba\right)}{-2Abc+4Bac}\right)}{2}+\frac{-2Aac+Bab+x^2\left(-Abc-2Bac+Bb^2\right)}{8a^2c^2-2ab^2c+x^4\left(8ac^3-2b^2c^2\right)+x^2\left(8abc^2-2b^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a)*log(x**2 + (-A*b**2 + 2*B*a*b -
16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a) + 8*a*b**2*c*sqrt(-1
/(4*a*c - b**2)**3)*(-A*b + 2*B*a) - b**4*sqrt(-1/(4*a*c - b**2)**3)*(-A*b
+ 2*B*a))/(-2*A*b*c + 4*B*a*c))/2 + sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*
a)*log(x**2 + (-A*b**2 + 2*B*a*b + 16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*
(-A*b + 2*B*a) - 8*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a) + b**
4*sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a))/(-2*A*b*c + 4*B*a*c))/2 + (-2*
A*a*c + B*a*b + x**2*(-A*b*c - 2*B*a*c + B*b**2))/(8*a**2*c**2 - 2*a*b**2*c
 + x**4*(8*a*c**3 - 2*b**2*c**2) + x**2*(8*a*b*c**2 - 2*b**3*c))
```

3.93 $\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=94

$$-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{-2aB-\left(x^2(bB-2Ac)\right)+Ab}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Rubi [A] time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1247, 638, 618, 206}

$$-\frac{-2aB+x^2(-(bB-2Ac))+Ab}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(A*b - 2*a*B - (b*B - 2*A*c)*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) -
((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)\\
&= -\frac{Ab-2aB-(bB-2Ac)x^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{(bB-2Ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{2\left(b^2-4ac\right)}\\
&= -\frac{Ab-2aB-(bB-2Ac)x^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{(bB-2Ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{b^2-4ac}\\
&= -\frac{Ab-2aB-(bB-2Ac)x^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 101, normalized size = 1.07

$$
\frac{\frac{2(bB-2Ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{B\left(2a+bx^2\right)-A\left(b+2cx^2\right)}{a+bx^2+cx^4}}{2\left(b^2-4ac\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((B*(2*a + b*x^2) - A*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + (2*(b*B - 2*A*c)
*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*
a*c))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2, x]
```

fricas [B] time = 0.69, size = 474, normalized size = 5.04

$$
\left[\frac{2Bab^2-Ab^3+\left(Bb^3+8Aac^2-2\left(2Bab+Ab^2\right)c\right)x^2+\left(\left(Bbc-2Ac^2\right)x^4+Bab-2Aac+\left(Bb^2-2Abc\right)x^2\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac-\left(2cx^2+b\right)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)-4\left(2Ba^2-Aab\right)c}{2\left(ab^4-8a^2b^2c+16a^3c^2+\left(b^4c-8ab^2c^2+16a^2c^3\right)x^4+\left(b^5-8ab^3c+16a^2bc^2\right)x^2\right)},\ \frac{2Bab^2-Ab^3+\left(Bb^3+8Aac^2-2\left(2Bab+Ab^2\right)c\right)x^2-2\left(\left(Bbc-2Ac^2\right)x^4+Bab-2Aac+\left(Bb^2-2Abc\right)x^2\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{\left(2cx^2+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)-4\left(2Ba^2-Aab\right)c}{2\left(ab^4-8a^2b^2c+16a^3c^2+\left(b^4c-8ab^2c^2+16a^2c^3\right)x^4+\left(b^5-8ab^3c+16a^2bc^2\right)x^2\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*a*b^2 - A*b^3 + (B*b^3 + 8*A*a*c^2 - 2*(2*B*a*b + A*b^2)*c)*x^2 +
 ((B*b*c - 2*A*c^2)*x^4 + B*a*b - 2*A*a*c + (B*b^2 - 2*A*b*c)*x^2)*sqrt(b^2
 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2
 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 4*(2*B*a^2 - A*a*b)*c)/(a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c
 + 16*a^2*b*c^2)*x^2), 1/2*(2*B*a*b^2 - A*b^3 + (B*b^3 + 8*A*a*c^2 - 2*(2*B
*a*b + A*b^2)*c)*x^2 - 2*((B*b*c - 2*A*c^2)*x^4 + B*a*b - 2*A*a*c + (B*b^2
- 2*A*b*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)
/(b^2 - 4*a*c)) - 4*(2*B*a^2 - A*a*b)*c)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
```

```
+ (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)
*x^2)]
```

giac [A] time = 1.37, size = 102, normalized size = 1.09

$$\frac{(Bb-2\,Ac)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}+\frac{Bbx^2-2\,Acx^2+2\,Ba-Ab}{2\left(cx^4+bx^2+a\right)\left(b^2-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] (B*b - 2*A*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(
-b^2 + 4*a*c)) + 1/2*(B*b*x^2 - 2*A*c*x^2 + 2*B*a - A*b)/((c*x^4 + b*x^2 +
a)*(b^2 - 4*a*c))
```

maple [A] time = 0.01, size = 127, normalized size = 1.35

$$\frac{2Ac\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}-\frac{Bb\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{Ab-2Ba+(2Ac-bB)\,x^2}{2\left(4ac-b^2\right)\left(c\,x^4+b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*((2*A*c-B*b)*x^2+A*b-2*a*B)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)^(
3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*c-1/(4*a*c-b^2)^(3/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.30, size = 264, normalized size = 2.81

$$\frac{\frac{A\,b-2\,B\,a}{2\left(4\,a\,c-b^2\right)}+\frac{x^2\,(2\,A\,c-B\,b)}{2\left(4\,a\,c-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\mathrm{atan}\left(\frac{\left(x^2\left(\frac{(2\,A\,c-B\,b)\left(2\,A\,c^3-B\,b\,c^2\right)}{a\left(4\,a\,c-b^2\right)^{7/2}}+\frac{\left(2\,b^3\,c^2-8\,a\,b\,c^3\right)(2\,A\,c-B\,b)^2\left(b^3-4\,a\,b\,c\right)}{2\,a\left(4\,a\,c-b^2\right)^{13/2}}\right)-\frac{2\,c^2\,(2\,A\,c-B\,b)^2\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{11/2}}\right)\left(4\,a\,c-b^2\right)^4}{8\,A^2\,c^4-8\,A\,B\,b\,c^3+2\,B^2\,b^2\,c^2}\right)(2\,A\,c-B\,b)}{\left(4\,a\,c-b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((A*b - 2*B*a)/(2*(4*a*c - b^2)) + (x^2*(2*A*c - B*b))/(2*(4*a*c - b^2)))/(
a + b*x^2 + c*x^4) + (atan(((x^2*(((2*A*c - B*b)*(2*A*c^3 - B*b*c^2))/(a*(4
*a*c - b^2)^(7/2)) + ((2*b^3*c^2 - 8*a*b*c^3)*(2*A*c - B*b)^2*(b^3 - 4*a*b*
c))/(2*a*(4*a*c - b^2)^(13/2))) - (2*c^2*(2*A*c - B*b)^2*(b^3 - 4*a*b*c))/(
4*a*c - b^2)^(11/2))*(4*a*c - b^2)^4)/(8*A^2*c^4 + 2*B^2*b^2*c^2 - 8*A*B*b*
c^3))*(2*A*c - B*b))/(4*a*c - b^2)^(3/2)
```

sympy [B] time = 3.36, size = 374, normalized size = 3.98

$$\frac{\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\,(-2Ac+Bb)\log\left(x^2+\frac{-2Abc+Bb^2-16a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\,(-2Ac+Bb)+8ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\,(-2Ac+Bb)-b^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\,(-2Ac+Bb)}{-4Ac^2+2Bbc}\right)}{2}-\frac{\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\,(-2Ac+Bb)\log\left(x^2+\frac{-2Abc+Bb^2+16a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\,(-2Ac+Bb)-8ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\,(-2Ac+Bb)+b^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\,(-2Ac+Bb)}{-4Ac^2+2Bbc}\right)}{2}+\frac{Ab-2Ba+x^2(2Ac-Bb)}{8a^2c-2ab^2+x^4\left(8ac^2-2b^2c\right)+x^2\left(8abc-2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b)*log(x**2 + (-2*A*b*c + B*b**2 - 1
6*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b) + 8*a*b**2*c*sqrt(-1/
(4*a*c - b**2)**3)*(-2*A*c + B*b) - b**4*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c
 + B*b))/(-4*A*c**2 + 2*B*b*c))/2 - sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*
b)*log(x**2 + (-2*A*b*c + B*b**2 + 16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*
(-2*A*c + B*b) - 8*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b) + b**
4*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b))/(-4*A*c**2 + 2*B*b*c))/2 + (A*
b - 2*B*a + x**2*(2*A*c - B*b))/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b
**2*c) + x**2*(8*a*b*c - 2*b**3))
```

3.94 $\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=150

$$\frac{\left(4a^2Bc+A\left(b^3-6abc\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{A\log(x)}{a^2}-\frac{-A\left(b^2-2ac\right)-\left(cx^2(Ab-2aB)\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Rubi [A] time = 0.33, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1251, 822, 800, 634, 618, 206, 628}

$$\frac{\left(4a^2Bc+A\left(b^3-6abc\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{A\log(x)}{a^2}-\frac{-A\left(b^2-2ac\right)+cx^2(-(Ab-2aB))+abB}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)) + ((4*a^2*B*c + A*(b^3 - 6*a*b*c))*ArcTanh[(b + 2*c*x^2)/Sqrt
[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)) + (A*Log[x])/a^2 - (A*Log[a + b
*x^2 + c*x^4])/(4*a^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-A\left(b^2-4ac\right)-(Ab-2aB)cx}{x\left(a+bx+cx^2\right)}\,dx,x,x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{A\left(-b^2+4ac\right)}{ax} + \frac{2a^2Bc+A\left(b^3-5abc\right)+Ac}{a\left(a+bx+cx^2\right)}\right.\right.}{2a\left(b^2-4ac\right)}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\log(x)}{a^2} - \frac{\text{Subst}\left(\int \frac{2a^2Bc+A\left(b^3-5abc\right)+Ac(b}{a+bx+cx^2}\right.}{2a^2\left(b^2-4ac\right)}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\log(x)}{a^2} - \frac{A\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a^2}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\log(x)}{a^2} - \frac{A\log\left(a+bx^2+cx^4\right)}{4a^2} + \frac{\left(4a^2\right.}{}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(4a^2Bc+A\left(b^3-6abc\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right.}{2a^2\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.33, size = 243, normalized size = 1.62

$$\frac{-\frac{\left(4a^2Bc+A\left(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}-6abc+b^3\right)\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{\left(4a^2Bc+A\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-6abc+b^3\right)\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{2a\left(aB\left(b+2cx^2\right)-A\left(-2ac+b^2+bcx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+4A\log(x)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*(a*B*(b + 2*c*x^2) - A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + 4*A*Log[x] - ((4*a^2*B*c + A*(b^3 - 6*a*b*c + b^2*Sqrt[b
```

```
^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2
])/(b^2 - 4*a*c)^(3/2) + ((4*a^2*B*c + A*(b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*
a*c] + 4*a*c*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2
 - 4*a*c)^(3/2))/(4*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2), x]
```

fricas [B] time = 1.82, size = 1014, normalized size = 6.76

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a^2*b^3 - 2*A*a*b^4 - 16*A*a^3*c^2 - 2*(4*(2*B*a^3 - A*a^2*b)*c^
2 - (2*B*a^2*b^2 - A*a*b^3)*c)*x^2 - (A*a*b^3 + (A*b^3*c + 2*(2*B*a^2 - 3*A
*a*b)*c^2)*x^4 + (A*b^4 + 2*(2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + 2*(2*B*a^3 - 3
*A*a^2*b)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (
2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 4*(2*B*a^3*b - 3*A*a
^2*b^2)*c + (A*a*b^4 - 8*A*a^2*b^2*c + 16*A*a^3*c^2 + (A*b^4*c - 8*A*a*b^2*
c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8*A*a*b^3*c + 16*A*a^2*b*c^2)*x^2)*log(c
*x^4 + b*x^2 + a) - 4*(A*a*b^4 - 8*A*a^2*b^2*c + 16*A*a^3*c^2 + (A*b^4*c -
8*A*a*b^2*c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8*A*a*b^3*c + 16*A*a^2*b*c^2)*
x^2)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c
^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), -1/4*(2
*B*a^2*b^3 - 2*A*a*b^4 - 16*A*a^3*c^2 - 2*(4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B
*a^2*b^2 - A*a*b^3)*c)*x^2 - 2*(A*a*b^3 + (A*b^3*c + 2*(2*B*a^2 - 3*A*a*b)*
c^2)*x^4 + (A*b^4 + 2*(2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + 2*(2*B*a^3 - 3*A*a^2
*b)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4
*a*c)) - 4*(2*B*a^3*b - 3*A*a^2*b^2)*c + (A*a*b^4 - 8*A*a^2*b^2*c + 16*A*a^
3*c^2 + (A*b^4*c - 8*A*a*b^2*c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8*A*a*b^3*c
 + 16*A*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) - 4*(A*a*b^4 - 8*A*a^2*b^2*c
 + 16*A*a^3*c^2 + (A*b^4*c - 8*A*a*b^2*c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8
*A*a*b^3*c + 16*A*a^2*b*c^2)*x^2)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c
^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c
+ 16*a^4*b*c^2)*x^2)]
```

giac [A] time = 1.71, size = 201, normalized size = 1.34

$$-\frac{\left(Ab^3+4\,Ba^2c-6\,Aabc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^2b^2-4\,a^3c\right)\sqrt{-b^2+4\,ac}}-\frac{A\log\left(cx^4+bx^2+a\right)}{4\,a^2}+\frac{A\log\left(x^2\right)}{2\,a^2}+\frac{Ab^2cx^4-4\,Aac^2x^4+Ab^3x^2-4\,Ba^2cx^2-2\,Aabcx^2-2\,Ba^2b+3\,Aab^2-8\,Aa^2c}{4\left(cx^4+bx^2+a\right)\left(a^2b^2-4\,a^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c
))/((a^2*b^2 - 4*a^3*c)*sqrt(-b^2 + 4*a*c)) - 1/4*A*log(c*x^4 + b*x^2 + a)/
a^2 + 1/2*A*log(x^2)/a^2 + 1/4*(A*b^2*c*x^4 - 4*A*a*c^2*x^4 + A*b^3*x^2 - 4
*B*a^2*c*x^2 - 2*A*a*b*c*x^2 - 2*B*a^2*b + 3*A*a*b^2 - 8*A*a^2*c)/((c*x^4 +
 b*x^2 + a)*(a^2*b^2 - 4*a^3*c))
```

maple [B] time = 0.02, size = 361, normalized size = 2.41

$$-\frac{Abc\,x^{2}}{2\left(c\,x^{4}+b\,x^{2}+a\right)\left(4ac-b^{2}\right)a}+\frac{Bc\,x^{2}}{\left(c\,x^{4}+b\,x^{2}+a\right)\left(4ac-b^{2}\right)}-\frac{3Abc\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}a}+\frac{A\,b^{3}\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{2\left(4ac-b^{2}\right)^{\frac{3}{2}}a^{2}}+\frac{2Bc\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}}-\frac{A\,b^{2}}{2\left(c\,x^{4}+b\,x^{2}+a\right)\left(4ac-b^{2}\right)a}-\frac{Ac\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{\left(4ac-b^{2}\right)a}+\frac{A\,b^{2}\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{4\left(4ac-b^{2}\right)a^{2}}+\frac{Ac}{\left(c\,x^{4}+b\,x^{2}+a\right)\left(4ac-b^{2}\right)}+\frac{Bb}{2\left(c\,x^{4}+b\,x^{2}+a\right)\left(4ac-b^{2}\right)}+\frac{A\ln(x)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] A*ln(x)/a^2-1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*A*b+1/(c*x^4+b*x^2+a)*c
/(4*a*c-b^2)*x^2*B+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A*c-1/2/a/(c*x^4+b*x^2+a)/
(4*a*c-b^2)*A*b^2+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b*B-1/a/(4*a*c-b^2)*c*ln(
c*x^4+b*x^2+a)*A+1/4/a^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*A*b^2-3/a/(4*a*c-b^2
)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b*c+1/2/a^2/(4*a*c-b^2)^(3/
2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^3+2/(4*a*c-b^2)^(3/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 7.88, size = 7119, normalized size = 47.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] ((2*A*a*c - A*b^2 + B*a*b)/(2*a*(4*a*c - b^2)) - (c*x^2*(A*b - 2*B*a))/(2*a
*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + (A*log(x))/a^2 - (log((((A + a^2*(-(
A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(((A + a^2*(
-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*((4*b*c^2*
(A*b^3 + 2*B*a^2*c - 5*A*a*b*c))/(a*(4*a*c - b^2)) - (b*c^2*(A + a^2*(-(A*b
^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*x^
2 - 10*a*c*x^2))/a^2 + (2*c^3*x^2*(A*b^3 + 8*B*a*b^2 - 20*B*a^2*c - 10*A*a*
b*c))/(a*(4*a*c - b^2))))/(4*a^2) + (c^3*(A*b - 2*B*a)*(4*A*b^3 + 2*B*a^2*c
 - 17*A*a*b*c))/(a^2*(4*a*c - b^2)^2) - (2*c^4*x^2*(A*b - 2*B*a)*(10*A*a*c
- 3*A*b^2 + B*a*b))/(a^2*(4*a*c - b^2)^2)))/(4*a^2) + (c^5*x^2*(A*b - 2*B*a
)^3)/(a^3*(4*a*c - b^2)^3) - (A*c^4*(A*b - 2*B*a)^2)/(a^3*(4*a*c - b^2)^2))
*(((A - a^2*(-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1/2
))*(((A - a^2*(-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1
/2))*((4*b*c^2*(A*b^3 + 2*B*a^2*c - 5*A*a*b*c))/(a*(4*a*c - b^2)) - (b*c^2*
(A - a^2*(-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*
(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^2 + (2*c^3*x^2*(A*b^3 + 8*B*a*b^2 - 20*B*
a^2*c - 10*A*a*b*c))/(a*(4*a*c - b^2))))/(4*a^2) + (c^3*(A*b - 2*B*a)*(4*A*
b^3 + 2*B*a^2*c - 17*A*a*b*c))/(a^2*(4*a*c - b^2)^2) - (2*c^4*x^2*(A*b - 2*
B*a)*(10*A*a*c - 3*A*b^2 + B*a*b))/(a^2*(4*a*c - b^2)^2)))/(4*a^2) + (c^5*x
^2*(A*b - 2*B*a)^3)/(a^3*(4*a*c - b^2)^3) - (A*c^4*(A*b - 2*B*a)^2)/(a^3*(4
*a*c - b^2)^2)))*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2
))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) - (atan((
x^2*((((A^3*b^3*c^5 - 8*B^3*a^3*c^5 + 12*A*B^2*a^2*b*c^5 - 6*A^2*B*a*b^2*c^
5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (((44*A^2*a^2*b
^3*c^5 - 4*B^2*a^3*b^3*c^4 + 160*A*B*a^4*c^6 - 6*A^2*a*b^5*c^4 - 80*A^2*a^3
*b*c^6 + 16*B^2*a^4*b*c^5 + 14*A*B*a^2*b^4*c^4 - 96*A*B*a^3*b^2*c^5)/(a^3*b
```

```
^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - (((640*B*a^6*c^6 + 320*A
*a^5*b*c^6 - 2*A*a^2*b^7*c^3 + 36*A*a^3*b^5*c^4 - 192*A*a^4*b^3*c^5 - 16*B*
a^3*b^6*c^3 + 168*B*a^4*b^4*c^4 - 576*B*a^5*b^2*c^5)/(a^3*b^6 - 64*a^6*c^3
 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c
 + 96*A*a^2*b^2*c^2)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1
056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*
c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c
^2)))*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^
2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A*b^6 - 128*A*a^
3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*
a^3*b^4*c + 192*a^4*b^2*c^2)) + (((((640*B*a^6*c^6 + 320*A*a^5*b*c^6 - 2*A*
a^2*b^7*c^3 + 36*A*a^3*b^5*c^4 - 192*A*a^4*b^3*c^5 - 16*B*a^3*b^6*c^3 + 168
*B*a^4*b^4*c^4 - 576*B*a^5*b^2*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c +
48*a^5*b^2*c^2) - ((2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c
^2)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 -
 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^
2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*
B*a^2*c - 6*A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((A*b^3 + 4*B*a^2*c - 6
*A*a*b*c)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2)*(2560
*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6
*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c
+ 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2
)))*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((A*b^3
+ 4*B*a^2*c - 6*A*a*b*c)^2*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a
^2*b^2*c^2)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b
^5*c^4 - 2688*a^6*b^3*c^5))/(32*a^4*(4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3 -
 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 1
92*a^4*b^2*c^2)))*(3*A*b^5 - 2*B*a^3*c^2 - 21*A*a*b^3*c + 33*A*a^2*b*c^2 +
2*B*a^2*b^2*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(400*A^2*a^3*c^3 - 6*A^2*b^6 + 4
*B^2*a^4*c^2 - 291*A^2*a^2*b^2*c^2 + 72*A^2*a*b^4*c + 2*A*B*a^2*b^3*c - 12*
A*B*a^3*b*c^2)) + (((((((640*B*a^6*c^6 + 320*A*a^5*b*c^6 - 2*A*a^2*b^7*c^3
+ 36*A*a^3*b^5*c^4 - 192*A*a^4*b^3*c^5 - 16*B*a^3*b^6*c^3 + 168*B*a^4*b^4*c
^4 - 576*B*a^5*b^2*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2) - ((2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2)*(2560*a^
7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^
3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^
6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*B*a^2*c - 6*
A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((A*b^3 + 4*B*a^2*c - 6*A*a*b*c)*(2
*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2)*(2560*a^7*b*c^6 +
 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(
8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2
*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A*b^6
 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 256*a^
5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) - (((44*A^2*a^2*b^3*c^5 - 4*B^2*a^
3*b^3*c^4 + 160*A*B*a^4*c^6 - 6*A^2*a*b^5*c^4 - 80*A^2*a^3*b*c^6 + 16*B^2*a
^4*b*c^5 + 14*A*B*a^2*b^4*c^4 - 96*A*B*a^3*b^2*c^5)/(a^3*b^6 - 64*a^6*c^3 -
 12*a^4*b^4*c + 48*a^5*b^2*c^2) - (((640*B*a^6*c^6 + 320*A*a^5*b*c^6 - 2*A*
a^2*b^7*c^3 + 36*A*a^3*b^5*c^4 - 192*A*a^4*b^3*c^5 - 16*B*a^3*b^6*c^3 + 168
*B*a^4*b^4*c^4 - 576*B*a^5*b^2*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c +
48*a^5*b^2*c^2) - ((2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c
^2)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 -
 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^
2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A*b^6 -
128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 256*a^5*c
^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c))/(4*
a^2*(4*a*c - b^2)^(3/2)) + ((A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^3*(2560*a^7*b*c
^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5
))/(64*a^6*(4*a*c - b^2)^(9/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^
5*b^2*c^2)))*(3*A*b^6 - 40*A*a^3*c^3 - 27*A*a*b^4*c + 2*B*a^2*b^3*c - 6*B*a
```

```
^3*b*c^2 + 69*A*a^2*b^2*c^2))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(400*A^2*a^3*c
^3 - 6*A^2*b^6 + 4*B^2*a^4*c^2 - 291*A^2*a^2*b^2*c^2 + 72*A^2*a*b^4*c + 2*A
*B*a^2*b^3*c - 12*A*B*a^3*b*c^2)))*(16*a^6*b^6*(4*a*c - b^2)^(9/2) - 1024*a
^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^8*b^
2*c^2*(4*a*c - b^2)^(9/2)))/(A^2*b^6*c^2 + 16*B^2*a^4*c^4 + 36*A^2*a^2*b^2*
c^4 - 12*A^2*a*b^4*c^3 + 8*A*B*a^2*b^3*c^3 - 48*A*B*a^3*b*c^4) - (((((((4*A
*a^2*b^6*c^2 - 32*B*a^5*b*c^4 - 36*A*a^3*b^4*c^3 + 80*A*a^4*b^2*c^4 + 8*B*a
^4*b^3*c^3)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5
*b^4*c^3 + 64*a^6*b^2*c^4)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a
^2*b^2*c^2))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c
^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c))/(4*
a^2*(4*a*c - b^2)^(3/2)) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^
4)*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c
 + 96*A*a^2*b^2*c^2))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^4 + 16*a^5*c^2 - 8*a
^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A
*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 25
6*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) + (((4*B^2*a^4*c^4 + 17*A^2*a^
2*b^2*c^4 - 4*A^2*a*b^4*c^3 + 8*A*B*a^2*b^3*c^3 - 36*A*B*a^3*b*c^4)/(a^3*b^
4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (((4*A*a^2*b^6*c^2 - 32*B*a^5*b*c^4 - 36*A*
a^3*b^4*c^3 + 80*A*a^4*b^2*c^4 + 8*B*a^4*b^3*c^3)/(a^3*b^4 + 16*a^5*c^2 - 8
*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*A*b^6 -
 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(a^3*b^4 + 16*a^5*c^2
 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))
)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^
6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*B*a^2*c - 6*
A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 +
64*a^6*b^2*c^4)*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^3)/(64*a^6*(4*a*c - b^2)^(9
/2)*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))*(16*a^6*b^6*(4*a*c - b^2)^(9/2)
 - 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2) + 76
8*a^8*b^2*c^2*(4*a*c - b^2)^(9/2))*(3*A*b^6 - 40*A*a^3*c^3 - 27*A*a*b^4*c +
 2*B*a^2*b^3*c - 6*B*a^3*b*c^2 + 69*A*a^2*b^2*c^2))/(8*a^3*c^2*(4*a*c - b^2
)^(7/2)*(A^2*b^6*c^2 + 16*B^2*a^4*c^4 + 36*A^2*a^2*b^2*c^4 - 12*A^2*a*b^4*c
^3 + 8*A*B*a^2*b^3*c^3 - 48*A*B*a^3*b*c^4)*(400*A^2*a^3*c^3 - 6*A^2*b^6 + 4
*B^2*a^4*c^2 - 291*A^2*a^2*b^2*c^2 + 72*A^2*a*b^4*c + 2*A*B*a^2*b^3*c - 12*
A*B*a^3*b*c^2)) + ((16*a^6*b^6*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*(4*a*c -
b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*(4*a*c - b
^2)^(9/2))*((A^3*b^2*c^4 + 4*A*B^2*a^2*c^4 - 4*A^2*B*a*b*c^4)/(a^3*b^4 + 16
*a^5*c^2 - 8*a^4*b^2*c) + (((4*B^2*a^4*c^4 + 17*A^2*a^2*b^2*c^4 - 4*A^2*a*b
^4*c^3 + 8*A*B*a^2*b^3*c^3 - 36*A*B*a^3*b*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^
4*b^2*c) + (((4*A*a^2*b^6*c^2 - 32*B*a^5*b*c^4 - 36*A*a^3*b^4*c^3 + 80*A*a^
4*b^2*c^4 + 8*B*a^4*b^3*c^3)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4
*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A
*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^
2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A*b^6 - 128*A*a^
3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*
a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96
*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c
^2)) - (((((4*A*a^2*b^6*c^2 - 32*B*a^5*b*c^4 - 36*A*a^3*b^4*c^3 + 80*A*a^4*
b^2*c^4 + 8*B*a^4*b^3*c^3)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b
^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a
*b^4*c + 96*A*a^2*b^2*c^2))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*
b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*B*a^2*c -
6*A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3
+ 64*a^6*b^2*c^4)*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)*(2*A*b^6 - 128*A*a^3*c^3
- 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*
b^2*c^2)))*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) - (
(4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(A*b^3 + 4*B*a^2*c - 6*A*
a*b*c)^2*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(32*a
```

```
^4*(4*a*c - b^2)^3*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^
5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(3*A*b^5 - 2*B*a^3*c^2 - 21*A*a*b
^3*c + 33*A*a^2*b*c^2 + 2*B*a^2*b^2*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(A^2*b^6
*c^2 + 16*B^2*a^4*c^4 + 36*A^2*a^2*b^2*c^4 - 12*A^2*a*b^4*c^3 + 8*A*B*a^2*b
^3*c^3 - 48*A*B*a^3*b*c^4)*(400*A^2*a^3*c^3 - 6*A^2*b^6 + 4*B^2*a^4*c^2 - 2
91*A^2*a^2*b^2*c^2 + 72*A^2*a*b^4*c + 2*A*B*a^2*b^3*c - 12*A*B*a^3*b*c^2)))
*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c))/(2*a^2*(4*a*c - b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.95 $\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=223

$$\frac{(2Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^3}-\frac{\log(x)(2Ab-aB)}{a^3}-\frac{-6aAc-abB+2Ab^2}{2a^2x^2\left(b^2-4ac\right)}+\frac{\left(abB\left(b^2-6ac\right)-2A\left(6a^2c^2-6a\right.\right.}{2a^3\left(b^2-4\right.}$$

Rubi [A] time = 0.42, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1251, 822, 800, 634, 618, 206, 628}

$$\frac{\left(abB\left(b^2-6ac\right)-2A\left(6a^2c^2-6ab^2c+b^4\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{3/2}}-\frac{-6aAc-abB+2Ab^2}{2a^2x^2\left(b^2-4ac\right)}+\frac{(2Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^3}-\frac{\log(x)(2Ab-aB)}{a^3}-\frac{-A\left(b^2-2ac\right)+cx^2(-(Ab-2aB))+abB}{2ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(2*A*b^2 - a*b*B - 6*a*A*c)/(2*a^2*(b^2 - 4*a*c)*x^2) - (a*b*B - A*(b^2 -
2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) +
 ((a*b*B*(b^2 - 6*a*c) - 2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2))*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(3/2)) - ((2*A*b - a*B)*Log
[x])/a^3 + ((2*A*b - a*B)*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-2Ab^2+abB+6aAc-2(Ab-2aB)cx}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{-2Ab^2+abB+6aAc}{ax^2}+\frac{(-2Ab+aB)\left(-b^2+4ac\right)}{a^2x}\right.\right.}{}$$

$$= -\frac{2Ab^2-abB-6aAc}{2a^2\left(b^2-4ac\right)x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{(2Ab-aB)\log(x)}{a^3} -$$

$$= -\frac{2Ab^2-abB-6aAc}{2a^2\left(b^2-4ac\right)x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{(2Ab-aB)\log(x)}{a^3} +$$

$$= -\frac{2Ab^2-abB-6aAc}{2a^2\left(b^2-4ac\right)x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{(2Ab-aB)\log(x)}{a^3} +$$

$$= -\frac{2Ab^2-abB-6aAc}{2a^2\left(b^2-4ac\right)x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} + \frac{\left(abB\left(b^2-6ac\right)-2A\right.}{}$$

Mathematica [A] time = 0.56, size = 379, normalized size = 1.70

$$\frac{\frac{\left(2A\left(6a^2c^2-6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)+aB\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}+6abc-b^3\right)\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{\left(2A\left(-6a^2c^2+6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}-b^4\right)+aB\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-6abc+b^3\right)\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{2a\left(A\left(-3abc-2ac^2x^2+b^3+b^2cx^2\right)+aB\left(2ac-b^2-bcx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+4\log(x)(aB-2Ab)-\frac{2aA}{x^2}}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*A)/x^2 - (2*a*(a*B*(-b^2 + 2*a*c - b*c*x^2) + A*(b^3 - 3*a*b*c + b^2
*c*x^2 - 2*a*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*(-2*A*b + a
```

```
*B)*Log[x] + ((a*B*(-b^3 + 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2
 - 4*a*c]) + 2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a
*b*c*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c
)^(3/2) + ((a*B*(b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4
*a*c]) + 2*A*(-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*
c*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(
3/2))/(4*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2), x]
```

fricas [B] time = 3.86, size = 1635, normalized size = 7.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*A*a^2*b^4 - 16*A*a^3*b^2*c + 32*A*a^4*c^2 + 2*(24*A*a^3*c^3 + 2*(2
*B*a^3*b - 7*A*a^2*b^2)*c^2 - (B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 - 2*(B*a^2*b^4
 - 2*A*a*b^5 + 4*(2*B*a^4 - 7*A*a^3*b)*c^2 - 3*(2*B*a^3*b^2 - 5*A*a^2*b^3)*
c)*x^2 + ((12*A*a^2*c^3 + 6*(B*a^2*b - 2*A*a*b^2)*c^2 - (B*a*b^3 - 2*A*b^4)
*c)*x^6 - (B*a*b^4 - 2*A*b^5 - 12*A*a^2*b*c^2 - 6*(B*a^2*b^2 - 2*A*a*b^3)*c
)*x^4 - (B*a^2*b^3 - 2*A*a*b^4 - 12*A*a^3*c^2 - 6*(B*a^3*b - 2*A*a^2*b^2)*c
)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^
2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((16*(B*a^3 - 2*A*a^2*b)*c
^3 - 8*(B*a^2*b^2 - 2*A*a*b^3)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5
- 2*A*b^6 + 16*(B*a^3*b - 2*A*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x
^4 + (B*a^2*b^4 - 2*A*a*b^5 + 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2
*A*a^2*b^3)*c)*x^2)*log(c*x^4 + b*x^2 + a) - 4*((16*(B*a^3 - 2*A*a^2*b)*c^3
 - 8*(B*a^2*b^2 - 2*A*a*b^3)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5 -
2*A*b^6 + 16*(B*a^3*b - 2*A*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x^4
 + (B*a^2*b^4 - 2*A*a*b^5 + 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2*A
*a^2*b^3)*c)*x^2)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (
a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6
*c^2)*x^2), -1/4*(2*A*a^2*b^4 - 16*A*a^3*b^2*c + 32*A*a^4*c^2 + 2*(24*A*a^3
*c^3 + 2*(2*B*a^3*b - 7*A*a^2*b^2)*c^2 - (B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 - 2
*(B*a^2*b^4 - 2*A*a*b^5 + 4*(2*B*a^4 - 7*A*a^3*b)*c^2 - 3*(2*B*a^3*b^2 - 5*
A*a^2*b^3)*c)*x^2 + 2*((12*A*a^2*c^3 + 6*(B*a^2*b - 2*A*a*b^2)*c^2 - (B*a*b
^3 - 2*A*b^4)*c)*x^6 - (B*a*b^4 - 2*A*b^5 - 12*A*a^2*b*c^2 - 6*(B*a^2*b^2 -
 2*A*a*b^3)*c)*x^4 - (B*a^2*b^3 - 2*A*a*b^4 - 12*A*a^3*c^2 - 6*(B*a^3*b - 2
*A*a^2*b^2)*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*
a*c)/(b^2 - 4*a*c)) + ((16*(B*a^3 - 2*A*a^2*b)*c^3 - 8*(B*a^2*b^2 - 2*A*a*b
^3)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5 - 2*A*b^6 + 16*(B*a^3*b - 2
*A*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 + (B*a^2*b^4 - 2*A*a*b^5
 + 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2*A*a^2*b^3)*c)*x^2)*log(c*x
^4 + b*x^2 + a) - 4*((16*(B*a^3 - 2*A*a^2*b)*c^3 - 8*(B*a^2*b^2 - 2*A*a*b^3
)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5 - 2*A*b^6 + 16*(B*a^3*b - 2*A
*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 + (B*a^2*b^4 - 2*A*a*b^5 +
 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2*A*a^2*b^3)*c)*x^2)*log(x))/(
(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*
a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2)]
```

giac [**A**] time = 1.65, size = 250, normalized size = 1.12

$$-\frac{\left(Bab^{3} - 2\,Ab^{4} - 6\,Ba^{2}bc + 12\,Aab^{2}c - 12\,Aa^{2}c^{2}\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\,\left(a^{3}b^{2} - 4\,a^{4}c\right)\sqrt{-b^{2} + 4\,ac}} + \frac{Babcx^{4} - 2\,Ab^{2}cx^{4} + 6\,Aac^{2}x^{4} + Bab^{2}x^{2} - 2\,Ab^{3}x^{2} - 2\,Ba^{2}cx^{2} + 7\,Aabcx^{2} - Aab^{2} + 4\,Aa^{2}c}{2\,\left(cx^{6} + bx^{4} + ax^{2}\right)\left(a^{2}b^{2} - 4\,a^{3}c\right)} - \frac{(Ba - 2\,Ab)\log\left(cx^{4} + bx^{2} + a\right)}{4\,a^{3}} + \frac{(Ba - 2\,Ab)\log\left(x^{2}\right)}{2\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*a*b^3 - 2*A*b^4 - 6*B*a^2*b*c + 12*A*a*b^2*c - 12*A*a^2*c^2)*arctan
((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c))
 + 1/2*(B*a*b*c*x^4 - 2*A*b^2*c*x^4 + 6*A*a*c^2*x^4 + B*a*b^2*x^2 - 2*A*b^3
*x^2 - 2*B*a^2*c*x^2 + 7*A*a*b*c*x^2 - A*a*b^2 + 4*A*a^2*c)/((c*x^6 + b*x^4
 + a*x^2)*(a^2*b^2 - 4*a^3*c)) - 1/4*(B*a - 2*A*b)*log(c*x^4 + b*x^2 + a)/a
^3 + 1/2*(B*a - 2*A*b)*log(x^2)/a^3
```

maple [**B**] time = 0.02, size = 622, normalized size = 2.79

$$-\frac{A c^{2} x^{2}}{\left(c x^{4}+b x^{2}+a\right)\left(4 a c-b^{2}\right) a}+\frac{A b^{2} c x^{2}}{2\left(c x^{4}+b x^{2}+a\right)\left(4 a c-b^{2}\right) a^{2}}-\frac{B b c x^{2}}{2\left(c x^{4}+b x^{2}+a\right)\left(4 a c-b^{2}\right) a}-\frac{6 A c^{2} \arctan\left(\frac{2 c x^{2}+b}{\sqrt{4 a c-b^{2}}}\right)}{\left(4 a c-b^{2}\right)^{\frac{3}{2}} a}+\frac{6 A b^{2} c \arctan\left(\frac{2 c x^{2}+b}{\sqrt{4 a c-b^{2}}}\right)}{\left(4 a c-b^{2}\right)^{\frac{3}{2}} a^{2}}-\frac{A b^{4} \arctan\left(\frac{2 c x^{2}+b}{\sqrt{4 a c-b^{2}}}\right)}{\left(4 a c-b^{2}\right)^{\frac{3}{2}} a^{3}}-\frac{3 B b c \arctan\left(\frac{2 c x^{2}+b}{\sqrt{4 a c-b^{2}}}\right)}{\left(4 a c-b^{2}\right)^{\frac{3}{2}} a}+\frac{B b^{3} \arctan\left(\frac{2 c x^{2}+b}{\sqrt{4 a c-b^{2}}}\right)}{2\left(4 a c-b^{2}\right)^{\frac{3}{2}} a^{2}}-\frac{3 A b c}{2\left(c x^{4}+b x^{2}+a\right)\left(4 a c-b^{2}\right) a}+\frac{A b^{3}}{2\left(c x^{4}+b x^{2}+a\right)\left(4 a c-b^{2}\right) a^{2}}+\frac{2 A b c \ln\left(c x^{4}+b x^{2}+a\right)}{\left(4 a c-b^{2}\right) a^{2}}-\frac{A b^{3} \ln\left(c x^{4}+b x^{2}+a\right)}{2\left(4 a c-b^{2}\right) a^{3}}-\frac{B b^{2}}{2\left(c x^{4}+b x^{2}+a\right)\left(4 a c-b^{2}\right) a}-\frac{B c \ln\left(c x^{4}+b x^{2}+a\right)}{\left(4 a c-b^{2}\right) a}+\frac{B b^{2} \ln\left(c x^{4}+b x^{2}+a\right)}{4\left(4 a c-b^{2}\right) a^{2}}+\frac{B c}{\left(c x^{4}+b x^{2}+a\right)\left(4 a c-b^{2}\right)}-\frac{2 A b \ln(x)}{a^{3}}+\frac{B \ln(x)}{a^{2}}-\frac{A}{2 a^{2} x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/2*A/a^2/x^2-2/a^3*ln(x)*A*b+1/a^2*ln(x)*B-1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c
-b^2)*x^2*A+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*A*b^2-1/2/a/(c*x^4+b*
x^2+a)*c/(4*a*c-b^2)*x^2*b*B-3/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A*b*c+1/2/a^
2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A*b^3+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*B*c-1/2/a
/(c*x^4+b*x^2+a)/(4*a*c-b^2)*B*b^2+2/a^2/(4*a*c-b^2)*c*ln(c*x^4+b*x^2+a)*A*
b-1/2/a^3/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*A*b^3-1/a/(4*a*c-b^2)*c*ln(c*x^4+b*
x^2+a)*B+1/4/a^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^2*B-6/a/(4*a*c-b^2)^(3/2)*
arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*c^2+6/a^2/(4*a*c-b^2)^(3/2)*arctan(
(2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^2*c-1/a^3/(4*a*c-b^2)^(3/2)*arctan((2*c*
x^2+b)/(4*a*c-b^2)^(1/2))*A*b^4-3/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4
*a*c-b^2)^(1/2))*b*B*c+1/2/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-
b^2)^(1/2))*B*b^3
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [**B**] time = 9.09, size = 10034, normalized size = 45.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] (log((((c^4*(2*A*b - B*a)*(6*A*a*c - 2*A*b^2 + B*a*b)^2)/(a^6*(4*a*c - b^2)^
2) - ((((B*a - 2*A*b + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2
*c + 6*B*a^2*b*c)^2/(a^6*(4*a*c - b^2)^3))^(1/2))*((b*c^2*(B*a - 2*A*b + a^
3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)^2/(a^6*
(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^3 + (4*b*c^2*(2*
A*b^4 + 6*A*a^2*c^2 - B*a*b^3 - 10*A*a*b^2*c + 5*B*a^2*b*c))/(a^2*(4*a*c -
```

```
b^2)) + (2*c^3*x^2*(2*A*b^4 - 60*A*a^2*c^2 - B*a*b^3 + 4*A*a*b^2*c + 10*B*a
^2*b*c))/(a^2*(4*a*c - b^2))))/(4*a^3) + (c^3*(36*A^2*a^3*c^3 - 16*A^2*b^6
- 4*B^2*a^2*b^4 + 16*A*B*a*b^5 - 216*A^2*a^2*b^2*c^2 + 116*A^2*a*b^4*c + 17
*B^2*a^3*b^2*c - 92*A*B*a^2*b^3*c + 108*A*B*a^3*b*c^2))/(a^4*(4*a*c - b^2)^
2) - (2*c^4*x^2*(12*A^2*b^5 + 3*B^2*a^2*b^3 - 12*A*B*a*b^4 - 60*A*B*a^3*c^2
 - 82*A^2*a*b^3*c - 10*B^2*a^3*b*c + 138*A^2*a^2*b*c^2 + 61*A*B*a^2*b^2*c))
/(a^4*(4*a*c - b^2)^2))*(B*a - 2*A*b + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*
b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)^2/(a^6*(4*a*c - b^2)^3))^(1/2)))/(4*a^3)
+ (c^5*x^2*(6*A*a*c - 2*A*b^2 + B*a*b)^3)/(a^6*(4*a*c - b^2)^3))*((c^4*(2*A
*b - B*a)*(6*A*a*c - 2*A*b^2 + B*a*b)^2)/(a^6*(4*a*c - b^2)^2) - ((((2*A*b
- B*a + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*
c)^2/(a^6*(4*a*c - b^2)^3))^(1/2))*((4*b*c^2*(2*A*b^4 + 6*A*a^2*c^2 - B*a*b
^3 - 10*A*a*b^2*c + 5*B*a^2*b*c))/(a^2*(4*a*c - b^2)) - (b*c^2*(2*A*b - B*a
 + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)^2/
(a^6*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^3 + (2*c^3*
x^2*(2*A*b^4 - 60*A*a^2*c^2 - B*a*b^3 + 4*A*a*b^2*c + 10*B*a^2*b*c))/(a^2*(
4*a*c - b^2))))/(4*a^3) - (c^3*(36*A^2*a^3*c^3 - 16*A^2*b^6 - 4*B^2*a^2*b^4
 + 16*A*B*a*b^5 - 216*A^2*a^2*b^2*c^2 + 116*A^2*a*b^4*c + 17*B^2*a^3*b^2*c
- 92*A*B*a^2*b^3*c + 108*A*B*a^3*b*c^2))/(a^4*(4*a*c - b^2)^2) + (2*c^4*x^2
*(12*A^2*b^5 + 3*B^2*a^2*b^3 - 12*A*B*a*b^4 - 60*A*B*a^3*c^2 - 82*A^2*a*b^3
*c - 10*B^2*a^3*b*c + 138*A^2*a^2*b*c^2 + 61*A*B*a^2*b^2*c))/(a^4*(4*a*c -
b^2)^2))*(2*A*b - B*a + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^
2*c + 6*B*a^2*b*c)^2/(a^6*(4*a*c - b^2)^3))^(1/2)))/(4*a^3) + (c^5*x^2*(6*A
*a*c - 2*A*b^2 + B*a*b)^3)/(a^6*(4*a*c - b^2)^3)))*(4*A*b^7 + 128*B*a^4*c^3
 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*
b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 1
92*a^5*b^2*c^2)) - (log(x)*(2*A*b - B*a))/a^3 - (A/(2*a) - (x^2*(2*A*b^3 -
B*a*b^2 + 2*B*a^2*c - 7*A*a*b*c))/(2*a^2*(4*a*c - b^2)) + (c*x^4*(6*A*a*c -
 2*A*b^2 + B*a*b))/(2*a^2*(4*a*c - b^2)))/(a*x^2 + b*x^4 + c*x^6) + (atan((
x^2*((((216*A^3*a^3*c^8 - 8*A^3*b^6*c^5 - 216*A^3*a^2*b^2*c^7 + B^3*a^3*b^3
*c^5 + 72*A^3*a*b^4*c^6 + 12*A^2*B*a*b^5*c^5 + 108*A^2*B*a^3*b*c^7 - 6*A*B^
2*a^2*b^4*c^5 + 18*A*B^2*a^3*b^2*c^6 - 72*A^2*B*a^2*b^3*c^6)/(a^6*b^6 - 64*
a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + (((24*A^2*a^2*b^7*c^4 - 260*A^2*
a^3*b^5*c^5 + 932*A^2*a^4*b^3*c^6 + 6*B^2*a^4*b^5*c^4 - 44*B^2*a^5*b^3*c^5
+ 480*A*B*a^6*c^7 - 1104*A^2*a^5*b*c^7 + 80*B^2*a^6*b*c^6 - 24*A*B*a^3*b^6*
c^4 + 218*A*B*a^4*b^4*c^5 - 608*A*B*a^5*b^2*c^6)/(a^6*b^6 - 64*a^9*c^3 - 12
*a^7*b^4*c + 48*a^8*b^2*c^2) + (((1920*A*a^8*c^7 - 320*B*a^8*b*c^6 - 4*A*a^
4*b^8*c^3 + 24*A*a^5*b^6*c^4 + 120*A*a^6*b^4*c^5 - 1088*A*a^7*b^2*c^6 + 2*B
*a^5*b^7*c^3 - 36*B*a^6*b^5*c^4 + 192*B*a^7*b^3*c^5)/(a^6*b^6 - 64*a^9*c^3
- 12*a^7*b^4*c + 48*a^8*b^2*c^2) - ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184
*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5)*(4*A*b^7 + 128*B*a^4*c^
3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2
*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*
a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(
4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B
*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 256*a^6
*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b
^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 -
96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2
*c^2)) - (((((1920*A*a^8*c^7 - 320*B*a^8*b*c^6 - 4*A*a^4*b^8*c^3 + 24*A*a^5
*b^6*c^4 + 120*A*a^6*b^4*c^5 - 1088*A*a^7*b^2*c^6 + 2*B*a^5*b^7*c^3 - 36*B*
a^6*b^5*c^4 + 192*B*a^7*b^3*c^5)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*
a^8*b^2*c^2) - ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*
a^8*b^5*c^4 - 2688*a^9*b^3*c^5)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A
*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*
b^2*c^2))/(2*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*
b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c
^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c))/(4*a^3*(4*a*c - b^2)^(3/2)) - (
(2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 26
```

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88*a^9*b^3*c^5)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*
b*c)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3
 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(8*a^3*(4*a*c - b
^2)^(3/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6
 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c^2
 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c))/(4*a^3*(4*a*c - b^2)^(3/2)) + ((25
60*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*
a^9*b^3*c^5)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c
)^2*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 +
 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(32*a^6*(4*a*c - b
^2)^3*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 2
56*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(6*A*a^3*c^3 - 6*A*b^6 + 3*B
*a*b^5 + 42*A*a*b^4*c - 21*B*a^2*b^3*c + 33*B*a^3*b*c^2 - 72*A*a^2*b^2*c^2)
)/(8*a^3*c^2*(4*a*c - b^2)^3*(36*A^2*a^4*c^4 - 24*A^2*b^8 - 6*B^2*a^2*b^6 +
 400*B^2*a^5*c^3 + 24*A*B*a*b^7 - 1152*A^2*a^2*b^4*c^2 + 1528*A^2*a^3*b^2*c
^3 - 291*B^2*a^4*b^2*c^2 + 288*A^2*a*b^6*c + 72*B^2*a^3*b^4*c + 1158*A*B*a^
3*b^3*c^2 - 288*A*B*a^2*b^5*c - 1564*A*B*a^4*b*c^3)) + (((((((1920*A*a^8*c^
7 - 320*B*a^8*b*c^6 - 4*A*a^4*b^8*c^3 + 24*A*a^5*b^6*c^4 + 120*A*a^6*b^4*c^
5 - 1088*A*a^7*b^2*c^6 + 2*B*a^5*b^7*c^3 - 36*B*a^6*b^5*c^4 + 192*B*a^7*b^3
*c^5)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - ((2560*a^10*
b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*
c^5)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3
 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(a^6*b^6 - 64*
a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*
b^4*c + 192*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c
 + 6*B*a^2*b*c))/(4*a^3*(4*a*c - b^2)^(3/2)) - ((2560*a^10*b*c^6 + 12*a^6*b
^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5)*(2*A*b^4 +
12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)*(4*A*b^7 + 128*B*a^4*c
^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^
2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(8*a^3*(4*a*c - b^2)^(3/2)*(a^6*b^6 - 64*a^9
*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4
*c + 192*a^5*b^2*c^2)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c
 - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2)
)/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)) + (((24*A^
2*a^2*b^7*c^4 - 260*A^2*a^3*b^5*c^5 + 932*A^2*a^4*b^3*c^6 + 6*B^2*a^4*b^5*c
^4 - 44*B^2*a^5*b^3*c^5 + 480*A*B*a^6*c^7 - 1104*A^2*a^5*b*c^7 + 80*B^2*a^6
*b*c^6 - 24*A*B*a^3*b^6*c^4 + 218*A*B*a^4*b^4*c^5 - 608*A*B*a^5*b^2*c^6)/(a
^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + (((1920*A*a^8*c^7 -
320*B*a^8*b*c^6 - 4*A*a^4*b^8*c^3 + 24*A*a^5*b^6*c^4 + 120*A*a^6*b^4*c^5 -
1088*A*a^7*b^2*c^6 + 2*B*a^5*b^7*c^3 - 36*B*a^6*b^5*c^4 + 192*B*a^7*b^3*c^5
)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - ((2560*a^10*b*c^
6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5)
*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24
*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(a^6*b^6 - 64*a^9*
c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*
c + 192*a^5*b^2*c^2)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c
- 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))
/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*A*b^4 +
 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c))/(4*a^3*(4*a*c - b^2)
^(3/2)) + ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b
^5*c^4 - 2688*a^9*b^3*c^5)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c
 + 6*B*a^2*b*c)^3)/(64*a^9*(4*a*c - b^2)^(9/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a
^7*b^4*c + 48*a^8*b^2*c^2)))*(768*A*b^7 + 5120*B*a^4*c^3 - 384*B*a*b^6 - 69
12*A*a*b^5*c - 12544*A*a^3*b*c^3 + 3456*B*a^2*b^4*c + 18432*A*a^2*b^3*c^2 -
 8832*B*a^3*b^2*c^2))/(1024*a^3*c^2*(4*a*c - b^2)^(7/2)*(36*A^2*a^4*c^4 - 2
4*A^2*b^8 - 6*B^2*a^2*b^6 + 400*B^2*a^5*c^3 + 24*A*B*a*b^7 - 1152*A^2*a^2*b
^4*c^2 + 1528*A^2*a^3*b^2*c^3 - 291*B^2*a^4*b^2*c^2 + 288*A^2*a*b^6*c + 72*
B^2*a^3*b^4*c + 1158*A*B*a^3*b^3*c^2 - 288*A*B*a^2*b^5*c - 1564*A*B*a^4*b*c
^3)))*(16*a^9*b^6*(4*a*c - b^2)^(9/2) - 1024*a^12*c^3*(4*a*c - b^2)^(9/2) -
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 192*a^10*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))
)/(144*A^2*a^4*c^6 + 4*A^2*b^8*c^2 + 192*A^2*a^2*b^4*c^4 - 288*A^2*a^3*b^2*
c^5 + B^2*a^2*b^6*c^2 - 12*B^2*a^3*b^4*c^3 + 36*B^2*a^4*b^2*c^4 - 48*A^2*a*
b^6*c^3 + 48*A*B*a^2*b^5*c^3 - 168*A*B*a^3*b^3*c^4 - 4*A*B*a*b^7*c^2 + 144*
A*B*a^4*b*c^5) + (((((((96*A*a^7*b*c^5 - 8*A*a^4*b^7*c^2 + 72*A*a^5*b^5*c^3
 - 184*A*a^6*b^3*c^4 + 4*B*a^5*b^6*c^2 - 36*B*a^6*b^4*c^3 + 80*B*a^7*b^2*c^
4)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3
+ 64*a^9*b^2*c^4)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256
*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(
a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c
 + 192*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*
B*a^2*b*c))/(4*a^3*(4*a*c - b^2)^(3/2)) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3
+ 64*a^9*b^2*c^4)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^
2*b*c)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^
3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(8*a^3*(4*a*c -
 b^2)^(3/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 -
 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 4
8*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a
^3*b^2*c^2))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2))
 - (((36*A^2*a^5*c^6 - 16*A^2*a^2*b^6*c^3 + 116*A^2*a^3*b^4*c^4 - 216*A^2*a
^4*b^2*c^5 - 4*B^2*a^4*b^4*c^3 + 17*B^2*a^5*b^2*c^4 + 16*A*B*a^3*b^5*c^3 -
92*A*B*a^4*b^3*c^4 + 108*A*B*a^5*b*c^5)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c
) - (((96*A*a^7*b*c^5 - 8*A*a^4*b^7*c^2 + 72*A*a^5*b^5*c^3 - 184*A*a^6*b^3*
c^4 + 4*B*a^5*b^6*c^2 - 36*B*a^6*b^4*c^3 + 80*B*a^7*b^2*c^4)/(a^6*b^4 + 16*
a^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)
*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24
*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(a^6*b^4 + 16*a^8*
c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^
2)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3
+ 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 2
56*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c^2 - B*
a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c))/(4*a^3*(4*a*c - b^2)^(3/2)) + ((4*a^7*
b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^
3 - 12*A*a*b^2*c + 6*B*a^2*b*c)^3)/(64*a^9*(4*a*c - b^2)^(9/2)*(a^6*b^4 + 1
6*a^8*c^2 - 8*a^7*b^2*c)))*(16*a^9*b^6*(4*a*c - b^2)^(9/2) - 1024*a^12*c^3*
(4*a*c - b^2)^(9/2) - 192*a^10*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^11*b^2*c^2
*(4*a*c - b^2)^(9/2))*(768*A*b^7 + 5120*B*a^4*c^3 - 384*B*a*b^6 - 6912*A*a*
b^5*c - 12544*A*a^3*b*c^3 + 3456*B*a^2*b^4*c + 18432*A*a^2*b^3*c^2 - 8832*B
*a^3*b^2*c^2))/(1024*a^3*c^2*(4*a*c - b^2)^(7/2)*(144*A^2*a^4*c^6 + 4*A^2*b
^8*c^2 + 192*A^2*a^2*b^4*c^4 - 288*A^2*a^3*b^2*c^5 + B^2*a^2*b^6*c^2 - 12*B
^2*a^3*b^4*c^3 + 36*B^2*a^4*b^2*c^4 - 48*A^2*a*b^6*c^3 + 48*A*B*a^2*b^5*c^3
 - 168*A*B*a^3*b^3*c^4 - 4*A*B*a*b^7*c^2 + 144*A*B*a^4*b*c^5)*(36*A^2*a^4*c
^4 - 24*A^2*b^8 - 6*B^2*a^2*b^6 + 400*B^2*a^5*c^3 + 24*A*B*a*b^7 - 1152*A^2
*a^2*b^4*c^2 + 1528*A^2*a^3*b^2*c^3 - 291*B^2*a^4*b^2*c^2 + 288*A^2*a*b^6*c
 + 72*B^2*a^3*b^4*c + 1158*A*B*a^3*b^3*c^2 - 288*A*B*a^2*b^5*c - 1564*A*B*a
^4*b*c^3)) + ((16*a^9*b^6*(4*a*c - b^2)^(9/2) - 1024*a^12*c^3*(4*a*c - b^2)
^(9/2) - 192*a^10*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^11*b^2*c^2*(4*a*c - b^2
)^(9/2))*((B^3*a^3*b^2*c^4 - 8*A^3*b^5*c^4 + 36*A^2*B*a^3*c^6 + 48*A^3*a*b^
3*c^5 - 72*A^3*a^2*b*c^6 + 12*A*B^2*a^3*b*c^5 + 12*A^2*B*a*b^4*c^4 - 6*A*B^
2*a^2*b^3*c^4 - 48*A^2*B*a^2*b^2*c^5)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)
- (((36*A^2*a^5*c^6 - 16*A^2*a^2*b^6*c^3 + 116*A^2*a^3*b^4*c^4 - 216*A^2*a^
4*b^2*c^5 - 4*B^2*a^4*b^4*c^3 + 17*B^2*a^5*b^2*c^4 + 16*A*B*a^3*b^5*c^3 - 9
2*A*B*a^4*b^3*c^4 + 108*A*B*a^5*b*c^5)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)
 - (((96*A*a^7*b*c^5 - 8*A*a^4*b^7*c^2 + 72*A*a^5*b^5*c^3 - 184*A*a^6*b^3*c
^4 + 4*B*a^5*b^6*c^2 - 36*B*a^6*b^4*c^3 + 80*B*a^7*b^2*c^4)/(a^6*b^4 + 16*a
^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*
(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*
B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(a^6*b^4 + 16*a^8*c
^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2
```

```
)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 +
 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 25
6*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*
B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c
^2 - 96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^
5*b^2*c^2)) - (((((96*A*a^7*b*c^5 - 8*A*a^4*b^7*c^2 + 72*A*a^5*b^5*c^3 - 18
4*A*a^6*b^3*c^4 + 4*B*a^5*b^6*c^2 - 36*B*a^6*b^4*c^3 + 80*B*a^7*b^2*c^4)/(a
^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*
a^9*b^2*c^4)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^
3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(a^6*b
^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 19
2*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2
*b*c))/(4*a^3*(4*a*c - b^2)^(3/2)) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*
a^9*b^2*c^4)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c
)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 2
4*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(8*a^3*(4*a*c - b^2)
^(3/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a
^4*b^4*c + 192*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^
2*c + 6*B*a^2*b*c))/(4*a^3*(4*a*c - b^2)^(3/2)) + ((4*a^7*b^6*c^2 - 32*a^8*
b^4*c^3 + 64*a^9*b^2*c^4)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c
+ 6*B*a^2*b*c)^2*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*
A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(32*a
^6*(4*a*c - b^2)^3*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^
6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(6*A*a^3*c^3 - 6*A*b^6 + 3*B*a*b^
5 + 42*A*a*b^4*c - 21*B*a^2*b^3*c + 33*B*a^3*b*c^2 - 72*A*a^2*b^2*c^2))/(8*
a^3*c^2*(4*a*c - b^2)^3*(144*A^2*a^4*c^6 + 4*A^2*b^8*c^2 + 192*A^2*a^2*b^4*
c^4 - 288*A^2*a^3*b^2*c^5 + B^2*a^2*b^6*c^2 - 12*B^2*a^3*b^4*c^3 + 36*B^2*a
^4*b^2*c^4 - 48*A^2*a*b^6*c^3 + 48*A*B*a^2*b^5*c^3 - 168*A*B*a^3*b^3*c^4 -
4*A*B*a*b^7*c^2 + 144*A*B*a^4*b*c^5)*(36*A^2*a^4*c^4 - 24*A^2*b^8 - 6*B^2*a
^2*b^6 + 400*B^2*a^5*c^3 + 24*A*B*a*b^7 - 1152*A^2*a^2*b^4*c^2 + 1528*A^2*a
^3*b^2*c^3 - 291*B^2*a^4*b^2*c^2 + 288*A^2*a*b^6*c + 72*B^2*a^3*b^4*c + 115
8*A*B*a^3*b^3*c^2 - 288*A*B*a^2*b^5*c - 1564*A*B*a^4*b*c^3)))*(2*A*b^4 + 12
*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c))/(2*a^3*(4*a*c - b^2)^(3
/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.96 $\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=425

$$-\frac{\left(-\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+3b^4B}{\sqrt{b^2-4ac}}+6aAc^2-13abBc-Ab^2c+3b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2Bc^2+8aAbc^2-19\ldots}{\sqrt{b^2-\ldots}}\right.}{}$$

Rubi [A] time = 3.67, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1275, 1279, 1166, 205}

$$-\frac{\left(-\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+3b^4B}{\sqrt{b^2-4ac}}+6aAc^2-13abBc-Ab^2c+3b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+3b^4B}{\sqrt{b^2-4ac}}+6aAc^2-13abBc-Ab^2c+3b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(-10aBc-Abc+3b^2B\right)}{2c^2\left(b^2-4ac\right)}-\frac{x^5\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{x^3(bB-2Ac)}{2c\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((3*b^2*B - A*b*c - 10*a*B*c)*x)/(2*c^2*(b^2 - 4*a*c)) - ((b*B - 2*A*c)*x^3
)/(2*c*(b^2 - 4*a*c)) - (x^5*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(2*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3*B - A*b^2*c - 13*a*b*B*c + 6*a*A*c^2 -
 (3*b^4*B - A*b^3*c - 19*a*b^2*B*c + 8*a*A*b*c^2 + 20*a^2*B*c^2)/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2
]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3*B - A*b^2*c
- 13*a*b*B*c + 6*a*A*c^2 + (3*b^4*B - A*b^3*c - 19*a*b^2*B*c + 8*a*A*b*c^2
+ 20*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c
]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
```

```
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\int \frac{x^4\left(5(Ab-2aB)-3(bB-2Ac)x^2\right)}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{(bB-2Ac)x^3}{2c\left(b^2-4ac\right)}-\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int \frac{x^2\left(-9a(bB-2Ac)-3\left(3b^2B-Abc-10aBc\right)x^2\right)}{a+bx^2+cx^4}\,d}{6c\left(b^2-4ac\right)}$$

$$= \frac{\left(3b^2B-Abc-10aBc\right)x}{2c^2\left(b^2-4ac\right)}-\frac{(bB-2Ac)x^3}{2c\left(b^2-4ac\right)}-\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\int\frac{-3a\left(3b^2B\right.}{}$$

$$= \frac{\left(3b^2B-Abc-10aBc\right)x}{2c^2\left(b^2-4ac\right)}-\frac{(bB-2Ac)x^3}{2c\left(b^2-4ac\right)}-\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(3b^3B-A\right.}{}$$

$$= \frac{\left(3b^2B-Abc-10aBc\right)x}{2c^2\left(b^2-4ac\right)}-\frac{(bB-2Ac)x^3}{2c\left(b^2-4ac\right)}-\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(3b^3B-A\right.}{}$$

Mathematica [A] time = 1.20, size = 455, normalized size = 1.07

$$\frac{\frac{2\sqrt{c}\,x\left(-2a^2Bc+a\left(-bc\left(A+3Bx^2\right)+2Ac^2x^2+b^2B\right)+b^2x^2(bB-Ac)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt{2}\left(2ac^2\left(3A\sqrt{b^2-4ac}-10aB\right)+b^2c\left(19aB-A\sqrt{b^2-4ac}\right)-abc\left(13B\sqrt{b^2-4ac}+8Ac\right)+b^3\left(3B\sqrt{b^2-4ac}+Ac\right)-3b^4B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(2ac^2\left(3A\sqrt{b^2-4ac}+10aB\right)-b^2c\left(A\sqrt{b^2-4ac}+19aB\right)+abc\left(8Ac-13B\sqrt{b^2-4ac}\right)+b^3\left(3B\sqrt{b^2-4ac}-Ac\right)+3b^4B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+4B\sqrt{c}\,x}{4c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*B*Sqrt[c]*x + (2*Sqrt[c]*x*(-2*a^2*B*c + b^2*(b*B - A*c)*x^2 + a*(b^2*B
+ 2*A*c^2*x^2 - b*c*(A + 3*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) -
(Sqrt[2]*(-3*b^4*B + b^2*c*(19*a*B - A*Sqrt[b^2 - 4*a*c]) + 2*a*c^2*(-10*a*
B + 3*A*Sqrt[b^2 - 4*a*c]) + b^3*(A*c + 3*B*Sqrt[b^2 - 4*a*c]) - a*b*c*(8*A
*c + 13*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(3
*b^4*B - b^2*c*(19*a*B + A*Sqrt[b^2 - 4*a*c]) + 2*a*c^2*(10*a*B + 3*A*Sqrt[
b^2 - 4*a*c]) + a*b*c*(8*A*c - 13*B*Sqrt[b^2 - 4*a*c]) + b^3*(-(A*c) + 3*B*
Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2, x]
```

fricas [B] time = 8.56, size = 7252, normalized size = 17.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(B*b^2*c - 4*B*a*c^2)*x^5 + 2*(3*B*b^3 + 2*A*a*c^2 - (11*B*a*b + A*b
^2)*c)*x^3 - sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (
b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4
 - 15*(28*B^2*a^3*b + 20*A*B*a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 +
80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c + (b^6*c^5 - 1
2*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^
6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2
200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6
*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^
3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*
a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^
3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log((189*
B^4*a^2*b^6 - 135*A*B^3*a*b^7 + 324*A^4*a^3*c^5 - 81*(28*A^3*B*a^3*b + A^4*
a^2*b^2)*c^4 - (2500*B^4*a^5 + 2500*A*B^3*a^4*b - 5016*A^2*B^2*a^3*b^2 - 64
7*A^3*B*a^2*b^3 - 5*A^4*a*b^4)*c^3 + 9*(625*B^4*a^4*b^2 - 303*A*B^3*a^3*b^3
 - 186*A^2*B^2*a^2*b^4 - 5*A^3*B*a*b^5)*c^2 - 27*(73*B^4*a^3*b^4 - 49*A*B^3
*a^2*b^5 - 5*A^2*B^2*a*b^6)*c)*x + 1/2*sqrt(1/2)*(27*B^3*b^10 + 144*(10*A^2
*B*a^4 + A^3*a^3*b)*c^6 - 8*(500*B^3*a^5 + 930*A*B^2*a^4*b + 252*A^2*B*a^3*
b^2 + 11*A^3*a^2*b^3)*c^5 + (11360*B^3*a^4*b^2 + 7608*A*B^2*a^3*b^3 + 882*A
^2*B*a^2*b^4 + 17*A^3*a*b^5)*c^4 - (8818*B^3*a^3*b^4 + 2841*A*B^2*a^2*b^5 +
 153*A^2*B*a*b^6 + A^3*b^7)*c^3 + 9*(329*B^3*a^2*b^6 + 51*A*B^2*a*b^7 + A^2
*B*b^8)*c^2 - 27*(17*B^3*a*b^8 + A*B^2*b^9)*c - (3*B*b^9*c^5 - 768*A*a^4*c^
10 + 128*(8*B*a^4*b + 5*A*a^3*b^2)*c^9 - 192*(5*B*a^3*b^3 + A*a^2*b^4)*c^8
+ 24*(14*B*a^2*b^5 + A*a*b^6)*c^7 - (52*B*a*b^7 + A*b^8)*c^6)*sqrt((81*B^4*
b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5
 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3
 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^
4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6
)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a
^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*
c^4 - 15*(28*B^2*a^3*b + 20*A*B*a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3
 + 80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c + (b^6*c^5
- 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2
*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4
+ 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4
- 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)
*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B
^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64
*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + sqr
t(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*
c^3)*x^2)*sqrt(-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^
3*b + 20*A*B*a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A
^2*b^5)*c^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c + (b^6*c^5 - 12*a*b^4*c^6 + 48
*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B
^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b
+ 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^
2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*
a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*
b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c
^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log((189*B^4*a^2*b^6 - 13
5*A*B^3*a*b^7 + 324*A^4*a^3*c^5 - 81*(28*A^3*B*a^3*b + A^4*a^2*b^2)*c^4 - (
2500*B^4*a^5 + 2500*A*B^3*a^4*b - 5016*A^2*B^2*a^3*b^2 - 647*A^3*B*a^2*b^3
- 5*A^4*a*b^4)*c^3 + 9*(625*B^4*a^4*b^2 - 303*A*B^3*a^3*b^3 - 186*A^2*B^2*a
```

```
^2*b^4 - 5*A^3*B*a*b^5)*c^2 - 27*(73*B^4*a^3*b^4 - 49*A*B^3*a^2*b^5 - 5*A^2
*B^2*a*b^6)*c)*x - 1/2*sqrt(1/2)*(27*B^3*b^10 + 144*(10*A^2*B*a^4 + A^3*a^3
*b)*c^6 - 8*(500*B^3*a^5 + 930*A*B^2*a^4*b + 252*A^2*B*a^3*b^2 + 11*A^3*a^2
*b^3)*c^5 + (11360*B^3*a^4*b^2 + 7608*A*B^2*a^3*b^3 + 882*A^2*B*a^2*b^4 + 1
7*A^3*a*b^5)*c^4 - (8818*B^3*a^3*b^4 + 2841*A*B^2*a^2*b^5 + 153*A^2*B*a*b^6
 + A^3*b^7)*c^3 + 9*(329*B^3*a^2*b^6 + 51*A*B^2*a*b^7 + A^2*B*b^8)*c^2 - 27
*(17*B^3*a*b^8 + A*B^2*b^9)*c - (3*B*b^9*c^5 - 768*A*a^4*c^10 + 128*(8*B*a^
4*b + 5*A*a^3*b^2)*c^9 - 192*(5*B*a^3*b^3 + A*a^2*b^4)*c^8 + 24*(14*B*a^2*b
^5 + A*a*b^6)*c^7 - (52*B*a*b^7 + A*b^8)*c^6)*sqrt((81*B^4*b^8 + 81*A^4*a^2
*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4
+ 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4
- 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)
*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B
^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64
*a^3*c^13)))*sqrt(-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2
*a^3*b + 20*A*B*a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4
+ A^2*b^5)*c^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c + (b^6*c^5 - 12*a*b^4*c^6 +
 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^
2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3
*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3
*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B
^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B
^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^
6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqrt(1/2)*(a*b^2*c^
2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-
(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^3*b + 20*A*B*a^2
*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*
(35*B^2*a*b^5 + 2*A*B*b^6)*c - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 6
4*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*
B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a
^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^
2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B
^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^1
0 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^
6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log((189*B^4*a^2*b^6 - 135*A*B^3*a*b^7 +
324*A^4*a^3*c^5 - 81*(28*A^3*B*a^3*b + A^4*a^2*b^2)*c^4 - (2500*B^4*a^5 + 2
500*A*B^3*a^4*b - 5016*A^2*B^2*a^3*b^2 - 647*A^3*B*a^2*b^3 - 5*A^4*a*b^4)*c
^3 + 9*(625*B^4*a^4*b^2 - 303*A*B^3*a^3*b^3 - 186*A^2*B^2*a^2*b^4 - 5*A^3*B
*a*b^5)*c^2 - 27*(73*B^4*a^3*b^4 - 49*A*B^3*a^2*b^5 - 5*A^2*B^2*a*b^6)*c)*x
 + 1/2*sqrt(1/2)*(27*B^3*b^10 + 144*(10*A^2*B*a^4 + A^3*a^3*b)*c^6 - 8*(500
*B^3*a^5 + 930*A*B^2*a^4*b + 252*A^2*B*a^3*b^2 + 11*A^3*a^2*b^3)*c^5 + (113
60*B^3*a^4*b^2 + 7608*A*B^2*a^3*b^3 + 882*A^2*B*a^2*b^4 + 17*A^3*a*b^5)*c^4
 - (8818*B^3*a^3*b^4 + 2841*A*B^2*a^2*b^5 + 153*A^2*B*a*b^6 + A^3*b^7)*c^3
+ 9*(329*B^3*a^2*b^6 + 51*A*B^2*a*b^7 + A^2*B*b^8)*c^2 - 27*(17*B^3*a*b^8 +
 A*B^2*b^9)*c + (3*B*b^9*c^5 - 768*A*a^4*c^10 + 128*(8*B*a^4*b + 5*A*a^3*b^
2)*c^9 - 192*(5*B*a^3*b^3 + A*a^2*b^4)*c^8 + 24*(14*B*a^2*b^5 + A*a*b^6)*c^
7 - (52*B*a*b^7 + A*b^8)*c^6)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^
2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3
*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3
*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B
^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B
^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqr
t(-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^3*b + 20*A*B*
a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A^2*b^5)*c^2 -
 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7
- 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A
^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^
2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3
*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*
```

```
A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*
c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4
*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 +
(b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-(9*B^2*b^7 + 60*
(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^3*b + 20*A*B*a^2*b^2 + A^2*a*b^3
)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*(35*B^2*a*b^5 +
2*A*B*b^6)*c - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt(
(81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*
b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3
*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*
B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2
*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^1
1 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c
^7 - 64*a^3*c^8))*log((189*B^4*a^2*b^6 - 135*A*B^3*a*b^7 + 324*A^4*a^3*c^5
- 81*(28*A^3*B*a^3*b + A^4*a^2*b^2)*c^4 - (2500*B^4*a^5 + 2500*A*B^3*a^4*b
- 5016*A^2*B^2*a^3*b^2 - 647*A^3*B*a^2*b^3 - 5*A^4*a*b^4)*c^3 + 9*(625*B^4*
a^4*b^2 - 303*A*B^3*a^3*b^3 - 186*A^2*B^2*a^2*b^4 - 5*A^3*B*a*b^5)*c^2 - 27
*(73*B^4*a^3*b^4 - 49*A*B^3*a^2*b^5 - 5*A^2*B^2*a*b^6)*c)*x - 1/2*sqrt(1/2)
*(27*B^3*b^10 + 144*(10*A^2*B*a^4 + A^3*a^3*b)*c^6 - 8*(500*B^3*a^5 + 930*A
*B^2*a^4*b + 252*A^2*B*a^3*b^2 + 11*A^3*a^2*b^3)*c^5 + (11360*B^3*a^4*b^2 +
 7608*A*B^2*a^3*b^3 + 882*A^2*B*a^2*b^4 + 17*A^3*a*b^5)*c^4 - (8818*B^3*a^3
*b^4 + 2841*A*B^2*a^2*b^5 + 153*A^2*B*a*b^6 + A^3*b^7)*c^3 + 9*(329*B^3*a^2
*b^6 + 51*A*B^2*a*b^7 + A^2*B*b^8)*c^2 - 27*(17*B^3*a*b^8 + A*B^2*b^9)*c +
(3*B*b^9*c^5 - 768*A*a^4*c^10 + 128*(8*B*a^4*b + 5*A*a^3*b^2)*c^9 - 192*(5*
B*a^3*b^3 + A*a^2*b^4)*c^8 + 24*(14*B*a^2*b^5 + A*a*b^6)*c^7 - (52*B*a*b^7
+ A*b^8)*c^6)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A
^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^
2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3
*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*
A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*
c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*B^2*b^7 +
60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^3*b + 20*A*B*a^2*b^2 + A^2*a*
b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*(35*B^2*a*b^5
 + 2*A*B*b^6)*c - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sq
rt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4
*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*
A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A
^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*
A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*
c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^
2*c^7 - 64*a^3*c^8))) + 2*(3*B*a*b^2 - (10*B*a^2 + A*a*b)*c)*x)/(a*b^2*c^2
- 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)
```

giac [B] time = 6.49, size = 5681, normalized size = 13.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] B*x/c^2 + 1/2*(B*b^3*x^3 - 3*B*a*b*c*x^3 - A*b^2*c*x^3 + 2*A*a*c^2*x^3 + B*
a*b^2*x - 2*B*a^2*c*x - A*a*b*c*x)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)
) + 1/16*((2*b^4*c^3 - 20*a*b^2*c^4 + 48*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 12*(b^2 - 4*a*c)*a*c^4)*(b^2*c^2 - 4
```

```
*a*c^3)^2*A - (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^3
)*(b^2*c^2 - 4*a*c^3)^2*B + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
5*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - 2*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 2*a*b^5*c^5 + 16*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^6 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 - 16*a^
2*b^3*c^6 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 + 32*a^3*b*
c^7 - 2*(b^2 - 4*a*c)*a*b^3*c^5 + 8*(b^2 - 4*a*c)*a^2*b*c^6)*A*abs(-b^2*c^2
 + 4*a*c^3) - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 6*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 6*a*b^6*c^4 + 128*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^3*b^2*c^5 + 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b^3*c^5 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 68*a^2*b^4
*c^5 - 160*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 22*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^2*c^6 + 256*a^3*b^2*c^6 + 40*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*c^7 - 320*a^4*c^7 - 6*(b^2 - 4*a*c)*a*b^4*c^4 + 44*(b^2 - 4*a*
c)*a^2*b^2*c^5 - 80*(b^2 - 4*a*c)*a^3*c^6)*B*abs(-b^2*c^2 + 4*a*c^3) - (2*b
^8*c^7 - 32*a*b^6*c^8 + 160*a^2*b^4*c^9 - 256*a^3*b^2*c^10 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^5 + 16*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 - 2*(b^2 - 4*a*c)*b^6*c^7 + 24*(b^2 - 4*
a*c)*a*b^4*c^8 - 64*(b^2 - 4*a*c)*a^2*b^2*c^9)*A + (6*b^9*c^6 - 86*a*b^7*c^
7 + 440*a^2*b^5*c^8 - 928*a^3*b^3*c^9 + 640*a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^5 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 6*(b^2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*
a*c)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*b^3*c^8 + 160*(b^2 - 4*a*c)*a^3*b*c^
9)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^2 - 4*a*b*c^3 + sqrt((b^3*c^2 - 4*a*
b*c^3)^2 - 4*(a*b^2*c^2 - 4*a^2*c^3)*(b^2*c^3 - 4*a*c^4)))/(b^2*c^3 - 4*a*c
^4)))/((a*b^6*c^5 - 12*a^2*b^4*c^6 - 2*a*b^5*c^6 + 48*a^3*b^2*c^7 + 16*a^2*
b^3*c^7 + a*b^4*c^7 - 64*a^4*c^8 - 32*a^3*b*c^8 - 8*a^2*b^2*c^8 + 16*a^3*c^
9)*abs(-b^2*c^2 + 4*a*c^3)*abs(c)) - 1/16*((2*b^4*c^3 - 20*a*b^2*c^4 + 48*a
^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c +
10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*
```

sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 24*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 12*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 12*
(b^2 - 4*a*c)*a*c^4)*(b^2*c^2 - 4*a*c^3)^2*A - (6*b^5*c^2 - 50*a*b^3*c^3 +
104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c
 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b
^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^3)*(b^2*c^2 - 4*a*c^3)^2*B - 2*(sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^3*c^5 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 -
 2*a*b^5*c^5 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 + 8*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^3*c^6 + 16*a^2*b^3*c^6 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b*c^7 - 32*a^3*b*c^7 + 2*(b^2 - 4*a*c)*a*b^3*c^5 - 8*(b^2 -
 4*a*c)*a^2*b*c^6)*A*abs(-b^2*c^2 + 4*a*c^3) + 2*(3*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^4*c^4 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 6*a*b^6*c^
4 + 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 44*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^4*c^5 + 68*a^2*b^4*c^5 - 160*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 2
2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 256*a^3*b^2*c^6 + 4
0*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^7 + 320*a^4*c^7 + 6*(b^2 -
4*a*c)*a*b^4*c^4 - 44*(b^2 - 4*a*c)*a^2*b^2*c^5 + 80*(b^2 - 4*a*c)*a^3*c^6)
*B*abs(-b^2*c^2 + 4*a*c^3) - (2*b^8*c^7 - 32*a*b^6*c^8 + 160*a^2*b^4*c^9 -
256*a^3*b^2*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^8*c^5 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^6*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c
^6 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c
^7 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 128*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 64*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 12*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - 32*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 - 2*(b^
2 - 4*a*c)*b^6*c^7 + 24*(b^2 - 4*a*c)*a*b^4*c^8 - 64*(b^2 - 4*a*c)*a^2*b^2*
c^9)*A + (6*b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8 - 928*a^3*b^3*c^9 + 64
0*a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
7*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^5
 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^
6 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6
- 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 464
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 19
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 3
1*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 320
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 160*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 96*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 80*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 6*(b^
2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*a*c)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*b^3
c^8 + 160(b^2 - 4*a*c)*a^3*b*c^9)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^2 -
 4*a*b*c^3 - sqrt((b^3*c^2 - 4*a*b*c^3)^2 - 4*(a*b^2*c^2 - 4*a^2*c^3)*(b^2*

```
c^3 - 4*a*c^4)))/(b^2*c^3 - 4*a*c^4)))/((a*b^6*c^5 - 12*a^2*b^4*c^6 - 2*a*b
^5*c^6 + 48*a^3*b^2*c^7 + 16*a^2*b^3*c^7 + a*b^4*c^7 - 64*a^4*c^8 - 32*a^3*
b*c^8 - 8*a^2*b^2*c^8 + 16*a^3*c^9)*abs(-b^2*c^2 + 4*a*c^3)*abs(c))
```

maple [B] time = 0.04, size = 1507, normalized size = 3.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*A*b^2+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-
b^2)*x*B-1/2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*b^3*B-3/2/(4*a*c-b^2)*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*c*x)*a*A+3/2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*A-1/2/c^2/(c*x^4
+b*x^2+a)*a/(4*a*c-b^2)*x*b^2*B+3/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a*b*B
+1/2/c/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*A*b+1/4/c/(4*a*c-b^2)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*A*b^2-3/4/c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*B-1/4/c/(4*a*c-
b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*c*x)*A*b^2+3/4/c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*B+
5/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*B+5/(4*a*c-b^2)/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*B-19/4/c/(4*a*c-b^2)/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2*B-19/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x)*a*b^2*B-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a*A+2/(4*a*c-b
^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*A*b+2/(4*a*c-b^2)/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*a*A*b+3/4/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x)*b^4*B-1/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*A*b^3+13/4/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*B+3/4/c^2/(4*a*c-
b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*B-1/4/c/(4*a*c-b^2)/(-4*a
*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^3-13/4/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*a*b*B+B/c^2*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*b^3 + 2*A*a*c^2 - (3*B*a*b + A*b^2)*c)*x^3 + (B*a*b^2 - (2*B*a^2 +
A*a*b)*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 -
4*a*b*c^3)*x^2) + B*x/c^2 - 1/2*integrate((3*B*a*b^2 + (3*B*b^3 + 6*A*a*c^2
 - (13*B*a*b + A*b^2)*c)*x^2 - (10*B*a^2 + A*a*b)*c)/(c*x^4 + b*x^2 + a), x
)/(b^2*c^2 - 4*a*c^3)
```

mupad [B] time = 4.36, size = 16604, normalized size = 39.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] (B*x)/c^2 - atan(((((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7 +
48*B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c^4
+ 4224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a
*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2
*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*
b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c
^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c -
 b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^
7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*
(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 806
4*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*
a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*
c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*
c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840
*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 102
4*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(
(9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^1
2*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2
077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800
*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-
(4*a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^
9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B
^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*
b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) -
 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2
*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 +
240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10))
)^(1/2) - (x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^2*a^4*c^4 -
6*A*B*b^7*c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^2*a^3*b^2*c^
3 - 114*B^2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 + 86*A*B*a*b^5
*c^2 + 472*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*B^
2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c -
 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B
^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*
a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*
c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3
 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^
6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c
^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*
A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4
*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a
^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/
2)*1i - (((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7 + 48*B*a*b^8
*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c^4 + 4224*B*a
^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 -
 48*a^2*b^2*c^5)) + (x*((9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2
- 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 -
3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240
*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^
2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c -
 b^2)^9)^(1/2) - 26880*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*
b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-
(4*a*c - b^2)^9)^(1/2) - 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9
```

)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^1
2*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c
^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^
8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*B^2*b^4
*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*
A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^
2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b
^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b
^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 38
40*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b*c
^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 -
30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a
*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c
- b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^
8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) +
(x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^2*a^4*c^4 - 6*A*B*b^7*
c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^2*a^3*b^2*c^3 - 114*B^
2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 + 86*A*B*a*b^5*c^2 + 472
*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*B^2*b^4*(-(4
*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a
^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9
*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^
5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9
)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^
2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b*c^6 +
1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720
*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a*b^10
*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2
)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7
 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i)/(((
(10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7 + 48*B*a*b^8*c^3 + 192
*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c^4 + 4224*B*a^3*b^4*c^5
 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^
2*c^5)) - (x*((9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^
13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a
^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b
^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*
B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c
 + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(
1/2) - 26880*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 +
22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b
^2)^9)^(1/2) - 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) +
44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24
*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*
a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^
2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*B^2*b^4*(-(4*a*c
- b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^
7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2
+ 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A
^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/
2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5
*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b*c^6 + 1548*
A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*
a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a*b^10*c^2
- 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^
(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 12
80*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*B^2*
b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^2*a^4*c^4 - 6*A*B*b^7*c + 74*A^2
*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^2*a^3*b^2*c^3 - 114*B^2*a*b^6*c

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- 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 + 86*A*B*a*b^5*c^2 + 472*A*B*a^3*b
*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*B^2*b^4*(-(4*a*c - b^2
)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4
 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 106
56*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^
2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) +
15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^
7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b*c^6 + 1548*A*B*a
^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b
^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a*b^10*c^2 - 6*A
*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2)
)/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^
3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (216*A^3*a^4*c^
4 - 63*B^3*a^3*b^5 + 5*A^3*a^2*b^4*c^2 - 66*A^3*a^3*b^2*c^3 + 45*A*B^2*a^2*
b^6 + 600*A*B^2*a^5*c^3 + 573*B^3*a^4*b^3*c - 1300*B^3*a^5*b*c^2 - 402*A*B^
2*a^3*b^4*c - 30*A^2*B*a^2*b^5*c - 924*A^2*B*a^4*b*c^3 + 762*A*B^2*a^4*b^2*
c^2 + 339*A^2*B*a^3*b^3*c^2)/(4*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a
^2*b^2*c^5)) + (((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7 + 48*
B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c^4 + 4
224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^
4*c^4 - 48*a^2*b^2*c^5)) + (x*((9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^
11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5
*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3
- 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^
2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 +
 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(
4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A
*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b
^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c -
 b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^1
1 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^
4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a
^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*
B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c
 - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077
*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^
2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*
a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c
^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*
a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4
*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 15
2*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-
(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240
*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(
1/2) + (x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^2*a^4*c^4 - 6*A
*B*b^7*c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^2*a^3*b^2*c^3 -
 114*B^2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 + 86*A*B*a*b^5*c^
2 + 472*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*B^2*b
^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 28
8*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*
a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5
*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c -
 b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 +
3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b
*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5
- 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B
*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*
c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*
b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2))
```

)*((9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*
b^12*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6
- 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44
800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2
*(-(4*a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a
*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 2688
0*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a
^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2
) - 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*
c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6
 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^1
0)))^(1/2)*2i - ((x^3*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c))/(2*(4*a*c
- b^2)) - (x*(2*B*a^2*c - B*a*b^2 + A*a*b*c))/(2*(4*a*c - b^2)))/(a*c^2 + c
^3*x^4 + b*c^2*x^2) - atan(((((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^
4*b*c^7 + 48*B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^
2*b^6*c^4 + 4224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*
c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9
*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 15
04*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^
2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2
*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360
*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9
*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^
8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^
6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^
3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32
*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6
*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^
5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b
^2*c^4)))*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2)
 - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4
*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5
*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^
2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c -
 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/
2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22
400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2
)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44
*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a
*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^
5*b^2*c^10)))^(1/2) - (x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^
2*a^4*c^4 - 6*A*B*b^7*c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^
2*a^3*b^2*c^3 - 114*B^2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 +
86*A*B*a*b^5*c^2 + 472*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c
^4)))*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6
*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3
*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4
 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^
2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*
A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) +
 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*
A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)
^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B
*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^1
0*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^
2*c^10)))^(1/2)*1i - (((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7
 + 48*B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c
^4 + 4224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 1
2*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^

```
4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*
a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b
^7*c^3 + 30240*B^2*a^4*b^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a
*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^
6*c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*
c^3*(-(4*a*c - b^2)^9)^(1/2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 +
 8064*A*B*a^3*b^6*c^4 - 22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*
B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(
4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*
a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 +
3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 -
 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)
))*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*
B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^
6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 -
44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c
^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*A^2
*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) + 26
880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*A*B
*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1
/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*
b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c
^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c
^10)))^(1/2) + (x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^2*a^4*c
^4 - 6*A*B*b^7*c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^2*a^3*b
^2*c^3 - 114*B^2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 + 86*A*B*
a*b^5*c^2 + 472*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(
-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*b^
12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^6 +
2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 - 4480
0*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(
-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b
^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) + 26880*
B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*A*B*a^4
*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2)
+ 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^
2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 +
 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)
))^(1/2)*1i)/((((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7 + 48*B
*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c^4 + 42
24*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4
*c^4 - 48*a^2*b^2*c^5)) - (x*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*
a*c - b^2)^9)^(1/2) - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5
*c^5 + 3840*A^2*a^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3
+ 30240*B^2*a^4*b^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^
2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 -
 213*B^2*a*b^11*c - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(
4*a*c - b^2)^9)^(1/2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A
*B*a^3*b^6*c^4 - 22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b
^2*c*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c -
 b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^1
1 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^
4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a
^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9
*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*b^12*
c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^6 + 207
7*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 - 44800*B
^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4
*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b^9*
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c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) + 26880*B^2
*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*A*B*a^4*b^
4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 1
52*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(
-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 24
0*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^
(1/2) - (x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^2*a^4*c^4 - 6*
A*B*b^7*c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^2*a^3*b^2*c^3
- 114*B^2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 + 86*A*B*a*b^5*c
^2 + 472*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*B^2
*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*b^12*c +
288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^6 + 2077*B^
2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 - 44800*B^2*a
^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c
 - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b^9*c^3
- 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) + 26880*B^2*a^6
*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*A*B*a^4*b^4*c^
5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 152*A
*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*
a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^
2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2
) - (216*A^3*a^4*c^4 - 63*B^3*a^3*b^5 + 5*A^3*a^2*b^4*c^2 - 66*A^3*a^3*b^2*
c^3 + 45*A*B^2*a^2*b^6 + 600*A*B^2*a^5*c^3 + 573*B^3*a^4*b^3*c - 1300*B^3*a
^5*b*c^2 - 402*A*B^2*a^3*b^4*c - 30*A^2*B*a^2*b^5*c - 924*A^2*B*a^4*b*c^3 +
 762*A*B^2*a^4*b^2*c^2 + 339*A^2*B*a^3*b^3*c^2)/(4*(64*a^3*c^6 - b^6*c^3 +
12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 102
4*A*a^4*b*c^7 + 48*B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 73
6*B*a^2*b^6*c^4 + 4224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6
- b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*(-(9*B^2*b^13 + A^2*b^11*c
^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^
4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10
656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b
^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c
^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*
a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*
b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*
A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2
))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a
^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 19
2*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 -
 8*a*b^2*c^4)))*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)
^(1/2) - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A
^2*a^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a
^4*b^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) +
 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^
11*c - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^
9)^(1/2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^
4 - 22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c
 - b^2)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2
) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5
- 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6
144*a^5*b^2*c^10)))^(1/2) + (x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 +
200*B^2*a^4*c^4 - 6*A*B*b^7*c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 -
718*B^2*a^3*b^2*c^3 - 114*B^2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*
c^3 + 86*A*B*a*b^5*c^2 + 472*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a
*b^2*c^4)))*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/
2) - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a
^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b
```

```
^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*
B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c
 - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(
1/2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 -
22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b
^2)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) +
44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24
*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*
a^5*b^2*c^10)))^(1/2)))*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c -
 b^2)^9)^(1/2) - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5
+ 3840*A^2*a^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 302
40*B^2*a^4*b^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)
^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*
B^2*a*b^11*c - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c
 - b^2)^9)^(1/2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^
3*b^6*c^4 - 22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*
(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)
^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b
^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4
*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.97 $\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=336

$$-\frac{x^3\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{x(bB-2Ac)}{2c\left(b^2-4ac\right)}+\frac{\left(-\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\cdots}{\sqrt{b-\sqrt{b^2-\cdots}}}\right.}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$

Rubi [A] time = 1.72, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1275, 1279, 1166, 205}

$$\frac{\left(-\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x^3\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{x(bB-2Ac)}{2c\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b*B - 2*A*c)*x)/(2*c*(b^2 - 4*a*c)) - (x^3*(A*b - 2*a*B - (b*B - 2*A*c)*
x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^2*B + A*b*c - 6*a*B*c - (
b^3*B + A*b^2*c - 8*a*b*B*c + 4*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*B + A*b*c - 6*a*B*c + (b^3*B + A*b^2*c
 - 8*a*b*B*c + 4*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b
^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
```

```
3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\int \frac{x^2\left(3(Ab-2aB)+(-bB+2Ac)x^2\right)}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{(bB-2Ac)x}{2c\left(b^2-4ac\right)}-\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int \frac{-a(bB-2Ac)+\left(-b^2B-Abc+6aBc\right)x^2}{a+bx^2+cx^4}\,dx}{2c\left(b^2-4ac\right)}$$

$$= -\frac{(bB-2Ac)x}{2c\left(b^2-4ac\right)}-\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b^2B+Abc-6aBc-\frac{b^3B+Ab^2c-8a\ldots}{\sqrt{b^2-\ldots}}\right.\ldots}{4c\left(b^2-\ldots\right.}$$

$$= -\frac{(bB-2Ac)x}{2c\left(b^2-4ac\right)}-\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b^2B+Abc-6aBc-\frac{b^3B+Ab^2c-8a\ldots}{\sqrt{b^2-\ldots}}\right.\ldots}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\ldots}$$

Mathematica [A] time = 0.85, size = 362, normalized size = 1.08

$$\frac{\frac{2\sqrt{c}\left(2acx\left(A+Bx^2\right)-abBx+bx^3(Ac-bB)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(b^2\left(B\sqrt{b^2-4ac}-Ac\right)+bc\left(A\sqrt{b^2-4ac}+8aB\right)-2ac\left(3B\sqrt{b^2-4ac}+2Ac\right)+b^3(-B)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b^2\left(B\sqrt{b^2-4ac}+Ac\right)+b\left(Ac\sqrt{b^2-4ac}-8aBc\right)+2ac\left(2Ac-3B\sqrt{b^2-4ac}\right)+b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*Sqrt[c]*(-(a*b*B*x) + b*(-(b*B) + A*c)*x^3 + 2*a*c*x*(A + B*x^2)))/((b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(b^3*B) + b*c*(8*a*B + A*Sqrt[
b^2 - 4*a*c]) + b^2*(-(A*c) + B*Sqrt[b^2 - 4*a*c]) - 2*a*c*(2*A*c + 3*B*Sqr
t[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3*B + 2*a*c*
(2*A*c - 3*B*Sqrt[b^2 - 4*a*c]) + b^2*(A*c + B*Sqrt[b^2 - 4*a*c]) + b*(-8*a
*B*c + A*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2, x]
```

fricas [B] time = 2.82, size = 4658, normalized size = 13.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(B*b^2 - (2*B*a + A*b)*c)*x^3 + sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4
+ a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(B^2*b^5 - 12*(4*A*B
*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*B^2
*a*b^3 - 2*A*B*b^4)*c + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c
^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a
^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(
b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4
*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log(-(5*B^4*a*b^4 - 3*A*B^3*b^5 - 4*A^
4*a*c^4 + (20*A^3*B*a*b - 3*A^4*b^2)*c^3 + 3*(108*B^4*a^3 - 108*A*B^3*a^2*b
 + 28*A^2*B^2*a*b^2 - 3*A^3*B*b^3)*c^2 - (81*B^4*a^2*b^2 - 65*A*B^3*a*b^3 +
 9*A^2*B^2*b^4)*c)*x + 1/2*sqrt(1/2)*(B^3*b^7 - 17*B^3*a*b^5*c - 32*A^3*a^2
*c^5 + 16*(18*A*B^2*a^3 - 3*A^2*B*a^2*b + A^3*a*b^2)*c^4 - 2*(72*B^3*a^3*b
+ 72*A*B^2*a^2*b^2 - 12*A^2*B*a*b^3 + A^3*b^4)*c^3 + (88*B^3*a^2*b^3 + 18*A
*B^2*a*b^4 - 3*A^2*B*b^5)*c^2 - (B*b^8*c^3 + 256*(3*B*a^4 - A*a^3*b)*c^7 -
64*(10*B*a^3*b^2 - 3*A*a^2*b^3)*c^6 + 48*(4*B*a^2*b^4 - A*a*b^5)*c^5 - 4*(6
*B*a*b^6 - A*b^7)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b
)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2
- 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*
sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^
2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c + (b^6*c^3 - 12*a*b^4*c^4 +
 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*
A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4
*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*
c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) - sqrt(1/2)
*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*
sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^
2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c + (b^6*c^3 - 12*a*b^4*c^4 +
 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*
A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4
*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*
c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log(-(5*B^4*
a*b^4 - 3*A*B^3*b^5 - 4*A^4*a*c^4 + (20*A^3*B*a*b - 3*A^4*b^2)*c^3 + 3*(108
*B^4*a^3 - 108*A*B^3*a^2*b + 28*A^2*B^2*a*b^2 - 3*A^3*B*b^3)*c^2 - (81*B^4*
a^2*b^2 - 65*A*B^3*a*b^3 + 9*A^2*B^2*b^4)*c)*x - 1/2*sqrt(1/2)*(B^3*b^7 - 1
7*B^3*a*b^5*c - 32*A^3*a^2*c^5 + 16*(18*A*B^2*a^3 - 3*A^2*B*a^2*b + A^3*a*b
^2)*c^4 - 2*(72*B^3*a^3*b + 72*A*B^2*a^2*b^2 - 12*A^2*B*a*b^3 + A^3*b^4)*c^
3 + (88*B^3*a^2*b^3 + 18*A*B^2*a*b^4 - 3*A^2*B*b^5)*c^2 - (B*b^8*c^3 + 256*
(3*B*a^4 - A*a^3*b)*c^7 - 64*(10*B*a^3*b^2 - 3*A*a^2*b^3)*c^6 + 48*(4*B*a^2
*b^4 - A*a*b^5)*c^5 - 4*(6*B*a*b^6 - A*b^7)*c^4)*sqrt((B^4*b^4 + A^4*c^4 -
2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*
b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^
2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (
60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c +
 (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4
*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A
^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7
+ 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 -
 64*a^3*c^6))) + sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 +
 (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (
60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c -
 (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4
*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A
^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7
+ 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 -
 64*a^3*c^6))*log(-(5*B^4*a*b^4 - 3*A*B^3*b^5 - 4*A^4*a*c^4 + (20*A^3*B*a*b
 - 3*A^4*b^2)*c^3 + 3*(108*B^4*a^3 - 108*A*B^3*a^2*b + 28*A^2*B^2*a*b^2 - 3
*A^3*B*b^3)*c^2 - (81*B^4*a^2*b^2 - 65*A*B^3*a*b^3 + 9*A^2*B^2*b^4)*c)*x +
1/2*sqrt(1/2)*(B^3*b^7 - 17*B^3*a*b^5*c - 32*A^3*a^2*c^5 + 16*(18*A*B^2*a^3
 - 3*A^2*B*a^2*b + A^3*a*b^2)*c^4 - 2*(72*B^3*a^3*b + 72*A*B^2*a^2*b^2 - 12
```

```
*A^2*B*a*b^3 + A^3*b^4)*c^3 + (88*B^3*a^2*b^3 + 18*A*B^2*a*b^4 - 3*A^2*B*b^
5)*c^2 + (B*b^8*c^3 + 256*(3*B*a^4 - A*a^3*b)*c^7 - 64*(10*B*a^3*b^2 - 3*A*
a^2*b^3)*c^6 + 48*(4*B*a^2*b^4 - A*a*b^5)*c^5 - 4*(6*B*a*b^6 - A*b^7)*c^4)*
sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 -
 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*
c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(B^2*b^5 - 12*(4*
A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*
B^2*a*b^3 - 2*A*B*b^4)*c - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^
3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^
4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c
)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*
b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x
^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(B^2*b^5 - 12*(4*
A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*
B^2*a*b^3 - 2*A*B*b^4)*c - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^
3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^
4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c
)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*
b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log(-(5*B^4*a*b^4 - 3*A*B^3*b^5 - 4
*A^4*a*c^4 + (20*A^3*B*a*b - 3*A^4*b^2)*c^3 + 3*(108*B^4*a^3 - 108*A*B^3*a^
2*b + 28*A^2*B^2*a*b^2 - 3*A^3*B*b^3)*c^2 - (81*B^4*a^2*b^2 - 65*A*B^3*a*b^
3 + 9*A^2*B^2*b^4)*c)*x - 1/2*sqrt(1/2)*(B^3*b^7 - 17*B^3*a*b^5*c - 32*A^3*
a^2*c^5 + 16*(18*A*B^2*a^3 - 3*A^2*B*a^2*b + A^3*a*b^2)*c^4 - 2*(72*B^3*a^3
*b + 72*A*B^2*a^2*b^2 - 12*A^2*B*a*b^3 + A^3*b^4)*c^3 + (88*B^3*a^2*b^3 + 1
8*A*B^2*a*b^4 - 3*A^2*B*b^5)*c^2 + (B*b^8*c^3 + 256*(3*B*a^4 - A*a^3*b)*c^7
 - 64*(10*B*a^3*b^2 - 3*A*a^2*b^3)*c^6 + 48*(4*B*a^2*b^4 - A*a*b^5)*c^5 - 4
*(6*B*a*b^6 - A*b^7)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*
B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b
^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)
))*sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a
*b^2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c - (b^6*c^3 - 12*a*b^4*c^
4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a -
 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*
B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a
^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) + 2*(B*a
*b - 2*A*a*c)*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c -
4*a*b*c^2)*x^2)
```

giac [B] time = 6.29, size = 4538, normalized size = 13.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*b^2*x^3 - 2*B*a*c*x^3 - A*b*c*x^3 + B*a*b*x - 2*A*a*c*x)/((c*x^4 +
b*x^2 + a)*(b^2*c - 4*a*c^2)) + 1/16*((2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(b^2*c - 4*a*c^2)
^2*A + (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^
3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2*B
 - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*
```

```
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^3*c^4 - 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 - 32*a^3*c^6 + 2*(b^2 - 4*a*c)*a*b^2*c^4 -
 8*(b^2 - 4*a*c)*a^2*c^5)*A*abs(b^2*c - 4*a*c^2) + 2*(sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c
^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^
2*b*c^4)*B*abs(b^2*c - 4*a*c^2) - (2*b^7*c^5 - 8*a*b^5*c^6 - 32*a^2*b^3*c^7
 + 128*a^3*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^7*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^5*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^
4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^
5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 64*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 32*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 16*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 - 2*(b^2
- 4*a*c)*b^5*c^5 + 32*(b^2 - 4*a*c)*a^2*b*c^7)*A - (2*b^8*c^4 - 32*a*b^6*c^
5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(
b^2 - 4*a*c)*a^2*b^2*c^6)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c - 4*a*b*c^2 +
 sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))
/(b^2*c^2 - 4*a*c^3)))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*
b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^
2*c^6 + 16*a^3*c^7)*abs(b^2*c - 4*a*c^2)*abs(c)) - 1/16*((2*b^3*c^3 - 8*a*b
*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3
)*(b^2*c - 4*a*c^2)^2*A + (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b
^2*c - 4*a*c^2)^2*B + 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3
- 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 16*a^2*b^2*c^5
- 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^6 + 32*a^3*c^6 - 2*(b^2 -
 4*a*c)*a*b^2*c^4 + 8*(b^2 - 4*a*c)*a^2*c^5)*A*abs(b^2*c - 4*a*c^2) - 2*(sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
```

```
b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^
4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 +
 8*(b^2 - 4*a*c)*a^2*b*c^4)*B*abs(b^2*c - 4*a*c^2) - (2*b^7*c^5 - 8*a*b^5*c
^6 - 32*a^2*b^3*c^7 + 128*a^3*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^5*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b^6*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^5*c^5 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^2*c^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^7 - 2*(b^2 - 4*a*c)*b^5*c^5 + 32*(b^2 - 4*a*c)*a^2*b*c^7)*A - (2*b^
8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*
c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6)*B)*arctan(2*sqrt(1/2)*x/sqrt((
b^3*c - 4*a*b*c^2 - sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b
^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3)))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*
a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a
^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*abs(b^2*c - 4*a*c^2)*abs(c))
```

maple [B] time = 0.04, size = 1030, normalized size = 3.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2*(A*b*c+2*B*a*c-B*b^2)/c/(4*a*c-b^2)*x^3-1/2*(2*A*c-B*b)/(4*a*c-b^2)*a
/c*x)/(c*x^4+b*x^2+a)+1/4/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b-1/(4*a*c-b^
2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*A-1/4/(4*a*c-b^2)/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2-3/2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*a*B+1/4/(4*a*c-b^2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*B+2/(4*a*c-b^2)/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*B-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)*b^3*B-1/4/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b-1/(4*a*c-
b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*A-1/4/(4*a*c-b^2)/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+3/2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*B-1/
4/(4*a*c-b^2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*B+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2
```

```
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*c*x)*a*b*B-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
c*x)*b^3*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((B*b^2 - (2*B*a + A*b)*c)*x^3 + (B*a*b - 2*A*a*c)*x)/((b^2*c^2 - 4*a*
c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2) + 1/2*integrate((
B*a*b - 2*A*a*c + (B*b^2 - (6*B*a - A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b
^2*c - 4*a*c^2)
```

mupad [B] time = 5.18, size = 12396, normalized size = 36.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((x^3*(A*b*c - B*b^2 + 2*B*a*c))/(2*c*(4*a*c - b^2)) + (x*(2*A*a*c - B*a*
b))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - atan(((((2048*A*a^4*c^6 - 32
*A*a*b^6*c^3 + 16*B*a*b^7*c^2 - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536
*A*a^3*b^2*c^5 - 192*B*a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*
c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*
c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^1
0*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504
*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c
 - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*
c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*
A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^
12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*
c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^
6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(B^2*b^11 +
 A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^9
)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2
*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c
^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b*c^
6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A
*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32
*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*
c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) - (x*(B^2*b^6 + 8*A^2*a^
2*c^4 + A^2*b^4*c^2 - 72*B^2*a^3*c^3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 1
6*B^2*a*b^4*c + 2*A^2*a*b^2*c^3 - 14*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3))/(2*(
b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 9
6*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^
3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^
2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 1
92*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c
*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3
 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6
144*a^5*b^2*c^8)))^(1/2)*1i - (((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b
^7*c^2 - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*
a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48
*a^2*b^2*c^3)) + (x*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^
(1/2) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^
```

```
4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840
*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c -
 b^2)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c
^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)
^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4
+ 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)
))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(
2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*
(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c
- 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2
*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9
*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5
+ 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*
b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c
^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7
- 6144*a^5*b^2*c^8)))^(1/2) + (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 - 7
2*B^2*a^3*c^3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*a
*b^2*c^3 - 14*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8*
a*b^2*c^2)))*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) +
 B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512
*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^
4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9
)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 12
8*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/
2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a
^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2
)*1i)/((((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b^7*c^2 - 1024*B*a^4*b*c
^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*a^2*b^5*c^3 + 768*B*a^3
*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*(-(
B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*
c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5
 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072
*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^
2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^
4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^
8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 128
0*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 -
192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 -
 8*a*b^2*c^2)))*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2
) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 +
512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2
*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2
)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 -
 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^
(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 24
0*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(
1/2) - (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 - 72*B^2*a^3*c^3 + 2*A*B*b
^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*a*b^2*c^3 - 14*A*B*a*b^3
*c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(B^2*b^1
1 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2
)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*
B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^
5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b
*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 153
6*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/
(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b
^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) - (3*A*B^2*a*b^5 - 21
6*B^3*a^4*c^2 - 5*B^3*a^2*b^4 - 24*A^2*B*a^3*c^3 + 3*A^3*a*b^3*c^2 + 4*A^3*
a^2*b*c^3 + 66*B^3*a^3*b^2*c - 51*A*B^2*a^2*b^3*c + 204*A*B^2*a^3*b*c^2 - 4
```

2*A^2*B*a^2*b^2*c^2 + 6*A^2*B*a*b^4*c)/(4*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^
2 + 48*a^2*b^2*c^3)) + (((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b^7*c^2
- 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*a^2*b^5
*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^
2*c^3)) + (x*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) +
B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512
*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^
4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9
)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 12
8*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/
2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a
^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2
)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*
c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*
c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^
2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^
5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*
c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A
*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(
4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24
*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*
a^5*b^2*c^8)))^(1/2) + (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 - 72*B^2*a
^3*c^3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*a*b^2*c^
3 - 14*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c
^2)))*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^
2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^
3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c
^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2)
 - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a
^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36
*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*
c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)))*(-(
B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*
c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5
 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072
*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^
2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^
4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^
8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 128
0*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*2i - atan(((((
2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b^7*c^2 - 1024*B*a^4*b*c^5 + 384*A
*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/
(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*((A^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)
^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*
a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^
6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6
 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*
B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*
(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c
^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c
^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^
2)))*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*
(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*
b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4
 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) +
 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3
*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) + 36*A
*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^

```
5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) - (x*(B
^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 - 72*B^2*a^3*c^3 + 2*A*B*b^5*c + 74*B^
2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*a*b^2*c^3 - 14*A*B*a*b^3*c^2 - 8*A*B
*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((A^2*c^2*(-(4*a*c - b
^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) -
2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c
^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^
2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*B
^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2
*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^6
*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840
*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*1i - (((2048*A*a^4*c^6 - 32*A*a*b^
6*c^3 + 16*B*a*b^7*c^2 - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*
b^2*c^5 - 192*B*a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 1
2*a*b^4*c^2 + 48*a^2*b^2*c^3)) + (x*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^
2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96
*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3
*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2
*a*c*(-(4*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 19
2*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*
(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 -
 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 61
44*a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*
a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((A^2*c^2*(-(4*a*c -
b^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) -
 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*
c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B
^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*
B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^
2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^
6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 384
0*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A
^2*b^4*c^2 - 72*B^2*a^3*c^3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b
^4*c + 2*A^2*a*b^2*c^3 - 14*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 1
6*a^2*c^3 - 8*a*b^2*c^2)))*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2
 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*
b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3
- 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4
*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2
*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c
- b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^1
0*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^
2*c^8)))^(1/2)*1i)/((((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b^7*c^2 - 1
024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*a^2*b^5*c^
3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c
^3)) - (x*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 + B^2
*b^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2
*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^
3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1
/2) + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*
B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) +
 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b
^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(1
6*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c +
16*a^2*c^3 - 8*a*b^2*c^2)))*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^
2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2
*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3
 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(
4*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^
```

```
2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c
 - b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^
10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b
^2*c^8)))^(1/2) - (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 - 72*B^2*a^3*c^
3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*a*b^2*c^3 - 1
4*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))
*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4
*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*
c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3
072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 768
*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4
*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a
*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 -
1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) - (3*A*B^2*
a*b^5 - 216*B^3*a^4*c^2 - 5*B^3*a^2*b^4 - 24*A^2*B*a^3*c^3 + 3*A^3*a*b^3*c^
2 + 4*A^3*a^2*b*c^3 + 66*B^3*a^3*b^2*c - 51*A*B^2*a^2*b^3*c + 204*A*B^2*a^3
*b*c^2 - 42*A^2*B*a^2*b^2*c^2 + 6*A^2*B*a*b^4*c)/(4*(b^6*c - 64*a^3*c^4 - 1
2*a*b^4*c^2 + 48*a^2*b^2*c^3)) + (((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*
a*b^7*c^2 - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192
*B*a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 +
 48*a^2*b^2*c^3)) + (x*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2 - B
^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*b^5*
c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 38
40*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c
 - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6
*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^
2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^
4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^
8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))
/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2)
 - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c
 + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^
2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c -
9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5
 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B
*b*c*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*
c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7
 - 6144*a^5*b^2*c^8)))^(1/2) + (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 -
72*B^2*a^3*c^3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*
a*b^2*c^3 - 14*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8
*a*b^2*c^2)))*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 +
 B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512
*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^
4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9
)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 12
8*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/
2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a
^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2
)))*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(
-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b
^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4
- 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) +
768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*
b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) + 36*A*
B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5
 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.98 $\int \frac{x^2\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=276

$$-\frac{x\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(-\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+b\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)}$$

Rubi [A] time = 0.55, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1275, 1166, 205}

$$-\frac{x\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(-\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(x*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
) + ((b*B - 2*A*c - (b^2*B - 4*A*b*c + 4*a*B*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 -
4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*B - 2*A*c + (b^2*B - 4*A*b*c + 4*
a*B*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^2\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{x\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\int \frac{Ab-2aB+(bB-2Ac)x^2}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{x\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(bB-2Ac-\frac{b^2B-4Abc+4aBc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4\left(b^2-4ac\right)} +$$

$$= -\frac{x\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(bB-2Ac-\frac{b^2B-4Abc+4aBc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} +$$

Mathematica [A] time = 0.66, size = 298, normalized size = 1.08

$$\frac{1}{4}\left(\frac{2x\left(B\left(2a+bx^2\right)-A\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(-2Ac\sqrt{b^2-4ac}+bB\sqrt{b^2-4ac}-4aBc+4Abc+b^2(-B)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(-2Ac\sqrt{b^2-4ac}+bB\sqrt{b^2-4ac}+4aBc-4Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(B*(2*a + b*x^2) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) + (Sqrt[2]*(-(b^2*B) + 4*A*b*c - 4*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - 2*A*
c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^
2*B - 4*A*b*c + 4*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - 2*A*c*Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*
a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2, x]
```

fricas [B] time = 1.92, size = 3467, normalized size = 12.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(B*b - 2*A*c)*x^3 - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2
*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2
+ (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*c^2 + 4
8*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b
^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*
b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4*a^2*b^2 - A*B^3*a*b^3 -
 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*a^3 - 12*A*B^3*a^2*b
+ A^3*B*b^3)*c)*x + 1/2*sqrt(1/2)*(2*B^3*a^2*b^4 - A*B^2*a*b^5 - 16*(2*A^2*
B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b + 2*A^2*B*a^2*b^2 - A
^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*A^2*B*a*b^4 - A^3*b^5
```

```
)*c + (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^5*b + A*a^4*b^2)*c^4
- 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a*b^8)*c)*sqrt((B^4*a^
2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4
 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2 + (12*B^2
*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2
*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 -
12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*b^4*c^2 +
 48*a^3*b^2*c^3 - 64*a^4*c^4))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2
- 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*
b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*
c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)
/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c -
12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4*a^2*b^2 - A*B^3*
a*b^3 - 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*a^3 - 12*A*B^3
*a^2*b + A^3*B*b^3)*c)*x - 1/2*sqrt(1/2)*(2*B^3*a^2*b^4 - A*B^2*a*b^5 - 16*
(2*A^2*B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b + 2*A^2*B*a^2*
b^2 - A^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*A^2*B*a*b^4 -
A^3*b^5)*c + (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^5*b + A*a^4*b^
2)*c^4 - 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a*b^8)*c)*sqrt(
(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*
b^2*c^4 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2 +
(12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*c^2 + 48*
a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6
*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*b^
4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 +
 a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3
*A^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c - 12*a
^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A
^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b
^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4*a^2*b^2 -
 A*B^3*a*b^3 - 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*a^3 - 1
2*A*B^3*a^2*b + A^3*B*b^3)*c)*x + 1/2*sqrt(1/2)*(2*B^3*a^2*b^4 - A*B^2*a*b^
5 - 16*(2*A^2*B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b + 2*A^2
*B*a^2*b^2 - A^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*A^2*B*a
*b^4 - A^3*b^5)*c - (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^5*b + A
*a^4*b^2)*c^4 - 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a*b^8)*c
)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 +
48*a^4*b^2*c^4 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)
*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c - 12*a^2*b^4*c^
2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(
a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12
*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))) + sqrt(1/2)*((b^2*c - 4*a*c^2
)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4*A*B*
a^2 - 3*A^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c
 - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*
a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)
))/(a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4*a^
2*b^2 - A*B^3*a*b^3 - 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*
a^3 - 12*A*B^3*a^2*b + A^3*B*b^3)*c)*x - 1/2*sqrt(1/2)*(2*B^3*a^2*b^4 - A*B
^2*a*b^5 - 16*(2*A^2*B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b
+ 2*A^2*B*a^2*b^2 - A^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*
A^2*B*a*b^4 - A^3*b^5)*c - (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^
5*b + A*a^4*b^2)*c^4 - 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a
*b^8)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4
*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A
^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c - 12*a^2
*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4
*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6
*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))) + 2*(2*B*a - A*b)*x)/(
```

```
(b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

giac [B] time = 4.78, size = 3776, normalized size = 13.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*b*x^3 - 2*A*c*x^3 + 2*B*a*x - A*b*x)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a
*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^
3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 - 8*a*b*c^3 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(b^2 - 4*a*c)^2*
B - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3
 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4
*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*
a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^
4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*B*abs(b^2 - 4*a*c)
 - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*a*c)*a*b^2*c^4)*A
+ (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*
b*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2
 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c -
12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3
- 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*
abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 - 8*a*b*c^3 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 -
```

```
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(b^2 - 4*a*c)
^2*B + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*
c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2
- 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b^4*c^2 + 16*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 -
16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32*a^3
*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*B*abs(b^2 - 4*a
*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*a*c)*a*b^2*c^4)
*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a
^2*b*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c
)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c
 - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c
^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*
c)*abs(c))
```

maple [B] time = 0.03, size = 733, normalized size = 2.66

$$\frac{\sqrt{2}\,A b c \operatorname{arctanh}\left(\frac{\sqrt{2}\,c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,A b c \arctan\left(\frac{\sqrt{2}\,c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,B a c \operatorname{arctanh}\left(\frac{\sqrt{2}\,c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,B a c \arctan\left(\frac{\sqrt{2}\,c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,B b^2 \operatorname{arctanh}\left(\frac{\sqrt{2}\,c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,B b^2 \arctan\left(\frac{\sqrt{2}\,c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,A c \operatorname{arctanh}\left(\frac{\sqrt{2}\,c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(4ac-b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,A c \arctan\left(\frac{\sqrt{2}\,c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(4ac-b^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,B b \operatorname{arctanh}\left(\frac{\sqrt{2}\,c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(4ac-b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,B b \arctan\left(\frac{\sqrt{2}\,c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(4ac-b^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\frac{(2Ac-bB)x^3}{8ac-2b^2}+\frac{(Ab-2Ba)x}{8ac-2b^2}}{c x^4+b x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (1/2*(2*A*c-B*b)/(4*a*c-b^2)*x^3+1/2*(A*b-2*B*a)/(4*a*c-b^2)*x)/(c*x^4+b*x^
2+a)-1/2/(4*a*c-b^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A+1/(4*a*c-b^2)*c/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b+1/4/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B-
1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*B-1/4/(4*a*c-b^2
)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*B+1/2/(4*a*c-b^2)*c*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*A+1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b-1/4/
(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4
```

```
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B-1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*c*x)*a*B-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2
*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(Bb-2\,Ac)x^3+(2\,Ba-Ab)x}{2\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)}-\frac{-\int\frac{(Bb-2\,Ac)x^2-2\,Ba+Ab}{cx^4+bx^2+a}\,dx}{2\left(b^2-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*b - 2*A*c)*x^3 + (2*B*a - A*b)*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 -
4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-((B*b - 2*A*c)*x^2 - 2*B*a
+ A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

mupad [B] time = 4.41, size = 9444, normalized size = 34.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((x*(A*b - 2*B*a))/(2*(4*a*c - b^2)) + (x^3*(2*A*c - B*b))/(2*(4*a*c - b^2)
))/(a + b*x^2 + c*x^4) - atan(((((16*A*b^7*c^2 + 2048*B*a^4*c^5 - 192*A*a*b
^5*c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*B*a^2*
b^4*c^3 - 1536*B*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*
b^4*c)) - (x*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^
2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 9
6*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*
c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*
B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4
*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*(16*b^7*
c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c
^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c
 + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^
4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a
^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 -
12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 128
0*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2) - (
x*(B^2*b^4*c - 8*A^2*a*c^4 + 10*A^2*b^2*c^3 + 8*B^2*a^2*c^3 - 6*A*B*b^3*c^2
 + 2*B^2*a*b^2*c^2 - 8*A*B*a*b*c^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-
(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c -
b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c
^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a
^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*
(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840
*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*1i - (((16*A*b^7*c^2 +
2048*B*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*c^2 + 768*
A*a^2*b^3*c^4 + 384*B*a^2*b^4*c^3 - 1536*B*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^
3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)
^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3
 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*
B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 3
84*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 +
240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 +
a*b^12*c)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^
3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c -
```

```
 b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^
5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 10
24*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^
2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c
^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c
^6 + a*b^12*c)))^(1/2) + (x*(B^2*b^4*c - 8*A^2*a*c^4 + 10*A^2*b^2*c^3 + 8*B
^2*a^2*c^3 - 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c^2 - 8*A*B*a*b*c^3))/(2*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2
*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*
b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768
*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*
c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3
 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/
2)*1i)/((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*B^2*b^4*c - 3*B^3*a*b^3*c + 8*A*B^
2*a^2*c^3 - 5*A^2*B*b^3*c^2 - 4*B^3*a^2*b*c^2 + 18*A*B^2*a*b^2*c^2 - 28*A^2
*B*a*b*c^3)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((16*A*
b^7*c^2 + 2048*B*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*
c^2 + 768*A*a^2*b^3*c^4 + 384*B*a^2*b^4*c^3 - 1536*B*a^3*b^2*c^4)/(8*(b^6 -
 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(-(B^2*a*b^9 - B^2*a*(-(4*
a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a
^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3
 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b
^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b
^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*
b^2*c^6 + a*b^12*c)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 +
768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2*a*
(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*
A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^
3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*
a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*
a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144
*a^6*b^2*c^6 + a*b^12*c)))^(1/2) - (x*(B^2*b^4*c - 8*A^2*a*c^4 + 10*A^2*b^2
*c^3 + 8*B^2*a^2*c^3 - 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c^2 - 8*A*B*a*b*c^3))/(2
*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(
1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 51
2*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5
*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*
B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a
^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^1
2*c)))^(1/2) + (((16*A*b^7*c^2 + 2048*B*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*
a^3*b*c^5 - 32*B*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*B*a^2*b^4*c^3 - 1536*B
*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(
B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b
^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^
2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^
5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(
4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*
a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*
c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
)*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*
c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b
^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B
^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/
(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 +
3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2) + (x*(B^2*b^4*c - 8
*A^2*a*c^4 + 10*A^2*b^2*c^3 + 8*B^2*a^2*c^3 - 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c
^2 - 8*A*B*a*b*c^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2
*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) -
96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4
```

```
*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A
*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 -
24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6
144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^
9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3
+ 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B
*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 38
4*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 2
40*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a
*b^12*c)))^(1/2)*2i - atan(((((16*A*b^7*c^2 + 2048*B*a^4*c^5 - 192*A*a*b^5*
c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*B*a^2*b^4
*c^3 - 1536*B*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4
*c)) - (x*(-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c
*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B
^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5
 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a
*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^
6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*(16*b^7*c^2
 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)))*(-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c -
A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 -
 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*
b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*
A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a
^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2) - (x*(
B^2*b^4*c - 8*A^2*a*c^4 + 10*A^2*b^2*c^3 + 8*B^2*a^2*c^3 - 6*A*B*b^3*c^2 +
2*B^2*a*b^2*c^2 - 8*A*B*a*b*c^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^
2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2
)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2
+ 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*
b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(40
96*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^
5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*1i - (((16*A*b^7*c^2 + 204
8*B*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*c^2 + 768*A*a
^2*b^3*c^4 + 384*B*a^2*b^4*c^3 - 1536*B*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)
^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 +
512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a
^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*
A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240
*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b
^12*c)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c
^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^
2)^9)^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c
^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*
A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 -
 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2
+ 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6
+ a*b^12*c)))^(1/2) + (x*(B^2*b^4*c - 8*A^2*a*c^4 + 10*A^2*b^2*c^3 + 8*B^2*
a^2*c^3 - 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c^2 - 8*A*B*a*b*c^3))/(2*(b^4 + 16*a^
2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^
9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3
*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^
2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3
 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 -
1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*
1i)/((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*B^2*b^4*c - 3*B^3*a*b^3*c + 8*A*B^2*a
^2*c^3 - 5*A^2*B*b^3*c^2 - 4*B^3*a^2*b*c^2 + 18*A*B^2*a*b^2*c^2 - 28*A^2*B*
a*b*c^3)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((16*A*b^7
```

```
*c^2 + 2048*B*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*c^2
 + 768*A*a^2*b^3*c^4 + 384*B*a^2*b^4*c^3 - 1536*B*a^3*b^2*c^4)/(8*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(-(B^2*a*b^9 + B^2*a*(-(4*a*c
 - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*
b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 +
1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*
c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10
*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2
*c^6 + a*b^12*c)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768
*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 + B^2*a*(-(
4*a*c - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2
*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c
^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2
*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2
*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^
6*b^2*c^6 + a*b^12*c)))^(1/2) - (x*(B^2*b^4*c - 8*A^2*a*c^4 + 10*A^2*b^2*c^
3 + 8*B^2*a^2*c^3 - 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c^2 - 8*A*B*a*b*c^3))/(2*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2
) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A
^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^
5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a
^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*
b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c
)))^(1/2) + (((16*A*b^7*c^2 + 2048*B*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3
*b*c^5 - 32*B*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*B*a^2*b^4*c^3 - 1536*B*a^
3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(B^2
*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)
^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 +
 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b
*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(409
6*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5
*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3
 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(
-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c -
 b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*
c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*
a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32
*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 384
0*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2) + (x*(B^2*b^4*c - 8*A^
2*a*c^4 + 10*A^2*b^2*c^3 + 8*B^2*a^2*c^3 - 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c^2
 - 8*A*B*a*b*c^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 + B^2*a*
(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*
A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^
3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*
a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*
a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144
*a^6*b^2*c^6 + a*b^12*c)))^(1/2)))*(-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^
(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 5
12*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^
5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A
*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*
a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^
12*c)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.99 $\int \frac{A+Bx^2}{\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=293

$$\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{A\left(b^2-12ac\right)+4abB}{\sqrt{b^2-4ac}}-2aB+Ab\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{-12aAc+4a}{\sqrt{b^2-4}}\right.}{2\sqrt{2}}$$

Rubi [A] time = 0.85, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1178, 1166, 205}

$$\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{A\left(b^2-12ac\right)+4abB}{\sqrt{b^2-4ac}}-2aB+Ab\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{-12aAc+4abB+Ab^2}{\sqrt{b^2-4ac}}-2aB+Ab\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2))/(2*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + (Sqrt[c]*(A*b - 2*a*B + (4*a*b*B + A*(b^2 - 12*a*c))/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2
*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(A*b - 2*a
*B - (A*b^2 + 4*a*b*B - 12*a*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt
[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{A+Bx^2}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(Ab^2-abB-2aAc+(Ab-2aB)cx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-Ab^2-abB+6aAc-(Ab-2aB)cx^2}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{x\left(Ab^2-abB-2aAc+(Ab-2aB)cx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(Ab-2aB-\frac{Ab^2+4abB-12aAc}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\cdots}{\frac{b}{2}+\frac{1}{2}\sqrt{b\cdots}}}{4a\left(b^2-4ac\right)}$$

$$= \frac{x\left(Ab^2-abB-2aAc+(Ab-2aB)cx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(2aB\left(2b-\sqrt{b^2-4ac}\right)+A\left(b^2-12a\cdots\right.\right.}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.79, size = 304, normalized size = 1.04

$$\frac{\frac{2x\left(A\left(-2ac+b^2+bcx^2\right)-aB\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(A\left(b\sqrt{b^2-4ac}-12ac+b^2\right)-2aB\left(\sqrt{b^2-4ac}-2b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(A\left(b\sqrt{b^2-4ac}+12ac-b^2\right)-2aB\left(\sqrt{b^2-4ac}+2b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(-(a*B*(b + 2*c*x^2)) + A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(-2*a*B*(-2*b + Sqrt[b^2 - 4*a*c]) +
A*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (
Sqrt[2]*Sqrt[c]*(-2*a*B*(2*b + Sqrt[b^2 - 4*a*c]) + A*(-b^2 + 12*a*c + b*Sq
rt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{\left(a+bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(a + b*x^2 + c*x^4)^2, x]
```

fricas [B] time = 3.72, size = 4885, normalized size = 16.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*B*a - A*b)*c*x^3 - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^
2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A
^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2
- 5*A^2*a*b^3)*c + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*s
qrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4
+ 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6
 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c +
48*a^5*b^2*c^2 - 64*a^6*c^3))*log((324*A^4*a^2*c^4 - 81*(4*A^3*B*a^2*b + A^
4*a*b^2)*c^3 - (4*B^4*a^4 - 20*A*B^3*a^3*b - 84*A^2*B^2*a^2*b^2 - 65*A^3*B*
```

```
a*b^3 - 5*A^4*b^4)*c^2 - 3*(B^4*a^3*b^2 + 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4
 + A^3*B*b^5)*c)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^5 + 3*A*B^2*a^2*b^6 + 3*A^2*B
*a*b^7 + A^3*b^8 + 864*A^3*a^4*c^4 - 48*(2*A*B^2*a^5 + 7*A^2*B*a^4*b + 14*A
^3*a^3*b^2)*c^3 + 2*(8*B^3*a^5*b + 48*A*B^2*a^4*b^2 + 108*A^2*B*a^3*b^3 + 9
5*A^3*a^2*b^4)*c^2 - (8*B^3*a^4*b^3 + 30*A*B^2*a^3*b^4 + 45*A^2*B*a^2*b^5 +
 23*A^3*a*b^6)*c - (B*a^4*b^8 + A*a^3*b^9 + 144*A*a^5*b^5*c^2 - 256*(B*a^8
- 2*A*a^7*b)*c^4 + 64*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^3 - 4*(2*B*a^5*b^6 + 5*
A*a^4*b^7)*c)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a
*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b
^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(B^2*
a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B
^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c + (a^3*b^6 - 12*a^4*b^4*c + 48*a^
5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 +
 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b
 + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(
a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(B
^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(
4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c + (a^3*b^6 - 12*a^4*b^4*c + 48
*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^
2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^
2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3))
)/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((324*A^4*a^2*
c^4 - 81*(4*A^3*B*a^2*b + A^4*a*b^2)*c^3 - (4*B^4*a^4 - 20*A*B^3*a^3*b - 84
*A^2*B^2*a^2*b^2 - 65*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 - 3*(B^4*a^3*b^2 + 3*A*B
^3*a^2*b^3 + 3*A^2*B^2*a*b^4 + A^3*B*b^5)*c)*x - 1/2*sqrt(1/2)*(B^3*a^3*b^5
 + 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 + A^3*b^8 + 864*A^3*a^4*c^4 - 48*(2*A*B^
2*a^5 + 7*A^2*B*a^4*b + 14*A^3*a^3*b^2)*c^3 + 2*(8*B^3*a^5*b + 48*A*B^2*a^4
*b^2 + 108*A^2*B*a^3*b^3 + 95*A^3*a^2*b^4)*c^2 - (8*B^3*a^4*b^3 + 30*A*B^2*
a^3*b^4 + 45*A^2*B*a^2*b^5 + 23*A^3*a*b^6)*c - (B*a^4*b^8 + A*a^3*b^9 + 144
*A*a^5*b^5*c^2 - 256*(B*a^8 - 2*A*a^7*b)*c^4 + 64*(2*B*a^7*b^2 - 7*A*a^6*b^
3)*c^3 - 4*(2*B*a^5*b^6 + 5*A*a^4*b^7)*c)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6
*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a
^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
 - 64*a^9*c^3)))*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3
 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c + (a^
3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3
*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*
(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^
6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^
3 - 4*a^2*b*c)*x^2)*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*
a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c -
(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*
B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 -
18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 4
8*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64
*a^6*c^3))*log((324*A^4*a^2*c^4 - 81*(4*A^3*B*a^2*b + A^4*a*b^2)*c^3 - (4*B
^4*a^4 - 20*A*B^3*a^3*b - 84*A^2*B^2*a^2*b^2 - 65*A^3*B*a*b^3 - 5*A^4*b^4)*
c^2 - 3*(B^4*a^3*b^2 + 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 + A^3*B*b^5)*c)*x
+ 1/2*sqrt(1/2)*(B^3*a^3*b^5 + 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 + A^3*b^8 +
864*A^3*a^4*c^4 - 48*(2*A*B^2*a^5 + 7*A^2*B*a^4*b + 14*A^3*a^3*b^2)*c^3 + 2
*(8*B^3*a^5*b + 48*A*B^2*a^4*b^2 + 108*A^2*B*a^3*b^3 + 95*A^3*a^2*b^4)*c^2
- (8*B^3*a^4*b^3 + 30*A*B^2*a^3*b^4 + 45*A^2*B*a^2*b^5 + 23*A^3*a*b^6)*c +
(B*a^4*b^8 + A*a^3*b^9 + 144*A*a^5*b^5*c^2 - 256*(B*a^8 - 2*A*a^7*b)*c^4 +
64*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^3 - 4*(2*B*a^5*b^6 + 5*A*a^4*b^7)*c)*sqrt(
(B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81
*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 1
2*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b
```

```
^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^
2*b^2 - 5*A^2*a*b^3)*c - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*
c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^
4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a
^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^
4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4
 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(B^2*a^2*b^3 + 2*A*B*
a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B
*a^2*b^2 - 5*A^2*a*b^3)*c - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a
^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 +
 A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)
/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((324*A^4*a^2*c^4 - 81*(4*A^3*B*a
^2*b + A^4*a*b^2)*c^3 - (4*B^4*a^4 - 20*A*B^3*a^3*b - 84*A^2*B^2*a^2*b^2 -
65*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 - 3*(B^4*a^3*b^2 + 3*A*B^3*a^2*b^3 + 3*A^2*
B^2*a*b^4 + A^3*B*b^5)*c)*x - 1/2*sqrt(1/2)*(B^3*a^3*b^5 + 3*A*B^2*a^2*b^6
+ 3*A^2*B*a*b^7 + A^3*b^8 + 864*A^3*a^4*c^4 - 48*(2*A*B^2*a^5 + 7*A^2*B*a^4
*b + 14*A^3*a^3*b^2)*c^3 + 2*(8*B^3*a^5*b + 48*A*B^2*a^4*b^2 + 108*A^2*B*a^
3*b^3 + 95*A^3*a^2*b^4)*c^2 - (8*B^3*a^4*b^3 + 30*A*B^2*a^3*b^4 + 45*A^2*B*
a^2*b^5 + 23*A^3*a*b^6)*c + (B*a^4*b^8 + A*a^3*b^9 + 144*A*a^5*b^5*c^2 - 25
6*(B*a^8 - 2*A*a^7*b)*c^4 + 64*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^3 - 4*(2*B*a^5
*b^6 + 5*A*a^4*b^7)*c)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 +
4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b
+ A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sq
rt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2
 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c - (a^3*b^6 - 12*a^4*b^4*
c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*
a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^
3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9
*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + 2*(B*a*b
 - A*b^2 + 2*A*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*
b^3 - 4*a^2*b*c)*x^2)
```

giac [B] time = 5.96, size = 4426, normalized size = 15.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*B*a*c*x^3 - A*b*c*x^3 + B*a*b*x - A*b^2*x + 2*A*a*c*x)/((c*x^4 + b*
x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*((2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2*A - 2
*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b
^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*B + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c + 64*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
2 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 4
8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*
(b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)*A*abs(a*b^2 - 4*a^2*c
```

) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*a^2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 16*a^3*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*a^4*b*c^3 + 2*(b^2 - 4*a*c)*a^2*b^3*c - 8*(b^2 - 4*a*c)*a^3*b*c^2)*B*abs(a*b^2 - 4*a^2*c) + (2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4)*A + 4*(2*a^3*b^6*c^2 - 16*a^4*b^4*c^3 + 32*a^5*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 8*(b^2 - 4*a*c)*a^4*b^2*c^3)*B)*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2 - 4*a^2*c)*abs(c)) - 1/16*((2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2*A - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*B - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 192*a^4*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c + 20*(b^2 - 4*a*c)*a^2*b^2*c^2 - 48*(b^2 - 4*a*c)*a^3*c^3)*A*abs(a*b^2 - 4*a^2*c) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c + 2*a^2*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 16*a^3*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 32*a^4*b*c^3 - 2*(b^2 - 4*a*c)*a^2*b^3*c + 8*(b^2 - 4*a*c)*a^3*b*c^2)*B*abs(a*b^2 - 4*a^2*c) + (2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*

```
c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)
*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4)*A + 4*(2*a^3*b^6*c^2 - 16*a^4*b^
4*c^3 + 32*a^5*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^3*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^4*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^3*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
5*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4
*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^
4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2
*c^3 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 8*(b^2 - 4*a*c)*a^4*b^2*c^3)*B)*arctan
(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c - sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^
2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6
- 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*b^4*c^
2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2 - 4*a
^2*c)*abs(c))
```

maple [B] time = 0.11, size = 1761, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/4/(4*a*c-b^2)*(-4*a*c+b^2)^(1/2)/a*x/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/
c)*A-1/4/(4*a*c-b^2)/a*x/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*A*b+1/2/(4*
a*c-b^2)*x/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*B-12*c^3/(4*a*c-b^2)/(-4*
a*c+b^2)^(1/2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*a-8*c^2/(4*a*c-b^2)/(-4*a
*c+b^2)^(1/2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+3/4*c/(4*a*c-b^2)/(-4*
a*c+b^2)^(1/2)/a/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^4-c^2/(4*a*c-b^2)/(4*
a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b-3/4*c/(4*a*c-b^2)/a/(4*a*c+3*b^2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*c*x)*A*b^3+2*c^2/(4*a*c-b^2)*a/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B+3/
2*c/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^2+4*c^2/(4*a*c-b^2)/(-
4*a*c+b^2)^(1/2)*a/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B+3*c/(4*a*c-b^2)/(-4
*a*c+b^2)^(1/2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^3-1/4/(4*a*c-b^2)*(-4*
a*c+b^2)^(1/2)/a*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*A-1/4/(4*a*c-b^2)
/a*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*A*b+1/2/(4*a*c-b^2)*x/(x^2+1/2*
b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*B-12*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)/(4*a*
c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*a-8*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)/(4*
a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+3/4*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)
/a/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^4+c^2/(4*a*c-b^2)/(4*a*c+3*b^2)*
```

```
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*A*b+3/4*c/(4*a*c-b^2)/a/(4*a*c+3*b^2)*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*A*b^3-2*c^2/(4*a*c-b^2)*a/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B-3/2*
c/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^2+4*c^2/(4*a*c-b^2)/(
-4*a*c+b^2)^(1/2)*a/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B+3*c/(4*a*c-b^2)
/(-4*a*c+b^2)^(1/2)/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{(2\,Ba-Ab)cx^3+\left(Bab-Ab^2+2\,Aac\right)x}{2\left(\left(ab^2c-4\,a^2c^2\right)x^4+a^2b^2-4\,a^3c+\left(ab^3-4\,a^2bc\right)x^2\right)}+\frac{-\int\frac{(2\,Ba-Ab)cx^2-Bab-Ab^2+6\,Aac}{cx^4+bx^2+a}\,dx}{2\left(ab^2-4\,a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*B*a - A*b)*c*x^3 + (B*a*b - A*b^2 + 2*A*a*c)*x)/((a*b^2*c - 4*a^2*
c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate(-((
2*B*a - A*b)*c*x^2 - B*a*b - A*b^2 + 6*A*a*c)/(c*x^4 + b*x^2 + a), x)/(a*b^
2 - 4*a^2*c)
```

mupad [B] time = 4.84, size = 12349, normalized size = 42.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] atan(((((6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^6
*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B*a
^3*b^5*c^3 + 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 4
8*a^4*b^2*c^2)) - (x*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)
^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*
c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 51
2*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c
- b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*
c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2
)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c
 + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5
)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*
c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 + B^2*a^2*b^9 +
 A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*
B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^
4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*
b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a
^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4
 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4
096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7
*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*A^2*a^2*c^5 + A^2*b^4*c^3 - 8
*B^2*a^3*c^4 + 10*B^2*a^2*b^2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 - 40
*A*B*a^2*b*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 + B^
2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(
1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2
*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5
- 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5
- 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*
```

```
a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a
^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3
 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*A*a^5*c^6 + 16
*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4 -
5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3*c
^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(A^2*
b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c -
b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 +
 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B
*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a
^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 -
1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*
c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^
6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 -
16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c
^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^
(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c
^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512
*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c -
 b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c
^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)
^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c
+ 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)
))^(1/2) - (x*(72*A^2*a^2*c^5 + A^2*b^4*c^3 - 8*B^2*a^3*c^4 + 10*B^2*a^2*b^
2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 - 40*A*B*a^2*b*c^4))/(2*(a^2*b^4
 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c
 - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^
2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^
5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c
*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*
B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4
*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*
a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a
^8*b^2*c^5)))^(1/2)*1i)/((((6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b*
c^5 - 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^
2*b^7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2
*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10
 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B
^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c -
9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4
 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B
*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*
c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4
 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5
*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b
^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b
^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 +
3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*
a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^
5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1
536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c
)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6
*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*A^2*a^2*c^
5 + A^2*b^4*c^3 - 8*B^2*a^3*c^4 + 10*B^2*a^2*b^2*c^3 - 14*A^2*a*b^2*c^4 + 2
*A*B*a*b^3*c^3 - 40*A*B*a^2*b*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)
))*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(
-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3
*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3
```

```
+ 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) -
3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*
b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*
B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2
 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (((614
4*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^6*c^3 + 1920*
A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B*a^3*b^5*c^3 +
 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2)) + (x*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2
*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A
^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^
3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1
/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*
B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) -
 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b
^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1
024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^
2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(
4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 2
88*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a
^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^
2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 1
92*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b
*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6
 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6
144*a^8*b^2*c^5)))^(1/2) - (x*(72*A^2*a^2*c^5 + A^2*b^4*c^3 - 8*B^2*a^3*c^4
 + 10*B^2*a^2*b^2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 - 40*A*B*a^2*b*c
^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 + B^2*a^2*b^9 +
A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B
*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4
 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b
^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^
6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4
+ 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 40
96*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*
b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (5*A^3*b^3*c^4 + 8*B^3*a^3*c^4 + 6*B^
3*a^2*b^2*c^3 - 36*A^3*a*b*c^5 + 72*A^2*B*a^2*c^5 - 3*A^2*B*b^4*c^3 + 3*A*B
^2*a*b^3*c^3 - 60*A*B^2*a^2*b*c^4 + 18*A^2*B*a*b^2*c^4)/(4*(a^2*b^6 - 64*a^
5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b
^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^
10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96
*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c
 - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c
^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A
*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^
9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c
^4 - 6144*a^8*b^2*c^5)))^(1/2)*2i + atan(((((6144*A*a^5*c^6 + 16*A*a*b^8*c^
2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*
b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3*c^4)/(8*(a^2
*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*((A^2*b^2*(-(4*a*c
 - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2
) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^
4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 2
7*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^5 + 7
68*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5
*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*(a^3*
b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 +
3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c
^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b
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^2*c)))*((A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a
^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2
*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*
c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2
) + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*
a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 3
6*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8
*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x
*(72*A^2*a^2*c^5 + A^2*b^4*c^3 - 8*B^2*a^3*c^4 + 10*B^2*a^2*b^2*c^3 - 14*A^
2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 - 40*A*B*a^2*b*c^4))/(2*(a^2*b^4 + 16*a^4*c^2
 - 8*a^3*b^2*c)))*((A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^
11 + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2
+ 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^
2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^
2)^9)^(1/2) + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2
+ 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)
^(1/2) + 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 2
40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^
(1/2)*1i - (((6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^
2*b^6*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 19
2*B*a^3*b^5*c^3 + 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*
c + 48*a^4*b^2*c^2)) + (x*((A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9
- A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*
b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2
- 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*
a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*
b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c -
 b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^
10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2
*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*
b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((A^2*b^2*(-(4*a*c - b^
2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) - 2
*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3
*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2
*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^5 + 768*B^
2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*
c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*(a^3*b^12
+ 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*
a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*A^2*a^2*c^5 + A^2*b^4*c^3
- 8*B^2*a^3*c^4 + 10*B^2*a^2*b^2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 -
 40*A*B*a^2*b*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((A^2*b^2*(-(
4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)
^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A
^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^
5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^
5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*
B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*
(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c
^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i)/((((6144*A*a^5*c^6 +
16*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4
- 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3
*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*((A^2
*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c -
 b^2)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3
- 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*
B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*
a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 +
 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8
*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a
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^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 -
 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*
c^2 - 8*a^3*b^2*c)))*((A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2
*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c
^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512
*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c -
 b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c
^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)
^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c
+ 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)
))^(1/2) + (x*(72*A^2*a^2*c^5 + A^2*b^4*c^3 - 8*B^2*a^3*c^4 + 10*B^2*a^2*b^
2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 - 40*A*B*a^2*b*c^4))/(2*(a^2*b^4
 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2
*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2
*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5
*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*
(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B
*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*
a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a
^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^
8*b^2*c^5)))^(1/2) + (((6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b*c^5
- 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^
7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*((A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^
2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*a*b^10 - 28
8*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^
4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2
*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 19
2*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*
(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 -
 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 61
44*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3
- 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((A^2*b^2*(-(
4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)
^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A
^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^
5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^
5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*
B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*
(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c
^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*A^2*a^2*c^5 + A^
2*b^4*c^3 - 8*B^2*a^3*c^4 + 10*B^2*a^2*b^2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a
*b^3*c^3 - 40*A*B*a^2*b*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((A
^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c
 - b^2)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^
3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*
A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^
2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3
 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b
^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280
*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (5*A^3*b^3*c^
4 + 8*B^3*a^3*c^4 + 6*B^3*a^2*b^2*c^3 - 36*A^3*a*b*c^5 + 72*A^2*B*a^2*c^5 -
 3*A^2*B*b^4*c^3 + 3*A*B^2*a*b^3*c^3 - 60*A*B^2*a^2*b*c^4 + 18*A^2*B*a*b^2*
c^4)/(4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*((A^2*b^2
*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2
)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 38
40*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^
6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*
b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 153
```

```
6*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/
(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b
^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*2i + ((x*(2*A*a*c - A
*b^2 + B*a*b))/(2*a*(4*a*c - b^2)) - (c*x^3*(A*b - 2*B*a))/(2*a*(4*a*c - b^
2)))/(a + b*x^2 + c*x^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.100 $\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=389

$$-\frac{-10aAc-abB+3Ab^2}{2a^2x\left(b^2-4ac\right)}+\frac{\sqrt{c}\left(aB\left(b\sqrt{b^2-4ac}-12ac+b^2\right)-A\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right.\right.}{2\sqrt{2}\,a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Rubi [A] time = 1.22, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1277, 1281, 1166, 205}

$$-\frac{-10aAc-abB+3Ab^2}{2a^2x\left(b^2-4ac\right)}+\frac{\sqrt{c}\left(aB\left(b\sqrt{b^2-4ac}-12ac+b^2\right)-A\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\frac{aB\left(b^2-12ac\right)-A\left(3b^3-16abc\right)}{\sqrt{b^2-4ac}}-10aAc-abB+3Ab^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a^2\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{-A\left(b^2-2ac\right)+cx^2(-(Ab-2aB))+abB}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(3*A*b^2 - a*b*B - 10*a*A*c)/(2*a^2*(b^2 - 4*a*c)*x) - (a*b*B - A*(b^2 - 2
*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) + (S
qrt[c]*(a*B*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]) - A*(3*b^3 - 16*a*b*c + 3*
b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*A*b^2 - a*b*B - 10*a*A*c + (a*B*(b^2 -
12*a*c) - A*(3*b^3 - 16*a*b*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)*Sqrt[b + Sqrt
[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
```

```
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-3Ab^2+abB+10aAc-3(Ab-2aB)cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx}{2a\left(b^2-4ac\right)}$$

$$= -\frac{3Ab^2-abB-10aAc}{2a^2\left(b^2-4ac\right)x} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{\int \frac{aB\left(b^2-6ac\right)-A\left(3b\right.}{\ldots}}{2\ldots}$$

$$= -\frac{3Ab^2-abB-10aAc}{2a^2\left(b^2-4ac\right)x} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(aB\left(b^2-12a\right.\right.\right.}{\ldots}$$

$$= -\frac{3Ab^2-abB-10aAc}{2a^2\left(b^2-4ac\right)x} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(aB\left(b^2-12\right.\right.}{\ldots}$$

Mathematica [A] time = 1.04, size = 382, normalized size = 0.98

$$\frac{\frac{2x\left(aB\left(-2ac+b^2+bcx^2\right)-A\left(-3abc-2ac^2x^2+b^3+b^2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(A\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)+aB\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(A\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}-16abc+3b^3\right)+aB\left(b\sqrt{b^2-4ac}+12ac-b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{4A}{x}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*A)/x + (2*x*(a*B*(b^2 - 2*a*c + b*c*x^2) - A*(b^3 - 3*a*b*c + b^2*c*x^
2 - 2*a*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(
a*B*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]) + A*(-3*b^3 + 16*a*b*c - 3*b^2*Sqr
t[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 + (Sqrt[2]*Sqrt[c]*(a*B*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c]) + A*(3*b^3 -
 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b
+ Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2), x]
```

fricas [B] time = 9.54, size = 7583, normalized size = 19.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(10*A*a*c^2 + (B*a*b - 3*A*b^2)*c)*x^4 - 4*A*a*b^2 + 16*A*a^2*c + 2*
(B*a*b^2 - 3*A*b^3 - (2*B*a^2 - 11*A*a*b)*c)*x^2 - sqrt(1/2)*((a^2*b^2*c -
4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(
B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 +
5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 -
 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^
2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 -
 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^
3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2
*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^
2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*
b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b
^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log((2500*A^4*a^3*c^6 + 6
25*(4*A^3*B*a^3*b - 9*A^4*a^2*b^2)*c^5 - 3*(108*B^4*a^5 - 756*A*B^3*a^4*b +
 1672*A^2*B^2*a^3*b^2 - 909*A^3*B*a^2*b^3 - 657*A^4*a*b^4)*c^4 + (81*B^4*a^
4*b^2 - 647*A*B^3*a^3*b^3 + 1674*A^2*B^2*a^2*b^4 - 1323*A^3*B*a*b^5 - 189*A
^4*b^6)*c^3 - 5*(B^4*a^3*b^4 - 9*A*B^3*a^2*b^5 + 27*A^2*B^2*a*b^6 - 27*A^3*
B*b^7)*c^2)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^8 - 9*A*B^2*a^2*b^9 + 27*A^2*B*a*b
^10 - 27*A^3*b^11 - 400*(6*A^2*B*a^6 - 13*A^3*a^5*b)*c^5 + 8*(108*B^3*a^7 -
 762*A*B^2*a^6*b + 1956*A^2*B*a^5*b^2 - 1801*A^3*a^4*b^3)*c^4 - (672*B^3*a^
6*b^2 - 4968*A*B^2*a^5*b^3 + 12414*A^2*B*a^4*b^4 - 10549*A^3*a^3*b^5)*c^3 +
 5*(38*B^3*a^5*b^4 - 297*A*B^2*a^4*b^5 + 771*A^2*B*a^3*b^6 - 666*A^3*a^2*b^
7)*c^2 - (23*B^3*a^4*b^6 - 192*A*B^2*a^3*b^7 + 531*A^2*B*a^2*b^8 - 486*A^3*
a*b^9)*c - (B*a^6*b^9 - 3*A*a^5*b^10 + 1280*A*a^10*c^5 + 128*(4*B*a^10*b -
17*A*a^9*b^2)*c^4 - 448*(B*a^9*b^3 - 3*A*a^8*b^4)*c^3 + 8*(18*B*a^8*b^5 - 4
9*A*a^7*b^6)*c^2 - 5*(4*B*a^7*b^7 - 11*A*a^6*b^8)*c)*sqrt((B^4*a^4*b^4 - 12
*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^
4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(2
7*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 10
17*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3
*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48
*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7
 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77
*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c + (
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 1
2*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A
^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(
27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1
017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^
3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 4
8*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 -
64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b
*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b
^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 +
77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c +
 (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 -
 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625
*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3
*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 +
 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*
a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c +
 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
- 64*a^8*c^3))*log((2500*A^4*a^3*c^6 + 625*(4*A^3*B*a^3*b - 9*A^4*a^2*b^2)*
c^5 - 3*(108*B^4*a^5 - 756*A*B^3*a^4*b + 1672*A^2*B^2*a^3*b^2 - 909*A^3*B*a
^2*b^3 - 657*A^4*a*b^4)*c^4 + (81*B^4*a^4*b^2 - 647*A*B^3*a^3*b^3 + 1674*A^
2*B^2*a^2*b^4 - 1323*A^3*B*a*b^5 - 189*A^4*b^6)*c^3 - 5*(B^4*a^3*b^4 - 9*A*
B^3*a^2*b^5 + 27*A^2*B^2*a*b^6 - 27*A^3*B*b^7)*c^2)*x - 1/2*sqrt(1/2)*(B^3*
a^3*b^8 - 9*A*B^2*a^2*b^9 + 27*A^2*B*a*b^10 - 27*A^3*b^11 - 400*(6*A^2*B*a^
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6 - 13*A^3*a^5*b)*c^5 + 8*(108*B^3*a^7 - 762*A*B^2*a^6*b + 1956*A^2*B*a^5*b
^2 - 1801*A^3*a^4*b^3)*c^4 - (672*B^3*a^6*b^2 - 4968*A*B^2*a^5*b^3 + 12414*
A^2*B*a^4*b^4 - 10549*A^3*a^3*b^5)*c^3 + 5*(38*B^3*a^5*b^4 - 297*A*B^2*a^4*
b^5 + 771*A^2*B*a^3*b^6 - 666*A^3*a^2*b^7)*c^2 - (23*B^3*a^4*b^6 - 192*A*B^
2*a^3*b^7 + 531*A^2*B*a^2*b^8 - 486*A^3*a*b^9)*c - (B*a^6*b^9 - 3*A*a^5*b^1
0 + 1280*A*a^10*c^5 + 128*(4*B*a^10*b - 17*A*a^9*b^2)*c^4 - 448*(B*a^9*b^3
- 3*A*a^8*b^4)*c^3 + 8*(18*B*a^8*b^5 - 49*A*a^7*b^6)*c^2 - 5*(4*B*a^7*b^7 -
 11*A*a^6*b^8)*c)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6
 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*
A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A
^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*
b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*
a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt
(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3
 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^
3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2
*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^
6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44
*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*
A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5
*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4
*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^
5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) - sqrt(1/2)*((a^2*b^2
*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sq
rt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c
^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*
b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b
^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*
b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 -
44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 96
8*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a
^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A
^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log((2500*A^4*a^3*c^
6 + 625*(4*A^3*B*a^3*b - 9*A^4*a^2*b^2)*c^5 - 3*(108*B^4*a^5 - 756*A*B^3*a^
4*b + 1672*A^2*B^2*a^3*b^2 - 909*A^3*B*a^2*b^3 - 657*A^4*a*b^4)*c^4 + (81*B
^4*a^4*b^2 - 647*A*B^3*a^3*b^3 + 1674*A^2*B^2*a^2*b^4 - 1323*A^3*B*a*b^5 -
189*A^4*b^6)*c^3 - 5*(B^4*a^3*b^4 - 9*A*B^3*a^2*b^5 + 27*A^2*B^2*a*b^6 - 27
*A^3*B*b^7)*c^2)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^8 - 9*A*B^2*a^2*b^9 + 27*A^2*
B*a*b^10 - 27*A^3*b^11 - 400*(6*A^2*B*a^6 - 13*A^3*a^5*b)*c^5 + 8*(108*B^3*
a^7 - 762*A*B^2*a^6*b + 1956*A^2*B*a^5*b^2 - 1801*A^3*a^4*b^3)*c^4 - (672*B
^3*a^6*b^2 - 4968*A*B^2*a^5*b^3 + 12414*A^2*B*a^4*b^4 - 10549*A^3*a^3*b^5)*
c^3 + 5*(38*B^3*a^5*b^4 - 297*A*B^2*a^4*b^5 + 771*A^2*B*a^3*b^6 - 666*A^3*a
^2*b^7)*c^2 - (23*B^3*a^4*b^6 - 192*A*B^2*a^3*b^7 + 531*A^2*B*a^2*b^8 - 486
*A^3*a*b^9)*c + (B*a^6*b^9 - 3*A*a^5*b^10 + 1280*A*a^10*c^5 + 128*(4*B*a^10
*b - 17*A*a^9*b^2)*c^4 - 448*(B*a^9*b^3 - 3*A*a^8*b^4)*c^3 + 8*(18*B*a^8*b^
5 - 49*A*a^7*b^6)*c^2 - 5*(4*B*a^7*b^7 - 11*A*a^6*b^8)*c)*sqrt((B^4*a^4*b^4
 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 6
25*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 +
 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3
 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^
2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c
 + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^
2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2
 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*
c - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^
4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 +
625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3
+ 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^
```

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3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B
^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*
c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c
^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*
a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*
A^2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b
^2 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5
)*c - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*
b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8
+ 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^
3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*
b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2
*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^
4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2
*c^2 - 64*a^8*c^3))*log((2500*A^4*a^3*c^6 + 625*(4*A^3*B*a^3*b - 9*A^4*a^2*
b^2)*c^5 - 3*(108*B^4*a^5 - 756*A*B^3*a^4*b + 1672*A^2*B^2*a^3*b^2 - 909*A^
3*B*a^2*b^3 - 657*A^4*a*b^4)*c^4 + (81*B^4*a^4*b^2 - 647*A*B^3*a^3*b^3 + 16
74*A^2*B^2*a^2*b^4 - 1323*A^3*B*a*b^5 - 189*A^4*b^6)*c^3 - 5*(B^4*a^3*b^4 -
 9*A*B^3*a^2*b^5 + 27*A^2*B^2*a*b^6 - 27*A^3*B*b^7)*c^2)*x - 1/2*sqrt(1/2)*
(B^3*a^3*b^8 - 9*A*B^2*a^2*b^9 + 27*A^2*B*a*b^10 - 27*A^3*b^11 - 400*(6*A^2
*B*a^6 - 13*A^3*a^5*b)*c^5 + 8*(108*B^3*a^7 - 762*A*B^2*a^6*b + 1956*A^2*B*
a^5*b^2 - 1801*A^3*a^4*b^3)*c^4 - (672*B^3*a^6*b^2 - 4968*A*B^2*a^5*b^3 + 1
2414*A^2*B*a^4*b^4 - 10549*A^3*a^3*b^5)*c^3 + 5*(38*B^3*a^5*b^4 - 297*A*B^2
*a^4*b^5 + 771*A^2*B*a^3*b^6 - 666*A^3*a^2*b^7)*c^2 - (23*B^3*a^4*b^6 - 192
*A*B^2*a^3*b^7 + 531*A^2*B*a^2*b^8 - 486*A^3*a*b^9)*c + (B*a^6*b^9 - 3*A*a^
5*b^10 + 1280*A*a^10*c^5 + 128*(4*B*a^10*b - 17*A*a^9*b^2)*c^4 - 448*(B*a^9
*b^3 - 3*A*a^8*b^4)*c^3 + 8*(18*B*a^8*b^5 - 49*A*a^7*b^6)*c^2 - 5*(4*B*a^7*
b^7 - 11*A*a^6*b^8)*c)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^
2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5
- 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b +
968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4
*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459
*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))
*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b
)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a
^3*b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c - (a^5*b^6 - 12*a^6*b^4*c + 48*a^
7*b^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a
^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5
 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b +
 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^
4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 45
9*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3))
)/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))))/((a^2*b^2*c - 4
*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)
```

giac [B] time = 6.16, size = 5408, normalized size = 13.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*a*b*c*x^4 - 3*A*b^2*c*x^4 + 10*A*a*c^2*x^4 + B*a*b^2*x^2 - 3*A*b^3*x
^2 - 2*B*a^2*c*x^2 + 11*A*a*b*c*x^2 - 2*A*a*b^2 + 8*A*a^2*c)/((c*x^5 + b*x^
3 + a*x)*(a^2*b^2 - 4*a^3*c)) - 1/16*((6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^
4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a
```

```
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 -
 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c)^2*A - (2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a^2*b
^2 - 4*a^3*c)^2*B + 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 -
37*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 6*a^2*b^7*c + 152*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 50*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b^4*c^2 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 74*a^3
*b^5*c^2 - 208*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 104*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 25*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 304*a^4*b^3*c^3 + 52*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^4*b*c^4 + 416*a^5*b*c^4 + 6*(b^2 - 4*a*c)*a^2*b^5*c - 5
0*(b^2 - 4*a*c)*a^3*b^3*c^2 + 104*(b^2 - 4*a*c)*a^4*b*c^3)*A*abs(a^2*b^2 -
4*a^3*c) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6 - 14*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*b^5*c - 2*a^3*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^5*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 +
 sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 28*a^4*b^4*c^2 - 96*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*c^3 - 48*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^5*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
4*b^2*c^3 - 128*a^5*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
5*c^4 + 192*a^6*c^4 + 2*(b^2 - 4*a*c)*a^3*b^4*c - 20*(b^2 - 4*a*c)*a^4*b^2*
c^2 + 48*(b^2 - 4*a*c)*a^5*c^3)*B*abs(a^2*b^2 - 4*a^3*c) + (6*a^4*b^8*c^2 -
 80*a^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^4*b^6*c^2 +
 56*(b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4)*A - (2*a^5*b
^7*c^2 - 40*a^6*b^5*c^3 + 224*a^7*b^3*c^4 - 384*a^8*b*c^5 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^7 + 20*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^4 - 2*(b^2 - 4*a*c)*a^5*b^5*c^2 + 32
*(b^2 - 4*a*c)*a^6*b^3*c^3 - 96*(b^2 - 4*a*c)*a^7*b*c^4)*B)*arctan(2*sqrt(1
/2)*x/sqrt((a^2*b^3 - 4*a^3*b*c + sqrt((a^2*b^3 - 4*a^3*b*c)^2 - 4*(a^3*b^2
 - 4*a^4*c)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*a^3*c^2)))/((a^5*b^6 -
 12*a^6*b^4*c - 2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6*b^3*c^2 + a^5*b^4*c^2
 - 64*a^8*c^3 - 32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7*c^4)*abs(a^2*b^2 - 4*
a^3*c)*abs(c)) + 1/16*((6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
```

```
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*
(a^2*b^2 - 4*a^3*c)^2*A - (2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a^2*b^2 - 4*a^3*c)^2
*B - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7 - 37*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b^6*c + 6*a^2*b^7*c + 152*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^4*b^3*c^2 + 50*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 3*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 74*a^3*b^5*c^2 - 208*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 104*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 25*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^3*b^3*c^3 + 304*a^4*b^3*c^3 + 52*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^4*b*c^4 - 416*a^5*b*c^4 - 6*(b^2 - 4*a*c)*a^2*b^5*c + 50*(b^2 - 4*a*c)
*a^3*b^3*c^2 - 104*(b^2 - 4*a*c)*a^4*b*c^3)*A*abs(a^2*b^2 - 4*a^3*c) + 2*(s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^4*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*
b^5*c + 2*a^3*b^6*c + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^
2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - 28*a^4*b^4*c^2 - 96*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^6*c^3 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^5*b*c^3 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 128
*a^5*b^2*c^3 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^4 - 192*a^6
*c^4 - 2*(b^2 - 4*a*c)*a^3*b^4*c + 20*(b^2 - 4*a*c)*a^4*b^2*c^2 - 48*(b^2 -
 4*a*c)*a^5*c^3)*B*abs(a^2*b^2 - 4*a^3*c) + (6*a^4*b^8*c^2 - 80*a^5*b^6*c^3
 + 352*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^4*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^4*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^4*b^6*c^2 + 56*(b^2 - 4*a*
c)*a^5*b^4*c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4)*A - (2*a^5*b^7*c^2 - 40*a^6
*b^5*c^3 + 224*a^7*b^3*c^4 - 384*a^8*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^7*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^5*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^8*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^6*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^7*b*c^4 - 2*(b^2 - 4*a*c)*a^5*b^5*c^2 + 32*(b^2 - 4*a*c)*
a^6*b^3*c^3 - 96*(b^2 - 4*a*c)*a^7*b*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^2
*b^3 - 4*a^3*b*c - sqrt((a^2*b^3 - 4*a^3*b*c)^2 - 4*(a^3*b^2 - 4*a^4*c)*(a^
2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*a^3*c^2)))/((a^5*b^6 - 12*a^6*b^4*c -
 2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6*b^3*c^2 + a^5*b^4*c^2 - 64*a^8*c^3 -
 32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7*c^4)*abs(a^2*b^2 - 4*a^3*c)*abs(c))
```

maple [B] time = 0.04, size = 1252, normalized size = 3.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -A/a^2/x-1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3*A+1/2/a^2/(c*x^4+b*x^2+a)*
c/(4*a*c-b^2)*x^3*A*b^2-1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*b*B-3/2/a/(
c*x^4+b*x^2+a)/(4*a*c-b^2)*x*A*b*c+1/2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*A*
b^3+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*B*c-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x
*B*b^2+5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A-3/4/a^2*c/(4*a*c-b^2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*A*b^2+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*c*x)*A*b-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x)*A*b^3+1/4/a*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B-3*c^2/(4*a*c-b^2)
/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B+1/4/a*c/(4*a*c-b^2)/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^2-5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*A+3/4/a^2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+4/a*c^2/(4*a*c-b^2)/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*A*b^3-1/4/a*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B-3*c
^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B+1/4/a*c/(4*a*c-b^2)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((10*A*a*c^2 + (B*a*b - 3*A*b^2)*c)*x^4 - 2*A*a*b^2 + 8*A*a^2*c + (B*a*
b^2 - 3*A*b^3 - (2*B*a^2 - 11*A*a*b)*c)*x^2)/((a^2*b^2*c - 4*a^3*c^2)*x^5 +
 (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x) + 1/2*integrate((B*a*b^
2 - 3*A*b^3 + (10*A*a*c^2 + (B*a*b - 3*A*b^2)*c)*x^2 - (6*B*a^2 - 13*A*a*b)
*c)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)
```

mupad [B] time = 5.38, size = 17591, normalized size = 45.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] - (A/a - (x^2*(3*A*b^3 - B*a*b^2 + 2*B*a^2*c - 11*A*a*b*c))/(2*a^2*(4*a*c -
 b^2)) + (c*x^4*(10*A*a*c - 3*A*b^2 + B*a*b))/(2*a^2*(4*a*c - b^2)))/(a*x +
 b*x^3 + c*x^5) - atan((((-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c
- b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c
^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*
c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*
```

c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213
*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5
 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a
^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c
*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*
a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*
a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^
4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(x*(-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*
b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*
A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^
2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288
*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*
a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*
B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2
 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 5
1*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/
2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5
*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3
 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a
^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^
5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 393216*B*a^15*c^8 + 85196
8*A*a^14*b*c^8 + 192*A*a^8*b^13*c^2 - 4672*A*a^9*b^11*c^3 + 47360*A*a^10*b^
9*c^4 - 256000*A*a^11*b^7*c^5 + 778240*A*a^12*b^5*c^6 - 1261568*A*a^13*b^3*
c^7 - 64*B*a^9*b^12*c^2 + 1664*B*a^10*b^10*c^3 - 17920*B*a^11*b^8*c^4 + 102
400*B*a^12*b^6*c^5 - 327680*B*a^13*b^4*c^6 + 557056*B*a^14*b^2*c^7) + x*(20
4800*A^2*a^12*c^9 - 73728*B^2*a^13*c^8 + 144*A^2*a^6*b^12*c^3 - 3264*A^2*a^
7*b^10*c^4 + 30112*A^2*a^8*b^8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^
10*b^4*c^7 - 458752*A^2*a^11*b^2*c^8 + 16*B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^
8*c^4 + 4608*B^2*a^10*b^6*c^5 - 25600*B^2*a^11*b^4*c^6 + 69632*B^2*a^12*b^2
*c^7 - 96*A*B*a^7*b^11*c^3 + 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 +
 107520*A*B*a^10*b^5*c^6 - 253952*A*B*a^11*b^3*c^7 + 237568*A*B*a^12*b*c^8)
)*(-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B
*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*
c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2
*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*
c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A
^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c
 - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*
a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/
2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2
*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*
c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c
^5)))^(1/2)*1i + ((-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^
9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30
240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2
)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1
504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*
b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^
2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*
c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a
*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^1
0*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^
6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 -
 6144*a^10*b^2*c^5)))^(1/2)*(x*(-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(
4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3
*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(
-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^
4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6
 - 213*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7

```
*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064
*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a
*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 15
2*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 +
 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*
a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^1
3*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983
040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) + 393216*B*a^15*c^8 - 851968*A*a^1
4*b*c^8 - 192*A*a^8*b^13*c^2 + 4672*A*a^9*b^11*c^3 - 47360*A*a^10*b^9*c^4 +
 256000*A*a^11*b^7*c^5 - 778240*A*a^12*b^5*c^6 + 1261568*A*a^13*b^3*c^7 + 6
4*B*a^9*b^12*c^2 - 1664*B*a^10*b^10*c^3 + 17920*B*a^11*b^8*c^4 - 102400*B*a
^12*b^6*c^5 + 327680*B*a^13*b^4*c^6 - 557056*B*a^14*b^2*c^7) + x*(204800*A^
2*a^12*c^9 - 73728*B^2*a^13*c^8 + 144*A^2*a^6*b^12*c^3 - 3264*A^2*a^7*b^10*
c^4 + 30112*A^2*a^8*b^8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^10*b^4*
c^7 - 458752*A^2*a^11*b^2*c^8 + 16*B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^8*c^4 +
 4608*B^2*a^10*b^6*c^5 - 25600*B^2*a^11*b^4*c^6 + 69632*B^2*a^12*b^2*c^7 -
96*A*B*a^7*b^11*c^3 + 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520
*A*B*a^10*b^5*c^6 - 253952*A*B*a^11*b^3*c^7 + 237568*A*B*a^12*b*c^8))*(-(9*
A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12
 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 4
4800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^
2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3
840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^2*a^6*
b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)
^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4
*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*
A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-
(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240
*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(
1/2)*1i)/(((-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2
) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2
*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1
/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2
*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c
 + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c
*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 2
2400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^
2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 4
4*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*
a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a
^10*b^2*c^5)))^(1/2)*(x*(-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c -
 b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^
3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c
 - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c
^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*
A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5
 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^
4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*
(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a
^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a
^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4
*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 -
 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^1
4*b^5*c^6 - 1572864*a^15*b^3*c^7) - 393216*B*a^15*c^8 + 851968*A*a^14*b*c^8
 + 192*A*a^8*b^13*c^2 - 4672*A*a^9*b^11*c^3 + 47360*A*a^10*b^9*c^4 - 256000
*A*a^11*b^7*c^5 + 778240*A*a^12*b^5*c^6 - 1261568*A*a^13*b^3*c^7 - 64*B*a^9
*b^12*c^2 + 1664*B*a^10*b^10*c^3 - 17920*B*a^11*b^8*c^4 + 102400*B*a^12*b^6
*c^5 - 327680*B*a^13*b^4*c^6 + 557056*B*a^14*b^2*c^7) + x*(204800*A^2*a^12*
c^9 - 73728*B^2*a^13*c^8 + 144*A^2*a^6*b^12*c^3 - 3264*A^2*a^7*b^10*c^4 + 3
```

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0112*A^2*a^8*b^8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^10*b^4*c^7 - 4
58752*A^2*a^11*b^2*c^8 + 16*B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^8*c^4 + 4608*B
^2*a^10*b^6*c^5 - 25600*B^2*a^11*b^4*c^6 + 69632*B^2*a^12*b^2*c^7 - 96*A*B*
a^7*b^11*c^3 + 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A*B*a^
10*b^5*c^6 - 253952*A*B*a^11*b^3*c^7 + 237568*A*B*a^12*b*c^8))*(-(9*A^2*b^1
3 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077
*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^
2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*
a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2
*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 -
 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/
2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 +
30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b
^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c
- b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^
8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) -
((-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*
a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c
^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*
a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c
^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^
2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c
- b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a
^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2
) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*
b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c
 + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^
5)))^(1/2)*(x*(-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(
1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*
A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)
^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*
B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11
*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^
3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3
- 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c -
 b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c
+ 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 -
24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 614
4*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11
*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6
- 1572864*a^15*b^3*c^7) + 393216*B*a^15*c^8 - 851968*A*a^14*b*c^8 - 192*A*a
^8*b^13*c^2 + 4672*A*a^9*b^11*c^3 - 47360*A*a^10*b^9*c^4 + 256000*A*a^11*b^
7*c^5 - 778240*A*a^12*b^5*c^6 + 1261568*A*a^13*b^3*c^7 + 64*B*a^9*b^12*c^2
- 1664*B*a^10*b^10*c^3 + 17920*B*a^11*b^8*c^4 - 102400*B*a^12*b^6*c^5 + 327
680*B*a^13*b^4*c^6 - 557056*B*a^14*b^2*c^7) + x*(204800*A^2*a^12*c^9 - 7372
8*B^2*a^13*c^8 + 144*A^2*a^6*b^12*c^3 - 3264*A^2*a^7*b^10*c^4 + 30112*A^2*a
^8*b^8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^10*b^4*c^7 - 458752*A^2*
a^11*b^2*c^8 + 16*B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^8*c^4 + 4608*B^2*a^10*b^
6*c^5 - 25600*B^2*a^11*b^4*c^6 + 69632*B^2*a^12*b^2*c^7 - 96*A*B*a^7*b^11*c
^3 + 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A*B*a^10*b^5*c^6
 - 253952*A*B*a^11*b^3*c^7 + 237568*A*B*a^12*b*c^8))*(-(9*A^2*b^13 + B^2*a^
2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b
^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*
c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)
^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c
^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^
3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*
A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*
a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*
```

c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) + 128000*A^3*a^10*c^9 + 504*A^3*a^6*b^8*c^5 - 8112*A^3*a^7*b^6*c^6 + 48704*A^3*a^8*b^4*c^7 - 129280*A^3*a^9*b^2*c^8 - 40*B^3*a^8*b^7*c^4 + 608*B^3*a^9*b^5*c^5 - 2944*B^3*a^10*b^3*c^6 + 46080*A*B^2*a^11*c^8 + 4608*B^3*a^11*b*c^7 - 84480*A^2*B*a^10*b*c^8 + 240*A*B^2*a^7*b^8*c^4 - 3792*A*B^2*a^8*b^6*c^5 + 21696*A*B^2*a^9*b^4*c^6 - 52992*A*B^2*a^10*b^2*c^7 - 360*A^2*B*a^6*b^9*c^4 + 5736*A^2*B*a^7*b^7*c^5 - 33888*A^2*B*a^8*b^5*c^6 + 87936*A^2*B*a^9*b^3*c^7))*(-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*2i - atan(((((9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(x*((9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 393216*B*a^15*c^8 + 851968*A*a^14*b*c^8 + 192*A*a^8*b^13*c^2 - 4672*A*a^9*b^11*c^3 + 47360*A*a^10*b^9*c^4 - 256000*A*a^11*b^7*c^5 + 778240*A*a^12*b^5*c^6 - 1261568*A*a^13*b^3*c^7 - 64*B*a^9*b^12*c^2 + 1664*B*a^10*b^10*c^3 - 17920*B*a^11*b^8*c^4 + 102400*B*a^12*b^6*c^5 - 327680*B*a^13*b^4*c^6 + 557056*B*a^14*b^2*c^7) + x*(204800*A^2*a^12*c^9 - 73728*B^2*a^13*c^8 + 144*A^2*a^6*b^12*c^3 - 3264*A^2*a^7*b^10*c^4 + 30112*A^2*a^8*b^8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^10*b^4*c^7 - 458752*A^2*a^11*b^2*c^8 + 16*B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^8*c^4 + 4608*B^2*a^10*b^6*c^5 - 25600*B^2*a^11*b^4*c^6 + 69632*B^2*a^12*b^2*c^7 - 96*A*B*a^7*b^11*c^3 + 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A*B*a^10*b^5*c^6 - 253952*A*B*a^11*b^3*c^7 + 237568*A*B*a^12*b*c^8))*((9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840

*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6*b*c
^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)
^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^
4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B
*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*
a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^
7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2
)*1i + (((9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 +
6*A*B*a*b^12 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4
*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2)
+ B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5
*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26
880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(
4*a*c - b^2)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400
*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9
)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*
B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*
b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*
b^2*c^5)))^(1/2)*(x*((9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9
*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 3
0240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^
2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 +
1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a
*b^11*c - 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B
^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6
*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*
a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^
10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c
^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4
- 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144
*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5
*c^6 - 1572864*a^15*b^3*c^7) + 393216*B*a^15*c^8 - 851968*A*a^14*b*c^8 - 19
2*A*a^8*b^13*c^2 + 4672*A*a^9*b^11*c^3 - 47360*A*a^10*b^9*c^4 + 256000*A*a^
11*b^7*c^5 - 778240*A*a^12*b^5*c^6 + 1261568*A*a^13*b^3*c^7 + 64*B*a^9*b^12
*c^2 - 1664*B*a^10*b^10*c^3 + 17920*B*a^11*b^8*c^4 - 102400*B*a^12*b^6*c^5
+ 327680*B*a^13*b^4*c^6 - 557056*B*a^14*b^2*c^7) + x*(204800*A^2*a^12*c^9 -
73728*B^2*a^13*c^8 + 144*A^2*a^6*b^12*c^3 - 3264*A^2*a^7*b^10*c^4 + 30112*
A^2*a^8*b^8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^10*b^4*c^7 - 458752
*A^2*a^11*b^2*c^8 + 16*B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^8*c^4 + 4608*B^2*a^
10*b^6*c^5 - 25600*B^2*a^11*b^4*c^6 + 69632*B^2*a^12*b^2*c^7 - 96*A*B*a^7*b
^11*c^3 + 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A*B*a^10*b^
5*c^6 - 253952*A*B*a^11*b^3*c^7 + 237568*A*B*a^12*b*c^8))*((9*A^2*b^4*(-(4*
a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2*a
^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*
b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c -
b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b
^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6*b*c^6 + 27*B^
2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) + 1
548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*
A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(
4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)
^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2
- 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i)/((((
9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^
12 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 +
44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*
b^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 -
3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^
6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^

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2)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b
^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) -
6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 2
40*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))
^(1/2)*(x*((9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13
+ 6*A*B*a*b^12 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a
^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2
) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a
^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c -
26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(
-(4*a*c - b^2)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 224
00*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)
^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*
A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^
6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^1
0*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11
*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 157
2864*a^15*b^3*c^7) - 393216*B*a^15*c^8 + 851968*A*a^14*b*c^8 + 192*A*a^8*b^
13*c^2 - 4672*A*a^9*b^11*c^3 + 47360*A*a^10*b^9*c^4 - 256000*A*a^11*b^7*c^5
 + 778240*A*a^12*b^5*c^6 - 1261568*A*a^13*b^3*c^7 - 64*B*a^9*b^12*c^2 + 166
4*B*a^10*b^10*c^3 - 17920*B*a^11*b^8*c^4 + 102400*B*a^12*b^6*c^5 - 327680*B
*a^13*b^4*c^6 + 557056*B*a^14*b^2*c^7) + x*(204800*A^2*a^12*c^9 - 73728*B^2
*a^13*c^8 + 144*A^2*a^6*b^12*c^3 - 3264*A^2*a^7*b^10*c^4 + 30112*A^2*a^8*b^
8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^10*b^4*c^7 - 458752*A^2*a^11*
b^2*c^8 + 16*B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^8*c^4 + 4608*B^2*a^10*b^6*c^5
 - 25600*B^2*a^11*b^4*c^6 + 69632*B^2*a^12*b^2*c^7 - 96*A*B*a^7*b^11*c^3 +
2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A*B*a^10*b^5*c^6 - 25
3952*A*B*a^11*b^3*c^7 + 237568*A*B*a^12*b*c^8))*((9*A^2*b^4*(-(4*a*c - b^2)
^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2*a^2*b^9*c^2
 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 +
25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1
/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 1
5360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*
c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) + 1548*A*B*a^
3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^
2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^
2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8
*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) - (((9*A^2*b^4*(-(
4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2
*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^
5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c
- b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6
*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6*b*c^6 + 27*
B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) +
 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 3072
0*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(
-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^
2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^
2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(x*((9
*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^1
2 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 +
44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b
^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 -
3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6
*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2
)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^
4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6
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*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(
-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 24
0*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^
(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*
a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3
*c^7) + 393216*B*a^15*c^8 - 851968*A*a^14*b*c^8 - 192*A*a^8*b^13*c^2 + 4672
*A*a^9*b^11*c^3 - 47360*A*a^10*b^9*c^4 + 256000*A*a^11*b^7*c^5 - 778240*A*a
^12*b^5*c^6 + 1261568*A*a^13*b^3*c^7 + 64*B*a^9*b^12*c^2 - 1664*B*a^10*b^10
*c^3 + 17920*B*a^11*b^8*c^4 - 102400*B*a^12*b^6*c^5 + 327680*B*a^13*b^4*c^6
 - 557056*B*a^14*b^2*c^7) + x*(204800*A^2*a^12*c^9 - 73728*B^2*a^13*c^8 + 1
44*A^2*a^6*b^12*c^3 - 3264*A^2*a^7*b^10*c^4 + 30112*A^2*a^8*b^8*c^5 - 14336
0*A^2*a^9*b^6*c^6 + 365568*A^2*a^10*b^4*c^7 - 458752*A^2*a^11*b^2*c^8 + 16*
B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^8*c^4 + 4608*B^2*a^10*b^6*c^5 - 25600*B^2*
a^11*b^4*c^6 + 69632*B^2*a^12*b^2*c^7 - 96*A*B*a^7*b^11*c^3 + 2336*A*B*a^8*
b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A*B*a^10*b^5*c^6 - 253952*A*B*a^11
*b^3*c^7 + 237568*A*B*a^12*b*c^8))*((9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B
^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*
a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^
2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2
*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*
c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*
a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8
064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^
2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) -
 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^1
2 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 38
40*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) + 128000*A^3*a^10*c^9 + 504*A^3
*a^6*b^8*c^5 - 8112*A^3*a^7*b^6*c^6 + 48704*A^3*a^8*b^4*c^7 - 129280*A^3*a^
9*b^2*c^8 - 40*B^3*a^8*b^7*c^4 + 608*B^3*a^9*b^5*c^5 - 2944*B^3*a^10*b^3*c^
6 + 46080*A*B^2*a^11*c^8 + 4608*B^3*a^11*b*c^7 - 84480*A^2*B*a^10*b*c^8 + 2
40*A*B^2*a^7*b^8*c^4 - 3792*A*B^2*a^8*b^6*c^5 + 21696*A*B^2*a^9*b^4*c^6 - 5
2992*A*B^2*a^10*b^2*c^7 - 360*A^2*B*a^6*b^9*c^4 + 5736*A^2*B*a^7*b^7*c^5 -
33888*A^2*B*a^8*b^5*c^6 + 87936*A^2*B*a^9*b^3*c^7))*((9*A^2*b^4*(-(4*a*c -
b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2*a^2*b^9
*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^
5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9
)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4
 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*
b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) + 1548*A*
B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*A*B*a^
6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c
- b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1
/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280
*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.101 $\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=522

$$-\frac{aB\left(3b^2-10ac\right)-A\left(5b^3-19abc\right)}{2a^3x\left(b^2-4ac\right)}-\frac{-14aAc-3abB+5Ab^2}{6a^2x^3\left(b^2-4ac\right)}-\frac{\sqrt{c}\left(aB\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16\right.\right.}{}$$

Rubi [A] time = 1.36, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1277, 1281, 1166, 205}

$$-\frac{\sqrt{c}\left(aB\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-A\left(28a^2c^2+5b^3\sqrt{b^2-4ac}-29ab^2c-19abc\sqrt{b^2-4ac}+5b^4\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(aB\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-A\left(28a^2c^2-5b^3\sqrt{b^2-4ac}-29ab^2c+19abc\sqrt{b^2-4ac}+5b^4\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{-14aAc-3abB+5Ab^2}{6a^2x^3\left(b^2-4ac\right)}-\frac{aB\left(3b^2-10ac\right)-A\left(5b^3-19abc\right)}{2a^3x\left(b^2-4ac\right)}-\frac{-A\left(b^2-2ac\right)+cx^2(-(Ab-2aB))+abB}{2ax^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(5*A*b^2 - 3*a*b*B - 14*a*A*c)/(6*a^2*(b^2 - 4*a*c)*x^3) - (a*B*(3*b^2 - 1
0*a*c) - A*(5*b^3 - 19*a*b*c))/(2*a^3*(b^2 - 4*a*c)*x) - (a*b*B - A*(b^2 -
2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x^3*(a + b*x^2 + c*x^4)) -
 (Sqrt[c]*(a*B*(3*b^3 - 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^
2 - 4*a*c]) - A*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c]
- 19*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]
) + (Sqrt[c]*(a*B*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt
[b^2 - 4*a*c]) - A*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - 5*b^3*Sqrt[b^2 - 4*a*
c] + 19*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*
c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-5Ab^2+3abB+14aAc-5(Ab-2aB)cx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx}{2a\left(b^2-4ac\right)}$$

$$= -\frac{5Ab^2-3abB-14aAc}{6a^2\left(b^2-4ac\right)x^3} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)} + \frac{\int \frac{-3\left(5Ab^3-3ab^2B-19a\right.}{}}{}$$

$$= -\frac{5Ab^2-3abB-14aAc}{6a^2\left(b^2-4ac\right)x^3} - \frac{aB\left(3b^2-10ac\right)-A\left(5b^3-19abc\right)}{2a^3\left(b^2-4ac\right)x} - \frac{abB-A\left(b^2-2ac\right)}{2a\left(b^2-4ac\right)x^3\left(\right.}$$

$$= -\frac{5Ab^2-3abB-14aAc}{6a^2\left(b^2-4ac\right)x^3} - \frac{aB\left(3b^2-10ac\right)-A\left(5b^3-19abc\right)}{2a^3\left(b^2-4ac\right)x} - \frac{abB-A\left(b^2-2ac\right)}{2a\left(b^2-4ac\right)x^3\left(\right.}$$

$$= -\frac{5Ab^2-3abB-14aAc}{6a^2\left(b^2-4ac\right)x^3} - \frac{aB\left(3b^2-10ac\right)-A\left(5b^3-19abc\right)}{2a^3\left(b^2-4ac\right)x} - \frac{abB-A\left(b^2-2ac\right)}{2a\left(b^2-4ac\right)x^3\left(\right.}$$

Mathematica [A] time = 1.20, size = 487, normalized size = 0.93

$$\frac{\frac{6x\left(A\left(2a^2c^2-4ab^2c-3abc^2x^2+b^4+b^3cx^2\right)+aB\left(3abc+2ac^2x^2-b^3-b^2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\sqrt{c}\left(A\left(28a^2c^2-29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}+5b^4\right)+aB\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{2}\sqrt{c}\left(A\left(28a^2c^2-29ab^2c+19abc\sqrt{b^2-4ac}-5b^3\sqrt{b^2-4ac}+5b^4\right)+aB\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}+16abc-3b^3\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{24Ab-12aB}{x}-\frac{4aA}{x^3}}{12a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*a*A)/x^3 + (24*A*b - 12*a*B)/x + (6*x*(a*B*(-b^3 + 3*a*b*c - b^2*c*x^2
 + 2*a*c^2*x^2) + A*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x^2 - 3*a*b*c^2*x^
2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(a*B*(-3*b^3
+ 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]) + A*(5*b^4
 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 -
4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4
*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(a*B*(-3*b^3
+ 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]) + A*(5*b^4
 - 29*a*b^2*c + 28*a^2*c^2 - 5*b^3*Sqrt[b^2 - 4*a*c] + 19*a*b*c*Sqrt[b^2 -
4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4
*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2), x]
```

fricas [B]   time = 22.20, size = 10190, normalized size = 19.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*((10*B*a^2 - 19*A*a*b)*c^2 - (3*B*a*b^2 - 5*A*b^3)*c)*x^6 - 4*A*a^2
*b^2 + 16*A*a^3*c - 2*(9*B*a*b^3 - 15*A*b^4 - 14*A*a^2*c^2 - (33*B*a^2*b -
62*A*a*b^2)*c)*x^4 - 4*(3*B*a^2*b^2 - 5*A*a*b^3 - 4*(3*B*a^3 - 5*A*a^2*b)*c
)*x^2 - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^
5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b
^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^4*b^2
+ 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2*b
^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c + (a^7*b^6 -
 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 - 540*A*
B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 24
01*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^
5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*
a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^
3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 +
3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^
3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*
b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14
*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b
^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*log((9604*A^4*a^4*c^8 + 7203*(4*A^3*B
*a^4*b - 7*A^4*a^3*b^2)*c^7 - (2500*B^4*a^6 - 22500*A*B^3*a^5*b + 43524*A^2
*B^2*a^4*b^2 + 4343*A^3*B*a^3*b^3 - 43410*A^4*a^2*b^4)*c^6 + (5625*B^4*a^5*
b^2 - 31137*A*B^3*a^4*b^3 + 52821*A^2*B^2*a^3*b^4 - 20190*A^3*B*a^2*b^5 - 1
2325*A^4*a*b^6)*c^5 - 3*(657*B^4*a^4*b^4 - 3351*A*B^3*a^3*b^5 + 5560*A^2*B^
2*a^2*b^6 - 2775*A^3*B*a*b^7 - 375*A^4*b^8)*c^4 + 7*(27*B^4*a^3*b^6 - 135*A
*B^3*a^2*b^7 + 225*A^2*B^2*a*b^8 - 125*A^3*B*b^9)*c^3)*x + 1/2*sqrt(1/2)*(2
7*B^3*a^3*b^11 - 135*A*B^2*a^2*b^12 + 225*A^2*B*a*b^13 - 125*A^3*b^14 + 109
76*A^3*a^7*c^7 - 112*(50*A*B^2*a^8 - 463*A^2*B*a^7*b + 709*A^3*a^6*b^2)*c^6
 - 2*(2600*B^3*a^8*b - 31256*A*B^2*a^7*b^2 + 96044*A^2*B*a^6*b^3 - 86495*A^
3*a^5*b^4)*c^5 + (14408*B^3*a^7*b^3 - 101006*A*B^2*a^6*b^4 + 224705*A^2*B*a
^5*b^5 - 160932*A^3*a^4*b^6)*c^4 - 7*(1507*B^3*a^6*b^5 - 8820*A*B^2*a^5*b^6
 + 16991*A^2*B*a^4*b^7 - 10797*A^3*a^3*b^8)*c^3 + (3330*B^3*a^5*b^7 - 17889
*A*B^2*a^4*b^8 + 31929*A^2*B*a^3*b^9 - 18940*A^3*a^2*b^10)*c^2 - (486*B^3*a
^4*b^9 - 2493*A*B^2*a^3*b^10 + 4260*A^2*B*a^2*b^11 - 2425*A^3*a*b^12)*c - (
3*B*a^8*b^10 - 5*A*a^7*b^11 - 256*(5*B*a^13 - 13*A*a^12*b)*c^5 + 64*(34*B*a
^12*b^2 - 73*A*a^11*b^3)*c^4 - 112*(12*B*a^11*b^4 - 23*A*a^10*b^5)*c^3 + 28
*(14*B*a^10*b^6 - 25*A*a^9*b^7)*c^2 - (55*B*a^9*b^8 - 94*A*a^8*b^9)*c)*sqrt
((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a
*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^
6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^
2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7
*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 +
41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A
^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*
a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 +
4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c
^3)))*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A*B*a^5 - 9
*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 +
 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(15*B^2*a^3
*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*
b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*
```

```
B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25
*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300
*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4
*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b
^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7
872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4
*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*
b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c +
48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 -
 64*a^10*c^3))) + 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a
^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8
+ 25*A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*
B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198
*A^2*a^2*b^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c +
(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^
8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4
*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a
^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109
544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*
B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b
^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6
- 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*
A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10
)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6
- 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*log((9604*A^4*a^4*c^8 + 720
3*(4*A^3*B*a^4*b - 7*A^4*a^3*b^2)*c^7 - (2500*B^4*a^6 - 22500*A*B^3*a^5*b +
 43524*A^2*B^2*a^4*b^2 + 4343*A^3*B*a^3*b^3 - 43410*A^4*a^2*b^4)*c^6 + (562
5*B^4*a^5*b^2 - 31137*A*B^3*a^4*b^3 + 52821*A^2*B^2*a^3*b^4 - 20190*A^3*B*a
^2*b^5 - 12325*A^4*a*b^6)*c^5 - 3*(657*B^4*a^4*b^4 - 3351*A*B^3*a^3*b^5 + 5
560*A^2*B^2*a^2*b^6 - 2775*A^3*B*a*b^7 - 375*A^4*b^8)*c^4 + 7*(27*B^4*a^3*b
^6 - 135*A*B^3*a^2*b^7 + 225*A^2*B^2*a*b^8 - 125*A^3*B*b^9)*c^3)*x - 1/2*sq
rt(1/2)*(27*B^3*a^3*b^11 - 135*A*B^2*a^2*b^12 + 225*A^2*B*a*b^13 - 125*A^3*
b^14 + 10976*A^3*a^7*c^7 - 112*(50*A*B^2*a^8 - 463*A^2*B*a^7*b + 709*A^3*a^
6*b^2)*c^6 - 2*(2600*B^3*a^8*b - 31256*A*B^2*a^7*b^2 + 96044*A^2*B*a^6*b^3
- 86495*A^3*a^5*b^4)*c^5 + (14408*B^3*a^7*b^3 - 101006*A*B^2*a^6*b^4 + 2247
05*A^2*B*a^5*b^5 - 160932*A^3*a^4*b^6)*c^4 - 7*(1507*B^3*a^6*b^5 - 8820*A*B
^2*a^5*b^6 + 16991*A^2*B*a^4*b^7 - 10797*A^3*a^3*b^8)*c^3 + (3330*B^3*a^5*b
^7 - 17889*A*B^2*a^4*b^8 + 31929*A^2*B*a^3*b^9 - 18940*A^3*a^2*b^10)*c^2 -
(486*B^3*a^4*b^9 - 2493*A*B^2*a^3*b^10 + 4260*A^2*B*a^2*b^11 - 2425*A^3*a*b
^12)*c - (3*B*a^8*b^10 - 5*A*a^7*b^11 - 256*(5*B*a^13 - 13*A*a^12*b)*c^5 +
64*(34*B*a^12*b^2 - 73*A*a^11*b^3)*c^4 - 112*(12*B*a^11*b^4 - 23*A*a^10*b^5
)*c^3 + 28*(14*B*a^10*b^6 - 25*A*a^9*b^7)*c^2 - (55*B*a^9*b^8 - 94*A*a^8*b^
9)*c)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 15
00*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 18
6*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51
894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(12
75*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*
a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5
 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2
*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*
a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 -
 64*a^17*c^3)))*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A
*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 23*A^2*a^3*
b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(
15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c + (a^7*b^6 - 12*a^8*b^4*c
 + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 +
 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^
6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*
a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 766
```

86*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2
*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a
^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 +
 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^
2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^1
5*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9
*b^2*c^2 - 64*a^10*c^3))) - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3
*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(9*B^2*a^2*b^7 - 30*
A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5
*b - 20*A*B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3
*b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a
*b^7)*c - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*
B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11
 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b +
 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6
*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2
- 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815
*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^
2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b
^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*
A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))
/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*log((9604*A^4*a^4
c^8 + 7203(4*A^3*B*a^4*b - 7*A^4*a^3*b^2)*c^7 - (2500*B^4*a^6 - 22500*A*B
^3*a^5*b + 43524*A^2*B^2*a^4*b^2 + 4343*A^3*B*a^3*b^3 - 43410*A^4*a^2*b^4)*
c^6 + (5625*B^4*a^5*b^2 - 31137*A*B^3*a^4*b^3 + 52821*A^2*B^2*a^3*b^4 - 201
90*A^3*B*a^2*b^5 - 12325*A^4*a*b^6)*c^5 - 3*(657*B^4*a^4*b^4 - 3351*A*B^3*a
^3*b^5 + 5560*A^2*B^2*a^2*b^6 - 2775*A^3*B*a*b^7 - 375*A^4*b^8)*c^4 + 7*(27
*B^4*a^3*b^6 - 135*A*B^3*a^2*b^7 + 225*A^2*B^2*a*b^8 - 125*A^3*B*b^9)*c^3)*
x + 1/2*sqrt(1/2)*(27*B^3*a^3*b^11 - 135*A*B^2*a^2*b^12 + 225*A^2*B*a*b^13
- 125*A^3*b^14 + 10976*A^3*a^7*c^7 - 112*(50*A*B^2*a^8 - 463*A^2*B*a^7*b +
709*A^3*a^6*b^2)*c^6 - 2*(2600*B^3*a^8*b - 31256*A*B^2*a^7*b^2 + 96044*A^2*
B*a^6*b^3 - 86495*A^3*a^5*b^4)*c^5 + (14408*B^3*a^7*b^3 - 101006*A*B^2*a^6*
b^4 + 224705*A^2*B*a^5*b^5 - 160932*A^3*a^4*b^6)*c^4 - 7*(1507*B^3*a^6*b^5
- 8820*A*B^2*a^5*b^6 + 16991*A^2*B*a^4*b^7 - 10797*A^3*a^3*b^8)*c^3 + (3330
*B^3*a^5*b^7 - 17889*A*B^2*a^4*b^8 + 31929*A^2*B*a^3*b^9 - 18940*A^3*a^2*b^
10)*c^2 - (486*B^3*a^4*b^9 - 2493*A*B^2*a^3*b^10 + 4260*A^2*B*a^2*b^11 - 24
25*A^3*a*b^12)*c + (3*B*a^8*b^10 - 5*A*a^7*b^11 - 256*(5*B*a^13 - 13*A*a^12
*b)*c^5 + 64*(34*B*a^12*b^2 - 73*A*a^11*b^3)*c^4 - 112*(12*B*a^11*b^4 - 23*
A*a^10*b^5)*c^3 + 28*(14*B*a^10*b^6 - 25*A*a^9*b^7)*c^2 - (55*B*a^9*b^8 - 9
4*A*a^8*b^9)*c)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2
*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^
2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*
a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c
^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77
424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B
^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^
8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9
550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16
*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9
- 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 2
3*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2*b^5)
c^2 - 7(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c - (a^7*b^6 - 12
*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3
*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*
A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 +
 (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5
*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 +
 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(
1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B

```
*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7
 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^
6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*
c + 48*a^9*b^2*c^2 - 64*a^10*c^3))) + 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*
x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(9*B^2*a^2
*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*
(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 2
10*A*B*a^3*b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6
+ 45*A^2*a*b^7)*c - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)
*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^
3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3
*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A
^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^
4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b
^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22
508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459
*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b
^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a
^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*log((96
04*A^4*a^4*c^8 + 7203*(4*A^3*B*a^4*b - 7*A^4*a^3*b^2)*c^7 - (2500*B^4*a^6 -
 22500*A*B^3*a^5*b + 43524*A^2*B^2*a^4*b^2 + 4343*A^3*B*a^3*b^3 - 43410*A^4
*a^2*b^4)*c^6 + (5625*B^4*a^5*b^2 - 31137*A*B^3*a^4*b^3 + 52821*A^2*B^2*a^3
*b^4 - 20190*A^3*B*a^2*b^5 - 12325*A^4*a*b^6)*c^5 - 3*(657*B^4*a^4*b^4 - 33
51*A*B^3*a^3*b^5 + 5560*A^2*B^2*a^2*b^6 - 2775*A^3*B*a*b^7 - 375*A^4*b^8)*c
^4 + 7*(27*B^4*a^3*b^6 - 135*A*B^3*a^2*b^7 + 225*A^2*B^2*a*b^8 - 125*A^3*B*
b^9)*c^3)*x - 1/2*sqrt(1/2)*(27*B^3*a^3*b^11 - 135*A*B^2*a^2*b^12 + 225*A^2
*B*a*b^13 - 125*A^3*b^14 + 10976*A^3*a^7*c^7 - 112*(50*A*B^2*a^8 - 463*A^2*
B*a^7*b + 709*A^3*a^6*b^2)*c^6 - 2*(2600*B^3*a^8*b - 31256*A*B^2*a^7*b^2 +
96044*A^2*B*a^6*b^3 - 86495*A^3*a^5*b^4)*c^5 + (14408*B^3*a^7*b^3 - 101006*
A*B^2*a^6*b^4 + 224705*A^2*B*a^5*b^5 - 160932*A^3*a^4*b^6)*c^4 - 7*(1507*B^
3*a^6*b^5 - 8820*A*B^2*a^5*b^6 + 16991*A^2*B*a^4*b^7 - 10797*A^3*a^3*b^8)*c
^3 + (3330*B^3*a^5*b^7 - 17889*A*B^2*a^4*b^8 + 31929*A^2*B*a^3*b^9 - 18940*
A^3*a^2*b^10)*c^2 - (486*B^3*a^4*b^9 - 2493*A*B^2*a^3*b^10 + 4260*A^2*B*a^2
*b^11 - 2425*A^3*a*b^12)*c + (3*B*a^8*b^10 - 5*A*a^7*b^11 - 256*(5*B*a^13 -
 13*A*a^12*b)*c^5 + 64*(34*B*a^12*b^2 - 73*A*a^11*b^3)*c^4 - 112*(12*B*a^11
*b^4 - 23*A*a^10*b^5)*c^3 + 28*(14*B*a^10*b^6 - 25*A*a^9*b^7)*c^2 - (55*B*a
^9*b^8 - 94*A*a^8*b^9)*c)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A
^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*
(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9
300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*
a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^
5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4
- 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*
A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a
^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c
 + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 2
5*A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a
^4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^
2*a^2*b^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c - (a^
7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 -
 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^
12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*
b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544
*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3
*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)
*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 2
8260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B
^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c
)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 1
```

```
2*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))))/((a^3*b^2*c - 4*a^4*c^2)*x^7
 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)
```

giac [B] time = 8.15, size = 6327, normalized size = 12.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*a*b^2*c*x^3 - A*b^3*c*x^3 - 2*B*a^2*c^2*x^3 + 3*A*a*b*c^2*x^3 + B*a
*b^3*x - A*b^4*x - 3*B*a^2*b*c*x + 4*A*a*b^2*c*x - 2*A*a^2*c^2*x)/((a^3*b^2
 - 4*a^4*c)*(c*x^4 + b*x^2 + a)) + 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a
^2*b*c^4 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5
+ 39*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*
c^2 + 38*(b^2 - 4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2*A - (6*a*b^4*c^2 - 44
*a^2*b^2*c^3 + 80*a^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 -
 6*(b^2 - 4*a*c)*a*b^2*c^2 + 20*(b^2 - 4*a*c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c)^
2*B + 2*(5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^8 - 64*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*b^7*c - 10*a^3*b^8*c + 286*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^5*b^4*c^2 + 88*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 +
 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^2 + 128*a^4*b^6*c^2 -
496*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 - 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^4*b^4*c^3 - 572*a^5*b^4*c^3 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^7*c^4 + 112*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 +
 110*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^4 + 992*a^6*b^2*c^4
- 56*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*c^5 - 448*a^7*c^5 + 10*(b^
2 - 4*a*c)*a^3*b^6*c - 88*(b^2 - 4*a*c)*a^4*b^4*c^2 + 220*(b^2 - 4*a*c)*a^5
*b^2*c^3 - 112*(b^2 - 4*a*c)*a^6*c^4)*A*abs(a^3*b^2 - 4*a^4*c) - 2*(3*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7 - 37*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^5*b^5*c - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*
c - 6*a^4*b^7*c + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^2 +
 50*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 3*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 74*a^5*b^5*c^2 - 208*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 104*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^6*b^2*c^3 - 25*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3
- 304*a^6*b^3*c^3 + 52*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 +
416*a^7*b*c^4 + 6*(b^2 - 4*a*c)*a^4*b^5*c - 50*(b^2 - 4*a*c)*a^5*b^3*c^2 +
104*(b^2 - 4*a*c)*a^6*b*c^3)*B*abs(a^3*b^2 - 4*a^4*c) + (10*a^6*b^9*c^2 - 1
38*a^7*b^7*c^3 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 - 5*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^9 + 69*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 10*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 340*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 98*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 5*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 + 688*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 288*sqrt(2)*
```

sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 49*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 - 448*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 224*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 144*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 + 112*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b*c^5 - 10*(b^2 -
4*a*c)*a^6*b^7*c^2 + 98*(b^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b^2 - 4*a*c)*a^8*b
^3*c^4 + 224*(b^2 - 4*a*c)*a^9*b*c^5)*A - (6*a^7*b^8*c^2 - 80*a^8*b^6*c^3 +
352*a^9*b^4*c^4 - 512*a^10*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^7*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^8*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^7*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^9*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^8*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^7*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^10*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^8*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 6*(b^2 - 4*a*c)*a^7*b^6*c^2 + 56*(b^2 - 4*a*
c)*a^8*b^4*c^3 - 128*(b^2 - 4*a*c)*a^9*b^2*c^4)*B)*arctan(2*sqrt(1/2)*x/sqr
t((a^3*b^3 - 4*a^4*b*c + sqrt((a^3*b^3 - 4*a^4*b*c)^2 - 4*(a^4*b^2 - 4*a^5*
c)*(a^3*b^2*c - 4*a^4*c^2)))/(a^3*b^2*c - 4*a^4*c^2)))/((a^7*b^6 - 12*a^8*b
^4*c - 2*a^7*b^5*c + 48*a^9*b^2*c^2 + 16*a^8*b^3*c^2 + a^7*b^4*c^2 - 64*a^1
0*c^3 - 32*a^9*b*c^3 - 8*a^8*b^2*c^3 + 16*a^9*c^4)*abs(a^3*b^2 - 4*a^4*c)*a
bs(c)) - 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a^2*b*c^4 - 5*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 39*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*c^2 + 38*(b^2 - 4*a*c)*a*b
c^3)(a^3*b^2 - 4*a^4*c)^2*A - (6*a*b^4*c^2 - 44*a^2*b^2*c^3 + 80*a^3*c^4
- 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 + 22*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 40*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 20*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b^2*c^2
+ 20*(b^2 - 4*a*c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c)^2*B - 2*(5*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b^8 - 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^4*b^6*c - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c + 10*a^
3*b^8*c + 286*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 88*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 5*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b^6*c^2 - 128*a^4*b^6*c^2 - 496*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^5*b^3*c^3 - 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 + 572
*a^5*b^4*c^3 + 224*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*c^4 + 112*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 + 110*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^5*b^2*c^4 - 992*a^6*b^2*c^4 - 56*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^6*c^5 + 448*a^7*c^5 - 10*(b^2 - 4*a*c)*a^3*b^6*c + 88*
(b^2 - 4*a*c)*a^4*b^4*c^2 - 220*(b^2 - 4*a*c)*a^5*b^2*c^3 + 112*(b^2 - 4*a*
c)*a^6*c^4)*A*abs(a^3*b^2 - 4*a^4*c) + 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^4*b^7 - 37*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c - 6
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c + 6*a^4*b^7*c + 152*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^2 + 50*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^4*b^5*c^2 - 74*a^5*b^5*c^2 - 208*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a

```
^7*b*c^3 - 104*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 25*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 + 304*a^6*b^3*c^3 + 52*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 - 416*a^7*b*c^4 - 6*(b^2 - 4
*a*c)*a^4*b^5*c + 50*(b^2 - 4*a*c)*a^5*b^3*c^2 - 104*(b^2 - 4*a*c)*a^6*b*c^
3)*B*abs(a^3*b^2 - 4*a^4*c) + (10*a^6*b^9*c^2 - 138*a^7*b^7*c^3 + 680*a^8*b
^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^9 + 69*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 340*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 98*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 + 688*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 288*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 49*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 - 448*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 144*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 + 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b*c^5 - 10*(b^2 - 4*a*c)*a^6*b^7*c^2 + 98*(b
^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b^2 - 4*a*c)*a^8*b^3*c^4 + 224*(b^2 - 4*a*c)
*a^9*b*c^5)*A - (6*a^7*b^8*c^2 - 80*a^8*b^6*c^3 + 352*a^9*b^4*c^4 - 512*a^1
0*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7
*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^6
*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c
- 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^4*c^2
 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2
 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2
+ 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^2*c^
3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c
^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c
^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c
^4 - 6*(b^2 - 4*a*c)*a^7*b^6*c^2 + 56*(b^2 - 4*a*c)*a^8*b^4*c^3 - 128*(b^2
- 4*a*c)*a^9*b^2*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^3*b^3 - 4*a^4*b*c - s
qrt((a^3*b^3 - 4*a^4*b*c)^2 - 4*(a^4*b^2 - 4*a^5*c)*(a^3*b^2*c - 4*a^4*c^2)
))/(a^3*b^2*c - 4*a^4*c^2)))/((a^7*b^6 - 12*a^8*b^4*c - 2*a^7*b^5*c + 48*a^
9*b^2*c^2 + 16*a^8*b^3*c^2 + a^7*b^4*c^2 - 64*a^10*c^3 - 32*a^9*b*c^3 - 8*a
^8*b^2*c^3 + 16*a^9*c^4)*abs(a^3*b^2 - 4*a^4*c)*abs(c)) - 1/3*(3*B*a*x^2 -
6*A*b*x^2 + A*a)/(a^3*x^3)
```

maple [B] time = 0.05, size = 1653, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*B*b^2+2/a^2/(c*x^4+b*x^2+a)/(4*a*
c-b^2)*x*A*b^2*c-3/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b*B*c+3/2/a^2/(c*x^4+b
*x^2+a)*c^2/(4*a*c-b^2)*x^3*A*b-1/2/a^3/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*A
*b^3+5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
h(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B-5/2/a*c^2/(4*a*c-b^2)*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x)*B+2/a^3/x*A*b+7/a*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x)*A-3/4/a^2*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^2+19/4/a
^2*c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b+7/a*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*c*x)*A+3/4/a^2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(
```

```
1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^2+5
/4/a^3*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^3-5/4/a^3*c/(4*a*c-b^2)*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*c*x)*A*b^3-19/4/a^2*c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b-1/3
*A/a^2/x^3-1/a^2/x*B-29/4/a^2*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*A*b^2+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*
B-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^3+5/4/a^3
*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^4+5/4/a^3*c/(4*
a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^4-29/4/a^2*c^2/(4*a*c-b^
2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+4/a*c^2/(4*a*c-b^2)/(-4*a
*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^3-1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3*B
-1/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*A*c^2+1/2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2
)*x*B*b^3-1/2/a^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*A*b^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*((10*B*a^2 - 19*A*a*b)*c^2 - (3*B*a*b^2 - 5*A*b^3)*c)*x^6 - 2*A*a^2*
b^2 + 8*A*a^3*c - (9*B*a*b^3 - 15*A*b^4 - 14*A*a^2*c^2 - (33*B*a^2*b - 62*A
*a*b^2)*c)*x^4 - 2*(3*B*a^2*b^2 - 5*A*a*b^3 - 4*(3*B*a^3 - 5*A*a^2*b)*c)*x^
2)/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*
a^5*c)*x^3) - 1/2*integrate((3*B*a*b^3 - 5*A*b^4 - 14*A*a^2*c^2 - ((10*B*a^
2 - 19*A*a*b)*c^2 - (3*B*a*b^2 - 5*A*b^3)*c)*x^2 - (13*B*a^2*b - 24*A*a*b^2
)*c)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c)
```

mupad [B] time = 5.70, size = 21554, normalized size = 41.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] - atan((((-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1
/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 11692
8*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^
2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2)
 + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 4
4800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*
a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 268
80*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*
a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*
a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(
1/2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c -
b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(
1/2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^1
3*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4
```

```
*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*A*a^19*c^9 + x*(-(25*A^2*b^15 + 9
*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*
A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*
A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9
)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 1
0656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B
^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c
- 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*
a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^
4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^
7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(
4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2
*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-
(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240
*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))
^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440
*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^
3*c^7) + 851968*B*a^19*b*c^8 - 320*A*a^12*b^14*c^2 + 7936*A*a^13*b^12*c^3 -
 82816*A*a^14*b^10*c^4 + 468480*A*a^15*b^8*c^5 - 1536000*A*a^16*b^6*c^6 + 2
867200*A*a^17*b^4*c^7 - 2719744*A*a^18*b^2*c^8 + 192*B*a^13*b^13*c^2 - 4672
*B*a^14*b^11*c^3 + 47360*B*a^15*b^9*c^4 - 256000*B*a^16*b^7*c^5 + 778240*B*
a^17*b^5*c^6 - 1261568*B*a^18*b^3*c^7) - x*(401408*A^2*a^16*c^10 - 204800*B
^2*a^17*c^9 - 400*A^2*a^9*b^14*c^3 + 9440*A^2*a^10*b^12*c^4 - 92816*A^2*a^1
1*b^10*c^5 + 488096*A^2*a^12*b^8*c^6 - 1458688*A^2*a^13*b^6*c^7 + 2401280*A
^2*a^14*b^4*c^8 - 1871872*A^2*a^15*b^2*c^9 - 144*B^2*a^11*b^12*c^3 + 3264*B
^2*a^12*b^10*c^4 - 30112*B^2*a^13*b^8*c^5 + 143360*B^2*a^14*b^6*c^6 - 36556
8*B^2*a^15*b^4*c^7 + 458752*B^2*a^16*b^2*c^8 + 480*A*B*a^10*b^13*c^3 - 1110
4*A*B*a^11*b^11*c^4 + 105824*A*B*a^12*b^9*c^5 - 530432*A*B*a^13*b^7*c^6 + 1
469440*A*B*a^14*b^5*c^7 - 2121728*A*B*a^15*b^3*c^8 + 1236992*A*B*a^16*b*c^9
))*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) -
30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*
a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*
c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 207
7*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B
^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^
7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2
*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^
10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^
4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) +
 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9
)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) +
 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6
- 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 -
 6144*a^12*b^2*c^5)))^(1/2)*1i - ((-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*
b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 3576
7*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 21504
0*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4
*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 +
30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b
^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 -
 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c -
b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*
a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b
^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) +
 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b
^3*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^1
0*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*A*a^19*c
^9 - x*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2
```

) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*
A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*
a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) +
2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 448
00*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^
8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880
*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^
3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^
6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/
2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^
2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/
2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*
c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c
^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6
144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*
b^5*c^6 - 1572864*a^20*b^3*c^7) + 851968*B*a^19*b*c^8 - 320*A*a^12*b^14*c^2
+ 7936*A*a^13*b^12*c^3 - 82816*A*a^14*b^10*c^4 + 468480*A*a^15*b^8*c^5 - 1
536000*A*a^16*b^6*c^6 + 2867200*A*a^17*b^4*c^7 - 2719744*A*a^18*b^2*c^8 + 1
92*B*a^13*b^13*c^2 - 4672*B*a^14*b^11*c^3 + 47360*B*a^15*b^9*c^4 - 256000*B
*a^16*b^7*c^5 + 778240*B*a^17*b^5*c^6 - 1261568*B*a^18*b^3*c^7) + x*(401408
*A^2*a^16*c^10 - 204800*B^2*a^17*c^9 - 400*A^2*a^9*b^14*c^3 + 9440*A^2*a^10
*b^12*c^4 - 92816*A^2*a^11*b^10*c^5 + 488096*A^2*a^12*b^8*c^6 - 1458688*A^2
*a^13*b^6*c^7 + 2401280*A^2*a^14*b^4*c^8 - 1871872*A^2*a^15*b^2*c^9 - 144*B
^2*a^11*b^12*c^3 + 3264*B^2*a^12*b^10*c^4 - 30112*B^2*a^13*b^8*c^5 + 143360
*B^2*a^14*b^6*c^6 - 365568*B^2*a^15*b^4*c^7 + 458752*B^2*a^16*b^2*c^8 + 480
*A*B*a^10*b^13*c^3 - 11104*A*B*a^11*b^11*c^4 + 105824*A*B*a^12*b^9*c^5 - 53
0432*A*B*a^13*b^7*c^6 + 1469440*A*B*a^14*b^5*c^7 - 2121728*A*B*a^15*b^3*c^8
+ 1236992*A*B*a^16*b*c^9))*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-
(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*
a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*
a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*
a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*
B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)
^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B
^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9
)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^
6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(
-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*
B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(
-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(
a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*
c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i)/(((-(25*A^2*b^15 +
9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 636
6*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 21974
4*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)
^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 -
10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25
*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*
c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^
2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*
a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*
a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(
-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a
^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 2
40*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)
))^(1/2)*(917504*A*a^19*c^9 + x*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^
6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*
A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*

A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(
-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30
240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2
)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 2
13*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^
2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^
5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4
c(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 3
0*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3
c(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(
32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*
b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^
8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^
18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 851968*B*a^19*b*
c^8 - 320*A*a^12*b^14*c^2 + 7936*A*a^13*b^12*c^3 - 82816*A*a^14*b^10*c^4 +
468480*A*a^15*b^8*c^5 - 1536000*A*a^16*b^6*c^6 + 2867200*A*a^17*b^4*c^7 - 2
719744*A*a^18*b^2*c^8 + 192*B*a^13*b^13*c^2 - 4672*B*a^14*b^11*c^3 + 47360*
B*a^15*b^9*c^4 - 256000*B*a^16*b^7*c^5 + 778240*B*a^17*b^5*c^6 - 1261568*B*
a^18*b^3*c^7) - x*(401408*A^2*a^16*c^10 - 204800*B^2*a^17*c^9 - 400*A^2*a^9
*b^14*c^3 + 9440*A^2*a^10*b^12*c^4 - 92816*A^2*a^11*b^10*c^5 + 488096*A^2*a
^12*b^8*c^6 - 1458688*A^2*a^13*b^6*c^7 + 2401280*A^2*a^14*b^4*c^8 - 1871872
*A^2*a^15*b^2*c^9 - 144*B^2*a^11*b^12*c^3 + 3264*B^2*a^12*b^10*c^4 - 30112*
B^2*a^13*b^8*c^5 + 143360*B^2*a^14*b^6*c^6 - 365568*B^2*a^15*b^4*c^7 + 4587
52*B^2*a^16*b^2*c^8 + 480*A*B*a^10*b^13*c^3 - 11104*A*B*a^11*b^11*c^4 + 105
824*A*B*a^12*b^9*c^5 - 530432*A*B*a^13*b^7*c^6 + 1469440*A*B*a^14*b^5*c^7 -
2121728*A*B*a^15*b^3*c^8 + 1236992*A*B*a^16*b*c^9))*(-(25*A^2*b^15 + 9*B^2
*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*
a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*
a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1
/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656
*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a
^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80
640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*
b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^
8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^
2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(4*a*
c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^1
2*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-(4*a
c - b^2)^9)^(1/2))/(32(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9
*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/
2) + ((-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2)
- 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A
^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a
^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) +
2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 4480
0*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8
*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*
B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3
*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6
*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2
) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2
)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2
) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c
^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^
4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*A*a^19*c^9 - x*(-(25*A^2*b^15 + 9*B^
2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2
*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2
*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(
1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 1065

6*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 851968*B*a^19*b*c^8 - 320*A*a^12*b^14*c^2 + 7936*A*a^13*b^12*c^3 - 82816*A*a^14*b^10*c^4 + 468480*A*a^15*b^8*c^5 - 1536000*A*a^16*b^6*c^6 + 2867200*A*a^17*b^4*c^7 - 2719744*A*a^18*b^2*c^8 + 192*B*a^13*b^13*c^2 - 4672*B*a^14*b^11*c^3 + 47360*B*a^15*b^9*c^4 - 256000*B*a^16*b^7*c^5 + 778240*B*a^17*b^5*c^6 - 1261568*B*a^18*b^3*c^7) + x*(401408*A^2*a^16*c^10 - 204800*B^2*a^17*c^9 - 400*A^2*a^9*b^14*c^3 + 9440*A^2*a^10*b^12*c^4 - 92816*A^2*a^11*b^10*c^5 + 488096*A^2*a^12*b^8*c^6 - 1458688*A^2*a^13*b^6*c^7 + 2401280*A^2*a^14*b^4*c^8 - 1871872*A^2*a^15*b^2*c^9 - 144*B^2*a^11*b^12*c^3 + 3264*B^2*a^12*b^10*c^4 - 30112*B^2*a^13*b^8*c^5 + 143360*B^2*a^14*b^6*c^6 - 365568*B^2*a^15*b^4*c^7 + 458752*B^2*a^16*b^2*c^8 + 480*A*B*a^10*b^13*c^3 - 11104*A*B*a^11*b^11*c^4 + 105824*A*B*a^12*b^9*c^5 - 530432*A*B*a^13*b^7*c^6 + 1469440*A*B*a^14*b^5*c^7 - 2121728*A*B*a^15*b^3*c^8 + 1236992*A*B*a^16*b*c^9))*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2) + 128000*B^3*a^15*c^9 - 1800*A^3*a^9*b^9*c^6 + 29080*A^3*a^10*b^7*c^7 - 176032*A^3*a^11*b^5*c^8 + 473216*A^3*a^12*b^3*c^9 + 504*B^3*a^11*b^8*c^5 - 8112*B^3*a^12*b^6*c^6 + 48704*B^3*a^13*b^4*c^7 - 129280*B^3*a^14*b^2*c^8 + 250880*A^2*B*a^14*c^10 - 476672*A^3*a^13*b*c^10 - 442880*A*B^2*a^14*b*c^9 - 1680*A*B^2*a^10*b^9*c^5 + 27176*A*B^2*a^11*b^7*c^6 - 164448*A*B^2*a^12*b^5*c^7 + 441216*A*B^2*a^13*b^3*c^8 + 1400*A^2*B*a^9*b^10*c^5 - 21680*A^2*B*a^10*b^8*c^6 + 121648*A^2*B*a^11*b^6*c^7 - 275264*A^2*B*a^12*b^4*c^8 + 121088*A^2*B*a^13*b^2*c^9))*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2

- 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*2i - a
tan((((-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2)
 - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A
^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a
^3*c^3*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) +
2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 4480
0*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8
*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*
B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3
*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6
*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2
) - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2
)^9)^(1/2) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2
) - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c
^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^
4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*A*a^19*c^9 + x*(-(25*A^2*b^15 + 9*B^
2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2
*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2
*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(
1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 1065
6*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*
a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 8
0640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2
*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b
^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b
^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a
*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^
12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*
a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^
9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1
/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^
17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c
^7) + 851968*B*a^19*b*c^8 - 320*A*a^12*b^14*c^2 + 7936*A*a^13*b^12*c^3 - 82
816*A*a^14*b^10*c^4 + 468480*A*a^15*b^8*c^5 - 1536000*A*a^16*b^6*c^6 + 2867
200*A*a^17*b^4*c^7 - 2719744*A*a^18*b^2*c^8 + 192*B*a^13*b^13*c^2 - 4672*B*
a^14*b^11*c^3 + 47360*B*a^15*b^9*c^4 - 256000*B*a^16*b^7*c^5 + 778240*B*a^1
7*b^5*c^6 - 1261568*B*a^18*b^3*c^7) - x*(401408*A^2*a^16*c^10 - 204800*B^2*
a^17*c^9 - 400*A^2*a^9*b^14*c^3 + 9440*A^2*a^10*b^12*c^4 - 92816*A^2*a^11*b
^10*c^5 + 488096*A^2*a^12*b^8*c^6 - 1458688*A^2*a^13*b^6*c^7 + 2401280*A^2*
a^14*b^4*c^8 - 1871872*A^2*a^15*b^2*c^9 - 144*B^2*a^11*b^12*c^3 + 3264*B^2*
a^12*b^10*c^4 - 30112*B^2*a^13*b^8*c^5 + 143360*B^2*a^14*b^6*c^6 - 365568*B
^2*a^15*b^4*c^7 + 458752*B^2*a^16*b^2*c^8 + 480*A*B*a^10*b^13*c^3 - 11104*A
*B*a^11*b^11*c^4 + 105824*A*B*a^12*b^9*c^5 - 530432*A*B*a^13*b^7*c^6 + 1469
440*A*B*a^14*b^5*c^7 - 2121728*A*B*a^15*b^3*c^8 + 1236992*A*B*a^16*b*c^9))*
(-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*
A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4
*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3
*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B
^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*
a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 -
 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^
8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*
c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c
^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51
*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(
1/2) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 18
6*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 2
4*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 61
44*a^12*b^2*c^5)))^(1/2)*1i - ((-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6

*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A
^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A
^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-
(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 302
40*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)
^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 21
3*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2
)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5
*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*
c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30
*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*
c*(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(3
2*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b
^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*A*a^19*c^9
- x*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) -
30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2
*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3
c^3(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 20
77*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*
B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c
^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^
2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b
^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b
^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2)
- 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^
9)^(1/2) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2)
- 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6
- 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4
- 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144
*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5
*c^6 - 1572864*a^20*b^3*c^7) + 851968*B*a^19*b*c^8 - 320*A*a^12*b^14*c^2 +
7936*A*a^13*b^12*c^3 - 82816*A*a^14*b^10*c^4 + 468480*A*a^15*b^8*c^5 - 1536
000*A*a^16*b^6*c^6 + 2867200*A*a^17*b^4*c^7 - 2719744*A*a^18*b^2*c^8 + 192*
B*a^13*b^13*c^2 - 4672*B*a^14*b^11*c^3 + 47360*B*a^15*b^9*c^4 - 256000*B*a^
16*b^7*c^5 + 778240*B*a^17*b^5*c^6 - 1261568*B*a^18*b^3*c^7) + x*(401408*A^
2*a^16*c^10 - 204800*B^2*a^17*c^9 - 400*A^2*a^9*b^14*c^3 + 9440*A^2*a^10*b^
12*c^4 - 92816*A^2*a^11*b^10*c^5 + 488096*A^2*a^12*b^8*c^6 - 1458688*A^2*a^
13*b^6*c^7 + 2401280*A^2*a^14*b^4*c^8 - 1871872*A^2*a^15*b^2*c^9 - 144*B^2*
a^11*b^12*c^3 + 3264*B^2*a^12*b^10*c^4 - 30112*B^2*a^13*b^8*c^5 + 143360*B^
2*a^14*b^6*c^6 - 365568*B^2*a^15*b^4*c^7 + 458752*B^2*a^16*b^2*c^8 + 480*A*
B*a^10*b^13*c^3 - 11104*A*B*a^11*b^11*c^4 + 105824*A*B*a^12*b^9*c^5 - 53043
2*A*B*a^13*b^7*c^6 + 1469440*A*B*a^14*b^5*c^7 - 2121728*A*B*a^15*b^3*c^8 +
1236992*A*B*a^16*b*c^9))*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*
a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3
*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6
*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c
- b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2
*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1
/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*
a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c
^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4
*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a
b^5(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4
*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7
*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3
+ 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i)/(((-(25*A^2*b^15 + 9*
B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A
^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A

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^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)
^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10
656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^
2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c -
 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a
^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4
*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7
*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4
*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*
b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(
4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*
a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^
(1/2)*(917504*A*a^19*c^9 + x*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(
-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2
*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2
*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4
*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240
*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9
)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*
B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^
9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b
^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*
(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A
*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*
(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*
(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6
*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 +
 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*
b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 851968*B*a^19*b*c^8
 - 320*A*a^12*b^14*c^2 + 7936*A*a^13*b^12*c^3 - 82816*A*a^14*b^10*c^4 + 468
480*A*a^15*b^8*c^5 - 1536000*A*a^16*b^6*c^6 + 2867200*A*a^17*b^4*c^7 - 2719
744*A*a^18*b^2*c^8 + 192*B*a^13*b^13*c^2 - 4672*B*a^14*b^11*c^3 + 47360*B*a
^15*b^9*c^4 - 256000*B*a^16*b^7*c^5 + 778240*B*a^17*b^5*c^6 - 1261568*B*a^1
8*b^3*c^7) - x*(401408*A^2*a^16*c^10 - 204800*B^2*a^17*c^9 - 400*A^2*a^9*b^
14*c^3 + 9440*A^2*a^10*b^12*c^4 - 92816*A^2*a^11*b^10*c^5 + 488096*A^2*a^12
*b^8*c^6 - 1458688*A^2*a^13*b^6*c^7 + 2401280*A^2*a^14*b^4*c^8 - 1871872*A^
2*a^15*b^2*c^9 - 144*B^2*a^11*b^12*c^3 + 3264*B^2*a^12*b^10*c^4 - 30112*B^2
*a^13*b^8*c^5 + 143360*B^2*a^14*b^6*c^6 - 365568*B^2*a^15*b^4*c^7 + 458752*
B^2*a^16*b^2*c^8 + 480*A*B*a^10*b^13*c^3 - 11104*A*B*a^11*b^11*c^4 + 105824
*A*B*a^12*b^9*c^5 - 530432*A*B*a^13*b^7*c^6 + 1469440*A*B*a^14*b^5*c^7 - 21
21728*A*B*a^15*b^3*c^8 + 1236992*A*B*a^16*b*c^9))*(-(25*A^2*b^15 + 9*B^2*a^
2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2
*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5
*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2)
 + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^
2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*
c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640
*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2
*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c
^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c
^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a*c -
 b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c
 + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*a*c
- b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^
8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)
+ ((-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) -
30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*
a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*
c^3*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 207
```

7*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B
^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^
7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2
*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^
10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^
4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) -
 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9
)^(1/2) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) -
 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6
 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 -
 6144*a^12*b^2*c^5)))^(1/2)*(917504*A*a^19*c^9 - x*(-(25*A^2*b^15 + 9*B^2*a
^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^
2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^
5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2
) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B
^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4
c^2(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 8064
0*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^
2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*
c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*
c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a*c
 - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*
c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b
^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)
*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*
b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7)
 + 851968*B*a^19*b*c^8 - 320*A*a^12*b^14*c^2 + 7936*A*a^13*b^12*c^3 - 82816
*A*a^14*b^10*c^4 + 468480*A*a^15*b^8*c^5 - 1536000*A*a^16*b^6*c^6 + 2867200
*A*a^17*b^4*c^7 - 2719744*A*a^18*b^2*c^8 + 192*B*a^13*b^13*c^2 - 4672*B*a^1
4*b^11*c^3 + 47360*B*a^15*b^9*c^4 - 256000*B*a^16*b^7*c^5 + 778240*B*a^17*b
^5*c^6 - 1261568*B*a^18*b^3*c^7) + x*(401408*A^2*a^16*c^10 - 204800*B^2*a^1
7*c^9 - 400*A^2*a^9*b^14*c^3 + 9440*A^2*a^10*b^12*c^4 - 92816*A^2*a^11*b^10
*c^5 + 488096*A^2*a^12*b^8*c^6 - 1458688*A^2*a^13*b^6*c^7 + 2401280*A^2*a^1
4*b^4*c^8 - 1871872*A^2*a^15*b^2*c^9 - 144*B^2*a^11*b^12*c^3 + 3264*B^2*a^1
2*b^10*c^4 - 30112*B^2*a^13*b^8*c^5 + 143360*B^2*a^14*b^6*c^6 - 365568*B^2*
a^15*b^4*c^7 + 458752*B^2*a^16*b^2*c^8 + 480*A*B*a^10*b^13*c^3 - 11104*A*B*
a^11*b^11*c^4 + 105824*A*B*a^12*b^9*c^5 - 530432*A*B*a^13*b^7*c^6 + 1469440
*A*B*a^14*b^5*c^7 - 2121728*A*B*a^15*b^3*c^8 + 1236992*A*B*a^16*b*c^9))*(-(
25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B
*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^
7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-
(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*
a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7
*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 61
5*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b
*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2
 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5
 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^
2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2
) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 186*A
*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a
^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*
a^12*b^2*c^5)))^(1/2) + 128000*B^3*a^15*c^9 - 1800*A^3*a^9*b^9*c^6 + 29080*
A^3*a^10*b^7*c^7 - 176032*A^3*a^11*b^5*c^8 + 473216*A^3*a^12*b^3*c^9 + 504*
B^3*a^11*b^8*c^5 - 8112*B^3*a^12*b^6*c^6 + 48704*B^3*a^13*b^4*c^7 - 129280*
B^3*a^14*b^2*c^8 + 250880*A^2*B*a^14*c^10 - 476672*A^3*a^13*b*c^10 - 442880
*A*B^2*a^14*b*c^9 - 1680*A*B^2*a^10*b^9*c^5 + 27176*A*B^2*a^11*b^7*c^6 - 16
4448*A*B^2*a^12*b^5*c^7 + 441216*A*B^2*a^13*b^3*c^8 + 1400*A^2*B*a^9*b^10*c

```
^5 - 21680*A^2*B*a^10*b^8*c^6 + 121648*A^2*B*a^11*b^6*c^7 - 275264*A^2*B*a^
12*b^4*c^8 + 121088*A^2*B*a^13*b^2*c^9))*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 +
25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2
 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5
+ 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*
a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7
*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*
a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*
b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4
*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 1196
16*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*
A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^
(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c + 184*A*
B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)
^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1
280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*2i - (A/(
3*a) - (x^2*(5*A*b - 3*B*a))/(3*a^2) + (x^4*(15*A*b^4 + 14*A*a^2*c^2 - 9*B*
a*b^3 - 62*A*a*b^2*c + 33*B*a^2*b*c))/(6*a^3*(4*a*c - b^2)) + (c*x^6*(5*A*b
^3 - 3*B*a*b^2 + 10*B*a^2*c - 19*A*a*b*c))/(2*a^3*(4*a*c - b^2)))/(a*x^3 +
b*x^5 + c*x^7)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.102 $\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=365

$$-\frac{x^4\left(x^2\left(20a^2Bc^2+10aAbc^2-20ab^2Bc-Ab^3c+3b^4B\right)+a\left(16aAc^2-18abBc-Ab^2c+3b^3B\right)\right)}{4c^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^2\left(30a^2Bc^2+\right.}{}$$

Rubi [A] time = 1.45, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1251, 818, 773, 634, 618, 206, 628}

$$-\frac{x^4\left(x^2\left(20a^2Bc^2+10aAbc^2-20ab^2Bc-Ab^3c+3b^4B\right)+a\left(16aAc^2-18abBc-Ab^2c+3b^3B\right)\right)}{4c^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^2\left(30a^2Bc^2+7aAbc^2-21ab^2Bc-Ab^3c+3b^4B\right)}{2c^3\left(b^2-4ac\right)^2}-\frac{\left(-30a^2Abc^3+90a^2b^2Bc^2-60a^3Bc^3+10aAb^3c^2-30ab^4Bc-Ab^5c+3b^6B\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^4\left(b^2-4ac\right)^{5/2}}-\frac{x^8\left(x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{(3bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((3*b^4*B - A*b^3*c - 21*a*b^2*B*c + 7*a*A*b*c^2 + 30*a^2*B*c^2)*x^2)/(2*c^
3*(b^2 - 4*a*c)^2) - (x^8*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)
)/(4*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^4*(a*(3*b^3*B - A*b^2*c -
18*a*b*B*c + 16*a*A*c^2) + (3*b^4*B - A*b^3*c - 20*a*b^2*B*c + 10*a*A*b*c^2
 + 20*a^2*B*c^2)*x^2))/(4*c^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((3*b^
6*B - A*b^5*c - 30*a*b^4*B*c + 10*a*A*b^3*c^2 + 90*a^2*b^2*B*c^2 - 30*a^2*A
*b*c^3 - 60*a^3*B*c^3)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^4*(b^
2 - 4*a*c)^(5/2)) - ((3*b*B - A*c)*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^{11}\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^5(A+Bx)}{\left(a+bx+cx^2\right)^3}\,dx,x,x^2\right)$$

$$= -\frac{x^8\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\text{Subst}\left(\int \frac{x^3\left(4a(bB-2Ac)+\left(3b^2B-Abc-10aBc\right.\right.}{\left(a+bx+cx^2\right)^2}\right.}{4c\left(b^2-4ac\right)}$$

$$= -\frac{x^8\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x^4\left(a\left(3b^3B-Ab^2c-18abBc+16aA\right.\right.}{4c^2\left(\right.}$$

$$= \frac{\left(3b^4B-Ab^3c-21ab^2Bc+7aAbc^2+30a^2Bc^2\right)x^2}{2c^3\left(b^2-4ac\right)^2} - \frac{x^8\left(a(bB-2Ac)+\left(b^2B-Abc-\right.\right.}{4c\left(b^2-4ac\right)\left(a+bx^2+c\right.}$$

$$= \frac{\left(3b^4B-Ab^3c-21ab^2Bc+7aAbc^2+30a^2Bc^2\right)x^2}{2c^3\left(b^2-4ac\right)^2} - \frac{x^8\left(a(bB-2Ac)+\left(b^2B-Abc-\right.\right.}{4c\left(b^2-4ac\right)\left(a+bx^2+c\right.}$$

$$= \frac{\left(3b^4B-Ab^3c-21ab^2Bc+7aAbc^2+30a^2Bc^2\right)x^2}{2c^3\left(b^2-4ac\right)^2} - \frac{x^8\left(a(bB-2Ac)+\left(b^2B-Abc-\right.\right.}{4c\left(b^2-4ac\right)\left(a+bx^2+c\right.}$$

$$= \frac{\left(3b^4B-Ab^3c-21ab^2Bc+7aAbc^2+30a^2Bc^2\right)x^2}{2c^3\left(b^2-4ac\right)^2} - \frac{x^8\left(a(bB-2Ac)+\left(b^2B-Abc-\right.\right.}{4c\left(b^2-4ac\right)\left(a+bx^2+c\right.}$$

Mathematica [A] time = 0.66, size = 435, normalized size = 1.19

$$\frac{\frac{a^3c^2\left(2c\left(A+Bx^2\right)-5bB\right)+a^2bc\left(-bc\left(4A+9Bx^2\right)+5Ac^2x^2+5b^2B\right)+ab^3\left(bc\left(A+6Bx^2\right)-5Ac^2x^2+b^2(-B)\right)+b^5x^2(Ac-bB)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{2c\left(60a^3Bc^3+30a^2Abc^3-90a^2b^2Bc^2-10aAb^3c^2+30ab^4Bc+Ab^5c-3b^6B\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}+\frac{4a^3c^4\left(8A+9Bx^2\right)-3a^2b^2c^3\left(13A+34Bx^2\right)+2a^2bc^3\left(25Acx^2-39aB\right)+2b^5c\left(2Acx^2-7aB\right)+ab^4c^2\left(11A+48Bx^2\right)+ab^3c^2\left(61aB-30Acx^2\right)-b^6c\left(A+6Bx^2\right)+b^7B}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+c(Ac-3bB)\log\left(a+bx^2+cx^4\right)+2Bc^2x^2}{4c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*B*c^2*x^2 + (b^7*B - b^6*c*(A + 6*B*x^2) + 4*a^3*c^4*(8*A + 9*B*x^2) - 3
*a^2*b^2*c^3*(13*A + 34*B*x^2) + a*b^4*c^2*(11*A + 48*B*x^2) + a*b^3*c^2*(6
1*a*B - 30*A*c*x^2) + 2*b^5*c*(-7*a*B + 2*A*c*x^2) + 2*a^2*b*c^3*(-39*a*B +
 25*A*c*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b^5*(-(b*B) + A*c)*x
^2 + a^3*c^2*(-5*b*B + 2*c*(A + B*x^2)) + a*b^3*(-(b^2*B) - 5*A*c^2*x^2 + b
*c*(A + 6*B*x^2)) + a^2*b*c*(5*b^2*B + 5*A*c^2*x^2 - b*c*(4*A + 9*B*x^2)))/
((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (2*c*(-3*b^6*B + A*b^5*c + 30*a*b^4
*B*c - 10*a*A*b^3*c^2 - 90*a^2*b^2*B*c^2 + 30*a^2*A*b*c^3 + 60*a^3*B*c^3)*A
rcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*(-3*b*B +
 A*c)*Log[a + b*x^2 + c*x^4])/(4*c^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 1.41, size = 3196, normalized size = 8.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(B*b^6*c^3 - 12*B*a*b^4*c^4 + 48*B*a^2*b^2*c^5 - 64*B*a^3*c^6)*x^10
 - 5*B*a^2*b^7 - 96*A*a^5*c^4 + 4*(B*b^7*c^2 - 12*B*a*b^5*c^3 + 48*B*a^2*b^
3*c^4 - 64*B*a^3*b*c^5)*x^8 - 2*(2*B*b^8*c + 100*(2*B*a^4 + A*a^3*b)*c^5 -
(254*B*a^3*b^2 + 85*A*a^2*b^3)*c^4 + (123*B*a^2*b^4 + 23*A*a*b^5)*c^3 - 2*(
13*B*a*b^6 + A*b^7)*c^2)*x^6 - (5*B*b^9 + 128*A*a^4*c^5 + 4*(22*B*a^4*b + 3
*A*a^3*b^2)*c^4 - (314*B*a^3*b^3 + 87*A*a^2*b^4)*c^3 + (225*B*a^2*b^5 + 31*
A*a*b^6)*c^2 - (58*B*a*b^7 + 3*A*b^8)*c)*x^4 + 4*(58*B*a^5*b + 27*A*a^4*b^2
)*c^3 - (202*B*a^4*b^3 + 33*A*a^3*b^4)*c^2 - 2*(5*B*a*b^8 + 4*(30*B*a^5 + 3
1*A*a^4*b)*c^4 - (346*B*a^4*b^2 + 119*A*a^3*b^3)*c^3 + (235*B*a^3*b^4 + 34*
A*a^2*b^5)*c^2 - (59*B*a^2*b^6 + 3*A*a*b^7)*c)*x^2 - (3*B*a^2*b^6 + (3*B*b^
6*c^2 - 30*(2*B*a^3 + A*a^2*b)*c^5 + 10*(9*B*a^2*b^2 + A*a*b^3)*c^4 - (30*B
*a*b^4 + A*b^5)*c^3)*x^8 + 2*(3*B*b^7*c - 30*(2*B*a^3*b + A*a^2*b^2)*c^4 +
10*(9*B*a^2*b^3 + A*a*b^4)*c^3 - (30*B*a*b^5 + A*b^6)*c^2)*x^6 + (3*B*b^8 -
 60*(2*B*a^4 + A*a^3*b)*c^4 + 10*(12*B*a^3*b^2 - A*a^2*b^3)*c^3 + 2*(15*B*a
^2*b^4 + 4*A*a*b^5)*c^2 - (24*B*a*b^6 + A*b^7)*c)*x^4 - 30*(2*B*a^5 + A*a^4
*b)*c^3 + 10*(9*B*a^4*b^2 + A*a^3*b^3)*c^2 + 2*(3*B*a*b^7 - 30*(2*B*a^4*b +
 A*a^3*b^2)*c^3 + 10*(9*B*a^3*b^3 + A*a^2*b^4)*c^2 - (30*B*a^2*b^5 + A*a*b^
6)*c)*x^2 - (30*B*a^3*b^4 + A*a^2*b^5)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4
+ 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
 + a)) + (56*B*a^3*b^5 + 3*A*a^2*b^6)*c - (3*B*a^2*b^7 + 64*A*a^5*c^4 + (3*
B*b^7*c^2 + 64*A*a^3*c^6 - 48*(4*B*a^3*b + A*a^2*b^2)*c^5 + 12*(12*B*a^2*b^
3 + A*a*b^4)*c^4 - (36*B*a*b^5 + A*b^6)*c^3)*x^8 + 2*(3*B*b^8*c + 64*A*a^3*
b*c^5 - 48*(4*B*a^3*b^2 + A*a^2*b^3)*c^4 + 12*(12*B*a^2*b^4 + A*a*b^5)*c^3
- (36*B*a*b^6 + A*b^7)*c^2)*x^6 + (3*B*b^9 + 128*A*a^4*c^5 - 32*(12*B*a^4*b
 + A*a^3*b^2)*c^4 + 24*(4*B*a^3*b^3 - A*a^2*b^4)*c^3 + 2*(36*B*a^2*b^5 + 5*
A*a*b^6)*c^2 - (30*B*a*b^7 + A*b^8)*c)*x^4 - 48*(4*B*a^5*b + A*a^4*b^2)*c^3
 + 12*(12*B*a^4*b^3 + A*a^3*b^4)*c^2 + 2*(3*B*a*b^8 + 64*A*a^4*b*c^4 - 48*(
4*B*a^4*b^2 + A*a^3*b^3)*c^3 + 12*(12*B*a^3*b^4 + A*a^2*b^5)*c^2 - (36*B*a^
2*b^6 + A*a*b^7)*c)*x^2 - (36*B*a^3*b^5 + A*a^2*b^6)*c)*log(c*x^4 + b*x^2 +
 a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7 + (b^6*c^6
```

```
 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^8 + 2*(b^7*c^5 - 12*a*b^5*
c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^6 + (b^8*c^4 - 10*a*b^6*c^5 + 24*a^2
*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^4 + 2*(a*b^7*c^4 - 12*a^2*b^5*c^
5 + 48*a^3*b^3*c^6 - 64*a^4*b*c^7)*x^2), 1/4*(2*(B*b^6*c^3 - 12*B*a*b^4*c^4
 + 48*B*a^2*b^2*c^5 - 64*B*a^3*c^6)*x^10 - 5*B*a^2*b^7 - 96*A*a^5*c^4 + 4*(
B*b^7*c^2 - 12*B*a*b^5*c^3 + 48*B*a^2*b^3*c^4 - 64*B*a^3*b*c^5)*x^8 - 2*(2*
B*b^8*c + 100*(2*B*a^4 + A*a^3*b)*c^5 - (254*B*a^3*b^2 + 85*A*a^2*b^3)*c^4
+ (123*B*a^2*b^4 + 23*A*a*b^5)*c^3 - 2*(13*B*a*b^6 + A*b^7)*c^2)*x^6 - (5*B
*b^9 + 128*A*a^4*c^5 + 4*(22*B*a^4*b + 3*A*a^3*b^2)*c^4 - (314*B*a^3*b^3 +
87*A*a^2*b^4)*c^3 + (225*B*a^2*b^5 + 31*A*a*b^6)*c^2 - (58*B*a*b^7 + 3*A*b^
8)*c)*x^4 + 4*(58*B*a^5*b + 27*A*a^4*b^2)*c^3 - (202*B*a^4*b^3 + 33*A*a^3*b
^4)*c^2 - 2*(5*B*a*b^8 + 4*(30*B*a^5 + 31*A*a^4*b)*c^4 - (346*B*a^4*b^2 + 1
19*A*a^3*b^3)*c^3 + (235*B*a^3*b^4 + 34*A*a^2*b^5)*c^2 - (59*B*a^2*b^6 + 3*
A*a*b^7)*c)*x^2 - 2*(3*B*a^2*b^6 + (3*B*b^6*c^2 - 30*(2*B*a^3 + A*a^2*b)*c^
5 + 10*(9*B*a^2*b^2 + A*a*b^3)*c^4 - (30*B*a*b^4 + A*b^5)*c^3)*x^8 + 2*(3*B
*b^7*c - 30*(2*B*a^3*b + A*a^2*b^2)*c^4 + 10*(9*B*a^2*b^3 + A*a*b^4)*c^3 -
(30*B*a*b^5 + A*b^6)*c^2)*x^6 + (3*B*b^8 - 60*(2*B*a^4 + A*a^3*b)*c^4 + 10*
(12*B*a^3*b^2 - A*a^2*b^3)*c^3 + 2*(15*B*a^2*b^4 + 4*A*a*b^5)*c^2 - (24*B*a
*b^6 + A*b^7)*c)*x^4 - 30*(2*B*a^5 + A*a^4*b)*c^3 + 10*(9*B*a^4*b^2 + A*a^3
*b^3)*c^2 + 2*(3*B*a*b^7 - 30*(2*B*a^4*b + A*a^3*b^2)*c^3 + 10*(9*B*a^3*b^3
 + A*a^2*b^4)*c^2 - (30*B*a^2*b^5 + A*a*b^6)*c)*x^2 - (30*B*a^3*b^4 + A*a^2
*b^5)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 -
 4*a*c)) + (56*B*a^3*b^5 + 3*A*a^2*b^6)*c - (3*B*a^2*b^7 + 64*A*a^5*c^4 + (
3*B*b^7*c^2 + 64*A*a^3*c^6 - 48*(4*B*a^3*b + A*a^2*b^2)*c^5 + 12*(12*B*a^2*
b^3 + A*a*b^4)*c^4 - (36*B*a*b^5 + A*b^6)*c^3)*x^8 + 2*(3*B*b^8*c + 64*A*a^
3*b*c^5 - 48*(4*B*a^3*b^2 + A*a^2*b^3)*c^4 + 12*(12*B*a^2*b^4 + A*a*b^5)*c^
3 - (36*B*a*b^6 + A*b^7)*c^2)*x^6 + (3*B*b^9 + 128*A*a^4*c^5 - 32*(12*B*a^4
*b + A*a^3*b^2)*c^4 + 24*(4*B*a^3*b^3 - A*a^2*b^4)*c^3 + 2*(36*B*a^2*b^5 +
5*A*a*b^6)*c^2 - (30*B*a*b^7 + A*b^8)*c)*x^4 - 48*(4*B*a^5*b + A*a^4*b^2)*c
^3 + 12*(12*B*a^4*b^3 + A*a^3*b^4)*c^2 + 2*(3*B*a*b^8 + 64*A*a^4*b*c^4 - 48
*(4*B*a^4*b^2 + A*a^3*b^3)*c^3 + 12*(12*B*a^3*b^4 + A*a^2*b^5)*c^2 - (36*B*
a^2*b^6 + A*a*b^7)*c)*x^2 - (36*B*a^3*b^5 + A*a^2*b^6)*c)*log(c*x^4 + b*x^2
 + a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7 + (b^6*c
^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^8 + 2*(b^7*c^5 - 12*a*b^
5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^6 + (b^8*c^4 - 10*a*b^6*c^5 + 24*a
^2*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^4 + 2*(a*b^7*c^4 - 12*a^2*b^5*
c^5 + 48*a^3*b^3*c^6 - 64*a^4*b*c^7)*x^2)]
```

giac [A] time = 5.88, size = 598, normalized size = 1.64

$$\frac{\left(3 Bb^6 - 30 Bab^4c - Ab^5c + 90 Ba^2b^2c^2 + 10 Aab^3c^2 - 60 Ba^3c^3 - 30 Aa^2bc^3\right)\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\left(b^4c^4 - 8ab^2c^5 + 16a^2c^6\right)\sqrt{-b^2+4ac}} + \frac{Bx^2}{2c^3} + \frac{9Bb^5c^2x^8 - 72Bab^3c^3x^8 - 3Ab^4c^3x^8 + 144Ba^2bc^4x^8 + 24Aab^2c^4x^8 - 48Aa^2c^5x^8 + 6Bb^6cx^6 - 48Bab^4c^2x^6 + 2Ab^5c^2x^6 + 84Ba^2b^2c^3x^6 - 12Aab^3c^3x^6 + 72Ba^3c^4x^6 + 4Aa^2bc^4x^6 - Bb^7x^4 + 14Bab^5cx^4 + 3Ab^6cx^4 - 82Ba^2b^3c^2x^4 - 20Aab^4c^2x^4 + 204Ba^3bc^3x^4 + 22Aa^2b^2c^3x^4 - 32Aa^3c^4x^4 - 2Bab^6x^2 + 8Ba^2b^4cx^2 + 6Aab^5cx^2 + 4Ba^3b^2c^2x^2 - 40Aa^2b^3c^2x^2 + 56Ba^4c^3x^2 + 28Aa^3bc^3x^2 - Ba^2b^5 + 3Aa^2b^4c + 28Ba^4bc^2 - 18Aa^3b^2c^2}{8\left(b^4c^4 - 8ab^2c^5 + 16a^2c^6\right)\left(cx^4+bx^2+a\right)^2} - \frac{(3Bb - Ac)\log\left(cx^4+bx^2+a\right)}{4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*(3*B*b^6 - 30*B*a*b^4*c - A*b^5*c + 90*B*a^2*b^2*c^2 + 10*A*a*b^3*c^2 -
 60*B*a^3*c^3 - 30*A*a^2*b*c^3)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((
b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*sqrt(-b^2 + 4*a*c)) + 1/2*B*x^2/c^3 + 1
/8*(9*B*b^5*c^2*x^8 - 72*B*a*b^3*c^3*x^8 - 3*A*b^4*c^3*x^8 + 144*B*a^2*b*c^
4*x^8 + 24*A*a*b^2*c^4*x^8 - 48*A*a^2*c^5*x^8 + 6*B*b^6*c*x^6 - 48*B*a*b^4*
c^2*x^6 + 2*A*b^5*c^2*x^6 + 84*B*a^2*b^2*c^3*x^6 - 12*A*a*b^3*c^3*x^6 + 72*
B*a^3*c^4*x^6 + 4*A*a^2*b*c^4*x^6 - B*b^7*x^4 + 14*B*a*b^5*c*x^4 + 3*A*b^6*
c*x^4 - 82*B*a^2*b^3*c^2*x^4 - 20*A*a*b^4*c^2*x^4 + 204*B*a^3*b*c^3*x^4 + 2
2*A*a^2*b^2*c^3*x^4 - 32*A*a^3*c^4*x^4 - 2*B*a*b^6*x^2 + 8*B*a^2*b^4*c*x^2
+ 6*A*a*b^5*c*x^2 + 4*B*a^3*b^2*c^2*x^2 - 40*A*a^2*b^3*c^2*x^2 + 56*B*a^4*c
^3*x^2 + 28*A*a^3*b*c^3*x^2 - B*a^2*b^5 + 3*A*a^2*b^4*c + 28*B*a^4*b*c^2 -
18*A*a^3*b^2*c^2)/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*(c*x^4 + b*x^2 + a)
^2) - 1/4*(3*B*b - A*c)*log(c*x^4 + b*x^2 + a)/c^4
```

maple [B] time = 0.03, size = 2054, normalized size = 5.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*B*x^2/c^3+6/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*B*a*b^3-1/
2/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*
c-b^2)^(1/2))*b^5*A-2/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*A*a*
b^2-12/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*B*a^2*b+1/c^2/(c*x^
4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*b^5-5/4/c^4/(c*x^4+b*x^2+a)^2
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*B*b^7+7/c/(c*x^4+b*x^2+a)^2*a^4/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^2*B-21/4/c^2/(c*x^4+b*x^2+a)^2*a^3/(16*a^2*c^2-8*a*b^2*c+
b^4)*A*b^2-29/2/c^2/(c*x^4+b*x^2+a)^2*a^4/(16*a^2*c^2-8*a*b^2*c+b^4)*B*b-5/
4/c^4/(c*x^4+b*x^2+a)^2*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*B*b^5+25/2/(c*x^4+b*
x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*a^2*b+3/2/c^4/(16*a^2*c^2-8*a*b^2
*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^6*B-30/c/
(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)
^(1/2))*a^3*B-3/2/c^3/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*B*b^
6+3/4/c^3/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*b^6+3/4/c^3/(c
*x^4+b*x^2+a)^2*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*A*b^4+9/c^3/(c*x^4+b*x^2+a)^
2*a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*B*b^3+3/2/c^3/(c*x^4+b*x^2+a)^2*a/(16*a^2*
c^2-8*a*b^2*c+b^4)*x^2*A*b^5-11/c^2/(c*x^4+b*x^2+a)^2*a^2/(16*a^2*c^2-8*a*b
^2*c+b^4)*x^2*A*b^3+11/4/c/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4
*A*a^2*b^2-19/4/c^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*a*b^
4-21/2/c/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*B*a^3*b-41/4/c^2/
(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*B*a^2*b^3+31/2/c/(c*x^4+b*
x^2+a)^2*a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b-71/2/c^2/(c*x^4+b*x^2+a)^2*
a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*b^2-5/2/c^4/(c*x^4+b*x^2+a)^2*a/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^2*B*b^6-15/2/c/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2
*c+b^4)*x^6*A*a*b^3+12/c^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6
*B*a*b^4-15/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+
b)/(4*a*c-b^2)^(1/2))*A*a^2*b+5/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^
(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*a*b^3+45/c^2/(16*a^2*c^2-8*a*
b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B*a^2*b^
2-15/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4
*a*c-b^2)^(1/2))*B*a*b^4+19/c^3/(c*x^4+b*x^2+a)^2*a^2/(16*a^2*c^2-8*a*b^2*c
+b^4)*x^2*B*b^4+17/2/c^3/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*B
*a*b^5-51/2/c/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*B*a^2*b^2+9/
(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*B*a^3+8/(c*x^4+b*x^2+a)^2/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*a^3+6/c/(c*x^4+b*x^2+a)^2*a^4/(16*a^2*c^2-
8*a*b^2*c+b^4)*A+1/4/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*A*b^4
+4/c/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*A*a^2-3/4/c^4/(16*a^2*c^2
-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*B*b^5
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.66, size = 4501, normalized size = 12.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^6*(18*B*a^3*c^3 - 3*B*b^6 + 2*A*b^5*c + 24*B*a*b^4*c - 15*A*a*b^3*c^2 +
 25*A*a^2*b*c^3 - 51*B*a^2*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (
a*(24*A*a^3*c^3 - 5*B*a*b^5 + 3*A*a*b^4*c + 36*B*a^2*b^3*c - 58*B*a^3*b*c^2
 - 21*A*a^2*b^2*c^2))/(4*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(14*B*a^4
*c^3 - 5*B*a*b^6 + 3*A*a*b^5*c + 31*A*a^3*b*c^3 + 38*B*a^2*b^4*c - 22*A*a^2
*b^3*c^2 - 71*B*a^3*b^2*c^2))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^4*(
5*B*b^7 - 32*A*a^3*c^4 - 3*A*b^6*c - 34*B*a*b^5*c + 19*A*a*b^4*c^2 + 42*B*a
^3*b*c^3 - 11*A*a^2*b^2*c^3 + 41*B*a^2*b^3*c^2))/(4*c*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)))/(a^2*c^3 + c^5*x^8 + x^4*(2*a*c^4 + b^2*c^3) + 2*b*c^4*x^6 + 2*
a*b*c^3*x^2) + (B*x^2)/(2*c^3) + (log(((a*(A*c - 3*B*b)^2)/c^6 - (((8*a*(A*
c - 3*B*b))/c^2 - (2*(2*a + b*x^2)*(A*c - 3*B*b + c^4*(-(60*B*a^3*c^3 - 3*B
*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*
b^2*c^2)^2/(c^8*(4*a*c - b^2)^5))^(1/2)))/c^2 + (2*x^2*(60*B*a^3*c^3 - 9*B*
b^6 + 3*A*b^5*c + 78*B*a*b^4*c - 26*A*a*b^3*c^2 + 62*A*a^2*b*c^3 - 186*B*a^
2*b^2*c^2))/(c^2*(4*a*c - b^2)^2))*(A*c - 3*B*b + c^4*(-(60*B*a^3*c^3 - 3*B
*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*
b^2*c^2)^2/(c^8*(4*a*c - b^2)^5))^(1/2)))/(4*c^4) + (x^2*(A*c - 3*B*b)*(30*
B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 27*B*a*b^4*c - 9*A*a*b^3*c^2 + 23*A*a^2*b*c
^3 - 69*B*a^2*b^2*c^2))/(c^6*(4*a*c - b^2)^2))*((a*(A*c - 3*B*b)^2)/c^6 + (
((2*(2*a + b*x^2)*(3*B*b - A*c + c^4*(-(60*B*a^3*c^3 - 3*B*b^6 + A*b^5*c +
30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*b^2*c^2)^2/(c^8*(
4*a*c - b^2)^5))^(1/2)))/c^2 + (8*a*(A*c - 3*B*b))/c^2 + (2*x^2*(60*B*a^3*c
^3 - 9*B*b^6 + 3*A*b^5*c + 78*B*a*b^4*c - 26*A*a*b^3*c^2 + 62*A*a^2*b*c^3 -
 186*B*a^2*b^2*c^2))/(c^2*(4*a*c - b^2)^2))*(3*B*b - A*c + c^4*(-(60*B*a^3*
c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 -
90*B*a^2*b^2*c^2)^2/(c^8*(4*a*c - b^2)^5))^(1/2)))/(4*c^4) + (x^2*(A*c - 3*
B*b)*(30*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 27*B*a*b^4*c - 9*A*a*b^3*c^2 + 23*
A*a^2*b*c^3 - 69*B*a^2*b^2*c^2))/(c^6*(4*a*c - b^2)^2)))*(6*B*b^11 + 2048*A
*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^5*b*c^5 -
 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960*B*a^2*b^
7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(2*(4096*a^5*c^9 - 4*b^10
*c^4 + 80*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8
)) - (atan(((32*a^2*c^8*(4*a*c - b^2)^5 + 2*b^4*c^6*(4*a*c - b^2)^5 - 16*a*
b^2*c^7*(4*a*c - b^2)^5)*(x^2*(((((6*A*b^5*c^5 + 120*B*a^3*c^7 - 18*B*b^6*c
^4 - 52*A*a*b^3*c^6 + 124*A*a^2*b*c^7 + 156*B*a*b^4*c^5 - 372*B*a^2*b^2*c^6
)/(16*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^7) - ((8*b^5*c^8 - 64*a*b^3*c^9 + 128*a
^2*b*c^10)*(6*B*b^11 + 2048*A*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a
*b^8*c^2 - 6144*B*a^5*b*c^5 - 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560
*A*a^4*b^2*c^5 + 960*B*a^2*b^7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^
4))/(2*(16*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^7)*(4096*a^5*c^9 - 4*b^10*c^4 + 80
*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8)))*(60*B
*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c
^3 - 90*B*a^2*b^2*c^2))/(8*c^4*(4*a*c - b^2)^(5/2)) - ((8*b^5*c^8 - 64*a*b^
3*c^9 + 128*a^2*b*c^10)*(60*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c -
10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*b^2*c^2)*(6*B*b^11 + 2048*A*a^5*
c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^5*b*c^5 - 320*
A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960*B*a^2*b^7*c^2
 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(16*c^4*(4*a*c - b^2)^(5/2)*(1
6*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^7)*(4096*a^5*c^9 - 4*b^10*c^4 + 80*a*b^8*c^
5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8)))/(a*(4*a*c - b^
2)^2) + (b*((((6*A*b^5*c^5 + 120*B*a^3*c^7 - 18*B*b^6*c^4 - 52*A*a*b^3*c^6
+ 124*A*a^2*b*c^7 + 156*B*a*b^4*c^5 - 372*B*a^2*b^2*c^6)/(16*a^2*c^8 + b^4*
c^6 - 8*a*b^2*c^7) - ((8*b^5*c^8 - 64*a*b^3*c^9 + 128*a^2*b*c^10)*(6*B*b^11
 + 2048*A*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^
5*b*c^5 - 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960
*B*a^2*b^7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(2*(16*a^2*c^8 +
 b^4*c^6 - 8*a*b^2*c^7)*(4096*a^5*c^9 - 4*b^10*c^4 + 80*a*b^8*c^5 - 640*a^2
*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8)))*(6*B*b^11 + 2048*A*a^5*c^
```

```
6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^5*b*c^5 - 320*A*
a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960*B*a^2*b^7*c^2 -
 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(2*(4096*a^5*c^9 - 4*b^10*c^4 +
80*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8)) - (9
*B^2*b^7 + A^2*b^5*c^2 - 6*A*B*b^6*c + 207*B^2*a^2*b^3*c^2 + 30*A*B*a^3*c^4
 - 81*B^2*a*b^5*c - 9*A^2*a*b^3*c^3 + 23*A^2*a^2*b*c^4 - 90*B^2*a^3*b*c^3 -
 138*A*B*a^2*b^2*c^3 + 54*A*B*a*b^4*c^2)/(16*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^
7) + (((b^5*c^8)/2 - 4*a*b^3*c^9 + 8*a^2*b*c^10)*(60*B*a^3*c^3 - 3*B*b^6 +
A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*b^2*c^2
)^2)/(c^8*(4*a*c - b^2)^5*(16*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^7))))/(2*a*(4*a
*c - b^2)^(5/2))) + ((((8*A*a*c^5 - 24*B*a*b*c^4)/c^6 - (8*a*c^2*(6*B*b^11
+ 2048*A*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^5
*b*c^5 - 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960*
B*a^2*b^7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(4096*a^5*c^9 - 4
*b^10*c^4 + 80*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^
2*c^8))*(60*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 +
 30*A*a^2*b*c^3 - 90*B*a^2*b^2*c^2))/(8*c^4*(4*a*c - b^2)^(5/2)) - (a*(60*B
*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c
^3 - 90*B*a^2*b^2*c^2)*(6*B*b^11 + 2048*A*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^
9*c + 40*A*a*b^8*c^2 - 6144*B*a^5*b*c^5 - 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^
4*c^4 - 2560*A*a^4*b^2*c^5 + 960*B*a^2*b^7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*
B*a^4*b^3*c^4))/(c^2*(4*a*c - b^2)^(5/2)*(4096*a^5*c^9 - 4*b^10*c^4 + 80*a*
b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8)))/(a*(4*a*
c - b^2)^2) + (b*((((8*A*a*c^5 - 24*B*a*b*c^4)/c^6 - (8*a*c^2*(6*B*b^11 + 2
048*A*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^5*b*
c^5 - 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960*B*a
^2*b^7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(4096*a^5*c^9 - 4*b^
10*c^4 + 80*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c
^8))*(6*B*b^11 + 2048*A*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c
^2 - 6144*B*a^5*b*c^5 - 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4
*b^2*c^5 + 960*B*a^2*b^7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(2
*(4096*a^5*c^9 - 4*b^10*c^4 + 80*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4
*c^7 - 5120*a^4*b^2*c^8)) - (A^2*a*c^2 + 9*B^2*a*b^2 - 6*A*B*a*b*c)/c^6 + (
a*(60*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*
a^2*b*c^3 - 90*B*a^2*b^2*c^2)^2)/(c^6*(4*a*c - b^2)^5)))/(2*a*(4*a*c - b^2)
^(5/2))))/(9*B^2*b^12 + A^2*b^10*c^2 + 3600*B^2*a^6*c^6 - 6*A*B*b^11*c + 16
0*A^2*a^2*b^6*c^4 - 600*A^2*a^3*b^4*c^5 + 900*A^2*a^4*b^2*c^6 + 1440*B^2*a^
2*b^8*c^2 - 5760*B^2*a^3*b^6*c^3 + 11700*B^2*a^4*b^4*c^4 - 10800*B^2*a^5*b^
2*c^5 - 180*B^2*a*b^10*c - 20*A^2*a*b^8*c^3 - 960*A*B*a^2*b^7*c^3 + 3720*A*
B*a^3*b^5*c^4 - 6600*A*B*a^4*b^3*c^5 + 120*A*B*a*b^9*c^2 + 3600*A*B*a^5*b*c
^6))*(60*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30
*A*a^2*b*c^3 - 90*B*a^2*b^2*c^2))/(2*c^4*(4*a*c - b^2)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.103 $\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=254

$$\frac{(-12a^2Ac^3+30a^2bBc^2-10ab^3Bc+b^5B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3(b^2-4ac)^{5/2}}-\frac{x^2\left(x^2\left(16a^2Bc^2+6aAbc^2-15ab^2Bc+2b^4B\right)+\cdots\right.}{4c^2(b^2-4ac)^2\left(a+bx^2+\cdots\right.}$$

Rubi [A] time = 0.40, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1251, 818, 634, 618, 206, 628}

$$-\frac{x^2\left(x^2\left(16a^2Bc^2+6aAbc^2-15ab^2Bc+2b^4B\right)+2a\left(6aAc^2-7abBc+b^3B\right)\right)}{4c^2(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{(-12a^2Ac^3+30a^2bBc^2-10ab^3Bc+b^5B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3(b^2-4ac)^{5/2}}-\frac{x^6\left(x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{4c(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{B\log(a+bx^2+cx^4)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x^6*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(4*c*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)^2) - (x^2*(2*a*(b^3*B - 7*a*b*B*c + 6*a*A*c^2) + (2*b^
4*B - 15*a*b^2*B*c + 6*a*A*b*c^2 + 16*a^2*B*c^2)*x^2))/(4*c^2*(b^2 - 4*a*c)
^2*(a + b*x^2 + c*x^4)) + ((b^5*B - 10*a*b^3*B*c + 30*a^2*b*B*c^2 - 12*a^2*
A*c^3)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)^(5/2)
) + (B*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
```

```
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^9\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4(A+Bx)}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{x^6\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2\left(3a(bB-2Ac)+2B\left(b^2-4ac\right)x\right)}{\left(a+bx+cx^2\right)^2}\,dx, x, \right.}{4c\left(b^2-4ac\right)}$$

$$= -\frac{x^6\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x^2\left(2a\left(b^3B-7abBc+6aAc^2\right)+\left(2b^4B\right.\right.}{4c^2\left(b^2-4ac\right)^2\left(a\right.}$$

$$= -\frac{x^6\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x^2\left(2a\left(b^3B-7abBc+6aAc^2\right)+\left(2b^4B\right.\right.}{4c^2\left(b^2-4ac\right)^2\left(a\right.}$$

$$= -\frac{x^6\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x^2\left(2a\left(b^3B-7abBc+6aAc^2\right)+\left(2b^4B\right.\right.}{4c^2\left(b^2-4ac\right)^2\left(a\right.}$$

$$= -\frac{x^6\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x^2\left(2a\left(b^3B-7abBc+6aAc^2\right)+\left(2b^4B\right.\right.}{4c^2\left(b^2-4ac\right)^2\left(a\right.}$$

Mathematica [A] time = 0.48, size = 354, normalized size = 1.39

$$\frac{-\frac{2c\left(-12a^2Ac^3+30a^2bBc^2-10ab^3Bc+b^5B\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}+\frac{2a^2bc^3\left(11A+25Bx^2\right)+4a^2c^3\left(8aB-5Acx^2\right)+b^4c\left(11aB-2Acx^2\right)-2ab^3c^2\left(4A+15Bx^2\right)+ab^2c^2\left(16Acx^2-39aB\right)+b^5c\left(A+4Bx^2\right)+b^6(-B)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{2a^3Bc^2+a^2c\left(bc\left(3A+5Bx^2\right)-2Ac^2x^2-4b^2B\right)+ab^2\left(-bc\left(A+5Bx^2\right)+4Ac^2x^2+b^2B\right)+b^4x^2(bB-Ac)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+Bc\log\left(a+bx^2+cx^4\right)}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-(b^6*B) + b^5*c*(A + 4*B*x^2) - 2*a*b^3*c^2*(4*A + 15*B*x^2) + 2*a^2*b*c
^3*(11*A + 25*B*x^2) + 4*a^2*c^3*(8*a*B - 5*A*c*x^2) + b^4*c*(11*a*B - 2*A*
c*x^2) + a*b^2*c^2*(-39*a*B + 16*A*c*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*
x^4)) + (2*a^3*B*c^2 + b^4*(b*B - A*c)*x^2 + a*b^2*(b^2*B + 4*A*c^2*x^2 - b
*c*(A + 5*B*x^2)) + a^2*c*(-4*b^2*B - 2*A*c^2*x^2 + b*c*(3*A + 5*B*x^2)))/(
(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (2*c*(b^5*B - 10*a*b^3*B*c + 30*a^2*
b*B*c^2 - 12*a^2*A*c^3)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4
*a*c)^(5/2) + B*c*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9 \left(A + Bx^2\right)}{\left(a + bx^2 + cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 1.16, size = 2167, normalized size = 8.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*B*a^2*b^6 + 2*(2*B*b^7*c + 40*A*a^3*c^5 - 2*(50*B*a^3*b + 21*A*a^2*
b^2)*c^4 + (85*B*a^2*b^3 + 12*A*a*b^4)*c^3 - (23*B*a*b^5 + A*b^6)*c^2)*x^6
+ (3*B*b^8 - 8*(16*B*a^4 + A*a^3*b)*c^4 - 6*(2*B*a^3*b^2 + 5*A*a^2*b^3)*c^3
 + 3*(29*B*a^2*b^4 + 4*A*a*b^5)*c^2 - (31*B*a*b^6 + A*b^7)*c)*x^4 - 8*(12*B
*a^5 + 5*A*a^4*b)*c^3 + 2*(54*B*a^4*b^2 + 7*A*a^3*b^3)*c^2 + 2*(3*B*a*b^7 +
 24*A*a^4*c^4 - 2*(62*B*a^4*b + 23*A*a^3*b^2)*c^3 + 7*(17*B*a^3*b^3 + 2*A*a
^2*b^4)*c^2 - (34*B*a^2*b^5 + A*a*b^6)*c)*x^2 - ((B*b^5*c^2 - 10*B*a*b^3*c^
3 + 30*B*a^2*b*c^4 - 12*A*a^2*c^5)*x^8 + B*a^2*b^5 - 10*B*a^3*b^3*c + 30*B*
a^4*b*c^2 - 12*A*a^4*c^3 + 2*(B*b^6*c - 10*B*a*b^4*c^2 + 30*B*a^2*b^2*c^3 -
 12*A*a^2*b*c^4)*x^6 + (B*b^7 - 8*B*a*b^5*c + 10*B*a^2*b^3*c^2 - 24*A*a^3*c
^4 + 12*(5*B*a^3*b - A*a^2*b^2)*c^3)*x^4 + 2*(B*a*b^6 - 10*B*a^2*b^4*c + 30
*B*a^3*b^2*c^2 - 12*A*a^3*b*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*
b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a
)) - (33*B*a^3*b^4 + A*a^2*b^5)*c + (B*a^2*b^6 - 12*B*a^3*b^4*c + 48*B*a^4*
b^2*c^2 - 64*B*a^5*c^3 + (B*b^6*c^2 - 12*B*a*b^4*c^3 + 48*B*a^2*b^2*c^4 - 6
4*B*a^3*c^5)*x^8 + 2*(B*b^7*c - 12*B*a*b^5*c^2 + 48*B*a^2*b^3*c^3 - 64*B*a^
3*b*c^4)*x^6 + (B*b^8 - 10*B*a*b^6*c + 24*B*a^2*b^4*c^2 + 32*B*a^3*b^2*c^3
- 128*B*a^4*c^4)*x^4 + 2*(B*a*b^7 - 12*B*a^2*b^5*c + 48*B*a^3*b^3*c^2 - 64*
B*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 4
8*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*
a^3*c^8)*x^8 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x
^6 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^
7)*x^4 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x^2
), 1/4*(3*B*a^2*b^6 + 2*(2*B*b^7*c + 40*A*a^3*c^5 - 2*(50*B*a^3*b + 21*A*a^
2*b^2)*c^4 + (85*B*a^2*b^3 + 12*A*a*b^4)*c^3 - (23*B*a*b^5 + A*b^6)*c^2)*x^
6 + (3*B*b^8 - 8*(16*B*a^4 + A*a^3*b)*c^4 - 6*(2*B*a^3*b^2 + 5*A*a^2*b^3)*c
^3 + 3*(29*B*a^2*b^4 + 4*A*a*b^5)*c^2 - (31*B*a*b^6 + A*b^7)*c)*x^4 - 8*(12
*B*a^5 + 5*A*a^4*b)*c^3 + 2*(54*B*a^4*b^2 + 7*A*a^3*b^3)*c^2 + 2*(3*B*a*b^7
 + 24*A*a^4*c^4 - 2*(62*B*a^4*b + 23*A*a^3*b^2)*c^3 + 7*(17*B*a^3*b^3 + 2*A
*a^2*b^4)*c^2 - (34*B*a^2*b^5 + A*a*b^6)*c)*x^2 + 2*((B*b^5*c^2 - 10*B*a*b^
3*c^3 + 30*B*a^2*b*c^4 - 12*A*a^2*c^5)*x^8 + B*a^2*b^5 - 10*B*a^3*b^3*c + 3
0*B*a^4*b*c^2 - 12*A*a^4*c^3 + 2*(B*b^6*c - 10*B*a*b^4*c^2 + 30*B*a^2*b^2*c
^3 - 12*A*a^2*b*c^4)*x^6 + (B*b^7 - 8*B*a*b^5*c + 10*B*a^2*b^3*c^2 - 24*A*a
^3*c^4 + 12*(5*B*a^3*b - A*a^2*b^2)*c^3)*x^4 + 2*(B*a*b^6 - 10*B*a^2*b^4*c
+ 30*B*a^3*b^2*c^2 - 12*A*a^3*b*c^3)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x
^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (33*B*a^3*b^4 + A*a^2*b^5)*c +
(B*a^2*b^6 - 12*B*a^3*b^4*c + 48*B*a^4*b^2*c^2 - 64*B*a^5*c^3 + (B*b^6*c^2
- 12*B*a*b^4*c^3 + 48*B*a^2*b^2*c^4 - 64*B*a^3*c^5)*x^8 + 2*(B*b^7*c - 12*B
*a*b^5*c^2 + 48*B*a^2*b^3*c^3 - 64*B*a^3*b*c^4)*x^6 + (B*b^8 - 10*B*a*b^6*c
 + 24*B*a^2*b^4*c^2 + 32*B*a^3*b^2*c^3 - 128*B*a^4*c^4)*x^4 + 2*(B*a*b^7 -
12*B*a^2*b^5*c + 48*B*a^3*b^3*c^2 - 64*B*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2
+ a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^
```

```
5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^8 + 2*(b^7*c^4 - 12*a*b^5
*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x^6 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^
2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^7)*x^4 + 2*(a*b^7*c^3 - 12*a^2*b^5*c
^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x^2)]
```

giac [A] time = 6.31, size = 466, normalized size = 1.83

$$-\frac{\left(Bb^{5}-10\,Bab^{3}c+30\,Ba^{2}bc^{2}-12\,Aa^{2}c^{3}\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\,\left(b^{4}c^{3}-8\,ab^{2}c^{4}+16\,a^{2}c^{5}\right)\sqrt{-b^{2}+4\,ac}}+\frac{B\log\left(cx^{4}+bx^{2}+a\right)}{4\,c^{3}}-\frac{3\,Bb^{4}c^{2}x^{8}-24\,Bab^{2}c^{3}x^{8}+48\,Ba^{2}c^{4}x^{8}-2\,Bb^{5}cx^{6}+12\,Bab^{3}c^{2}x^{6}+4\,Ab^{4}c^{2}x^{6}-4\,Ba^{2}bc^{3}x^{6}-32\,Aab^{2}c^{3}x^{6}+40\,Aa^{2}c^{4}x^{6}-3\,Bb^{6}x^{4}+20\,Bab^{4}cx^{4}+2\,Ab^{5}cx^{4}-22\,Ba^{2}b^{2}c^{2}x^{4}-16\,Aab^{3}c^{2}x^{4}+32\,Ba^{3}c^{3}x^{4}-4\,Aa^{2}bc^{3}x^{4}-6\,Bab^{5}x^{2}+40\,Ba^{2}b^{3}cx^{2}+4\,Aab^{4}cx^{2}-28\,Ba^{3}bc^{2}x^{2}-40\,Aa^{2}b^{2}c^{2}x^{2}+24\,Aa^{3}c^{3}x^{2}-3\,Ba^{2}b^{4}+18\,Ba^{3}b^{2}c+2\,Aa^{2}b^{3}c-20\,Aa^{3}bc^{2}}{8\,\left(b^{4}c^{3}-8\,ab^{2}c^{4}+16\,a^{2}c^{5}\right)\left(cx^{4}+bx^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(B*b^5 - 10*B*a*b^3*c + 30*B*a^2*b*c^2 - 12*A*a^2*c^3)*arctan((2*c*x^2
 + b)/sqrt(-b^2 + 4*a*c))/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(-b^2 +
 4*a*c)) + 1/4*B*log(c*x^4 + b*x^2 + a)/c^3 - 1/8*(3*B*b^4*c^2*x^8 - 24*B*a
*b^2*c^3*x^8 + 48*B*a^2*c^4*x^8 - 2*B*b^5*c*x^6 + 12*B*a*b^3*c^2*x^6 + 4*A*
b^4*c^2*x^6 - 4*B*a^2*b*c^3*x^6 - 32*A*a*b^2*c^3*x^6 + 40*A*a^2*c^4*x^6 - 3
*B*b^6*x^4 + 20*B*a*b^4*c*x^4 + 2*A*b^5*c*x^4 - 22*B*a^2*b^2*c^2*x^4 - 16*A
*a*b^3*c^2*x^4 + 32*B*a^3*c^3*x^4 - 4*A*a^2*b*c^3*x^4 - 6*B*a*b^5*x^2 + 40*
B*a^2*b^3*c*x^2 + 4*A*a*b^4*c*x^2 - 28*B*a^3*b*c^2*x^2 - 40*A*a^2*b^2*c^2*x
^2 + 24*A*a^3*c^3*x^2 - 3*B*a^2*b^4 + 18*B*a^3*b^2*c + 2*A*a^2*b^3*c - 20*A
*a^3*b*c^2)/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*(c*x^4 + b*x^2 + a)^2)
```

maple [B] time = 0.03, size = 723, normalized size = 2.85

$$\frac{6A\,a^{2}\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}}-\frac{15B\,a^{2}b\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}\,c}+\frac{5Ba\,b^{3}\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}\,c^{2}}-\frac{B\,b^{5}\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}\,c^{3}}+\frac{4B\,a^{2}\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c}-\frac{2Ba\,b^{2}\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c^{2}}+\frac{B\,b^{4}\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{4\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c^{3}}+\frac{-\frac{\left(10A\,a^{2}c^{3}-8Aa\,b^{2}c^{2}+A\,b^{4}c-25B\,a^{2}b\,c^{2}+15Ba\,b^{3}c-2B\,b^{5}\right)x^{6}}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c^{2}}+\frac{\left(2A\,a^{2}b\,c^{3}+8Aa\,b^{3}c^{2}-A\,b^{5}c+32B\,a^{3}c^{3}+11B\,a^{2}b^{2}c^{2}-19Ba\,b^{4}c+3B\,b^{6}\right)x^{4}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c^{3}}-\frac{\left(6A\,a^{2}c^{3}-10Aa\,b^{2}c^{2}+A\,b^{4}c-31B\,a^{2}b\,c^{2}+22Ba\,b^{3}c-3B\,b^{5}\right)a\,x^{2}}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c^{3}}+\frac{\left(10aAb\,c^{2}-A\,b^{3}c+24a^{2}B\,c^{2}-21a\,b^{2}Bc+3b^{4}B\right)a^{2}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c^{3}}}{2\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(-1/c^2*(10*A*a^2*c^3-8*A*a*b^2*c^2+A*b^4*c-25*B*a^2*b*c^2+15*B*a*b^3*c
-2*B*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2*(2*A*a^2*b*c^3+8*A*a*b^3*c^2-A
*b^5*c+32*B*a^3*c^3+11*B*a^2*b^2*c^2-19*B*a*b^4*c+3*B*b^6)/c^3/(16*a^2*c^2-
8*a*b^2*c+b^4)*x^4-a*(6*A*a^2*c^3-10*A*a*b^2*c^2+A*b^4*c-31*B*a^2*b*c^2+22*
B*a*b^3*c-3*B*b^5)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+1/2*a^2*(10*A*a*b*c^2
-A*b^3*c+24*B*a^2*c^2-21*B*a*b^2*c+3*B*b^4)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4))
/(c*x^4+b*x^2+a)^2+4/c/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*a^2*B-2
/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*a*b^2*B+1/4/c^3/(16*a^2*c
^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*b^4*B+6/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a
*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*a^2-15/c/(16*a^2*c^2-
8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*
b*B+5/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(
4*a*c-b^2)^(1/2))*B*a*b^3-1/2/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1
/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^5*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 5.15, size = 3062, normalized size = 12.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^4*(3*B*b^6 + 32*B*a^3*c^3 - A*b^5*c - 19*B*a*b^4*c + 8*A*a*b^3*c^2 + 2*
A*a^2*b*c^3 + 11*B*a^2*b^2*c^2))/(4*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (
x^6*(2*B*b^5 - 10*A*a^2*c^3 - A*b^4*c - 15*B*a*b^3*c + 8*A*a*b^2*c^2 + 25*B
*a^2*b*c^2))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*(24*B*a^3*c^2 + 3*
B*a*b^4 - A*a*b^3*c + 10*A*a^2*b*c^2 - 21*B*a^2*b^2*c))/(4*c^3*(b^4 + 16*a^
2*c^2 - 8*a*b^2*c)) - (x^2*(6*A*a^3*c^3 - 3*B*a*b^5 + A*a*b^4*c + 22*B*a^2*
b^3*c - 31*B*a^3*b*c^2 - 10*A*a^2*b^2*c^2))/(2*c^3*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - (log
(((B^2*a)/c^4 - ((B + c^3*(-(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2
*b*c^2)^2/(c^6*(4*a*c - b^2)^5))^(1/2))*((8*B*a)/c - (2*(B + c^3*(-(B*b^5 -
 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2)^2/(c^6*(4*a*c - b^2)^5))^(1/
2))*(2*a + b*x^2))/c + (2*x^2*(3*B*b^5 - 12*A*a^2*c^3 - 26*B*a*b^3*c + 62*B
*a^2*b*c^2))/(c*(4*a*c - b^2)^2)))/(4*c^3) + (B*x^2*(B*b^5 - 6*A*a^2*c^3 -
9*B*a*b^3*c + 23*B*a^2*b*c^2))/(c^4*(4*a*c - b^2)^2))*((B^2*a)/c^4 - ((B -
c^3*(-(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2)^2/(c^6*(4*a*c
- b^2)^5))^(1/2))*((8*B*a)/c - (2*(B - c^3*(-(B*b^5 - 12*A*a^2*c^3 - 10*B*a
*b^3*c + 30*B*a^2*b*c^2)^2/(c^6*(4*a*c - b^2)^5))^(1/2))*(2*a + b*x^2))/c +
 (2*x^2*(3*B*b^5 - 12*A*a^2*c^3 - 26*B*a*b^3*c + 62*B*a^2*b*c^2))/(c*(4*a*c
 - b^2)^2)))/(4*c^3) + (B*x^2*(B*b^5 - 6*A*a^2*c^3 - 9*B*a*b^3*c + 23*B*a^2
*b*c^2))/(c^4*(4*a*c - b^2)^2)))*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c
+ 320*B*a^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))/(2*(4096*a^
5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 51
20*a^4*b^2*c^7)) + (atan(((32*a^2*c^6*(4*a*c - b^2)^5 + 2*b^4*c^4*(4*a*c -
b^2)^5 - 16*a*b^2*c^5*(4*a*c - b^2)^5)*(x^2*(((((24*A*a^2*c^6 - 6*B*b^5*c^3
 + 52*B*a*b^3*c^4 - 124*B*a^2*b*c^5)/(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5) -
 ((8*b^5*c^6 - 64*a*b^3*c^7 + 128*a^2*b*c^8)*(2*B*b^10 - 2048*B*a^5*c^5 - 4
0*B*a*b^8*c + 320*B*a^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))
/(2*(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5)*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*
b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)))*(B*b^5 -
 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2))/(8*c^3*(4*a*c - b^2)^(5/2))
 - ((8*b^5*c^6 - 64*a*b^3*c^7 + 128*a^2*b*c^8)*(B*b^5 - 12*A*a^2*c^3 - 10*B
*a*b^3*c + 30*B*a^2*b*c^2)*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*
B*a^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))/(16*c^3*(4*a*c -
b^2)^(5/2)*(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5)*(4096*a^5*c^8 - 4*b^10*c^3
+ 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)))/(
a*(4*a*c - b^2)^2) - (b*((((24*A*a^2*c^6 - 6*B*b^5*c^3 + 52*B*a*b^3*c^4 - 1
24*B*a^2*b*c^5)/(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5) - ((8*b^5*c^6 - 64*a*b
^3*c^7 + 128*a^2*b*c^8)*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*B*a
^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))/(2*(16*a^2*c^6 + b^4
*c^4 - 8*a*b^2*c^5)*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6
*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)))*(2*B*b^10 - 2048*B*a^5*c^5 -
40*B*a*b^8*c + 320*B*a^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4)
)/(2*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3
*b^4*c^6 - 5120*a^4*b^2*c^7)) - (B^2*b^5 - 6*A*B*a^2*c^3 - 9*B^2*a*b^3*c +
23*B^2*a^2*b*c^2)/(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5) + (((b^5*c^6)/2 - 4*
a*b^3*c^7 + 8*a^2*b*c^8)*(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*
c^2)^2)/(c^6*(4*a*c - b^2)^5*(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5))))/(2*a*(
4*a*c - b^2)^(5/2))) - ((((8*B*a)/c + (8*a*c^2*(2*B*b^10 - 2048*B*a^5*c^5 -
 40*B*a*b^8*c + 320*B*a^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4
))/(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b
^4*c^6 - 5120*a^4*b^2*c^7))*(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2
*b*c^2))/(8*c^3*(4*a*c - b^2)^(5/2)) + (a*(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^
3*c + 30*B*a^2*b*c^2)*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*B*a^2
*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))/(c*(4*a*c - b^2)^(5/2)
*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4
*c^6 - 5120*a^4*b^2*c^7)))/(a*(4*a*c - b^2)^2) + (b*((B^2*a)/c^4 + (((8*B*a
```

```
)/c + (8*a*c^2*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*B*a^2*b^6*c^
2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))/(4096*a^5*c^8 - 4*b^10*c^3 +
80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7))*(2*B
*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*B*a^2*b^6*c^2 - 1280*B*a^3*b^4*
c^3 + 2560*B*a^4*b^2*c^4))/(2*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 6
40*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)) - (a*(B*b^5 - 12*A*a
^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2)^2)/(c^4*(4*a*c - b^2)^5)))/(2*a*(4*
a*c - b^2)^(5/2))))/(B^2*b^10 + 144*A^2*a^4*c^6 + 160*B^2*a^2*b^6*c^2 - 600
*B^2*a^3*b^4*c^3 + 900*B^2*a^4*b^2*c^4 - 20*B^2*a*b^8*c - 24*A*B*a^2*b^5*c^
3 + 240*A*B*a^3*b^3*c^4 - 720*A*B*a^4*b*c^5))*(B*b^5 - 12*A*a^2*c^3 - 10*B*
a*b^3*c + 30*B*a^2*b*c^2))/(2*c^3*(4*a*c - b^2)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.104 $\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=146

$$\frac{3a(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{3x^2\left(2a+bx^2\right)(Ab-2aB)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^6\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Rubi [A] time = 0.14, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1251, 804, 722, 618, 206}

$$-\frac{x^6\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3x^2\left(2a+bx^2\right)(Ab-2aB)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3a(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x^6*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)^2) + (3*(A*b - 2*a*B)*x^2*(2*a + b*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 +
 c*x^4)) + (3*a*(A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^
2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 804

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*
(b*f - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(m*(b*(
e*f + d*g) - 2*(c*d*f + a*e*g)))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m
- 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0] && LtQ[p, -1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3(A+Bx)}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{x^6\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{(3(Ab-2aB))\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4\left(b^2-4ac\right)}$$

$$= -\frac{x^6\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3(Ab-2aB)x^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{(3a(Ab-2aB))\,\text{Sub}}{2\left(b^2\right.}$$

$$= -\frac{x^6\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3(Ab-2aB)x^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{(3a(Ab-2aB))\,\text{Sub}}{(}$$

$$= -\frac{x^6\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3(Ab-2aB)x^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3a(Ab-2aB)\tanh^{-}}{\left(b^2-4ac\right)}$$

Mathematica [A] time = 0.27, size = 261, normalized size = 1.79

$$\frac{1}{4}\left(\frac{a^2c\left(2c\left(A+Bx^2\right)-3bB\right)+ab\left(-bc\left(A+4Bx^2\right)+3Ac^2x^2+b^2B\right)+b^3x^2(bB-Ac)}{c^3\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)^2}+\frac{-4a^2c^3\left(4A+5Bx^2\right)+ab^2c^2\left(5A+16Bx^2\right)+2abc^2\left(11aB-3Acx^2\right)-8ab^3Bc-b^4c\left(A+2Bx^2\right)+b^5B}{c^3\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{12a(Ab-2aB)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b^5*B - 8*a*b^3*B*c - b^4*c*(A + 2*B*x^2) - 4*a^2*c^3*(4*A + 5*B*x^2) + a
*b^2*c^2*(5*A + 16*B*x^2) + 2*a*b*c^2*(11*a*B - 3*A*c*x^2))/(c^3*(b^2 - 4*a
*c)^2*(a + b*x^2 + c*x^4)) + (b^3*(b*B - A*c)*x^2 + a^2*c*(-3*b*B + 2*c*(A
+ B*x^2)) + a*b*(b^2*B + 3*A*c^2*x^2 - b*c*(A + 4*B*x^2)))/(c^3*(-b^2 + 4*a
*c)*(a + b*x^2 + c*x^4)^2) - (12*a*(A*b - 2*a*B)*ArcTan[(b + 2*c*x^2)/Sqrt[
-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 0.95, size = 1378, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(B*a^2*b^5 - 32*A*a^4*c^3 + 2*(B*b^6*c - 12*B*a*b^4*c^2 - 4*(10*B*a^3
 + 3*A*a^2*b)*c^4 + 3*(14*B*a^2*b^2 + A*a*b^3)*c^3)*x^6 + (B*b^7 - 64*A*a^3
*c^4 + 4*(2*B*a^3*b + 3*A*a^2*b^2)*c^3 + 3*(10*B*a^2*b^3 - A*a*b^4)*c^2 - (
12*B*a*b^5 - A*b^6)*c)*x^4 + 4*(10*B*a^4*b + A*a^3*b^2)*c^2 + 2*(B*a*b^6 -
4*(6*B*a^4 + 5*A*a^3*b)*c^3 + (46*B*a^3*b^2 + A*a^2*b^3)*c^2 - (14*B*a^2*b^
4 - A*a*b^5)*c)*x^2 + 6*((2*B*a^2 - A*a*b)*c^4*x^8 + 2*(2*B*a^2*b - A*a*b^2
)*c^3*x^6 + 2*(2*B*a^3*b - A*a^2*b^2)*c^2*x^2 + (2*(2*B*a^3 - A*a^2*b)*c^3
+ (2*B*a^2*b^2 - A*a*b^3)*c^2)*x^4 + (2*B*a^4 - A*a^3*b)*c^2)*sqrt(b^2 - 4*
a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4 + b*x^2 + a)) - (14*B*a^3*b^3 - A*a^2*b^4)*c)/(a^2*b^6*c^2 - 1
2*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*
a^2*b^2*c^6 - 64*a^3*c^7)*x^8 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5
- 64*a^3*b*c^6)*x^6 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2
*c^5 - 128*a^4*c^6)*x^4 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 -
64*a^4*b*c^5)*x^2), -1/4*(B*a^2*b^5 - 32*A*a^4*c^3 + 2*(B*b^6*c - 12*B*a*b^
4*c^2 - 4*(10*B*a^3 + 3*A*a^2*b)*c^4 + 3*(14*B*a^2*b^2 + A*a*b^3)*c^3)*x^6
+ (B*b^7 - 64*A*a^3*c^4 + 4*(2*B*a^3*b + 3*A*a^2*b^2)*c^3 + 3*(10*B*a^2*b^3
 - A*a*b^4)*c^2 - (12*B*a*b^5 - A*b^6)*c)*x^4 + 4*(10*B*a^4*b + A*a^3*b^2)*
c^2 + 2*(B*a*b^6 - 4*(6*B*a^4 + 5*A*a^3*b)*c^3 + (46*B*a^3*b^2 + A*a^2*b^3)
*c^2 - (14*B*a^2*b^4 - A*a*b^5)*c)*x^2 + 12*((2*B*a^2 - A*a*b)*c^4*x^8 + 2*
(2*B*a^2*b - A*a*b^2)*c^3*x^6 + 2*(2*B*a^3*b - A*a^2*b^2)*c^2*x^2 + (2*(2*B
*a^3 - A*a^2*b)*c^3 + (2*B*a^2*b^2 - A*a*b^3)*c^2)*x^4 + (2*B*a^4 - A*a^3*b
)*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4
*a*c)) - (14*B*a^3*b^3 - A*a^2*b^4)*c)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a
^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3
*c^7)*x^8 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x^6
+ (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 128*a^4*c^6)*
x^4 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b*c^5)*x^2)]
```

giac [B] time = 6.46, size = 318, normalized size = 2.18

$$\frac{3\left(2\,Ba^2 - Aab\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4 - 8\,ab^2c + 16\,a^2c^2\right)\sqrt{-b^2 + 4\,ac}} - \frac{2\,Bb^4cx^6 - 16\,Bab^2c^2x^6 + 20\,Ba^2c^3x^6 + 6\,Aabc^3x^6 + Bb^5x^4 - 8\,Bab^3cx^4 + Ab^4cx^4 - 2\,Ba^2bc^2x^4 + Aab^2c^2x^4 + 16\,Aa^2c^3x^4 + 2\,Bab^4x^2 - 20\,Ba^2b^2cx^2 + 2\,Aab^3cx^2 + 12\,Ba^3c^2x^2 + 10\,Aa^2bc^2x^2 + Ba^2b^3 - 10\,Ba^3bc + Aa^2b^2c + 8\,Aa^3c^2}{4\left(b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4\right)\left(cx^4 + bx^2 + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3*(2*B*a^2 - A*a*b)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^
2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(2*B*b^4*c*x^6 - 16*B*a*b^2*c^2
*x^6 + 20*B*a^2*c^3*x^6 + 6*A*a*b*c^3*x^6 + B*b^5*x^4 - 8*B*a*b^3*c*x^4 + A
*b^4*c*x^4 - 2*B*a^2*b*c^2*x^4 + A*a*b^2*c^2*x^4 + 16*A*a^2*c^3*x^4 + 2*B*a
*b^4*x^2 - 20*B*a^2*b^2*c*x^2 + 2*A*a*b^3*c*x^2 + 12*B*a^3*c^2*x^2 + 10*A*a
^2*b*c^2*x^2 + B*a^2*b^3 - 10*B*a^3*b*c + A*a^2*b^2*c + 8*A*a^3*c^2)/((b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*(c*x^4 + b*x^2 + a)^2)
```

maple [B] time = 0.02, size = 398, normalized size = 2.73

$$-\frac{3Aab\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2 - 8ab^2c + b^4\right)\sqrt{4ac - b^2}} + \frac{6Ba^2\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2 - 8ab^2c + b^4\right)\sqrt{4ac - b^2}} + \frac{-\frac{\left(3aAbc^2+10a^2Bc^2-8ab^2Bc+b^4B\right)x^6}{\left(16a^2c^2-8ab^2c+b^4\right)c} - \frac{\left(16Aa^2c^3+Aab^2c^2+Ab^4c-2Ba^2bc^2-8Bab^3c+Bb^5\right)x^4}{2\left(16a^2c^2-8ab^2c+b^4\right)c^2} - \frac{\left(5aAbc^2+Ab^3c+6a^2Bc^2-10ab^2Bc+b^4B\right)ax^2}{\left(16a^2c^2-8ab^2c+b^4\right)c^2} - \frac{\left(8aAc^2+Ab^2c-10abBc+b^3B\right)a^2}{2\left(16a^2c^2-8ab^2c+b^4\right)c^2}}{2\left(cx^4 + bx^2 + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(-(3*A*a*b*c^2+10*B*a^2*c^2-8*B*a*b^2*c+B*b^4)/c/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^6-1/2*(16*A*a^2*c^3+A*a*b^2*c^2+A*b^4*c-2*B*a^2*b*c^2-8*B*a*b^3*c+B*
b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x^4-a*(5*A*a*b*c^2+A*b^3*c+6*B*a^2*c^2-
10*B*a*b^2*c+B*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x^2-1/2*a^2/c^2*(8*A*a*c
^2+A*b^2*c-10*B*a*b*c+B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2-
3*a/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-
```

```
b^2)^(1/2))*A*b+6*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.69, size = 593, normalized size = 4.06

$$\frac{3\,a\,\mathrm{atan}\left(\frac{\left(x^2\left(\frac{3\,(A\,b-2\,B\,a)\left(6\,B\,a^2\,c^2-3\,A\,a\,b\,c^2\right)}{\left(4\,a\,c-b^2\right)^{9/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}-\frac{9\,a\,b\,(A\,b-2\,B\,a)^2\left(32\,a^2\,b\,c^4-16\,a\,b^3\,c^3+2\,b^5\,c^2\right)}{2\left(4\,a\,c-b^2\right)^{15/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)-\frac{18\,a^2\,b\,c^2\,(A\,b-2\,B\,a)^2}{\left(4\,a\,c-b^2\right)^{15/2}}\right)\left(b^4\left(4\,a\,c-b^2\right)^5+16\,a^2\,c^2\left(4\,a\,c-b^2\right)^5-8\,a\,b^2\,c\left(4\,a\,c-b^2\right)^5\right)}{18\,A^2\,a^2\,b^2\,c^2-72\,A\,B\,a^3\,b\,c^2+72\,B^2\,a^4\,c^2}\right)(A\,b-2\,B\,a)}{\left(4\,a\,c-b^2\right)^{5/2}}-\frac{\frac{x^4\left(-2\,B\,a^2\,b\,c^2+16\,A\,a^2\,c^3-8\,B\,a\,b^3\,c+A\,a\,b^2\,c^2+B\,b^5+A\,b^4\,c\right)}{4\,c^2\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a^2\left(B\,b^3+A\,b^2\,c-10\,B\,a\,b\,c+8\,A\,a\,c^2\right)}{4\,c^2\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x^6\left(10\,B\,a^2\,c^2-8\,B\,a\,b^2\,c+3\,A\,a\,b\,c^2+B\,b^4\right)}{2\,c\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\,x^2\left(6\,B\,a^2\,c^2-10\,B\,a\,b^2\,c+5\,A\,a\,b\,c^2+B\,b^4+A\,b^3\,c\right)}{2\,c^2\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^4\left(b^2+2\,a\,c\right)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (3*a*atan(((x^2*((3*(A*b - 2*B*a)*(6*B*a^2*c^2 - 3*A*a*b*c^2))/((4*a*c - b^
2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (9*a*b*(A*b - 2*B*a)^2*(2*b^5*c^
2 - 16*a*b^3*c^3 + 32*a^2*b*c^4))/(2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c))) - (18*a^2*b*c^2*(A*b - 2*B*a)^2)/(4*a*c - b^2)^(15/2))*(b^4
*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))
/(72*B^2*a^4*c^2 + 18*A^2*a^2*b^2*c^2 - 72*A*B*a^3*b*c^2))*(A*b - 2*B*a))/(
4*a*c - b^2)^(5/2) - ((x^4*(B*b^5 + 16*A*a^2*c^3 + A*b^4*c - 8*B*a*b^3*c +
A*a*b^2*c^2 - 2*B*a^2*b*c^2))/(4*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2
*(B*b^3 + 8*A*a*c^2 + A*b^2*c - 10*B*a*b*c))/(4*c^2*(b^4 + 16*a^2*c^2 - 8*a
*b^2*c)) + (x^6*(B*b^4 + 10*B*a^2*c^2 + 3*A*a*b*c^2 - 8*B*a*b^2*c))/(2*c*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x^2*(B*b^4 + 6*B*a^2*c^2 + A*b^3*c + 5*A
*a*b*c^2 - 10*B*a*b^2*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a
*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

sympy [B] time = 102.04, size = 775, normalized size = 5.31

$$-\frac{3a\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\,(-Ab+2Ba)\log\left(x^2+\frac{-3Aab^2+6Ba^2b-192a^4c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}(-Ab+2Ba)+144a^3b^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}(-Ab+2Ba)-36a^2b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}(-Ab+2Ba)+3ab^6\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}(-Ab+2Ba)}{-6Aabc+12Ba^2c}\right)}{2}+\frac{3a\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\,(-Ab+2Ba)\log\left(x^2+\frac{-3Aab^2+6Ba^2b+192a^4c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}(-Ab+2Ba)-144a^3b^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}(-Ab+2Ba)+36a^2b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}(-Ab+2Ba)-3ab^6\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}(-Ab+2Ba)}{-6Aabc+12Ba^2c}\right)}{2}+\frac{-8Aa^3c^2-Aa^2b^2c+10Ba^3bc-Ba^2b^3+x^6\left(-6Aabc^3-20Ba^2c^3+16Bab^2c^2-2Bb^4c\right)+x^4\left(-16Aa^2c^3-Aab^2c^2-Ab^4c+2Ba^2bc^2+8Bab^3c-Bb^5\right)+x^2\left(-10Aa^2bc^2-2Aab^3c-12Ba^3c^2+20Ba^2b^2c-2Bab^4\right)}{64a^4c^4-32a^3b^2c^3+4a^2b^4c^2+x^8\left(64a^2c^6-32ab^2c^5+4b^4c^4\right)+x^6\left(128a^2bc^5-64ab^3c^4+8b^5c^3\right)+x^4\left(128a^3c^5-24ab^4c^3+4b^6c^2\right)+x^2\left(128a^3bc^4-64a^2b^3c^3+8ab^5c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -3*a*sqrt(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a)*log(x**2 + (-3*A*a*b**2 + 6*
B*a**2*b - 192*a**4*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a) + 144*a*
*3*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a) - 36*a**2*b**4*c*sqr
t(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a) + 3*a*b**6*sqrt(-1/(4*a*c - b**2)**5
)*(-A*b + 2*B*a))/(-6*A*a*b*c + 12*B*a**2*c))/2 + 3*a*sqrt(-1/(4*a*c - b**2
)**5)*(-A*b + 2*B*a)*log(x**2 + (-3*A*a*b**2 + 6*B*a**2*b + 192*a**4*c**3*s
qrt(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a) - 144*a**3*b**2*c**2*sqrt(-1/(4*a*
c - b**2)**5)*(-A*b + 2*B*a) + 36*a**2*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(-
A*b + 2*B*a) - 3*a*b**6*sqrt(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a))/(-6*A*a*
b*c + 12*B*a**2*c))/2 + (-8*A*a**3*c**2 - A*a**2*b**2*c + 10*B*a**3*b*c - B
*a**2*b**3 + x**6*(-6*A*a*b*c**3 - 20*B*a**2*c**3 + 16*B*a*b**2*c**2 - 2*B*
b**4*c) + x**4*(-16*A*a**2*c**3 - A*a*b**2*c**2 - A*b**4*c + 2*B*a**2*b*c**
2 + 8*B*a*b**3*c - B*b**5) + x**2*(-10*A*a**2*b*c**2 - 2*A*a*b**3*c - 12*B*
a**3*c**2 + 20*B*a**2*b**2*c - 2*B*a*b**4))/(64*a**4*c**4 - 32*a**3*b**2*c*
*3 + 4*a**2*b**4*c**2 + x**8*(64*a**2*c**6 - 32*a*b**2*c**5 + 4*b**4*c**4)
```

```
+ x**6*(128*a**2*b*c**5 - 64*a*b**3*c**4 + 8*b**5*c**3) + x**4*(128*a**3*c*
*5 - 24*a*b**4*c**3 + 4*b**6*c**2) + x**2*(128*a**3*b*c**4 - 64*a**2*b**3*c
**3 + 8*a*b**5*c**2))
```

3.105 $\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$

Optimal. Leaf size=185

$$\frac{\left(3abB - A\left(2ac + b^2\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2 - 4ac\right)^{5/2}} - \frac{x^4\left(-2aB - \left(x^2(bB - 2Ac)\right) + Ab\right)}{4\left(b^2 - 4ac\right)\left(a + bx^2 + cx^4\right)^2} - \frac{a\left(8aBc - 6Abc + b^2B\right) + x^2\left(4aAc\right.}{4c\left(b^2 - 4ac\right)^2\left(a + \right.}$$

Rubi [A] time = 0.26, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{1251, 820, 777, 618, 206\}$

$$-\frac{x^2\left(4aAc^2 + 2abBc - 4Ab^2c + b^3B\right) + a\left(8aBc - 6Abc + b^2B\right)}{4c\left(b^2 - 4ac\right)^2\left(a + bx^2 + cx^4\right)} - \frac{x^4\left(-2aB + x^2(-(bB - 2Ac)) + Ab\right)}{4\left(b^2 - 4ac\right)\left(a + bx^2 + cx^4\right)^2} + \frac{\left(3abB - A\left(2ac + b^2\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2 - 4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x^4*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)^2) - (a*(b^2*B - 6*A*b*c + 8*a*B*c) + (b^3*B - 4*A*b^2*c + 2*a*b*B*c + 4
*a*A*c^2)*x^2)/(4*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((3*a*b*B - A*(b
^2 + 2*a*c))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*
(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p +
 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g*
(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(
m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (I
ntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^5\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{\left(a+bx+cx^2\right)^3}\,dx,x,x^2\right)$$

$$=-\frac{x^4\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\text{Subst}\left(\int \frac{x(-2(Ab-2aB)-(bB-2Ac)x)}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)}{4\left(b^2-4ac\right)}$$

$$=-\frac{x^4\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{a\left(b^2B-6Abc+8aBc\right)+\left(b^3B-4Ab^2c+2abBc\right.}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$=-\frac{x^4\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{a\left(b^2B-6Abc+8aBc\right)+\left(b^3B-4Ab^2c+2abBc\right.}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$=-\frac{x^4\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{a\left(b^2B-6Abc+8aBc\right)+\left(b^3B-4Ab^2c+2abBc\right.}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Mathematica [A] time = 0.24, size = 233, normalized size = 1.26

$$\frac{1}{4}\left(\frac{2a^2Bc+a\left(bc\left(A+3Bx^2\right)-2Ac^2x^2+b^2(-B)\right)+b^2x^2(Ac-bB)}{c^2\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)^2}+\frac{4\left(A\left(2ac+b^2\right)-3abB\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}+\frac{b^2c\left(5aB+2Acx^2\right)+2abc^2\left(A-3Bx^2\right)+4ac^2\left(Acx^2-4aB\right)+Ab^3c+b^4(-B)}{c^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-(b^4*B) + A*b^3*c + 2*a*b*c^2*(A - 3*B*x^2) + 4*a*c^2*(-4*a*B + A*c*x^2)
 + b^2*c*(5*a*B + 2*A*c*x^2))/(c^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (
2*a^2*B*c + b^2*(-(b*B) + A*c)*x^2 + a*(-(b^2*B) - 2*A*c^2*x^2 + b*c*(A + 3
*B*x^2)))/(c^2*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (4*(-3*a*b*B + A*(b^
2 + 2*a*c))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))
/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 0.95, size = 1369, normalized size = 7.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(B*a^2*b^4 + 2*(8*A*a^2*c^4 - 2*(6*B*a^2*b - A*a*b^2)*c^3 + (3*B*a*b^
3 - A*b^4)*c^2)*x^6 + (B*b^6 - 8*(8*B*a^3 - 3*A*a^2*b)*c^3 + 6*(2*B*a^2*b^2
 + A*a*b^3)*c^2 - 3*(B*a*b^4 + A*b^5)*c)*x^4 - 8*(4*B*a^4 - 3*A*a^3*b)*c^2
+ 2*(B*a*b^5 - 8*A*a^3*c^3 - 2*(10*B*a^3*b - 11*A*a^2*b^2)*c^2 + (B*a^2*b^3
 - 5*A*a*b^4)*c)*x^2 - 2*((2*A*a*c^4 - (3*B*a*b - A*b^2)*c^3)*x^8 + 2*(2*A*
a*b*c^3 - (3*B*a*b^2 - A*b^3)*c^2)*x^6 + 2*A*a^3*c^2 + (4*A*a^2*c^3 - 2*(3*
B*a^2*b - 2*A*a*b^2)*c^2 - (3*B*a*b^3 - A*b^4)*c)*x^4 + 2*(2*A*a^2*b*c^2 -
(3*B*a^2*b^2 - A*a*b^3)*c)*x^2 - (3*B*a^3*b - A*a^2*b^2)*c)*sqrt(b^2 - 4*a*
c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*
c))/(c*x^4 + b*x^2 + a)) + 2*(2*B*a^3*b^2 - 3*A*a^2*b^3)*c)/(a^2*b^6*c - 12
*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a
^2*b^2*c^5 - 64*a^3*c^6)*x^8 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 -
 64*a^3*b*c^5)*x^6 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 + 32*a^3*b^2*c^
4 - 128*a^4*c^5)*x^4 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^3*c^3 - 64*a^
4*b*c^4)*x^2), -1/4*(B*a^2*b^4 + 2*(8*A*a^2*c^4 - 2*(6*B*a^2*b - A*a*b^2)*c
^3 + (3*B*a*b^3 - A*b^4)*c^2)*x^6 + (B*b^6 - 8*(8*B*a^3 - 3*A*a^2*b)*c^3 +
6*(2*B*a^2*b^2 + A*a*b^3)*c^2 - 3*(B*a*b^4 + A*b^5)*c)*x^4 - 8*(4*B*a^4 - 3
*A*a^3*b)*c^2 + 2*(B*a*b^5 - 8*A*a^3*c^3 - 2*(10*B*a^3*b - 11*A*a^2*b^2)*c^
2 + (B*a^2*b^3 - 5*A*a*b^4)*c)*x^2 + 4*((2*A*a*c^4 - (3*B*a*b - A*b^2)*c^3)
*x^8 + 2*(2*A*a*b*c^3 - (3*B*a*b^2 - A*b^3)*c^2)*x^6 + 2*A*a^3*c^2 + (4*A*a
^2*c^3 - 2*(3*B*a^2*b - 2*A*a*b^2)*c^2 - (3*B*a*b^3 - A*b^4)*c)*x^4 + 2*(2*
A*a^2*b*c^2 - (3*B*a^2*b^2 - A*a*b^3)*c)*x^2 - (3*B*a^3*b - A*a^2*b^2)*c)*s
qrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) +
 2*(2*B*a^3*b^2 - 3*A*a^2*b^3)*c)/(a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*
c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x
^8 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^6 + (b^8*
c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 + 32*a^3*b^2*c^4 - 128*a^4*c^5)*x^4 + 2*(
a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^3*c^3 - 64*a^4*b*c^4)*x^2)]
```

giac [A] time = 6.59, size = 268, normalized size = 1.45

$$-\frac{\left(3\,Bab - Ab^2 - 2\,Aac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4 - 8\,ab^2c + 16\,a^2c^2\right)\sqrt{-b^2 + 4\,ac}} - \frac{6\,Babc^2x^6 - 2\,Ab^2c^2x^6 - 4\,Aac^3x^6 + Bb^4x^4 + Bab^2cx^4 - 3\,Ab^3cx^4 + 16\,Ba^2c^2x^4 - 6\,Aabc^2x^4 + 2\,Bab^3x^2 + 10\,Ba^2bcx^2 - 10\,Aab^2cx^2 + 4\,Aa^2c^2x^2 + Ba^2b^2 + 8\,Ba^3c - 6\,Aa^2bc}{4\left(b^4c - 8\,ab^2c^2 + 16\,a^2c^3\right)\left(cx^4 + bx^2 + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -(3*B*a*b - A*b^2 - 2*A*a*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4
 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*B*a*b*c^2*x^6 - 2*A
*b^2*c^2*x^6 - 4*A*a*c^3*x^6 + B*b^4*x^4 + B*a*b^2*c*x^4 - 3*A*b^3*c*x^4 +
16*B*a^2*c^2*x^4 - 6*A*a*b*c^2*x^4 + 2*B*a*b^3*x^2 + 10*B*a^2*b*c*x^2 - 10*
A*a*b^2*c*x^2 + 4*A*a^2*c^2*x^2 + B*a^2*b^2 + 8*B*a^3*c - 6*A*a^2*b*c)/((b^
4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*(c*x^4 + b*x^2 + a)^2)
```

maple [B] time = 0.02, size = 411, normalized size = 2.22

$$\frac{2Aac\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2 - 8a\,b^2c + b^4\right)\sqrt{4ac-b^2}} + \frac{A\,b^2\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2 - 8a\,b^2c + b^4\right)\sqrt{4ac-b^2}} - \frac{3Bab\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2 - 8a\,b^2c + b^4\right)\sqrt{4ac-b^2}} + \frac{\frac{\left(2aAc+A\,b^2-3abB\right)c\,x^6}{16a^2c^2-8a\,b^2c+b^4} + \frac{\left(6aAb\,c^2+3A\,b^3c-16a^2B\,c^2-a\,b^2Bc-b^4B\right)x^4}{2\left(16a^2c^2-8a\,b^2c+b^4\right)c} - \frac{\left(2aA\,c^2-5A\,b^2c+5abBc+b^3B\right)a\,x^2}{\left(16a^2c^2-8a\,b^2c+b^4\right)c} + \frac{\left(6Abc-8aBc-b^2B\right)a^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)c}}{2\left(c\,x^4 + b\,x^2 + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(c*(2*A*a*c+A*b^2-3*B*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2*(6*A*a*b*
c^2+3*A*b^3*c-16*B*a^2*c^2-B*a*b^2*c-B*b^4)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^
4-a/c*(2*A*a*c^2-5*A*b^2*c+5*B*a*b*c+B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+
1/2*a^2*(6*A*b*c-8*B*a*c-B*b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+
a)^2+2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a
*c-b^2)^(1/2))*a*A*c+1/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan(
```

```
(2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^2-3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.68, size = 625, normalized size = 3.38

$$\frac{\operatorname{atan}\left(\frac{\left(x^2\left(\frac{(Ab^2c^2-3Babc^2+2Aac^3)(Ab^2-3Bab+2Aac)}{a(4ac-b^2)^{9/2}(16a^2c^2-8ab^2c+b^4)}+\frac{b(Ab^2-3Bab+2Aac)^2(32a^2bc^4-16ab^3c^3+2b^5c^2)}{2a(4ac-b^2)^{15/2}(16a^2c^2-8ab^2c+b^4)}\right)+\frac{2bc^2(Ab^2-3Bab+2Aac)^2}{(4ac-b^2)^{15/2}}\right)\left(b^4(4ac-b^2)^5+16a^2c^2(4ac-b^2)^5-8ab^2c(4ac-b^2)^5\right)}{8A^2a^2c^4+8A^2ab^2c^3+2A^2b^4c^2-24ABa^2bc^3-12ABab^3c^2+18B^2a^2b^2c^2}\right)(Ab^2-3Bab+2Aac)}{(4ac-b^2)^{5/2}}-\frac{\frac{x^4(16Ba^2c^2+Bab^2c-6Aabc^2+Bb^4-3Ab^3c)}{4c(16a^2c^2-8ab^2c+b^4)}-\frac{cx^6(Ab^2-3Bab+2Aac)}{2(16a^2c^2-8ab^2c+b^4)}+\frac{a(8Bca^2+Bab^2-6Acab)}{4c(16a^2c^2-8ab^2c+b^4)}+\frac{x^2(5Ba^2bc+2Aa^2c^2+Bab^3-5Aab^2c)}{2c(16a^2c^2-8ab^2c+b^4)}}{x^4(b^2+2ac)+a^2+c^2x^8+2abx^2+2bcx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (atan(((x^2*(((A*b^2*c^2 + 2*A*a*c^3 - 3*B*a*b*c^2)*(A*b^2 + 2*A*a*c - 3*B*
a*b))/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*(A*b^2 +
2*A*a*c - 3*B*a*b)^2*(2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4))/(2*a*(4*a*c
 - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (2*b*c^2*(A*b^2 + 2*A*a*c
 - 3*B*a*b)^2)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a
*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(8*A^2*a^2*c^4 + 2*A^2*b^4*c^2 +
18*B^2*a^2*b^2*c^2 + 8*A^2*a*b^2*c^3 - 12*A*B*a*b^3*c^2 - 24*A*B*a^2*b*c^3)
)*(A*b^2 + 2*A*a*c - 3*B*a*b))/(4*a*c - b^2)^(5/2) - ((x^4*(B*b^4 + 16*B*a^
2*c^2 - 3*A*b^3*c - 6*A*a*b*c^2 + B*a*b^2*c))/(4*c*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c)) - (c*x^6*(A*b^2 + 2*A*a*c - 3*B*a*b))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^
2*c)) + (a*(B*a*b^2 + 8*B*a^2*c - 6*A*a*b*c))/(4*c*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c)) + (x^2*(2*A*a^2*c^2 + B*a*b^3 - 5*A*a*b^2*c + 5*B*a^2*b*c))/(2*c*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x
^2 + 2*b*c*x^6)
```

sympy [B] time = 44.84, size = 833, normalized size = 4.50

$$\frac{\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Aac-Ab^2+3Bab)\log\left(x^2+\frac{-2Aabc-Ab^3+3Bab^2-64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Aac-Ab^2+3Bab)+48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Aac-Ab^2+3Bab)-12ab^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Aac-Ab^2+3Bab)+b^6\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Aac-Ab^2+3Bab)}{-4Aac^2-2Ab^2c+6Babc}\right)}{2}-\frac{\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Aac-Ab^2+3Bab)\log\left(x^2+\frac{-2Aabc-Ab^3+3Bab^2+64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Aac-Ab^2+3Bab)-48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Aac-Ab^2+3Bab)+12ab^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Aac-Ab^2+3Bab)-b^6\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Aac-Ab^2+3Bab)}{-4Aac^2-2Ab^2c+6Babc}\right)}{2}+\frac{6Aa^2bc-8Ba^3c-Ba^2b^2+x^6(4Aac^3+2Ab^2c^2-6Babc^2)+x^4(6Aabc^2+3Ab^3c-16Ba^2c^2-Bab^2c-Bb^4)+x^2(-4Aa^2c^2+10Aab^2c-10Ba^2bc-2Bab^3)}{64a^4c^3-32a^3b^2c^2+4a^2b^4c+x^8(64a^2c^5-32ab^2c^4+4b^4c^3)+x^6(128a^2bc^4-64ab^3c^3+8b^5c^2)+x^4(128a^3c^4-24ab^4c^2+4b^6c)+x^2(128a^3bc^3-64a^2b^3c^2+8ab^5c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*c - A*b**2 + 3*B*a*b)*log(x**2 + (-2*A*a
*b*c - A*b**3 + 3*B*a*b**2 - 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*
a*c - A*b**2 + 3*B*a*b) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-2*
A*a*c - A*b**2 + 3*B*a*b) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*
c - A*b**2 + 3*B*a*b) + b**6*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*c - A*b**2
+ 3*B*a*b))/(-4*A*a*c**2 - 2*A*b**2*c + 6*B*a*b*c))/2 - sqrt(-1/(4*a*c - b*
*2)**5)*(-2*A*a*c - A*b**2 + 3*B*a*b)*log(x**2 + (-2*A*a*b*c - A*b**3 + 3*B
*a*b**2 + 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*c - A*b**2 + 3*B*
a*b) - 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*c - A*b**2 + 3*
B*a*b) + 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*c - A*b**2 + 3*B*a*
b) - b**6*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*c - A*b**2 + 3*B*a*b))/(-4*A*a
*c**2 - 2*A*b**2*c + 6*B*a*b*c))/2 + (6*A*a**2*b*c - 8*B*a**3*c - B*a**2*b*
*2 + x**6*(4*A*a*c**3 + 2*A*b**2*c**2 - 6*B*a*b*c**2) + x**4*(6*A*a*b*c**2
+ 3*A*b**3*c - 16*B*a**2*c**2 - B*a*b**2*c - B*b**4) + x**2*(-4*A*a**2*c**2
```

```
 + 10*A*a*b**2*c - 10*B*a**2*b*c - 2*B*a*b**3))/(64*a**4*c**3 - 32*a**3*b**
2*c**2 + 4*a**2*b**4*c + x**8*(64*a**2*c**5 - 32*a*b**2*c**4 + 4*b**4*c**3)
 + x**6*(128*a**2*b*c**4 - 64*a*b**3*c**3 + 8*b**5*c**2) + x**4*(128*a**3*c
**4 - 24*a*b**4*c**2 + 4*b**6*c) + x**2*(128*a**3*b*c**3 - 64*a**2*b**3*c**
2 + 8*a*b**5*c))
```

3.106 $\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=170

$$-\frac{(2aBc-3Abc+b^2B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{(b+2cx^2)(2aBc-3Abc+b^2B)}{4c(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{x^2(-2aBc-Abc+b^2B)+a(bB}{4c(b^2-4ac)(a+bx^2+cx^4}$$

Rubi [A] time = 0.16, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1251, 777, 614, 618, 206}

$$\frac{(b+2cx^2)(2aBc-3Abc+b^2B)}{4c(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{x^2(-2aBc-Abc+b^2B)+a(bB-2Ac)}{4c(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{(2aBc-3Abc+b^2B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)/(4*c*(b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)^2) + ((b^2*B - 3*A*b*c + 2*a*B*c)*(b + 2*c*x^2))/(4*c*(b^2 -
 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((b^2*B - 3*A*b*c + 2*a*B*c)*ArcTanh[(b +
2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
```

```
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3 (A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx+cx^2)^3}\,dx, x, x^2\right)$$

$$= -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)x^2}{4c(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{(b^2B-3Abc+2aBc)\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^2}\,dx\right.}{4c(b^2-4ac)}$$

$$= -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)x^2}{4c(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{(b^2B-3Abc+2aBc)(b+2cx^2)}{4c(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{(b^2B-3}{}$$

$$= -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)x^2}{4c(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{(b^2B-3Abc+2aBc)(b+2cx^2)}{4c(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{(b^2B-3}{}$$

$$= -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)x^2}{4c(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{(b^2B-3Abc+2aBc)(b+2cx^2)}{4c(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{(b^2B-3}{}$$

Mathematica [A] time = 0.20, size = 172, normalized size = 1.01

$$\frac{1}{4}\left(\frac{4(2aBc-3Abc+b^2B)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{5/2}} + \frac{(b+2cx^2)(2aBc-3Abc+b^2B)}{c(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{-2ac(A+Bx^2)+abB+bx^2(bB-Ac)}{c(4ac-b^2)(a+bx^2+cx^4)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (((b^2*B - 3*A*b*c + 2*a*B*c)*(b + 2*c*x^2))/(c*(b^2 - 4*a*c)^2*(a + b*x^2
+ c*x^4)) + (a*b*B + b*(b*B - A*c)*x^2 - 2*a*c*(A + B*x^2))/(c*(-b^2 + 4*a*
c)*(a + b*x^2 + c*x^4)^2) + (4*(b^2*B - 3*A*b*c + 2*a*B*c)*ArcTan[(b + 2*c*
x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 0.93, size = 1226, normalized size = 7.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(B*b^4*c - 4*(2*B*a^2 - 3*A*a*b)*c^3 - (2*B*a*b^2 + 3*A*b^3)*c^2)*x
^6 + 6*B*a^2*b^3 - A*a*b^4 + 32*A*a^3*c^2 + 3*(B*b^5 - 4*(2*B*a^2*b - 3*A*a
*b^2)*c^2 - (2*B*a*b^3 + 3*A*b^4)*c)*x^4 + 2*(5*B*a*b^4 - A*b^5 + 4*(2*B*a^
3 + 5*A*a^2*b)*c^2 - (22*B*a^2*b^2 + A*a*b^3)*c)*x^2 - 2*((B*b^2*c^2 + (2*B
*a - 3*A*b)*c^3)*x^8 + 2*(B*b^3*c + (2*B*a*b - 3*A*b^2)*c^2)*x^6 + B*a^2*b^
2 + (B*b^4 + 2*(2*B*a^2 - 3*A*a*b)*c^2 + (4*B*a*b^2 - 3*A*b^3)*c)*x^4 + 2*(
B*a*b^3 + (2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + (2*B*a^3 - 3*A*a^2*b)*c)*sqrt(b^
2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^
2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 4*(6*B*a^3*b + A*a^2*b^2)*c)/((b^6*c^2 -
 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 6
4*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 12
8*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x
^2), 1/4*(2*(B*b^4*c - 4*(2*B*a^2 - 3*A*a*b)*c^3 - (2*B*a*b^2 + 3*A*b^3)*c^
2)*x^6 + 6*B*a^2*b^3 - A*a*b^4 + 32*A*a^3*c^2 + 3*(B*b^5 - 4*(2*B*a^2*b - 3
*A*a*b^2)*c^2 - (2*B*a*b^3 + 3*A*b^4)*c)*x^4 + 2*(5*B*a*b^4 - A*b^5 + 4*(2*
B*a^3 + 5*A*a^2*b)*c^2 - (22*B*a^2*b^2 + A*a*b^3)*c)*x^2 - 4*((B*b^2*c^2 +
(2*B*a - 3*A*b)*c^3)*x^8 + 2*(B*b^3*c + (2*B*a*b - 3*A*b^2)*c^2)*x^6 + B*a^
2*b^2 + (B*b^4 + 2*(2*B*a^2 - 3*A*a*b)*c^2 + (4*B*a*b^2 - 3*A*b^3)*c)*x^4 +
 2*(B*a*b^3 + (2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + (2*B*a^3 - 3*A*a^2*b)*c)*sqr
t(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 4
*(6*B*a^3*b + A*a^2*b^2)*c)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^
7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b
^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)]
```

giac [A] time = 6.06, size = 228, normalized size = 1.34

$$\frac{\left(Bb^{2}+2\,Bac-3\,Abc\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)\sqrt{-b^{2}+4\,ac}}+\frac{2\,Bb^{2}cx^{6}+4\,Bac^{2}x^{6}-6\,Abc^{2}x^{6}+3\,Bb^{3}x^{4}+6\,Babcx^{4}-9\,Ab^{2}cx^{4}+10\,Bab^{2}x^{2}-2\,Ab^{3}x^{2}-4\,Ba^{2}cx^{2}-10\,Aabcx^{2}+6\,Ba^{2}b-Aab^{2}-8\,Aa^{2}c}{4\,\left(cx^{4}+bx^{2}+a\right)^{2}\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] (B*b^2 + 2*B*a*c - 3*A*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4
- 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(2*B*b^2*c*x^6 + 4*B*a*
c^2*x^6 - 6*A*b*c^2*x^6 + 3*B*b^3*x^4 + 6*B*a*b*c*x^4 - 9*A*b^2*c*x^4 + 10*
B*a*b^2*x^2 - 2*A*b^3*x^2 - 4*B*a^2*c*x^2 - 10*A*a*b*c*x^2 + 6*B*a^2*b - A*
a*b^2 - 8*A*a^2*c)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

maple [B] time = 0.02, size = 379, normalized size = 2.23

$$-\frac{3Abc\arctan\left(\frac{2cx^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}}+\frac{2Bac\arctan\left(\frac{2cx^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}}+\frac{B\,b^{2}\arctan\left(\frac{2cx^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}}+\frac{-\frac{\left(3Abc-2aBc-b^{2}B\right)c\,x^{6}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{3\left(3Abc-2aBc-b^{2}B\right)b\,x^{4}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(5Aabc+A\,b^{3}+2a^{2}Bc-5Ba\,b^{2}\right)x^{2}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{\left(8aAc+A\,b^{2}-6abB\right)a}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{2\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(-c*(3*A*b*c-2*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-3/2*b*(3*A*b
*c-2*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-(5*A*a*b*c+A*b^3+2*B*a^2*c
-5*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/2*a*(8*A*a*c+A*b^2-6*B*a*b)/(1
6*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2-3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4
*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b*c+2/(16*a^2*c^2-8
*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*B*c
+1/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b
^2)^(1/2))*b^2*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.66, size = 587, normalized size = 3.45

$$\frac{\operatorname{atan}\left(\frac{\left(x^2\left(\frac{\left(B b^2 c^2-3 A b c^3+2 B a c^3\right)\left(B b^2-3 A c b+2 B a c\right)}{a\left(4 a c-b^2\right)^{9/2}\left(16 a^2 c^2-8 a b^2 c+b^4\right)}+\frac{b\left(B b^2-3 A c b+2 B a c\right)^2\left(32 a^2 b c^4-16 a b^3 c^3+2 b^5 c^2\right)}{2 a\left(4 a c-b^2\right)^{15/2}\left(16 a^2 c^2-8 a b^2 c+b^4\right)}\right)+\frac{2 b c^2\left(B b^2-3 A c b+2 B a c\right)^2}{\left(4 a c-b^2\right)^{15/2}}\right)\left(b^4\left(4 a c-b^2\right)^5+16 a^2 c^2\left(4 a c-b^2\right)^5-8 a b^2 c\left(4 a c-b^2\right)^5\right)}{18 A^2 b^2 c^4-24 A B a b c^4-12 A B b^3 c^3+8 B^2 a^2 c^4+8 B^2 a b^2 c^3+2 B^2 b^4 c^2}\right)\left(B b^2-3 A c b+2 B a c\right)}{\left(4 a c-b^2\right)^{5/2}}-\frac{\frac{-6 B a^2 b+8 A c a^2+A a b^2}{4\left(16 a^2 c^2-8 a b^2 c+b^4\right)}+\frac{x^2\left(2 B c a^2-5 B a b^2+5 A c a b+A b^3\right)}{2\left(16 a^2 c^2-8 a b^2 c+b^4\right)}-\frac{3 b x^4\left(B b^2-3 A c b+2 B a c\right)}{4\left(16 a^2 c^2-8 a b^2 c+b^4\right)}-\frac{c x^6\left(B b^2-3 A c b+2 B a c\right)}{2\left(16 a^2 c^2-8 a b^2 c+b^4\right)}}{x^4\left(b^2+2 a c\right)+a^2+c^2 x^8+2 a b x^2+2 b c x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (atan(((x^2*(((B*b^2*c^2 - 3*A*b*c^3 + 2*B*a*c^3)*(B*b^2 - 3*A*b*c + 2*B*a*
c))/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*(B*b^2 - 3*
A*b*c + 2*B*a*c)^2*(2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4))/(2*a*(4*a*c -
 b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (2*b*c^2*(B*b^2 - 3*A*b*c +
 2*B*a*c)^2)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c
 - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(18*A^2*b^2*c^4 + 8*B^2*a^2*c^4 + 2
*B^2*b^4*c^2 - 12*A*B*b^3*c^3 + 8*B^2*a*b^2*c^3 - 24*A*B*a*b*c^4))*(B*b^2 -
 3*A*b*c + 2*B*a*c))/(4*a*c - b^2)^(5/2) - ((A*a*b^2 + 8*A*a^2*c - 6*B*a^2*
b)/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(A*b^3 - 5*B*a*b^2 + 2*B*a^2*c
 + 5*A*a*b*c))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (3*b*x^4*(B*b^2 - 3*A*b
*c + 2*B*a*c))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (c*x^6*(B*b^2 - 3*A*b*c
 + 2*B*a*c))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 +
 c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

sympy [B] time = 21.37, size = 789, normalized size = 4.64

$$-\frac{\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(-3Abc+2Bac+Bb^2\right)\log\left(x^2+\frac{-3Ab^2c+2Babc+Bb^3-64a^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(-3Abc+2Bac+Bb^2\right)+48a^2b^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(-3Abc+2Bac+Bb^2\right)-12ab^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(-3Abc+2Bac+Bb^2\right)+b^6\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(-3Abc+2Bac+Bb^2\right)}{-6Abc^2+4Bac^2+2Bb^2c}\right)}{2}+\frac{\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(-3Abc+2Bac+Bb^2\right)\log\left(x^2+\frac{-3Ab^2c+2Babc+Bb^3+64a^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(-3Abc+2Bac+Bb^2\right)-48a^2b^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(-3Abc+2Bac+Bb^2\right)+12ab^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(-3Abc+2Bac+Bb^2\right)-b^6\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(-3Abc+2Bac+Bb^2\right)}{-6Abc^2+4Bac^2+2Bb^2c}\right)}{2}+\frac{-8Aa^2c-Aab^2+6Ba^2b+x^6\left(-6Abc^2+4Bac^2+2Bb^2c\right)+x^4\left(-9Ab^2c+6Babc+3Bb^3\right)+x^2\left(-10Aabc-2Ab^3-4Ba^2c+10Bab^2\right)}{64a^4c^2-32a^3b^2c+4a^2b^4+x^8\left(64a^2c^4-32ab^2c^3+4b^4c^2\right)+x^6\left(128a^2bc^3-64ab^3c^2+8b^5c\right)+x^4\left(128a^3c^3-24ab^4c+4b^6\right)+x^2\left(128a^3bc^2-64a^2b^3c+8ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b**2)*log(x**2 + (-3*A*
b**2*c + 2*B*a*b*c + B*b**3 - 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-3*A
*b*c + 2*B*a*c + B*b**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-3
*A*b*c + 2*B*a*c + B*b**2) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b
*c + 2*B*a*c + B*b**2) + b**6*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*
c + B*b**2))/(-6*A*b*c**2 + 4*B*a*c**2 + 2*B*b**2*c))/2 + sqrt(-1/(4*a*c -
b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b**2)*log(x**2 + (-3*A*b**2*c + 2*B*a*b*c
 + B*b**3 + 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B
*b**2) - 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c +
 B*b**2) + 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b
**2) - b**6*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b**2))/(-6*A
*b*c**2 + 4*B*a*c**2 + 2*B*b**2*c))/2 + (-8*A*a**2*c - A*a*b**2 + 6*B*a**2*
b + x**6*(-6*A*b*c**2 + 4*B*a*c**2 + 2*B*b**2*c) + x**4*(-9*A*b**2*c + 6*B*
a*b*c + 3*B*b**3) + x**2*(-10*A*a*b*c - 2*A*b**3 - 4*B*a**2*c + 10*B*a*b**2
))/(64*a**4*c**2 - 32*a**3*b**2*c + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a
*b**2*c**3 + 4*b**4*c**2) + x**6*(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5
*c) + x**4*(128*a**3*c**3 - 24*a*b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2 -
 64*a**2*b**3*c + 8*a*b**5))
```

3.107 $\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$

Optimal. Leaf size=139

$$\frac{3c(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}-\frac{3(b+2cx^2)(bB-2Ac)}{4(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{-2aB-(x^2(bB-2Ac))+Ab}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

Rubi [A] time = 0.12, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1247, 638, 614, 618, 206}

$$-\frac{3(b+2cx^2)(bB-2Ac)}{4(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{-2aB+x^2(-(bB-2Ac))+Ab}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3c(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(A*b - 2*a*B - (b*B - 2*A*c)*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2)
- (3*(b*B - 2*A*c)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) +
 (3*c*(b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)
^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx=\frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{\left(a+bx+cx^2\right)^3}\,dx,x,x^2\right)$$

$$=-\frac{Ab-2aB-(bB-2Ac)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{(3(bB-2Ac))\operatorname{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)}{4\left(b^2-4ac\right)}$$

$$=-\frac{Ab-2aB-(bB-2Ac)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3(bB-2Ac)\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{(3c(bB-2Ac))\operatorname{Subst}}{2\left(b^2-\right.}$$

$$=-\frac{Ab-2aB-(bB-2Ac)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3(bB-2Ac)\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{(3c(bB-2Ac))\operatorname{Subst}}{\left(b\right.}$$

$$=-\frac{Ab-2aB-(bB-2Ac)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3(bB-2Ac)\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3c(bB-2Ac)\tanh^{-1}}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.13, size = 142, normalized size = 1.02

$$\frac{-\frac{12c(bB-2Ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{\left(b^2-4ac\right)\left(B\left(2a+bx^2\right)-A\left(b+2cx^2\right)\right)}{\left(a+bx^2+cx^4\right)^2}-\frac{3\left(b+2cx^2\right)(bB-2Ac)}{a+bx^2+cx^4}}{4\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-3*(b*B - 2*A*c)*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + ((b^2 - 4*a*c)*(B*(
2*a + b*x^2) - A*(b + 2*c*x^2)))/(a + b*x^2 + c*x^4)^2 - (12*c*(b*B - 2*A*c
)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(4*(b^2 - 4
*a*c)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 0.69, size = 1109, normalized size = 7.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*(B*b^3*c^2 + 8*A*a*c^4 - 2*(2*B*a*b + A*b^2)*c^3)*x^6 + B*a*b^4 +
A*b^5 + 9*(B*b^4*c + 8*A*a*b*c^3 - 2*(2*B*a*b^2 + A*b^3)*c^2)*x^4 - 8*(4*B*
```

```
a^3 - 5*A*a^2*b)*c^2 + 2*(B*b^5 + 40*A*a^2*c^3 - 2*(10*B*a^2*b + A*a*b^2)*c
^2 + (B*a*b^3 - 2*A*b^4)*c)*x^2 + 6*((B*b*c^3 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2
 - 2*A*b*c^3)*x^6 + B*a^2*b*c - 2*A*a^2*c^2 + (B*b^3*c - 4*A*a*c^3 + 2*(B*a
*b - A*b^2)*c^2)*x^4 + 2*(B*a*b^2*c - 2*A*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*l
og((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/
(c*x^4 + b*x^2 + a)) + 2*(2*B*a^2*b^2 - 7*A*a*b^3)*c)/((b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^
4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)
*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2), -1/4*
(6*(B*b^3*c^2 + 8*A*a*c^4 - 2*(2*B*a*b + A*b^2)*c^3)*x^6 + B*a*b^4 + A*b^5
+ 9*(B*b^4*c + 8*A*a*b*c^3 - 2*(2*B*a*b^2 + A*b^3)*c^2)*x^4 - 8*(4*B*a^3 -
5*A*a^2*b)*c^2 + 2*(B*b^5 + 40*A*a^2*c^3 - 2*(10*B*a^2*b + A*a*b^2)*c^2 + (
B*a*b^3 - 2*A*b^4)*c)*x^2 - 12*((B*b*c^3 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2 - 2*
A*b*c^3)*x^6 + B*a^2*b*c - 2*A*a^2*c^2 + (B*b^3*c - 4*A*a*c^3 + 2*(B*a*b -
A*b^2)*c^2)*x^4 + 2*(B*a*b^2*c - 2*A*a*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arcta
n(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + 2*(2*B*a^2*b^2 - 7*A*a
*b^3)*c)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2
+ 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 +
 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c
^2 - 64*a^4*b*c^3)*x^2)]
```

giac [A] time = 5.57, size = 208, normalized size = 1.50

$$-\frac{3\left(Bbc-2\,Ac^{2}\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)\sqrt{-b^{2}+4\,ac}}-\frac{6\,Bbc^{2}x^{6}-12\,Ac^{3}x^{6}+9\,Bb^{2}cx^{4}-18\,Abc^{2}x^{4}+2\,Bb^{3}x^{2}+10\,Babcx^{2}-4\,Ab^{2}cx^{2}-20\,Aac^{2}x^{2}+Bab^{2}+Ab^{3}+8\,Ba^{2}c-10\,Aabc}{4\left(cx^{4}+bx^{2}+a\right)^{2}\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3*(B*b*c - 2*A*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b
^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*B*b*c^2*x^6 - 12*A*c^3*x^6
+ 9*B*b^2*c*x^4 - 18*A*b*c^2*x^4 + 2*B*b^3*x^2 + 10*B*a*b*c*x^2 - 4*A*b^2*c
*x^2 - 20*A*a*c^2*x^2 + B*a*b^2 + A*b^3 + 8*B*a^2*c - 10*A*a*b*c)/((c*x^4 +
 b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

maple [A] time = 0.01, size = 262, normalized size = 1.88

$$\frac{3A\,c^{2}x^{2}}{\left(4ac-b^{2}\right)^{2}\left(c\,x^{4}+b\,x^{2}+a\right)}-\frac{3Bbc\,x^{2}}{2\left(4ac-b^{2}\right)^{2}\left(c\,x^{4}+b\,x^{2}+a\right)}+\frac{6A\,c^{2}\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{5}{2}}}-\frac{3Bbc\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{5}{2}}}+\frac{3Abc}{2\left(4ac-b^{2}\right)^{2}\left(c\,x^{4}+b\,x^{2}+a\right)}-\frac{3B\,b^{2}}{4\left(4ac-b^{2}\right)^{2}\left(c\,x^{4}+b\,x^{2}+a\right)}+\frac{Ab-2Ba+\left(2Ac-bB\right)x^{2}}{4\left(4ac-b^{2}\right)\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/4*(A*b-2*B*a+(2*A*c-B*b)*x^2)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^2+3/(4*a*c-b^2)
^2/(c*x^4+b*x^2+a)*c^2*x^2*A-3/2/(4*a*c-b^2)^2/(c*x^4+b*x^2+a)*c*x^2*b*B+3/
2/(4*a*c-b^2)^2/(c*x^4+b*x^2+a)*b*A*c-3/4/(4*a*c-b^2)^2/(c*x^4+b*x^2+a)*b^2
*B+6/(4*a*c-b^2)^(5/2)*c^2*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A-3/(4*a*c
-b^2)^(5/2)*c*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.59, size = 517, normalized size = 3.72

$$\frac{3c\,\mathrm{atan}\left(\frac{\left(x^2\left(\frac{3c(2Ac-Bb)(6Ac^4-3Bbc^3)}{a(4ac-b^2)^{9/2}(16a^2c^2-8ab^2c+b^4)}+\frac{9bc^2(2Ac-Bb)^2(32a^2bc^4-16ab^3c^3+2b^5c^2)}{2a(4ac-b^2)^{15/2}(16a^2c^2-8ab^2c+b^4)}\right)+\frac{18bc^4(2Ac-Bb)^2}{(4ac-b^2)^{15/2}}\right)\left(b^4(4ac-b^2)^5+16a^2c^2(4ac-b^2)^5-8ab^2c(4ac-b^2)^5\right)}{72A^2c^6-72ABbc^5+18B^2b^2c^4}(2Ac-Bb)\right)}{(4ac-b^2)^{5/2}}-\frac{\frac{8Bca^2+Bab^2-10Acab+Ab^3}{4(16a^2c^2-8ab^2c+b^4)}-\frac{9x^4(2Abc^2-Bb^2c)}{4(16a^2c^2-8ab^2c+b^4)}+\frac{x^2(Bb^3-2Ab^2c+5Babc-10Aac^2)}{2(16a^2c^2-8ab^2c+b^4)}-\frac{3c^2x^6(2Ac-Bb)}{2(16a^2c^2-8ab^2c+b^4)}}{x^4(b^2+2ac)+a^2+c^2x^8+2abx^2+2bcx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (3*c*atan(((x^2*((3*c*(2*A*c - B*b)*(6*A*c^4 - 3*B*b*c^3))/(a*(4*a*c - b^2)
^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*(2*A*c - B*b)^2*(2*b^5*c^
2 - 16*a*b^3*c^3 + 32*a^2*b*c^4))/(2*a*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c
^2 - 8*a*b^2*c))) + (18*b*c^4*(2*A*c - B*b)^2)/(4*a*c - b^2)^(15/2))*(b^4*(
4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(
72*A^2*c^6 + 18*B^2*b^2*c^4 - 72*A*B*b*c^5))*(2*A*c - B*b))/(4*a*c - b^2)^(
5/2) - ((A*b^3 + B*a*b^2 + 8*B*a^2*c - 10*A*a*b*c)/(4*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)) - (9*x^4*(2*A*b*c^2 - B*b^2*c))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)) + (x^2*(B*b^3 - 10*A*a*c^2 - 2*A*b^2*c + 5*B*a*b*c))/(2*(b^4 + 16*a^2*c^
2 - 8*a*b^2*c)) - (3*c^2*x^6*(2*A*c - B*b))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

sympy [B] time = 12.40, size = 661, normalized size = 4.76

$$\frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)\log\left(x^2+\frac{-6Abc^2+3Bb^2c-192a^3c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)+144a^2b^2c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)-36ab^4c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)+3b^6c\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)}{-12Ac^3+6Bbc^2}\right)}{2}-\frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)\log\left(x^2+\frac{-6Abc^2+3Bb^2c+192a^3c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)-144a^2b^2c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)+36ab^4c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)-3b^6c\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)}{-12Ac^3+6Bbc^2}\right)}{2}+\frac{10Aabc-Ab^3-8Ba^2c-Bab^2+x^6(12Ac^3-6Bbc^2)+x^4(18Abc^2-9Bb^2c)+x^2(20Aac^2+4Ab^2c-10Babc-2Bb^3)}{64a^4c^2-32a^3b^2c+4a^2b^4+x^8(64a^2c^4-32ab^2c^3+4b^4c^2)+x^6(128a^2bc^3-64ab^3c^2+8b^5c)+x^4(128a^3c^3-24ab^4c+4b^6)+x^2(128a^3bc^2-64a^2b^3c+8ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 3*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b)*log(x**2 + (-6*A*b*c**2 + 3*B
*b**2*c - 192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) + 144*a**
2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) - 36*a*b**4*c**2*sqrt
(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) + 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)
*(-2*A*c + B*b))/(-12*A*c**3 + 6*B*b*c**2))/2 - 3*c*sqrt(-1/(4*a*c - b**2)*
*5)*(-2*A*c + B*b)*log(x**2 + (-6*A*b*c**2 + 3*B*b**2*c + 192*a**3*c**4*sqr
t(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) - 144*a**2*b**2*c**3*sqrt(-1/(4*a*c
- b**2)**5)*(-2*A*c + B*b) + 36*a*b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-2*
A*c + B*b) - 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b))/(-12*A*c**
3 + 6*B*b*c**2))/2 + (10*A*a*b*c - A*b**3 - 8*B*a**2*c - B*a*b**2 + x**6*(1
2*A*c**3 - 6*B*b*c**2) + x**4*(18*A*b*c**2 - 9*B*b**2*c) + x**2*(20*A*a*c**
2 + 4*A*b**2*c - 10*B*a*b*c - 2*B*b**3))/(64*a**4*c**2 - 32*a**3*b**2*c + 4
*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*c**3 + 4*b**4*c**2) + x**6*(128
*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5*c) + x**4*(128*a**3*c**3 - 24*a*b**4
*c + 4*b**6) + x**2*(128*a**3*b*c**2 - 64*a**2*b**3*c + 8*a*b**5))
```

3.108 $\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=252

$$-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{A\log(x)}{a^3}+\frac{2cx^2\left(6a^2Bc+A\left(b^3-7abc\right)\right)+A\left(16a^2c^2-15ab^2c+2b^4\right)+6a^2bBc}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{(12\cdots}{\cdots}$$

Rubi [A] time = 0.54, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1251, 822, 800, 634, 618, 206, 628}

$$\frac{2cx^2\left(6a^2Bc+A\left(b^3-7abc\right)\right)+A\left(16a^2c^2-15ab^2c+2b^4\right)+6a^2bBc}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\left(12a^3Bc^2-A\left(30a^2bc^2-10ab^3c+b^5\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{5/2}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{A\log(x)}{a^3}-\frac{-A\left(b^2-2ac\right)+cx^2(-(Ab-2aB))+abB}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] -(a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(4*a*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)^2) + (6*a^2*b*B*c + A*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2) + 2*c*
(6*a^2*B*c + A*(b^3 - 7*a*b*c))*x^2)/(4*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*
x^4)) - ((12*a^3*B*c^2 - A*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2))*ArcTanh[(b +
2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)) + (A*Log[x])/a^3 -
 (A*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{-2A\left(b^2-4ac\right)-3(Ab-2aB)cx}{x\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4a\left(b^2-4ac\right)}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A\left(2b^4-15ab^2c+16a^2c^2\right)+2c\left(6\right.}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2\right.}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A\left(2b^4-15ab^2c+16a^2c^2\right)+2c\left(6\right.}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2\right.}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A\left(2b^4-15ab^2c+16a^2c^2\right)+2c\left(6\right.}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2\right.}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A\left(2b^4-15ab^2c+16a^2c^2\right)+2c\left(6\right.}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2\right.}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A\left(2b^4-15ab^2c+16a^2c^2\right)+2c\left(6\right.}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2\right.}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A\left(2b^4-15ab^2c+16a^2c^2\right)+2c\left(6\right.}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2\right.}$$

Mathematica [A] time = 0.69, size = 396, normalized size = 1.57

$$\frac{\frac{a^2\left(A\left(-2ac+b^2+bcx^2\right)-aB\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{a\left(2a^2c\left(8Ac+3bB+6Bcx^2\right)-aAbc\left(15b+14cx^2\right)+2Ab^3\left(b+cx^2\right)\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\left(A\left(16a^2c^2\sqrt{b^2-4ac}+30a^2bc^2-10ab^3c-8ab^2c\sqrt{b^2-4ac}+b^4\sqrt{b^2-4ac}+b^5\right)-12a^3Bc^2\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{5/2}}-\frac{\left(12a^3Bc^2+A\left(16a^2c^2\sqrt{b^2-4ac}-30a^2bc^2+10ab^3c-8ab^2c\sqrt{b^2-4ac}+b^4\sqrt{b^2-4ac}-b^5\right)\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{5/2}}+4A\log(x)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((a^2*(-(a*B*(b + 2*c*x^2)) + A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)^2) + (a*(2*A*b^3*(b + c*x^2) - a*A*b*c*(15*b + 14*c*x^2)
+ 2*a^2*c*(3*b*B + 8*A*c + 6*B*c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4
)) + 4*A*Log[x] - ((-12*a^3*B*c^2 + A*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^
4*Sqrt[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4
*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2) - ((12*a^
3*B*c^2 + A*(-b^5 + 10*a*b^3*c - 30*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c] - 8*a
*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2
- 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/(4*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3), x]
```

fricas [B] time = 7.45, size = 2494, normalized size = 9.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(B*a^3*b^5 - 3*A*a^2*b^6 + 96*A*a^5*c^3 - 2*(A*a*b^5*c^2 - 4*(6*B*a^4
 - 7*A*a^3*b)*c^4 + (6*B*a^3*b^2 - 11*A*a^2*b^3)*c^3)*x^6 - (4*A*a*b^6*c -
64*A*a^4*c^4 - 12*(6*B*a^4*b - 11*A*a^3*b^2)*c^3 + 9*(2*B*a^3*b^3 - 5*A*a^2
*b^4)*c^2)*x^4 + 4*(10*B*a^5*b - 27*A*a^4*b^2)*c^2 - 2*(A*a*b^7 - 4*(10*B*a
^5 - A*a^4*b)*c^3 + (2*B*a^4*b^2 + 23*A*a^3*b^3)*c^2 + 2*(B*a^3*b^4 - 5*A*a
^2*b^5)*c)*x^2 - ((A*b^5*c^2 - 10*A*a*b^3*c^3 - 6*(2*B*a^3 - 5*A*a^2*b)*c^4
)*x^8 + A*a^2*b^5 - 10*A*a^3*b^3*c + 2*(A*b^6*c - 10*A*a*b^4*c^2 - 6*(2*B*a
^3*b - 5*A*a^2*b^2)*c^3)*x^6 + (A*b^7 - 8*A*a*b^5*c - 12*(2*B*a^4 - 5*A*a^3
*b)*c^3 - 2*(6*B*a^3*b^2 - 5*A*a^2*b^3)*c^2)*x^4 - 6*(2*B*a^5 - 5*A*a^4*b)*
c^2 + 2*(A*a*b^6 - 10*A*a^2*b^4*c - 6*(2*B*a^4*b - 5*A*a^3*b^2)*c^2)*x^2)*s
qrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*s
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (14*B*a^4*b^3 - 33*A*a^3*b^4)*c +
(A*a^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A*b^6*c^2
- 12*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7*c - 12*A
*a*b^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*A*a*b^6*c
 + 24*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A*a*b^7 -
12*A*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2
+ a) - 4*(A*a^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A
*b^6*c^2 - 12*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7
*c - 12*A*a*b^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*
A*a*b^6*c + 24*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A
*a*b^7 - 12*A*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*log(x))/(
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^
4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^
2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*x^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b
^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*x^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48
*a^6*b^3*c^2 - 64*a^7*b*c^3)*x^2), -1/4*(B*a^3*b^5 - 3*A*a^2*b^6 + 96*A*a^5
*c^3 - 2*(A*a*b^5*c^2 - 4*(6*B*a^4 - 7*A*a^3*b)*c^4 + (6*B*a^3*b^2 - 11*A*a
^2*b^3)*c^3)*x^6 - (4*A*a*b^6*c - 64*A*a^4*c^4 - 12*(6*B*a^4*b - 11*A*a^3*b
```

```
^2)*c^3 + 9*(2*B*a^3*b^3 - 5*A*a^2*b^4)*c^2)*x^4 + 4*(10*B*a^5*b - 27*A*a^4
*b^2)*c^2 - 2*(A*a*b^7 - 4*(10*B*a^5 - A*a^4*b)*c^3 + (2*B*a^4*b^2 + 23*A*a
^3*b^3)*c^2 + 2*(B*a^3*b^4 - 5*A*a^2*b^5)*c)*x^2 - 2*((A*b^5*c^2 - 10*A*a*b
^3*c^3 - 6*(2*B*a^3 - 5*A*a^2*b)*c^4)*x^8 + A*a^2*b^5 - 10*A*a^3*b^3*c + 2*
(A*b^6*c - 10*A*a*b^4*c^2 - 6*(2*B*a^3*b - 5*A*a^2*b^2)*c^3)*x^6 + (A*b^7 -
 8*A*a*b^5*c - 12*(2*B*a^4 - 5*A*a^3*b)*c^3 - 2*(6*B*a^3*b^2 - 5*A*a^2*b^3)
*c^2)*x^4 - 6*(2*B*a^5 - 5*A*a^4*b)*c^2 + 2*(A*a*b^6 - 10*A*a^2*b^4*c - 6*(
2*B*a^4*b - 5*A*a^3*b^2)*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)
*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (14*B*a^4*b^3 - 33*A*a^3*b^4)*c + (A*a
^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A*b^6*c^2 - 12
*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7*c - 12*A*a*b
^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*A*a*b^6*c + 2
4*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A*a*b^7 - 12*A
*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a)
 - 4*(A*a^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A*b^6
*c^2 - 12*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7*c -
 12*A*a*b^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*A*a*
b^6*c + 24*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A*a*b
^7 - 12*A*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*log(x))/(a^5*
b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^
4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 +
48*a^5*b^3*c^3 - 64*a^6*b*c^4)*x^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c
^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*x^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6
*b^3*c^2 - 64*a^7*b*c^3)*x^2)]
```

giac [**A**] time = 6.44, size = 421, normalized size = 1.67

$$-\frac{\left(Ab^{5} - 10\,Aab^{3}c - 12\,Ba^{3}c^{2} + 30\,Aa^{2}bc^{2}\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\left(a^{3}b^{4} - 8\,a^{4}b^{2}c + 16\,a^{5}c^{2}\right)\sqrt{-b^{2}+4\,ac}} - \frac{A\log\left(cx^{4}+bx^{2}+a\right)}{4\,a^{3}} + \frac{A\log\left(x^{2}\right)}{2\,a^{3}} + \frac{3\,Ab^{4}c^{2}x^{8} - 24\,Aab^{2}c^{3}x^{8} + 48\,Aa^{2}c^{4}x^{8} + 6\,Ab^{5}cx^{6} - 44\,Aab^{3}c^{2}x^{6} + 24\,Ba^{3}c^{3}x^{6} + 68\,Aa^{2}bc^{3}x^{6} + 3\,Ab^{6}x^{4} - 10\,Aab^{4}cx^{4} + 36\,Ba^{3}bc^{2}x^{4} - 58\,Aa^{2}b^{2}c^{2}x^{4} + 128\,Aa^{3}c^{3}x^{4} + 10\,Aab^{5}x^{2} + 8\,Ba^{3}b^{2}cx^{2} - 72\,Aa^{2}b^{3}cx^{2} + 40\,Ba^{4}c^{2}x^{2} + 92\,Aa^{3}bc^{2}x^{2} - 2\,Ba^{3}b^{3} + 9\,Aa^{2}b^{4} + 20\,Ba^{4}bc - 66\,Aa^{3}b^{2}c + 96\,Aa^{4}c^{2}}{8\left(a^{3}b^{4} - 8\,a^{4}b^{2}c + 16\,a^{5}c^{2}\right)\left(cx^{4}+bx^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(A*b^5 - 10*A*a*b^3*c - 12*B*a^3*c^2 + 30*A*a^2*b*c^2)*arctan((2*c*x^2
 + b)/sqrt(-b^2 + 4*a*c))/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt(-b^2 +
 4*a*c)) - 1/4*A*log(c*x^4 + b*x^2 + a)/a^3 + 1/2*A*log(x^2)/a^3 + 1/8*(3*A
*b^4*c^2*x^8 - 24*A*a*b^2*c^3*x^8 + 48*A*a^2*c^4*x^8 + 6*A*b^5*c*x^6 - 44*A
*a*b^3*c^2*x^6 + 24*B*a^3*c^3*x^6 + 68*A*a^2*b*c^3*x^6 + 3*A*b^6*x^4 - 10*A
*a*b^4*c*x^4 + 36*B*a^3*b*c^2*x^4 - 58*A*a^2*b^2*c^2*x^4 + 128*A*a^3*c^3*x^
4 + 10*A*a*b^5*x^2 + 8*B*a^3*b^2*c*x^2 - 72*A*a^2*b^3*c*x^2 + 40*B*a^4*c^2*
x^2 + 92*A*a^3*b*c^2*x^2 - 2*B*a^3*b^3 + 9*A*a^2*b^4 + 20*B*a^4*b*c - 66*A*
a^3*b^2*c + 96*A*a^4*c^2)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c*x^4 + b*
x^2 + a)^2)
```

maple [**B**] time = 0.03, size = 1161, normalized size = 4.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 9/2/(c*x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b*B-1/2/(c*x^4+b*x
^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b*c^2+1/(c*x^4+b*x^2+a)^2/(16*a^2*
c^2-8*a*b^2*c+b^4)*x^2*B*b^2*c+1/2/a^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^
2*c+b^4)*x^2*A*b^5+5*a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*c
^2+5/2*a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b*B*c+2/a^2/(16*a^2*c
^2-8*a*b^2*c+b^4)*c*ln(c*x^4+b*x^2+a)*A*b^2-1/2/a^3/(16*a^2*c^2-8*a*b^2*c+b
^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^5+A*ln(x)/a
^3-1/4/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*B*b^3-7/2/a/(c*x^4+b*x^
2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*b+1/2/a^2/(c*x^4+b*x^2+a)^2*c^2
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*b^3-29/4/a/(c*x^4+b*x^2+a)^2*c^2/(16*a^2*
```

```
c^2-8*a*b^2*c+b^4)*x^4*A*b^2+1/a^2/(c*x^4+b*x^2+a)^2*c/(16*a^2*c^2-8*a*b^2*
c+b^4)*x^4*A*b^4-3/a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b^3
*c-15/a/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*
a*c-b^2)^(1/2))*A*b*c^2+5/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*
arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^3*c-21/4/(c*x^4+b*x^2+a)^2/(16*a^
2*c^2-8*a*b^2*c+b^4)*A*b^2*c+6*a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^
4)*A*c^2+3/4/a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*A*b^4-4/a/(16*a
^2*c^2-8*a*b^2*c+b^4)*c^2*ln(c*x^4+b*x^2+a)*A-1/4/a^3/(16*a^2*c^2-8*a*b^2*c
+b^4)*ln(c*x^4+b*x^2+a)*A*b^4+6/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2
)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B*c^2+3/(c*x^4+b*x^2+a)^2*c^3/(16*a
^2*c^2-8*a*b^2*c+b^4)*x^6*B+4/(c*x^4+b*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^4*A
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 11.57, size = 11674, normalized size = 46.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3),x)
```

```
[Out] ((3*A*b^4 + 24*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 10*B*a^2*b*c)/(4*a*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(A*b^5 + 10*B*a^3*c^2 - 6*A*a*b^3*c - A*
a^2*b*c^2 + 2*B*a^2*b^2*c))/(2*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^4*(
16*A*a^2*c^3 + 4*A*b^4*c - 29*A*a*b^2*c^2 + 18*B*a^2*b*c^2))/(4*a^2*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) + (c^2*x^6*(A*b^3 + 6*B*a^2*c - 7*A*a*b*c))/(2*a^2
*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*
b*x^2 + 2*b*c*x^6) + (A*log(x))/a^3 - (log(((c^5*x^2*(A*b^3 + 6*B*a^2*c - 7
*A*a*b*c)^3)/(a^6*(4*a*c - b^2)^6) - ((A + a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10
*A*a*b^3*c + 30*A*a^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*((c^3*(4*A^2*b
^8 - 36*B^2*a^5*c^3 + 302*A^2*a^2*b^4*c^2 - 497*A^2*a^3*b^2*c^3 - 61*A^2*a*
b^6*c - 204*A*B*a^3*b^3*c^2 + 24*A*B*a^2*b^5*c + 468*A*B*a^4*b*c^3))/(a^4*(
4*a*c - b^2)^4) - ((A + a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a
^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*((2*c^3*x^2*(A*b^5 + 60*B*a^3*c^2
 - 2*A*a*b^3*c + 10*A*a^2*b*c^2 - 24*B*a^2*b^2*c))/(a^2*(4*a*c - b^2)^2) +
(4*b*c^2*(A*b^5 - 6*B*a^3*c^2 - 9*A*a*b^3*c + 23*A*a^2*b*c^2))/(a^2*(4*a*c
- b^2)^2) + (b*c^2*(A + a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a
^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a
^3))/(4*a^3) + (c^4*x^2*(6*A^2*b^7 + 409*A^2*a^2*b^3*c^2 + 480*A*B*a^4*c^3
- 89*A^2*a*b^5*c - 560*A^2*a^3*b*c^3 + 36*B^2*a^4*b*c^2 - 324*A*B*a^3*b^2*c
^2 + 42*A*B*a^2*b^4*c))/(a^4*(4*a*c - b^2)^4)))/(4*a^3) + (A*c^4*(A*b^3 + 6
*B*a^2*c - 7*A*a*b*c)^2)/(a^6*(4*a*c - b^2)^4))*((c^5*x^2*(A*b^3 + 6*B*a^2*
c - 7*A*a*b*c)^3)/(a^6*(4*a*c - b^2)^6) - ((A - a^3*(-(A*b^5 - 12*B*a^3*c^2
 - 10*A*a*b^3*c + 30*A*a^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*((c^3*(4*
A^2*b^8 - 36*B^2*a^5*c^3 + 302*A^2*a^2*b^4*c^2 - 497*A^2*a^3*b^2*c^3 - 61*A
^2*a*b^6*c - 204*A*B*a^3*b^3*c^2 + 24*A*B*a^2*b^5*c + 468*A*B*a^4*b*c^3))/(
a^4*(4*a*c - b^2)^4) - ((A - a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 3
0*A*a^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*((2*c^3*x^2*(A*b^5 + 60*B*a^
3*c^2 - 2*A*a*b^3*c + 10*A*a^2*b*c^2 - 24*B*a^2*b^2*c))/(a^2*(4*a*c - b^2)^
2) + (4*b*c^2*(A*b^5 - 6*B*a^3*c^2 - 9*A*a*b^3*c + 23*A*a^2*b*c^2))/(a^2*(4
```

```
*a*c - b^2)^2) + (b*c^2*(A - a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 3
0*A*a^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^
2))/a^3))/(4*a^3) + (c^4*x^2*(6*A^2*b^7 + 409*A^2*a^2*b^3*c^2 + 480*A*B*a^4
*c^3 - 89*A^2*a*b^5*c - 560*A^2*a^3*b*c^3 + 36*B^2*a^4*b*c^2 - 324*A*B*a^3*
b^2*c^2 + 42*A*B*a^2*b^4*c))/(a^4*(4*a*c - b^2)^4)))/(4*a^3) + (A*c^4*(A*b^
3 + 6*B*a^2*c - 7*A*a*b*c)^2)/(a^6*(4*a*c - b^2)^4)))*(2*A*b^10 - 2048*A*a^
5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*
b^2*c^4))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 -
2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)) - (atan((x^2*((((((((30720*B*a^11*c^9
 + 5120*A*a^10*b*c^9 + 2*A*a^4*b^13*c^3 - 36*A*a^5*b^11*c^4 + 276*A*a^6*b^9
*c^5 - 1216*A*a^7*b^7*c^6 + 3456*A*a^8*b^5*c^7 - 6144*A*a^9*b^3*c^8 - 48*B*
a^6*b^10*c^4 + 888*B*a^7*b^8*c^5 - 6528*B*a^8*b^6*c^6 + 23808*B*a^9*b^4*c^7
 - 43008*B*a^10*b^2*c^8)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^
8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2
*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^
4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*
b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 + 227
328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80
*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^
12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3
840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*
c + 30*A*a^2*b*c^2))/(4*a^3*(4*a*c - b^2)^(5/2)) - ((A*b^5 - 12*B*a^3*c^2 -
 10*A*a*b^3*c + 30*A*a^2*b*c^2)*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c +
 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^13*
b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*
b^9*c^5 - 97280*a^10*b^7*c^6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/
(8*a^3*(4*a*c - b^2)^(5/2)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*
a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^12*c^
6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4
- 6144*a^11*b^2*c^5)))*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^
2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(4*a^3*b^10 - 4096
*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2
*c^4)) - (((6*A^2*a^2*b^11*c^4 - 137*A^2*a^3*b^9*c^5 + 1217*A^2*a^4*b^7*c^6
 - 5256*A^2*a^5*b^5*c^7 + 11024*A^2*a^6*b^3*c^8 + 36*B^2*a^6*b^5*c^6 - 288*
B^2*a^7*b^3*c^7 + 7680*A*B*a^8*c^9 - 8960*A^2*a^7*b*c^9 + 576*B^2*a^8*b*c^8
 + 42*A*B*a^4*b^8*c^5 - 660*A*B*a^5*b^6*c^6 + 3744*A*B*a^6*b^4*c^7 - 9024*A
*B*a^7*b^2*c^8)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2
 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - (((30720*B*a
^11*c^9 + 5120*A*a^10*b*c^9 + 2*A*a^4*b^13*c^3 - 36*A*a^5*b^11*c^4 + 276*A*
a^6*b^9*c^5 - 1216*A*a^7*b^7*c^6 + 3456*A*a^8*b^5*c^7 - 6144*A*a^9*b^3*c^8
- 48*B*a^6*b^10*c^4 + 888*B*a^7*b^8*c^5 - 6528*B*a^8*b^6*c^6 + 23808*B*a^9*
b^4*c^7 - 43008*B*a^10*b^2*c^8)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c +
 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5
) - ((2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A
*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 3
28*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^
6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(2*(4*a^3*b^10 - 4096*a^8*c
^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*
(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*
c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*A*b^10 - 2048*A*a^5*c^5 -
 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4
))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^
6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A
*a^2*b*c^2))/(4*a^3*(4*a*c - b^2)^(5/2)) + ((A*b^5 - 12*B*a^3*c^2 - 10*A*a*
b^3*c + 30*A*a^2*b*c^2)^3*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*b^
13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 + 22732
8*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(64*a^9*(4*a*c - b^2)^(15/2)*(a^6*b^
12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3
840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(3*A*b^8 + 160*A*a^4*c^4 - 39*A*a*b
```

```
^6*c + 18*B*a^4*b*c^3 + 180*A*a^2*b^4*c^2 - 325*A*a^3*b^2*c^3 - 6*B*a^3*b^3
*c^2))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(6*A^2*b^10 - 6400*A^2*a^5*c^5 - 36*
B^2*a^6*c^4 + 960*A^2*a^2*b^6*c^2 - 3850*A^2*a^3*b^4*c^3 + 7775*A^2*a^4*b^2
*c^4 - 120*A^2*a*b^8*c + 6*A*B*a^3*b^5*c^2 - 60*A*B*a^4*b^3*c^3 + 180*A*B*a
^5*b*c^4)) + (((A^3*b^9*c^5 + 216*B^3*a^6*c^8 + 147*A^3*a^2*b^5*c^7 - 343*A
^3*a^3*b^3*c^8 - 21*A^3*a*b^7*c^6 - 756*A*B^2*a^5*b*c^8 + 108*A*B^2*a^4*b^3
*c^7 + 18*A^2*B*a^2*b^6*c^6 - 252*A^2*B*a^3*b^4*c^7 + 882*A^2*B*a^4*b^2*c^8
)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^
6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - (((6*A^2*a^2*b^11*c^4 - 13
7*A^2*a^3*b^9*c^5 + 1217*A^2*a^4*b^7*c^6 - 5256*A^2*a^5*b^5*c^7 + 11024*A^2
*a^6*b^3*c^8 + 36*B^2*a^6*b^5*c^6 - 288*B^2*a^7*b^3*c^7 + 7680*A*B*a^8*c^9
- 8960*A^2*a^7*b*c^9 + 576*B^2*a^8*b*c^8 + 42*A*B*a^4*b^8*c^5 - 660*A*B*a^5
*b^6*c^6 + 3744*A*B*a^6*b^4*c^7 - 9024*A*B*a^7*b^2*c^8)/(a^6*b^12 + 4096*a^
12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4
*c^4 - 6144*a^11*b^2*c^5) - (((30720*B*a^11*c^9 + 5120*A*a^10*b*c^9 + 2*A*a
^4*b^13*c^3 - 36*A*a^5*b^11*c^4 + 276*A*a^6*b^9*c^5 - 1216*A*a^7*b^7*c^6 +
3456*A*a^8*b^5*c^7 - 6144*A*a^9*b^3*c^8 - 48*B*a^6*b^10*c^4 + 888*B*a^7*b^8
*c^5 - 6528*B*a^8*b^6*c^6 + 23808*B*a^9*b^4*c^7 - 43008*B*a^10*b^2*c^8)/(a^
6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3
 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2*A*b^10 - 2048*A*a^5*c^5 - 4
0*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*
(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4
 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 + 227328*a^11*b^5*c^7 - 294912*a^
12*b^3*c^8))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2
 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*
b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11
*b^2*c^5)))*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 -
 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(4*a^3*b^10 - 4096*a^8*c^5 -
80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(2*
A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4
*c^3 + 2560*A*a^4*b^2*c^4))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c +
640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)) - (((((30720*B*a^11
*c^9 + 5120*A*a^10*b*c^9 + 2*A*a^4*b^13*c^3 - 36*A*a^5*b^11*c^4 + 276*A*a^6
*b^9*c^5 - 1216*A*a^7*b^7*c^6 + 3456*A*a^8*b^5*c^7 - 6144*A*a^9*b^3*c^8 - 4
8*B*a^6*b^10*c^4 + 888*B*a^7*b^8*c^5 - 6528*B*a^8*b^6*c^6 + 23808*B*a^9*b^4
*c^7 - 43008*B*a^10*b^2*c^8)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 24
0*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) -
 ((2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^
3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*
a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 +
 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(2*(4*a^3*b^10 - 4096*a^8*c^5
- 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^
6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3
 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*
b^3*c + 30*A*a^2*b*c^2))/(4*a^3*(4*a*c - b^2)^(5/2)) - ((A*b^5 - 12*B*a^3*c
^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2)*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8
*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a
^13*b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*
a^9*b^9*c^5 - 97280*a^10*b^7*c^6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^
8))/(8*a^3*(4*a*c - b^2)^(5/2)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c +
640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^1
2*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*
c^4 - 6144*a^11*b^2*c^5)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*
b*c^2))/(4*a^3*(4*a*c - b^2)^(5/2)) + ((A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c
 + 30*A*a^2*b*c^2)^2*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*
b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^13*b*c^9 - 12*
a^6*b^15*c^2 + 328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 9
7280*a^10*b^7*c^6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(32*a^6*(4*
a*c - b^2)^5*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 -
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2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^1
0*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^
2*c^5)))*(3*A*b^7 + 6*B*a^4*c^3 - 33*A*a*b^5*c - 135*A*a^3*b*c^3 + 120*A*a^
2*b^3*c^2 - 6*B*a^3*b^2*c^2))/(8*a^3*c^2*(4*a*c - b^2)^6*(6*A^2*b^10 - 6400
*A^2*a^5*c^5 - 36*B^2*a^6*c^4 + 960*A^2*a^2*b^6*c^2 - 3850*A^2*a^3*b^4*c^3
+ 7775*A^2*a^4*b^2*c^4 - 120*A^2*a*b^8*c + 6*A*B*a^3*b^5*c^2 - 60*A*B*a^4*b
^3*c^3 + 180*A*B*a^5*b*c^4)))*(16*a^9*b^12*(4*a*c - b^2)^(15/2) + 65536*a^1
5*c^6*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^
11*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(15/2) +
 61440*a^13*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c - b^2)
^(15/2)))/(A^2*b^10*c^2 + 144*B^2*a^6*c^6 + 160*A^2*a^2*b^6*c^4 - 600*A^2*a
^3*b^4*c^5 + 900*A^2*a^4*b^2*c^6 - 20*A^2*a*b^8*c^3 - 24*A*B*a^3*b^5*c^4 +
240*A*B*a^4*b^3*c^5 - 720*A*B*a^5*b*c^6) - (((((((384*B*a^9*b*c^6 - 4*A*a^4
*b^10*c^2 + 68*A*a^5*b^8*c^3 - 444*A*a^6*b^6*c^4 + 1312*A*a^7*b^4*c^5 - 147
2*A*a^8*b^2*c^6 + 24*B*a^7*b^5*c^4 - 192*B*a^8*b^3*c^5)/(a^6*b^8 + 256*a^10
*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3) - ((4*a^7*b^10*c^2
- 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)
*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3
*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c +
 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*
c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(A*b^5 - 12*B*
a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2))/(4*a^3*(4*a*c - b^2)^(5/2)) - ((A
*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2)*(4*a^7*b^10*c^2 - 64*a
^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*A*
b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c
^3 + 2560*A*a^4*b^2*c^4))/(8*a^3*(4*a*c - b^2)^(5/2)*(a^6*b^8 + 256*a^10*c^
4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8
*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4
)))*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*
a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b
^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)) - (((36*B^2*
a^7*c^6 - 4*A^2*a^2*b^8*c^3 + 61*A^2*a^3*b^6*c^4 - 302*A^2*a^4*b^4*c^5 + 49
7*A^2*a^5*b^2*c^6 - 24*A*B*a^4*b^5*c^4 + 204*A*B*a^5*b^3*c^5 - 468*A*B*a^6*
b*c^6)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^
2*c^3) - (((384*B*a^9*b*c^6 - 4*A*a^4*b^10*c^2 + 68*A*a^5*b^8*c^3 - 444*A*a
^6*b^6*c^4 + 1312*A*a^7*b^4*c^5 - 1472*A*a^8*b^2*c^6 + 24*B*a^7*b^5*c^4 - 1
92*B*a^8*b^3*c^5)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 -
 256*a^9*b^2*c^3) - ((4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1
024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b
^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(a^
6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*
a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3
 + 5120*a^7*b^2*c^4)))*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^
2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(4*a^3*b^10 - 4096
*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2
*c^4)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2))/(4*a^3*(4*a
*c - b^2)^(5/2)) + ((A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2)^
3*(4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 +
1024*a^11*b^2*c^6))/(64*a^9*(4*a*c - b^2)^(15/2)*(a^6*b^8 + 256*a^10*c^4 -
16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)))*(16*a^9*b^12*(4*a*c - b^
2)^(15/2) + 65536*a^15*c^6*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*(4*a*c -
b^2)^(15/2) + 3840*a^11*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*(
4*a*c - b^2)^(15/2) + 61440*a^13*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^14*
b^2*c^5*(4*a*c - b^2)^(15/2))*(3*A*b^8 + 160*A*a^4*c^4 - 39*A*a*b^6*c + 18*
B*a^4*b*c^3 + 180*A*a^2*b^4*c^2 - 325*A*a^3*b^2*c^3 - 6*B*a^3*b^3*c^2))/(8*
a^3*c^2*(4*a*c - b^2)^(13/2)*(A^2*b^10*c^2 + 144*B^2*a^6*c^6 + 160*A^2*a^2*
b^6*c^4 - 600*A^2*a^3*b^4*c^5 + 900*A^2*a^4*b^2*c^6 - 20*A^2*a*b^8*c^3 - 24
*A*B*a^3*b^5*c^4 + 240*A*B*a^4*b^3*c^5 - 720*A*B*a^5*b*c^6)*(6*A^2*b^10 - 6
400*A^2*a^5*c^5 - 36*B^2*a^6*c^4 + 960*A^2*a^2*b^6*c^2 - 3850*A^2*a^3*b^4*c
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^3 + 7775*A^2*a^4*b^2*c^4 - 120*A^2*a*b^8*c + 6*A*B*a^3*b^5*c^2 - 60*A*B*a^
4*b^3*c^3 + 180*A*B*a^5*b*c^4)) + (((A^3*b^6*c^4 + 49*A^3*a^2*b^2*c^6 + 36*
A*B^2*a^4*c^6 - 14*A^3*a*b^4*c^5 - 84*A^2*B*a^3*b*c^6 + 12*A^2*B*a^2*b^3*c^
5)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^
3) + (((36*B^2*a^7*c^6 - 4*A^2*a^2*b^8*c^3 + 61*A^2*a^3*b^6*c^4 - 302*A^2*a
^4*b^4*c^5 + 497*A^2*a^5*b^2*c^6 - 24*A*B*a^4*b^5*c^4 + 204*A*B*a^5*b^3*c^5
 - 468*A*B*a^6*b*c^6)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c
^2 - 256*a^9*b^2*c^3) - (((384*B*a^9*b*c^6 - 4*A*a^4*b^10*c^2 + 68*A*a^5*b^
8*c^3 - 444*A*a^6*b^6*c^4 + 1312*A*a^7*b^4*c^5 - 1472*A*a^8*b^2*c^6 + 24*B*
a^7*b^5*c^4 - 192*B*a^8*b^3*c^5)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 9
6*a^8*b^4*c^2 - 256*a^9*b^2*c^3) - ((4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*
a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*A*b^10 - 2048*A*a^5
*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b
^2*c^4))/(2*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a
^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2
560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b
^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(4*
a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3
 + 5120*a^7*b^2*c^4)))*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^
2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(4*a^3*b^10 - 4096
*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2
*c^4)) + (((((384*B*a^9*b*c^6 - 4*A*a^4*b^10*c^2 + 68*A*a^5*b^8*c^3 - 444*A
*a^6*b^6*c^4 + 1312*A*a^7*b^4*c^5 - 1472*A*a^8*b^2*c^6 + 24*B*a^7*b^5*c^4 -
 192*B*a^8*b^3*c^5)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2
 - 256*a^9*b^2*c^3) - ((4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 -
 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a
*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(
a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(
4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c
^3 + 5120*a^7*b^2*c^4)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*
c^2))/(4*a^3*(4*a*c - b^2)^(5/2)) - ((A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c +
 30*A*a^2*b*c^2)*(4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*
a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c
 + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(8*a^3*(4*
a*c - b^2)^(5/2)*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 -
256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^
2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b
^3*c + 30*A*a^2*b*c^2))/(4*a^3*(4*a*c - b^2)^(5/2)) - ((A*b^5 - 12*B*a^3*c^
2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2)^2*(4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384
*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*A*b^10 - 2048*A*a^
5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*
b^2*c^4))/(32*a^6*(4*a*c - b^2)^5*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c +
96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c
 + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(3*A*b^7 + 6*B*
a^4*c^3 - 33*A*a*b^5*c - 135*A*a^3*b*c^3 + 120*A*a^2*b^3*c^2 - 6*B*a^3*b^2*
c^2)*(16*a^9*b^12*(4*a*c - b^2)^(15/2) + 65536*a^15*c^6*(4*a*c - b^2)^(15/2
) - 384*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^11*b^8*c^2*(4*a*c - b^2)^
(15/2) - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^13*b^4*c^4*(4*a*
c - b^2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c - b^2)^(15/2)))/(8*a^3*c^2*(4*a
*c - b^2)^6*(A^2*b^10*c^2 + 144*B^2*a^6*c^6 + 160*A^2*a^2*b^6*c^4 - 600*A^2
*a^3*b^4*c^5 + 900*A^2*a^4*b^2*c^6 - 20*A^2*a*b^8*c^3 - 24*A*B*a^3*b^5*c^4
+ 240*A*B*a^4*b^3*c^5 - 720*A*B*a^5*b*c^6)*(6*A^2*b^10 - 6400*A^2*a^5*c^5 -
 36*B^2*a^6*c^4 + 960*A^2*a^2*b^6*c^2 - 3850*A^2*a^3*b^4*c^3 + 7775*A^2*a^4
*b^2*c^4 - 120*A^2*a*b^8*c + 6*A*B*a^3*b^5*c^2 - 60*A*B*a^4*b^3*c^3 + 180*A
*B*a^5*b*c^4)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2))/(2*
a^3*(4*a*c - b^2)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.109 $\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=363

$$\frac{(3Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^4}-\frac{\log(x)(3Ab-aB)}{a^4}-\frac{-A\left(20a^2c^2-20ab^2c+3b^4\right)+cx^2\left(aB\left(b^2-16ac\right)-3A\left(\right.\right.}{4a^2x^2\left(b^2-4ac\right)^2\left(a+bx^2+c\right.}$$

Rubi [A] time = 0.77, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1251, 822, 800, 634, 618, 206, 628}

$$\frac{abB\left(b^2-7ac\right)-3A\left(10a^2c^2-7ab^2c+b^4\right)}{2a^3x^2\left(b^2-4ac\right)^2}-\frac{-A\left(20a^2c^2-20ab^2c+3b^4\right)+cx^2\left(aB\left(b^2-16ac\right)-3A\left(b^3-6abc\right)\right)+abB\left(b^2-10ac\right)}{4a^2x^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\left(abB\left(30a^2c^2-10ab^2c+b^4\right)-3A\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\left(b^2-4ac\right)^{5/2}}+\frac{(3Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^4}-\frac{\log(x)(3Ab-aB)}{a^4}-\frac{-A\left(b^2-2ac\right)+cx^2(-(Ab-2aB))+abB}{4ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (a*b*B*(b^2 - 7*a*c) - 3*A*(b^4 - 7*a*b^2*c + 10*a^2*c^2))/(2*a^3*(b^2 - 4*
a*c)^2*x^2) - (a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(4*a*(b^2 - 4
*a*c)*x^2*(a + b*x^2 + c*x^4)^2) - (a*b*B*(b^2 - 10*a*c) - A*(3*b^4 - 20*a*
b^2*c + 20*a^2*c^2) + c*(a*B*(b^2 - 16*a*c) - 3*A*(b^3 - 6*a*b*c))*x^2)/(4*
a^2*(b^2 - 4*a*c)^2*x^2*(a + b*x^2 + c*x^4)) + ((a*b*B*(b^4 - 10*a*b^2*c +
30*a^2*c^2) - 3*A*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3))*ArcTanh
[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2)) - ((3*A*b -
a*B)*Log[x])/a^4 + ((3*A*b - a*B)*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{\mathrm{Subst}\left(\int \frac{-3Ab^2+abB+10aAc-4(Ab-2aB)cx}{x^2\left(a+bx+cx^2\right)^2}\,dx, x, x\ldots\right.}{4a\left(b^2-4ac\right)}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{abB\left(b^2-10ac\right)-A\left(3b^4-20ab^2c+20a^2c\ldots\right.}{4a^2\left(b^2-4ac\right)^2x\ldots}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{abB\left(b^2-10ac\right)-A\left(3b^4-20ab^2c+20a^2c\ldots\right.}{4a^2\left(b^2-4ac\right)^2x\ldots}$$

$$= \frac{abB\left(b^2-7ac\right)-3A\left(b^4-7ab^2c+10a^2c^2\right)}{2a^3\left(b^2-4ac\right)^2x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \ldots$$

$$= \frac{abB\left(b^2-7ac\right)-3A\left(b^4-7ab^2c+10a^2c^2\right)}{2a^3\left(b^2-4ac\right)^2x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \ldots$$

$$= \frac{abB\left(b^2-7ac\right)-3A\left(b^4-7ab^2c+10a^2c^2\right)}{2a^3\left(b^2-4ac\right)^2x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \ldots$$

$$= \frac{abB\left(b^2-7ac\right)-3A\left(b^4-7ab^2c+10a^2c^2\right)}{2a^3\left(b^2-4ac\right)^2x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \ldots$$

Mathematica [A] time = 1.50, size = 642, normalized size = 1.77

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-2*a*A)/x^2 - (a^2*(a*B*(-b^2 + 2*a*c - b*c*x^2) + A*(b^3 - 3*a*b*c + b^2
*c*x^2 - 2*a*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (a*(a*B*(2*
b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*x^2 - 14*a*b*c^2*x^2) - A*(4*b^5 -
29*a*b^3*c + 46*a^2*b*c^2 + 4*b^4*c*x^2 - 26*a*b^2*c^2*x^2 + 28*a^2*c^3*x^2
)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + 4*(-3*A*b + a*B)*Log[x] + ((-(a
*B*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c] - 8*a*b^2*c*Sqr
t[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c])) + 3*A*(b^6 - 10*a*b^4*c + 3
0*a^2*b^2*c^2 - 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4
*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^
2])/(b^2 - 4*a*c)^(5/2) + ((a*B*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt
[b^2 - 4*a*c] + 8*a*b^2*c*Sqrt[b^2 - 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])
 + 3*A*(-b^6 + 10*a*b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*
a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c]))*Log[b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/(4*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3), x]
```

fricas [B] time = 14.44, size = 3956, normalized size = 10.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*A*a^3*b^6 - 24*A*a^4*b^4*c + 96*A*a^5*b^2*c^2 - 128*A*a^6*c^3 - 2*
(120*A*a^4*c^5 + 2*(14*B*a^4*b - 57*A*a^3*b^2)*c^4 - 11*(B*a^3*b^3 - 3*A*a^
2*b^4)*c^3 + (B*a^2*b^5 - 3*A*a*b^6)*c^2)*x^8 + (8*(8*B*a^5 - 69*A*a^4*b)*c
^4 - 6*(22*B*a^4*b^2 - 81*A*a^3*b^3)*c^3 + 45*(B*a^3*b^4 - 3*A*a^2*b^5)*c^2
 - 4*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^8 + 200*A*a^5*
c^4 + 2*(2*B*a^5*b - 11*A*a^4*b^2)*c^3 + (23*B*a^4*b^3 - 79*A*a^3*b^4)*c^2
- 10*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (3*B*a^3*b^6 - 9*A*a^2*b^7 - 8*(12*
B*a^6 - 61*A*a^5*b)*c^3 + 2*(54*B*a^5*b^2 - 197*A*a^4*b^3)*c^2 - (33*B*a^4*
b^4 - 104*A*a^3*b^5)*c)*x^2 - ((60*A*a^3*c^5 + 30*(B*a^3*b - 3*A*a^2*b^2)*c
^4 - 10*(B*a^2*b^3 - 3*A*a*b^4)*c^3 + (B*a*b^5 - 3*A*b^6)*c^2)*x^10 + 2*(60
*A*a^3*b*c^4 + 30*(B*a^3*b^2 - 3*A*a^2*b^3)*c^3 - 10*(B*a^2*b^4 - 3*A*a*b^5
)*c^2 + (B*a*b^6 - 3*A*b^7)*c)*x^8 + (B*a*b^7 - 3*A*b^8 + 120*A*a^4*c^4 + 6
0*(B*a^4*b - 2*A*a^3*b^2)*c^3 + 10*(B*a^3*b^3 - 3*A*a^2*b^4)*c^2 - 8*(B*a^2
*b^5 - 3*A*a*b^6)*c)*x^6 + 2*(B*a^2*b^6 - 3*A*a*b^7 + 60*A*a^4*b*c^3 + 30*(
B*a^4*b^2 - 3*A*a^3*b^3)*c^2 - 10*(B*a^3*b^4 - 3*A*a^2*b^5)*c)*x^4 + (B*a^3
*b^5 - 3*A*a^2*b^6 + 60*A*a^5*c^3 + 30*(B*a^5*b - 3*A*a^4*b^2)*c^2 - 10*(B*
a^4*b^3 - 3*A*a^3*b^4)*c)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2
 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((
64*(B*a^4 - 3*A*a^3*b)*c^5 - 48*(B*a^3*b^2 - 3*A*a^2*b^3)*c^4 + 12*(B*a^2*b
^4 - 3*A*a*b^5)*c^3 - (B*a*b^6 - 3*A*b^7)*c^2)*x^10 + 2*(64*(B*a^4*b - 3*A*
a^3*b^2)*c^4 - 48*(B*a^3*b^3 - 3*A*a^2*b^4)*c^3 + 12*(B*a^2*b^5 - 3*A*a*b^6
)*c^2 - (B*a*b^7 - 3*A*b^8)*c)*x^8 - (B*a*b^8 - 3*A*b^9 - 128*(B*a^5 - 3*A*
a^4*b)*c^4 + 32*(B*a^4*b^2 - 3*A*a^3*b^3)*c^3 + 24*(B*a^3*b^4 - 3*A*a^2*b^5
)*c^2 - 10*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^8 - 64*(
```

```
B*a^5*b - 3*A*a^4*b^2)*c^3 + 48*(B*a^4*b^3 - 3*A*a^3*b^4)*c^2 - 12*(B*a^3*b
^5 - 3*A*a^2*b^6)*c)*x^4 - (B*a^3*b^6 - 3*A*a^2*b^7 - 64*(B*a^6 - 3*A*a^5*b
)*c^3 + 48*(B*a^5*b^2 - 3*A*a^4*b^3)*c^2 - 12*(B*a^4*b^4 - 3*A*a^3*b^5)*c)*
x^2)*log(c*x^4 + b*x^2 + a) + 4*((64*(B*a^4 - 3*A*a^3*b)*c^5 - 48*(B*a^3*b^
2 - 3*A*a^2*b^3)*c^4 + 12*(B*a^2*b^4 - 3*A*a*b^5)*c^3 - (B*a*b^6 - 3*A*b^7)
*c^2)*x^10 + 2*(64*(B*a^4*b - 3*A*a^3*b^2)*c^4 - 48*(B*a^3*b^3 - 3*A*a^2*b^
4)*c^3 + 12*(B*a^2*b^5 - 3*A*a*b^6)*c^2 - (B*a*b^7 - 3*A*b^8)*c)*x^8 - (B*a
*b^8 - 3*A*b^9 - 128*(B*a^5 - 3*A*a^4*b)*c^4 + 32*(B*a^4*b^2 - 3*A*a^3*b^3)
*c^3 + 24*(B*a^3*b^4 - 3*A*a^2*b^5)*c^2 - 10*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6
 - 2*(B*a^2*b^7 - 3*A*a*b^8 - 64*(B*a^5*b - 3*A*a^4*b^2)*c^3 + 48*(B*a^4*b^
3 - 3*A*a^3*b^4)*c^2 - 12*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (B*a^3*b^6 - 3
*A*a^2*b^7 - 64*(B*a^6 - 3*A*a^5*b)*c^3 + 48*(B*a^5*b^2 - 3*A*a^4*b^3)*c^2
- 12*(B*a^4*b^4 - 3*A*a^3*b^5)*c)*x^2)*log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*c
^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*x^10 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48
*a^6*b^3*c^3 - 64*a^7*b*c^4)*x^8 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2
 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*x^6 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b
^3*c^2 - 64*a^8*b*c^3)*x^4 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*
a^9*c^3)*x^2), -1/4*(2*A*a^3*b^6 - 24*A*a^4*b^4*c + 96*A*a^5*b^2*c^2 - 128*
A*a^6*c^3 - 2*(120*A*a^4*c^5 + 2*(14*B*a^4*b - 57*A*a^3*b^2)*c^4 - 11*(B*a^
3*b^3 - 3*A*a^2*b^4)*c^3 + (B*a^2*b^5 - 3*A*a*b^6)*c^2)*x^8 + (8*(8*B*a^5 -
 69*A*a^4*b)*c^4 - 6*(22*B*a^4*b^2 - 81*A*a^3*b^3)*c^3 + 45*(B*a^3*b^4 - 3*
A*a^2*b^5)*c^2 - 4*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^
8 + 200*A*a^5*c^4 + 2*(2*B*a^5*b - 11*A*a^4*b^2)*c^3 + (23*B*a^4*b^3 - 79*A
*a^3*b^4)*c^2 - 10*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (3*B*a^3*b^6 - 9*A*a^
2*b^7 - 8*(12*B*a^6 - 61*A*a^5*b)*c^3 + 2*(54*B*a^5*b^2 - 197*A*a^4*b^3)*c^
2 - (33*B*a^4*b^4 - 104*A*a^3*b^5)*c)*x^2 - 2*((60*A*a^3*c^5 + 30*(B*a^3*b
- 3*A*a^2*b^2)*c^4 - 10*(B*a^2*b^3 - 3*A*a*b^4)*c^3 + (B*a*b^5 - 3*A*b^6)*c
^2)*x^10 + 2*(60*A*a^3*b*c^4 + 30*(B*a^3*b^2 - 3*A*a^2*b^3)*c^3 - 10*(B*a^2
*b^4 - 3*A*a*b^5)*c^2 + (B*a*b^6 - 3*A*b^7)*c)*x^8 + (B*a*b^7 - 3*A*b^8 + 1
20*A*a^4*c^4 + 60*(B*a^4*b - 2*A*a^3*b^2)*c^3 + 10*(B*a^3*b^3 - 3*A*a^2*b^4
)*c^2 - 8*(B*a^2*b^5 - 3*A*a*b^6)*c)*x^6 + 2*(B*a^2*b^6 - 3*A*a*b^7 + 60*A*
a^4*b*c^3 + 30*(B*a^4*b^2 - 3*A*a^3*b^3)*c^2 - 10*(B*a^3*b^4 - 3*A*a^2*b^5)
*c)*x^4 + (B*a^3*b^5 - 3*A*a^2*b^6 + 60*A*a^5*c^3 + 30*(B*a^5*b - 3*A*a^4*b
^2)*c^2 - 10*(B*a^4*b^3 - 3*A*a^3*b^4)*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(
2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((64*(B*a^4 - 3*A*a^3*b)*c
^5 - 48*(B*a^3*b^2 - 3*A*a^2*b^3)*c^4 + 12*(B*a^2*b^4 - 3*A*a*b^5)*c^3 - (B
*a*b^6 - 3*A*b^7)*c^2)*x^10 + 2*(64*(B*a^4*b - 3*A*a^3*b^2)*c^4 - 48*(B*a^3
*b^3 - 3*A*a^2*b^4)*c^3 + 12*(B*a^2*b^5 - 3*A*a*b^6)*c^2 - (B*a*b^7 - 3*A*b
^8)*c)*x^8 - (B*a*b^8 - 3*A*b^9 - 128*(B*a^5 - 3*A*a^4*b)*c^4 + 32*(B*a^4*b
^2 - 3*A*a^3*b^3)*c^3 + 24*(B*a^3*b^4 - 3*A*a^2*b^5)*c^2 - 10*(B*a^2*b^6 -
3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^8 - 64*(B*a^5*b - 3*A*a^4*b^2)*c
^3 + 48*(B*a^4*b^3 - 3*A*a^3*b^4)*c^2 - 12*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4
 - (B*a^3*b^6 - 3*A*a^2*b^7 - 64*(B*a^6 - 3*A*a^5*b)*c^3 + 48*(B*a^5*b^2 -
3*A*a^4*b^3)*c^2 - 12*(B*a^4*b^4 - 3*A*a^3*b^5)*c)*x^2)*log(c*x^4 + b*x^2 +
 a) + 4*((64*(B*a^4 - 3*A*a^3*b)*c^5 - 48*(B*a^3*b^2 - 3*A*a^2*b^3)*c^4 + 1
2*(B*a^2*b^4 - 3*A*a*b^5)*c^3 - (B*a*b^6 - 3*A*b^7)*c^2)*x^10 + 2*(64*(B*a^
4*b - 3*A*a^3*b^2)*c^4 - 48*(B*a^3*b^3 - 3*A*a^2*b^4)*c^3 + 12*(B*a^2*b^5 -
 3*A*a*b^6)*c^2 - (B*a*b^7 - 3*A*b^8)*c)*x^8 - (B*a*b^8 - 3*A*b^9 - 128*(B*
a^5 - 3*A*a^4*b)*c^4 + 32*(B*a^4*b^2 - 3*A*a^3*b^3)*c^3 + 24*(B*a^3*b^4 - 3
*A*a^2*b^5)*c^2 - 10*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*
b^8 - 64*(B*a^5*b - 3*A*a^4*b^2)*c^3 + 48*(B*a^4*b^3 - 3*A*a^3*b^4)*c^2 - 1
2*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (B*a^3*b^6 - 3*A*a^2*b^7 - 64*(B*a^6 -
 3*A*a^5*b)*c^3 + 48*(B*a^5*b^2 - 3*A*a^4*b^3)*c^2 - 12*(B*a^4*b^4 - 3*A*a^
3*b^5)*c)*x^2)*log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64
*a^7*c^5)*x^10 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*
c^4)*x^8 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*
a^8*c^4)*x^6 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*x
^4 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*x^2)]
```

giac [A] time = 6.37, size = 648, normalized size = 1.79

$$-\frac{\left(Bab^5 - 3\,Ab^6 - 10\,Ba^2b^3c + 30\,Aab^4c + 30\,Ba^3bc^2 - 90\,Aa^2b^2c^2 + 60\,Aa^3c^3\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^4b^4 - 8\,a^5b^2c + 16\,a^6c^2\right)\sqrt{-b^2+4\,ac}} + \frac{3\,Bab^4c^2x^8 - 9\,Ab^5c^2x^8 - 24\,Ba^2b^2c^3x^8 + 72\,Aab^3c^3x^8 + 48\,Ba^3c^4x^8 - 144\,Aa^2bc^4x^8 + 6\,Bab^5cx^6 - 18\,Ab^6cx^6 - 44\,Ba^2b^3c^2x^6 + 136\,Aab^4c^2x^6 + 68\,Ba^3bc^3x^6 - 236\,Aa^2b^2c^3x^6 - 56\,Aa^3c^4x^6 + 3\,Bab^6x^4 - 9\,Ab^7x^4 - 10\,Ba^2b^4cx^4 + 38\,Aab^5cx^4 - 58\,Ba^3b^2c^2x^4 + 110\,Aa^2b^3c^2x^4 + 128\,Ba^4c^3x^4 - 436\,Aa^3bc^3x^4 + 10\,Ba^2b^5x^2 - 26\,Aab^6x^2 - 72\,Ba^3b^3cx^2 + 192\,Aa^2b^4cx^2 + 92\,Ba^4bc^2x^2 - 316\,Aa^3b^2c^2x^2 - 72\,Aa^4c^3x^2 + 9\,Ba^3b^4 - 19\,Aa^2b^5 - 66\,Ba^4b^2c + 144\,Aa^3b^3c + 96\,Ba^5c^2 - 260\,Aa^4bc^2}{8\left(a^4b^4 - 8\,a^5b^2c + 16\,a^6c^2\right)\left(cx^4+bx^2+a\right)^2} - \frac{(Ba - 3\,Ab)\log\left(cx^4+bx^2+a\right)}{4\,a^4} + \frac{(Ba - 3\,Ab)\log\left(x^2\right)}{2\,a^4} - \frac{Bax^2 - 3\,Abx^2 + Aa}{2\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(B*a*b^5 - 3*A*b^6 - 10*B*a^2*b^3*c + 30*A*a*b^4*c + 30*B*a^3*b*c^2 -
90*A*a^2*b^2*c^2 + 60*A*a^3*c^3)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(
(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b^2 + 4*a*c)) + 1/8*(3*B*a*b^4*c
^2*x^8 - 9*A*b^5*c^2*x^8 - 24*B*a^2*b^2*c^3*x^8 + 72*A*a*b^3*c^3*x^8 + 48*B
*a^3*c^4*x^8 - 144*A*a^2*b*c^4*x^8 + 6*B*a*b^5*c*x^6 - 18*A*b^6*c*x^6 - 44*
B*a^2*b^3*c^2*x^6 + 136*A*a*b^4*c^2*x^6 + 68*B*a^3*b*c^3*x^6 - 236*A*a^2*b^
2*c^3*x^6 - 56*A*a^3*c^4*x^6 + 3*B*a*b^6*x^4 - 9*A*b^7*x^4 - 10*B*a^2*b^4*c
*x^4 + 38*A*a*b^5*c*x^4 - 58*B*a^3*b^2*c^2*x^4 + 110*A*a^2*b^3*c^2*x^4 + 12
8*B*a^4*c^3*x^4 - 436*A*a^3*b*c^3*x^4 + 10*B*a^2*b^5*x^2 - 26*A*a*b^6*x^2 -
 72*B*a^3*b^3*c*x^2 + 192*A*a^2*b^4*c*x^2 + 92*B*a^4*b*c^2*x^2 - 316*A*a^3*
b^2*c^2*x^2 - 72*A*a^4*c^3*x^2 + 9*B*a^3*b^4 - 19*A*a^2*b^5 - 66*B*a^4*b^2*
c + 144*A*a^3*b^3*c + 96*B*a^5*c^2 - 260*A*a^4*b*c^2)/((a^4*b^4 - 8*a^5*b^2
*c + 16*a^6*c^2)*(c*x^4 + b*x^2 + a)^2) - 1/4*(B*a - 3*A*b)*log(c*x^4 + b*x
^2 + a)/a^4 + 1/2*(B*a - 3*A*b)*log(x^2)/a^4 - 1/2*(B*a*x^2 - 3*A*b*x^2 + A
*a)/(a^4*x^2)
```

maple [B] time = 0.04, size = 1862, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 6/a^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b^4*c-3/a/(c*x^4+b
*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*b^3*c+13/2/a^2/(c*x^4+b*x^2+a)^2
*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*b^2-1/a^3/(c*x^4+b*x^2+a)^2*c^2/(16*a
^2*c^2-8*a*b^2*c+b^4)*x^6*A*b^4-7/2/a/(c*x^4+b*x^2+a)^2*c^3/(16*a^2*c^2-8*a
*b^2*c+b^4)*x^6*b*B+1/2/a^2/(c*x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4
)*x^6*B*b^3-37/2/a/(c*x^4+b*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*b
+55/4/a^2/(c*x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*b^3+45/a^2
/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2
)^(1/2))*A*b^2*c^2-15/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arct
an((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^4*c-15/a/(16*a^2*c^2-8*a*b^2*c+b^4)/(
4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B*b*c^2+5/a^2/(16*a^
2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)
)*B*b^3*c-2/a^3/(c*x^4+b*x^2+a)^2*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*b^5-29
/4/a/(c*x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*B*b^2+1/a^2/(c*x^
4+b*x^2+a)^2*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*B*b^4-7/2/a/(c*x^4+b*x^2+a)^2
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b^2*c^2-3/a^4*ln(x)*A*b+3/4/a/(c*x^4+b*x^
2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*B*b^4+3/4/a^4/(16*a^2*c^2-8*a*b^2*c+b^4)*
ln(c*x^4+b*x^2+a)*A*b^5-4/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*ln(c*x^4+b*x^2+a
)*B-1/4/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*B*b^4+4/(c*x^4+b*x
^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*B-9/(c*x^4+b*x^2+a)^2/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^2*A*c^3-29/2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4
)*A*b*c^2-21/4/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*B*b^2*c-1/2/(c*
x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*b*c^2-7/a/(c*x^4+b*x^2+a)^2
*c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A-1/a^3/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^2*A*b^6+1/2/a^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4
)*x^2*B*b^5+9/a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*A*b^3*c+12/a^2
/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*ln(c*x^4+b*x^2+a)*A*b-6/a^3/(16*a^2*c^2-8*a
*b^2*c+b^4)*c*ln(c*x^4+b*x^2+a)*A*b^3+2/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c*ln
(c*x^4+b*x^2+a)*B*b^2-30/a/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*c^3-1/2/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)
```

```
/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B*b^5+3/2/a^4/(16*
a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/
2))*A*b^6-5/4/a^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*A*b^5+6*a/(c
*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*B*c^2-1/2*A/a^3/x^2+1/a^3*ln(x)*
B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 15.91, size = 16265, normalized size = 44.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3),x)
```

```
[Out] (log(((c^5*x^2*(3*A*b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 7*B*a^2*b
*c)^3)/(a^9*(4*a*c - b^2)^6) - ((B*a - 3*A*b + a^4*(-(60*A*a^3*c^3 - 3*A*b^
6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2
*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(((B*a - 3*A*b + a^4*(-(60*A*a^3*c^3
- 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A
*a^2*b^2*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*((4*b*c^2*(30*A*a^3*c^3 - 3*A
*b^6 + B*a*b^5 + 27*A*a*b^4*c - 9*B*a^2*b^3*c + 23*B*a^3*b*c^2 - 69*A*a^2*b
^2*c^2))/(a^3*(4*a*c - b^2)^2) + (2*c^3*x^2*(B*a*b^5 - 300*A*a^3*c^3 - 3*A*
b^6 + 6*A*a*b^4*c - 2*B*a^2*b^3*c + 10*B*a^3*b*c^2 + 90*A*a^2*b^2*c^2))/(a^
3*(4*a*c - b^2)^2) + (b*c^2*(B*a - 3*A*b + a^4*(-(60*A*a^3*c^3 - 3*A*b^6 +
B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2
)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^4))/(4*
a^4) + (c^3*(900*A^2*a^5*c^5 - 36*A^2*b^10 - 4*B^2*a^2*b^8 + 24*A*B*a*b^9 -
 3078*A^2*a^2*b^6*c^2 + 7533*A^2*a^3*b^4*c^3 - 7020*A^2*a^4*b^2*c^4 - 302*B
^2*a^4*b^4*c^2 + 497*B^2*a^5*b^2*c^3 + 549*A^2*a*b^8*c + 61*B^2*a^3*b^6*c +
 1932*A*B*a^3*b^5*c^2 - 4002*A*B*a^4*b^3*c^3 - 366*A*B*a^2*b^7*c + 2340*A*B
*a^5*b*c^4))/(a^6*(4*a*c - b^2)^4) - (c^4*x^2*(54*A^2*b^9 + 6*B^2*a^2*b^7 -
 36*A*B*a*b^8 + 4311*A^2*a^2*b^5*c^2 - 9900*A^2*a^3*b^3*c^3 + 409*B^2*a^4*b
^3*c^2 - 2400*A*B*a^5*c^4 - 801*A^2*a*b^7*c + 8100*A^2*a^4*b*c^4 - 89*B^2*a
^3*b^5*c - 560*B^2*a^5*b*c^3 - 2664*A*B*a^3*b^4*c^2 + 4980*A*B*a^4*b^2*c^3
+ 534*A*B*a^2*b^6*c))/(a^6*(4*a*c - b^2)^4)))/(4*a^4) + (c^4*(3*A*b - B*a)*
(3*A*b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 7*B*a^2*b*c)^2)/(a^9*(4*
a*c - b^2)^4))*((c^5*x^2*(3*A*b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c +
 7*B*a^2*b*c)^3)/(a^9*(4*a*c - b^2)^6) - ((3*A*b - B*a + a^4*(-(60*A*a^3*c^
3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90
*A*a^2*b^2*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(((3*A*b - B*a + a^4*(-(60*
A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*
c^2 - 90*A*a^2*b^2*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*((4*b*c^2*(30*A*a^3
*c^3 - 3*A*b^6 + B*a*b^5 + 27*A*a*b^4*c - 9*B*a^2*b^3*c + 23*B*a^3*b*c^2 -
69*A*a^2*b^2*c^2))/(a^3*(4*a*c - b^2)^2) + (2*c^3*x^2*(B*a*b^5 - 300*A*a^3*
c^3 - 3*A*b^6 + 6*A*a*b^4*c - 2*B*a^2*b^3*c + 10*B*a^3*b*c^2 + 90*A*a^2*b^2
*c^2))/(a^3*(4*a*c - b^2)^2) - (b*c^2*(3*A*b - B*a + a^4*(-(60*A*a^3*c^3 -
3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a
^2*b^2*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))
/a^4))/(4*a^4) - (c^3*(900*A^2*a^5*c^5 - 36*A^2*b^10 - 4*B^2*a^2*b^8 + 24*A
*B*a*b^9 - 3078*A^2*a^2*b^6*c^2 + 7533*A^2*a^3*b^4*c^3 - 7020*A^2*a^4*b^2*c
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^4 - 302*B^2*a^4*b^4*c^2 + 497*B^2*a^5*b^2*c^3 + 549*A^2*a*b^8*c + 61*B^2*a
^3*b^6*c + 1932*A*B*a^3*b^5*c^2 - 4002*A*B*a^4*b^3*c^3 - 366*A*B*a^2*b^7*c
+ 2340*A*B*a^5*b*c^4))/(a^6*(4*a*c - b^2)^4) + (c^4*x^2*(54*A^2*b^9 + 6*B^2
*a^2*b^7 - 36*A*B*a*b^8 + 4311*A^2*a^2*b^5*c^2 - 9900*A^2*a^3*b^3*c^3 + 409
*B^2*a^4*b^3*c^2 - 2400*A*B*a^5*c^4 - 801*A^2*a*b^7*c + 8100*A^2*a^4*b*c^4
- 89*B^2*a^3*b^5*c - 560*B^2*a^5*b*c^3 - 2664*A*B*a^3*b^4*c^2 + 4980*A*B*a^
4*b^2*c^3 + 534*A*B*a^2*b^6*c))/(a^6*(4*a*c - b^2)^4)))/(4*a^4) + (c^4*(3*A
*b - B*a)*(3*A*b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 7*B*a^2*b*c)^2
)/(a^9*(4*a*c - b^2)^4)))*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a
*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3
*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 25
60*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b
^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)) - (log(x)*(3*A*b - B*a))/a^4
 - (A/(2*a) + (x^2*(9*A*b^5 - 24*B*a^3*c^2 - 3*B*a*b^4 - 68*A*a*b^3*c + 122
*A*a^2*b*c^2 + 21*B*a^2*b^2*c))/(4*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x
^6*(16*B*a^3*c^3 - 12*A*b^5*c + 4*B*a*b^4*c + 87*A*a*b^3*c^2 - 138*A*a^2*b*
c^3 - 29*B*a^2*b^2*c^2))/(4*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^4*(3*A
*b^6 + 50*A*a^3*c^3 - B*a*b^5 - 18*A*a*b^4*c + 6*B*a^2*b^3*c + B*a^3*b*c^2
+ 7*A*a^2*b^2*c^2))/(2*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c^2*x^8*(3*A*
b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 7*B*a^2*b*c))/(2*a^3*(b^4 + 1
6*a^2*c^2 - 8*a*b^2*c)))/(x^6*(2*a*c + b^2) + a^2*x^2 + c^2*x^10 + 2*a*b*x^
4 + 2*b*c*x^8) - (atan((x^2*((((((((153600*A*a^13*c^10 - 5120*B*a^13*b*c^9
+ 6*A*a^6*b^14*c^3 - 108*A*a^7*b^12*c^4 + 588*A*a^8*b^10*c^5 + 792*A*a^9*b^
8*c^6 - 22272*A*a^10*b^6*c^7 + 100608*A*a^11*b^4*c^8 - 199680*A*a^12*b^2*c^
9 - 2*B*a^7*b^13*c^3 + 36*B*a^8*b^11*c^4 - 276*B*a^9*b^9*c^5 + 1216*B*a^10*
b^7*c^6 - 3456*B*a^11*b^5*c^7 + 6144*B*a^12*b^3*c^8)/(a^9*b^12 + 4096*a^15*
c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4
*c^4 - 6144*a^14*b^2*c^5) - ((163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^1
0*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 +
 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*
B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^
7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*
B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5
*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 +
 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 38
40*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 +
30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2))/(4*a^4*
(4*a*c - b^2)^(5/2)) - ((60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c -
10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2)*(163840*a^16*b*c^9 - 12
*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5
 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8)*(6*A*b^1
1 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a
^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 32
0*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(8*a^4*(4*a*c -
 b^2)^(5/2)*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2
560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^1
0*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*
b^2*c^5)))*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A
*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680
*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^
4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a
^7*b^4*c^3 + 5120*a^8*b^2*c^4)) - (((54*A^2*a^3*b^13*c^4 - 1233*A^2*a^4*b^1
1*c^5 + 11583*A^2*a^5*b^9*c^6 - 57204*A^2*a^6*b^7*c^7 + 156276*A^2*a^7*b^5*
c^8 - 223200*A^2*a^8*b^3*c^9 + 6*B^2*a^5*b^11*c^4 - 137*B^2*a^6*b^9*c^5 + 1
217*B^2*a^7*b^7*c^6 - 5256*B^2*a^8*b^5*c^7 + 11024*B^2*a^9*b^3*c^8 - 38400*
A*B*a^10*c^10 + 129600*A^2*a^9*b*c^10 - 8960*B^2*a^10*b*c^9 - 36*A*B*a^4*b^
12*c^4 + 822*A*B*a^5*b^10*c^5 - 7512*A*B*a^6*b^8*c^6 + 34836*A*B*a^7*b^6*c^
7 - 84864*A*B*a^8*b^4*c^8 + 98880*A*B*a^9*b^2*c^9)/(a^9*b^12 + 4096*a^15*c^
6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c
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^4 - 6144*a^14*b^2*c^5) - (((153600*A*a^13*c^10 - 5120*B*a^13*b*c^9 + 6*A*a
^6*b^14*c^3 - 108*A*a^7*b^12*c^4 + 588*A*a^8*b^10*c^5 + 792*A*a^9*b^8*c^6 -
 22272*A*a^10*b^6*c^7 + 100608*A*a^11*b^4*c^8 - 199680*A*a^12*b^2*c^9 - 2*B
*a^7*b^13*c^3 + 36*B*a^8*b^11*c^4 - 276*B*a^9*b^9*c^5 + 1216*B*a^10*b^7*c^6
 - 3456*B*a^11*b^5*c^7 + 6144*B*a^12*b^3*c^8)/(a^9*b^12 + 4096*a^15*c^6 - 2
4*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 -
6144*a^14*b^2*c^5) - ((163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*
c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328
*a^14*b^5*c^7 - 294912*a^15*b^3*c^8)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^1
0 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 -
 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b
^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c
+ 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 4096*a
^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13
*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 1
20*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840
*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^
3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640
*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(60*A*a^3*c^3 - 3*A*b
^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^
2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) + ((60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 +
30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2)^3*(16384
0*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^11*c^4 + 2
4960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 294912*a^15*
b^3*c^8))/(64*a^12*(4*a*c - b^2)^(15/2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10
*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a
^14*b^2*c^5)))*(9*A*b^9 - 160*B*a^5*c^4 - 3*B*a*b^8 - 117*A*a*b^7*c + 570*A
*a^4*b*c^4 + 39*B*a^2*b^6*c + 540*A*a^2*b^5*c^2 - 1005*A*a^3*b^3*c^3 - 180*
B*a^3*b^4*c^2 + 325*B*a^4*b^2*c^3))/(8*(4*a*c - b^2)^(13/2)*(900*A^2*a^9*c^
8 + 6400*B^2*a^10*c^7 - 54*A^2*a^3*b^12*c^2 - 960*a^6*b^6*c^4*(B^2*a - 36*A
^2*c) + 120*a^5*b^8*c^3*(B^2*a - 72*A^2*c) - 6*a^4*b^10*c^2*(B^2*a - 180*A^
2*c) - 25*a^8*b^2*c^6*(311*B^2*a - 2196*A^2*c) + 25*a^7*b^4*c^5*(154*B^2*a
- 2763*A^2*c) + 36*A*B*a^4*b^11*c^2 - 720*A*B*a^5*b^9*c^3 + 5760*A*B*a^6*b^
7*c^4 - 23070*A*B*a^7*b^5*c^5 + 46350*A*B*a^8*b^3*c^6 - 37500*A*B*a^9*b*c^7
)) - (((27000*A^3*a^6*c^11 + 27*A^3*b^12*c^5 + 4779*A^3*a^2*b^8*c^7 - 20601
*A^3*a^3*b^6*c^8 + 47790*A^3*a^4*b^4*c^9 - 56700*A^3*a^5*b^2*c^10 - B^3*a^3
*b^9*c^5 + 21*B^3*a^4*b^7*c^6 - 147*B^3*a^5*b^5*c^7 + 343*B^3*a^6*b^3*c^8 -
 567*A^3*a*b^10*c^6 - 27*A^2*B*a*b^11*c^5 + 18900*A^2*B*a^6*b*c^10 + 9*A*B^
2*a^2*b^10*c^5 - 189*A*B^2*a^3*b^8*c^6 + 1413*A*B^2*a^4*b^6*c^7 - 4347*A*B^
2*a^5*b^4*c^8 + 4410*A*B^2*a^6*b^2*c^9 + 567*A^2*B*a^2*b^9*c^6 - 4509*A^2*B
*a^3*b^7*c^7 + 16821*A^2*B*a^4*b^5*c^8 - 29160*A^2*B*a^5*b^3*c^9)/(a^9*b^12
 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 +
3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((54*A^2*a^3*b^13*c^4 - 1233*A^2*
a^4*b^11*c^5 + 11583*A^2*a^5*b^9*c^6 - 57204*A^2*a^6*b^7*c^7 + 156276*A^2*a
^7*b^5*c^8 - 223200*A^2*a^8*b^3*c^9 + 6*B^2*a^5*b^11*c^4 - 137*B^2*a^6*b^9*
c^5 + 1217*B^2*a^7*b^7*c^6 - 5256*B^2*a^8*b^5*c^7 + 11024*B^2*a^9*b^3*c^8 -
 38400*A*B*a^10*c^10 + 129600*A^2*a^9*b*c^10 - 8960*B^2*a^10*b*c^9 - 36*A*B
*a^4*b^12*c^4 + 822*A*B*a^5*b^10*c^5 - 7512*A*B*a^6*b^8*c^6 + 34836*A*B*a^7
*b^6*c^7 - 84864*A*B*a^8*b^4*c^8 + 98880*A*B*a^9*b^2*c^9)/(a^9*b^12 + 4096*
a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^1
3*b^4*c^4 - 6144*a^14*b^2*c^5) - (((153600*A*a^13*c^10 - 5120*B*a^13*b*c^9
+ 6*A*a^6*b^14*c^3 - 108*A*a^7*b^12*c^4 + 588*A*a^8*b^10*c^5 + 792*A*a^9*b^
8*c^6 - 22272*A*a^10*b^6*c^7 + 100608*A*a^11*b^4*c^8 - 199680*A*a^12*b^2*c^
9 - 2*B*a^7*b^13*c^3 + 36*B*a^8*b^11*c^4 - 276*B*a^9*b^9*c^5 + 1216*B*a^10*
b^7*c^6 - 3456*B*a^11*b^5*c^7 + 6144*B*a^12*b^3*c^8)/(a^9*b^12 + 4096*a^15*
c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4
*c^4 - 6144*a^14*b^2*c^5) - ((163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^1
0*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 +
 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*
```

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B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^
7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*
B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5
*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 +
 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 38
40*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b
^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2
 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4
*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*
c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*A*b^11 + 20
48*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8
*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^
3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096
*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2
*c^4)) - (((((153600*A*a^13*c^10 - 5120*B*a^13*b*c^9 + 6*A*a^6*b^14*c^3 - 1
08*A*a^7*b^12*c^4 + 588*A*a^8*b^10*c^5 + 792*A*a^9*b^8*c^6 - 22272*A*a^10*b
^6*c^7 + 100608*A*a^11*b^4*c^8 - 199680*A*a^12*b^2*c^9 - 2*B*a^7*b^13*c^3 +
 36*B*a^8*b^11*c^4 - 276*B*a^9*b^9*c^5 + 1216*B*a^10*b^7*c^6 - 3456*B*a^11*
b^5*c^7 + 6144*B*a^12*b^3*c^8)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c +
 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c
^5) - ((163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11
*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 -
 294912*a^15*b^3*c^8)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9
*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5
*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B
*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c
^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^
10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144
*a^14*b^2*c^5)))*(60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^
2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) -
 ((60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*
a^3*b*c^2 - 90*A*a^2*b^2*c^2)*(163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^
10*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6
+ 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8)*(6*A*b^11 + 2048*B*a^6*c^5 - 2
*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b
^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280
*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(8*a^4*(4*a*c - b^2)^(5/2)*(4*a^4*b^1
0 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120
*a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2
 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(60*A*a^3*c
^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 9
0*A*a^2*b^2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) + ((60*A*a^3*c^3 - 3*A*b^6 +
B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2
)^2*(163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^
11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 29
4912*a^15*b^3*c^8)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c
- 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^
3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^
5*b^2*c^4))/(32*a^8*(4*a*c - b^2)^5*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8
*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 409
6*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a
^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(9*A*b^8 + 30*A*a^4*c^4 - 3*B*a*b^7 - 99
*A*a*b^6*c + 33*B*a^2*b^5*c + 135*B*a^4*b*c^3 + 360*A*a^2*b^4*c^2 - 435*A*a
^3*b^2*c^3 - 120*B*a^3*b^3*c^2))/(8*a^3*c^2*(4*a*c - b^2)^6*(900*A^2*a^6*c^
6 - 54*A^2*b^12 - 6*B^2*a^2*b^10 + 6400*B^2*a^7*c^5 + 36*A*B*a*b^11 - 8640*
A^2*a^2*b^8*c^2 + 34560*A^2*a^3*b^6*c^3 - 69075*A^2*a^4*b^4*c^4 + 54900*A^2
*a^5*b^2*c^5 - 960*B^2*a^4*b^6*c^2 + 3850*B^2*a^5*b^4*c^3 - 7775*B^2*a^6*b^
2*c^4 + 1080*A^2*a*b^10*c + 120*B^2*a^3*b^8*c + 5760*A*B*a^3*b^7*c^2 - 2307
0*A*B*a^4*b^5*c^3 + 46350*A*B*a^5*b^3*c^4 - 720*A*B*a^2*b^9*c - 37500*A*B*a
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^6*b*c^5)))*(16*a^12*b^12*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*(4*a*c - b^
2)^(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*(4*a*c
 - b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c
^4*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*a*c - b^2)^(15/2)))/(3600*A
^2*a^6*c^8 + 9*A^2*b^12*c^2 + 1440*A^2*a^2*b^8*c^4 - 5760*A^2*a^3*b^6*c^5 +
 11700*A^2*a^4*b^4*c^6 - 10800*A^2*a^5*b^2*c^7 + B^2*a^2*b^10*c^2 - 20*B^2*
a^3*b^8*c^3 + 160*B^2*a^4*b^6*c^4 - 600*B^2*a^5*b^4*c^5 + 900*B^2*a^6*b^2*c
^6 - 180*A^2*a*b^10*c^3 + 120*A*B*a^2*b^9*c^3 - 960*A*B*a^3*b^7*c^4 + 3720*
A*B*a^4*b^5*c^5 - 6600*A*B*a^5*b^3*c^6 - 6*A*B*a*b^11*c^2 + 3600*A*B*a^6*b*
c^7) - (((((((1920*A*a^11*b*c^7 - 12*A*a^6*b^11*c^2 + 204*A*a^7*b^9*c^3 - 1
332*A*a^8*b^7*c^4 + 4056*A*a^9*b^5*c^5 - 5376*A*a^10*b^3*c^6 + 4*B*a^7*b^10
*c^2 - 68*B*a^8*b^8*c^3 + 444*B*a^9*b^6*c^4 - 1312*B*a^10*b^4*c^5 + 1472*B*
a^11*b^2*c^6)/(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 2
56*a^12*b^2*c^3) - ((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 -
 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^6)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*
b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^
2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^
4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c
 + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*
b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(60*A*a^3*
c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 -
90*A*a^2*b^2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) - ((4*a^10*b^10*c^2 - 64*a^1
1*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^6)*(60*A
*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c
^2 - 90*A*a^2*b^2*c^2)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^
9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^
5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*
B*a^5*b^2*c^4))/(8*a^4*(4*a*c - b^2)^(5/2)*(a^9*b^8 + 256*a^13*c^4 - 16*a^1
0*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^9*c^5 -
80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*
A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 4
0*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4
 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*
b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5
120*a^8*b^2*c^4)) - (((900*A^2*a^8*c^8 - 36*A^2*a^3*b^10*c^3 + 549*A^2*a^4*
b^8*c^4 - 3078*A^2*a^5*b^6*c^5 + 7533*A^2*a^6*b^4*c^6 - 7020*A^2*a^7*b^2*c^
7 - 4*B^2*a^5*b^8*c^3 + 61*B^2*a^6*b^6*c^4 - 302*B^2*a^7*b^4*c^5 + 497*B^2*
a^8*b^2*c^6 + 24*A*B*a^4*b^9*c^3 - 366*A*B*a^5*b^7*c^4 + 1932*A*B*a^6*b^5*c
^5 - 4002*A*B*a^7*b^3*c^6 + 2340*A*B*a^8*b*c^7)/(a^9*b^8 + 256*a^13*c^4 - 1
6*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) - (((1920*A*a^11*b*c^7 -
 12*A*a^6*b^11*c^2 + 204*A*a^7*b^9*c^3 - 1332*A*a^8*b^7*c^4 + 4056*A*a^9*b^
5*c^5 - 5376*A*a^10*b^3*c^6 + 4*B*a^7*b^10*c^2 - 68*B*a^8*b^8*c^3 + 444*B*a
^9*b^6*c^4 - 1312*B*a^10*b^4*c^5 + 1472*B*a^11*b^2*c^6)/(a^9*b^8 + 256*a^13
*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) - ((4*a^10*b^10*
c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^
2*c^6)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5
*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a
^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/
(2*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2
*c^3)*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^
7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 1
20*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840
*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^
3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640
*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(60*A*a^3*c^3 - 3*A*b
^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^
2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) + ((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 +
 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^6)*(60*A*a^3*c^3 -
3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a
```

^2*b^2*c^2)^3)/(64*a^12*(4*a*c - b^2)^(15/2)*(a^9*b^8 + 256*a^13*c^4 - 16*a
^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))*(16*a^12*b^12*(4*a*c - b^
2)^(15/2) + 65536*a^18*c^6*(4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*(4*a*c -
b^2)^(15/2) + 3840*a^14*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*(
4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^17*
b^2*c^5*(4*a*c - b^2)^(15/2))*(9*A*b^9 - 160*B*a^5*c^4 - 3*B*a*b^8 - 117*A*
a*b^7*c + 570*A*a^4*b*c^4 + 39*B*a^2*b^6*c + 540*A*a^2*b^5*c^2 - 1005*A*a^3
*b^3*c^3 - 180*B*a^3*b^4*c^2 + 325*B*a^4*b^2*c^3))/(8*(4*a*c - b^2)^(13/2)*
(900*A^2*a^9*c^8 + 6400*B^2*a^10*c^7 - 54*A^2*a^3*b^12*c^2 - 960*a^6*b^6*c^
4*(B^2*a - 36*A^2*c) + 120*a^5*b^8*c^3*(B^2*a - 72*A^2*c) - 6*a^4*b^10*c^2*
(B^2*a - 180*A^2*c) - 25*a^8*b^2*c^6*(311*B^2*a - 2196*A^2*c) + 25*a^7*b^4*
c^5*(154*B^2*a - 2763*A^2*c) + 36*A*B*a^4*b^11*c^2 - 720*A*B*a^5*b^9*c^3 +
5760*A*B*a^6*b^7*c^4 - 23070*A*B*a^7*b^5*c^5 + 46350*A*B*a^8*b^3*c^6 - 3750
0*A*B*a^9*b*c^7)*(3600*A^2*a^6*c^8 + 9*A^2*b^12*c^2 + 1440*A^2*a^2*b^8*c^4
- 5760*A^2*a^3*b^6*c^5 + 11700*A^2*a^4*b^4*c^6 - 10800*A^2*a^5*b^2*c^7 + B^
2*a^2*b^10*c^2 - 20*B^2*a^3*b^8*c^3 + 160*B^2*a^4*b^6*c^4 - 600*B^2*a^5*b^4
*c^5 + 900*B^2*a^6*b^2*c^6 - 180*A^2*a*b^10*c^3 + 120*A*B*a^2*b^9*c^3 - 960
*A*B*a^3*b^7*c^4 + 3720*A*B*a^4*b^5*c^5 - 6600*A*B*a^5*b^3*c^6 - 6*A*B*a*b^
11*c^2 + 3600*A*B*a^6*b*c^7)) + (((3780*A^3*a^3*b^3*c^7 - 1863*A^3*a^2*b^5*
c^6 - 27*A^3*b^9*c^4 + B^3*a^3*b^6*c^4 - 14*B^3*a^4*b^4*c^5 + 49*B^3*a^5*b^
2*c^6 + 900*A^2*B*a^5*c^8 + 378*A^3*a*b^7*c^5 - 2700*A^3*a^4*b*c^8 + 420*A*
B^2*a^5*b*c^7 + 27*A^2*B*a*b^8*c^4 - 9*A*B^2*a^2*b^7*c^4 + 126*A*B^2*a^3*b^
5*c^5 - 501*A*B^2*a^4*b^3*c^6 - 378*A^2*B*a^2*b^6*c^5 + 1683*A^2*B*a^3*b^4*
c^6 - 2520*A^2*B*a^4*b^2*c^7)/(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*
a^11*b^4*c^2 - 256*a^12*b^2*c^3) - (((900*A^2*a^8*c^8 - 36*A^2*a^3*b^10*c^3
 + 549*A^2*a^4*b^8*c^4 - 3078*A^2*a^5*b^6*c^5 + 7533*A^2*a^6*b^4*c^6 - 7020
*A^2*a^7*b^2*c^7 - 4*B^2*a^5*b^8*c^3 + 61*B^2*a^6*b^6*c^4 - 302*B^2*a^7*b^4
*c^5 + 497*B^2*a^8*b^2*c^6 + 24*A*B*a^4*b^9*c^3 - 366*A*B*a^5*b^7*c^4 + 193
2*A*B*a^6*b^5*c^5 - 4002*A*B*a^7*b^3*c^6 + 2340*A*B*a^8*b*c^7)/(a^9*b^8 + 2
56*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) - (((1920
*A*a^11*b*c^7 - 12*A*a^6*b^11*c^2 + 204*A*a^7*b^9*c^3 - 1332*A*a^8*b^7*c^4
+ 4056*A*a^9*b^5*c^5 - 5376*A*a^10*b^3*c^6 + 4*B*a^7*b^10*c^2 - 68*B*a^8*b^
8*c^3 + 444*B*a^9*b^6*c^4 - 1312*B*a^10*b^4*c^5 + 1472*B*a^11*b^2*c^6)/(a^9
*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) -
 ((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5
 + 1024*a^14*b^2*c^6)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9
*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5
*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B
*a^5*b^2*c^4))/(2*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2
 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^
6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*A*b^11 + 2048*B*a^6*c^5 -
 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2
*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 12
80*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*
a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*A*b
^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B
*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 -
320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^1
0 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120
*a^8*b^2*c^4)) - (((((1920*A*a^11*b*c^7 - 12*A*a^6*b^11*c^2 + 204*A*a^7*b^9
*c^3 - 1332*A*a^8*b^7*c^4 + 4056*A*a^9*b^5*c^5 - 5376*A*a^10*b^3*c^6 + 4*B*
a^7*b^10*c^2 - 68*B*a^8*b^8*c^3 + 444*B*a^9*b^6*c^4 - 1312*B*a^10*b^4*c^5 +
 1472*B*a^11*b^2*c^6)/(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4
*c^2 - 256*a^12*b^2*c^3) - ((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b
^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^6)*(6*A*b^11 + 2048*B*a^6*c^5
- 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^
2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1
280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(a^9*b^8 + 256*a^13*c^4 - 16*a^
10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^9*c^5 -

```
 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6
0*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*
b*c^2 - 90*A*a^2*b^2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) - ((4*a^10*b^10*c^2
- 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^
6)*(60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B
*a^3*b*c^2 - 90*A*a^2*b^2*c^2)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 12
0*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*
A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3
 - 2560*B*a^5*b^2*c^4))/(8*a^4*(4*a*c - b^2)^(5/2)*(a^9*b^8 + 256*a^13*c^4
- 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^
9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^
4)))*(60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30
*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) + ((4*a^10*b^
10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14
*b^2*c^6)*(60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c
 + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2)^2*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*
b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^
2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^
4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(32*a^8*(4*a*c - b^2)^5*(a^9*b^8 + 256*a^1
3*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4
096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*
b^2*c^4)))*(16*a^12*b^12*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*(4*a*c - b^2
)^(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*(4*a*c
- b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^
4*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*a*c - b^2)^(15/2))*(9*A*b^8
+ 30*A*a^4*c^4 - 3*B*a*b^7 - 99*A*a*b^6*c + 33*B*a^2*b^5*c + 135*B*a^4*b*c^
3 + 360*A*a^2*b^4*c^2 - 435*A*a^3*b^2*c^3 - 120*B*a^3*b^3*c^2))/(8*a^3*c^2*
(4*a*c - b^2)^6*(3600*A^2*a^6*c^8 + 9*A^2*b^12*c^2 + 1440*A^2*a^2*b^8*c^4 -
 5760*A^2*a^3*b^6*c^5 + 11700*A^2*a^4*b^4*c^6 - 10800*A^2*a^5*b^2*c^7 + B^2
*a^2*b^10*c^2 - 20*B^2*a^3*b^8*c^3 + 160*B^2*a^4*b^6*c^4 - 600*B^2*a^5*b^4*
c^5 + 900*B^2*a^6*b^2*c^6 - 180*A^2*a*b^10*c^3 + 120*A*B*a^2*b^9*c^3 - 960*
A*B*a^3*b^7*c^4 + 3720*A*B*a^4*b^5*c^5 - 6600*A*B*a^5*b^3*c^6 - 6*A*B*a*b^1
1*c^2 + 3600*A*B*a^6*b*c^7)*(900*A^2*a^6*c^6 - 54*A^2*b^12 - 6*B^2*a^2*b^10
 + 6400*B^2*a^7*c^5 + 36*A*B*a*b^11 - 8640*A^2*a^2*b^8*c^2 + 34560*A^2*a^3*
b^6*c^3 - 69075*A^2*a^4*b^4*c^4 + 54900*A^2*a^5*b^2*c^5 - 960*B^2*a^4*b^6*c
^2 + 3850*B^2*a^5*b^4*c^3 - 7775*B^2*a^6*b^2*c^4 + 1080*A^2*a*b^10*c + 120*
B^2*a^3*b^8*c + 5760*A*B*a^3*b^7*c^2 - 23070*A*B*a^4*b^5*c^3 + 46350*A*B*a^
5*b^3*c^4 - 720*A*B*a^2*b^9*c - 37500*A*B*a^6*b*c^5)))*(60*A*a^3*c^3 - 3*A*
b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b
^2*c^2))/(2*a^4*(4*a*c - b^2)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.110 $\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=554

$$\frac{\left(-\frac{-40a^2Ac^3+132a^2bBc^2-18aAb^2c^2-33ab^3Bc+Ab^4c+3b^5B}{\sqrt{b^2-4ac}}+84a^2Bc^2-16aAbc^2-27ab^2Bc+Ab^3c+3b^4B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\ }}{\sqrt{b-\sqrt{b^2}}}\right.}{8\sqrt{2}c^{5/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}$$

Rubi [A] time = 11.19, antiderivative size = 554, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1275, 1279, 1166, 205}

$$\frac{\left(-\frac{-40a^2Ac^3+132a^2bBc^2-18aAb^2c^2-33ab^3Bc+Ab^4c+3b^5B}{\sqrt{b^2-4ac}}+84a^2Bc^2-16aAbc^2-27ab^2Bc+Ab^3c+3b^4B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{5/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2Ac^3+132a^2bBc^2-18aAb^2c^2-33ab^3Bc+Ab^4c+3b^5B}{\sqrt{b^2-4ac}}+84a^2Bc^2-16aAbc^2-27ab^2Bc+Ab^3c+3b^4B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}c^{5/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x\left(20aAc^2-24abBc+Ab^2c+3b^3B\right)}{8c^2\left(b^2-4ac\right)^2}-\frac{x^7\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x^5\left(x^2\left(-28aBc+12Abc+b^2B\right)-4aAc-12abB+7Ab^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^3\left(-28aBc+12Abc+b^2B\right)}{8c\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -((3*b^3*B + A*b^2*c - 24*a*b*B*c + 20*a*A*c^2)*x)/(8*c^2*(b^2 - 4*a*c)^2)
+ ((b^2*B + 12*A*b*c - 28*a*B*c)*x^3)/(8*c*(b^2 - 4*a*c)^2) - (x^7*(A*b - 2
*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^5*(
7*A*b^2 - 12*a*b*B - 4*a*A*c + (b^2*B + 12*A*b*c - 28*a*B*c)*x^2))/(8*(b^2
- 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((3*b^4*B + A*b^3*c - 27*a*b^2*B*c - 16*a
*A*b*c^2 + 84*a^2*B*c^2 - (3*b^5*B + A*b^4*c - 33*a*b^3*B*c - 18*a*A*b^2*c^
2 + 132*a^2*b*B*c^2 - 40*a^2*A*c^3)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)^2*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) + ((3*b^4*B + A*b^3*c - 27*a*b^2*B*c - 16*a*A*b*c
^2 + 84*a^2*B*c^2 + (3*b^5*B + A*b^4*c - 33*a*b^3*B*c - 18*a*A*b^2*c^2 + 13
2*a^2*b*B*c^2 - 40*a^2*A*c^3)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b +
Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^8\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx = -\frac{x^7\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\int \frac{x^6\left(7(Ab-2aB)+(-bB+2Ac)x^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= -\frac{x^7\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x^5\left(7Ab^2-12abB-4aAc+\left(b^2B+12Abc-28aBc\right)\right.}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= \frac{\left(b^2B+12Abc-28aBc\right)x^3}{8c\left(b^2-4ac\right)^2}-\frac{x^7\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x^5\left(7Ab^2-12abB-4aA\right.}{8\left(b^2-4ac\right.}$$

$$= -\frac{\left(3b^3B+Ab^2c-24abBc+20aAc^2\right)x}{8c^2\left(b^2-4ac\right)^2}+\frac{\left(b^2B+12Abc-28aBc\right)x^3}{8c\left(b^2-4ac\right)^2}-\frac{x^7\left(Ab-2aB-\right.}{4\left(b^2-4ac\right)\left(a\right.}$$

$$= -\frac{\left(3b^3B+Ab^2c-24abBc+20aAc^2\right)x}{8c^2\left(b^2-4ac\right)^2}+\frac{\left(b^2B+12Abc-28aBc\right)x^3}{8c\left(b^2-4ac\right)^2}-\frac{x^7\left(Ab-2aB-\right.}{4\left(b^2-4ac\right)\left(a\right.}$$

$$= -\frac{\left(3b^3B+Ab^2c-24abBc+20aAc^2\right)x}{8c^2\left(b^2-4ac\right)^2}+\frac{\left(b^2B+12Abc-28aBc\right)x^3}{8c\left(b^2-4ac\right)^2}-\frac{x^7\left(Ab-2aB-\right.}{4\left(b^2-4ac\right)\left(a\right.}$$

Mathematica [A] time = 2.40, size = 644, normalized size = 1.16

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*x*(2*b^5*B - b^4*c*(2*A + 5*B*x^2) - 4*a^2*c^3*(9*A + 11*B*x^2) + a*b^2
*c^2*(11*A + 37*B*x^2) + 16*a*b*c^2*(3*a*B - A*c*x^2) + b^3*c*(-17*a*B + A*
c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (4*x*(b^3*(b*B - A*c)*x^2
+ a^2*c*(-3*b*B + 2*c*(A + B*x^2)) + a*b*(b^2*B + 3*A*c^2*x^2 - b*c*(A + 4*
B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[2]*Sqrt[c]*(-3*b^5*
B + b^3*c*(33*a*B + A*Sqrt[b^2 - 4*a*c]) - 4*a*b*c^2*(33*a*B + 4*A*Sqrt[b^2
 - 4*a*c]) + 9*a*b^2*c*(2*A*c - 3*B*Sqrt[b^2 - 4*a*c]) + b^4*(-(A*c) + 3*B*
Sqrt[b^2 - 4*a*c]) + 4*a^2*c^2*(10*A*c + 21*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b
 - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^5*B + 4*a*b*c^2*(33*a*B - 4*
A*Sqrt[b^2 - 4*a*c]) + b^4*(A*c + 3*B*Sqrt[b^2 - 4*a*c]) - 9*a*b^2*c*(2*A*c
 + 3*B*Sqrt[b^2 - 4*a*c]) + 4*a^2*c^2*(-10*A*c + 21*B*Sqrt[b^2 - 4*a*c]) +
b^3*(-33*a*B*c + A*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(
16*c^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 \left(A + Bx^2\right)}{\left(a + bx^2 + cx^4\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 20.78, size = 9636, normalized size = 17.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*(5*B*b^4*c + 4*(11*B*a^2 + 4*A*a*b)*c^3 - (37*B*a*b^2 + A*b^3)*c^2
)*x^7 + 2*(3*B*b^5 + 36*A*a^2*c^3 - (4*B*a^2*b - 5*A*a*b^2)*c^2 - (20*B*a*b
^3 - A*b^4)*c)*x^5 + 2*(6*B*a*b^4 + 28*(B*a^3 + A*a^2*b)*c^2 - (49*B*a^2*b^
2 - 2*A*a*b^3)*c)*x^3 - sqrt(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8
 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16
*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 -
 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a^4 - A^
2*a^3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c^4 - 35*(
216*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5 - 168
*A*B*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c + (b^10*c^5 - 20
*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^
5*c^10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*
B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*
B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*
A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4
- 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)
/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^
4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 -
640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(-(1701*B^4*a^2*b^8
 - 945*A*B^3*a*b^9 - 10000*A^4*a^4*c^6 + 15000*(6*A^3*B*a^4*b - A^4*a^3*b^2
)*c^5 + 3*(1037232*B^4*a^6 - 1037232*A*B^3*a^5*b + 287712*A^2*B^2*a^4*b^2 -
 32952*A^3*B*a^3*b^3 + 497*A^4*a^2*b^4)*c^4 - (1555848*B^4*a^5*b^2 - 129837
6*A*B^3*a^4*b^3 + 238464*A^2*B^2*a^3*b^4 - 11277*A^3*B*a^2*b^5 + 35*A^4*a*b
^6)*c^3 + 9*(37701*B^4*a^4*b^4 - 26973*A*B^3*a^3*b^5 + 3066*A^2*B^2*a^2*b^6
 - 35*A^3*B*a*b^7)*c^2 - 27*(1341*B^4*a^3*b^6 - 819*A*B^3*a^2*b^7 + 35*A^2*
B^2*a*b^8)*c)*x + 1/2*sqrt(1/2)*(27*B^3*b^13 + 32000*A^3*a^5*c^8 - 640*(882
*A*B^2*a^6 - 156*A^2*B*a^5*b + 37*A^3*a^4*b^2)*c^7 + 64*(10584*B^3*a^6*b +
5562*A*B^2*a^5*b^2 - 1083*A^2*B*a^4*b^3 + 89*A^3*a^3*b^4)*c^6 - 8*(93096*B^
3*a^5*b^3 + 3816*A*B^2*a^4*b^4 - 1746*A^2*B*a^3*b^5 + 49*A^3*a^2*b^6)*c^5 +
 (337392*B^3*a^4*b^5 - 24120*A*B^2*a^3*b^6 - 84*A^2*B*a^2*b^7 - 17*A^3*a*b^
8)*c^4 - (81324*B^3*a^3*b^7 - 6993*A*B^2*a^2*b^8 + 195*A^2*B*a*b^9 - A^3*b^
10)*c^3 + 9*(1239*B^3*a^2*b^9 - 79*A*B^2*a*b^10 + A^2*B*b^11)*c^2 - 27*(31*
B^3*a*b^11 - A*B^2*b^12)*c - (3*B*b^14*c^5 - 4096*(42*B*a^7 - 13*A*a^6*b)*c
^12 + 6144*(40*B*a^6*b^2 - 11*A*a^5*b^3)*c^11 - 768*(194*B*a^5*b^4 - 45*A*a
^4*b^5)*c^10 + 1280*(39*B*a^4*b^6 - 7*A*a^3*b^7)*c^9 - 240*(42*B*a^3*b^8 -
5*A*a^2*b^9)*c^8 + 24*(52*B*a^2*b^10 - 3*A*a*b^11)*c^7 - (90*B*a*b^12 - A*b
^13)*c^6)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^
3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^
2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 444
6*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^
4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*
c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*
```

```
a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a^4 - A^2*a^
3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c^4 - 35*(216*
B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5 - 168*A*B
*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c + (b^10*c^5 - 20*a*b
^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^
10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^
2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2*
a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^
3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4 - 11
6*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^
10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^
2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*
a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) + sqrt(1/2)*((b^4*c^4 - 8
*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2
*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a
^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(9*B^
2*b^9 - 1680*(4*A*B*a^4 - A^2*a^3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b
^2 + A^2*a^2*b^3)*c^4 - 35*(216*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c
^3 + (1701*B^2*a^2*b^5 - 168*A*B*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2
*A*B*b^8)*c + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8
 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50
*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95
256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 -
6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5
)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33
*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12
 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*
b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c
^10))*log(-(1701*B^4*a^2*b^8 - 945*A*B^3*a*b^9 - 10000*A^4*a^4*c^6 + 15000*
(6*A^3*B*a^4*b - A^4*a^3*b^2)*c^5 + 3*(1037232*B^4*a^6 - 1037232*A*B^3*a^5*
b + 287712*A^2*B^2*a^4*b^2 - 32952*A^3*B*a^3*b^3 + 497*A^4*a^2*b^4)*c^4 - (
1555848*B^4*a^5*b^2 - 1298376*A*B^3*a^4*b^3 + 238464*A^2*B^2*a^3*b^4 - 1127
7*A^3*B*a^2*b^5 + 35*A^4*a*b^6)*c^3 + 9*(37701*B^4*a^4*b^4 - 26973*A*B^3*a^
3*b^5 + 3066*A^2*B^2*a^2*b^6 - 35*A^3*B*a*b^7)*c^2 - 27*(1341*B^4*a^3*b^6 -
 819*A*B^3*a^2*b^7 + 35*A^2*B^2*a*b^8)*c)*x - 1/2*sqrt(1/2)*(27*B^3*b^13 +
32000*A^3*a^5*c^8 - 640*(882*A*B^2*a^6 - 156*A^2*B*a^5*b + 37*A^3*a^4*b^2)*
c^7 + 64*(10584*B^3*a^6*b + 5562*A*B^2*a^5*b^2 - 1083*A^2*B*a^4*b^3 + 89*A^
3*a^3*b^4)*c^6 - 8*(93096*B^3*a^5*b^3 + 3816*A*B^2*a^4*b^4 - 1746*A^2*B*a^3
*b^5 + 49*A^3*a^2*b^6)*c^5 + (337392*B^3*a^4*b^5 - 24120*A*B^2*a^3*b^6 - 84
*A^2*B*a^2*b^7 - 17*A^3*a*b^8)*c^4 - (81324*B^3*a^3*b^7 - 6993*A*B^2*a^2*b^
8 + 195*A^2*B*a*b^9 - A^3*b^10)*c^3 + 9*(1239*B^3*a^2*b^9 - 79*A*B^2*a*b^10
 + A^2*B*b^11)*c^2 - 27*(31*B^3*a*b^11 - A*B^2*b^12)*c - (3*B*b^14*c^5 - 40
96*(42*B*a^7 - 13*A*a^6*b)*c^12 + 6144*(40*B*a^6*b^2 - 11*A*a^5*b^3)*c^11 -
 768*(194*B*a^5*b^4 - 45*A*a^4*b^5)*c^10 + 1280*(39*B*a^4*b^6 - 7*A*a^3*b^7
)*c^9 - 240*(42*B*a^3*b^8 - 5*A*a^2*b^9)*c^8 + 24*(52*B*a^2*b^10 - 3*A*a*b^
11)*c^7 - (90*B*a*b^12 - A*b^13)*c^6)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 -
50*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 -
95256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4
- 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b
^5)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(
33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^1
2 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(9*B^2*b^
9 - 1680*(4*A*B*a^4 - A^2*a^3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 +
 A^2*a^2*b^3)*c^4 - 35*(216*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^3 +
 (1701*B^2*a^2*b^5 - 168*A*B*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B
*b^8)*c + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 12
80*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(44
1*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*
A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(1
```

```
4553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^
3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4
*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 64
0*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*
c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)
)) - sqrt(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*
a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (
b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16
*a^3*b*c^4)*x^2)*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a^4 - A^2*a^3*b)*c^5 + 280*
(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c^4 - 35*(216*B^2*a^3*b^3 - 3
6*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5 - 168*A*B*a*b^6 + A^2*b^
7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c - (b^10*c^5 - 20*a*b^8*c^6 + 160*a^
2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((81*B^
4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2
)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A
^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324
*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 +
 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*a*
b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a
^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1
280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(-(1701*B^4*a^2*b^8 - 945*A*B^3*a*b^9
 - 10000*A^4*a^4*c^6 + 15000*(6*A^3*B*a^4*b - A^4*a^3*b^2)*c^5 + 3*(1037232*
B^4*a^6 - 1037232*A*B^3*a^5*b + 287712*A^2*B^2*a^4*b^2 - 32952*A^3*B*a^3*b^
3 + 497*A^4*a^2*b^4)*c^4 - (1555848*B^4*a^5*b^2 - 1298376*A*B^3*a^4*b^3 + 2
38464*A^2*B^2*a^3*b^4 - 11277*A^3*B*a^2*b^5 + 35*A^4*a*b^6)*c^3 + 9*(37701*
B^4*a^4*b^4 - 26973*A*B^3*a^3*b^5 + 3066*A^2*B^2*a^2*b^6 - 35*A^3*B*a*b^7)*
c^2 - 27*(1341*B^4*a^3*b^6 - 819*A*B^3*a^2*b^7 + 35*A^2*B^2*a*b^8)*c)*x + 1
/2*sqrt(1/2)*(27*B^3*b^13 + 32000*A^3*a^5*c^8 - 640*(882*A*B^2*a^6 - 156*A^
2*B*a^5*b + 37*A^3*a^4*b^2)*c^7 + 64*(10584*B^3*a^6*b + 5562*A*B^2*a^5*b^2
 - 1083*A^2*B*a^4*b^3 + 89*A^3*a^3*b^4)*c^6 - 8*(93096*B^3*a^5*b^3 + 3816*A*
B^2*a^4*b^4 - 1746*A^2*B*a^3*b^5 + 49*A^3*a^2*b^6)*c^5 + (337392*B^3*a^4*b^
5 - 24120*A*B^2*a^3*b^6 - 84*A^2*B*a^2*b^7 - 17*A^3*a*b^8)*c^4 - (81324*B^3
*a^3*b^7 - 6993*A*B^2*a^2*b^8 + 195*A^2*B*a*b^9 - A^3*b^10)*c^3 + 9*(1239*B
^3*a^2*b^9 - 79*A*B^2*a*b^10 + A^2*B*b^11)*c^2 - 27*(31*B^3*a*b^11 - A*B^2*
b^12)*c + (3*B*b^14*c^5 - 4096*(42*B*a^7 - 13*A*a^6*b)*c^12 + 6144*(40*B*a^
6*b^2 - 11*A*a^5*b^3)*c^11 - 768*(194*B*a^5*b^4 - 45*A*a^4*b^5)*c^10 + 1280
*(39*B*a^4*b^6 - 7*A*a^3*b^7)*c^9 - 240*(42*B*a^3*b^8 - 5*A*a^2*b^9)*c^8 +
24*(52*B*a^2*b^10 - 3*A*a*b^11)*c^7 - (90*B*a*b^12 - A*b^13)*c^6)*sqrt((81*
B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b
^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516
*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 3
24*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5
 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*
a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024
*a^5*c^15)))*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a^4 - A^2*a^3*b)*c^5 + 280*(54*
B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c^4 - 35*(216*B^2*a^3*b^3 - 36*A*
B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5 - 168*A*B*a*b^6 + A^2*b^7)*c
^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c - (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^
6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((81*B^4*b^
8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^
5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B
*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2
*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A
^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*a*b^8*
c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c
^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*
a^4*b^2*c^9 - 1024*a^5*c^10))) + sqrt(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2
*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3
*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*
```

```
b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(9*B^2*b^9 - 1680*(4*A*B
*a^4 - A^2*a^3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c
^4 - 35*(216*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*
b^5 - 168*A*B*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c - (b^10
*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9
- 1024*a^5*c^10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 -
 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 1
7496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^
2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4
*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^
3*b^7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13
+ 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b
^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(-(1701*B^
4*a^2*b^8 - 945*A*B^3*a*b^9 - 10000*A^4*a^4*c^6 + 15000*(6*A^3*B*a^4*b - A^
4*a^3*b^2)*c^5 + 3*(1037232*B^4*a^6 - 1037232*A*B^3*a^5*b + 287712*A^2*B^2*
a^4*b^2 - 32952*A^3*B*a^3*b^3 + 497*A^4*a^2*b^4)*c^4 - (1555848*B^4*a^5*b^2
 - 1298376*A*B^3*a^4*b^3 + 238464*A^2*B^2*a^3*b^4 - 11277*A^3*B*a^2*b^5 + 3
5*A^4*a*b^6)*c^3 + 9*(37701*B^4*a^4*b^4 - 26973*A*B^3*a^3*b^5 + 3066*A^2*B^
2*a^2*b^6 - 35*A^3*B*a*b^7)*c^2 - 27*(1341*B^4*a^3*b^6 - 819*A*B^3*a^2*b^7
+ 35*A^2*B^2*a*b^8)*c)*x - 1/2*sqrt(1/2)*(27*B^3*b^13 + 32000*A^3*a^5*c^8 -
 640*(882*A*B^2*a^6 - 156*A^2*B*a^5*b + 37*A^3*a^4*b^2)*c^7 + 64*(10584*B^3
*a^6*b + 5562*A*B^2*a^5*b^2 - 1083*A^2*B*a^4*b^3 + 89*A^3*a^3*b^4)*c^6 - 8*
(93096*B^3*a^5*b^3 + 3816*A*B^2*a^4*b^4 - 1746*A^2*B*a^3*b^5 + 49*A^3*a^2*b
^6)*c^5 + (337392*B^3*a^4*b^5 - 24120*A*B^2*a^3*b^6 - 84*A^2*B*a^2*b^7 - 17
*A^3*a*b^8)*c^4 - (81324*B^3*a^3*b^7 - 6993*A*B^2*a^2*b^8 + 195*A^2*B*a*b^9
 - A^3*b^10)*c^3 + 9*(1239*B^3*a^2*b^9 - 79*A*B^2*a*b^10 + A^2*B*b^11)*c^2
- 27*(31*B^3*a*b^11 - A*B^2*b^12)*c + (3*B*b^14*c^5 - 4096*(42*B*a^7 - 13*A
*a^6*b)*c^12 + 6144*(40*B*a^6*b^2 - 11*A*a^5*b^3)*c^11 - 768*(194*B*a^5*b^4
 - 45*A*a^4*b^5)*c^10 + 1280*(39*B*a^4*b^6 - 7*A*a^3*b^7)*c^9 - 240*(42*B*a
^3*b^8 - 5*A*a^2*b^9)*c^8 + 24*(52*B*a^2*b^10 - 3*A*a*b^11)*c^7 - (90*B*a*b
^12 - A*b^13)*c^6)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3
 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b +
 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*
b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B
^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*
B^3*b^7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^1
3 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a^4
 - A^2*a^3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c^4 -
 35*(216*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5
- 168*A*B*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c - (b^10*c^5
 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 10
24*a^5*c^10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108
*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496
*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 -
4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2
*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^
7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 12
80*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c
^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) + 2*(3*B*a^2*b^3
 + 20*A*a^3*c^2 - (24*B*a^3*b - A*a^2*b^2)*c)*x)/((b^4*c^4 - 8*a*b^2*c^5 +
16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8
*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 +
 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)
```

giac [B] time = 8.28, size = 3987, normalized size = 7.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 12*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^5*c^2 - 2*b^6*c^2 - 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^3 - 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 24*a*b^4*c^3 - 2*b^5*c^3 + 320*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 160*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^
4 + 288*a^2*b^2*c^4 + 112*a*b^3*c^4 - 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*c^5 - 640*a^3*c^5 - 416*a^2*b*c^5 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c^2 + 208*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^3 + 104*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^3*c^3 - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b*c^4 + 2*(b^2 - 4*a*c)*b^4*c^2 + 32*(b^2 - 4*a*c)*a*b^2*c^3 + 2*(
b^2 - 4*a*c)*b^3*c^3 - 160*(b^2 - 4*a*c)*a^2*c^4 - 104*(b^2 - 4*a*c)*a*b*c^
4)*A + 3*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 - 16*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^6*c - 2*b^7*c + 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 +
 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 32*a*b^5*c^2 - 2*b^6*c^2 - 128*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^2*c^3 - 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 -
 160*a^2*b^3*c^3 + 28*a*b^4*c^3 + 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^4 + 256*a^3*b*c^4 - 192*a^2*b^2*c^4 + 448*a^3*c^5 + sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 14*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*c^3 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b*c^3 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^2*c^3 + 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*c^4 + 2*(b^2 - 4*a*c)*b^5*c - 24*(b^2 - 4*a*c)*a*b^3*c^2 + 2
*(b^2 - 4*a*c)*b^4*c^2 + 64*(b^2 - 4*a*c)*a^2*b*c^3 - 20*(b^2 - 4*a*c)*a*b^
2*c^3 + 112*(b^2 - 4*a*c)*a^2*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^5*c^2 -
8*a*b^3*c^3 + 16*a^2*b*c^4 + sqrt((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)^2
- 4*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^
2*c^5)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/((b^8*c^2 - 16*a*b^6*c^3 -
2*b^7*c^3 + 96*a^2*b^4*c^4 + 24*a*b^5*c^4 + b^6*c^4 - 256*a^3*b^2*c^5 - 96*
a^2*b^3*c^5 - 12*a*b^4*c^5 + 256*a^4*c^6 + 128*a^3*b*c^6 + 48*a^2*b^2*c^6 -
 64*a^3*c^7)*abs(c)) + 1/32*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c
 + 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 2*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 2*b^6*c^2 - 144*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^3*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 24*a*b^4*c^3 +
2*b^5*c^3 + 320*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 160*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^4 - 288*a^2*b^2*c^4 - 112*a*b^3*c^4 - 80*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 640*a^3*c^5 + 416*a^2*b*c^5 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 56*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 208*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 104*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 - 32*(b^2 -
```

```
4*a*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^3*c^3 + 160*(b^2 - 4*a*c)*a^2*c^4 + 1
04*(b^2 - 4*a*c)*a*b*c^4)*A + 3*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
7 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^6*c + 2*b^7*c + 80*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*b^3*c^2 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 32*a*b^5*c^2 + 2*b^6*
c^2 - 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 64*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 12*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^3*c^3 + 160*a^2*b^3*c^3 - 28*a*b^4*c^3 + 32*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 256*a^3*b*c^4 + 192*a^2*b^2*c^4 - 448*
a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 1
4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 96*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 20*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 224*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 112*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 10*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 56*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^5*c + 24*(b^2
 - 4*a*c)*a*b^3*c^2 - 2*(b^2 - 4*a*c)*b^4*c^2 - 64*(b^2 - 4*a*c)*a^2*b*c^3
+ 20*(b^2 - 4*a*c)*a*b^2*c^3 - 112*(b^2 - 4*a*c)*a^2*c^4)*B)*arctan(2*sqrt(
1/2)*x/sqrt((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4 - sqrt((b^5*c^2 - 8*a*b^3
*c^3 + 16*a^2*b*c^4)^2 - 4*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*(b^4*c^
3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/((b^
8*c^2 - 16*a*b^6*c^3 - 2*b^7*c^3 + 96*a^2*b^4*c^4 + 24*a*b^5*c^4 + b^6*c^4
- 256*a^3*b^2*c^5 - 96*a^2*b^3*c^5 - 12*a*b^4*c^5 + 256*a^4*c^6 + 128*a^3*b
*c^6 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*abs(c)) - 1/8*(5*B*b^4*c*x^7 - 37*B*a*b
^2*c^2*x^7 - A*b^3*c^2*x^7 + 44*B*a^2*c^3*x^7 + 16*A*a*b*c^3*x^7 + 3*B*b^5*
x^5 - 20*B*a*b^3*c*x^5 + A*b^4*c*x^5 - 4*B*a^2*b*c^2*x^5 + 5*A*a*b^2*c^2*x^
5 + 36*A*a^2*c^3*x^5 + 6*B*a*b^4*x^3 - 49*B*a^2*b^2*c*x^3 + 2*A*a*b^3*c*x^3
 + 28*B*a^3*c^2*x^3 + 28*A*a^2*b*c^2*x^3 + 3*B*a^2*b^3*x - 24*B*a^3*b*c*x +
 A*a^2*b^2*c*x + 20*A*a^3*c^2*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*(c*x
^4 + b*x^2 + a)^2)
```

maple [B] time = 0.07, size = 2015, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (-1/8*(16*A*a*b*c^2-A*b^3*c+44*B*a^2*c^2-37*B*a*b^2*c+5*B*b^4)/(16*a^2*c^2-
8*a*b^2*c+b^4)/c*x^7-1/8*(36*A*a^2*c^3+5*A*a*b^2*c^2+A*b^4*c-4*B*a^2*b*c^2-
20*B*a*b^3*c+3*B*b^5)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*a/c^2*(28*A*a*
b*c^2+2*A*b^3*c+28*B*a^2*c^2-49*B*a*b^2*c+6*B*b^4)/(16*a^2*c^2-8*a*b^2*c+b^
4)*x^3-1/8*a^2*(20*A*a*c^2+A*b^2*c-24*B*a*b*c+3*B*b^3)/c^2/(16*a^2*c^2-8*a*
b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+1/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*a*A*b-1/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^3
-5/2*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*
A*a^2-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*
x)*A*a*b^2+1/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*A*b^4-21/4/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*
B+27/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
```

```
/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2*B-3/16/c^2
/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*B+33/4/(16*a^2*c^2-8*
a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*a^2*b-33/16/c/(16*a
^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*a*b^3+3/16
/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*
b^5-1/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*A*b+1/16/c/(16*a^2*c^
2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^3-5/2*c/(16*a^2*c^2-8*a*b^2*c+b^4)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*a^2-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*a*b^2+1/16/c/(16*a^2*c^2-8*a*b^2*c+
b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^4+21/4/(16*a^2*c^2-8*a*b^2*c
+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*c*x)*a^2*B-27/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*c*x)*a*b^2*B+3/16/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^
4*B+33/4/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B
*a^2*b-33/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
c*x)*B*a*b^3+3/16/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*c*x)*B*b^5
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(5\,Bb^4c+4\,\left(11\,Ba^2+4\,Aab\right)c^3-\left(37\,Bab^2+Ab^3\right)c^2\right)x^7+\left(3\,Bb^5+36\,Aa^2c^3-\left(4\,Ba^2b-5\,Aab^2\right)c^2-\left(20\,Bab^3-Ab^4\right)c\right)x^5+\left(6\,Bab^4+28\,\left(Ba^3+Aa^2b\right)c^2-\left(49\,Ba^2b^2-2\,Aab^3\right)c\right)x^3+\left(3\,Ba^2b^3+20\,Aa^3c^2-\left(24\,Ba^3b-Aa^2b^2\right)c\right)x}{8\left(\left(b^4c^4-8\,ab^2c^5+16\,a^2c^6\right)x^8+a^2b^4c^2-8\,a^3b^2c^3+16\,a^4c^4+2\left(b^5c^3-8\,ab^3c^4+16\,a^2bc^5\right)x^6+\left(b^6c^2-6\,ab^4c^3+32\,a^3c^5\right)x^4+2\left(ab^5c^2-8\,a^2b^3c^3+16\,a^3bc^4\right)x^2\right)}-\frac{-\int\frac{3\,Bab^3+20\,Aa^2c^2+\left(3\,Bb^4+4\,\left(21\,Ba^2-4\,Aab\right)c^2-\left(27\,Bab^2-Ab^3\right)c\right)x^2-\left(24\,Ba^2b-Aab^2\right)c}{cx^4+bx^2+a}\,dx}{8\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*B*b^4*c + 4*(11*B*a^2 + 4*A*a*b)*c^3 - (37*B*a*b^2 + A*b^3)*c^2)*x
^7 + (3*B*b^5 + 36*A*a^2*c^3 - (4*B*a^2*b - 5*A*a*b^2)*c^2 - (20*B*a*b^3 -
A*b^4)*c)*x^5 + (6*B*a*b^4 + 28*(B*a^3 + A*a^2*b)*c^2 - (49*B*a^2*b^2 - 2*A
*a*b^3)*c)*x^3 + (3*B*a^2*b^3 + 20*A*a^3*c^2 - (24*B*a^3*b - A*a^2*b^2)*c)*
x)/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3
+ 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*
a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*
x^2) - 1/8*integrate(-(3*B*a*b^3 + 20*A*a^2*c^2 + (3*B*b^4 + 4*(21*B*a^2 -
4*A*a*b)*c^2 - (27*B*a*b^2 - A*b^3)*c)*x^2 - (24*B*a^2*b - A*a*b^2)*c)/(c*x
^4 + b*x^2 + a), x)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
```

mupad [B]   time = 5.05, size = 22911, normalized size = 41.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] - ((x^5*(3*B*b^5 + 36*A*a^2*c^3 + A*b^4*c - 20*B*a*b^3*c + 5*A*a*b^2*c^2 -
4*B*a^2*b*c^2))/(8*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^7*(5*B*b^4 + 44
*B*a^2*c^2 - A*b^3*c + 16*A*a*b*c^2 - 37*B*a*b^2*c))/(8*c*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c)) + (x^3*(28*B*a^3*c^2 + 6*B*a*b^4 + 2*A*a*b^3*c + 28*A*a^2*b*
```

c^2 - 49*B*a^2*b^2*c))/(8*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*x*(3*B
*b^3 + 20*A*a*c^2 + A*b^2*c - 24*B*a*b*c))/(8*c^2*(b^4 + 16*a^2*c^2 - 8*a*b
^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - atan(
((((256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B*a^
7*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^
8 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5 - 1
474560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*
(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c
^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*(-(9*B^2*b^19 + A^2*b^17*c^
2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*
c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^
7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^
2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*
c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8
*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c
 - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15
*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15
482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377
280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 -
1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c
 - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1
/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 +
b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4
*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^
12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5
120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8
 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b
^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a
*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*
b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b
^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^
13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^
6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c
^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) +
6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a
^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9
 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5
 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*
c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) -
288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^
2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18
*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a
^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4
*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) - (x*(9*B^2*b^10 + 800*A^2*a^4*c^6 +
A^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A
^2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^
4*b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 446
4*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*c^7
 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^
19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 11
40*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 4377
6*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921
*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851
776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 +
27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*
a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c
 - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2
)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^
3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*

```
B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*
a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a
*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048
576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14
*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 19
66080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*1
i - (((256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B
*a^7*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4
*c^8 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5
- 1474560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(5
12*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^
6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) + (x*(-(9*B^2*b^19 + A^2*b^17
*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^
13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7
*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15
*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b
^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*
a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*
a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b
^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) -
 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 +
377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7
 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*
a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)
^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15
 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*
a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6
*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5
- 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c
^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^
2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(
4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a
^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^
6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3
*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2
*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^
2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
 + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^
2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*
c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*
c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b
^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)
 - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b
*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b
^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 25804
8*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*
b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) + (x*(9*B^2*b^10 + 800*A^2*a^4*c^6
 + A^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 20
8*A^2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2
*a^4*b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 -
4464*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*
c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2
*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c +
 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 4
3776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6
921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2
851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7
 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B
^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^1
```

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7*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c -
b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B
*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032
*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B
^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(
4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1
048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b
^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 -
 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2
)*1i)/((((256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 629145
6*B*a^7*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*
b^4*c^8 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c
^5 - 1474560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)
/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3
*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*(-(9*B^2*b^19 + A^2*b
^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2
*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*
b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b
^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^
5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B
^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-
(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*
a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2
) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4
 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*
c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-
(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^
15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c
^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 537
60*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*
b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c
^5 - 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^
5*c^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96
*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*
(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^
2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2
*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*
a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*
B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2
*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1
/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320
*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9
*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^
10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^
7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1
/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*
a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*
a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 25
8048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a
^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) - (x*(9*B^2*b^10 + 800*A^2*a^4*
c^6 + A^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 +
 208*A^2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*
B^2*a^4*b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3
 - 4464*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a
^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*
B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*
c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6
+ 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9
+ 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4
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- 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) + (((256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B*a^7*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^8 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5 - 1474560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) + (x*(-(9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) + (x*(9*B^2*b^10 + 800*A^2*a^4*c^6 + A^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A^2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^4*b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 4464*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*

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c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6
+ 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9
+ 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4
- 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*
c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 44
1*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*
b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c
 - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*
A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860
032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 9
9*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*
(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512
*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^
3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^1
1 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(
1/2) + (35*A^3*a^2*b^7*c^2 - 592704*B^3*a^7*c^4 - 567*B^3*a^3*b^8 - 1176*A^
3*a^3*b^5*c^3 + 9456*A^3*a^4*b^3*c^4 - 89532*B^3*a^5*b^4*c^2 + 353808*B^3*a
^6*b^2*c^3 + 315*A*B^2*a^2*b^9 - 33600*A^2*B*a^6*c^5 + 6400*A^3*a^5*b*c^5 +
 10935*B^3*a^4*b^6*c - 6552*A*B^2*a^3*b^7*c + 560448*A*B^2*a^6*b*c^4 + 210*
A^2*B*a^2*b^8*c + 61524*A*B^2*a^4*b^5*c^2 - 280800*A*B^2*a^5*b^3*c^3 - 5649
*A^2*B*a^3*b^6*c^2 + 42516*A^2*B*a^4*b^4*c^3 - 126192*A^2*B*a^5*b^2*c^4)/(2
56*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^
6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))))*(-(9*B^2*b^19 + A^2*b^17*c^
2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*
c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^
7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^
2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*
c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8
*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c
 - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15
*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15
482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377
280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 -
1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c
 - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1
/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 +
b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4
*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^
12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*2i - atan(((((256
*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B*a^7*b*c^9
 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^8 + 629
1456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5 - 1474560*
B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*(4096*a
^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 38
40*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B
^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 1
0160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680
960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 775
80*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9
628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^
8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)
^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 -
1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*
B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B
*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240
*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)
^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 1
08*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^
5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c
```

^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 29
49120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5120*a*b
^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + 32768
0*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5
- 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c - b^
2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^
5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8
+ 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3
+ 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c
^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4
*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280
*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^
10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580
*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430
784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5
160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B
*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 108*A*B*a*b*c^2*(-(4*
a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 +
720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10
*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 -
2621440*a^9*b^2*c^14)))^(1/2) - (x*(9*B^2*b^10 + 800*A^2*a^4*c^6 + A^2*b^8
*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A^2*a^3*
b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^4*b^2*c
^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 4464*A*B*a
^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*c^7 + b^8*
c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^
2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*
a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a
^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^
2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2
*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 2709504
0*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2
*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A
^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(
1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*
c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b
^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c
*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^
2)^15)^(1/2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^1
0*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 +
53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a
^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*1i - (((
256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B*a^7*b*
c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^8 +
6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5 - 14745
60*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*(409
6*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 +
3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) + (x*(-(9*B^2*b^19 + A^2*b^17*c^2 -
9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4
- 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 -
680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 -
77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5
+ 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3
*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b
^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3
- 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 154828
80*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*
A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290
240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b

```
^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2)
+ 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20
*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^1
2*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 +
 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5120*
a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + 32
7680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c
^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c -
 b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11
*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c
^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c
^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^
7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(
-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881
280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b
*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5
580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1
430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8
 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*
A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 108*A*B*a*b*c^2*(-
(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6
 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b
^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^1
3 - 2621440*a^9*b^2*c^14)))^(1/2) + (x*(9*B^2*b^10 + 800*A^2*a^4*c^6 + A^2*
b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A^2*a
^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^4*b^
2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 4464*A*
B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*c^7 + b
^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 +
 A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A
^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^
2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2
*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*
B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 2709
5040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*
c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 5
5*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15
)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^
12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^
6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^
2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c -
 b^2)^15)^(1/2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*
a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8
 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 196608
0*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*1i)/(
(((256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B*a^7
*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^8
 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5 - 14
74560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*(
4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^
6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*(-(9*B^2*b^19 + A^2*b^17*c^2
 - 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c
^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7
 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2
 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c
^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*
b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c
 - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*
c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 154
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82880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) - (x*(9*B^2*b^10 + 800*A^2*a^4*c^6 + A^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A^2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^4*b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 4464*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) + (((256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B*a^7*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^8 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5 - 1474560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) + (x*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8* b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) + (x*(9*B^2*b^10 + 800*A^2*a^4*c^6 + A^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A^2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^4*b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 4464*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) + (35*A^3*a^2*b^7*c^2 - 592704*B^3*a^7*c^4 - 567*B^3*a^3*b^8 - 1176*A^3*a^3*b^5*c^3 + 9456*A^3*a^4*b^3*c^4 - 89532*B^3*a^5*b^4*c^2 + 353808*B^3*a^6*b^2*c^3 + 315*A*B^2*a^2*b^9 - 33600*A^2*B*a^6*c^5 + 6400*A^3*a^5*b*c^5 + 10935*B^3*a^4*b^6*c - 6552*A*B^2*a^3*b^7*c + 560448*A*B^2*a^6*b*c^4 + 210*A^2*B*a^2*b^8*c + 61524*A*B^2*a^4*b^5*c^2 - 280800*A*B^2*a^5*b^3*c^3 - 5649*A^2*B*a^3*b^6*c^2 + 42516*A^2*B*a^4*b^4*c^3 - 126192*A^2*B*a^5*b^2*c^4)/(256*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))))*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B

```
^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 1
0160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680
960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 775
80*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9
628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^
8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)
^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 -
1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*
B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B
*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240
*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)
^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 1
08*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^
5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c
^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 29
49120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.111 $\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=461

$$\frac{\left(-\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4B}{\sqrt{b^2-4ac}}+12aAc^2-16abBc+3Ab^2c+b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{3/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2Bc^2+36aAbc^2-1\ldots}{\sqrt{b^2-\ldots}}\right.}{}$$

Rubi [A] time = 4.62, antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1275, 1279, 1166, 205}

$$\frac{\left(-\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4B}{\sqrt{b^2-4ac}}+12aAc^2-16abBc+3Ab^2c+b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{3/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4B}{\sqrt{b^2-4ac}}+12aAc^2-16abBc+3Ab^2c+b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}c^{3/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x^5\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x^3\left(-x^2\left(20aBc-12Abc+b^2B\right)+4aAc-12abB+5Ab^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x\left(20aBc-12Abc+b^2B\right)}{8c\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -((b^2*B - 12*A*b*c + 20*a*B*c)*x)/(8*c*(b^2 - 4*a*c)^2) - (x^5*(A*b - 2*a*
B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^3*(5*A
*b^2 - 12*a*b*B + 4*a*A*c - (b^2*B - 12*A*b*c + 20*a*B*c)*x^2))/(8*(b^2 - 4
*a*c)^2*(a + b*x^2 + c*x^4)) + ((b^3*B + 3*A*b^2*c - 16*a*b*B*c + 12*a*A*c^
2 - (b^4*B + 3*A*b^3*c - 18*a*b^2*B*c + 36*a*A*b*c^2 - 40*a^2*B*c^2)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sq
rt[2]*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3*B + 3*A*
b^2*c - 16*a*b*B*c + 12*a*A*c^2 + (b^4*B + 3*A*b^3*c - 18*a*b^2*B*c + 36*a*
A*b*c^2 - 40*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b
^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
```

```
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx = -\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\int \frac{x^4\left(5(Ab-2aB)+(bB-2Ac)x^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= -\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x^3\left(5Ab^2-12abB+4aAc-\left(b^2B-12Abc+20a\ldots\right.\right.}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= -\frac{\left(b^2B-12Abc+20aBc\right)x}{8c\left(b^2-4ac\right)^2}-\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x^3\left(5Ab^2-12abB+\ldots\right.}{8\left(b^2-\ldots\right.}$$

$$= -\frac{\left(b^2B-12Abc+20aBc\right)x}{8c\left(b^2-4ac\right)^2}-\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x^3\left(5Ab^2-12abB+\ldots\right.}{8\left(b^2-\ldots\right.}$$

$$= -\frac{\left(b^2B-12Abc+20aBc\right)x}{8c\left(b^2-4ac\right)^2}-\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x^3\left(5Ab^2-12abB+\ldots\right.}{8\left(b^2-\ldots\right.}$$

Mathematica [A] time = 2.02, size = 543, normalized size = 1.18

$$\frac{-\frac{4x\left(2a^2Bc+a\left(bc\left(A+3Bx^2\right)-2Ac^2x^2+b^2(-B)\right)+b^2x^2(Ac-bB)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{2x\left(b^2c\left(11aB+3Acx^2\right)+4abc^2\left(A-4Bx^2\right)+12ac^2\left(Acx^2-3aB\right)+b^3c\left(2A+Bx^2\right)-2b^4B\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(4ac^2\left(3A\sqrt{b^2-4ac}+10aB\right)+3b^2c\left(A\sqrt{b^2-4ac}+6aB\right)-4abc\left(4B\sqrt{b^2-4ac}+9Ac\right)+b^3\left(B\sqrt{b^2-4ac}-3Ac\right)+b^4(-B)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(4ac^2\left(3A\sqrt{b^2-4ac}-10aB\right)+3b^2c\left(A\sqrt{b^2-4ac}-6aB\right)+4abc\left(9Ac-4B\sqrt{b^2-4ac}\right)+b^3\left(B\sqrt{b^2-4ac}+3Ac\right)+b^4B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}}{16c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*x*(-2*b^4*B + 4*a*b*c^2*(A - 4*B*x^2) + b^3*c*(2*A + B*x^2) + 12*a*c^2*
(-3*a*B + A*c*x^2) + b^2*c*(11*a*B + 3*A*c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x
^2 + c*x^4)) - (4*x*(2*a^2*B*c + b^2*(-(b*B) + A*c)*x^2 + a*(-(b^2*B) - 2*A
*c^2*x^2 + b*c*(A + 3*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sq
rt[2]*Sqrt[c]*(-(b^4*B) + 3*b^2*c*(6*a*B + A*Sqrt[b^2 - 4*a*c]) + 4*a*c^2*(
10*a*B + 3*A*Sqrt[b^2 - 4*a*c]) + b^3*(-3*A*c + B*Sqrt[b^2 - 4*a*c]) - 4*a*
b*c*(9*A*c + 4*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqr
t[2]*Sqrt[c]*(b^4*B + 3*b^2*c*(-6*a*B + A*Sqrt[b^2 - 4*a*c]) + 4*a*c^2*(-10
*a*B + 3*A*Sqrt[b^2 - 4*a*c]) + 4*a*b*c*(9*A*c - 4*B*Sqrt[b^2 - 4*a*c]) + b
^3*(3*A*c + B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 6.94, size = 7060, normalized size = 15.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(B*b^3*c + 12*A*a*c^3 - (16*B*a*b - 3*A*b^2)*c^2)*x^7 - 2*(B*b^4 +
4*(9*B*a^2 - 4*A*a*b)*c^2 + 5*(B*a*b^2 - A*b^3)*c)*x^5 - 2*(2*B*a*b^3 + 4*A
*a^2*c^2 + (28*B*a^2*b - 19*A*a*b^2)*c)*x^3 - sqrt(1/2)*((b^4*c^3 - 8*a*b^2
*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^
2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^
4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(B^2*b^7 - 240*(4
*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*A^2*a*
b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5
 - 6*A*B*b^6)*c + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*
c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*
A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)
*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2
*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3
 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 10
24*a^5*c^8))*log(-(35*B^4*a*b^6 - 15*A*B^3*b^7 - 1296*A^4*a^2*c^5 + 648*(14
*A^3*B*a^2*b - 5*A^4*a*b^2)*c^4 + (10000*B^4*a^4 - 30000*A*B^3*a^3*b + 9936
*A^2*B^2*a^2*b^2 + 1080*A^3*B*a*b^3 - 405*A^4*b^4)*c^3 + 3*(5000*B^4*a^3*b^
2 - 3864*A*B^3*a^2*b^3 + 1080*A^2*B^2*a*b^4 - 135*A^3*B*b^5)*c^2 - 3*(497*B
^4*a^2*b^4 - 315*A*B^3*a*b^5 + 45*A^2*B^2*b^6)*c)*x + 1/2*sqrt(1/2)*(B^3*b^
10 - 2304*(5*A^2*B*a^4 - 3*A^3*a^3*b)*c^6 + 64*(500*B^3*a^5 - 420*A*B^2*a^4
*b + 198*A^2*B*a^3*b^2 - 81*A^3*a^2*b^3)*c^5 - 16*(1480*B^3*a^4*b^2 - 1284*
A*B^2*a^3*b^3 + 324*A^2*B*a^2*b^4 - 81*A^3*a*b^5)*c^4 + 4*(1424*B^3*a^3*b^4
 - 1332*A*B^2*a^2*b^5 + 234*A^2*B*a*b^6 - 27*A^3*b^7)*c^3 - (392*B^3*a^2*b^
6 - 492*A*B^2*a*b^7 + 63*A^2*B*b^8)*c^2 - (17*B^3*a*b^8 + 6*A*B^2*b^9)*c -
(B*b^13*c^3 - 24576*A*a^6*c^10 + 4096*(13*B*a^6*b + 3*A*a^5*b^2)*c^9 - 1536
*(44*B*a^5*b^3 - 5*A*a^4*b^4)*c^8 + 3840*(9*B*a^4*b^5 - 2*A*a^3*b^6)*c^7 -
160*(56*B*a^3*b^7 - 15*A*a^2*b^8)*c^6 + 48*(25*B*a^2*b^9 - 7*A*a*b^10)*c^5
- 18*(4*B*a*b^11 - A*b^12)*c^4)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2
*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 -
2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^
8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(B^2*b^7 -
 240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3
*A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^
2*a*b^5 - 6*A*B*b^6)*c + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a
^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*c^4 -
18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B
^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 +
160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b
^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c
^7 - 1024*a^5*c^8))) + sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8
+ a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^
2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*
b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(B^2*b^7 - 240*(4*A*B*a^3 - 3*A^2*a^2*b)
*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*A^2*a*b^3)*c^3 + (280*B^2*a^2
*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5 - 6*A*B*b^6)*c + (b^10
*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7
- 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*
c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2
- 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*
c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a
```

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^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log(-(35*B
^4*a*b^6 - 15*A*B^3*b^7 - 1296*A^4*a^2*c^5 + 648*(14*A^3*B*a^2*b - 5*A^4*a*
b^2)*c^4 + (10000*B^4*a^4 - 30000*A*B^3*a^3*b + 9936*A^2*B^2*a^2*b^2 + 1080
*A^3*B*a*b^3 - 405*A^4*b^4)*c^3 + 3*(5000*B^4*a^3*b^2 - 3864*A*B^3*a^2*b^3
+ 1080*A^2*B^2*a*b^4 - 135*A^3*B*b^5)*c^2 - 3*(497*B^4*a^2*b^4 - 315*A*B^3*
a*b^5 + 45*A^2*B^2*b^6)*c)*x - 1/2*sqrt(1/2)*(B^3*b^10 - 2304*(5*A^2*B*a^4
- 3*A^3*a^3*b)*c^6 + 64*(500*B^3*a^5 - 420*A*B^2*a^4*b + 198*A^2*B*a^3*b^2
- 81*A^3*a^2*b^3)*c^5 - 16*(1480*B^3*a^4*b^2 - 1284*A*B^2*a^3*b^3 + 324*A^2
*B*a^2*b^4 - 81*A^3*a*b^5)*c^4 + 4*(1424*B^3*a^3*b^4 - 1332*A*B^2*a^2*b^5 +
 234*A^2*B*a*b^6 - 27*A^3*b^7)*c^3 - (392*B^3*a^2*b^6 - 492*A*B^2*a*b^7 + 6
3*A^2*B*b^8)*c^2 - (17*B^3*a*b^8 + 6*A*B^2*b^9)*c - (B*b^13*c^3 - 24576*A*a
^6*c^10 + 4096*(13*B*a^6*b + 3*A*a^5*b^2)*c^9 - 1536*(44*B*a^5*b^3 - 5*A*a^
4*b^4)*c^8 + 3840*(9*B*a^4*b^5 - 2*A*a^3*b^6)*c^7 - 160*(56*B*a^3*b^7 - 15*
A*a^2*b^8)*c^6 + 48*(25*B*a^2*b^9 - 7*A*a*b^10)*c^5 - 18*(4*B*a*b^11 - A*b^
12)*c^4)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (
625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B
^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1
280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(B^2*b^7 - 240*(4*A*B*a^3 - 3*A^2
*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*A^2*a*b^3)*c^3 + (280*
B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5 - 6*A*B*b^6)*c
+ (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b
^2*c^7 - 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^
3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4
*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a
^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4
+ 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))) -
sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2
*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c -
 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)
*x^2)*sqrt(-(B^2*b^7 - 240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*
b - 16*A*B*a^2*b^2 + 3*A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9
*A^2*b^5)*c^2 - (35*B^2*a*b^5 - 6*A*B*b^6)*c - (b^10*c^3 - 20*a*b^8*c^4 + 1
60*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((B
^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 30
0*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10
*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10
 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^
4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log(-(35*B^4*a*b^6 - 15*A*B^3*b^7
 - 1296*A^4*a^2*c^5 + 648*(14*A^3*B*a^2*b - 5*A^4*a*b^2)*c^4 + (10000*B^4*a
^4 - 30000*A*B^3*a^3*b + 9936*A^2*B^2*a^2*b^2 + 1080*A^3*B*a*b^3 - 405*A^4*
b^4)*c^3 + 3*(5000*B^4*a^3*b^2 - 3864*A*B^3*a^2*b^3 + 1080*A^2*B^2*a*b^4 -
135*A^3*B*b^5)*c^2 - 3*(497*B^4*a^2*b^4 - 315*A*B^3*a*b^5 + 45*A^2*B^2*b^6)
*c)*x + 1/2*sqrt(1/2)*(B^3*b^10 - 2304*(5*A^2*B*a^4 - 3*A^3*a^3*b)*c^6 + 64
*(500*B^3*a^5 - 420*A*B^2*a^4*b + 198*A^2*B*a^3*b^2 - 81*A^3*a^2*b^3)*c^5 -
 16*(1480*B^3*a^4*b^2 - 1284*A*B^2*a^3*b^3 + 324*A^2*B*a^2*b^4 - 81*A^3*a*b
^5)*c^4 + 4*(1424*B^3*a^3*b^4 - 1332*A*B^2*a^2*b^5 + 234*A^2*B*a*b^6 - 27*A
^3*b^7)*c^3 - (392*B^3*a^2*b^6 - 492*A*B^2*a*b^7 + 63*A^2*B*b^8)*c^2 - (17*
B^3*a*b^8 + 6*A*B^2*b^9)*c + (B*b^13*c^3 - 24576*A*a^6*c^10 + 4096*(13*B*a^
6*b + 3*A*a^5*b^2)*c^9 - 1536*(44*B*a^5*b^3 - 5*A*a^4*b^4)*c^8 + 3840*(9*B*
a^4*b^5 - 2*A*a^3*b^6)*c^7 - 160*(56*B*a^3*b^7 - 15*A*a^2*b^8)*c^6 + 48*(25
*B*a^2*b^9 - 7*A*a*b^10)*c^5 - 18*(4*B*a*b^11 - A*b^12)*c^4)*sqrt((B^4*b^4
+ 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3
*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 -
20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024
*a^5*c^11)))*sqrt(-(B^2*b^7 - 240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B
^2*a^3*b - 16*A*B*a^2*b^2 + 3*A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*
b^4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5 - 6*A*B*b^6)*c - (b^10*c^3 - 20*a*b^8*
c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*
sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a
```

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^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c
)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b
^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640
*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))) + sqrt(1/2)*((b^4*c^3 - 8
*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(
b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c
^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(B^2*b^7 -
240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*
A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^2
*a*b^5 - 6*A*B*b^6)*c - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^
3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*c^4 - 1
8*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^
2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 1
60*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^
10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^
7 - 1024*a^5*c^8))*log(-(35*B^4*a*b^6 - 15*A*B^3*b^7 - 1296*A^4*a^2*c^5 + 6
48*(14*A^3*B*a^2*b - 5*A^4*a*b^2)*c^4 + (10000*B^4*a^4 - 30000*A*B^3*a^3*b
+ 9936*A^2*B^2*a^2*b^2 + 1080*A^3*B*a*b^3 - 405*A^4*b^4)*c^3 + 3*(5000*B^4*
a^3*b^2 - 3864*A*B^3*a^2*b^3 + 1080*A^2*B^2*a*b^4 - 135*A^3*B*b^5)*c^2 - 3*
(497*B^4*a^2*b^4 - 315*A*B^3*a*b^5 + 45*A^2*B^2*b^6)*c)*x - 1/2*sqrt(1/2)*(
B^3*b^10 - 2304*(5*A^2*B*a^4 - 3*A^3*a^3*b)*c^6 + 64*(500*B^3*a^5 - 420*A*B
^2*a^4*b + 198*A^2*B*a^3*b^2 - 81*A^3*a^2*b^3)*c^5 - 16*(1480*B^3*a^4*b^2 -
 1284*A*B^2*a^3*b^3 + 324*A^2*B*a^2*b^4 - 81*A^3*a*b^5)*c^4 + 4*(1424*B^3*a
^3*b^4 - 1332*A*B^2*a^2*b^5 + 234*A^2*B*a*b^6 - 27*A^3*b^7)*c^3 - (392*B^3*
a^2*b^6 - 492*A*B^2*a*b^7 + 63*A^2*B*b^8)*c^2 - (17*B^3*a*b^8 + 6*A*B^2*b^9
)*c + (B*b^13*c^3 - 24576*A*a^6*c^10 + 4096*(13*B*a^6*b + 3*A*a^5*b^2)*c^9
- 1536*(44*B*a^5*b^3 - 5*A*a^4*b^4)*c^8 + 3840*(9*B*a^4*b^5 - 2*A*a^3*b^6)*
c^7 - 160*(56*B*a^3*b^7 - 15*A*a^2*b^8)*c^6 + 48*(25*B*a^2*b^9 - 7*A*a*b^10
)*c^5 - 18*(4*B*a*b^11 - A*b^12)*c^4)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A
^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*
c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*
b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(B^2
*b^7 - 240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b
^2 + 3*A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 -
(35*B^2*a*b^5 - 6*A*B*b^6)*c - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 -
 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*
c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54
*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*
c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11
)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4
*b^2*c^7 - 1024*a^5*c^8))) - 2*(B*a^2*b^2 + 4*(5*B*a^3 - 3*A*a^2*b)*c)*x)/(
(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a
^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^
2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)
```

giac [B] time = 11.66, size = 7578, normalized size = 16.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/64*(3*(2*b^4*c^3 - 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^3*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^
3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(
b^2 - 4*a*c)*b^2*c^3 - 8*(b^2 - 4*a*c)*a*c^4)*(b^4*c - 8*a*b^2*c^2 + 16*a^2
*c^3)^2*A + (2*b^5*c^2 - 40*a*b^3*c^3 + 128*a^2*b*c^4 - sqrt(2)*sqrt(b^2 -
```

4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^3*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 32*(b^2 - 4*a*c)*a*b*c^3)*(b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)^2*B - 24*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^7*c^3 - 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^4 -
2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 - 2*a*b^7*c^4 + 48*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 16*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b^4*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^5*c^5 + 24*a^2*b^5*c^5 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*
c^6 - 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 - 8*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^6 - 96*a^3*b^3*c^6 + 16*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^7 + 128*a^4*b*c^7 + 2*(b^2 - 4*a*c)*a*
b^5*c^4 - 16*(b^2 - 4*a*c)*a^2*b^3*c^5 + 32*(b^2 - 4*a*c)*a^3*b*c^6)*A*abs(
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^8*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^3 - 2*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 - 2*a*b^8*c^3 - 192*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^4 - 24*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^5*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6
*c^4 - 16*a^2*b^6*c^4 + 896*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2
*c^5 + 288*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 12*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 + 384*a^3*b^4*c^5 - 1280*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^6 - 640*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^4*b*c^6 - 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b
^2*c^6 - 1792*a^4*b^2*c^6 + 320*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4
*c^7 + 2560*a^5*c^7 + 2*(b^2 - 4*a*c)*a*b^6*c^3 + 24*(b^2 - 4*a*c)*a^2*b^4*
c^4 - 288*(b^2 - 4*a*c)*a^3*b^2*c^5 + 640*(b^2 - 4*a*c)*a^4*c^6)*B*abs(b^4*
c - 8*a*b^2*c^2 + 16*a^2*c^3) - 3*(2*b^12*c^5 - 8*a*b^10*c^6 - 192*a^2*b^8*
c^7 + 1792*a^3*b^6*c^8 - 5632*a^4*b^4*c^9 + 6144*a^5*b^2*c^10 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^12*c^3 + 4*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^10*c^4 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^11*c^4 + 96*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^10*c^5 - 896*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^6 - 192*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 + 2816*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 + 1024*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 + 96*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^7 - 3072*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 - 1536*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 - 512*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^8 + 768*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^9 - 2*(b^2 - 4*a*c)*b
^10*c^5 + 192*(b^2 - 4*a*c)*a^2*b^6*c^7 - 1024*(b^2 - 4*a*c)*a^3*b^4*c^8 +
1536*(b^2 - 4*a*c)*a^4*b^2*c^9)*A - (2*b^13*c^4 - 68*a*b^11*c^5 + 688*a^2*b
^9*c^6 - 2688*a^3*b^7*c^7 + 2048*a^4*b^5*c^8 + 11264*a^5*b^3*c^9 - 20480*a^
6*b*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^13*c
^2 + 34*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^11*c^
3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^12*c^3 -
344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^4 -
 60*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^10*c^4 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^11*c^4 + 1344*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^5 + 448*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 + 30*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^9*c^5 - 1024*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^6 - 896

*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^6 - 22
4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 - 5
632*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^7 -
 1536*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7
 + 448*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^
7 + 10240*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c
^8 + 5120*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2
*c^8 + 768*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^
3*c^8 - 2560*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*
b*c^9 - 2*(b^2 - 4*a*c)*b^11*c^4 + 60*(b^2 - 4*a*c)*a*b^9*c^5 - 448*(b^2 -
4*a*c)*a^2*b^7*c^6 + 896*(b^2 - 4*a*c)*a^3*b^5*c^7 + 1536*(b^2 - 4*a*c)*a^4
*b^3*c^8 - 5120*(b^2 - 4*a*c)*a^5*b*c^9)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^5*
c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + sqrt((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)^
2 - 4*(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^
2*c^4)))/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)))/((a*b^10*c^3 - 20*a^2*b^8*c
^4 - 2*a*b^9*c^4 + 160*a^3*b^6*c^5 + 32*a^2*b^7*c^5 + a*b^8*c^5 - 640*a^4*b
^4*c^6 - 192*a^3*b^5*c^6 - 16*a^2*b^6*c^6 + 1280*a^5*b^2*c^7 + 512*a^4*b^3*
c^7 + 96*a^3*b^4*c^7 - 1024*a^6*c^8 - 512*a^5*b*c^8 - 256*a^4*b^2*c^8 + 256
*a^5*c^9)*abs(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*abs(c)) - 1/64*(3*(2*b^4*c^
3 - 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2
 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 8
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*
c^3 - 8*(b^2 - 4*a*c)*a*c^4)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)^2*A + (2*b^
5*c^2 - 40*a*b^3*c^3 + 128*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^5 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^4*c - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*
c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3
 - 2*(b^2 - 4*a*c)*b^3*c^2 + 32*(b^2 - 4*a*c)*a*b*c^3)*(b^4*c - 8*a*b^2*c^2
 + 16*a^2*c^3)^2*B + 24*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 -
 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^4 - 2*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 + 2*a*b^7*c^4 + 48*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*b^4*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^5 - 24*a^2*b
^5*c^5 - 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^6 - 32*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c^6 + 96*a^3*b^3*c^6 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^3*b*c^7 - 128*a^4*b*c^7 - 2*(b^2 - 4*a*c)*a*b^5*c^4 + 16*(b^2
 - 4*a*c)*a^2*b^3*c^5 - 32*(b^2 - 4*a*c)*a^3*b*c^6)*A*abs(b^4*c - 8*a*b^2*c
^2 + 16*a^2*c^3) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^8*c^2 + 8
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^3 - 2*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 + 2*a*b^8*c^3 - 192*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b^4*c^4 - 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^5*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 + 16*a^2*b^6
*c^4 + 896*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^5 + 288*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 12*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b^4*c^5 - 384*a^3*b^4*c^5 - 1280*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^5*c^6 - 640*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4
*b*c^6 - 144*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 + 1792*a^4
*b^2*c^6 + 320*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^7 - 2560*a^5*c
^7 - 2*(b^2 - 4*a*c)*a*b^6*c^3 - 24*(b^2 - 4*a*c)*a^2*b^4*c^4 + 288*(b^2 -
4*a*c)*a^3*b^2*c^5 - 640*(b^2 - 4*a*c)*a^4*c^6)*B*abs(b^4*c - 8*a*b^2*c^2 +
 16*a^2*c^3) - 3*(2*b^12*c^5 - 8*a*b^10*c^6 - 192*a^2*b^8*c^7 + 1792*a^3*b^
6*c^8 - 5632*a^4*b^4*c^9 + 6144*a^5*b^2*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq

```
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^12*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^10*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^11*c^4 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^10*c^5 - 896*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^3*b^6*c^6 - 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b^7*c^6 + 2816*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 + 1024*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^7 - 3072*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 - 1536*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 - 512*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^8 + 768*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^9 - 2*(b^2 - 4*a*c)*b^10*c^5 + 192*(b^
2 - 4*a*c)*a^2*b^6*c^7 - 1024*(b^2 - 4*a*c)*a^3*b^4*c^8 + 1536*(b^2 - 4*a*c
)*a^4*b^2*c^9)*A - (2*b^13*c^4 - 68*a*b^11*c^5 + 688*a^2*b^9*c^6 - 2688*a^3
*b^7*c^7 + 2048*a^4*b^5*c^8 + 11264*a^5*b^3*c^9 - 20480*a^6*b*c^10 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^13*c^2 + 34*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^11*c^3 + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^12*c^3 - 344*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^4 - 60*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^10*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^11*c^4 + 1344*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^5 + 448*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 + 30*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^9*c^5 - 1024*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^6 - 896*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^6 - 224*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 - 5632*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^7 - 1536*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 + 448*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 + 10240*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^8 + 5120*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 + 768*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 - 2560*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^9 - 2*(b^2 -
4*a*c)*b^11*c^4 + 60*(b^2 - 4*a*c)*a*b^9*c^5 - 448*(b^2 - 4*a*c)*a^2*b^7*c^
6 + 896*(b^2 - 4*a*c)*a^3*b^5*c^7 + 1536*(b^2 - 4*a*c)*a^4*b^3*c^8 - 5120*(
b^2 - 4*a*c)*a^5*b*c^9)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^5*c - 8*a*b^3*c^2 +
 16*a^2*b*c^3 - sqrt((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)^2 - 4*(a*b^4*c -
8*a^2*b^2*c^2 + 16*a^3*c^3)*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)))/(b^4*c^2
 - 8*a*b^2*c^3 + 16*a^2*c^4)))/((a*b^10*c^3 - 20*a^2*b^8*c^4 - 2*a*b^9*c^4
+ 160*a^3*b^6*c^5 + 32*a^2*b^7*c^5 + a*b^8*c^5 - 640*a^4*b^4*c^6 - 192*a^3*
b^5*c^6 - 16*a^2*b^6*c^6 + 1280*a^5*b^2*c^7 + 512*a^4*b^3*c^7 + 96*a^3*b^4*
c^7 - 1024*a^6*c^8 - 512*a^5*b*c^8 - 256*a^4*b^2*c^8 + 256*a^5*c^9)*abs(b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)*abs(c)) + 1/8*(B*b^3*c*x^7 - 16*B*a*b*c^2*x^
7 + 3*A*b^2*c^2*x^7 + 12*A*a*c^3*x^7 - B*b^4*x^5 - 5*B*a*b^2*c*x^5 + 5*A*b^
3*c*x^5 - 36*B*a^2*c^2*x^5 + 16*A*a*b*c^2*x^5 - 2*B*a*b^3*x^3 - 28*B*a^2*b*
c*x^3 + 19*A*a*b^2*c*x^3 - 4*A*a^2*c^2*x^3 - B*a^2*b^2*x - 20*B*a^3*c*x + 1
2*A*a^2*b*c*x)/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*(c*x^4 + b*x^2 + a)^2)
```

maple [B] time = 0.05, size = 1631, normalized size = 3.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (1/8*(12*A*a*c^2+3*A*b^2*c-16*B*a*b*c+B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7
+1/8*(16*A*a*b*c^2+5*A*b^3*c-36*B*a^2*c^2-5*B*a*b^2*c-B*b^4)/c/(16*a^2*c^2-
```

```
8*a*b^2*c+b^4)*x^5-1/8*a/c*(4*A*a*c^2-19*A*b^2*c+28*B*a*b*c+2*B*b^3)/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^3+1/8*a^2*(12*A*b*c-20*B*a*c-B*b^2)/c/(16*a^2*c^2-8*
a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2-3/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*c*x)*a*A-3/16/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b
^2+9/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*a*A*b+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x)*A*b^3+1/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*B-1/16/(
16*a^2*c^2-8*a*b^2*c+b^4)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*B-5/2/(16*a^2*c^2-8*a
*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*B-9/8/(16*a^2*c^
2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2*B+1/16/(16*
a^2*c^2-8*a*b^2*c+b^4)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*B+3/4
/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*A+3/16/(16*a^2*c^2-8*a*b
^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+9/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*A*b+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*
c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^3-1/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*a*b*B+1/16/(16*a^2*c^2-8*a*b^2*c+b^4)/c*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*B
-5/2/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2
*B-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b
^2*B+1/16/(16*a^2*c^2-8*a*b^2*c+b^4)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*b^4*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((B*b^3*c + 12*A*a*c^3 - (16*B*a*b - 3*A*b^2)*c^2)*x^7 - (B*b^4 + 4*(9*
B*a^2 - 4*A*a*b)*c^2 + 5*(B*a*b^2 - A*b^3)*c)*x^5 - (2*B*a*b^3 + 4*A*a^2*c^
2 + (28*B*a^2*b - 19*A*a*b^2)*c)*x^3 - (B*a^2*b^2 + 4*(5*B*a^3 - 3*A*a^2*b)
*c)*x)/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^
2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*
a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^
2) + 1/8*integrate((B*a*b^2 + (B*b^3 + 12*A*a*c^2 - (16*B*a*b - 3*A*b^2)*c)
*x^2 + 4*(5*B*a^2 - 3*A*a*b)*c)/(c*x^4 + b*x^2 + a), x)/(b^4*c - 8*a*b^2*c^
2 + 16*a^2*c^3)
```

mupad [B] time = 3.95, size = 19041, normalized size = 41.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] atan(((((5242880*B*a^7*c^8 + 3072*A*a*b^11*c^3 - 3145728*A*a^6*b*c^8 - 256*
B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^3*b^7*c^5 - 1966080*A*a^4*b
^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8*c^4 - 655360*B*a^4*b^6*c^5
 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c
^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5
- 6144*a^5*b^2*c^6)) - (x*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c
 - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040
*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A
^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2
*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*
a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*
c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*
a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^
10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*
b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 18
0*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*
a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8
 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 26214
40*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8
+ 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c
+ 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b
^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a
*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*
b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^
3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*
c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*
c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)
^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8
+ 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 +
992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 +
 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^
10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 +
 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^
7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) - (x*(B^2
*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 + 800*B^2*a^4*c^4 + 6*A*B*b^7*c + 57
6*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 + 208*B^2*a^3*b^2*c^3 - 36*B^2*a*b^
6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b^3*c^3 - 66*A*B*a*b^5*c^2 - 672*A*B*
a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*
a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^15)
^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^
11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*
c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c
^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6
 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c - 25*B^2*
a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 -
 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 24
1920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 7
37280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^1
4*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^
5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a
^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*
c^12)))^(1/2)*1i - (((5242880*B*a^7*c^8 + 3072*A*a*b^11*c^3 - 3145728*A*a^6
*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^3*b^7*c^5 - 19
66080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8*c^4 - 655360*
B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7)/(512*(b^12*c
 + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840
*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + (x*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2
*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*
```

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b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7
*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^
2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5
- 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 5
5*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3
- 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 240
00*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 178
1760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^1
5)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^
18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048
*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4
*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4 - 2621
44*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7)
)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^
4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B
^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 3
7440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 5529
60*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880
*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680
*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a
*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B
^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a
^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*
a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(51
2*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a
^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9
 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1
/2) + (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 + 800*B^2*a^4*c^4 + 6*A
*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 + 208*B^2*a^3*b^2*c^3
- 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b^3*c^3 - 66*A*B*a*b^5*c
^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b
^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a
*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 50
40*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216
*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B
^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^
2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^1
5*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^
2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*
b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^
6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) -
180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 72
0*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c
^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 262
1440*a^9*b^2*c^12)))^(1/2)*1i)/((((5242880*B*a^7*c^8 + 3072*A*a*b^11*c^3 -
3145728*A*a^6*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^3
*b^7*c^5 - 1966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8*
c^4 - 655360*B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7)
/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b
^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - (x*(-(B^2*b^17 + 9*A^2*b^1
5*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(
1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 10368
0*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^
2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2
*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*
B*a^8*c^9 - 55*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^
2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b
^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*
b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4
```

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*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*
c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12
*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 29
49120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b
^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680
*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 2
56*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^
15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2
*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b
^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^1
1*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*
c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c - 25*B
^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^
9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 -
 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7
+ 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*
b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16
*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 86016
0*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b
^2*c^12)))^(1/2) - (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 + 800*B^2*
a^4*c^4 + 6*A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 + 208*B^2
*a^3*b^2*c^3 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b^3*c^3 - 6
6*A*B*a*b^5*c^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c
^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A
^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*
B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b
^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*
c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^
5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 -
 55*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^
3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 2
4000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1
781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)
^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*
b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 2580
48*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b
^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) + (((5242880*B*a^7*c^8 + 3072*A*a*b
^11*c^3 - 3145728*A*a^6*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 49
1520*A*a^3*b^7*c^5 - 1966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*
B*a^3*b^8*c^4 - 655360*B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^
6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 -
1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + (x*(-(B^2*b^17 +
 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c -
b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c
^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8
 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 +
 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 +
 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2
) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240
*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 99225
6*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*
B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^
13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 5376
0*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6
*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3
 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^
6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b
^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a
*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 50
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```
40*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216
*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B
^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^
2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^1
5*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^
2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*
b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^
6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) -
180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 72
0*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c
^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 262
1440*a^9*b^2*c^12)))^(1/2) + (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2
+ 800*B^2*a^4*c^4 + 6*A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2
 + 208*B^2*a^3*b^2*c^3 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b
^3*c^3 - 66*A*B*a*b^5*c^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 -
16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15
*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1
/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680
*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2
*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*
a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B
*a^8*c^9 - 55*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2
*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^
12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b
^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*
a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c
^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*
c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 294
9120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) + (1728*A^3*a^4*c^5 - 35*
B^3*a^2*b^7 + 1620*A^3*a^2*b^4*c^3 + 4752*A^3*a^3*b^2*c^4 - 9456*B^3*a^4*b^
3*c^2 + 15*A*B^2*a*b^8 + 4800*A*B^2*a^5*c^4 + 135*A^3*a*b^6*c^2 + 1176*B^3*
a^3*b^5*c - 6400*B^3*a^5*b*c^3 - 705*A*B^2*a^2*b^6*c - 15552*A^2*B*a^4*b*c^
4 + 6084*A*B^2*a^3*b^4*c^2 + 26256*A*B^2*a^4*b^2*c^3 - 1260*A^2*B*a^2*b^5*c
^2 - 13248*A^2*B*a^3*b^3*c^3 + 90*A^2*B*a*b^7*c)/(256*(b^12*c + 4096*a^6*c^
7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 -
 6144*a^5*b^2*c^6))))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c - b
^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2
*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a
^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3
*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*
b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c -
25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*
b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c
^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*
c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*
B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*
b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 8
60160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a
^9*b^2*c^12)))^(1/2)*2i - ((x^5*(B*b^4 + 36*B*a^2*c^2 - 5*A*b^3*c - 16*A*a*
b*c^2 + 5*B*a*b^2*c))/(8*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^7*(B*b^3 +
12*A*a*c^2 + 3*A*b^2*c - 16*B*a*b*c))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) +
(x^3*(4*A*a^2*c^2 + 2*B*a*b^3 - 19*A*a*b^2*c + 28*B*a^2*b*c))/(8*c*(b^4 + 1
6*a^2*c^2 - 8*a*b^2*c)) + (a^2*x*(B*b^2 - 12*A*b*c + 20*B*a*c))/(8*c*(b^4 +
 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 +
 2*b*c*x^6) + atan(((((5242880*B*a^7*c^8 + 3072*A*a*b^11*c^3 - 3145728*A*a^
6*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^3*b^7*c^5 - 1
966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8*c^4 - 655360
*B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7)/(512*(b^12*
c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 384
```

```
0*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - (x*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^
2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B
*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^
7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c
^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5
 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 -
55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3
 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24
000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 17
81760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^
15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b
^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 25804
8*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^
4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4 - 262
144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7
))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c
^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) -
B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 +
37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552
960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 3488
0*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 186368
0*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*
a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*
B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*
a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B
*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(5
12*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*
a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^
9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(
1/2) - (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 + 800*B^2*a^4*c^4 + 6*
A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 + 208*B^2*a^3*b^2*c^3
 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b^3*c^3 - 66*A*B*a*b^5*
c^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*
b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*
a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5
040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 921
6*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*
B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B
^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^
15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A
^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3
*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a
^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) -
 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 7
20*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*
c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 26
21440*a^9*b^2*c^12)))^(1/2)*1i - (((5242880*B*a^7*c^8 + 3072*A*a*b^11*c^3 -
 3145728*A*a^6*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^
3*b^7*c^5 - 1966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8
*c^4 - 655360*B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7
)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*
b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + (x*(-(B^2*b^17 + 9*A^2*b^
15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^
(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 1036
80*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B
^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^
2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A
*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A
^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*
```

b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5
*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(
4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20
*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^1
2*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2
949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*
b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 32768
0*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 -
256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)
^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^
2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*
b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^
11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5
*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*
B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c
^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4
- 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7
+ 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a
*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^1
6*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 8601
60*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*
b^2*c^12)))^(1/2) + (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 + 800*B^2
*a^4*c^4 + 6*A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 + 208*B^
2*a^3*b^2*c^3 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b^3*c^3 -
66*A*B*a*b^5*c^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*
c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*
A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A
*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*
b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13
*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c
^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9
- 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c
^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 +
24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 -
1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2
)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a
*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258
048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*
b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*1i)/((((5242880*B*a^7*c^8 + 3072*A
*a*b^11*c^3 - 3145728*A*a^6*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4
+ 491520*A*a^3*b^7*c^5 - 1966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61
440*B*a^3*b^8*c^4 - 655360*B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*
B*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^
3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - (x*(-(B^2*b^
17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*
c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b
^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3
*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c
^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c
^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^
(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 +
240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 9
92256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 -
6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^1
0*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 +
53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7
*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*
c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^
5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a

```
^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-
(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c
- 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 -
9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 101
60*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 68096
0*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a
*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 73728
0*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*
a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*
B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2
) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4
+ 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^
10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 -
 2621440*a^9*b^2*c^12)))^(1/2) - (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*
c^2 + 800*B^2*a^4*c^4 + 6*A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4
*c^2 + 208*B^2*a^3*b^2*c^3 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a
^2*b^3*c^3 - 66*A*B*a*b^5*c^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^
5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*
b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15
)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 10
3680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140
*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*
B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040
*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180
*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^
2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a
^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(
-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^
20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b
^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 +
 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) + (((5242880*B*a^7*c^
8 + 3072*A*a*b^11*c^3 - 3145728*A*a^6*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^
2*b^9*c^4 + 491520*A*a^3*b^7*c^5 - 1966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^
3*c^7 + 61440*B*a^3*b^8*c^4 - 655360*B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6
- 6291456*B*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*
a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + (x
*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b
^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440
*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A
^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2
*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2
*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c -
 b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a
^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b
^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*
b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1
048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b
^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1
966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*
(256*b^11*c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163
840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*
c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*
A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A
*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*
b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13
*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c
^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9
- 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c
^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 +
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24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 -
1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2
)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a
*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258
048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*
b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) + (x*(B^2*b^8 - 288*A^2*a^3*c^5 +
9*A^2*b^6*c^2 + 800*B^2*a^4*c^4 + 6*A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B
^2*a^2*b^4*c^2 + 208*B^2*a^3*b^2*c^3 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 -
 816*A*B*a^2*b^3*c^3 - 66*A*B*a*b^5*c^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c +
256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^1
7 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c
 - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^
9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*
c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^
4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^
7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(
1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 +
240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 99
2256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6
*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10
*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 5
3760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*
b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) + (1728*A^3
*a^4*c^5 - 35*B^3*a^2*b^7 + 1620*A^3*a^2*b^4*c^3 + 4752*A^3*a^3*b^2*c^4 - 9
456*B^3*a^4*b^3*c^2 + 15*A*B^2*a*b^8 + 4800*A*B^2*a^5*c^4 + 135*A^3*a*b^6*c
^2 + 1176*B^3*a^3*b^5*c - 6400*B^3*a^5*b*c^3 - 705*A*B^2*a^2*b^6*c - 15552*
A^2*B*a^4*b*c^4 + 6084*A*B^2*a^3*b^4*c^2 + 26256*A*B^2*a^4*b^2*c^3 - 1260*A
^2*B*a^2*b^5*c^2 - 13248*A^2*B*a^3*b^3*c^3 + 90*A^2*B*a*b^7*c)/(256*(b^12*c
 + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840
*a^4*b^4*c^5 - 6144*a^5*b^2*c^6))))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^
2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^1
6*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^
6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 -
 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 6
80960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B
^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 7
37280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*
A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 178176
0*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^
(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*
c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^
5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^
11 - 2621440*a^9*b^2*c^12)))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
[Out] Timed out
```

3.112 $\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=380

$$\frac{3x\left(x^2\left(4aBc-4Abc+b^2B\right)-A\left(4ac+b^2\right)+4abB\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^3\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3\left(-\frac{-8aAc^2+12abBc-6A}{\sqrt{b^2-4ac}}\right.}{8\sqrt{}}$$

Rubi [A] time = 1.41, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1275, 1166, 205}

$$\frac{3\left(-\frac{-8aAc^2+12abBc-6Ab^2c+b^3B}{\sqrt{b^2-4ac}}+4aBc-4Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{-8aAc^2+12abBc-6Ab^2c+b^3B}{\sqrt{b^2-4ac}}+4aBc-4Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x^3\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3x\left(x^2\left(4aBc-4Abc+b^2B\right)-A\left(4ac+b^2\right)+4abB\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x^3*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)^2) + (3*x*(4*a*b*B - A*(b^2 + 4*a*c) + (b^2*B - 4*A*b*c + 4*a*B*c)*x^2))
/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*(b^2*B - 4*A*b*c + 4*a*B*c -
(b^3*B - 6*A*b^2*c + 12*a*b*B*c - 8*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(b^2 - 4*a
*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*(b^2*B - 4*A*b*c + 4*a*B*c + (b^3*B
 - 6*A*b^2*c + 12*a*b*B*c - 8*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^2*
Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx = -\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\int \frac{x^2\left(3(Ab-2aB)+3(bB-2Ac)x^2\right)}{\left(a+bx^2+cx^4\right)^2}dx}{4\left(b^2-4ac\right)}$$

$$= -\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x\left(4abB-A\left(b^2+4ac\right)+\left(b^2B-4Abc+4aBc\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= -\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x\left(4abB-A\left(b^2+4ac\right)+\left(b^2B-4Abc+4aBc\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= -\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x\left(4abB-A\left(b^2+4ac\right)+\left(b^2B-4Abc+4aBc\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Mathematica [A] time = 1.70, size = 447, normalized size = 1.18

$$\frac{\frac{8acx\left(A+Bx^2\right)-4abBx+4bx^3(Ac-bB)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{2x\left(4bc\left(aB-3Acx^2\right)+4ac^2\left(A+3Bx^2\right)+b^2\left(3Bcx^2-7Ac\right)+2b^3B\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\sqrt{c}\left(b^2\left(B\sqrt{b^2-4ac}+6Ac\right)-4bc\left(A\sqrt{b^2-4ac}+3aB\right)+4ac\left(B\sqrt{b^2-4ac}+2Ac\right)+b^3(-B)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(b^2\left(B\sqrt{b^2-4ac}-6Ac\right)+4bc\left(3aB-A\sqrt{b^2-4ac}\right)+4ac\left(B\sqrt{b^2-4ac}-2Ac\right)+b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}}{16c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*a*b*B*x + 4*b*(-(b*B) + A*c)*x^3 + 8*a*c*x*(A + B*x^2))/((b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)^2) + (2*x*(2*b^3*B + 4*a*c^2*(A + 3*B*x^2) + 4*b*c*(a*
B - 3*A*c*x^2) + b^2*(-7*A*c + 3*B*c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c
*x^4)) + (3*Sqrt[2]*Sqrt[c]*(-(b^3*B) - 4*b*c*(3*a*B + A*Sqrt[b^2 - 4*a*c])
 + 4*a*c*(2*A*c + B*Sqrt[b^2 - 4*a*c]) + b^2*(6*A*c + B*Sqrt[b^2 - 4*a*c]))
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5
/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(b^3*B + 4*b*c*(3*a*B
 - A*Sqrt[b^2 - 4*a*c]) + b^2*(-6*A*c + B*Sqrt[b^2 - 4*a*c]) + 4*a*c*(-2*A*
c + B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 5.87, size = 5650, normalized size = 14.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(B*b^2*c + 4*(B*a - A*b)*c^2)*x^7 + 2*(5*B*b^3 + 4*A*a*c^2 + (16*B*
a*b - 19*A*b^2)*c)*x^5 + 2*(19*B*a*b^2 - 5*A*b^3 - 4*(B*a^2 + 4*A*a*b)*c)*x
^3 - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a
```

```
*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 -
6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*s
qrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*
B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c +
(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b
^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c
^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6
- 1024*a^7*c^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^
4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))*log(-27*(5*B^4*a^2*b^4 - A*B^3*a*
b^5 - 16*A^4*a^2*c^4 + 40*(2*A^3*B*a^2*b - A^4*a*b^2)*c^3 + (16*B^4*a^4 - 8
0*A*B^3*a^3*b + 40*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 + (40*B^4*a^3*b^2 - 40*A*B^
3*a^2*b^3 + A^3*B*b^5)*c)*x + 27/2*sqrt(1/2)*(4*B^3*a^2*b^7 - A*B^2*a*b^8 -
 256*A^3*a^4*c^5 + 128*(2*A*B^2*a^5 + 2*A^2*B*a^4*b + A^3*a^3*b^2)*c^4 - 64
*(4*B^3*a^5*b + 2*A*B^2*a^4*b^2 + 3*A^2*B*a^3*b^3)*c^3 + 8*(24*B^3*a^4*b^3
+ 6*A^2*B*a^2*b^5 - A^3*a*b^6)*c^2 - (48*B^3*a^3*b^5 - 8*A*B^2*a^2*b^6 + 4*
A^2*B*a*b^7 - A^3*b^8)*c - (4096*(2*B*a^8 - 3*A*a^7*b)*c^7 - 2048*(2*B*a^7*
b^2 - 7*A*a^6*b^3)*c^6 - 1280*(2*B*a^6*b^4 + 5*A*a^5*b^5)*c^5 + 1280*(2*B*a
^5*b^6 + A*a^4*b^7)*c^4 - 80*(10*B*a^4*b^8 + A*a^3*b^9)*c^3 + 8*(14*B*a^3*b
^10 - A*a^2*b^11)*c^2 - (6*B*a^2*b^12 - A*a*b^13)*c)*sqrt((B^4*a^2 - 2*A^2*
B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a
^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))*sqrt(-(B^2*a*b^5 - 16*(4*A*
B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2
 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c + (a*b^10*c - 20*a^2*b^8*c^2
 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)*sqr
t((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*
a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^10*
c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 -
 1024*a^6*c^6))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 +
2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4
*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3
*b*c^2)*x^2)*sqrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^
2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 +
 A^2*b^5)*c + (a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^
4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^
2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 128
0*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^
3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))*log(-27*(5*B^4*a^2*
b^4 - A*B^3*a*b^5 - 16*A^4*a^2*c^4 + 40*(2*A^3*B*a^2*b - A^4*a*b^2)*c^3 + (
16*B^4*a^4 - 80*A*B^3*a^3*b + 40*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 + (40*B^4*a^3
*b^2 - 40*A*B^3*a^2*b^3 + A^3*B*b^5)*c)*x - 27/2*sqrt(1/2)*(4*B^3*a^2*b^7 -
 A*B^2*a*b^8 - 256*A^3*a^4*c^5 + 128*(2*A*B^2*a^5 + 2*A^2*B*a^4*b + A^3*a^3
*b^2)*c^4 - 64*(4*B^3*a^5*b + 2*A*B^2*a^4*b^2 + 3*A^2*B*a^3*b^3)*c^3 + 8*(2
4*B^3*a^4*b^3 + 6*A^2*B*a^2*b^5 - A^3*a*b^6)*c^2 - (48*B^3*a^3*b^5 - 8*A*B^
2*a^2*b^6 + 4*A^2*B*a*b^7 - A^3*b^8)*c - (4096*(2*B*a^8 - 3*A*a^7*b)*c^7 -
2048*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^6 - 1280*(2*B*a^6*b^4 + 5*A*a^5*b^5)*c^5
 + 1280*(2*B*a^5*b^6 + A*a^4*b^7)*c^4 - 80*(10*B*a^4*b^8 + A*a^3*b^9)*c^3 +
 8*(14*B*a^3*b^10 - A*a^2*b^11)*c^2 - (6*B*a^2*b^12 - A*a*b^13)*c)*sqrt((B^
4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b
^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))*sqrt(-(B^2*a*
b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 +
A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c + (a*b^10*c -
20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 102
4*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*
b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c
^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280
*a^5*b^2*c^5 - 1024*a^6*c^6))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a
^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*
b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*
b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*
```

```
c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 -
20*A*B*a*b^4 + A^2*b^5)*c - (a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 -
640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^
2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5
*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^10*c - 20*a^2*b^8*c^2 +
160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))*log(-
27*(5*B^4*a^2*b^4 - A*B^3*a*b^5 - 16*A^4*a^2*c^4 + 40*(2*A^3*B*a^2*b - A^4*
a*b^2)*c^3 + (16*B^4*a^4 - 80*A*B^3*a^3*b + 40*A^3*B*a*b^3 - 5*A^4*b^4)*c^2
 + (40*B^4*a^3*b^2 - 40*A*B^3*a^2*b^3 + A^3*B*b^5)*c)*x + 27/2*sqrt(1/2)*(4
*B^3*a^2*b^7 - A*B^2*a*b^8 - 256*A^3*a^4*c^5 + 128*(2*A*B^2*a^5 + 2*A^2*B*a
^4*b + A^3*a^3*b^2)*c^4 - 64*(4*B^3*a^5*b + 2*A*B^2*a^4*b^2 + 3*A^2*B*a^3*b
^3)*c^3 + 8*(24*B^3*a^4*b^3 + 6*A^2*B*a^2*b^5 - A^3*a*b^6)*c^2 - (48*B^3*a^
3*b^5 - 8*A*B^2*a^2*b^6 + 4*A^2*B*a*b^7 - A^3*b^8)*c + (4096*(2*B*a^8 - 3*A
*a^7*b)*c^7 - 2048*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^6 - 1280*(2*B*a^6*b^4 + 5*
A*a^5*b^5)*c^5 + 1280*(2*B*a^5*b^6 + A*a^4*b^7)*c^4 - 80*(10*B*a^4*b^8 + A*
a^3*b^9)*c^3 + 8*(14*B*a^3*b^10 - A*a^2*b^11)*c^2 - (6*B*a^2*b^12 - A*a*b^1
3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c
^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))
*sqrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*
A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c
- (a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5
*b^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10
*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^
6 - 1024*a^7*c^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*
b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*
b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^
2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(
a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 -
 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*
B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c - (a*b^10*c - 20*a^2*b^8*c^2 + 160*
a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*
a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6
*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^10*c - 20*
a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a
^6*c^6))*log(-27*(5*B^4*a^2*b^4 - A*B^3*a*b^5 - 16*A^4*a^2*c^4 + 40*(2*A^3*
B*a^2*b - A^4*a*b^2)*c^3 + (16*B^4*a^4 - 80*A*B^3*a^3*b + 40*A^3*B*a*b^3 -
5*A^4*b^4)*c^2 + (40*B^4*a^3*b^2 - 40*A*B^3*a^2*b^3 + A^3*B*b^5)*c)*x - 27/
2*sqrt(1/2)*(4*B^3*a^2*b^7 - A*B^2*a*b^8 - 256*A^3*a^4*c^5 + 128*(2*A*B^2*a
^5 + 2*A^2*B*a^4*b + A^3*a^3*b^2)*c^4 - 64*(4*B^3*a^5*b + 2*A*B^2*a^4*b^2 +
 3*A^2*B*a^3*b^3)*c^3 + 8*(24*B^3*a^4*b^3 + 6*A^2*B*a^2*b^5 - A^3*a*b^6)*c^
2 - (48*B^3*a^3*b^5 - 8*A*B^2*a^2*b^6 + 4*A^2*B*a*b^7 - A^3*b^8)*c + (4096*
(2*B*a^8 - 3*A*a^7*b)*c^7 - 2048*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^6 - 1280*(2*
B*a^6*b^4 + 5*A*a^5*b^5)*c^5 + 1280*(2*B*a^5*b^6 + A*a^4*b^7)*c^4 - 80*(10*
B*a^4*b^8 + A*a^3*b^9)*c^3 + 8*(14*B*a^3*b^10 - A*a^2*b^11)*c^2 - (6*B*a^2*
b^12 - A*a*b^13)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2
- 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1
024*a^7*c^7)))*sqrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*
B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4
 + A^2*b^5)*c - (a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*
c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*
c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1
280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*
c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))) + 6*(4*B*a^2*b -
 A*a*b^2 - 4*A*a^2*c)*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 +
 (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2
)*x^2)
```

giac [B] time = 8.26, size = 3162, normalized size = 8.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/32*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 4*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5
*c - 2*b^6*c - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*a*b^4*c^2 + 2*b^5*c^2 + 64
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 32*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^2*b^2*c^3 + 16*a*b^3*c^3 - 16*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 128*a^3*c^4 - 96*a^2*b*c^4 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 48*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*(b^2 - 4*a*c)*b^4*c - 2*(b^2 - 4*a*c)*b^3
*c^2 - 32*(b^2 - 4*a*c)*a^2*c^3 - 24*(b^2 - 4*a*c)*a*b*c^3)*A - 2*(2*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^3*c - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 4
*a*b^5*c + 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 16*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 2*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^3*c^2 + 32*a^2*b^3*c^2 + 6*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 64*a^3*b*c^3 - 16*a^2*b^2*c^3 - 32*a^3
*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 +
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 16*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 8*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 4*(b^2 - 4*a*c)*a*b^3*c
 - 16*(b^2 - 4*a*c)*a^2*b*c^2 - 6*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)
*a^2*c^3)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + sq
rt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2)^2 - 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))
/((a*b^8 - 16*a^2*b^6*c - 2*a*b^7*c + 96*a^3*b^4*c^2 + 24*a^2*b^5*c^2 + a*b
^6*c^2 - 256*a^4*b^2*c^3 - 96*a^3*b^3*c^3 - 12*a^2*b^4*c^3 + 256*a^5*c^4 +
128*a^4*b*c^4 + 48*a^3*b^2*c^4 - 64*a^4*c^5)*abs(c)) + 3/32*((sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 2*b^6*c - 16*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^4*c^2 - 8*a*b^4*c^2 - 2*b^5*c^2 + 64*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b*c^3 - 32*a^2*b^2*c^3 - 16*a*b^3*c^3 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*c^4 + 128*a^3*c^4 + 96*a^2*b*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^4*c - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^3*c^2 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^4*c + 2*(b^2 - 4*a*c)*b^3*c^2 + 32*(b^2 - 4*a
*c)*a^2*c^3 + 24*(b^2 - 4*a*c)*a*b*c^3)*A - 2*(2*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^5 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c
- 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*a*b^5*c + 32*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b^2*c^2 + 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3
```

```
*c^2 - 32*a^2*b^3*c^2 - 6*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^3 + 64*a^3*b*c^3 + 16*a^2*b^2*c^3 + 32*a^3*c^4 + 3*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 - 8*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - 6*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 4*(b^2 - 4*a*c)*a*b^3*c + 16*(b^2 - 4*a*c)*
a^2*b*c^2 + 6*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*B)*arctan(
2*sqrt(1/2)*x/sqrt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - sqrt((b^5 - 8*a*b^3*c
+ 16*a^2*b*c^2)^2 - 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*(b^4*c - 8*a*b^2*c
^2 + 16*a^2*c^3)))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/((a*b^8 - 16*a^2*b^
6*c - 2*a*b^7*c + 96*a^3*b^4*c^2 + 24*a^2*b^5*c^2 + a*b^6*c^2 - 256*a^4*b^2
*c^3 - 96*a^3*b^3*c^3 - 12*a^2*b^4*c^3 + 256*a^5*c^4 + 128*a^4*b*c^4 + 48*a
^3*b^2*c^4 - 64*a^4*c^5)*abs(c)) + 1/8*(3*B*b^2*c*x^7 + 12*B*a*c^2*x^7 - 12
*A*b*c^2*x^7 + 5*B*b^3*x^5 + 16*B*a*b*c*x^5 - 19*A*b^2*c*x^5 + 4*A*a*c^2*x^
5 + 19*B*a*b^2*x^3 - 5*A*b^3*x^3 - 4*B*a^2*c*x^3 - 16*A*a*b*c*x^3 + 12*B*a^
2*b*x - 3*A*a*b^2*x - 12*A*a^2*c*x)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c
 + 16*a^2*c^2))
```

maple [B] time = 0.05, size = 1283, normalized size = 3.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (-3/8*c*(4*A*b*c-4*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8*(4*A*a*c
^2-19*A*b^2*c+16*B*a*b*c+5*B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*(16*A*
a*b*c+5*A*b^3+4*B*a^2*c-19*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-3/8*a*(4
*A*a*c+A*b^2-4*B*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+3/4/(
16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b-3/2/(16*a^2*c^2-8*a*b
^2*c+b^4)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*A-9/8/(16*a^2*c^2-
8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2-3/4/(16*a^2
*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*B-3/16/(16*a^2*c^2-8*a*b^2*c+
b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*B+9/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*B+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a
*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*B-3/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x)*A*b-3/2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)*a*A-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)*A*b^2+3/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*a*B+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*B+9/4/(16*a^
2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*B+3/16/(16
*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(B*b^2*c + 4*(B*a - A*b)*c^2)*x^7 + (5*B*b^3 + 4*A*a*c^2 + (16*B*a*b
 - 19*A*b^2)*c)*x^5 + (19*B*a*b^2 - 5*A*b^3 - 4*(B*a^2 + 4*A*a*b)*c)*x^3 +
3*(4*B*a^2*b - A*a*b^2 - 4*A*a^2*c)*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c
+ 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c
+ 16*a^3*b*c^2)*x^2) - 3/8*integrate((4*B*a*b - A*b^2 - 4*A*a*c - (B*b^2 +
4*(B*a - A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^4 - 8*a*b^2*c + 16*a^2*c^2
)
```

mupad [B] time = 3.49, size = 16688, normalized size = 43.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] atan(((((3*(1048576*A*a^6*c^8 - 256*A*b^12*c^2 + 4096*A*a*b^10*c^3 + 1024*B
*a*b^11*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*
c^6 - 1048576*A*a^5*b^2*c^7 - 20480*B*a^2*b^9*c^3 + 163840*B*a^3*b^7*c^4 -
655360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 +
240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 -
24*a*b^10*c)) - (x*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2
*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2
*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6
*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*
c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20
*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b
*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 +
 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 2
0*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c
^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*
a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*
c^10 + a*b^20*c)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7
+ 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 +
256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*
b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)
^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^
7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2
 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61
440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7
*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 6
4*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360
*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*
a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*
a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c
^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2) - (x*(
9*B^2*b^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2
*a^2*b^2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720
*A*B*a*b^3*c^3 - 288*A*B*a^2*b*c^4))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^
2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2
)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^
11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^
6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 1
1520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536
*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c
```

```
- 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*
A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*
B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2
 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b
^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 -
2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*1i - (((3*(1048576*A*a^6*c^8 - 25
6*A*b^12*c^2 + 4096*A*a*b^10*c^3 + 1024*B*a*b^11*c^2 - 1048576*B*a^6*b*c^7
- 20480*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 - 2048
0*B*a^2*b^9*c^3 + 163840*B*a^3*b^7*c^4 - 655360*B*a^4*b^5*c^5 + 1310720*B*a
^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3
 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(-(9*(B^2*a*b^1
5 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15
)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c
^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 +
4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440
*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*
c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A
*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*
B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^1
1*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5
*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8
+ 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*(256*b^11
*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b
^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256
*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(
1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3
+ 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 614
40*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^
2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^
8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920
*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4
*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b
^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*
a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6
 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440
*a^10*b^2*c^10 + a*b^20*c)))^(1/2) + (x*(9*B^2*b^6*c + 288*A^2*a^2*c^5 + 23
4*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2*a^2*b^2*c^3 - 90*A*B*b^5*c^2 + 14
4*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720*A*B*a*b^3*c^3 - 288*A*B*a^2*b*c^4
))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c))
)*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(
-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11
520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^
2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^
6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 -
 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^
2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^
6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/
(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^1
4*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 196
6080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c))
)^(1/2)*1i)/((((3*(1048576*A*a^6*c^8 - 256*A*b^12*c^2 + 4096*A*a*b^10*c^3 +
 1024*B*a*b^11*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2*b^8*c^4 + 327680*A*a
^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 - 20480*B*a^2*b^9*c^3 + 163840*B*a^3*b^7
*c^4 - 655360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6
*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2
*c^5 - 24*a*b^10*c)) - (x*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2
) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4
160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*
A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a
```

```
^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c
^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^
2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^
8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*
c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3
*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 +
860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^
10*b^2*c^10 + a*b^20*c)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5
*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*
(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*
(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c
 - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2
*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b
^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c
^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*
A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*
c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 -
 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1
048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 +
 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^
8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)
 - (x*(9*B^2*b^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 +
576*B^2*a^2*b^2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^
2 - 720*A*B*a*b^3*c^3 - 288*A*B*a^2*b*c^4))/(32*(b^8 + 256*a^4*c^4 + 96*a^2
*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*
c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2
*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5
*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*
c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6
+ 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*
b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 -
 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 8
1920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b
^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 25804
8*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4
*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2) + (((3*(1048576*A*a^6*c^8
- 256*A*b^12*c^2 + 4096*A*a*b^10*c^3 + 1024*B*a*b^11*c^2 - 1048576*B*a^6*b*
c^7 - 20480*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 -
20480*B*a^2*b^9*c^3 + 163840*B*a^3*b^7*c^4 - 655360*B*a^4*b^5*c^5 + 1310720
*B*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6
*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(-(9*(B^2*a
*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2
)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b
^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^
2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 6
1440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^
7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 -
64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 14336
0*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576
*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760
*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*
c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*(256*
b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a
^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 -
 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^1
5)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*
c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 +
 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 1152
0*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*
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B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 8
1920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B
*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a
^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 +
720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10
*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 262
1440*a^10*b^2*c^10 + a*b^20*c)))^(1/2) + (x*(9*B^2*b^6*c + 288*A^2*a^2*c^5
+ 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2*a^2*b^2*c^3 - 90*A*B*b^5*c^2
+ 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720*A*B*a*b^3*c^3 - 288*A*B*a^2*b
*c^4))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6
*c)))*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2
*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4
- 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 56
0*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^
2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c
^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*
B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^
5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*
c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4
*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 -
 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20
*c)))^(1/2) + (3*(576*B^3*a^4*c^4 - 180*A^3*b^5*c^3 + 540*B^3*a^2*b^4*c^2 +
 1584*B^3*a^3*b^2*c^3 - 9*A*B^2*b^7*c + 45*B^3*a*b^6*c + 576*A^2*B*a^3*c^5
+ 81*A^2*B*b^6*c^2 - 1440*A^3*a*b^3*c^4 - 576*A^3*a^2*b*c^5 - 576*A*B^2*a*b
^5*c^2 - 3456*A*B^2*a^3*b*c^4 + 1980*A^2*B*a*b^4*c^3 - 3600*A*B^2*a^2*b^3*c
^3 + 4464*A^2*B*a^2*b^2*c^4))/(256*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 -
 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c))))*(
-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4
*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520
*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a
^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b
^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81
920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b
^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c
^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(51
2*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c
^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 196608
0*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(
1/2)*2i - ((x^3*(5*A*b^3 - 19*B*a*b^2 + 4*B*a^2*c + 16*A*a*b*c))/(8*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) - (x^5*(5*B*b^3 + 4*A*a*c^2 - 19*A*b^2*c + 16*B*a*
b*c))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a*x*(A*b^2 + 4*A*a*c - 4*B*a*
b))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (3*c*x^7*(B*b^2 - 4*A*b*c + 4*B*a*
c))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8
+ 2*a*b*x^2 + 2*b*c*x^6) + atan(((((3*(1048576*A*a^6*c^8 - 256*A*b^12*c^2 +
 4096*A*a*b^10*c^3 + 1024*B*a*b^11*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2*
b^8*c^4 + 327680*A*a^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 - 20480*B*a^2*b^9*c^
3 + 163840*B*a^3*b^7*c^4 - 655360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6))/(
512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^
4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - (x*(-(9*(B^2*a*b^15 - B^2*a*(-(4
*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*
A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*
a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b
^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c
^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a
^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^
3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6
+ 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^
2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 25
8048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*
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b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*(256*b^11*c^2 - 5120*a*
b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 32768
0*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 -
 16*a*b^6*c)))*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^1
5*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3
*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3
*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4
- 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2
*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7
 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 665
60*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*
B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 -
 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*
b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10
 + a*b^20*c)))^(1/2) - (x*(9*B^2*b^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3
- 288*B^2*a^3*c^4 + 576*B^2*a^2*b^2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^
4 + 126*B^2*a*b^4*c^2 - 720*A*B*a*b^3*c^3 - 288*A*B*a^2*b*c^4))/(32*(b^8 +
256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*
b^15 - B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)
^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^
7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2
 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61
440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7
*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 6
4*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360
*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*
a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*
a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c
^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*1i - (
((3*(1048576*A*a^6*c^8 - 256*A*b^12*c^2 + 4096*A*a*b^10*c^3 + 1024*B*a*b^11
*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*c^6 - 1
048576*A*a^5*b^2*c^7 - 20480*B*a^2*b^9*c^3 + 163840*B*a^3*b^7*c^4 - 655360*
B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2
*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^
10*c)) + (x*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c
 + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^
9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^
7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1
024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*
b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 +
240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*
A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a
*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 76
80*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8
*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 +
a*b^20*c)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960
*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4
*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 -
B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1
/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 -
 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160
*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2
*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8
+ 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a
^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^
6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^
11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^1
2*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 29
49120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2) + (x*(9*B^2*b
```

^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2*a^2*b^
2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720*A*B*a*
b^3*c^3 - 288*A*B*a^2*b*c^4))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256
*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^(
1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3
+ 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 614
40*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^
2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^
8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920
*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4
*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b
^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*
a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6
 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440
*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*1i)/((((3*(1048576*A*a^6*c^8 - 256*A*b^1
2*c^2 + 4096*A*a*b^10*c^3 + 1024*B*a*b^11*c^2 - 1048576*B*a^6*b*c^7 - 20480
*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 - 20480*B*a^2
*b^9*c^3 + 163840*B*a^3*b^7*c^4 - 655360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*
c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840
*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - (x*(-(9*(B^2*a*b^15 - B^2
a(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2)
 - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 10
24*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^
2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^
7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 2
0*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*
b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b
^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11
- 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c
^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 29491
20*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*(256*b^11*c^2 -
5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5
+ 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^
2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^(1/2) +
A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*
A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*
a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b
^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 +
 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^
8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^
4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7
- 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^1
6*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 8601
60*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b
^2*c^10 + a*b^20*c)))^(1/2) - (x*(9*B^2*b^6*c + 288*A^2*a^2*c^5 + 234*A^2*b
^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2*a^2*b^2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a
*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720*A*B*a*b^3*c^3 - 288*A*B*a^2*b*c^4))/(32*
(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*
(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c
 - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2
*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b
^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c
^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*
A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*
c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 -
 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1
048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 +
 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^
8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)

```
 + (((3*(1048576*A*a^6*c^8 - 256*A*b^12*c^2 + 4096*A*a*b^10*c^3 + 1024*B*a*
b^11*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*c^6
 - 1048576*A*a^5*b^2*c^7 - 20480*B*a^2*b^9*c^3 + 163840*B*a^3*b^7*c^4 - 655
360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240
*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*
a*b^10*c)) + (x*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^
15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^
3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^
3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4
 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^
2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^
7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66
560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A
*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3
 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7
*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^1
0 + a*b^20*c)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 4
0960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256
*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^1
5 - B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15
)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c
^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 +
4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440
*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*
c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A
*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*
B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^1
1*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5
*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8
 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2) + (x*(9*B
^2*b^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2*a^
2*b^2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720*A*
B*a*b^3*c^3 - 288*A*B*a^2*b*c^4))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 -
 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^1
5)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*
c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 +
 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 1152
0*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*
B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 8
1920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B
*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a
^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 +
720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10
*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 262
1440*a^10*b^2*c^10 + a*b^20*c)))^(1/2) + (3*(576*B^3*a^4*c^4 - 180*A^3*b^5*
c^3 + 540*B^3*a^2*b^4*c^2 + 1584*B^3*a^3*b^2*c^3 - 9*A*B^2*b^7*c + 45*B^3*a
*b^6*c + 576*A^2*B*a^3*c^5 + 81*A^2*B*b^6*c^2 - 1440*A^3*a*b^3*c^4 - 576*A^
3*a^2*b*c^5 - 576*A*B^2*a*b^5*c^2 - 3456*A*B^2*a^3*b*c^4 + 1980*A^2*B*a*b^4
*c^3 - 3600*A*B^2*a^2*b^3*c^3 + 4464*A^2*B*a^2*b^2*c^4))/(256*(b^12 + 4096*
a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*
b^2*c^5 - 24*a*b^10*c))))*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^(1/2
) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4
160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*
A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a
^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c
^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^
2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^
8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*
c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3
```

```
*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 +
860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^
10*b^2*c^10 + a*b^20*c)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.113 $\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=438

$$-\frac{x\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(-A\left(8abc+b^3\right)+cx^2\left(12abB-A\left(20ac+b^2\right)\right)+aB\left(7b^2-4ac\right)\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}}{}$$

Rubi [A] time = 1.09, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1275, 1178, 1166, 205}

$$-\frac{x\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(cx^2\left(12abB-A\left(20ac+b^2\right)\right)-A\left(8abc+b^3\right)+aB\left(7b^2-4ac\right)\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(A\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}-52abc+b^3\right)+6aB\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A\left(-b^2\sqrt{b^2-4ac}-20ac\sqrt{b^2-4ac}-52abc+b^3\right)+6aB\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
^2) - (x*(a*B*(7*b^2 - 4*a*c) - A*(b^3 + 8*a*b*c) + c*(12*a*b*B - A*(b^2 +
20*a*c))*x^2))/(8*a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(6*a*B*
(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c]) + A*(b^3 - 52*a*b*c + b^2*Sqrt[b^2
- 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) - (Sqrt[c]*(6*a*B*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c]) + A*(b^3 -
 52*a*b*c - b^2*Sqrt[b^2 - 4*a*c] - 20*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^(5/
2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
```

```
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^2 \left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3} dx = -\frac{x\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\int \frac{Ab-2aB+5(bB-2Ac)x^2}{\left(a+bx^2+cx^4\right)^2} dx}{4\left(b^2-4ac\right)}$$

$$= -\frac{x\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x\left(aB\left(7b^2-4ac\right)-A\left(b^3+8abc\right)+c\left(12abB-A\left(b^2 \right.\right.\right.}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= -\frac{x\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x\left(aB\left(7b^2-4ac\right)-A\left(b^3+8abc\right)+c\left(12abB-A\left(b^2 \right.\right.\right.}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= -\frac{x\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x\left(aB\left(7b^2-4ac\right)-A\left(b^3+8abc\right)+c\left(12abB-A\left(b^2 \right.\right.\right.}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Mathematica [A] time = 1.65, size = 436, normalized size = 1.00

$$\frac{1}{16}\left(\frac{4x\left(B\left(2a+bx^2\right)-A\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{2x\left(A\left(8abc+20ac^2x^2+b^3+b^2cx^2\right)+aB\left(4ac-7b^2-12bcx^2\right)\right)}{a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(A\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}-52abc+b^3\right)+6aB\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(A\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}+52abc-b^3\right)-6aB\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*x*(B*(2*a + b*x^2) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)^2) + (2*x*(a*B*(-7*b^2 + 4*a*c - 12*b*c*x^2) + A*(b^3 + 8*a*b*c + b^2*c*
x^2 + 20*a*c^2*x^2)))/(a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sq
rt[c]*(6*a*B*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c]) + A*(b^3 - 52*a*b*c +
b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-6*a*B*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c]
) + A*(-b^3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]))
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^
(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 9.42, size = 7270, normalized size = 16.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(20*A*a*c^3 - (12*B*a*b - A*b^2)*c^2)*x^7 + 2*(4*(B*a^2 + 7*A*a*b)*
c^2 - (19*B*a*b^2 - 2*A*b^3)*c)*x^5 - 2*(5*B*a*b^3 - A*b^4 - 36*A*a^2*c^2 +
 (16*B*a^2*b - 5*A*a*b^2)*c)*x^3 + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 +
16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b
^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^
2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6
 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*
a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a
*b^5)*c + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 12
80*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*
B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^
3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^
2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a
^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*
c^5))*log((10000*A^4*a^3*c^5 - 15000*(2*A^3*B*a^3*b - A^4*a^2*b^2)*c^4 - 3*
(432*B^4*a^5 - 3024*A*B^3*a^4*b - 3312*A^2*B^2*a^3*b^2 + 3864*A^3*B*a^2*b^3
 + 497*A^4*a*b^4)*c^3 - 5*(648*B^4*a^4*b^2 - 216*A*B^3*a^3*b^3 - 648*A^2*B^
2*a^2*b^4 - 189*A^3*B*a*b^5 - 7*A^4*b^6)*c^2 - 15*(27*B^4*a^3*b^4 + 27*A*B^
3*a^2*b^5 + 9*A^2*B^2*a*b^6 + A^3*B*b^7)*c)*x + 1/2*sqrt(1/2)*(27*B^3*a^3*b
^8 + 27*A*B^2*a^2*b^9 + 9*A^2*B*a*b^10 + A^3*b^11 + 6400*(3*A^2*B*a^6 - 4*A
^3*a^5*b)*c^5 - 64*(108*B^3*a^7 - 72*A*B^2*a^6*b + 66*A^2*B*a^5*b^2 - 341*A
^3*a^4*b^3)*c^4 + 16*(216*B^3*a^6*b^2 - 324*A*B^2*a^5*b^3 - 288*A^2*B*a^4*b
^4 - 427*A^3*a^3*b^5)*c^3 + 20*(108*A*B^2*a^4*b^5 + 102*A^2*B*a^3*b^6 + 47*
A^3*a^2*b^7)*c^2 - (216*B^3*a^4*b^6 + 396*A*B^2*a^3*b^7 + 267*A^2*B*a^2*b^8
 + 53*A^3*a*b^9)*c - (3*B*a^4*b^13 + A*a^3*b^14 + 40960*A*a^10*c^7 - 4096*(
9*B*a^10*b + 8*A*a^9*b^2)*c^6 + 1536*(28*B*a^9*b^3 + A*a^8*b^4)*c^5 - 6400*
(3*B*a^8*b^5 - A*a^7*b^6)*c^4 + 160*(24*B*a^7*b^7 - 17*A*a^6*b^8)*c^3 - 240
*(B*a^6*b^9 - 2*A*a^5*b^10)*c^2 - 2*(12*B*a^5*b^11 + 19*A*a^4*b^12)*c)*sqrt
((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*
b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(
a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2
*c^4 - 1024*a^11*c^5)))*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*
(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a
^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c + (a^3*b^
10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 -
1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*
A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b
 + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c
^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^
5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))) - sqrt(1/2
)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 1
6*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2
*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sq
rt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*
c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^
3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c + (a^3*b^10 - 20*a^4*b^8*c + 160*a^
5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*B^4
*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 62
5*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10
 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1
024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^
3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((10000*A^4*a^3*c^5 - 15000*(2*A^3
*B*a^3*b - A^4*a^2*b^2)*c^4 - 3*(432*B^4*a^5 - 3024*A*B^3*a^4*b - 3312*A^2*
B^2*a^3*b^2 + 3864*A^3*B*a^2*b^3 + 497*A^4*a*b^4)*c^3 - 5*(648*B^4*a^4*b^2
 - 216*A*B^3*a^3*b^3 - 648*A^2*B^2*a^2*b^4 - 189*A^3*B*a*b^5 - 7*A^4*b^6)*c^
2 - 15*(27*B^4*a^3*b^4 + 27*A*B^3*a^2*b^5 + 9*A^2*B^2*a*b^6 + A^3*B*b^7)*c)
```

*x - 1/2*sqrt(1/2)*(27*B^3*a^3*b^8 + 27*A*B^2*a^2*b^9 + 9*A^2*B*a*b^10 + A^3*b^11 + 6400*(3*A^2*B*a^6 - 4*A^3*a^5*b)*c^5 - 64*(108*B^3*a^7 - 72*A*B^2*a^6*b + 66*A^2*B*a^5*b^2 - 341*A^3*a^4*b^3)*c^4 + 16*(216*B^3*a^6*b^2 - 324*A*B^2*a^5*b^3 - 288*A^2*B*a^4*b^4 - 427*A^3*a^3*b^5)*c^3 + 20*(108*A*B^2*a^4*b^5 + 102*A^2*B*a^3*b^6 + 47*A^3*a^2*b^7)*c^2 - (216*B^3*a^4*b^6 + 396*A*B^2*a^3*b^7 + 267*A^2*B*a^2*b^8 + 53*A^3*a*b^9)*c - (3*B*a^4*b^13 + A*a^3*b^14 + 40960*A*a^10*c^7 - 4096*(9*B*a^10*b + 8*A*a^9*b^2)*c^6 + 1536*(28*B*a^9*b^3 + A*a^8*b^4)*c^5 - 6400*(3*B*a^8*b^5 - A*a^7*b^6)*c^4 + 160*(24*B*a^7*b^7 - 17*A*a^6*b^8)*c^3 - 240*(B*a^6*b^9 - 2*A*a^5*b^10)*c^2 - 2*(12*B*a^5*b^11 + 19*A*a^4*b^12)*c)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))) + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((10000*A^4*a^3*c^5 - 15000*(2*A^3*B*a^3*b - A^4*a^2*b^2)*c^4 - 3*(432*B^4*a^5 - 3024*A*B^3*a^4*b - 3312*A^2*B^2*a^3*b^2 + 3864*A^3*B*a^2*b^3 + 497*A^4*a*b^4)*c^3 - 5*(648*B^4*a^4*b^2 - 216*A*B^3*a^3*b^3 - 648*A^2*B^2*a^2*b^4 - 189*A^3*B*a*b^5 - 7*A^4*b^6)*c^2 - 15*(27*B^4*a^3*b^4 + 27*A*B^3*a^2*b^5 + 9*A^2*B^2*a*b^6 + A^3*B*b^7)*c)*x + 1/2*sqrt(1/2)*(27*B^3*a^3*b^8 + 27*A*B^2*a^2*b^9 + 9*A^2*B*a*b^10 + A^3*b^11 + 6400*(3*A^2*B*a^6 - 4*A^3*a^5*b)*c^5 - 64*(108*B^3*a^7 - 72*A*B^2*a^6*b + 66*A^2*B*a^5*b^2 - 341*A^3*a^4*b^3)*c^4 + 16*(216*B^3*a^6*b^2 - 324*A*B^2*a^5*b^3 - 288*A^2*B*a^4*b^4 - 427*A^3*a^3*b^5)*c^3 + 20*(108*A*B^2*a^4*b^5 + 102*A^2*B*a^3*b^6 + 47*A^3*a^2*b^7)*c^2 - (216*B^3*a^4*b^6 + 396*A*B^2*a^3*b^7 + 267*A^2*B*a^2*b^8 + 53*A^3*a*b^9)*c + (3*B*a^4*b^13 + A*a^3*b^14 + 40960*A*a^10*c^7 - 4096*(9*B*a^10*b + 8*A*a^9*b^2)*c^6 + 1536*(28*B*a^9*b^3 + A*a^8*b^4)*c^5 - 6400*(3*B*a^8*b^5 - A*a^7*b^6)*c^4 + 160*(24*B*a^7*b^7 - 17*A*a^6*b^8)*c^3 - 240*(B*a^6*b^9 - 2*A*a^5*b^10)*c^2 - 2*(12*B*a^5*b^11 + 19*A*a^4*b^12)*c)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))) - sqrt(1/2)*((a*b^4*c

```
^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 +
 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32
*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(9*B^2*
a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(1
8*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*
A*B*a^2*b^4 - 7*A^2*a*b^5)*c - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 -
 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*
A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c
^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b
^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^
5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^
7*b^2*c^4 - 1024*a^8*c^5))*log((10000*A^4*a^3*c^5 - 15000*(2*A^3*B*a^3*b -
A^4*a^2*b^2)*c^4 - 3*(432*B^4*a^5 - 3024*A*B^3*a^4*b - 3312*A^2*B^2*a^3*b^2
 + 3864*A^3*B*a^2*b^3 + 497*A^4*a*b^4)*c^3 - 5*(648*B^4*a^4*b^2 - 216*A*B^3
*a^3*b^3 - 648*A^2*B^2*a^2*b^4 - 189*A^3*B*a*b^5 - 7*A^4*b^6)*c^2 - 15*(27*
B^4*a^3*b^4 + 27*A*B^3*a^2*b^5 + 9*A^2*B^2*a*b^6 + A^3*B*b^7)*c)*x - 1/2*sq
rt(1/2)*(27*B^3*a^3*b^8 + 27*A*B^2*a^2*b^9 + 9*A^2*B*a*b^10 + A^3*b^11 + 64
00*(3*A^2*B*a^6 - 4*A^3*a^5*b)*c^5 - 64*(108*B^3*a^7 - 72*A*B^2*a^6*b + 66*
A^2*B*a^5*b^2 - 341*A^3*a^4*b^3)*c^4 + 16*(216*B^3*a^6*b^2 - 324*A*B^2*a^5*
b^3 - 288*A^2*B*a^4*b^4 - 427*A^3*a^3*b^5)*c^3 + 20*(108*A*B^2*a^4*b^5 + 10
2*A^2*B*a^3*b^6 + 47*A^3*a^2*b^7)*c^2 - (216*B^3*a^4*b^6 + 396*A*B^2*a^3*b^
7 + 267*A^2*B*a^2*b^8 + 53*A^3*a*b^9)*c + (3*B*a^4*b^13 + A*a^3*b^14 + 4096
0*A*a^10*c^7 - 4096*(9*B*a^10*b + 8*A*a^9*b^2)*c^6 + 1536*(28*B*a^9*b^3 + A
*a^8*b^4)*c^5 - 6400*(3*B*a^8*b^5 - A*a^7*b^6)*c^4 + 160*(24*B*a^7*b^7 - 17
*A*a^6*b^8)*c^3 - 240*(B*a^6*b^9 - 2*A*a^5*b^10)*c^2 - 2*(12*B*a^5*b^11 + 1
9*A*a^4*b^12)*c)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 +
12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^
2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^
4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a
*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*
A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A
^2*a*b^5)*c - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3
+ 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*
A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^
2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^
6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 -
20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*
a^8*c^5))) - 2*(3*B*a^2*b^2 + A*a*b^3 + 4*(3*B*a^3 - 4*A*a^2*b)*c)*x)/((a*b
^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c
^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c
+ 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)
```

giac [B] time = 11.77, size = 7267, normalized size = 16.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/64*((2*b^4*c^2 + 32*a*b^2*c^3 - 160*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^3*c + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*c^2 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c
^3 - 2*(b^2 - 4*a*c)*b^2*c^2 - 40*(b^2 - 4*a*c)*a*c^3)*(a*b^4 - 8*a^2*b^2*c
 + 16*a^3*c^2)^2*A - 12*(2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
```

+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a*b^4 - 8*a^2*b^2*c +
16*a^3*c^2)^2*B + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^9 - 28*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 2*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^8*c - 2*a*b^9*c + 240*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b^5*c^2 + 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2
+ sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^2 + 56*a^2*b^7*c^2 - 832
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 288*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^5*c^3 - 480*a^3*b^5*c^3 + 1024*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^5*b*c^4 + 512*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4
+ 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + 1664*a^4*b^3*c
^4 - 256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 2048*a^5*b*c^5
+ 2*(b^2 - 4*a*c)*a*b^7*c - 48*(b^2 - 4*a*c)*a^2*b^5*c^2 + 288*(b^2 - 4*a*
c)*a^3*b^3*c^3 - 512*(b^2 - 4*a*c)*a^4*b*c^4)*A*abs(a*b^4 - 8*a^2*b^2*c + 1
6*a^3*c^2) + 6*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8 - 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^7*c - 2*a^2*b^8*c + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^3*b^5*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 + 16
*a^3*b^6*c^2 + 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 + 32
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 4*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^6*c^4 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 16*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 256*a^5*b^2*c^4 + 64*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^5 + 512*a^6*c^5 + 2*(b^2 - 4*a
*c)*a^2*b^6*c - 8*(b^2 - 4*a*c)*a^3*b^4*c^2 - 32*(b^2 - 4*a*c)*a^4*b^2*c^3
+ 128*(b^2 - 4*a*c)*a^5*c^4)*B*abs(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2) + (2*a
^2*b^12*c^2 - 136*a^3*b^10*c^3 + 1856*a^4*b^8*c^4 - 10496*a^5*b^6*c^5 + 271
36*a^6*b^4*c^6 - 26624*a^7*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^12 + 68*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b^10*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^11*c - 928*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^4*b^8*c^2 - 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^9*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^10*c^2 + 5248*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^5*b^6*c^3 + 1344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^4*b^7*c^3 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b^8*c^3 - 13568*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 - 5120*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^4 - 672*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^4 + 13312*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^5 + 6656*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 + 2560*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^5 - 3328*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^6 - 2*(b^2 - 4*a*c)*a^2*b^10*c^2
+ 128*(b^2 - 4*a*c)*a^3*b^8*c^3 - 1344*(b^2 - 4*a*c)*a^4*b^6*c^4 + 5120*(b
^2 - 4*a*c)*a^5*b^4*c^5 - 6656*(b^2 - 4*a*c)*a^6*b^2*c^6)*A + 6*(6*a^3*b^11
*c^2 - 88*a^4*b^9*c^3 + 448*a^5*b^7*c^4 - 768*a^6*b^5*c^5 - 512*a^7*b^3*c^6
+ 2048*a^8*b*c^7 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^3*b^11 + 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^4*b^9*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b^10*c - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^5*b^7*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^4*b^8*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*b^9*c^2 + 384*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^6*b^5*c^3 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^5*b^6*c^3 + 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^4*b^7*c^3 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^7*b^3*c^4 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)

```
*a^5*b^5*c^4 - 1024*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^8*b*c^5 - 512*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^7*b^2*c^5 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^7*b*c^6 - 6*(b^2 - 4*a*c)*a^3*b^9*c^2 + 64*(b^2 - 4*a*c)*a^4*b^7*c^3 -
 192*(b^2 - 4*a*c)*a^5*b^5*c^4 + 512*(b^2 - 4*a*c)*a^7*b*c^6)*B)*arctan(2*s
qrt(1/2)*x/sqrt((a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + sqrt((a*b^5 - 8*a^2*b
^3*c + 16*a^3*b*c^2)^2 - 4*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*(a*b^4*c -
8*a^2*b^2*c^2 + 16*a^3*c^3)))/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)))/((a^
3*b^10 - 20*a^4*b^8*c - 2*a^3*b^9*c + 160*a^5*b^6*c^2 + 32*a^4*b^7*c^2 + a^
3*b^8*c^2 - 640*a^6*b^4*c^3 - 192*a^5*b^5*c^3 - 16*a^4*b^6*c^3 + 1280*a^7*b
^2*c^4 + 512*a^6*b^3*c^4 + 96*a^5*b^4*c^4 - 1024*a^8*c^5 - 512*a^7*b*c^5 -
256*a^6*b^2*c^5 + 256*a^7*c^6)*abs(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*abs(c)
) - 1/64*((2*b^4*c^2 + 32*a*b^2*c^3 - 160*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^3*c + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*c^2 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 - 40*(b^2 - 4*a*c)*a*c^3)*(a*b^4 - 8*a^2*b
^2*c + 16*a^3*c^2)^2*A - 12*(2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a*b^4 - 8*a^2*b^2*
c + 16*a^3*c^2)^2*B - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^9 - 28
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 2*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^8*c + 2*a*b^9*c + 240*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^3*b^5*c^2 + 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6
*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^2 - 56*a^2*b^7*c^2 -
 832*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 288*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 24*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*b^5*c^3 + 480*a^3*b^5*c^3 + 1024*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^5*b*c^4 + 512*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2
*c^4 + 144*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 1664*a^4*b
^3*c^4 - 256*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + 2048*a^5*b
*c^5 - 2*(b^2 - 4*a*c)*a*b^7*c + 48*(b^2 - 4*a*c)*a^2*b^5*c^2 - 288*(b^2 -
4*a*c)*a^3*b^3*c^3 + 512*(b^2 - 4*a*c)*a^4*b*c^4)*A*abs(a*b^4 - 8*a^2*b^2*c
 + 16*a^3*c^2) - 6*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^8 - 8*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c - 2*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b^7*c + 2*a^2*b^8*c + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*b^5*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2
- 16*a^3*b^6*c^2 + 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3
+ 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 4*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 256*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^6*c^4 - 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 -
16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + 256*a^5*b^2*c^4 +
64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^5 - 512*a^6*c^5 - 2*(b^2 -
 4*a*c)*a^2*b^6*c + 8*(b^2 - 4*a*c)*a^3*b^4*c^2 + 32*(b^2 - 4*a*c)*a^4*b^2*
c^3 - 128*(b^2 - 4*a*c)*a^5*c^4)*B*abs(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2) +
(2*a^2*b^12*c^2 - 136*a^3*b^10*c^3 + 1856*a^4*b^8*c^4 - 10496*a^5*b^6*c^5 +
 27136*a^6*b^4*c^6 - 26624*a^7*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b^12 + 68*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b^10*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b^11*c - 928*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^4*b^8*c^2 - 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*b^9*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b^10*c^2 + 5248*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^5*b^6*c^3 + 1344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
```

```
 sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^3 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^3 - 13568*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 - 5120*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^4 - 672*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^4 + 13312*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^5 + 6656*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 + 2560*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^5 - 3328*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^6 - 2*(b^2 - 4*a*c)*a^2*b^10
*c^2 + 128*(b^2 - 4*a*c)*a^3*b^8*c^3 - 1344*(b^2 - 4*a*c)*a^4*b^6*c^4 + 512
0*(b^2 - 4*a*c)*a^5*b^4*c^5 - 6656*(b^2 - 4*a*c)*a^6*b^2*c^6)*A + 6*(6*a^3*
b^11*c^2 - 88*a^4*b^9*c^3 + 448*a^5*b^7*c^4 - 768*a^6*b^5*c^5 - 512*a^7*b^3
*c^6 + 2048*a^8*b*c^7 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*b^11 + 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^4*b^9*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^3*b^10*c - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^5*b^7*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^4*b^8*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b^9*c^2 + 384*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^6*b^5*c^3 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^5*b^6*c^3 + 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^4*b^7*c^3 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^7*b^3*c^4 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^5*b^5*c^4 - 1024*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^8*b*c^5 - 512*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^7*b^2*c^5 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^7*b*c^6 - 6*(b^2 - 4*a*c)*a^3*b^9*c^2 + 64*(b^2 - 4*a*c)*a^4*b^7*c
^3 - 192*(b^2 - 4*a*c)*a^5*b^5*c^4 + 512*(b^2 - 4*a*c)*a^7*b*c^6)*B)*arctan
(2*sqrt(1/2)*x/sqrt((a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 - sqrt((a*b^5 - 8*a
^2*b^3*c + 16*a^3*b*c^2)^2 - 4*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*(a*b^4*
c - 8*a^2*b^2*c^2 + 16*a^3*c^3)))/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)))/
((a^3*b^10 - 20*a^4*b^8*c - 2*a^3*b^9*c + 160*a^5*b^6*c^2 + 32*a^4*b^7*c^2
+ a^3*b^8*c^2 - 640*a^6*b^4*c^3 - 192*a^5*b^5*c^3 - 16*a^4*b^6*c^3 + 1280*a
^7*b^2*c^4 + 512*a^6*b^3*c^4 + 96*a^5*b^4*c^4 - 1024*a^8*c^5 - 512*a^7*b*c^
5 - 256*a^6*b^2*c^5 + 256*a^7*c^6)*abs(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*ab
s(c)) - 1/8*(12*B*a*b*c^2*x^7 - A*b^2*c^2*x^7 - 20*A*a*c^3*x^7 + 19*B*a*b^2
*c*x^5 - 2*A*b^3*c*x^5 - 4*B*a^2*c^2*x^5 - 28*A*a*b*c^2*x^5 + 5*B*a*b^3*x^3
 - A*b^4*x^3 + 16*B*a^2*b*c*x^3 - 5*A*a*b^2*c*x^3 - 36*A*a^2*c^2*x^3 + 3*B*
a^2*b^2*x + A*a*b^3*x + 12*B*a^3*c*x - 16*A*a^2*b*c*x)/((a*b^4 - 8*a^2*b^2*
c + 16*a^3*c^2)*(c*x^4 + b*x^2 + a)^2)
```

maple [B] time = 0.05, size = 1335, normalized size = 3.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (1/8*c^2*(20*A*a*c+A*b^2-12*B*a*b)/a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8/a*c
*(28*A*a*b*c+2*A*b^3+4*B*a^2*c-19*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+1
/8*(36*A*a^2*c^2+5*A*a*b^2*c+A*b^4-16*B*a^2*b*c-5*B*a*b^3)/a/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^3+1/8*(16*A*a*b*c-A*b^3-12*B*a^2*c-3*B*a*b^2)/(16*a^2*c^2-8*
a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2-5/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*c*x)*A-1/16/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*A*b^2+13/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*A*b-1/16/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
```

```
*c)^(1/2)*c*x)*A*b^3+3/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*b*B-3/2*a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*B-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*B*b^2+5/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A+1/16
/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+13/4/(16*a^2*c^2-8
*a*b^2*c+b^4)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b-1/16/a/(16*a^2*
c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^3-3/4/(16*a^
2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B-3/2*a/(16*a^2*c^2-8*a*b^2*c+b
^4)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B-9/8/(16*a^2*c^2-8*a*b^2*c+b
^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((20*A*a*c^3 - (12*B*a*b - A*b^2)*c^2)*x^7 + (4*(B*a^2 + 7*A*a*b)*c^2 -
 (19*B*a*b^2 - 2*A*b^3)*c)*x^5 - (5*B*a*b^3 - A*b^4 - 36*A*a^2*c^2 + (16*B*
a^2*b - 5*A*a*b^2)*c)*x^3 - (3*B*a^2*b^2 + A*a*b^3 + 4*(3*B*a^3 - 4*A*a^2*b
)*c)*x)/((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2
*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 -
 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x
^2) + 1/8*integrate((3*B*a*b^2 + A*b^3 + (20*A*a*c^2 - (12*B*a*b - A*b^2)*c
)*x^2 + 4*(3*B*a^2 - 4*A*a*b)*c)/(c*x^4 + b*x^2 + a), x)/(a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2)
```

mupad [B] time = 3.92, size = 18992, normalized size = 43.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^3*(A*b^4 + 36*A*a^2*c^2 - 5*B*a*b^3 + 5*A*a*b^2*c - 16*B*a^2*b*c))/(8*a
*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x*(A*b^3 + 3*B*a*b^2 + 12*B*a^2*c - 16*
A*a*b*c))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5*(4*B*a^2*c^2 + 2*A*b^3*
c + 28*A*a*b*c^2 - 19*B*a*b^2*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c
*x^7*(20*A*a*c^2 + A*b^2*c - 12*B*a*b*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + atan((((
(256*A*a*b^13*c^2 - 3145728*B*a^8*c^8 + 4194304*A*a^7*b*c^8 - 9216*A*a^2*b^
11*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A*a^4*b^7*c^5 + 2949120*A*a^5*b^5*c^
6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^12*c^2 - 12288*B*a^3*b^10*c^3 + 614
40*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 + 3145728*B*a^7*b^2*c^7)/(512*(a^2*
b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 +
3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (x*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A
^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*
A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4
*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7
*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b
```

```
^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8
 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8
*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 +
 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 -
 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^
2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*
a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 25
8048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^1
1*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*
c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^
6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 25
6*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)
^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*
b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b
^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^
11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*
c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^
2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*
c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 -
241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 +
 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2
*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*
c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160
*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b
^2*c^9)))^(1/2) + (x*(A^2*b^6*c^3 - 800*A^2*a^3*c^6 + 288*B^2*a^4*c^5 + 147
2*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 + 144*B^2*a^3*b^2*c^4 - 34*A^2*a*b^
4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^5*c^3 - 288*A*B*a^3*b*c^5))/(32*(a
^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(
-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2
*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A
^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^
2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2
*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a
^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b
^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9
*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8
*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^
2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3
*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^1
4*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 196
6080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*1i
 - (((256*A*a*b^13*c^2 - 3145728*B*a^8*c^8 + 4194304*A*a^7*b*c^8 - 9216*A*a
^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A*a^4*b^7*c^5 + 2949120*A*a^5*b
^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^12*c^2 - 12288*B*a^3*b^10*c^3
+ 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 + 3145728*B*a^7*b^2*c^7)/(512*
(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c
^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(-(A^2*b^17 + 9*B^2*a^2*b^1
5 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2)
 + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^
2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^
2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*
a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^
9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^
2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*
c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*
c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c
 - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10
- 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4
 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 294912
```

0*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) - (x*(A^2*b^6*c^3 - 800*A^2*a^3*c^6 + 288*B^2*a^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 + 144*B^2*a^3*b^2*c^4 - 34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^5*c^3 - 288*A*B*a^3*b*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*1i)/((((256*A*a*b^13*c^2 - 3145728*B*a^8*c^8 + 4194304*A*a^7*b*c^8 - 9216*A*a^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A*a^4*b^7*c^5 + 2949120*A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^12*c^2 - 12288*B*a^3*b^10*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 + 3145728*B*a^7*b^2*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (x*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^

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2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15
*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2
*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b
^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7
*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 1
80*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720
*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^
5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621
440*a^12*b^2*c^9)))^(1/2) + (x*(A^2*b^6*c^3 - 800*A^2*a^3*c^6 + 288*B^2*a^4
*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 + 144*B^2*a^3*b^2*c^4 - 3
4*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^5*c^3 - 288*A*B*a^3*b*c^
5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^
2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) +
 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2
 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 -
 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 +
 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 55
2960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-
(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 7372
80*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A
*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*
A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)
/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 76
80*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8
*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9))
)^(1/2) + (((256*A*a*b^13*c^2 - 3145728*B*a^8*c^8 + 4194304*A*a^7*b*c^8 - 9
216*A*a^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A*a^4*b^7*c^5 + 2949120*
A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^12*c^2 - 12288*B*a^3*b^
10*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 + 3145728*B*a^7*b^2*c^7
)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^
5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(-(A^2*b^17 + 9*B^2*
a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15
)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 3
4880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 186
3680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 1036
80*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040
*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 172
0320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^
3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a
^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(
-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^1
3*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b
^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 +
 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 2
56*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5
 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*
b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*
c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1
140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 437
76*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 504
0*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*
B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^
15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B
^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4
*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a
^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) -
 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 7
20*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*
c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 26
```

21440*a^12*b^2*c^9)))^(1/2) - (x*(A^2*b^6*c^3 - 800*A^2*a^3*c^6 + 288*B^2*a^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 + 144*B^2*a^3*b^2*c^4 - 34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^5*c^3 - 288*A*B*a^3*b*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (35*A^3*b^6*c^4 - 8000*A^3*a^3*c^7 - 12720*A^3*a^2*b^2*c^6 + 540*B^3*a^2*b^5*c^3 + 4320*B^3*a^3*b^3*c^4 - 2880*A*B^2*a^4*c^6 - 15*A^2*B*b^7*c^3 + 84*A^3*a*b^4*c^5 + 1728*B^3*a^4*b*c^5 + 135*A*B^2*a*b^6*c^3 - 360*A^2*B*a*b^5*c^4 + 26880*A^2*B*a^3*b*c^6 - 5580*A*B^2*a^2*b^4*c^4 - 20592*A*B^2*a^3*b^2*c^5 + 15696*A^2*B*a^2*b^3*c^5)/(256*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5))))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*2i + atan(((((256*A*a*b^13*c^2 - 3145728*B*a^8*c^8 + 4194304*A*a^7*b*c^8 - 9216*A*a^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A*a^4*b^7*c^5 + 2949120*A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^12*c^2 - 12288*B*a^3*b^10*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 + 3145728*B*a^7*b^2*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (x*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) +

```
6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*
a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*
a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^
6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*
c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*
a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^
2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^
5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c -
 b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 -
40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 -
 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*
a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (x*(A^2*b^6*c^3 - 800*A^2*a^
3*c^6 + 288*B^2*a^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 + 144*
B^2*a^3*b^2*c^4 - 34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^5*c^3
 - 288*A*B*a^3*b*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b
^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(4*a*c
 - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 11
40*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 4377
6*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040
*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B
^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^1
5*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^
2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*
b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^
7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) -
180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 72
0*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c
^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 262
1440*a^12*b^2*c^9)))^(1/2)*1i - (((256*A*a*b^13*c^2 - 3145728*B*a^8*c^8 + 4
194304*A*a^7*b*c^8 - 9216*A*a^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A*
a^4*b^7*c^5 + 2949120*A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^1
2*c^2 - 12288*B*a^3*b^10*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 +
 3145728*B*a^7*b^2*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240
*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (
x*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*
a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 1016
0*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960
*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*
B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^
2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c
- b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*
a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*
b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8
*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(
a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*
b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 -
1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)
*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^
4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 -
16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*
b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1
/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880
*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680
*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B
^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B
*a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320
*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^
12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b
^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*
```

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a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^
10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*
c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 294
9120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) - (x*(A^2*b^6*c^3 - 800*A
^2*a^3*c^6 + 288*B^2*a^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 +
 144*B^2*a^3*b^2*c^4 - 34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^
5*c^3 - 288*A*B*a^3*b*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*
a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(
4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16
 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 +
 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 -
 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9
216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*
a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 1
80*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B
*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A
*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/
2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c
 + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b
^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8
- 2621440*a^12*b^2*c^9)))^(1/2)*1i)/((((256*A*a*b^13*c^2 - 3145728*B*a^8*c^
8 + 4194304*A*a^7*b*c^8 - 9216*A*a^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 8192
00*A*a^4*b^7*c^5 + 2949120*A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^
2*b^12*c^2 - 12288*B*a^3*b^10*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*
c^6 + 3145728*B*a^7*b^2*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c
+ 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)
) - (x*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9
*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 -
 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 6
80960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 3
7440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 5529
60*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4
*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280
*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B
*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*
B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(
512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680
*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c
^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^
(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b
^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c
^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2
*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^1
5)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 +
34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 18
63680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103
680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 98304
0*A*B*a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 17
20320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a
^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*
a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*
(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^
13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*
b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7
+ 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (x*(A^2*b^6*c^3 -
800*A^2*a^3*c^6 + 288*B^2*a^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*
c^3 + 144*B^2*a^3*b^2*c^4 - 34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B
*a*b^5*c^3 - 288*A*B*a^3*b*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c
+ 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^
```

2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a
*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*
c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*
c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^
4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55
*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^
8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 2400
0*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781
760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15
)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b
^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*
a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4
*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (((256*A*a*b^13*c^2 - 3145728*B*a^8*
c^8 + 4194304*A*a^7*b*c^8 - 9216*A*a^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 81
9200*A*a^4*b^7*c^5 + 2949120*A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*
a^2*b^12*c^2 - 12288*B*a^3*b^10*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^
4*c^6 + 3145728*B*a^7*b^2*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*
c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^
5)) + (x*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) -
 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2
 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 -
 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 +
 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 55
2960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-
(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 7372
80*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A
*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*
A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)
/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 76
80*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8
*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9))
)^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4
*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6
*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B
^2*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)
^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3
+ 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 +
1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 1
03680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983
040*A*B*a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) -
1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B
*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*
B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*
b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*
a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^
7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^
7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) - (x*(A^2*b^6*c^3
- 800*A^2*a^3*c^6 + 288*B^2*a^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^
4*c^3 + 144*B^2*a^3*b^2*c^4 - 34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A
*B*a*b^5*c^3 - 288*A*B*a^3*b*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*
c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*
b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B
*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^
9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^
3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*
c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 -
55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*
c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24
000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 17

```
81760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^
15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4
*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 25804
8*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b
^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (35*A^3*b^6*c^4 - 8000*A^3*a^3*c^7
 - 12720*A^3*a^2*b^2*c^6 + 540*B^3*a^2*b^5*c^3 + 4320*B^3*a^3*b^3*c^4 - 288
0*A*B^2*a^4*c^6 - 15*A^2*B*b^7*c^3 + 84*A^3*a*b^4*c^5 + 1728*B^3*a^4*b*c^5
+ 135*A*B^2*a*b^6*c^3 - 360*A^2*B*a*b^5*c^4 + 26880*A^2*B*a^3*b*c^6 - 5580*
A*B^2*a^2*b^4*c^4 - 20592*A*B^2*a^3*b^2*c^5 + 15696*A^2*B*a^2*b^3*c^5)/(256
*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*
c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5))))*(-(A^2*b^17 + 9*B^2*a^2*b^15
- A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) +
 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*
a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*
a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^
6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*
c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*
a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^
2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^
5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c -
 b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 -
40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 -
 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*
a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.114 $\int \frac{A+Bx^2}{\left(a+bx^2+cx^4\right)^3} \, dx$

Optimal. Leaf size=460

$$\frac{x\left(A\left(28a^2c^2-25ab^2c+3b^4\right)+cx^2\left(3A\left(b^3-8abc\right)+aB\left(20ac+b^2\right)\right)+abB\left(8ac+b^2\right)\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{3A\left(56a^2c^2-10ab^2c+b^4\right)}{\sqrt{b^2-4a}}\right.}{}$$

Rubi [A] time = 1.35, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1178, 1166, 205}

$$\frac{x\left(A\left(28a^2c^2-25ab^2c+3b^4\right)+cx^2\left(3A\left(b^3-8abc\right)+aB\left(20ac+b^2\right)\right)+abB\left(8ac+b^2\right)\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{3A\left(56a^2c^2-10ab^2c+b^4\right)+abB\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+3A\left(b^3-8abc\right)+aB\left(20ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{3A\left(56a^2c^2-10ab^2c+b^4\right)+abB\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+3A\left(b^3-8abc\right)+aB\left(20ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2))/(4*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)^2) + (x*(a*b*B*(b^2 + 8*a*c) + A*(3*b^4 - 25*a*b^2*c + 28*a
^2*c^2) + c*(a*B*(b^2 + 20*a*c) + 3*A*(b^3 - 8*a*b*c))*x^2))/(8*a^2*(b^2 -
4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(a*B*(b^2 + 20*a*c) + 3*A*(b^3 - 8
*a*b*c) + (a*b*B*(b^2 - 52*a*c) + 3*A*(b^4 - 10*a*b^2*c + 56*a^2*c^2))/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*
Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(a*B*(b
^2 + 20*a*c) + 3*A*(b^3 - 8*a*b*c) - (a*b*B*(b^2 - 52*a*c) + 3*A*(b^4 - 10*
a*b^2*c + 56*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 -
4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{A+Bx^2}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{x\left(Ab^2-abB-2aAc+(Ab-2aB)cx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{-3Ab^2-abB+14aAc-5(Ab-2aB)cx^2}{\left(a+bx^2+cx^4\right)^2}\,dx}{4a\left(b^2-4ac\right)}$$

$$= \frac{x\left(Ab^2-abB-2aAc+(Ab-2aB)cx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(abB\left(b^2+8ac\right)+A\left(3b^4-25ab^2c+28\right.\right.}{8a^2\left(b^2-4a\right.}$$

$$= \frac{x\left(Ab^2-abB-2aAc+(Ab-2aB)cx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(abB\left(b^2+8ac\right)+A\left(3b^4-25ab^2c+28\right.\right.}{8a^2\left(b^2-4a\right.}$$

$$= \frac{x\left(Ab^2-abB-2aAc+(Ab-2aB)cx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(abB\left(b^2+8ac\right)+A\left(3b^4-25ab^2c+28\right.\right.}{8a^2\left(b^2-4a\right.}$$

Mathematica [A] time = 2.19, size = 516, normalized size = 1.12

$$\frac{\frac{2x\left(A\left(28a^2c^2-25ab^2c-24abc^2x^2+3b^4+3b^3cx^2\right)+aB\left(8abc+20ac^2x^2+b^3+b^2cx^2\right)\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(3A\left(56a^2c^2-10ab^2c-8abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)+aB\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}-52abc+b^3\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(3A\left(-56a^2c^2+10ab^2c-8abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}-b^4\right)+aB\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}+52abc-b^3\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{4ax\left(aB\left(b+2cx^2\right)-A\left(-2ac+b^2+bcx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*a*x*(a*B*(b + 2*c*x^2) - A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)^2) + (2*x*(a*B*(b^3 + 8*a*b*c + b^2*c*x^2 + 20*a*c^2*x^2)
 + A*(3*b^4 - 25*a*b^2*c + 28*a^2*c^2 + 3*b^3*c*x^2 - 24*a*b*c^2*x^2)))/((b
^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(a*B*(b^3 - 52*a*b*c
+ b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]) + 3*A*(b^4 - 10*a*b^2*c
 + 56*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt
[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(a*B*(-b^3 + 52*a*b*c + b^2*Sqr
t[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]) + 3*A*(-b^4 + 10*a*b^2*c - 56*a^
2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqr
t[b^2 - 4*a*c]]))/(16*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{\left(a+bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(a + b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 23.68, size = 9909, normalized size = 21.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(4*(5*B*a^2 - 6*A*a*b)*c^3 + (B*a*b^2 + 3*A*b^3)*c^2)*x^7 + 2*(28*A
*a^2*c^3 + 7*(4*B*a^2*b - 7*A*a*b^2)*c^2 + 2*(B*a*b^3 + 3*A*b^4)*c)*x^5 + 2
```

```
*(B*a*b^4 + 3*A*b^5 + 4*(9*B*a^3 - A*a^2*b)*c^2 + 5*(B*a^2*b^2 - 4*A*a*b^3)
*c)*x^3 - sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b
^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3
)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c
 + 16*a^5*b*c^2)*x^2)*sqrt(-(B^2*a^2*b^7 + 6*A*B*a*b^8 + 9*A^2*b^9 - 1680*(
4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3
*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*
(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7)*c + (a^5*b^10 - 20*a^6*b^8*
c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*s
qrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7
+ 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b +
 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2*B^2*a^4*
b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 25
8*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)
*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280
*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2
 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log((3111696*A^4*a^
4*c^7 - 1555848*(2*A^3*B*a^4*b + A^4*a^3*b^2)*c^6 - (10000*B^4*a^6 - 90000*
A*B^3*a^5*b - 863136*A^2*B^2*a^4*b^2 - 1298376*A^3*B*a^3*b^3 - 339309*A^4*a
^2*b^4)*c^5 - 3*(5000*B^4*a^5*b^2 + 32952*A*B^3*a^4*b^3 + 79488*A^2*B^2*a^3
*b^4 + 80919*A^3*B*a^2*b^5 + 12069*A^4*a*b^6)*c^4 + 21*(71*B^4*a^4*b^4 + 53
7*A*B^3*a^3*b^5 + 1314*A^2*B^2*a^2*b^6 + 1053*A^3*B*a*b^7 + 81*A^4*b^8)*c^3
 - 35*(B^4*a^3*b^6 + 9*A*B^3*a^2*b^7 + 27*A^2*B^2*a*b^8 + 27*A^3*B*b^9)*c^2
)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^11 + 9*A*B^2*a^2*b^12 + 27*A^2*B*a*b^13 + 27
*A^3*b^14 - 2370816*A^3*a^7*c^7 + 2688*(50*A*B^2*a^8 + 384*A^2*B*a^7*b + 11
43*A^3*a^6*b^2)*c^6 - 64*(400*B^3*a^8*b + 4062*A*B^2*a^7*b^2 + 17541*A^2*B*
a^6*b^3 + 26865*A^3*a^5*b^4)*c^5 + 8*(2728*B^3*a^7*b^3 + 20520*A*B^2*a^6*b^
4 + 62694*A^2*B*a^5*b^5 + 67797*A^3*a^4*b^6)*c^4 - 7*(976*B^3*a^6*b^5 + 674
4*A*B^2*a^5*b^6 + 16884*A^2*B*a^4*b^7 + 14985*A^3*a^3*b^8)*c^3 + (940*B^3*a
^5*b^7 + 6591*A*B^2*a^4*b^8 + 15489*A^2*B*a^3*b^9 + 12528*A^3*a^2*b^10)*c^2
 - (53*B^3*a^4*b^9 + 414*A*B^2*a^3*b^10 + 1053*A^2*B*a^2*b^11 + 864*A^3*a*b
^12)*c - (B*a^6*b^14 + 3*A*a^5*b^15 + 4096*(10*B*a^13 - 33*A*a^12*b)*c^7 -
2048*(16*B*a^12*b^2 - 99*A*a^11*b^3)*c^6 + 768*(2*B*a^11*b^4 - 169*A*a^10*b
^5)*c^5 + 1280*(5*B*a^10*b^6 + 36*A*a^9*b^7)*c^4 - 80*(34*B*a^9*b^8 + 123*A
*a^8*b^9)*c^3 + 24*(20*B*a^8*b^10 + 53*A*a^7*b^11)*c^2 - (38*B*a^7*b^12 + 9
3*A*a^6*b^13)*c)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6
+ 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 +
 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b +
17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(2
5*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^
5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a
^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(B^2*a^2*b^7 + 6*A
*B*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*
b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^
4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7
)*c + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a
^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B
^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^
2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B
^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)
*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A
^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*
c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 2
0*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a
^10*c^5))) + sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^
4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*
c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^
3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(B^2*a^2*b^7 + 6*A*B*a*b^8 + 9*A^2*b^9 - 168
0*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*
```

```
a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5)*c^2 -
 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7)*c + (a^5*b^10 - 20*a^6*b
^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5
)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b
^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*B*a^4*
b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2*B^2*a
^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 +
 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b
^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1
280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*
c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log((3111696*A^4
*a^4*c^7 - 1555848*(2*A^3*B*a^4*b + A^4*a^3*b^2)*c^6 - (10000*B^4*a^6 - 900
00*A*B^3*a^5*b - 863136*A^2*B^2*a^4*b^2 - 1298376*A^3*B*a^3*b^3 - 339309*A^
4*a^2*b^4)*c^5 - 3*(5000*B^4*a^5*b^2 + 32952*A*B^3*a^4*b^3 + 79488*A^2*B^2*
a^3*b^4 + 80919*A^3*B*a^2*b^5 + 12069*A^4*a*b^6)*c^4 + 21*(71*B^4*a^4*b^4 +
 537*A*B^3*a^3*b^5 + 1314*A^2*B^2*a^2*b^6 + 1053*A^3*B*a*b^7 + 81*A^4*b^8)*
c^3 - 35*(B^4*a^3*b^6 + 9*A*B^3*a^2*b^7 + 27*A^2*B^2*a*b^8 + 27*A^3*B*b^9)*
c^2)*x - 1/2*sqrt(1/2)*(B^3*a^3*b^11 + 9*A*B^2*a^2*b^12 + 27*A^2*B*a*b^13 +
 27*A^3*b^14 - 2370816*A^3*a^7*c^7 + 2688*(50*A*B^2*a^8 + 384*A^2*B*a^7*b +
 1143*A^3*a^6*b^2)*c^6 - 64*(400*B^3*a^8*b + 4062*A*B^2*a^7*b^2 + 17541*A^2
*B*a^6*b^3 + 26865*A^3*a^5*b^4)*c^5 + 8*(2728*B^3*a^7*b^3 + 20520*A*B^2*a^6
*b^4 + 62694*A^2*B*a^5*b^5 + 67797*A^3*a^4*b^6)*c^4 - 7*(976*B^3*a^6*b^5 +
6744*A*B^2*a^5*b^6 + 16884*A^2*B*a^4*b^7 + 14985*A^3*a^3*b^8)*c^3 + (940*B^
3*a^5*b^7 + 6591*A*B^2*a^4*b^8 + 15489*A^2*B*a^3*b^9 + 12528*A^3*a^2*b^10)*
c^2 - (53*B^3*a^4*b^9 + 414*A*B^2*a^3*b^10 + 1053*A^2*B*a^2*b^11 + 864*A^3*
a*b^12)*c - (B*a^6*b^14 + 3*A*a^5*b^15 + 4096*(10*B*a^13 - 33*A*a^12*b)*c^7
 - 2048*(16*B*a^12*b^2 - 99*A*a^11*b^3)*c^6 + 768*(2*B*a^11*b^4 - 169*A*a^1
0*b^5)*c^5 + 1280*(5*B*a^10*b^6 + 36*A*a^9*b^7)*c^4 - 80*(34*B*a^9*b^8 + 12
3*A*a^8*b^9)*c^3 + 24*(20*B*a^8*b^10 + 53*A*a^7*b^11)*c^2 - (38*B*a^7*b^12
+ 93*A*a^6*b^13)*c)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b
^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^
5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b
 + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2
*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2
*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 64
0*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(B^2*a^2*b^7 +
6*A*B*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a
^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3
*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*
b^7)*c + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 128
0*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^
2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25
*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*
A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b
^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 156
6*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b
^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10
 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 102
4*a^10*c^5))) - sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 +
 a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4
*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4
*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(B^2*a^2*b^7 + 6*A*B*a*b^8 + 9*A^2*b^9 -
1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*b - 4*A*B*a^4*b^2 - 9*A
^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5)*c^
2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7)*c - (a^5*b^10 - 20*a^
6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*
c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3*B*
a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*B*a
^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2*B^
```

2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^
2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*
a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3
+ 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b
^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log((3111696*
A^4*a^4*c^7 - 1555848*(2*A^3*B*a^4*b + A^4*a^3*b^2)*c^6 - (10000*B^4*a^6 -
90000*A*B^3*a^5*b - 863136*A^2*B^2*a^4*b^2 - 1298376*A^3*B*a^3*b^3 - 339309
*A^4*a^2*b^4)*c^5 - 3*(5000*B^4*a^5*b^2 + 32952*A*B^3*a^4*b^3 + 79488*A^2*B
^2*a^3*b^4 + 80919*A^3*B*a^2*b^5 + 12069*A^4*a*b^6)*c^4 + 21*(71*B^4*a^4*b^
4 + 537*A*B^3*a^3*b^5 + 1314*A^2*B^2*a^2*b^6 + 1053*A^3*B*a*b^7 + 81*A^4*b^
8)*c^3 - 35*(B^4*a^3*b^6 + 9*A*B^3*a^2*b^7 + 27*A^2*B^2*a*b^8 + 27*A^3*B*b^
9)*c^2)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^11 + 9*A*B^2*a^2*b^12 + 27*A^2*B*a*b^1
3 + 27*A^3*b^14 - 2370816*A^3*a^7*c^7 + 2688*(50*A*B^2*a^8 + 384*A^2*B*a^7*
b + 1143*A^3*a^6*b^2)*c^6 - 64*(400*B^3*a^8*b + 4062*A*B^2*a^7*b^2 + 17541*
A^2*B*a^6*b^3 + 26865*A^3*a^5*b^4)*c^5 + 8*(2728*B^3*a^7*b^3 + 20520*A*B^2*
a^6*b^4 + 62694*A^2*B*a^5*b^5 + 67797*A^3*a^4*b^6)*c^4 - 7*(976*B^3*a^6*b^5
 + 6744*A*B^2*a^5*b^6 + 16884*A^2*B*a^4*b^7 + 14985*A^3*a^3*b^8)*c^3 + (940
*B^3*a^5*b^7 + 6591*A*B^2*a^4*b^8 + 15489*A^2*B*a^3*b^9 + 12528*A^3*a^2*b^1
0)*c^2 - (53*B^3*a^4*b^9 + 414*A*B^2*a^3*b^10 + 1053*A^2*B*a^2*b^11 + 864*A
^3*a*b^12)*c + (B*a^6*b^14 + 3*A*a^5*b^15 + 4096*(10*B*a^13 - 33*A*a^12*b)*
c^7 - 2048*(16*B*a^12*b^2 - 99*A*a^11*b^3)*c^6 + 768*(2*B*a^11*b^4 - 169*A*
a^10*b^5)*c^5 + 1280*(5*B*a^10*b^6 + 36*A*a^9*b^7)*c^4 - 80*(34*B*a^9*b^8 +
 123*A*a^8*b^9)*c^3 + 24*(20*B*a^8*b^10 + 53*A*a^7*b^11)*c^2 - (38*B*a^7*b^
12 + 93*A*a^6*b^13)*c)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^
2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2
*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^
5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2
 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*
a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 -
 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(B^2*a^2*b^7
 + 6*A*B*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^
2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*
a^3*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2
*a*b^7)*c - (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 +
1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54
*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*
(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 54
00*A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^
2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 +
1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^1
2*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^
10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 -
1024*a^10*c^5))) + sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^
8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*
a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*
a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(B^2*a^2*b^7 + 6*A*B*a*b^8 + 9*A^2*b^9
 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*b - 4*A*B*a^4*b^2 -
9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5)
c^2 - 7(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7)*c - (a^5*b^10 - 20
*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^
10*c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3
*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*
B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2
*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5
*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A
^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c
^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^
7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log((31116
96*A^4*a^4*c^7 - 1555848*(2*A^3*B*a^4*b + A^4*a^3*b^2)*c^6 - (10000*B^4*a^6

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 - 90000*A*B^3*a^5*b - 863136*A^2*B^2*a^4*b^2 - 1298376*A^3*B*a^3*b^3 - 339
309*A^4*a^2*b^4)*c^5 - 3*(5000*B^4*a^5*b^2 + 32952*A*B^3*a^4*b^3 + 79488*A^
2*B^2*a^3*b^4 + 80919*A^3*B*a^2*b^5 + 12069*A^4*a*b^6)*c^4 + 21*(71*B^4*a^4
*b^4 + 537*A*B^3*a^3*b^5 + 1314*A^2*B^2*a^2*b^6 + 1053*A^3*B*a*b^7 + 81*A^4
*b^8)*c^3 - 35*(B^4*a^3*b^6 + 9*A*B^3*a^2*b^7 + 27*A^2*B^2*a*b^8 + 27*A^3*B
*b^9)*c^2)*x - 1/2*sqrt(1/2)*(B^3*a^3*b^11 + 9*A*B^2*a^2*b^12 + 27*A^2*B*a*
b^13 + 27*A^3*b^14 - 2370816*A^3*a^7*c^7 + 2688*(50*A*B^2*a^8 + 384*A^2*B*a
^7*b + 1143*A^3*a^6*b^2)*c^6 - 64*(400*B^3*a^8*b + 4062*A*B^2*a^7*b^2 + 175
41*A^2*B*a^6*b^3 + 26865*A^3*a^5*b^4)*c^5 + 8*(2728*B^3*a^7*b^3 + 20520*A*B
^2*a^6*b^4 + 62694*A^2*B*a^5*b^5 + 67797*A^3*a^4*b^6)*c^4 - 7*(976*B^3*a^6*
b^5 + 6744*A*B^2*a^5*b^6 + 16884*A^2*B*a^4*b^7 + 14985*A^3*a^3*b^8)*c^3 + (
940*B^3*a^5*b^7 + 6591*A*B^2*a^4*b^8 + 15489*A^2*B*a^3*b^9 + 12528*A^3*a^2*
b^10)*c^2 - (53*B^3*a^4*b^9 + 414*A*B^2*a^3*b^10 + 1053*A^2*B*a^2*b^11 + 86
4*A^3*a*b^12)*c + (B*a^6*b^14 + 3*A*a^5*b^15 + 4096*(10*B*a^13 - 33*A*a^12*
b)*c^7 - 2048*(16*B*a^12*b^2 - 99*A*a^11*b^3)*c^6 + 768*(2*B*a^11*b^4 - 169
*A*a^10*b^5)*c^5 + 1280*(5*B*a^10*b^6 + 36*A*a^9*b^7)*c^4 - 80*(34*B*a^9*b^
8 + 123*A*a^8*b^9)*c^3 + 24*(20*B*a^8*b^10 + 53*A*a^7*b^11)*c^2 - (38*B*a^7
*b^12 + 93*A*a^6*b^13)*c)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2
*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*
B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3
*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c
^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3
*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^
2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(B^2*a^2*
b^7 + 6*A*B*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2
*B^2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A
*B*a^3*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*
A^2*a*b^7)*c - (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3
 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 +
 54*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 8
82*(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 +
 5400*A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4
*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4
 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*
a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5
*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4
 - 1024*a^10*c^5))) - 2*(B*a^2*b^3 - 5*A*a*b^4 - 44*A*a^3*c^2 - (16*B*a^3*b
 - 37*A*a^2*b^2)*c)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^
4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*
c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^
3*c + 16*a^5*b*c^2)*x^2)
```

giac [B] time = 8.54, size = 4609, normalized size = 10.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(3*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8 - 17*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^7*c - 2*b^8*c + 116*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 +
 26*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 34*a*b^6*c^2 + 2*b^7*c^2 - 368*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
3 - 232*a^2*b^4*c^3 - 30*a*b^5*c^3 + 448*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 64*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 736*a^3*b^2*c^4 + 176
*a^2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 - 896*a^
```

4*c^5 - 352*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c - 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^6*c - 26*(b^2 - 4*a*c)*a*b^4*c^2 - 2*(b^2 - 4*a*c)*b^5*c^2 + 128*(b^2 - 4*a*c)*a^2*b^2*c^3 + 22*(b^2 - 4*a*c)*a*b^3*c^3 - 224*(b^2 - 4*a*c)*a^3*c^4 - 88*(b^2 - 4*a*c)*a^2*b*c^4)*A + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7 - 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*a*b^7*c + 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + 48*a^2*b^5*c^2 + 2*a*b^6*c^2 - 256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 288*a^3*b^3*c^3 - 44*a^2*b^4*c^3 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 512*a^4*b*c^4 + 64*a^3*b^2*c^4 + 320*a^4*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 36*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^5*c - 40*(b^2 - 4*a*c)*a^2*b^3*c^2 - 2*(b^2 - 4*a*c)*a*b^4*c^2 + 128*(b^2 - 4*a*c)*a^3*b*c^3 + 36*(b^2 - 4*a*c)*a^2*b^2*c^3 + 80*(b^2 - 4*a*c)*a^3*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)^2 - 4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/((a^3*b^8 - 16*a^4*b^6*c - 2*a^3*b^7*c + 96*a^5*b^4*c^2 + 24*a^4*b^5*c^2 + a^3*b^6*c^2 - 256*a^6*b^2*c^3 - 96*a^5*b^3*c^3 - 12*a^4*b^4*c^3 + 256*a^7*c^4 + 128*a^6*b*c^4 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*abs(c)) + 1/32*(3*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8 - 17*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c + 2*b^8*c + 116*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 26*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 - 34*a*b^6*c^2 - 2*b^7*c^2 - 368*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 232*a^2*b^4*c^3 + 30*a*b^5*c^3 + 448*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 736*a^3*b^2*c^4 - 176*a^2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 896*a^4*c^5 + 352*a^3*b*c^5 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 - 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4

```
*a*c)*c)*a*b^3*c^3 + 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^6*c + 26*(b^2 - 4*a*c)*a*b^4*c^2 + 2*(
b^2 - 4*a*c)*b^5*c^2 - 128*(b^2 - 4*a*c)*a^2*b^2*c^3 - 22*(b^2 - 4*a*c)*a*b
^3*c^3 + 224*(b^2 - 4*a*c)*a^3*c^4 + 88*(b^2 - 4*a*c)*a^2*b*c^4)*A + (sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7 - 24*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b^5*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c +
2*a*b^7*c + 144*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 40*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^5*c^2 - 48*a^2*b^5*c^2 - 2*a*b^6*c^2 - 256*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 - 128*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^3*b^2*c^3 - 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*
c^3 + 288*a^3*b^3*c^3 + 44*a^2*b^4*c^3 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*b*c^4 - 512*a^4*b*c^4 - 64*a^3*b^2*c^4 - 320*a^4*c^5 + sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6 - 22*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 32*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 36*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 160*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^5*c + 40*(b^2 - 4
*a*c)*a^2*b^3*c^2 + 2*(b^2 - 4*a*c)*a*b^4*c^2 - 128*(b^2 - 4*a*c)*a^3*b*c^3
 - 36*(b^2 - 4*a*c)*a^2*b^2*c^3 - 80*(b^2 - 4*a*c)*a^3*c^4)*B)*arctan(2*sqr
t(1/2)*x/sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 - sqrt((a^2*b^5 - 8*a^3
*b^3*c + 16*a^4*b*c^2)^2 - 4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*
c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3))
)/((a^3*b^8 - 16*a^4*b^6*c - 2*a^3*b^7*c + 96*a^5*b^4*c^2 + 24*a^4*b^5*c^2
+ a^3*b^6*c^2 - 256*a^6*b^2*c^3 - 96*a^5*b^3*c^3 - 12*a^4*b^4*c^3 + 256*a^7
*c^4 + 128*a^6*b*c^4 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*abs(c)) + 1/8*(B*a*b^2*
c^2*x^7 + 3*A*b^3*c^2*x^7 + 20*B*a^2*c^3*x^7 - 24*A*a*b*c^3*x^7 + 2*B*a*b^3
*c*x^5 + 6*A*b^4*c*x^5 + 28*B*a^2*b*c^2*x^5 - 49*A*a*b^2*c^2*x^5 + 28*A*a^2
*c^3*x^5 + B*a*b^4*x^3 + 3*A*b^5*x^3 + 5*B*a^2*b^2*c*x^3 - 20*A*a*b^3*c*x^3
 + 36*B*a^3*c^2*x^3 - 4*A*a^2*b*c^2*x^3 - B*a^2*b^3*x + 5*A*a*b^4*x + 16*B*
a^3*b*c*x - 37*A*a^2*b^2*c*x + 44*A*a^3*c^2*x)/((a^2*b^4 - 8*a^3*b^2*c + 16
*a^4*c^2)*(c*x^4 + b*x^2 + a)^2)
```

maple [B] time = 0.28, size = 11936, normalized size = 25.95

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(4\left(5\,Ba^2-6\,Aab\right)c^3+\left(Bab^2+3\,Ab^3\right)c^2\right)x^7+\left(28\,Aa^2c^3+7\left(4\,Ba^2b-7\,Aab^2\right)c^2+2\left(Bab^3+3\,Ab^4\right)c\right)x^5+\left(Bab^4+3\,Ab^5+4\left(9\,Ba^3-Aa^2b\right)c^2+5\left(Ba^2b^2-4\,Aab^3\right)c\right)x^3-\left(Ba^2b^3-5\,Aab^4-44\,Aa^3c^2-\left(16\,Ba^3b-37\,Aa^2b^2\right)c\right)x}{8\left(\left(a^2b^4c^2-8\,a^3b^2c^3+16\,a^4c^4\right)x^8+a^4b^4-8\,a^5b^2c+16\,a^6c^2+2\left(a^2b^5c-8\,a^3b^3c^2+16\,a^4bc^3\right)x^6+\left(a^2b^6-6\,a^3b^4c+32\,a^5c^3\right)x^4+2\left(a^3b^5-8\,a^4b^3c+16\,a^5bc^2\right)x^2\right)}-\frac{-\int\frac{Bab^3+3\,Ab^4+84\,Aa^2c^2+\left(4\left(5\,Ba^2-6\,Aab\right)c^2+\left(Bab^2+3\,Ab^3\right)c\right)x^2-\left(16\,Ba^2b+27\,Aab^2\right)c}{cx^4+bx^2+a}\,dx}{8\left(a^2b^4-8\,a^3b^2c+16\,a^4c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((4*(5*B*a^2 - 6*A*a*b)*c^3 + (B*a*b^2 + 3*A*b^3)*c^2)*x^7 + (28*A*a^2*
c^3 + 7*(4*B*a^2*b - 7*A*a*b^2)*c^2 + 2*(B*a*b^3 + 3*A*b^4)*c)*x^5 + (B*a*b
^4 + 3*A*b^5 + 4*(9*B*a^3 - A*a^2*b)*c^2 + 5*(B*a^2*b^2 - 4*A*a*b^3)*c)*x^3
 - (B*a^2*b^3 - 5*A*a*b^4 - 44*A*a^3*c^2 - (16*B*a^3*b - 37*A*a^2*b^2)*c)*x
)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c +
```

```
 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 -
 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x
^2) - 1/8*integrate(-(B*a*b^3 + 3*A*b^4 + 84*A*a^2*c^2 + (4*(5*B*a^2 - 6*A*
a*b)*c^2 + (B*a*b^2 + 3*A*b^3)*c)*x^2 - (16*B*a^2*b + 27*A*a*b^2)*c)/(c*x^4
 + b*x^2 + a), x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)
```

mupad [B] time = 4.61, size = 22914, normalized size = 49.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^3*(3*A*b^5 + 36*B*a^3*c^2 + B*a*b^4 - 20*A*a*b^3*c - 4*A*a^2*b*c^2 + 5*
B*a^2*b^2*c))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5*(28*A*a^2*c^3 +
 6*A*b^4*c + 2*B*a*b^3*c - 49*A*a*b^2*c^2 + 28*B*a^2*b*c^2))/(8*a^2*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) + (x*(5*A*b^4 + 44*A*a^2*c^2 - B*a*b^3 - 37*A*a*b^
2*c + 16*B*a^2*b*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^7*(20*B*a^
2*c^2 + 3*A*b^3*c - 24*A*a*b*c^2 + B*a*b^2*c))/(8*a^2*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + a
tan(((((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*b^14*c^2 - 222
72*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 2027520*A*a^5*b^8*c^5 + 8847360
*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*c^8 + 256*B*a^
3*b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5*b^9*c^4 - 819200*B*a^6*b^7*
c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^4*b^12 + 4096*
a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^
4*c^4 - 6144*a^9*b^2*c^5)) - (x*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-
(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*
a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*
A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441
*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1
/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^
4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^
7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B
^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(
1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10
*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*
b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2
) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^
2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^
6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 2580
48*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^1
3*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*
c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^
8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 25
6*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-(4*a*c - b^2)^1
5)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 +
570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6
 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(-
(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*
a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a
^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B
*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c -
 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B
*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*
A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 51609
60*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^3
*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^2*b*c*(-(4*a*c
- b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*
a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^
```

5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 262
1440*a^14*b^2*c^9)))^(1/2) + (x*(14112*A^2*a^4*c^7 + 9*A^2*b^8*c^3 - 800*B^
2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2*a^2*b^6*c^3 -
 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^4 - 162*A*B*a^
2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B*a^4*b*c^6))/(
32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3
)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*
A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^
4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A
^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)
^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2
- 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 -
680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 3
69*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*
a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2
 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c
^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^
2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^3*(-(4*a*c - b
^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1
/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2
- 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^
11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2
*c^9)))^(1/2)*1i - (((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*
b^14*c^2 - 22272*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 2027520*A*a^5*b^8
*c^5 + 8847360*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*
c^8 + 256*B*a^3*b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5*b^9*c^4 - 819
200*B*a^6*b^7*c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^
4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3
 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(-(9*A^2*b^19 + B^2*a^2*b^17
+ 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^
2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*
c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8
*b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c
 - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*
B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*
B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^
9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*
c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 37728
0*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 12
90240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c -
 b^2)^15)^(1/2) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*
c - 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^1
5*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b
^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7
 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 -
 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c
^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^
6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-(
4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a
^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A
^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*
A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/
2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4
 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7
 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^
2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1
/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*
c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b
^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)

```
 + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^2
*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6
*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 25804
8*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13
*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) - (x*(14112*A^2*a^4*c^7 + 9*A^2*b^
8*c^3 - 800*B^2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2
*a^2*b^6*c^3 - 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^
4 - 162*A*B*a^2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B
*a^4*b*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 2
56*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-(4*a*c - b^2)^
15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 +
 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^
6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(
-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2
*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*
a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*
B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c
 - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*
B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784
*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160
960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^
3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^2*b*c*(-(4*a*c
 - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720
*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c
^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 26
21440*a^14*b^2*c^9)))^(1/2)*1i)/((((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^
9 + 768*A*a^2*b^14*c^2 - 22272*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 202
7520*A*a^5*b^8*c^5 + 8847360*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603
008*A*a^8*b^2*c^8 + 256*B*a^3*b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5
*b^9*c^4 - 819200*B*a^6*b^7*c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3
*c^7)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 12
80*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (x*(-(9*A^2*b^19 +
 B^2*a^2*b^17 + 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A
^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 285177
6*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27
095040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^
2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^1
1*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*
c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 1
5482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2
*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^
12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^
7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^
2*c*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288
*A*B*a^2*b^16*c - 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20
 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3
 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 196608
0*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(2621
44*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 16
3840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5
*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 +
 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2
 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c
^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*
b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c
 - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B
^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B
^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9
*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c
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 - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280
*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 129
0240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c -
b^2)^15)^(1/2) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c
 - 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15
*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^
12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7
+ 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) + (x*(14112*A^2*a^4*
c^7 + 9*A^2*b^8*c^3 - 800*B^2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3
*b^2*c^6 + B^2*a^2*b^6*c^3 - 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 18
0*A^2*a*b^6*c^4 - 162*A*B*a^2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*
c^3 - 6816*A*B*a^4*b*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a
^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-
(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*
a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*
A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441
*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1
/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^
4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^
7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B
^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(
1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10
*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*
b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2
) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^
2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^
6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 2580
48*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^1
3*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) - (567*A^3*b^7*c^5 + 8000*B^3*a^5
*c^7 + 67824*A^3*a^2*b^3*c^7 - 35*B^3*a^2*b^6*c^4 - 84*B^3*a^3*b^4*c^5 + 12
720*B^3*a^4*b^2*c^6 + 141120*A^2*B*a^4*c^8 - 315*A^2*B*b^8*c^4 - 10368*A^3*
a*b^5*c^6 - 169344*A^3*a^3*b*c^8 - 210*A*B^2*a*b^7*c^4 - 116160*A*B^2*a^4*b
*c^7 + 6237*A^2*B*a*b^6*c^5 + 1764*A*B^2*a^2*b^5*c^5 + 4608*A*B^2*a^3*b^3*c
^6 - 42372*A^2*B*a^2*b^4*c^6 + 96048*A^2*B*a^3*b^2*c^7)/(256*(a^4*b^12 + 40
96*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8
*b^4*c^4 - 6144*a^9*b^2*c^5)) + (((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9
 + 768*A*a^2*b^14*c^2 - 22272*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 2027
520*A*a^5*b^8*c^5 + 8847360*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 346030
08*A*a^8*b^2*c^8 + 256*B*a^3*b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5*
b^9*c^4 - 819200*B*a^6*b^7*c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*
c^7)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 128
0*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(-(9*A^2*b^19 +
B^2*a^2*b^17 + 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^
2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776
*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 270
95040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2
*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11
*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c
^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15
482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*
a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^1
2*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7
*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2
*c*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*
A*B*a^2*b^16*c - 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20
+ 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3
+ 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080
*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(26214
4*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163
```

840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*
b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 +
9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2
- 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^
5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b
^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c -
b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^
2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^
2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*
b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c
- b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*
A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290
240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b
^2)^15)^(1/2) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c
- 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*
c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^1
2*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 +
2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) - (x*(14112*A^2*a^4*c
^7 + 9*A^2*b^8*c^3 - 800*B^2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*
b^2*c^6 + B^2*a^2*b^6*c^3 - 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180
*A^2*a*b^6*c^4 - 162*A*B*a^2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c
^3 - 6816*A*B*a^4*b*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^
6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-(
4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a
^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A
^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*
A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/
2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4
+ 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7
+ 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^
2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1
/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*
c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b
^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)
+ 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^2
*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6
*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 25804
8*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13
*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9
*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 -
77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5
+ 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^
3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c -
b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2
*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2
*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b
*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c -
b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A
*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 12902
40*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^
2)^15)^(1/2) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c -
108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c
^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12
*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 +
2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*2i + atan(((((4194304*
B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*b^14*c^2 - 22272*A*a^3*b^12*c^
3 + 282624*A*a^4*b^10*c^4 - 2027520*A*a^5*b^8*c^5 + 8847360*A*a^6*b^6*c^6 -
23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*c^8 + 256*B*a^3*b^13*c^2 - 921
6*B*a^4*b^11*c^3 + 122880*B*a^5*b^9*c^4 - 819200*B*a^6*b^7*c^5 + 2949120*B*

a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (x*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3))) *(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) + (x*(14112*A^2*a^4*c^7 + 9*A^2*b^8*c^3 - 800*B^2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2*a^2*b^6*c^3 - 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^4 - 162*A*B*a^2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B*a^4*b*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*1i - (((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*b^14*c^2 - 22272

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*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 2027520*A*a^5*b^8*c^5 + 8847360*A
*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*c^8 + 256*B*a^3*
b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5*b^9*c^4 - 819200*B*a^6*b^7*c^
5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^4*b^12 + 4096*a^
10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*
c^4 - 6144*a^9*b^2*c^5)) + (x*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4
*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^
3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^
2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A
^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2
) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4
+ 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7
+ 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2
*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/
2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c
^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^
4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)
- 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*
b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*
b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048
*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*
b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*c^
2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8*
b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*
a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)
^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 57
0960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 -
 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4
*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^
4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7
*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a
^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1
720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a
^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*
B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960
*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(
-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c -
b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^
7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5
+ 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 26214
40*a^14*b^2*c^9)))^(1/2) - (x*(14112*A^2*a^4*c^7 + 9*A^2*b^8*c^3 - 800*B^2*
a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2*a^2*b^6*c^3 - 3
4*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^4 - 162*A*B*a^2*
b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B*a^4*b*c^6))/(32
*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3))
)*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*
B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*
b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2
*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^1
5)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 -
10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 68
0960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369
*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^
10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 -
 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5
 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*
c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2
)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2
))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 -
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7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11
*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c
^9)))^(1/2)*1i)/((((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*b^
14*c^2 - 22272*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 2027520*A*a^5*b^8*c
^5 + 8847360*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*c^
8 + 256*B*a^3*b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5*b^9*c^4 - 81920
0*B*a^6*b^7*c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^4*
b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 +
 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (x*(-(9*A^2*b^19 + B^2*a^2*b^17 -
9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2
- 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^
5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b
^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c -
 b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^
2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^
2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*
b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c
- b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*
A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290
240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b
^2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c
+ 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*
c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^1
2*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 +
 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 2
56*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5
 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*
b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*
a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3
*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2
*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^
2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2)
 + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 +
 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 +
 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*
a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2
) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^
4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4
*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) -
 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b
*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b
^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*
a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b
^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) + (x*(14112*A^2*a^4*c^7 + 9*A^2*b^8*
c^3 - 800*B^2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2*a
^2*b^6*c^3 - 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^4
- 162*A*B*a^2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B*a
^4*b*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256
*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15
)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 5
70960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6
- 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(
4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a
^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^
7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*
a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c -
1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*
a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A
*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 516096
```

```
0*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*
(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c -
 b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a
^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5
 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621
440*a^14*b^2*c^9)))^(1/2) - (567*A^3*b^7*c^5 + 8000*B^3*a^5*c^7 + 67824*A^3
*a^2*b^3*c^7 - 35*B^3*a^2*b^6*c^4 - 84*B^3*a^3*b^4*c^5 + 12720*B^3*a^4*b^2*
c^6 + 141120*A^2*B*a^4*c^8 - 315*A^2*B*b^8*c^4 - 10368*A^3*a*b^5*c^6 - 1693
44*A^3*a^3*b*c^8 - 210*A*B^2*a*b^7*c^4 - 116160*A*B^2*a^4*b*c^7 + 6237*A^2*
B*a*b^6*c^5 + 1764*A*B^2*a^2*b^5*c^5 + 4608*A*B^2*a^3*b^3*c^6 - 42372*A^2*B
*a^2*b^4*c^6 + 96048*A^2*B*a^3*b^2*c^7)/(256*(a^4*b^12 + 4096*a^10*c^6 - 24
*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*
a^9*b^2*c^5)) + (((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*b^1
4*c^2 - 22272*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 2027520*A*a^5*b^8*c^
5 + 8847360*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*c^8
 + 256*B*a^3*b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5*b^9*c^4 - 819200
*B*a^6*b^7*c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^4*b
^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 +
3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9
*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 -
 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5
 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^
3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c -
b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2
*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2
*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b
*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c -
 b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A
*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 12902
40*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^
2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c +
 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c
^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12
*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 +
2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 25
6*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5
- 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b
^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a
*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*
b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*
a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2
*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2)
+ 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 +
43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 +
6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a
^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2)
 + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4
 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*
c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) -
6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*
c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^
18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a
^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^
4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) - (x*(14112*A^2*a^4*c^7 + 9*A^2*b^8*c
^3 - 800*B^2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2*a^
2*b^6*c^3 - 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^4 -
 162*A*B*a^2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B*a^
4*b*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*
a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)
```

```
^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 57
0960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 -
 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4
*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^
4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7
*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a
^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1
720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a
^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*
B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960
*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(
-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c -
b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^
7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5
+ 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 26214
40*a^14*b^2*c^9)))^(1/2))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*
c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b
^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a
^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*
a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) +
 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 4
3776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6
881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^
3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2)
+ 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4
- 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c
^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6
*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c
*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^1
8*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^
10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4
*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.115 $\int \frac{x(-7+4x^2)}{4-5x^2+x^4} \, dx$

Optimal. Leaf size=25

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1247, 632, 31}

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(-7 + 4*x^2))/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(-7+4x^2\right)}{4-5x^2+x^4} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{-7+4x}{4-5x+x^2} \, dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x} \, dx, x, x^2\right)+\frac{3}{2}\text{Subst}\left(\int \frac{1}{-4+x} \, dx, x, x^2\right)\\ &= \frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(-7 + 4*x^2))/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(-7+4x^2\right)}{4-5x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(-7 + 4*x^2))/(4 - 5*x^2 + x^4),x]
[Out] IntegrateAlgebraic[(x*(-7 + 4*x^2))/(4 - 5*x^2 + x^4), x]
```

fricas [A] time = 1.14, size = 17, normalized size = 0.68

$$\frac{1}{2}\,\log\left(x^2-1\right)+\frac{3}{2}\,\log\left(x^2-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2-7)/(x^4-5*x^2+4),x, algorithm="fricas")
[Out] 1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)
```

giac [A] time = 0.31, size = 19, normalized size = 0.76

$$\frac{1}{2}\,\log\left(\left|x^2-1\right|\right)+\frac{3}{2}\,\log\left(\left|x^2-4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2-7)/(x^4-5*x^2+4),x, algorithm="giac")
[Out] 1/2*log(abs(x^2 - 1)) + 3/2*log(abs(x^2 - 4))
```

maple [A] time = 0.01, size = 18, normalized size = 0.72

$$\frac{3\ln\left(x^2-4\right)}{2}+\frac{\ln\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(4*x^2-7)/(x^4-5*x^2+4),x)
[Out] 3/2*ln(x^2-4)+1/2*ln(x^2-1)
```

maxima [A] time = 0.72, size = 17, normalized size = 0.68

$$\frac{1}{2}\,\log\left(x^2-1\right)+\frac{3}{2}\,\log\left(x^2-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2-7)/(x^4-5*x^2+4),x, algorithm="maxima")
[Out] 1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)
```

mupad [B] time = 0.06, size = 17, normalized size = 0.68

$$\frac{\ln\left(x^2-1\right)}{2}+\frac{3\,\ln\left(x^2-4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(4*x^2 - 7))/(x^4 - 5*x^2 + 4),x)
```

```
[Out] log(x^2 - 1)/2 + (3*log(x^2 - 4))/2
```

sympy [A] time = 0.12, size = 17, normalized size = 0.68

$$\frac{3\log\left(x^2-4\right)}{2}+\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x**2-7)/(x**4-5*x**2+4),x)
[Out] 3*log(x**2 - 4)/2 + log(x**2 - 1)/2
```

3.116 $\int \frac{-7x+4x^3}{4-5x^2+x^4} \, dx$

Optimal. Leaf size=25

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 1247, 632, 31}

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-7*x + 4*x^3)/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{-7x+4x^3}{4-5x^2+x^4} \, dx &= \int \frac{x\left(-7+4x^2\right)}{4-5x^2+x^4} \, dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{-7+4x}{4-5x+x^2} \, dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-1+x} \, dx, x, x^2\right)+\frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{-4+x} \, dx, x, x^2\right)\\
&= \frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-7*x + 4*x^3)/(4 - 5*x^2 + x^4),x]
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-7x+4x^3}{4-5x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-7*x + 4*x^3)/(4 - 5*x^2 + x^4),x]
[Out] IntegrateAlgebraic[(-7*x + 4*x^3)/(4 - 5*x^2 + x^4), x]
```

fricas [A] time = 0.66, size = 17, normalized size = 0.68

$$\frac{1}{2}\log\left(x^2-1\right)+\frac{3}{2}\log\left(x^2-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-7*x)/(x^4-5*x^2+4),x, algorithm="fricas")
[Out] 1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)
```

giac [A] time = 0.28, size = 19, normalized size = 0.76

$$\frac{1}{2}\log\left(\left|x^2-1\right|\right)+\frac{3}{2}\log\left(\left|x^2-4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-7*x)/(x^4-5*x^2+4),x, algorithm="giac")
[Out] 1/2*log(abs(x^2 - 1)) + 3/2*log(abs(x^2 - 4))
```

maple [A] time = 0.00, size = 18, normalized size = 0.72

$$\frac{3\ln\left(x^2-4\right)}{2}+\frac{\ln\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3-7*x)/(x^4-5*x^2+4),x)
[Out] 3/2*ln(x^2-4)+1/2*ln(x^2-1)
```

maxima [A] time = 0.74, size = 25, normalized size = 1.00

$$\frac{3}{2}\log\left(x+2\right)+\frac{1}{2}\log\left(x+1\right)+\frac{1}{2}\log\left(x-1\right)+\frac{3}{2}\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-7*x)/(x^4-5*x^2+4),x, algorithm="maxima")
[Out] 3/2*log(x + 2) + 1/2*log(x + 1) + 1/2*log(x - 1) + 3/2*log(x - 2)
```

mupad [B] time = 0.03, size = 17, normalized size = 0.68

$$\frac{\ln\left(x^2-1\right)}{2}+\frac{3\ \ln\left(x^2-4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(7*x - 4*x^3)/(x^4 - 5*x^2 + 4),x)
[Out] log(x^2 - 1)/2 + (3*log(x^2 - 4))/2
```

sympy [A] time = 0.11, size = 17, normalized size = 0.68

$$\frac{3\log\left(x^2-4\right)}{2}+\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3-7*x)/(x**4-5*x**2+4),x)
[Out] 3*log(x**2 - 4)/2 + log(x**2 - 1)/2
```

3.117 $\int \frac{x(2+x^2)}{1+x^2+x^4} \, dx$

Optimal. Leaf size=37

$$\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1247, 634, 618, 204, 628}

$$\frac{1}{4}\log\left(x^4+x^2+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2 + x^2))/(1 + x^2 + x^4),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(2+x^2\right)}{1+x^2+x^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{2+x}{1+x+x^2}\,dx,x,x^2\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,x^2\right)+\frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^2\right)\\ &= \frac{1}{4}\log\left(1+x^2+x^4\right)-\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^2\right)\\ &= \frac{1}{2}\sqrt{3}\,\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)+\frac{1}{4}\log\left(1+x^2+x^4\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{1}{2}\sqrt{3}\,\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2 + x^2))/(1 + x^2 + x^4),x]
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(2+x^2\right)}{1+x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(2 + x^2))/(1 + x^2 + x^4),x]
[Out] IntegrateAlgebraic[(x*(2 + x^2))/(1 + x^2 + x^4), x]
```

fricas [A] time = 0.94, size = 30, normalized size = 0.81

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2)/(x^4+x^2+1),x, algorithm="fricas")
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

giac [A] time = 0.28, size = 30, normalized size = 0.81

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2)/(x^4+x^2+1),x, algorithm="giac")
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

maple [A] time = 0.00, size = 31, normalized size = 0.84

$$\frac{\sqrt{3}\arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)}{2}+\frac{\ln\left(x^4+x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+2)/(x^4+x^2+1),x)
```

```
[Out] 1/4*ln(x^4+x^2+1)+1/2*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

maxima [A] time = 1.40, size = 30, normalized size = 0.81

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2)/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

mupad [B] time = 0.21, size = 32, normalized size = 0.86

$$\frac{\ln\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x^2 + 2))/(x^2 + x^4 + 1),x)
```

```
[Out] log(x^2 + x^4 + 1)/4 + (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/3))/2
```

sympy [A] time = 0.12, size = 37, normalized size = 1.00

$$\frac{\log\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+2)/(x**4+x**2+1),x)
```

```
[Out] log(x**4 + x**2 + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/2
```

3.118 $\int \frac{2x+x^3}{1+x^2+x^4}\,dx$

Optimal. Leaf size=37

$$\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1593, 1247, 634, 618, 204, 628}

$$\frac{1}{4}\log\left(x^4+x^2+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + x^3)/(1 + x^2 + x^4),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2x+x^3}{1+x^2+x^4}\,dx &= \int \frac{x\left(2+x^2\right)}{1+x^2+x^4}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{2+x}{1+x+x^2}\,dx, x, x^2\right) \\
&= \frac{1}{4}\,\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^2\right) + \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right) \\
&= \frac{1}{4}\log\left(1+x^2+x^4\right) - \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^2\right) \\
&= \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right) + \frac{1}{4}\log\left(1+x^2+x^4\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + \frac{1}{4}\log\left(x^4+x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + x^3)/(1 + x^2 + x^4),x]
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+x^3}{1+x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2*x + x^3)/(1 + x^2 + x^4),x]
[Out] IntegrateAlgebraic[(2*x + x^3)/(1 + x^2 + x^4), x]
```

fricas [A] time = 1.19, size = 30, normalized size = 0.81

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right) + \frac{1}{4}\log\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)/(x^4+x^2+1),x, algorithm="fricas")
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

giac [A] time = 0.37, size = 30, normalized size = 0.81

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right) + \frac{1}{4}\log\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)/(x^4+x^2+1),x, algorithm="giac")
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

maple [A] time = 0.00, size = 31, normalized size = 0.84

$$\frac{\sqrt{3}\arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)}{2} + \frac{\ln\left(x^4+x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+2*x)/(x^4+x^2+1),x)
```

```
[Out] 1/2*3^(1/2)*arctan(1/3*(2*x^2+1)*3^(1/2))+1/4*ln(x^4+x^2+1)
```

maxima [A] time = 1.60, size = 53, normalized size = 1.43

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)+\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{4}\log\left(x^2+x+1\right)+\frac{1}{4}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/4*log(x^2 + x + 1) + 1/4*log(x^2 - x + 1)
```

mupad [B] time = 0.03, size = 32, normalized size = 0.86

$$\frac{\ln\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^3)/(x^2 + x^4 + 1),x)
```

```
[Out] log(x^2 + x^4 + 1)/4 + (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/3))/2
```

sympy [A] time = 0.12, size = 37, normalized size = 1.00

$$\frac{\log\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+2*x)/(x**4+x**2+1),x)
```

```
[Out] log(x**4 + x**2 + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/2
```

3.119 $\int \frac{11x+2x^3}{\left(3+2x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=45

$$\frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{9x^2+5}{8\left(x^4+2x^2+3\right)}$$

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1593, 1247, 638, 618, 204}

$$\frac{9x^2+5}{8\left(x^4+2x^2+3\right)}+\frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(11*x + 2*x^3)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5 + 9*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{11x+2x^3}{\left(3+2x^2+x^4\right)^2}\,dx &= \int \frac{x\left(11+2x^2\right)}{\left(3+2x^2+x^4\right)^2}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{11+2x}{\left(3+2x+x^2\right)^2}\,dx,x,x^2\right) \\
&= \frac{5+9x^2}{8\left(3+2x^2+x^4\right)}+\frac{9}{8}\,\text{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx,x,x^2\right) \\
&= \frac{5+9x^2}{8\left(3+2x^2+x^4\right)}-\frac{9}{4}\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,2\left(1+x^2\right)\right) \\
&= \frac{5+9x^2}{8\left(3+2x^2+x^4\right)}+\frac{9\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 45, normalized size = 1.00

$$
\frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{9x^2+5}{8\left(x^4+2x^2+3\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(11*x + 2*x^3)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5 + 9*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{11x+2x^3}{\left(3+2x^2+x^4\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(11*x + 2*x^3)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(11*x + 2*x^3)/(3 + 2*x^2 + x^4)^2, x]
```

fricas [A] time = 0.83, size = 47, normalized size = 1.04

$$
\frac{9\sqrt{2}\left(x^4+2x^2+3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+18x^2+10}{16\left(x^4+2x^2+3\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+11*x)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(9*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 18*x^2 +
10)/(x^4 + 2*x^2 + 3)
```

giac [A] time = 0.94, size = 38, normalized size = 0.84

$$
\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+\frac{9x^2+5}{8\left(x^4+2x^2+3\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+11*x)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/8*(9*x^2 + 5)/(x^4 + 2*x^2 +
 3)
```

maple [A] time = 0.01, size = 41, normalized size = 0.91

$$\frac{9\sqrt{2}\,\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16}+\frac{18x^2+10}{16x^4+32x^2+48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+11*x)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 1/16*(18*x^2+10)/(x^4+2*x^2+3)+9/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{9\,x^2+5}{8\left(x^4+2\,x^2+3\right)}+\frac{9}{4}\int\frac{x}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+11*x)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(9*x^2 + 5)/(x^4 + 2*x^2 + 3) + 9/4*integrate(x/(x^4 + 2*x^2 + 3), x)
```

mupad [B] time = 0.05, size = 41, normalized size = 0.91

$$\frac{\frac{9\,x^2}{8}+\frac{5}{8}}{x^4+2\,x^2+3}+\frac{9\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((11*x + 2*x^3)/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] ((9*x^2)/8 + 5/8)/(2*x^2 + x^4 + 3) + (9*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*
x^2)/2))/16
```

sympy [A] time = 0.15, size = 44, normalized size = 0.98

$$\frac{9x^2+5}{8x^4+16x^2+24}+\frac{9\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+11*x)/(x**4+2*x**2+3)**2,x)
```

```
[Out] (9*x**2 + 5)/(8*x**4 + 16*x**2 + 24) + 9*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt
(2)/2)/16
```

3.120 $\int x^5 \left(2+3x^2\right) \sqrt{3+5x^2+x^4}\, dx$

Optimal. Leaf size=102

$$\frac{3}{10}\left(x^4+5x^2+3\right)^{3/2} x^4+\frac{1}{480}\left(1837-510x^2\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{1633}{256}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}+\frac{21229}{512}\tanh^{-1}$$

Rubi [A] time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1251, 832, 779, 612, 621, 206}

$$\frac{3}{10}\left(x^4+5x^2+3\right)^{3/2} x^4+\frac{1}{480}\left(1837-510x^2\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{1633}{256}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}+\frac{21229}{512}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-1633*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/256 + (3*x^4*(3 + 5*x^2 + x^4)^(3
/2))/10 + ((1837 - 510*x^2)*(3 + 5*x^2 + x^4)^(3/2))/480 + (21229*ArcTanh[(
5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/512
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int x^5\left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx = \frac{1}{2}\text{Subst}\left(\int x^2(2+3x)\sqrt{3+5x+x^2}\,dx,x,x^2\right)$$
$$= \frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{10}\text{Subst}\left(\int\left(-18-\frac{85x}{2}\right)x\sqrt{3+5x+x^2}\,dx,x,x^2\right)$$
$$= \frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{480}\left(1837-510x^2\right)\left(3+5x^2+x^4\right)^{3/2}-\frac{1633}{64}\text{Subs}$$
$$= -\frac{1633}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}+\frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{480}\left(1837-510\right.$$
$$= -\frac{1633}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}+\frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{480}\left(1837-510\right.$$
$$= -\frac{1633}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}+\frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{480}\left(1837-510\right.$$

Mathematica [A] time = 0.03, size = 71, normalized size = 0.70

$$\frac{318435\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)+2\sqrt{x^4+5x^2+3}\left(1152x^8+1680x^6-2248x^4+12250x^2-78387\right)}{7680}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(-78387 + 12250*x^2 - 2248*x^4 + 1680*x^6 + 1152*x
^8) + 318435*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/7680
```

IntegrateAlgebraic [A] time = 0.18, size = 69, normalized size = 0.68

$$\frac{\sqrt{x^4+5x^2+3}\left(1152x^8+1680x^6-2248x^4+12250x^2-78387\right)}{3840}-\frac{21229}{512}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-78387 + 12250*x^2 - 2248*x^4 + 1680*x^6 + 1152*x^8
))/3840 - (21229*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/512
```

fricas [A] time = 1.36, size = 61, normalized size = 0.60

$$\frac{1}{3840}\left(1152x^8+1680x^6-2248x^4+12250x^2-78387\right)\sqrt{x^4+5x^2+3}-\frac{21229}{512}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3840*(1152*x^8 + 1680*x^6 - 2248*x^4 + 12250*x^2 - 78387)*sqrt(x^4 + 5*x^
2 + 3) - 21229/512*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

giac [A] time = 0.46, size = 102, normalized size = 1.00

$$\frac{1}{1280}\sqrt{x^4+5x^2+3}\left(2\left(4\left(6\left(8x^2+5\right)x^2-127\right)x^2+2635\right)x^2-33429\right)+\frac{1}{192}\sqrt{x^4+5x^2+3}\left(2\left(4\left(6x^2+5\right)x^2-89\right)x^2+1095\right)-\frac{21229}{512}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1280*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 + 5)*x^2 - 127)*x^2 + 2635)*x^
2 - 33429) + 1/192*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x^2 +
1095) - 21229/512*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.03, size = 91, normalized size = 0.89

$$\frac{3\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^4}{10}-\frac{17\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2}{16}+\frac{21229\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{512}+\frac{1837\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{480}-\frac{1633\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 3/10*x^4*(x^4+5*x^2+3)^(3/2)-17/16*x^2*(x^4+5*x^2+3)^(3/2)+1837/480*(x^4+5*
x^2+3)^(3/2)-1633/256*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)+21229/512*ln(5/2+x^2+(x
^4+5*x^2+3)^(1/2))
```

maxima [A] time = 0.67, size = 104, normalized size = 1.02

$$\frac{3}{10}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^4-\frac{17}{16}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2-\frac{1633}{128}\sqrt{x^4+5x^2+3}x^2+\frac{1837}{480}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{8165}{256}\sqrt{x^4+5x^2+3}+\frac{21229}{512}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/10*(x^4 + 5*x^2 + 3)^(3/2)*x^4 - 17/16*(x^4 + 5*x^2 + 3)^(3/2)*x^2 - 1633
/128*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1837/480*(x^4 + 5*x^2 + 3)^(3/2) - 8165/25
6*sqrt(x^4 + 5*x^2 + 3) + 21229/512*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5
)
```

mupad [B] time = 0.49, size = 102, normalized size = 1.00

$$\frac{21229\ln\left(\sqrt{x^4+5x^2+3}+x^2+\frac{5}{2}\right)}{512}-\frac{17x^2\left(x^4+5x^2+3\right)^{3/2}}{16}+\frac{3x^4\left(x^4+5x^2+3\right)^{3/2}}{10}+\frac{51\left(\frac{x^2}{2}+\frac{5}{4}\right)\sqrt{x^4+5x^2+3}}{16}+\frac{1837\sqrt{x^4+5x^2+3}\left(8x^4+10x^2-51\right)}{3840}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] (21229*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/512 - (17*x^2*(5*x^2 + x^4
 + 3)^(3/2))/16 + (3*x^4*(5*x^2 + x^4 + 3)^(3/2))/10 + (51*(x^2/2 + 5/4)*(5
*x^2 + x^4 + 3)^(1/2))/16 + (1837*(5*x^2 + x^4 + 3)^(1/2)*(10*x^2 + 8*x^4 -
 51))/3840
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(3x^2+2\right)\sqrt{x^4+5x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**5*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

3.121 $\int x^3 \left(2+3x^2\right) \sqrt{3+5x^2+x^4}\, dx$

Optimal. Leaf size=81

$$-\frac{1}{48}\left(59-18x^2\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{259}{128}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}-\frac{3367}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1251, 779, 612, 621, 206}

$$-\frac{1}{48}\left(59-18x^2\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{259}{128}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}-\frac{3367}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (259*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/128 - ((59 - 18*x^2)*(3 + 5*x^2 + x
^4)^(3/2))/48 - (3367*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/256
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int x^3\left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx = \frac{1}{2}\text{Subst}\left(\int x(2+3x)\sqrt{3+5x+x^2}\,dx,x,x^2\right)$$
$$= -\frac{1}{48}\left(59-18x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{259}{32}\text{Subst}\left(\int\sqrt{3+5x+x^2}\,dx,x,x^2\right.$$
$$= \frac{259}{128}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}-\frac{1}{48}\left(59-18x^2\right)\left(3+5x^2+x^4\right)^{3/2}-\frac{3367}{256}$$
$$= \frac{259}{128}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}-\frac{1}{48}\left(59-18x^2\right)\left(3+5x^2+x^4\right)^{3/2}-\frac{3367}{128}$$
$$= \frac{259}{128}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}-\frac{1}{48}\left(59-18x^2\right)\left(3+5x^2+x^4\right)^{3/2}-\frac{3367}{256}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 0.81

$$\frac{1}{768}\left(2\sqrt{x^4+5x^2+3}\left(144x^6+248x^4-374x^2+2469\right)-10101\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(2469 - 374*x^2 + 248*x^4 + 144*x^6) - 10101*ArcTa
nh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/768
```

IntegrateAlgebraic [A] time = 0.15, size = 64, normalized size = 0.79

$$\frac{3367}{256}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)+\frac{1}{384}\sqrt{x^4+5x^2+3}\left(144x^6+248x^4-374x^2+2469\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(2469 - 374*x^2 + 248*x^4 + 144*x^6))/384 + (3367*Lo
g[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/256
```

fricas [A] time = 1.28, size = 56, normalized size = 0.69

$$\frac{1}{384}\left(144\,x^6+248\,x^4-374\,x^2+2469\right)\sqrt{x^4+5\,x^2+3}+\frac{3367}{256}\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/384*(144*x^6 + 248*x^4 - 374*x^2 + 2469)*sqrt(x^4 + 5*x^2 + 3) + 3367/256
*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

giac [A] time = 0.36, size = 88, normalized size = 1.09

$$\frac{1}{128}\sqrt{x^4+5\,x^2+3}\left(2\left(4\left(6\,x^2+5\right)x^2-89\right)x^2+1095\right)+\frac{1}{24}\sqrt{x^4+5\,x^2+3}\left(2\left(4\,x^2+5\right)x^2-51\right)+\frac{3367}{256}\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/128*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x^2 + 1095) + 1/24*
sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 5)*x^2 - 51) + 3367/256*log(2*x^2 - 2*sqr
t(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.02, size = 74, normalized size = 0.91

$$\frac{3\left(x^{4}+5x^{2}+3\right)^{\frac{3}{2}} x^{2}}{8}-\frac{3367\ln\left(x^{2}+\frac{5}{2}+\sqrt{x^{4}+5x^{2}+3}\right)}{256}-\frac{59\left(x^{4}+5x^{2}+3\right)^{\frac{3}{2}}}{48}+\frac{259\left(2x^{2}+5\right)\sqrt{x^{4}+5x^{2}+3}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 3/8*(x^4+5*x^2+3)^(3/2)*x^2-59/48*(x^4+5*x^2+3)^(3/2)+259/128*(2*x^2+5)*(x^
4+5*x^2+3)^(1/2)-3367/256*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))
```

maxima [A] time = 0.59, size = 87, normalized size = 1.07

$$\frac{3}{8}\left(x^{4}+5x^{2}+3\right)^{\frac{3}{2}} x^{2}+\frac{259}{64}\sqrt{x^{4}+5x^{2}+3}\,x^{2}-\frac{59}{48}\left(x^{4}+5x^{2}+3\right)^{\frac{3}{2}}+\frac{1295}{128}\sqrt{x^{4}+5x^{2}+3}-\frac{3367}{256}\log\left(2x^{2}+2\sqrt{x^{4}+5x^{2}+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/8*(x^4 + 5*x^2 + 3)^(3/2)*x^2 + 259/64*sqrt(x^4 + 5*x^2 + 3)*x^2 - 59/48*
(x^4 + 5*x^2 + 3)^(3/2) + 1295/128*sqrt(x^4 + 5*x^2 + 3) - 3367/256*log(2*x
^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [B] time = 0.43, size = 85, normalized size = 1.05

$$\frac{3x^{2}\left(x^{4}+5x^{2}+3\right)^{3/2}}{8}-\frac{3367\ln\left(\sqrt{x^{4}+5x^{2}+3}+x^{2}+\frac{5}{2}\right)}{256}-\frac{9\left(\frac{x^{2}}{2}+\frac{5}{4}\right)\sqrt{x^{4}+5x^{2}+3}}{8}-\frac{59\sqrt{x^{4}+5x^{2}+3}\left(8x^{4}+10x^{2}-51\right)}{384}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] (3*x^2*(5*x^2 + x^4 + 3)^(3/2))/8 - (3367*log((5*x^2 + x^4 + 3)^(1/2) + x^2
 + 5/2))/256 - (9*(x^2/2 + 5/4)*(5*x^2 + x^4 + 3)^(1/2))/8 - (59*(5*x^2 + x
^4 + 3)^(1/2)*(10*x^2 + 8*x^4 - 51))/384
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\left(3x^{2}+2\right)\sqrt{x^{4}+5x^{2}+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**3*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

3.122 $\int x \left(2+3x^2\right) \sqrt{3+5x^2+x^4} \, dx$

Optimal. Leaf size=74

$$\frac{1}{2}\left(x^4+5x^2+3\right)^{3/2}-\frac{11}{16}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}+\frac{143}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1247, 640, 612, 621, 206}

$$\frac{1}{2}\left(x^4+5x^2+3\right)^{3/2}-\frac{11}{16}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}+\frac{143}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-11*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/16 + (3 + 5*x^2 + x^4)^(3/2)/2 + (1
43*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int x\left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx = \frac{1}{2}\text{Subst}\left(\int(2+3x)\sqrt{3+5x+x^2}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\left(3+5x^2+x^4\right)^{3/2} - \frac{11}{4}\text{Subst}\left(\int\sqrt{3+5x+x^2}\,dx,x,x^2\right)$$
$$= -\frac{11}{16}\left(5+2x^2\right)\sqrt{3+5x^2+x^4} + \frac{1}{2}\left(3+5x^2+x^4\right)^{3/2} + \frac{143}{32}\text{Subst}\left(\int\frac{1}{\sqrt{3+5x}}\right.$$
$$= -\frac{11}{16}\left(5+2x^2\right)\sqrt{3+5x^2+x^4} + \frac{1}{2}\left(3+5x^2+x^4\right)^{3/2} + \frac{143}{16}\text{Subst}\left(\int\frac{1}{4-x^2}\,d\right.$$
$$= -\frac{11}{16}\left(5+2x^2\right)\sqrt{3+5x^2+x^4} + \frac{1}{2}\left(3+5x^2+x^4\right)^{3/2} + \frac{143}{32}\tanh^{-1}\left(\frac{5+2}{2\sqrt{3+5x}}\right.$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.82

$$\frac{1}{32}\left(2\sqrt{x^4+5x^2+3}\left(8x^4+18x^2-31\right)+143\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(-31 + 18*x^2 + 8*x^4) + 143*ArcTanh[(5 + 2*x^2)/(
2*Sqrt[3 + 5*x^2 + x^4])])/32
```

IntegrateAlgebraic [A] time = 0.15, size = 59, normalized size = 0.80

$$\frac{1}{16}\sqrt{x^4+5x^2+3}\left(8x^4+18x^2-31\right)-\frac{143}{32}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-31 + 18*x^2 + 8*x^4))/16 - (143*Log[-5 - 2*x^2 + 2
*Sqrt[3 + 5*x^2 + x^4]])/32
```

fricas [A] time = 1.28, size = 51, normalized size = 0.69

$$\frac{1}{16}\left(8x^4+18x^2-31\right)\sqrt{x^4+5x^2+3}-\frac{143}{32}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(8*x^4 + 18*x^2 - 31)*sqrt(x^4 + 5*x^2 + 3) - 143/32*log(-2*x^2 + 2*sq
rt(x^4 + 5*x^2 + 3) - 5)
```

giac [A] time = 0.34, size = 74, normalized size = 1.00

$$\frac{1}{16}\sqrt{x^4+5x^2+3}\left(2\left(4x^2+5\right)x^2-51\right)+\frac{1}{4}\sqrt{x^4+5x^2+3}\left(2x^2+5\right)-\frac{143}{32}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 5)*x^2 - 51) + 1/4*sqrt(x^4 + 5*x^2
+ 3)*(2*x^2 + 5) - 143/32*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.01, size = 57, normalized size = 0.77

$$\frac{143\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{32}+\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{2}-\frac{11\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 1/2*(x^4+5*x^2+3)^(3/2)-11/16*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)+143/32*ln(x^2+5
/2+(x^4+5*x^2+3)^(1/2))
```

maxima [A] time = 0.65, size = 70, normalized size = 0.95

$$-\frac{11}{8}\sqrt{x^4+5\,x^2+3}\,x^2+\frac{1}{2}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}-\frac{55}{16}\sqrt{x^4+5\,x^2+3}+\frac{143}{32}\log\left(2\,x^2+2\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -11/8*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1/2*(x^4 + 5*x^2 + 3)^(3/2) - 55/16*sqrt(
x^4 + 5*x^2 + 3) + 143/32*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [B] time = 0.29, size = 67, normalized size = 0.91

$$\frac{143\,\ln\left(\sqrt{x^4+5\,x^2+3}+x^2+\frac{5}{2}\right)}{32}+\left(\frac{x^2}{2}+\frac{5}{4}\right)\sqrt{x^4+5\,x^2+3}+\frac{\sqrt{x^4+5\,x^2+3}\,\left(8\,x^4+10\,x^2-51\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] (143*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/32 + (x^2/2 + 5/4)*(5*x^2 +
x^4 + 3)^(1/2) + ((5*x^2 + x^4 + 3)^(1/2)*(10*x^2 + 8*x^4 - 51))/16
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(3x^2+2\right)\sqrt{x^4+5x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

3.123 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x}\,dx$

Optimal. Leaf size=94

$$\frac{1}{8}\sqrt{x^4+5x^2+3}\,\left(6x^2+23\right)+\frac{1}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\sqrt{3}\,\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\,\sqrt{x^4+5x^2+3}}\right)$$

Rubi [A] time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1251, 814, 843, 621, 206, 724}

$$\frac{1}{8}\sqrt{x^4+5x^2+3}\,\left(6x^2+23\right)+\frac{1}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\sqrt{3}\,\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\,\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x,x]
```

```
[Out] ((23 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 + ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*
x^2 + x^4])]/16 - Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x
^4])]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
```

```
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(2+3x^2\right)\sqrt{3+5x^2+x^4}}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x}\,dx,x,x^2\right)\\
&= \frac{1}{8}\left(23+6x^2\right)\sqrt{3+5x^2+x^4}-\frac{1}{8}\text{Subst}\left(\int \frac{-24-\frac{x}{2}}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= \frac{1}{8}\left(23+6x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{16}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)+3\,\text{Su}\\
&= \frac{1}{8}\left(23+6x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{8}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)-\\
&= \frac{1}{8}\left(23+6x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{16}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)-\sqrt{3}\tanh^{-1}\Bigg(
\end{aligned}$$

Mathematica [A] time = 0.04, size = 92, normalized size = 0.98

$$\frac{1}{16}\left(2\sqrt{x^4+5x^2+3}\left(6x^2+23\right)+\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-16\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x,x]
```

```
[Out] (2*(23 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4] + ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x
^2 + x^4])] - 16*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^
4])])/16
```

IntegrateAlgebraic [A] time = 0.20, size = 93, normalized size = 0.99

$$\frac{1}{8}\sqrt{x^4+5x^2+3}\left(6x^2+23\right)-\frac{1}{16}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)+2\sqrt{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{3}}-\frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x,x]
```

```
[Out] ((23 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 + 2*Sqrt[3]*ArcTanh[x^2/Sqrt[3] - Sq
rt[3 + 5*x^2 + x^4]/Sqrt[3]] - Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]]/16
```

fricas [A] time = 1.26, size = 95, normalized size = 1.01

$$\frac{1}{8}\sqrt{x^4+5x^2+3}\left(6x^2+23\right)+\sqrt{3}\log\left(\frac{25x^2-2\sqrt{3}\left(5x^2+6\right)-2\sqrt{x^4+5x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)-\frac{1}{16}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 + 23) + sqrt(3)*log((25*x^2 - 2*sqrt(3)*(5
*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 1/16*log(-
2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

giac [A] time = 0.48, size = 98, normalized size = 1.04

$$\frac{1}{8}\sqrt{x^4+5x^2+3}\left(6x^2+23\right)+\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)-\frac{1}{16}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 + 23) + sqrt(3)*log((x^2 + sqrt(3) - sqrt(
x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))) - 1/16*log(2*x^2
 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.02, size = 85, normalized size = 0.90

$$-\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)+\frac{\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{16}+\frac{3\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{8}+\sqrt{x^4+5x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x)
```

```
[Out] 3/8*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)+1/16*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))+(x^4
+5*x^2+3)^(1/2)-arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

maxima [A] time = 1.50, size = 89, normalized size = 0.95

$$\frac{3}{4}\sqrt{x^4+5x^2+3}\,x^2-\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{23}{8}\sqrt{x^4+5x^2+3}+\frac{1}{16}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(x^4 + 5*x^2 + 3)*x^2 - sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)
/x^2 + 6/x^2 + 5) + 23/8*sqrt(x^4 + 5*x^2 + 3) + 1/16*log(2*x^2 + 2*sqrt(x^
4 + 5*x^2 + 3) + 5)
```

mupad [B] time = 0.43, size = 86, normalized size = 0.91

$$\frac{\ln\left(\sqrt{x^4+5x^2+3}+x^2+\frac{5}{2}\right)}{16}-\sqrt{3}\,\ln\left(\frac{3}{x^2}+\frac{\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{5}{2}\right)+\frac{3\left(\frac{x^2}{2}+\frac{5}{4}\right)\sqrt{x^4+5x^2+3}}{2}+\sqrt{x^4+5x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x,x)
```

```
[Out] log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2)/16 - 3^(1/2)*log(3/x^2 + (3^(1/2)*
(5*x^2 + x^4 + 3)^(1/2))/x^2 + 5/2) + (3*(x^2/2 + 5/4)*(5*x^2 + x^4 + 3)^(1
/2))/2 + (5*x^2 + x^4 + 3)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x, x)
```

3.124 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt{x^4+5x^2+3}\,(2-3x^2)}{2x^2}+\frac{19}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{7\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 812, 843, 621, 206, 724}

$$-\frac{\sqrt{x^4+5x^2+3}\,(2-3x^2)}{2x^2}+\frac{19}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{7\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^3,x]
```

```
[Out] -((2 - 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/(2*x^2) + (19*ArcTanh[(5 + 2*x^2)/(2*S
qrt[3 + 5*x^2 + x^4])])/4 - (7*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^
2 + x^4])])/Sqrt[3]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
```

```
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(2+3x^2\right)\sqrt{3+5x^2+x^4}}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{\left(2-3x^2\right)\sqrt{3+5x^2+x^4}}{2x^2} - \frac{1}{4}\text{Subst}\left(\int \frac{-28-19x}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$

$$= -\frac{\left(2-3x^2\right)\sqrt{3+5x^2+x^4}}{2x^2} + \frac{19}{4}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right) + 7\,\text{Subst}\Big($$

$$= -\frac{\left(2-3x^2\right)\sqrt{3+5x^2+x^4}}{2x^2} + \frac{19}{2}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right) - 14\,\text{S}$$

$$= -\frac{\left(2-3x^2\right)\sqrt{3+5x^2+x^4}}{2x^2} + \frac{19}{4}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \frac{7\tanh^{-1}\Big(\frac{6+}{2\sqrt{3}\sqrt{3}}}{\sqrt{3}}$$

Mathematica [A] time = 0.04, size = 97, normalized size = 1.00

$$\frac{\sqrt{x^4+5x^2+3}\left(3x^2-2\right)}{2x^2} + \frac{19}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - \frac{7\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^3,x]
```

```
[Out] ((-2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/(2*x^2) + (19*ArcTanh[(5 + 2*x^2)/(2*S
qrt[3 + 5*x^2 + x^4])])/4 - (7*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^
2 + x^4])])/Sqrt[3]
```

IntegrateAlgebraic [A] time = 0.22, size = 96, normalized size = 0.99

$$\frac{\sqrt{x^4+5x^2+3}\left(3x^2-2\right)}{2x^2} - \frac{19}{4}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right) + \frac{14\tanh^{-1}\left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^3,x]
```

```
[Out] ((-2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/(2*x^2) + (14*ArcTanh[x^2/Sqrt[3] - Sq
rt[3 + 5*x^2 + x^4]/Sqrt[3]])/Sqrt[3] - (19*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x
^2 + x^4]])/4
```

fricas [A] time = 0.71, size = 112, normalized size = 1.15

$$\frac{56 \sqrt{3} x^2 \log\left(\frac{25 x^2-2 \sqrt{3}\left(5 x^2+6\right)-2 \sqrt{x^4+5 x^2+3}\left(5 \sqrt{3}-6\right)+30}{x^2}\right)-114 x^2 \log\left(-2 x^2+2 \sqrt{x^4+5 x^2+3}-5\right)+21 x^2+12 \sqrt{x^4+5 x^2+3}\left(3 x^2-2\right)}{24 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(56*sqrt(3)*x^2*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x
^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 114*x^2*log(-2*x^2 + 2*sqrt(x^4 + 5*x^
2 + 3) - 5) + 21*x^2 + 12*sqrt(x^4 + 5*x^2 + 3)*(3*x^2 - 2))/x^2
```

giac [A] time = 0.51, size = 138, normalized size = 1.42

$$\frac{7}{3} \sqrt{3} \log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5 x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5 x^2+3}}\right)+\frac{3}{2} \sqrt{x^4+5 x^2+3}+\frac{5 x^2-5 \sqrt{x^4+5 x^2+3}+6}{\left(x^2-\sqrt{x^4+5 x^2+3}\right)^2-3}-\frac{19}{4} \log\left(2 x^2-2 \sqrt{x^4+5 x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 7/3*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sq
rt(x^4 + 5*x^2 + 3))) + 3/2*sqrt(x^4 + 5*x^2 + 3) + (5*x^2 - 5*sqrt(x^4 + 5
*x^2 + 3) + 6)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3) - 19/4*log(2*x^2 - 2*s
qrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.02, size = 104, normalized size = 1.07

$$-\frac{7\sqrt{3}\ \operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{3}+\frac{19\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{4}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{3x^2}+\frac{7\sqrt{x^4+5x^2+3}}{3}+\frac{\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x)
```

```
[Out] 7/3*(x^4+5*x^2+3)^(1/2)+19/4*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))-7/3*arctanh(1/
6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/3/x^2*(x^4+5*x^2+3)^(3/2
)+1/6*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

maxima [A] time = 1.35, size = 89, normalized size = 0.92

$$-\frac{7}{3} \sqrt{3} \log\left(\frac{2 \sqrt{3} \sqrt{x^4+5 x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{3}{2} \sqrt{x^4+5 x^2+3}-\frac{\sqrt{x^4+5 x^2+3}}{x^2}+\frac{19}{4} \log\left(2 x^2+2 \sqrt{x^4+5 x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -7/3*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 3/2*sqr
t(x^4 + 5*x^2 + 3) - sqrt(x^4 + 5*x^2 + 3)/x^2 + 19/4*log(2*x^2 + 2*sqrt(x^
4 + 5*x^2 + 3) + 5)
```

mupad [B] time = 0.88, size = 84, normalized size = 0.87

$$\frac{19 \ln\left(\sqrt{x^4+5 x^2+3}+x^2+\frac{5}{2}\right)}{4}-\frac{\sqrt{x^4+5 x^2+3}}{x^2}-\frac{7 \sqrt{3} \ln\left(\frac{3}{x^2}+\frac{\sqrt{3} \sqrt{x^4+5 x^2+3}}{x^2}+\frac{5}{2}\right)}{3}+\frac{3 \sqrt{x^4+5 x^2+3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^3,x)
```

```
[Out] (19*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/4 - (5*x^2 + x^4 + 3)^(1/2)/x
^2 - (7*3^(1/2)*log(3/x^2 + (3^(1/2)*(5*x^2 + x^4 + 3)^(1/2))/x^2 + 5/2))/3
 + (3*(5*x^2 + x^4 + 3)^(1/2))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**3,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**3, x)
```

3.125 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5}\,dx$

Optimal. Leaf size=99

$$-\frac{\sqrt{x^4+5x^2+3}\,(23x^2+6)}{12x^4}+\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{77\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}$$

Rubi [A] time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1251, 810, 843, 621, 206, 724}

$$-\frac{\sqrt{x^4+5x^2+3}\,(23x^2+6)}{12x^4}+\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{77\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^5,x]
```

```
[Out] -((6 + 23*x^2)*Sqrt[3 + 5*x^2 + x^4])/(12*x^4) + (3*ArcTanh[(5 + 2*x^2)/(2*
Sqrt[3 + 5*x^2 + x^4])])/2 - (77*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*
x^2 + x^4])])/(24*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} - \frac{1}{24}\text{Subst}\left(\int \frac{-77-36x}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right) + \frac{77}{24}\text{Subs}$$
$$= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} + 3\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right) - \frac{77}{12}$$
$$= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} + \frac{3}{2}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \frac{77\tanh^{-1}\left(\frac{6}{2\sqrt{3}}\right.}{24\sqrt{3}}$$

Mathematica [A] time = 0.04, size = 97, normalized size = 0.98

$$\frac{1}{72}\left(-\frac{6\sqrt{x^4+5x^2+3}\,(23x^2+6)}{x^4} + 108\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - 77\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^5,x]
```

```
[Out] ((-6*(6 + 23*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4 + 108*ArcTanh[(5 + 2*x^2)/(2*S
qrt[3 + 5*x^2 + x^4])] - 77*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 +
 5*x^2 + x^4])])/72
```

IntegrateAlgebraic [A] time = 0.27, size = 98, normalized size = 0.99

$$\frac{\sqrt{x^4+5x^2+3}\,(-23x^2-6)}{12x^4} - \frac{3}{2}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right) + \frac{77\tanh^{-1}\left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}}\right)}{12\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^5,x]
```

```
[Out] ((-6 - 23*x^2)*Sqrt[3 + 5*x^2 + x^4])/(12*x^4) + (77*ArcTanh[x^2/Sqrt[3] -
Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/(12*Sqrt[3]) - (3*Log[-5 - 2*x^2 + 2*Sqrt[3
 + 5*x^2 + x^4]])/2
```

fricas [A] time = 0.92, size = 112, normalized size = 1.13

$$\frac{77\sqrt{3}x^4\log\left(\frac{25x^2-2\sqrt{3}(5x^2+6)-2\sqrt{x^4+5x^2+3}(5\sqrt{3}-6)+30}{x^2}\right)-108x^4\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)-138x^4-6\sqrt{x^4+5x^2+3}(23x^2+6)}{72x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/72*(77*sqrt(3)*x^4*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x
^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 108*x^4*log(-2*x^2 + 2*sqrt(x^4 + 5*x^
2 + 3) - 5) - 138*x^4 - 6*sqrt(x^4 + 5*x^2 + 3)*(23*x^2 + 6))/x^4
```

giac [B] time = 0.57, size = 169, normalized size = 1.71

$$\frac{77}{72}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)+\frac{127\left(x^2-\sqrt{x^4+5x^2+3}\right)^3+228\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-159x^2+159\sqrt{x^4+5x^2+3}-324}{12\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)^2}-\frac{3}{2}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 77/72*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) -
sqrt(x^4 + 5*x^2 + 3))) + 1/12*(127*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 228*(
x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 159*x^2 + 159*sqrt(x^4 + 5*x^2 + 3) - 324)
/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^2 - 3/2*log(2*x^2 - 2*sqrt(x^4 + 5*x
^2 + 3) + 5)
```

maple [A] time = 0.02, size = 121, normalized size = 1.22

$$-\frac{77\sqrt{3}\,\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{72}+\frac{3\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{2}-\frac{13\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{36x^2}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{6x^4}+\frac{77\sqrt{x^4+5x^2+3}}{72}+\frac{13\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x)
```

```
[Out] -1/6/x^4*(x^4+5*x^2+3)^(3/2)-13/36*(x^4+5*x^2+3)^(3/2)/x^2+77/72*(x^4+5*x^2
+3)^(1/2)-77/72*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+
13/72*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)+3/2*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))
```

maxima [A] time = 1.43, size = 106, normalized size = 1.07

$$-\frac{77}{72}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{1}{6}\sqrt{x^4+5x^2+3}-\frac{13\sqrt{x^4+5x^2+3}}{12x^2}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{6x^4}+\frac{3}{2}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -77/72*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 1/6*s
qrt(x^4 + 5*x^2 + 3) - 13/12*sqrt(x^4 + 5*x^2 + 3)/x^2 - 1/6*(x^4 + 5*x^2 +
 3)^(3/2)/x^4 + 3/2*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^5,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**5,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**5, x)
```

3.126 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx$

Optimal. Leaf size=90

$$-\frac{(5x^2+6)\sqrt{x^4+5x^2+3}}{18x^4}+\frac{13\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{36\sqrt{3}}-\frac{(x^4+5x^2+3)^{3/2}}{9x^6}$$

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1251, 806, 720, 724, 206}

$$-\frac{(x^4+5x^2+3)^{3/2}}{9x^6}-\frac{(5x^2+6)\sqrt{x^4+5x^2+3}}{18x^4}+\frac{13\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{36\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^7,x]
```

```
[Out] -((6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(18*x^4) - (3 + 5*x^2 + x^4)^(3/2)/(9*
x^6) + (13*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(36*Sqrt
[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^4}\,dx, x, x^2\right)$$
$$= -\frac{(3+5x^2+x^4)^{3/2}}{9x^6} + \frac{2}{3}\text{Subst}\left(\int \frac{\sqrt{3+5x+x^2}}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{(6+5x^2)\sqrt{3+5x^2+x^4}}{18x^4} - \frac{(3+5x^2+x^4)^{3/2}}{9x^6} - \frac{13}{36}\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\right.$$
$$= -\frac{(6+5x^2)\sqrt{3+5x^2+x^4}}{18x^4} - \frac{(3+5x^2+x^4)^{3/2}}{9x^6} + \frac{13}{18}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, -\right.$$
$$= -\frac{(6+5x^2)\sqrt{3+5x^2+x^4}}{18x^4} - \frac{(3+5x^2+x^4)^{3/2}}{9x^6} + \frac{13\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{36\sqrt{3}}$$

Mathematica [A] time = 0.03, size = 74, normalized size = 0.82

$$\frac{1}{108}\left(13\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right) - \frac{6\sqrt{x^4+5x^2+3}\,(7x^4+16x^2+6)}{x^6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^7,x]
```

```
[Out] ((-6*Sqrt[3 + 5*x^2 + x^4]*(6 + 16*x^2 + 7*x^4))/x^6 + 13*Sqrt[3]*ArcTanh[(
6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/108
```

IntegrateAlgebraic [A] time = 0.33, size = 75, normalized size = 0.83

$$\frac{(-7x^4-16x^2-6)\sqrt{x^4+5x^2+3}}{18x^6} - \frac{13\tanh^{-1}\left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}}\right)}{18\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^7,x]
```

```
[Out] ((-6 - 16*x^2 - 7*x^4)*Sqrt[3 + 5*x^2 + x^4])/(18*x^6) - (13*ArcTanh[x^2/Sq
rt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/(18*Sqrt[3])
```

fricas [A] time = 0.75, size = 90, normalized size = 1.00

$$\frac{13\sqrt{3}\,x^6\log\left(\frac{25\,x^2+2\,\sqrt{3}\,(5\,x^2+6)+2\,\sqrt{x^4+5\,x^2+3}\,(5\,\sqrt{3}+6)+30}{x^2}\right) - 42\,x^6 - 6\,(7\,x^4+16\,x^2+6)\sqrt{x^4+5\,x^2+3}}{108\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/108*(13*sqrt(3)*x^6*log((25*x^2 + 2*sqrt(3)*(5*x^2 + 6) + 2*sqrt(x^4 + 5*
x^2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) - 42*x^6 - 6*(7*x^4 + 16*x^2 + 6)*sqrt(
x^4 + 5*x^2 + 3))/x^6
```

giac [B] time = 0.53, size = 189, normalized size = 2.10

$$-\frac{13}{108}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)+\frac{67\left(x^2-\sqrt{x^4+5x^2+3}\right)^5+306\left(x^2-\sqrt{x^4+5x^2+3}\right)^4+430\left(x^2-\sqrt{x^4+5x^2+3}\right)^3+90\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-63x^2+63\sqrt{x^4+5x^2+3}+108}{18\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] -13/108*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3)
- sqrt(x^4 + 5*x^2 + 3))) + 1/18*(67*(x^2 - sqrt(x^4 + 5*x^2 + 3))^5 + 306*
(x^2 - sqrt(x^4 + 5*x^2 + 3))^4 + 430*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 90*
(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 63*x^2 + 63*sqrt(x^4 + 5*x^2 + 3) + 108)/
((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^3
```

maple [A] time = 0.02, size = 118, normalized size = 1.31

$$\frac{13\sqrt{3}\,\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{108}+\frac{5\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{54x^2}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{9x^4}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{9x^6}-\frac{13\sqrt{x^4+5x^2+3}}{108}-\frac{5\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{108}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x)
```

```
[Out] -1/9*(x^4+5*x^2+3)^(3/2)/x^6-1/9*(x^4+5*x^2+3)^(3/2)/x^4+5/54*(x^4+5*x^2+3)
^(3/2)/x^2-13/108*(x^4+5*x^2+3)^(1/2)+13/108*arctanh(1/6*(5*x^2+6)*3^(1/2)/
(x^4+5*x^2+3)^(1/2))*3^(1/2)-5/108*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

maxima [A] time = 1.45, size = 99, normalized size = 1.10

$$\frac{13}{108}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{1}{9}\sqrt{x^4+5x^2+3}+\frac{5\sqrt{x^4+5x^2+3}}{18x^2}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{9x^4}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] 13/108*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 1/9*s
qrt(x^4 + 5*x^2 + 3) + 5/18*sqrt(x^4 + 5*x^2 + 3)/x^2 - 1/9*(x^4 + 5*x^2 +
3)^(3/2)/x^4 - 1/9*(x^4 + 5*x^2 + 3)^(3/2)/x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^7,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**7,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**7, x)
```

3.127 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx$

Optimal. Leaf size=111

$$\frac{67\left(5x^2+6\right)\sqrt{x^4+5x^2+3}}{1728x^4}-\frac{871\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3456\sqrt{3}}-\frac{\left(x^4+5x^2+3\right)^{3/2}}{12x^8}-\frac{11\left(x^4+5x^2+3\right)^{3/2}}{216x^6}$$

Rubi [A] time = 0.09, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1251, 834, 806, 720, 724, 206}

$$-\frac{11\left(x^4+5x^2+3\right)^{3/2}}{216x^6}-\frac{\left(x^4+5x^2+3\right)^{3/2}}{12x^8}+\frac{67\left(5x^2+6\right)\sqrt{x^4+5x^2+3}}{1728x^4}-\frac{871\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3456\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^9,x]
```

```
[Out] (67*(6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(1728*x^4) - (3 + 5*x^2 + x^4)^(3/2)
/(12*x^8) - (11*(3 + 5*x^2 + x^4)^(3/2))/(216*x^6) - (871*ArcTanh[(6 + 5*x^
2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(3456*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
```

```
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(2+3x^2\right)\sqrt{3+5x^2+x^4}}{x^9}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^5}\,dx, x, x^2\right)$$
$$= -\frac{\left(3+5x^2+x^4\right)^{3/2}}{12x^8} - \frac{1}{24}\text{Subst}\left(\int \frac{(-11+2x)\sqrt{3+5x+x^2}}{x^4}\,dx, x, x^2\right)$$
$$= -\frac{\left(3+5x^2+x^4\right)^{3/2}}{12x^8} - \frac{11\left(3+5x^2+x^4\right)^{3/2}}{216x^6} - \frac{67}{144}\text{Subst}\left(\int \frac{\sqrt{3+5x+x^2}}{x^3}\,d\right.$$
$$= \frac{67\left(6+5x^2\right)\sqrt{3+5x^2+x^4}}{1728x^4} - \frac{\left(3+5x^2+x^4\right)^{3/2}}{12x^8} - \frac{11\left(3+5x^2+x^4\right)^{3/2}}{216x^6} + \frac{8}{}$$
$$= \frac{67\left(6+5x^2\right)\sqrt{3+5x^2+x^4}}{1728x^4} - \frac{\left(3+5x^2+x^4\right)^{3/2}}{12x^8} - \frac{11\left(3+5x^2+x^4\right)^{3/2}}{216x^6} - \frac{8}{}$$
$$= \frac{67\left(6+5x^2\right)\sqrt{3+5x^2+x^4}}{1728x^4} - \frac{\left(3+5x^2+x^4\right)^{3/2}}{12x^8} - \frac{11\left(3+5x^2+x^4\right)^{3/2}}{216x^6} - \frac{8}{}$$

Mathematica [A] time = 0.03, size = 82, normalized size = 0.74

$$\frac{6\sqrt{x^4+5x^2+3}\left(247x^6-182x^4-984x^2-432\right)-871\sqrt{3}\,x^8\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{10368x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^9,x]
```

```
[Out] (6*Sqrt[3 + 5*x^2 + x^4]*(-432 - 984*x^2 - 182*x^4 + 247*x^6) - 871*Sqrt[3]
*x^8*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(10368*x^8)
```

IntegrateAlgebraic [A] time = 0.37, size = 80, normalized size = 0.72

$$\frac{871\tanh^{-1}\left(\frac{x^2}{\sqrt{3}}-\frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}}\right)}{1728\sqrt{3}}+\frac{\sqrt{x^4+5x^2+3}\left(247x^6-182x^4-984x^2-432\right)}{1728x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^9,x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-432 - 984*x^2 - 182*x^4 + 247*x^6))/(1728*x^8) + (
871*ArcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/(1728*Sqrt[3])
```

fricas [A] time = 1.22, size = 95, normalized size = 0.86

$$\frac{871\sqrt{3}\,x^8\log\left(\frac{25\,x^2-2\sqrt{3}\left(5\,x^2+6\right)-2\sqrt{x^4+5\,x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)+1482\,x^8+6\left(247\,x^6-182\,x^4-984\,x^2-432\right)\sqrt{x^4+5\,x^2+3}}{10368\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/10368*(871*sqrt(3)*x^8*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 +
 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) + 1482*x^8 + 6*(247*x^6 - 182*x^4 -
984*x^2 - 432)*sqrt(x^4 + 5*x^2 + 3))/x^8
```

giac [B] time = 0.46, size = 233, normalized size = 2.10

$$\frac{871}{10368}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right)-\frac{871\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^7-5184\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^6-57389\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^5-165888\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^4-204807\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^3-93312\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^2-2403\,x^2+2403\sqrt{x^4+5\,x^2+3}-5184}{1728\left(\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^2-3\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] 871/10368*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3
) - sqrt(x^4 + 5*x^2 + 3))) - 1/1728*(871*(x^2 - sqrt(x^4 + 5*x^2 + 3))^7 -
 5184*(x^2 - sqrt(x^4 + 5*x^2 + 3))^6 - 57389*(x^2 - sqrt(x^4 + 5*x^2 + 3))
^5 - 165888*(x^2 - sqrt(x^4 + 5*x^2 + 3))^4 - 204807*(x^2 - sqrt(x^4 + 5*x^
2 + 3))^3 - 93312*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 2403*x^2 + 2403*sqrt(x^
4 + 5*x^2 + 3) - 5184)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^4
```

maple [A] time = 0.02, size = 135, normalized size = 1.22

$$-\frac{871\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{10368}-\frac{335\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{5184x^2}+\frac{67\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{864x^4}-\frac{11\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{216x^6}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{12x^8}+\frac{871\sqrt{x^4+5x^2+3}}{10368}+\frac{335\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{10368}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x)
```

```
[Out] -11/216*(x^4+5*x^2+3)^(3/2)/x^6+67/864*(x^4+5*x^2+3)^(3/2)/x^4-335/5184*(x^
4+5*x^2+3)^(3/2)/x^2+871/10368*(x^4+5*x^2+3)^(1/2)-871/10368*arctanh(1/6*(5
*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+335/10368*(2*x^2+5)*(x^4+5*x^2
+3)^(1/2)-1/12*(x^4+5*x^2+3)^(3/2)/x^8
```

maxima [A] time = 1.74, size = 116, normalized size = 1.05

$$-\frac{871}{10368}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{67}{864}\sqrt{x^4+5\,x^2+3}-\frac{335\sqrt{x^4+5\,x^2+3}}{1728\,x^2}+\frac{67\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}}{864\,x^4}-\frac{11\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}}{216\,x^6}-\frac{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}}{12\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] -871/10368*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) - 6
7/864*sqrt(x^4 + 5*x^2 + 3) - 335/1728*sqrt(x^4 + 5*x^2 + 3)/x^2 + 67/864*(
x^4 + 5*x^2 + 3)^(3/2)/x^4 - 11/216*(x^4 + 5*x^2 + 3)^(3/2)/x^6 - 1/12*(x^4
 + 5*x^2 + 3)^(3/2)/x^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3\,x^2+2\right)\,\sqrt{x^4+5\,x^2+3}}{x^9} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^9,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^9, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**9,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**9, x)
```

3.128 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx$

Optimal. Leaf size=132

$$-\frac{161\left(5x^2+6\right)\sqrt{x^4+5x^2+3}}{5184x^4}+\frac{2093\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{10368\sqrt{3}}-\frac{\left(x^4+5x^2+3\right)^{3/2}}{15x^{10}}-\frac{\left(x^4+5x^2+3\right)^{3/2}}{36x^8}+\frac{173\left(x^4+5\right.}{3240}$$

Rubi [A] time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1251, 834, 806, 720, 724, 206}

$$\frac{173\left(x^4+5x^2+3\right)^{3/2}}{3240x^6}-\frac{\left(x^4+5x^2+3\right)^{3/2}}{36x^8}-\frac{\left(x^4+5x^2+3\right)^{3/2}}{15x^{10}}-\frac{161\left(5x^2+6\right)\sqrt{x^4+5x^2+3}}{5184x^4}+\frac{2093\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{10368\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^11,x]
```

```
[Out] (-161*(6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(5184*x^4) - (3 + 5*x^2 + x^4)^(3/
2)/(15*x^10) - (3 + 5*x^2 + x^4)^(3/2)/(36*x^8) + (173*(3 + 5*x^2 + x^4)^(3
/2))/(3240*x^6) + (2093*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4
])])/(10368*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(2+3x^2\right)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^6}\,dx,x,x^2\right)$$

$$= -\frac{\left(3+5x^2+x^4\right)^{3/2}}{15x^{10}} - \frac{1}{30}\text{Subst}\left(\int \frac{(-10+4x)\sqrt{3+5x+x^2}}{x^5}\,dx,x,x^2\right)$$

$$= -\frac{\left(3+5x^2+x^4\right)^{3/2}}{15x^{10}} - \frac{\left(3+5x^2+x^4\right)^{3/2}}{36x^8} + \frac{1}{360}\text{Subst}\left(\int \frac{(-173-10x)\sqrt{3+}}{x^4}\right.$$

$$= -\frac{\left(3+5x^2+x^4\right)^{3/2}}{15x^{10}} - \frac{\left(3+5x^2+x^4\right)^{3/2}}{36x^8} + \frac{173\left(3+5x^2+x^4\right)^{3/2}}{3240x^6} + \frac{161}{432}\text{Subs}$$

$$= -\frac{161\left(6+5x^2\right)\sqrt{3+5x^2+x^4}}{5184x^4} - \frac{\left(3+5x^2+x^4\right)^{3/2}}{15x^{10}} - \frac{\left(3+5x^2+x^4\right)^{3/2}}{36x^8} + \frac{17}{}$$

$$= -\frac{161\left(6+5x^2\right)\sqrt{3+5x^2+x^4}}{5184x^4} - \frac{\left(3+5x^2+x^4\right)^{3/2}}{15x^{10}} - \frac{\left(3+5x^2+x^4\right)^{3/2}}{36x^8} + \frac{17}{}$$

$$= -\frac{161\left(6+5x^2\right)\sqrt{3+5x^2+x^4}}{5184x^4} - \frac{\left(3+5x^2+x^4\right)^{3/2}}{15x^{10}} - \frac{\left(3+5x^2+x^4\right)^{3/2}}{36x^8} + \frac{17}{}$$

Mathematica [A] time = 0.04, size = 84, normalized size = 0.64

$$\frac{10465\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right) - \frac{6\sqrt{x^4+5x^2+3}\left(2641x^8-1370x^6+1176x^4+10800x^2+5184\right)}{x^{10}}}{155520}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^11,x]
```

```
[Out] ((-6*Sqrt[3 + 5*x^2 + x^4]*(5184 + 10800*x^2 + 1176*x^4 - 1370*x^6 + 2641*x
^8))/x^10 + 10465*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x
^4])])/155520
```

IntegrateAlgebraic [A] time = 0.45, size = 85, normalized size = 0.64

$$\frac{\sqrt{x^4+5x^2+3}\left(-2641x^8+1370x^6-1176x^4-10800x^2-5184\right)}{25920x^{10}} - \frac{2093\tanh^{-1}\left(\frac{x^2}{\sqrt{3}}-\frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}}\right)}{5184\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^11,x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-5184 - 10800*x^2 - 1176*x^4 + 1370*x^6 - 2641*x^8)
)/(25920*x^10) - (2093*ArcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]]
)/(5184*Sqrt[3])
```

fricas [A] time = 0.93, size = 100, normalized size = 0.76

$$\frac{10465\sqrt{3}x^{10}\log\left(\frac{25x^2+2\sqrt{3}(5x^2+6)+2\sqrt{x^4+5x^2+3}(5\sqrt{3}+6)+30}{x^2}\right)-15846x^{10}-6\left(2641x^8-1370x^6+1176x^4+10800x^2+5184\right)\sqrt{x^4+5x^2+3}}{155520x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/155520*(10465*sqrt(3)*x^10*log((25*x^2 + 2*sqrt(3)*(5*x^2 + 6) + 2*sqrt(x
^4 + 5*x^2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) - 15846*x^10 - 6*(2641*x^8 - 137
0*x^6 + 1176*x^4 + 10800*x^2 + 5184)*sqrt(x^4 + 5*x^2 + 3))/x^10
```

giac [B] time = 0.56, size = 255, normalized size = 1.93

$$-\frac{2093}{31104}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)+\frac{10465\left(x^2-\sqrt{x^4+5x^2+3}\right)^9-42830\left(x^2-\sqrt{x^4+5x^2+3}\right)^7+1270080\left(x^2-\sqrt{x^4+5x^2+3}\right)^6+7060800\left(x^2-\sqrt{x^4+5x^2+3}\right)^5+15310080\left(x^2-\sqrt{x^4+5x^2+3}\right)^4+16095870\left(x^2-\sqrt{x^4+5x^2+3}\right)^3+7568640\left(x^2-\sqrt{x^4+5x^2+3}\right)^2+1096335x^2-1096335\sqrt{x^4+5x^2+3}+202176}{25920\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] -2093/31104*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt
(3) - sqrt(x^4 + 5*x^2 + 3))) + 1/25920*(10465*(x^2 - sqrt(x^4 + 5*x^2 + 3)
)^9 - 42830*(x^2 - sqrt(x^4 + 5*x^2 + 3))^7 + 1270080*(x^2 - sqrt(x^4 + 5*x
^2 + 3))^6 + 7060800*(x^2 - sqrt(x^4 + 5*x^2 + 3))^5 + 15310080*(x^2 - sqrt
(x^4 + 5*x^2 + 3))^4 + 16095870*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 7568640*(
x^2 - sqrt(x^4 + 5*x^2 + 3))^2 + 1096335*x^2 - 1096335*sqrt(x^4 + 5*x^2 + 3
) + 202176)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^5
```

maple [A] time = 0.02, size = 152, normalized size = 1.15

$$\frac{2093\sqrt{3}\,\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{31104}+\frac{805\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{15552x^2}-\frac{161\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{2592x^4}+\frac{173\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{3240x^6}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{36x^8}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{15x^{10}}-\frac{2093\sqrt{x^4+5x^2+3}}{31104}-\frac{805\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{31104}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x)
```

```
[Out] -1/15*(x^4+5*x^2+3)^(3/2)/x^10-1/36*(x^4+5*x^2+3)^(3/2)/x^8+173/3240*(x^4+5
*x^2+3)^(3/2)/x^6-161/2592*(x^4+5*x^2+3)^(3/2)/x^4+805/15552*(x^4+5*x^2+3)^
(3/2)/x^2-2093/31104*(x^4+5*x^2+3)^(1/2)+2093/31104*arctanh(1/6*(5*x^2+6)*3
^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-805/31104*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

maxima [A] time = 1.73, size = 133, normalized size = 1.01

$$\frac{2093}{31104}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{161}{2592}\sqrt{x^4+5x^2+3}+\frac{805\sqrt{x^4+5x^2+3}}{5184x^2}-\frac{161\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{2592x^4}+\frac{173\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{3240x^6}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{36x^8}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{15x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] 2093/31104*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 1
61/2592*sqrt(x^4 + 5*x^2 + 3) + 805/5184*sqrt(x^4 + 5*x^2 + 3)/x^2 - 161/25
92*(x^4 + 5*x^2 + 3)^(3/2)/x^4 + 173/3240*(x^4 + 5*x^2 + 3)^(3/2)/x^6 - 1/3
6*(x^4 + 5*x^2 + 3)^(3/2)/x^8 - 1/15*(x^4 + 5*x^2 + 3)^(3/2)/x^10
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3\,x^2+2\right)\,\sqrt{x^4+5\,x^2+3}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^11,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^11, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**11,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**11, x)
```

3.129 $\int x^5 \left(2+3x^2\right) \left(3+5x^2+x^4\right)^{3/2} \, dx$

Optimal. Leaf size=127

$$\frac{3}{14}\left(x^4+5x^2+3\right)^{5/2}x^4+\frac{\left(3313-1070x^2\right)\left(x^4+5x^2+3\right)^{5/2}}{1680}-\frac{2183}{768}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{28379\left(2x^2+5\right)\sqrt{\ldots}}{2048\ldots}$$

Rubi [A] time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 832, 779, 612, 621, 206}

$$\frac{3}{14}\left(x^4+5x^2+3\right)^{5/2}x^4+\frac{\left(3313-1070x^2\right)\left(x^4+5x^2+3\right)^{5/2}}{1680}-\frac{2183}{768}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{28379\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{2048}-\frac{368927\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{4096}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (28379*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/2048 - (2183*(5 + 2*x^2)*(3 + 5*x
^2 + x^4)^(3/2))/768 + (3*x^4*(3 + 5*x^2 + x^4)^(5/2))/14 + ((3313 - 1070*x
^2)*(3 + 5*x^2 + x^4)^(5/2))/1680 - (368927*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 +
 5*x^2 + x^4])])/4096
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int x^5 \left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2} dx = \frac{1}{2}\text{Subst}\left(\int x^2(2+3x)\left(3+5x+x^2\right)^{3/2} dx, x, x^2\right)$$
$$= \frac{3}{14}x^4\left(3+5x^2+x^4\right)^{5/2} + \frac{1}{14}\text{Subst}\left(\int\left(-18-\frac{107x}{2}\right)x\left(3+5x+x^2\right)^{3/2}\right.$$
$$= \frac{3}{14}x^4\left(3+5x^2+x^4\right)^{5/2} + \frac{\left(3313-1070x^2\right)\left(3+5x^2+x^4\right)^{5/2}}{1680} - \frac{2183}{96}\text{Su}$$
$$= -\frac{2183}{768}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2} + \frac{3}{14}x^4\left(3+5x^2+x^4\right)^{5/2} + \frac{(3313-}{}$$
$$= \frac{28379\left(5+2x^2\right)\sqrt{3+5x^2+x^4}}{2048} - \frac{2183}{768}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2} + \frac{}{1}$$
$$= \frac{28379\left(5+2x^2\right)\sqrt{3+5x^2+x^4}}{2048} - \frac{2183}{768}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2} + \frac{}{1}$$
$$= \frac{28379\left(5+2x^2\right)\sqrt{3+5x^2+x^4}}{2048} - \frac{2183}{768}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2} + \frac{}{1}$$

Mathematica [A] time = 0.04, size = 81, normalized size = 0.64

$$\frac{2\sqrt{x^4+5x^2+3}\left(46080x^{12}+323840x^{10}+482944x^8+154800x^6+283304x^4-1499570x^2+9546951\right)-38737335\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{430080}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(9546951 - 1499570*x^2 + 283304*x^4 + 154800*x^6 +
 482944*x^8 + 323840*x^10 + 46080*x^12) - 38737335*ArcTanh[(5 + 2*x^2)/(2*S
qrt[3 + 5*x^2 + x^4])])/430080
```

IntegrateAlgebraic [A] time = 0.30, size = 79, normalized size = 0.62

$$\frac{368927\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)}{4096}+\frac{\sqrt{x^4+5x^2+3}\left(46080x^{12}+323840x^{10}+482944x^8+154800x^6+283304x^4-1499570x^2+9546951\right)}{215040}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(9546951 - 1499570*x^2 + 283304*x^4 + 154800*x^6 + 4
82944*x^8 + 323840*x^10 + 46080*x^12))/215040 + (368927*Log[-5 - 2*x^2 + 2*
Sqrt[3 + 5*x^2 + x^4]])/4096
```

fricas [A] time = 1.00, size = 71, normalized size = 0.56

$$\frac{1}{215040}\left(46080\,x^{12}+323840\,x^{10}+482944\,x^8+154800\,x^6+283304\,x^4-1499570\,x^2+9546951\right)\sqrt{x^4+5\,x^2+3}+\frac{368927}{4096}\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/215040*(46080*x^12 + 323840*x^10 + 482944*x^8 + 154800*x^6 + 283304*x^4 -
 1499570*x^2 + 9546951)*sqrt(x^4 + 5*x^2 + 3) + 368927/4096*log(-2*x^2 + 2*
sqrt(x^4 + 5*x^2 + 3) - 5)
```

giac [A] time = 0.54, size = 207, normalized size = 1.63

$$\frac{1}{71680}\sqrt{x^4+5x^2+3}\left(2\left(4\left(2\left(8\left(10\left(12x^2+5\right)x^2-203\right)x^2+7635\right)x^2-76083\right)x^2+1627215\right)x^2-20756241\right)+\frac{17}{3072}\sqrt{x^4+5x^2+3}\left(2\left(4\left(2\left(8\left(2x^2+1\right)x^2-33\right)x^2+321\right)x^2-6837\right)x^2+87147\right)+\frac{19}{3840}\sqrt{x^4+5x^2+3}\left(2\left(4\left(6\left(8x^2+5\right)x^2-127\right)x^2+2635\right)x^2-33429\right)+\frac{1}{64}\sqrt{x^4+5x^2+3}\left(2\left(4\left(6x^2+5\right)x^2-89\right)x^2+1095\right)+\frac{368927}{4096}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/71680*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(2*(8*(10*(12*x^2 + 5)*x^2 - 203)*x^2 +
 7635)*x^2 - 76083)*x^2 + 1627215)*x^2 - 20756241) + 17/3072*sqrt(x^4 + 5*x
^2 + 3)*(2*(4*(2*(8*(2*x^2 + 1)*x^2 - 33)*x^2 + 321)*x^2 - 6837)*x^2 + 8714
7) + 19/3840*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 + 5)*x^2 - 127)*x^2 + 26
35)*x^2 - 33429) + 1/64*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x
^2 + 1095) + 368927/4096*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.03, size = 138, normalized size = 1.09

$$\frac{3\sqrt{x^4+5x^2+3}\,x^{12}}{14}+\frac{253\sqrt{x^4+5x^2+3}\,x^{10}}{168}+\frac{539\sqrt{x^4+5x^2+3}\,x^{8}}{240}+\frac{645\sqrt{x^4+5x^2+3}\,x^{6}}{896}+\frac{5059\sqrt{x^4+5x^2+3}\,x^{4}}{3840}-\frac{149957\sqrt{x^4+5x^2+3}\,x^{2}}{21504}-\frac{368927\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{4096}+\frac{3182317\sqrt{x^4+5x^2+3}}{71680}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 645/896*x^6*(x^4+5*x^2+3)^(1/2)-149957/21504*x^2*(x^4+5*x^2+3)^(1/2)-368927
/4096*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))+3/14*x^12*(x^4+5*x^2+3)^(1/2)+253/168
*x^10*(x^4+5*x^2+3)^(1/2)+539/240*x^8*(x^4+5*x^2+3)^(1/2)+3182317/71680*(x^
4+5*x^2+3)^(1/2)+5059/3840*x^4*(x^4+5*x^2+3)^(1/2)
```

maxima [A] time = 0.59, size = 135, normalized size = 1.06

$$\frac{3}{14}\left(x^4+5x^2+3\right)^{\frac{5}{2}}x^4-\frac{107}{168}\left(x^4+5x^2+3\right)^{\frac{5}{2}}x^2-\frac{2183}{384}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2+\frac{3313}{1680}\left(x^4+5x^2+3\right)^{\frac{5}{2}}+\frac{28379}{1024}\sqrt{x^4+5x^2+3}\,x^2-\frac{10915}{768}\left(x^4+5x^2+3\right)^{\frac{3}{2}}+\frac{141895}{2048}\sqrt{x^4+5x^2+3}-\frac{368927}{4096}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/14*(x^4 + 5*x^2 + 3)^(5/2)*x^4 - 107/168*(x^4 + 5*x^2 + 3)^(5/2)*x^2 - 21
83/384*(x^4 + 5*x^2 + 3)^(3/2)*x^2 + 3313/1680*(x^4 + 5*x^2 + 3)^(5/2) + 28
379/1024*sqrt(x^4 + 5*x^2 + 3)*x^2 - 10915/768*(x^4 + 5*x^2 + 3)^(3/2) + 14
1895/2048*sqrt(x^4 + 5*x^2 + 3) - 368927/4096*log(2*x^2 + 2*sqrt(x^4 + 5*x^
2 + 3) + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\,\left(3\,x^2+2\right)\,{\left(x^4+5\,x^2+3\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] int(x^5*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{5}\left(3x^{2}+2\right)\left(x^{4}+5x^{2}+3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**5*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.130 $\int x^3 \left(2+3x^2\right) \left(3+5x^2+x^4\right)^{3/2} dx$

Optimal. Leaf size=106

$$-\frac{1}{40}\left(27-10x^2\right)\left(x^4+5x^2+3\right)^{5/2}+\frac{123}{128}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{4797\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{1024}+\frac{62361\tanh$$

Rubi [A] time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1251, 779, 612, 621, 206}

$$-\frac{1}{40}\left(27-10x^2\right)\left(x^4+5x^2+3\right)^{5/2}+\frac{123}{128}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{4797\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{1024}+\frac{62361\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{2048}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (-4797*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/1024 + (123*(5 + 2*x^2)*(3 + 5*x^
2 + x^4)^(3/2))/128 - ((27 - 10*x^2)*(3 + 5*x^2 + x^4)^(5/2))/40 + (62361*A
rcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2048
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int x^3 \left(2+3x^2\right) \left(3+5x^2+x^4\right)^{3/2} dx = \frac{1}{2} \text{Subst}\left(\int x(2+3x) \left(3+5x+x^2\right)^{3/2} dx, x, x^2\right)$$
$$= -\frac{1}{40} \left(27-10x^2\right) \left(3+5x^2+x^4\right)^{5/2} + \frac{123}{16} \text{Subst}\left(\int \left(3+5x+x^2\right)^{3/2} dx\right.$$
$$= \frac{123}{128} \left(5+2x^2\right) \left(3+5x^2+x^4\right)^{3/2} - \frac{1}{40} \left(27-10x^2\right) \left(3+5x^2+x^4\right)^{5/2} -$$
$$= -\frac{4797 \left(5+2x^2\right) \sqrt{3+5x^2+x^4}}{1024} + \frac{123}{128} \left(5+2x^2\right) \left(3+5x^2+x^4\right)^{3/2} -$$
$$= -\frac{4797 \left(5+2x^2\right) \sqrt{3+5x^2+x^4}}{1024} + \frac{123}{128} \left(5+2x^2\right) \left(3+5x^2+x^4\right)^{3/2} -$$
$$= -\frac{4797 \left(5+2x^2\right) \sqrt{3+5x^2+x^4}}{1024} + \frac{123}{128} \left(5+2x^2\right) \left(3+5x^2+x^4\right)^{3/2} -$$

Mathematica [A] time = 0.03, size = 76, normalized size = 0.72

$$\frac{311805 \tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) + 2\sqrt{x^4+5x^2+3} \left(1280x^{10}+9344x^8+14960x^6+5064x^4+12390x^2-77229\right)}{10240}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(-77229 + 12390*x^2 + 5064*x^4 + 14960*x^6 + 9344*
x^8 + 1280*x^10) + 311805*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/1
0240
```

IntegrateAlgebraic [A] time = 0.24, size = 74, normalized size = 0.70

$$\frac{\sqrt{x^4+5x^2+3} \left(1280x^{10}+9344x^8+14960x^6+5064x^4+12390x^2-77229\right)}{5120} - \frac{62361 \log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)}{2048}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-77229 + 12390*x^2 + 5064*x^4 + 14960*x^6 + 9344*x^
8 + 1280*x^10))/5120 - (62361*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/20
48
```

fricas [A] time = 1.02, size = 66, normalized size = 0.62

$$\frac{1}{5120} \left(1280\,x^{10}+9344\,x^8+14960\,x^6+5064\,x^4+12390\,x^2-77229\right)\sqrt{x^4+5\,x^2+3} - \frac{62361}{2048} \log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5120*(1280*x^10 + 9344*x^8 + 14960*x^6 + 5064*x^4 + 12390*x^2 - 77229)*sq
rt(x^4 + 5*x^2 + 3) - 62361/2048*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

giac [B] time = 0.59, size = 179, normalized size = 1.69

$$\frac{1}{1024} \sqrt{x^4+5\,x^2+3}\left(2\left(4\left(2\left(8\left(2\,x^2+1\right)x^2-33\right)x^2+321\right)x^2-6837\right)x^2+87147\right) + \frac{17}{3840} \sqrt{x^4+5\,x^2+3}\left(2\left(4\left(6\left(8\,x^2+5\right)x^2-127\right)x^2+2635\right)x^2-33429\right) + \frac{19}{384} \sqrt{x^4+5\,x^2+3}\left(2\left(4\left(6\,x^2+5\right)x^2-89\right)x^2+1095\right) + \frac{1}{8} \sqrt{x^4+5\,x^2+3}\left(2\left(4\,x^2+5\right)x^2-51\right) - \frac{62361}{2048} \log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1024*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(2*(8*(2*x^2 + 1)*x^2 - 33)*x^2 + 321)*x
^2 - 6837)*x^2 + 87147) + 17/3840*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 + 5
)*x^2 - 127)*x^2 + 2635)*x^2 - 33429) + 19/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*
(6*x^2 + 5)*x^2 - 89)*x^2 + 1095) + 1/8*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 5
)*x^2 - 51) - 62361/2048*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.02, size = 121, normalized size = 1.14

$$\frac{\sqrt{x^4+5x^2+3}\,x^{10}}{4}+\frac{73\sqrt{x^4+5x^2+3}\,x^8}{40}+\frac{187\sqrt{x^4+5x^2+3}\,x^6}{64}+\frac{633\sqrt{x^4+5x^2+3}\,x^4}{640}+\frac{1239\sqrt{x^4+5x^2+3}\,x^2}{512}+\frac{62361\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{2048}-\frac{77229\sqrt{x^4+5x^2+3}}{5120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 187/64*(x^4+5*x^2+3)^(1/2)*x^6+1239/512*(x^4+5*x^2+3)^(1/2)*x^2+62361/2048*
ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))+1/4*(x^4+5*x^2+3)^(1/2)*x^10+73/40*(x^4+5*x
^2+3)^(1/2)*x^8-77229/5120*(x^4+5*x^2+3)^(1/2)+633/640*(x^4+5*x^2+3)^(1/2)*
x^4
```

maxima [A] time = 0.88, size = 118, normalized size = 1.11

$$\frac{1}{4}\left(x^4+5\,x^2+3\right)^{\frac{5}{2}}x^2+\frac{123}{64}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^2-\frac{27}{40}\left(x^4+5\,x^2+3\right)^{\frac{5}{2}}-\frac{4797}{512}\sqrt{x^4+5\,x^2+3}\,x^2+\frac{615}{128}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}-\frac{23985}{1024}\sqrt{x^4+5\,x^2+3}+\frac{62361}{2048}\log\left(2\,x^2+2\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(x^4 + 5*x^2 + 3)^(5/2)*x^2 + 123/64*(x^4 + 5*x^2 + 3)^(3/2)*x^2 - 27/4
0*(x^4 + 5*x^2 + 3)^(5/2) - 4797/512*sqrt(x^4 + 5*x^2 + 3)*x^2 + 615/128*(x
^4 + 5*x^2 + 3)^(3/2) - 23985/1024*sqrt(x^4 + 5*x^2 + 3) + 62361/2048*log(2
*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(3\,x^2+2\right)\,{\left(x^4+5\,x^2+3\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] int(x^3*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\left(3x^{2}+2\right)\left(x^{4}+5x^{2}+3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**3*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.131 $\int x\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=99

$$\frac{3}{10}\left(x^4+5x^2+3\right)^{5/2}-\frac{11}{32}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{429}{256}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}-\frac{5577}{512}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Rubi [A] time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1247, 640, 612, 621, 206}

$$\frac{3}{10}\left(x^4+5x^2+3\right)^{5/2}-\frac{11}{32}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{429}{256}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}-\frac{5577}{512}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (429*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/256 - (11*(5 + 2*x^2)*(3 + 5*x^2 +
x^4)^(3/2))/32 + (3*(3 + 5*x^2 + x^4)^(5/2))/10 - (5577*ArcTanh[(5 + 2*x^2)
/(2*Sqrt[3 + 5*x^2 + x^4])])/512
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\int x\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx &= \frac{1}{2}\text{Subst}\left(\int (2+3x)\left(3+5x+x^2\right)^{3/2}\,dx,x,x^2\right)\\
&= \frac{3}{10}\left(3+5x^2+x^4\right)^{5/2}-\frac{11}{4}\text{Subst}\left(\int \left(3+5x+x^2\right)^{3/2}\,dx,x,x^2\right)\\
&= -\frac{11}{32}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{3}{10}\left(3+5x^2+x^4\right)^{5/2}+\frac{429}{64}\text{Subst}\left(\int \sqrt{3}\right.\\
&= \frac{429}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}-\frac{11}{32}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{3}{10}\left(3+5\right.\\
&= \frac{429}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}-\frac{11}{32}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{3}{10}\left(3+5\right.\\
&= \frac{429}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}-\frac{11}{32}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{3}{10}\left(3+5\right.
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 71, normalized size = 0.72

$$
\frac{2\sqrt{x^4+5x^2+3}\left(384x^8+2960x^6+5304x^4+2170x^2+7581\right)-27885\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{2560}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(7581 + 2170*x^2 + 5304*x^4 + 2960*x^6 + 384*x^8)
- 27885*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2560
```

IntegrateAlgebraic [A] time = 0.24, size = 69, normalized size = 0.70

$$
\frac{5577}{512}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)+\frac{\sqrt{x^4+5x^2+3}\left(384x^8+2960x^6+5304x^4+2170x^2+7581\right)}{1280}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(7581 + 2170*x^2 + 5304*x^4 + 2960*x^6 + 384*x^8))/1
280 + (5577*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/512
```

fricas [A] time = 0.76, size = 61, normalized size = 0.62

$$
\frac{1}{1280}\left(384\,x^8+2960\,x^6+5304\,x^4+2170\,x^2+7581\right)\sqrt{x^4+5\,x^2+3}+\frac{5577}{512}\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1280*(384*x^8 + 2960*x^6 + 5304*x^4 + 2170*x^2 + 7581)*sqrt(x^4 + 5*x^2 +
 3) + 5577/512*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

giac [A] time = 0.45, size = 151, normalized size = 1.53

$$
\frac{1}{1280}\sqrt{x^4+5\,x^2+3}\left(2\left(4\left(6\left(8\,x^2+5\right)x^2-127\right)x^2+2635\right)x^2-33429\right)+\frac{17}{384}\sqrt{x^4+5\,x^2+3}\left(2\left(4\left(6\,x^2+5\right)x^2-89\right)x^2+1095\right)+\frac{19}{48}\sqrt{x^4+5\,x^2+3}\left(2\left(4\,x^2+5\right)x^2-51\right)+\frac{3}{4}\sqrt{x^4+5\,x^2+3}\left(2\,x^2+5\right)+\frac{5577}{512}\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1280*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 + 5)*x^2 - 127)*x^2 + 2635)*x^
2 - 33429) + 17/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x^2 +
```

```
1095) + 19/48*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 5)*x^2 - 51) + 3/4*sqrt(x^
4 + 5*x^2 + 3)*(2*x^2 + 5) + 5577/512*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) +
 5)
```

maple [A] time = 0.02, size = 104, normalized size = 1.05

$$\frac{3\sqrt{x^4+5x^2+3}\,x^8}{10}+\frac{37\sqrt{x^4+5x^2+3}\,x^6}{16}+\frac{663\sqrt{x^4+5x^2+3}\,x^4}{160}+\frac{217\sqrt{x^4+5x^2+3}\,x^2}{128}-\frac{5577\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{512}+\frac{7581\sqrt{x^4+5x^2+3}}{1280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 3/10*(x^4+5*x^2+3)^(1/2)*x^8+37/16*(x^4+5*x^2+3)^(1/2)*x^6+663/160*(x^4+5*x
^2+3)^(1/2)*x^4+217/128*(x^4+5*x^2+3)^(1/2)*x^2+7581/1280*(x^4+5*x^2+3)^(1/
2)-5577/512*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))
```

maxima [A] time = 0.67, size = 101, normalized size = 1.02

$$-\frac{11}{16}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2+\frac{3}{10}\left(x^4+5x^2+3\right)^{\frac{5}{2}}+\frac{429}{128}\sqrt{x^4+5x^2+3}\,x^2-\frac{55}{32}\left(x^4+5x^2+3\right)^{\frac{3}{2}}+\frac{2145}{256}\sqrt{x^4+5x^2+3}-\frac{5577}{512}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] -11/16*(x^4 + 5*x^2 + 3)^(3/2)*x^2 + 3/10*(x^4 + 5*x^2 + 3)^(5/2) + 429/128
*sqrt(x^4 + 5*x^2 + 3)*x^2 - 55/32*(x^4 + 5*x^2 + 3)^(3/2) + 2145/256*sqrt(
x^4 + 5*x^2 + 3) - 5577/512*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [B] time = 0.53, size = 127, normalized size = 1.28

$$\frac{\left(x^2+\frac{5}{2}\right)\left(x^4+5x^2+3\right)^{3/2}}{4}-\frac{15x^2\left(x^4+5x^2+3\right)^{3/2}}{16}-\frac{5577\ln\left(\sqrt{x^4+5x^2+3}+x^2+\frac{5}{2}\right)}{512}+\frac{585\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{256}-\frac{39\left(\frac{x^2}{2}+\frac{5}{4}\right)\sqrt{x^4+5x^2+3}}{16}-\frac{75\left(x^4+5x^2+3\right)^{3/2}}{32}+\frac{3\left(x^4+5x^2+3\right)^{5/2}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] ((x^2 + 5/2)*(5*x^2 + x^4 + 3)^(3/2))/4 - (15*x^2*(5*x^2 + x^4 + 3)^(3/2))/
16 - (5577*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/512 + (585*(2*x^2 + 5)
*(5*x^2 + x^4 + 3)^(1/2))/256 - (39*(x^2/2 + 5/4)*(5*x^2 + x^4 + 3)^(1/2))/
16 - (75*(5*x^2 + x^4 + 3)^(3/2))/32 + (3*(5*x^2 + x^4 + 3)^(5/2))/10
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.132 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x}\,dx$

Optimal. Leaf size=119

$$\frac{1}{48}\left(18x^2+61\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{1}{128}\left(199-74x^2\right)\sqrt{x^4+5x^2+3}+\frac{2401}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-3\sqrt{3}\tanh$$

Rubi [A] time = 0.11, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1251, 814, 843, 621, 206, 724}

$$\frac{1}{48}\left(18x^2+61\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{1}{128}\left(199-74x^2\right)\sqrt{x^4+5x^2+3}+\frac{2401}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-3\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x,x]
```

```
[Out] ((199 - 74*x^2)*Sqrt[3 + 5*x^2 + x^4])/128 + ((61 + 18*x^2)*(3 + 5*x^2 + x^
4)^(3/2))/48 + (2401*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/256 -
3*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
```

```
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)(3+5x+x^2)^{3/2}}{x}\,dx, x, x^2\right)$$

$$= \frac{1}{48}(61+18x^2)(3+5x^2+x^4)^{3/2} - \frac{1}{16}\text{Subst}\left(\int \frac{\left(-48+\frac{37x}{2}\right)\sqrt{3+5x+x}}{x}\right.$$

$$= \frac{1}{128}(199-74x^2)\sqrt{3+5x^2+x^4} + \frac{1}{48}(61+18x^2)(3+5x^2+x^4)^{3/2} + \frac{1}{64}$$

$$= \frac{1}{128}(199-74x^2)\sqrt{3+5x^2+x^4} + \frac{1}{48}(61+18x^2)(3+5x^2+x^4)^{3/2} + 9\,\text{S}$$

$$= \frac{1}{128}(199-74x^2)\sqrt{3+5x^2+x^4} + \frac{1}{48}(61+18x^2)(3+5x^2+x^4)^{3/2} - 18$$

$$= \frac{1}{128}(199-74x^2)\sqrt{3+5x^2+x^4} + \frac{1}{48}(61+18x^2)(3+5x^2+x^4)^{3/2} + \frac{24}{25}$$

Mathematica [A] time = 0.06, size = 104, normalized size = 0.87

$$\frac{2401}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - 3\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right) + \frac{1}{384}\sqrt{x^4+5x^2+3}\left(144x^6+1208x^4+2650x^2+2061\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x,x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(2061 + 2650*x^2 + 1208*x^4 + 144*x^6))/384 + (2401*
ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/256 - 3*Sqrt[3]*ArcTanh[(6
+ 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]
```

IntegrateAlgebraic [A] time = 0.32, size = 103, normalized size = 0.87

$$-\frac{2401}{256}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right) + 6\sqrt{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}}\right) + \frac{1}{384}\sqrt{x^4+5x^2+3}\left(144x^6+1208x^4+2650x^2+2061\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x,x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(2061 + 2650*x^2 + 1208*x^4 + 144*x^6))/384 + 6*Sqrt
[3]*ArcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]] - (2401*Log[-5 - 2
*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/256
```

fricas [A] time = 1.23, size = 106, normalized size = 0.89

$$\frac{1}{384}\left(144x^6+1208x^4+2650x^2+2061\right)\sqrt{x^4+5x^2+3}+3\sqrt{3}\log\left(\frac{25x^2-2\sqrt{3}\left(5x^2+6\right)-2\sqrt{x^4+5x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)-\frac{2401}{256}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/384*(144*x^6 + 1208*x^4 + 2650*x^2 + 2061)*sqrt(x^4 + 5*x^2 + 3) + 3*sqrt
(3)*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3
) - 6) + 30)/x^2) - 2401/256*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

giac [A] time = 0.50, size = 113, normalized size = 0.95

$$\frac{1}{384}\sqrt{x^4+5x^2+3}\left(2\left(4\left(18x^2+151\right)x^2+1325\right)x^2+2061\right)+3\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)-\frac{2401}{256}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(18*x^2 + 151)*x^2 + 1325)*x^2 + 2061) +
3*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt
(x^4 + 5*x^2 + 3))) - 2401/256*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.02, size = 117, normalized size = 0.98

$$\frac{3\sqrt{x^4+5x^2+3}\,x^6}{8}+\frac{151\sqrt{x^4+5x^2+3}\,x^4}{48}+\frac{1325\sqrt{x^4+5x^2+3}\,x^2}{192}-3\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)+\frac{2401\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{256}+\frac{687\sqrt{x^4+5x^2+3}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x)
```

```
[Out] 3/8*(x^4+5*x^2+3)^(1/2)*x^6+151/48*(x^4+5*x^2+3)^(1/2)*x^4+1325/192*(x^4+5*
x^2+3)^(1/2)*x^2+687/128*(x^4+5*x^2+3)^(1/2)+2401/256*ln(x^2+5/2+(x^4+5*x^2
+3)^(1/2))-3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

maxima [A] time = 1.40, size = 120, normalized size = 1.01

$$\frac{3}{8}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2-\frac{37}{64}\sqrt{x^4+5x^2+3}\,x^2+\frac{61}{48}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-3\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{199}{128}\sqrt{x^4+5x^2+3}+\frac{2401}{256}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 3/8*(x^4 + 5*x^2 + 3)^(3/2)*x^2 - 37/64*sqrt(x^4 + 5*x^2 + 3)*x^2 + 61/48*(
x^4 + 5*x^2 + 3)^(3/2) - 3*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2
+ 6/x^2 + 5) + 199/128*sqrt(x^4 + 5*x^2 + 3) + 2401/256*log(2*x^2 + 2*sqrt(
x^4 + 5*x^2 + 3) + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2+2)(x^4+5x^2+3)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x, x)
```

3.133 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=122

$$-\frac{(2-x^2)(x^4+5x^2+3)^{3/2}}{2x^2}+\frac{3}{16}(18x^2+109)\sqrt{x^4+5x^2+3}+\frac{609}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-12\sqrt{3}\tanh^{-1}\left(\frac{\quad}{2\sqrt{\quad}}\right.$$

Rubi [A] time = 0.11, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1251, 812, 814, 843, 621, 206, 724}

$$-\frac{(2-x^2)(x^4+5x^2+3)^{3/2}}{2x^2}+\frac{3}{16}(18x^2+109)\sqrt{x^4+5x^2+3}+\frac{609}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-12\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^3,x]
```

```
[Out] (3*(109 + 18*x^2)*Sqrt[3 + 5*x^2 + x^4])/16 - ((2 - x^2)*(3 + 5*x^2 + x^4)^
(3/2))/(2*x^2) + (609*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32 -
12*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
```

```
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)(3+5x+x^2)^{3/2}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{(2-x^2)(3+5x^2+x^4)^{3/2}}{2x^2} - \frac{1}{4}\text{Subst}\left(\int \frac{(-48-27x)\sqrt{3+5x+x^2}}{x}\,dx, x\right.$$

$$= \frac{3}{16}(109+18x^2)\sqrt{3+5x^2+x^4} - \frac{(2-x^2)(3+5x^2+x^4)^{3/2}}{2x^2} + \frac{1}{16}\text{Subst}$$

$$= \frac{3}{16}(109+18x^2)\sqrt{3+5x^2+x^4} - \frac{(2-x^2)(3+5x^2+x^4)^{3/2}}{2x^2} + \frac{609}{32}\text{Subst}$$

$$= \frac{3}{16}(109+18x^2)\sqrt{3+5x^2+x^4} - \frac{(2-x^2)(3+5x^2+x^4)^{3/2}}{2x^2} + \frac{609}{16}\text{Subst}$$

$$= \frac{3}{16}(109+18x^2)\sqrt{3+5x^2+x^4} - \frac{(2-x^2)(3+5x^2+x^4)^{3/2}}{2x^2} + \frac{609}{32}\tanh^{-}$$

Mathematica [A] time = 0.05, size = 107, normalized size = 0.88

$$\frac{609}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - 12\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right) + \frac{\sqrt{x^4+5x^2+3}\,(8x^6+78x^4+271x^2-48)}{16x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^3,x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-48 + 271*x^2 + 78*x^4 + 8*x^6))/(16*x^2) + (609*Ar
cTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32 - 12*Sqrt[3]*ArcTanh[(6 +
5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]
```

IntegrateAlgebraic [A] time = 0.35, size = 106, normalized size = 0.87

$$-\frac{609}{32}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)+24\sqrt{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{3}}-\frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}}\right)+\frac{\sqrt{x^4+5x^2+3}\left(8x^6+78x^4+271x^2-48\right)}{16x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^3,x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-48 + 271*x^2 + 78*x^4 + 8*x^6))/(16*x^2) + 24*Sqrt
[3]*ArcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]] - (609*Log[-5 - 2*
x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/32
```

fricas [A] time = 1.04, size = 122, normalized size = 1.00

$$\frac{1536\sqrt{3}x^2\log\left(\frac{25x^2-2\sqrt{3}(5x^2+6)-2\sqrt{x^4+5x^2+3}(5\sqrt{3}-6)+30}{x^2}\right)-2436x^2\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)+1541x^2+8\left(8x^6+78x^4+271x^2-48\right)\sqrt{x^4+5x^2+3}}{128x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/128*(1536*sqrt(3)*x^2*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 +
5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 2436*x^2*log(-2*x^2 + 2*sqrt(x^4 +
5*x^2 + 3) - 5) + 1541*x^2 + 8*(8*x^6 + 78*x^4 + 271*x^2 - 48)*sqrt(x^4 + 5
*x^2 + 3))/x^2
```

giac [A] time = 0.56, size = 153, normalized size = 1.25

$$\frac{1}{16}\sqrt{x^4+5x^2+3}\left(2\left(4x^2+39\right)x^2+271\right)+12\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)+\frac{3\left(5x^2-5\sqrt{x^4+5x^2+3}+6\right)}{\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3}-\frac{609}{32}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 39)*x^2 + 271) + 12*sqrt(3)*log((x^2
 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))
) + 3*(5*x^2 - 5*sqrt(x^4 + 5*x^2 + 3) + 6)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^
2 - 3) - 609/32*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.02, size = 117, normalized size = 0.96

$$\frac{\sqrt{x^4+5x^2+3}\,x^4}{2}+\frac{39\sqrt{x^4+5x^2+3}\,x^2}{8}-12\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)+\frac{609\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{32}-\frac{3\sqrt{x^4+5x^2+3}}{x^2}+\frac{271\sqrt{x^4+5x^2+3}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x)
```

```
[Out] 1/2*(x^4+5*x^2+3)^(1/2)*x^4+39/8*(x^4+5*x^2+3)^(1/2)*x^2+271/16*(x^4+5*x^2+
3)^(1/2)+609/32*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))-12*arctanh(1/6*(5*x^2+6)*3^
(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-3*(x^4+5*x^2+3)^(1/2)/x^2
```

maxima [A] time = 1.75, size = 120, normalized size = 0.98

$$\frac{27}{8}\sqrt{x^4+5x^2+3}\,x^2+\frac{1}{2}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-12\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{327}{16}\sqrt{x^4+5x^2+3}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{x^2}+\frac{609}{32}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 27/8*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1/2*(x^4 + 5*x^2 + 3)^(3/2) - 12*sqrt(3)*l
og(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 327/16*sqrt(x^4 + 5*x
```

```
^2 + 3) - (x^4 + 5*x^2 + 3)^(3/2)/x^2 + 609/32*log(2*x^2 + 2*sqrt(x^4 + 5*x
^2 + 3) + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3\,x^2+2\right)\,{\left(x^4+5\,x^2+3\right)}^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^3,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^{2} + 2\right)\left(x^{4} + 5x^{2} + 3\right)^{\frac{3}{2}}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**3,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**3, x)
```

3.134 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=127

$$-\frac{(2-3x^2)(x^4+5x^2+3)^{3/2}}{4x^4}-\frac{3(28-19x^2)\sqrt{x^4+5x^2+3}}{8x^2}+\frac{453}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{127}{8}\sqrt{3}\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.$$

Rubi [A] time = 0.11, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1251, 812, 843, 621, 206, 724}

$$-\frac{(2-3x^2)(x^4+5x^2+3)^{3/2}}{4x^4}-\frac{3(28-19x^2)\sqrt{x^4+5x^2+3}}{8x^2}+\frac{453}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{127}{8}\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^5,x]
```

```
[Out] (-3*(28 - 19*x^2)*Sqrt[3 + 5*x^2 + x^4])/(8*x^2) - ((2 - 3*x^2)*(3 + 5*x^2
+ x^4)^(3/2))/(4*x^4) + (453*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])]
)/16 - (127*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])
/8
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
```

```
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{x^5}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(2+3x)\left(3+5x+x^2\right)^{3/2}}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{\left(2-3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{4x^4} - \frac{3}{16}\,\text{Subst}\left(\int \frac{(-56-38x)\sqrt{3+5x+x^2}}{x^2}\,d\right.$$

$$= -\frac{3\left(28-19x^2\right)\sqrt{3+5x^2+x^4}}{8x^2} - \frac{\left(2-3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{4x^4} + \frac{3}{32}\,\text{Subst}$$

$$= -\frac{3\left(28-19x^2\right)\sqrt{3+5x^2+x^4}}{8x^2} - \frac{\left(2-3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{4x^4} + \frac{453}{16}\,\text{Subs}$$

$$= -\frac{3\left(28-19x^2\right)\sqrt{3+5x^2+x^4}}{8x^2} - \frac{\left(2-3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{4x^4} + \frac{453}{8}\,\text{Subs}$$

$$= -\frac{3\left(28-19x^2\right)\sqrt{3+5x^2+x^4}}{8x^2} - \frac{\left(2-3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{4x^4} + \frac{453}{16}\,\tanh$$

Mathematica [A] time = 0.06, size = 107, normalized size = 0.84

$$\frac{1}{16}\left(453\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - 254\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right) + \frac{2\sqrt{x^4+5x^2+3}\left(6x^6+83x^4-86x^2-12\right)}{x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^5,x]
```

```
[Out] ((2*Sqrt[3 + 5*x^2 + x^4]*(-12 - 86*x^2 + 83*x^4 + 6*x^6))/x^4 + 453*ArcTan
h[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])] - 254*Sqrt[3]*ArcTanh[(6 + 5*x^2)/
(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/16
```

IntegrateAlgebraic [A] time = 0.39, size = 108, normalized size = 0.85

$$-\frac{453}{16}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right) + \frac{127}{4}\sqrt{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}}\right) + \frac{\sqrt{x^4+5x^2+3}\left(6x^6+83x^4-86x^2-12\right)}{8x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^5,x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-12 - 86*x^2 + 83*x^4 + 6*x^6))/(8*x^4) + (127*Sqrt
[3]*ArcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/4 - (453*Log[-5 -
 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/16
```

fricas [A] time = 1.15, size = 122, normalized size = 0.96

$$\frac{1016 \sqrt{3} x^4 \log\left(\frac{25 x^2 - 2 \sqrt{3}\left(5 x^2+6\right) - 2 \sqrt{x^4+5 x^2+3}\left(5 \sqrt{3}-6\right)+30}{x^2}\right) - 1812 x^4 \log\left(-2 x^2 + 2 \sqrt{x^4+5 x^2+3} - 5\right) + 67 x^4 + 8\left(6 x^6 + 83 x^4 - 86 x^2 - 12\right)\sqrt{x^4+5 x^2+3}}{64 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/64*(1016*sqrt(3)*x^4*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5
*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 1812*x^4*log(-2*x^2 + 2*sqrt(x^4 + 5
*x^2 + 3) - 5) + 67*x^4 + 8*(6*x^6 + 83*x^4 - 86*x^2 - 12)*sqrt(x^4 + 5*x^2
 + 3))/x^4
```

giac [A] time = 0.65, size = 190, normalized size = 1.50

$$\frac{1}{8} \sqrt{x^4+5 x^2+3}\left(6 x^2+83\right) + \frac{127}{8} \sqrt{3} \log\left(\frac{\left|x^2+\sqrt{3}-\sqrt{x^4+5 x^2+3}\right|}{\left|x^2-\sqrt{3}-\sqrt{x^4+5 x^2+3}\right|}\right) + \frac{227\left(x^2-\sqrt{x^4+5 x^2+3}\right)^3 + 348\left(x^2-\sqrt{x^4+5 x^2+3}\right)^2 - 459 x^2 + 459 \sqrt{x^4+5 x^2+3} - 684}{4\left(\left(x^2-\sqrt{x^4+5 x^2+3}\right)^2-3\right)^2} - \frac{453}{16} \log\left(2 x^2 - 2 \sqrt{x^4+5 x^2+3} + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 + 83) + 127/8*sqrt(3)*log((x^2 + sqrt(3) -
 sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))) + 1/4*(227
*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 348*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 45
9*x^2 + 459*sqrt(x^4 + 5*x^2 + 3) - 684)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 -
 3)^2 - 453/16*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.02, size = 117, normalized size = 0.92

$$\frac{3 \sqrt{x^4+5 x^2+3}\, x^2}{4} - \frac{127 \sqrt{3}\, \operatorname{arctanh}\left(\frac{\left(5 x^2+6\right)\sqrt{3}}{6 \sqrt{x^4+5 x^2+3}}\right)}{8} + \frac{453 \ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5 x^2+3}\right)}{16} - \frac{43 \sqrt{x^4+5 x^2+3}}{4 x^2} - \frac{3 \sqrt{x^4+5 x^2+3}}{2 x^4} + \frac{83 \sqrt{x^4+5 x^2+3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x)
```

```
[Out] 83/8*(x^4+5*x^2+3)^(1/2)+453/16*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))-127/8*arcta
nh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-43/4*(x^4+5*x^2+3)^(1
/2)/x^2-3/2*(x^4+5*x^2+3)^(1/2)/x^4+3/4*(x^4+5*x^2+3)^(1/2)*x^2
```

maxima [A] time = 1.46, size = 137, normalized size = 1.08

$$\frac{7}{2} \sqrt{x^4+5 x^2+3}\, x^2 + \frac{1}{6}\left(x^4+5 x^2+3\right)^{\frac{3}{2}} - \frac{127}{8} \sqrt{3} \log\left(\frac{2 \sqrt{3} \sqrt{x^4+5 x^2+3}}{x^2} + \frac{6}{x^2} + 5\right) + \frac{197}{8} \sqrt{x^4+5 x^2+3} - \frac{23\left(x^4+5 x^2+3\right)^{\frac{3}{2}}}{12 x^2} - \frac{\left(x^4+5 x^2+3\right)^{\frac{5}{2}}}{6 x^4} + \frac{453}{16} \log\left(2 x^2 + 2 \sqrt{x^4+5 x^2+3} + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 7/2*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1/6*(x^4 + 5*x^2 + 3)^(3/2) - 127/8*sqrt(3)
*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 197/8*sqrt(x^4 + 5*
x^2 + 3) - 23/12*(x^4 + 5*x^2 + 3)^(3/2)/x^2 - 1/6*(x^4 + 5*x^2 + 3)^(5/2)/
x^4 + 453/16*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3 x^2+2\right)\left(x^4+5 x^2+3\right)^{3/2}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^5,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2+2)(x^4+5x^2+3)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**5,x)
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**5, x)
```

3.135 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=127

$$-\frac{(67-32x^2)\sqrt{x^4+5x^2+3}}{12x^2}+\frac{49}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{527\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}-\frac{(7x^2+2)(x^4+5x^2+}{6x^6}$$

Rubi [A] time = 0.11, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1251, 810, 812, 843, 621, 206, 724}

$$-\frac{(7x^2+2)(x^4+5x^2+3)^{3/2}}{6x^6}-\frac{(67-32x^2)\sqrt{x^4+5x^2+3}}{12x^2}+\frac{49}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{527\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^7,x]
```

```
[Out] -((67 - 32*x^2)*Sqrt[3 + 5*x^2 + x^4])/(12*x^2) - ((2 + 7*x^2)*(3 + 5*x^2 +
 x^4)^(3/2))/(6*x^6) + (49*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/
4 - (527*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(24*Sqrt[3
])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{x^7}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(2+3x)\left(3+5x+x^2\right)^{3/2}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{\left(2+7x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{6x^6} - \frac{1}{24}\,\text{Subst}\left(\int \frac{(-134-64x)\sqrt{3+5x+x^2}}{x^2}\right.$$

$$= -\frac{\left(67-32x^2\right)\sqrt{3+5x^2+x^4}}{12x^2} - \frac{\left(2+7x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{6x^6} + \frac{1}{48}\,\text{Subst}\Big($$

$$= -\frac{\left(67-32x^2\right)\sqrt{3+5x^2+x^4}}{12x^2} - \frac{\left(2+7x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{6x^6} + \frac{49}{4}\,\text{Subst}\Big($$

$$= -\frac{\left(67-32x^2\right)\sqrt{3+5x^2+x^4}}{12x^2} - \frac{\left(2+7x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{6x^6} + \frac{49}{2}\,\text{Subst}\Big($$

$$= -\frac{\left(67-32x^2\right)\sqrt{3+5x^2+x^4}}{12x^2} - \frac{\left(2+7x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{6x^6} + \frac{49}{4}\tanh^{-1}\Big($$

Mathematica [A] time = 0.05, size = 107, normalized size = 0.84

$$\frac{1}{72}\left(882\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-527\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)+\frac{6\sqrt{x^4+5x^2+3}\left(18x^6-141x^4-62x^2-12\right)}{x^6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^7,x]
```

```
[Out] ((6*Sqrt[3 + 5*x^2 + x^4]*(-12 - 62*x^2 - 141*x^4 + 18*x^6))/x^6 + 882*ArcT
anh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])] - 527*Sqrt[3]*ArcTanh[(6 + 5*x^2
)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/72
```

IntegrateAlgebraic [A] time = 0.53, size = 108, normalized size = 0.85

$$-\frac{49}{4}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)+\frac{527\tanh^{-1}\left(\frac{x^2}{\sqrt{3}}-\frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{\sqrt{x^4+5x^2+3}\left(18x^6-141x^4-62x^2-12\right)}{12x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^7,x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-12 - 62*x^2 - 141*x^4 + 18*x^6))/(12*x^6) + (527*A
rcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/(12*Sqrt[3]) - (49*Log
[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/4
```

fricas [A] time = 1.02, size = 122, normalized size = 0.96

$$\frac{527\sqrt{3}x^6\log\left(\frac{25x^2-2\sqrt{3}(5x^2+6)-2\sqrt{x^4+5x^2+3}(5\sqrt{3}-6)+30}{x^2}\right)-882x^6\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)-711x^6+6\left(18x^6-141x^4-62x^2-12\right)\sqrt{x^4+5x^2+3}}{72x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/72*(527*sqrt(3)*x^6*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*
x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 882*x^6*log(-2*x^2 + 2*sqrt(x^4 + 5*x
^2 + 3) - 5) - 711*x^6 + 6*(18*x^6 - 141*x^4 - 62*x^2 - 12)*sqrt(x^4 + 5*x^
2 + 3))/x^6
```

giac [B] time = 0.68, size = 227, normalized size = 1.79

$$\frac{527}{72}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)+\frac{3}{2}\sqrt{x^4+5x^2+3}+\frac{829\left(x^2-\sqrt{x^4+5x^2+3}\right)^5+1824\left(x^2-\sqrt{x^4+5x^2+3}\right)^4-2200\left(x^2-\sqrt{x^4+5x^2+3}\right)^3-5292\left(x^2-\sqrt{x^4+5x^2+3}\right)^2+2799x^2-2799\sqrt{x^4+5x^2+3}+5724}{12\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)^3}-\frac{49}{4}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 527/72*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) -
 sqrt(x^4 + 5*x^2 + 3))) + 3/2*sqrt(x^4 + 5*x^2 + 3) + 1/12*(829*(x^2 - sqr
t(x^4 + 5*x^2 + 3))^5 + 1824*(x^2 - sqrt(x^4 + 5*x^2 + 3))^4 - 2200*(x^2 -
sqrt(x^4 + 5*x^2 + 3))^3 - 5292*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 + 2799*x^2
- 2799*sqrt(x^4 + 5*x^2 + 3) + 5724)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^
3 - 49/4*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.02, size = 117, normalized size = 0.92

$$-\frac{527\sqrt{3}\,\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{72}+\frac{49\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{4}-\frac{47\sqrt{x^4+5x^2+3}}{4x^2}-\frac{31\sqrt{x^4+5x^2+3}}{6x^4}-\frac{\sqrt{x^4+5x^2+3}}{x^6}+\frac{3\sqrt{x^4+5x^2+3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x)
```

```
[Out] 49/4*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))-527/72*arctanh(1/6*(5*x^2+6)*3^(1/2)/(
x^4+5*x^2+3)^(1/2))*3^(1/2)-47/4*(x^4+5*x^2+3)^(1/2)/x^2-(x^4+5*x^2+3)^(1/2
)/x^6-31/6*(x^4+5*x^2+3)^(1/2)/x^4+3/2*(x^4+5*x^2+3)^(1/2)
```

maxima [A] time = 1.58, size = 154, normalized size = 1.21

$$\frac{67}{36}\sqrt{x^4+5x^2+3}\,x^2+\frac{11}{54}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{527}{72}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{431}{36}\sqrt{x^4+5x^2+3}-\frac{79\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{108x^2}-\frac{11\left(x^4+5x^2+3\right)^{\frac{5}{2}}}{54x^4}-\frac{\left(x^4+5x^2+3\right)^{\frac{5}{2}}}{9x^6}+\frac{49}{4}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 67/36*sqrt(x^4 + 5*x^2 + 3)*x^2 + 11/54*(x^4 + 5*x^2 + 3)^(3/2) - 527/72*sq
rt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 431/36*sqrt(x^
4 + 5*x^2 + 3) - 79/108*(x^4 + 5*x^2 + 3)^(3/2)/x^2 - 11/54*(x^4 + 5*x^2 +
3)^(5/2)/x^4 - 1/9*(x^4 + 5*x^2 + 3)^(5/2)/x^6 + 49/4*log(2*x^2 + 2*sqrt(x^
4 + 5*x^2 + 3) + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3\,x^2+2\right)\,{\left(x^4+5\,x^2+3\right)}^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^7,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^{2} + 2\right)\left(x^{4} + 5x^{2} + 3\right)^{\frac{3}{2}}}{x^{7}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**7,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**7, x)
```

3.136 $\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=153

$$\frac{\sqrt{a+bx^2+cx^4}\left(-16aBc-2cx^2(5bB-6Ac)-18Abc+15b^2B\right)}{48c^3}-\frac{\left(8aAc^2-12abBc-6Ab^2c+5b^3B\right)\tanh^{-1}\left(\frac{}{2\sqrt{c}}\right.}{32c^{7/2}}$$

Rubi [A] time = 0.20, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1251, 832, 779, 621, 206}

$$\frac{\sqrt{a+bx^2+cx^4}\left(-16aBc-2cx^2(5bB-6Ac)-18Abc+15b^2B\right)}{48c^3}-\frac{\left(8aAc^2-12abBc-6Ab^2c+5b^3B\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{7/2}}+\frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*x^4*Sqrt[a + b*x^2 + c*x^4])/(6*c) + ((15*b^2*B - 18*A*b*c - 16*a*B*c -
2*c*(5*b*B - 6*A*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(48*c^3) - ((5*b^3*B - 6*
A*b^2*c - 12*a*b*B*c + 8*a*A*c^2)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
 b*x^2 + c*x^4])])/(32*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
```

```
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5\left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)\\ &= \frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c}+\frac{\text{Subst}\left(\int \frac{x\left(-2aB-\frac{1}{2}(5bB-6Ac)x\right)}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{6c}\\ &= \frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c}+\frac{\left(15b^2B-18Abc-16aBc-2c(5bB-6Ac)x^2\right)\sqrt{a+bx^2+cx^4}}{48c^3}\\ &= \frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c}+\frac{\left(15b^2B-18Abc-16aBc-2c(5bB-6Ac)x^2\right)\sqrt{a+bx^2+cx^4}}{48c^3}\\ &= \frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c}+\frac{\left(15b^2B-18Abc-16aBc-2c(5bB-6Ac)x^2\right)\sqrt{a+bx^2+cx^4}}{48c^3}\end{aligned}$$

Mathematica [A] time = 0.11, size = 139, normalized size = 0.91

$$\frac{2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(4c\left(-4aB+3Acx^2+2Bcx^4\right)-2bc\left(9A+5Bx^2\right)+15b^2B\right)-3\left(8aAc^2-12abBc-6Ab^2c+5b^3B\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{96c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(15*b^2*B - 2*b*c*(9*A + 5*B*x^2) + 4*c*
(-4*a*B + 3*A*c*x^2 + 2*B*c*x^4)) - 3*(5*b^3*B - 6*A*b^2*c - 12*a*b*B*c + 8
*a*A*c^2)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(96*c
^(7/2))
```

IntegrateAlgebraic [A] time = 0.54, size = 135, normalized size = 0.88

$$\frac{\sqrt{a+bx^2+cx^4}\left(-16aBc-18Abc+12Ac^2x^2+15b^2B-10bBcx^2+8Bc^2x^4\right)}{48c^3}+\frac{\left(8aAc^2-12abBc-6Ab^2c+5b^3B\right)\log\left(-2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{32c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(15*b^2*B - 18*A*b*c - 16*a*B*c - 10*b*B*c*x^2 + 1
2*A*c^2*x^2 + 8*B*c^2*x^4))/(48*c^3) + ((5*b^3*B - 6*A*b^2*c - 12*a*b*B*c +
 8*a*A*c^2)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(32*c^(7/
2))
```

fricas [A] time = 0.73, size = 315, normalized size = 2.06

$$\left[\frac{3\left(5Bb^3+8Aac^2-6\left(2Bab+Ab^2\right)c\right)\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2+4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c}-4ac\right)+4\left(8Bc^3x^4+15Bb^2c-2(8Ba+9Ab)c^2-2\left(5Bbc^2-6Ac^3\right)x^2\right)\sqrt{cx^4+bx^2+a}}{192c^4},\ \frac{3\left(5Bb^3+8Aac^2-6\left(2Bab+Ab^2\right)c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right)+2\left(8Bc^3x^4+15Bb^2c-2(8Ba+9Ab)c^2-2\left(5Bbc^2-6Ac^3\right)x^2\right)\sqrt{cx^4+bx^2+a}}{96c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(5*B*b^3 + 8*A*a*c^2 - 6*(2*B*a*b + A*b^2)*c)*sqrt(c)*log(-8*c^2*
x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4
```

```
*a*c) + 4*(8*B*c^3*x^4 + 15*B*b^2*c - 2*(8*B*a + 9*A*b)*c^2 - 2*(5*B*b*c^2
- 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^4, 1/96*(3*(5*B*b^3 + 8*A*a*c^2
- 6*(2*B*a*b + A*b^2)*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x
^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(8*B*c^3*x^4 + 15*B*b^2*c -
 2*(8*B*a + 9*A*b)*c^2 - 2*(5*B*b*c^2 - 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2 +
a))/c^4]
```

giac [A] time = 0.54, size = 138, normalized size = 0.90

$$\frac{1}{48}\sqrt{cx^4+bx^2+a}\left(2\left(\frac{4Bx^2}{c}-\frac{5Bbc-6Ac^2}{c^3}\right)x^2+\frac{15Bb^2-16Bac-18Abc}{c^3}\right)+\frac{\left(5Bb^3-12Babc-6Ab^2c+8Aac^2\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{32c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c*x^4 + b*x^2 + a)*(2*(4*B*x^2/c - (5*B*b*c - 6*A*c^2)/c^3)*x^2 +
 (15*B*b^2 - 16*B*a*c - 18*A*b*c)/c^3) + 1/32*(5*B*b^3 - 12*B*a*b*c - 6*A*b
^2*c + 8*A*a*c^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c
) - b))/c^(7/2)
```

maple [B] time = 0.03, size = 286, normalized size = 1.87

$$\frac{\sqrt{cx^4+bx^2+a}\,Bx^4}{6c}+\frac{\sqrt{cx^4+bx^2+a}\,Ax^2}{4c}-\frac{5\sqrt{cx^4+bx^2+a}\,Bbx^2}{24c^2}-\frac{Aa\ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{4c^{\frac{3}{2}}}+\frac{3Ab^2\ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{16c^{\frac{5}{2}}}+\frac{3Bab\ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{8c^{\frac{5}{2}}}-\frac{5Bb^3\ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{32c^{\frac{7}{2}}}-\frac{3\sqrt{cx^4+bx^2+a}\,Ab}{8c^2}-\frac{\sqrt{cx^4+bx^2+a}\,Ba}{3c^2}+\frac{5\sqrt{cx^4+bx^2+a}\,Bb^2}{16c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*B*x^4*(c*x^4+b*x^2+a)^(1/2)/c-5/24*B*b/c^2*x^2*(c*x^4+b*x^2+a)^(1/2)+5/
16*B*b^2/c^3*(c*x^4+b*x^2+a)^(1/2)-5/32*B*b^3/c^(7/2)*ln((1/2*b+c*x^2)/c^(1
/2)+(c*x^4+b*x^2+a)^(1/2))+3/8*B*b/c^(5/2)*a*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^
4+b*x^2+a)^(1/2))-1/3*B*a/c^2*(c*x^4+b*x^2+a)^(1/2)+1/4*A*x^2/c*(c*x^4+b*x^
2+a)^(1/2)-3/8*A*b/c^2*(c*x^4+b*x^2+a)^(1/2)+3/16*A*b^2/c^(5/2)*ln((1/2*b+c
*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/4*A*a/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/
2)+(c*x^4+b*x^2+a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \left(A + Bx^2\right)}{\sqrt{a + bx^2 + cx^4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

3.137 $\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=100

$$\frac{(-4aBc-4Abc+3b^2B)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{\sqrt{a+bx^2+cx^4}\,(-4Ac+3bB-2Bcx^2)}{8c^2}$$

Rubi [A] time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1251, 779, 621, 206}

$$\frac{(-4aBc-4Abc+3b^2B)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{\sqrt{a+bx^2+cx^4}\,(-4Ac+3bB-2Bcx^2)}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] -((3*b*B - 4*A*c - 2*B*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c^2) + ((3*b^2*B
- 4*A*b*c - 4*a*B*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(16*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3 \left(A + Bx^2\right)}{\sqrt{a + bx^2 + cx^4}}\, dx = \frac{1}{2} \text{Subst}\left(\int \frac{x(A + Bx)}{\sqrt{a + bx + cx^2}}\, dx, x, x^2\right)$$
$$= -\frac{\left(3bB - 4Ac - 2Bcx^2\right)\sqrt{a + bx^2 + cx^4}}{8c^2} + \frac{\left(3b^2B - 4Abc - 4aBc\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\right.}{16c^2}$$
$$= -\frac{\left(3bB - 4Ac - 2Bcx^2\right)\sqrt{a + bx^2 + cx^4}}{8c^2} + \frac{\left(3b^2B - 4Abc - 4aBc\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\, dx, x\right.}{8c^2}$$
$$= -\frac{\left(3bB - 4Ac - 2Bcx^2\right)\sqrt{a + bx^2 + cx^4}}{8c^2} + \frac{\left(3b^2B - 4Abc - 4aBc\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+}}\right.}{16c^{5/2}}$$

Mathematica [A] time = 0.05, size = 101, normalized size = 1.01

$$\frac{\left(-4aBc - 4Abc + 3b^2B\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right) + 2\sqrt{c}\sqrt{a + bx^2 + cx^4}\left(4Ac - 3bB + 2Bcx^2\right)}{16c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*(-3*b*B + 4*A*c + 2*B*c*x^2)*Sqrt[a + b*x^2 + c*x^4] + (3*b^2*B
- 4*A*b*c - 4*a*B*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(16*c^(5/2))
```

IntegrateAlgebraic [A] time = 0.47, size = 104, normalized size = 1.04

$$\frac{\left(4aBc + 4Abc - 3b^2B\right)\log\left(-2c^{5/2}\sqrt{a + bx^2 + cx^4} + bc^2 + 2c^3x^2\right)}{16c^{5/2}} + \frac{\sqrt{a + bx^2 + cx^4}\left(4Ac - 3bB + 2Bcx^2\right)}{8c^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((-3*b*B + 4*A*c + 2*B*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c^2) + ((-3*b^2*B
 + 4*A*b*c + 4*a*B*c)*Log[b*c^2 + 2*c^3*x^2 - 2*c^(5/2)*Sqrt[a + b*x^2 + c*
x^4]])/(16*c^(5/2))
```

fricas [A] time = 1.09, size = 233, normalized size = 2.33

$$\left[-\frac{\left(3Bb^2 - 4(Ba + Ab)c\right)\sqrt{c}\log\left(-8c^2x^4 - 8bcx^2 - b^2 + 4\sqrt{cx^4 + bx^2 + a}\left(2cx^2 + b\right)\sqrt{c} - 4ac\right) - 4\left(2Bc^2x^2 - 3Bbc + 4Ac^2\right)\sqrt{cx^4 + bx^2 + a}}{32c^3}, -\frac{\left(3Bb^2 - 4(Ba + Ab)c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right) - 2\left(2Bc^2x^2 - 3Bbc + 4Ac^2\right)\sqrt{cx^4 + bx^2 + a}}{16c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*B*b^2 - 4*(B*a + A*b)*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^
2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*(2*B*c^2*x
^2 - 3*B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^3, -1/16*((3*B*b^2 - 4*(
B*a + A*b)*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqr
t(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*(2*B*c^2*x^2 - 3*B*b*c + 4*A*c^2)*sqrt
(c*x^4 + b*x^2 + a))/c^3]
```

giac [A] time = 0.45, size = 98, normalized size = 0.98

$$\frac{1}{8}\sqrt{cx^4 + bx^2 + a}\left(\frac{2Bx^2}{c} - \frac{3Bb - 4Ac}{c^2}\right) - \frac{\left(3Bb^2 - 4Bac - 4Abc\right)\log\left(\left|-2\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)\sqrt{c} - b\right|\right)}{16c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*B*x^2/c - (3*B*b - 4*A*c)/c^2) - 1/16*(3*B*b
^2 - 4*B*a*c - 4*A*b*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*
sqrt(c) - b))/c^(5/2)
```

maple [B] time = 0.02, size = 176, normalized size = 1.76

$$\frac{\sqrt{c x^4+b x^2+a}\, B x^2}{4c}-\frac{A b \ln\left(\frac{c x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{4 c^{\frac{3}{2}}}-\frac{B a \ln\left(\frac{c x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{4 c^{\frac{3}{2}}}+\frac{3 B b^2 \ln\left(\frac{c x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{16 c^{\frac{5}{2}}}+\frac{\sqrt{c x^4+b x^2+a}\, A}{2c}-\frac{3\sqrt{c x^4+b x^2+a}\, B b}{8 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*B*x^2/c*(c*x^4+b*x^2+a)^(1/2)-3/8*B*b/c^2*(c*x^4+b*x^2+a)^(1/2)+3/16*B*
b^2/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/4*B*a/c^(3/2)
*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/2*A/c*(c*x^4+b*x^2+a)^(1
/2)-1/4*A*b/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\left(B x^2+A\right)}{\sqrt{c x^4+b x^2+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

3.138 $\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}} \, dx$

Optimal. Leaf size=76

$$\frac{B\sqrt{a+bx^2+cx^4}}{2c}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1247, 640, 621, 206}

$$\frac{B\sqrt{a+bx^2+cx^4}}{2c}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*Sqrt[a + b*x^2 + c*x^4])/(2*c) - ((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x\left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$
$$= \frac{B\sqrt{a+bx^2+cx^4}}{2c} + \frac{(-bB+2Ac)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4c}$$
$$= \frac{B\sqrt{a+bx^2+cx^4}}{2c} + \frac{(-bB+2Ac)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2c}$$
$$= \frac{B\sqrt{a+bx^2+cx^4}}{2c} - \frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

Mathematica [A] time = 0.03, size = 78, normalized size = 1.03

$$\frac{1}{2}\left(\frac{(2Ac-bB)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}} + \frac{B\sqrt{a+bx^2+cx^4}}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((B*Sqrt[a + b*x^2 + c*x^4])/c + ((-(b*B) + 2*A*c)*ArcTanh[(b + 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*c^(3/2)))/2
```

IntegrateAlgebraic [A] time = 0.38, size = 78, normalized size = 1.03

$$\frac{(bB-2Ac)\log\left(-2c^{3/2}\sqrt{a+bx^2+cx^4}+bc+2c^2x^2\right)}{4c^{3/2}} + \frac{B\sqrt{a+bx^2+cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*Sqrt[a + b*x^2 + c*x^4])/(2*c) + ((b*B - 2*A*c)*Log[b*c + 2*c^2*x^2 - 2*
c^(3/2)*Sqrt[a + b*x^2 + c*x^4]])/(4*c^(3/2))
```

fricas [A] time = 1.30, size = 178, normalized size = 2.34

$$\left[\frac{4\sqrt{cx^4+bx^2+a}\,Bc-(Bb-2\,Ac)\sqrt{c}\,\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)}{8\,c^2}, \frac{2\,\sqrt{cx^4+bx^2+a}\,Bc+(Bb-2\,Ac)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right)}{4\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(c*x^4 + b*x^2 + a)*B*c - (B*b - 2*A*c)*sqrt(c)*log(-8*c^2*x^4
- 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c
))/c^2, 1/4*(2*sqrt(c*x^4 + b*x^2 + a)*B*c + (B*b - 2*A*c)*sqrt(-c)*arctan(
1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c
)))/c^2]
```

giac [A] time = 0.46, size = 69, normalized size = 0.91

$$\frac{\sqrt{cx^4+bx^2+a}\,B}{2\,c} + \frac{(Bb-2\,Ac)\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{4\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^4 + b*x^2 + a)*B/c + 1/4*(B*b - 2*A*c)*log(abs(-2*(sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3/2)
```

maple [A] time = 0.01, size = 93, normalized size = 1.22

$$\frac{A\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{2\sqrt{c}}-\frac{Bb\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{4c^{\frac{3}{2}}}+\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\;B}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*B*(c*x^4+b*x^2+a)^(1/2)/c-1/4*B*b/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x
^4+b*x^2+a)^(1/2))+1/2*A*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^
(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 1.05, size = 92, normalized size = 1.21

$$\frac{A\,\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{2\,\sqrt{c}}+\frac{B\,\sqrt{c\,x^4+b\,x^2+a}}{2\,c}-\frac{B\,b\,\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{4\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (A*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2)))/(2*c^(1/2)) + (B
*(a + b*x^2 + c*x^4)^(1/2))/(2*c) - (B*b*log((a + b*x^2 + c*x^4)^(1/2) + (b
/2 + c*x^2)/c^(1/2)))/(4*c^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\left(A+Bx^{2}\right)}{\sqrt{a+bx^{2}+cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

3.139 $\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=90

$$\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}-\frac{A\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Rubi [A] time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1251, 843, 621, 206, 724}

$$\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}-\frac{A\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(A*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a])
 + (B*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[c
])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}A\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right) + \frac{1}{2}B\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= -\left(A\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)\right) + B\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b \cdots}{\sqrt{a+\cdots}}\right.\\
&= -\frac{A\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}} + \frac{B\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 89, normalized size = 0.99

$$
\frac{1}{2}\left(\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}} - \frac{A\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-((A*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[a])
+ (B*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[c])/2
```

IntegrateAlgebraic [A] time = 0.33, size = 89, normalized size = 0.99

$$
\frac{A\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}} - \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{B\log\left(-2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{2\sqrt{c}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (A*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a] - Sqrt[a + b*x^2 + c*x^4]/Sqrt[a]])/Sqrt[a
] - (B*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(2*Sqrt[c])
```

fricas [A] time = 1.25, size = 517, normalized size = 5.74

$$
\left[\frac{Ba\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2-4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c}-4ac\right)+A\sqrt{a}c\log\left(-\frac{\left(b^2+4ac\right)x^4+8abx^2-4\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{a}+8a^2}{x^4}\right)}{4ac}, -\frac{2Ba\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right)-A\sqrt{a}c\log\left(-\frac{\left(b^2+4ac\right)x^4+8abx^2-4\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{a}+8a^2}{x^4}\right)}{4ac}, \frac{2A\sqrt{-a}c\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)+Ba\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2-4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c}-4ac\right)}{4ac}, \frac{A\sqrt{-a}c\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)-Ba\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right)}{2ac}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(B*a*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 +
 a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + A*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^4 +
8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4))/
(a*c), -1/4*(2*B*a*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b
)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - A*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^4
 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4
))/(a*c), 1/4*(2*A*sqrt(-a)*c*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2
*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + B*a*sqrt(c)*log(-8*c^2*x^4 - 8*b*
c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c))/(a*
c), 1/2*(A*sqrt(-a)*c*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt
(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - B*a*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^
2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)))/(a*c)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Value
```

maple [A] time = 0.01, size = 76, normalized size = 0.84

$$-\frac{A \ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{2\sqrt{a}}+\frac{B \ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*B*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-1/2*A/a^(1/2)
*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 0.76, size = 81, normalized size = 0.90

$$\frac{B\,\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{2\,\sqrt{c}}-\frac{A\,\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a}}-\frac{A\,\ln\left(2\,a+2\,\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}+b\,x^2\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] (B*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2)))/(2*c^(1/2)) - (A
*log(1/x^2))/(2*a^(1/2)) - (A*log(2*a + 2*a^(1/2)*(a + b*x^2 + c*x^4)^(1/2)
 + b*x^2))/(2*a^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x*sqrt(a + b*x**2 + c*x**4)), x)
```

3.140 $\int \frac{A+Bx^2}{x^3 \sqrt{a+bx^2+cx^4}} \, dx$

Optimal. Leaf size=80

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}-\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2}$$

Rubi [A] time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1251, 806, 724, 206}

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}-\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(2*a*x^2) + ((A*b - 2*a*B)*ArcTanh[(2*a + b*x^
2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2} - \frac{(Ab-2aB)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4a}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2} + \frac{(Ab-2aB)\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2a}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2} + \frac{(Ab-2aB)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}$$

Mathematica [A] time = 0.04, size = 82, normalized size = 1.02

$$\frac{1}{2}\left(\frac{(Ab-2aB)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}} - \frac{A\sqrt{a+bx^2+cx^4}}{ax^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-((A*Sqrt[a + b*x^2 + c*x^4])/(a*x^2)) + ((A*b - 2*a*B)*ArcTanh[(2*a + b*x
^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*a^(3/2)))/2
```

IntegrateAlgebraic [A] time = 0.39, size = 81, normalized size = 1.01

$$\frac{(2aB-Ab)\tanh^{-1}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{2a^{3/2}} - \frac{A\sqrt{a+bx^2+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*(A*Sqrt[a + b*x^2 + c*x^4])/(a*x^2) + ((-(A*b) + 2*a*B)*ArcTanh[(Sqrt[
c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(2*a^(3/2))
```

fricas [A] time = 1.33, size = 197, normalized size = 2.46

$$\left[-\frac{(2Ba-Ab)\sqrt{a}x^2\log\left(-\frac{(b^2+4ac)x^4+8abx^2+4\sqrt{cx^4+bx^2+a}(bx^2+2a)\sqrt{a}+8a^2}{x^4}\right)+4\sqrt{cx^4+bx^2+a}Aa}{8a^2x^2}, \frac{(2Ba-Ab)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{cx^4+bx^2+a}(bx^2+2a)\sqrt{-a}}{2(acx^4+abx^2+a^2)}\right)-2\sqrt{cx^4+bx^2+a}Aa}{4a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((2*B*a - A*b)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sq
rt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*sqrt(c*x^4 +
b*x^2 + a)*A*a)/(a^2*x^2), 1/4*((2*B*a - A*b)*sqrt(-a)*x^2*arctan(1/2*sqrt(
c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*sq
rt(c*x^4 + b*x^2 + a)*A*a)/(a^2*x^2)]
```

giac [A] time = 0.54, size = 124, normalized size = 1.55

$$\frac{(2Ba-Ab)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}a} + \frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)Ab+2Aa\sqrt{c}}{2\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*B*a - A*b)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))
/(sqrt(-a)*a) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*A*b + 2*A*a*sq
rt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*a)
```

maple [A] time = 0.02, size = 104, normalized size = 1.30

$$\frac{Ab\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{4a^{\frac{3}{2}}}-\frac{B\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{2\sqrt{a}}-\frac{\sqrt{c\,x^4+b\,x^2+a}\,A}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*B/a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-1/2*A*(c
*x^4+b*x^2+a)^(1/2)/a/x^2+1/4*A*b/a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(
1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 0.78, size = 103, normalized size = 1.29

$$\frac{A\,b\,\mathrm{atanh}\left(\frac{\frac{b\,x^2}{2}+a}{\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}\right)}{4\,a^{3/2}}-\frac{B\,\ln\left(2\,a+2\,\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}\,+b\,x^2\right)}{2\,\sqrt{a}}-\frac{A\,\sqrt{c\,x^4+b\,x^2+a}}{2\,a\,x^2}-\frac{B\,\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] (A*b*atanh((a + (b*x^2)/2)/(a^(1/2)*(a + b*x^2 + c*x^4)^(1/2))))/(4*a^(3/2)
) - (B*log(2*a + 2*a^(1/2)*(a + b*x^2 + c*x^4)^(1/2) + b*x^2))/(2*a^(1/2))
- (A*(a + b*x^2 + c*x^4)^(1/2))/(2*a*x^2) - (B*log(1/x^2))/(2*a^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**3*sqrt(a + b*x**2 + c*x**4)), x)
```

3.141 $\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=124

$$-\frac{(-4aAc-4abB+3Ab^2)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4}$$

Rubi [A] time = 0.15, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1251, 834, 806, 724, 206}

$$-\frac{(-4aAc-4abB+3Ab^2)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(4*a*x^4) + ((3*A*b - 4*a*B)*Sqrt[a + b*x^2 +
c*x^4])/(8*a^2*x^2) - ((3*A*b^2 - 4*a*b*B - 4*a*A*c)*ArcTanh[(2*a + b*x^2)/
(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(3Ab-4aB)+Acx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4a}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4} + \frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2} + \frac{\left(3Ab^2-4abB-4aAc\right)\text{Subs}}{16a^{\cdot}}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4} + \frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2} - \frac{\left(3Ab^2-4abB-4aAc\right)\text{Subs}}{8a}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4} + \frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2} - \frac{\left(3Ab^2-4abB-4aAc\right)\tanh}{16a^{5/2}}$$

Mathematica [A] time = 0.08, size = 107, normalized size = 0.86

$$\frac{\left(4aAc+4abB-3Ab^2\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}} + \frac{\sqrt{a+bx^2+cx^4}\left(3Abx^2-2a\left(A+2Bx^2\right)\right)}{8a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(3*A*b*x^2 - 2*a*(A + 2*B*x^2)))/(8*a^2*x^4) + ((-
3*A*b^2 + 4*a*b*B + 4*a*A*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^
2 + c*x^4])])/(16*a^(5/2))
```

IntegrateAlgebraic [A] time = 0.64, size = 148, normalized size = 1.19

$$\frac{3Ab^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}-\frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{8a^{5/2}} + \frac{(Ac+bB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2+cx^4}-\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{\sqrt{a+bx^2+cx^4}\left(-2aA-4aBx^2+3Abx^2\right)}{8a^2x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-2*a*A + 3*A*b*x^2 - 4*a*B*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*a^2*x^4) + ((
b*B + A*c)*ArcTanh[(-(Sqrt[c]*x^2) + Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(2*
a^(3/2)) + (3*A*b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a] - Sqrt[a + b*x^2 + c*x^4]
/Sqrt[a]])/(8*a^(5/2))
```

fricas [A] time = 1.59, size = 255, normalized size = 2.06

$$\left[\frac{\left(4\,Bab-3\,Ab^2+4\,Aac\right)\sqrt{a}\,x^4\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)-4\,\sqrt{cx^4+bx^2+a}\left(2\,Aa^2+\left(4\,Ba^2-3\,Aab\right)x^2\right)}{32\,a^3x^4}, -\frac{\left(4\,Bab-3\,Ab^2+4\,Aac\right)\sqrt{-a}\,x^4\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)+2\,\sqrt{cx^4+bx^2+a}\left(2\,Aa^2+\left(4\,Ba^2-3\,Aab\right)x^2\right)}{16\,a^3x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((4*B*a*b - 3*A*b^2 + 4*A*a*c)*sqrt(a)*x^4*log(-((b^2 + 4*a*c)*x^4 +
8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) -
 4*sqrt(c*x^4 + b*x^2 + a)*(2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x^2))/(a^3*x^4),
-1/16*((4*B*a*b - 3*A*b^2 + 4*A*a*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b
*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*sqrt(c*x^4
+ b*x^2 + a)*(2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x^2))/(a^3*x^4)]
```

giac [B] time = 0.52, size = 339, normalized size = 2.73

$$-\frac{\left(4\,Bab-3\,Ab^2+4\,Aac\right)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}\,a^2}+\frac{4\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3Bab-3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3Ab^2+4\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3Aac+8\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2Ba^2\sqrt{c}-4\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)Ba^2b+5\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)Aab^2+4\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)Aa^2c-8\,Ba^3\sqrt{c}+8\,Aa^2b\sqrt{c}}{8\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(4*B*a*b - 3*A*b^2 + 4*A*a*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^
2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + 1/8*(4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
 + a))^3*B*a*b - 3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*A*b^2 + 4*(sqr
t(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*A*a*c + 8*(sqrt(c)*x^2 - sqrt(c*x^4 +
 b*x^2 + a))^2*B*a^2*sqrt(c) - 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*B*
a^2*b + 5*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*A*a*b^2 + 4*(sqrt(c)*x^2
- sqrt(c*x^4 + b*x^2 + a))*A*a^2*c - 8*B*a^3*sqrt(c) + 8*A*a^2*b*sqrt(c))/(
((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^2*a^2)
```

maple [A] time = 0.02, size = 194, normalized size = 1.56

$$\frac{Ac\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{4a^{\frac{3}{2}}}-\frac{3A\,b^2\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{16a^{\frac{5}{2}}}+\frac{Bb\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{4a^{\frac{3}{2}}}+\frac{3\sqrt{c\,x^4+b\,x^2+a}\,Ab}{8a^2x^2}-\frac{\sqrt{c\,x^4+b\,x^2+a}\,B}{2a\,x^2}-\frac{\sqrt{c\,x^4+b\,x^2+a}\,A}{4a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/4*A*(c*x^4+b*x^2+a)^(1/2)/a/x^4+3/8*A*b/a^2/x^2*(c*x^4+b*x^2+a)^(1/2)-3/
16*A*b^2/a^(5/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)+1/4*A*
c/a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-1/2*B/a/x^2*(
c*x^4+b*x^2+a)^(1/2)+1/4*B*b/a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*
a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,x^2+A}{x^5\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^5*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((A + B*x^2)/(x^5*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^5\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**5*sqrt(a + b*x**2 + c*x**4)), x)
```

3.142 $\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=177

$$-\frac{\sqrt{a+bx^2+cx^4}\left(-16aAc-18abB+15Ab^2\right)}{48a^3x^2}+\frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4}+\frac{\left(8a^2Bc-12aAbc-6ab^2B+5Ab^3\right)}{32a^{7/2}}$$

Rubi [A] time = 0.24, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1251, 834, 806, 724, 206}

$$-\frac{\sqrt{a+bx^2+cx^4}\left(-16aAc-18abB+15Ab^2\right)}{48a^3x^2}+\frac{\left(8a^2Bc-12aAbc-6ab^2B+5Ab^3\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}+\frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4}-\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(6*a*x^6) + ((5*A*b - 6*a*B)*Sqrt[a + b*x^2 +
c*x^4])/(24*a^2*x^4) - ((15*A*b^2 - 18*a*b*B - 16*a*A*c)*Sqrt[a + b*x^2 + c
*x^4])/(48*a^3*x^2) + ((5*A*b^3 - 6*a*b^2*B - 12*a*A*b*c + 8*a^2*B*c)*ArcTa
nh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(32*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^4\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(5Ab-6aB)+2Acx}{x^3\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{6a}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} + \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{4}\left(15Ab^2-18abB-16aAc\right.}{x^2\sqrt{a+bx+}}\right.}{12a^2}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{\left(15Ab^2-18abB-16aAc\right)\sqrt{}}{48a^3x^2}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{\left(15Ab^2-18abB-16aAc\right)\sqrt{}}{48a^3x^2}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{\left(15Ab^2-18abB-16aAc\right)\sqrt{}}{48a^3x^2}$$

Mathematica [A] time = 0.11, size = 148, normalized size = 0.84

$$\frac{\left(8a^2Bc-12aAbc-6ab^2B+5Ab^3\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}+\frac{\sqrt{a+bx^2+cx^4}\left(-4a^2\left(2A+3Bx^2\right)+2a\left(5Abx^2+8Acx^4+9bBx^4\right)-15Ab^2x^4\right)}{48a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-15*A*b^2*x^4 - 4*a^2*(2*A + 3*B*x^2) + 2*a*(5*A*
b*x^2 + 9*b*B*x^4 + 8*A*c*x^4)))/(48*a^3*x^6) + ((5*A*b^3 - 6*a*b^2*B - 12*
a*A*b*c + 8*a^2*B*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^
4])])/(32*a^(7/2))
```

IntegrateAlgebraic [A] time = 0.91, size = 188, normalized size = 1.06

$$-\frac{3\left(2Abc+b^2B\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2+cx^4}-\sqrt{c}x^2}{\sqrt{a}}\right)}{8a^{5/2}}+\frac{\left(-8a^2Bc-5Ab^3\right)\tanh^{-1}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{16a^{7/2}}+\frac{\sqrt{a+bx^2+cx^4}\left(-8a^2A-12a^2Bx^2+10aAbx^2+16aAcx^4+18abBx^4-15Ab^2x^4\right)}{48a^3x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-8*a^2*A + 10*a*A*b*x^2 - 12*a^2*B*x^2 - 15*A*b^2
*x^4 + 18*a*b*B*x^4 + 16*a*A*c*x^4))/(48*a^3*x^6) + ((-5*A*b^3 - 8*a^2*B*c)
*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(16*a^(7/2)) - (
3*(b^2*B + 2*A*b*c)*ArcTanh[(-(Sqrt[c]*x^2) + Sqrt[a + b*x^2 + c*x^4])/Sqrt
[a]])/(8*a^(5/2))
```

fricas [A] time = 1.88, size = 339, normalized size = 1.92

$$\left[\frac{3\left(6Bab^2-5Ab^3-4\left(2Ba^2-3Aab\right)c\right)\sqrt{a}x^6\log\left(-\frac{\left(b^2+4ac\right)x^4+8abx^2-4\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{a}+8a^2}{x^4}\right)+4\left(\left(18Ba^2b-15Aab^2+16Aa^2c\right)x^4-8Aa^3-2\left(6Ba^3-5Aa^2b\right)x^2\right)\sqrt{cx^4+bx^2+a}}{192a^4x^6}, \frac{3\left(6Bab^2-5Ab^3-4\left(2Ba^2-3Aab\right)c\right)\sqrt{-a}x^6\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)+2\left(\left(18Ba^2b-15Aab^2+16Aa^2c\right)x^4-8Aa^3-2\left(6Ba^3-5Aa^2b\right)x^2\right)\sqrt{cx^4+bx^2+a}}{96a^4x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(6*B*a*b^2 - 5*A*b^3 - 4*(2*B*a^2 - 3*A*a*b)*c)*sqrt(a)*x^6*log(-
((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sq
rt(a) + 8*a^2)/x^4) + 4*((18*B*a^2*b - 15*A*a*b^2 + 16*A*a^2*c)*x^4 - 8*A*a
^3 - 2*(6*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^6), 1/96*
(3*(6*B*a*b^2 - 5*A*b^3 - 4*(2*B*a^2 - 3*A*a*b)*c)*sqrt(-a)*x^6*arctan(1/2*
sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) +
 2*((18*B*a^2*b - 15*A*a*b^2 + 16*A*a^2*c)*x^4 - 8*A*a^3 - 2*(6*B*a^3 - 5*A
*a^2*b)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^6)]
```

giac [B] time = 0.64, size = 571, normalized size = 3.23

$$\frac{\left(6 Bab^2 - 5 Ab^3 - 8 Ba^2c + 12 Aabc\right)\arctan\left(-\frac{\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{16\sqrt{-a}a^3} - \frac{18\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right)^5 Bab^2 - 15\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right)^5 Ab^3 - 24\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right)^5 Ba^2c + 36\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right)^5 Aabc - 48\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right)^3 Ba^2b^2 + 40\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right)^3 Aab^3 - 96\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right)^3 Aa^2bc - 48\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right)^2 Ba^3b\sqrt{c} - 96\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right)^2 Aa^3c^{\frac{3}{2}} + 30\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right) Ba^3b^2 - 33\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right) Aa^2b^3 + 24\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right) Ba^4c - 36\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right) Aa^3bc + 48 Ba^4b\sqrt{c} - 48 Aa^3b^2\sqrt{c} + 32 Aa^4c^{\frac{3}{2}}}{48\left(\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right)^2 - a\right)^3 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(6*B*a*b^2 - 5*A*b^3 - 8*B*a^2*c + 12*A*a*b*c)*arctan(-(sqrt(c)*x^2 -
sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^3) - 1/48*(18*(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2 + a))^5*B*a*b^2 - 15*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))^5*A*b^3 - 24*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*B*a^2*c + 36*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*A*a*b*c - 48*(sqrt(c)*x^2 - sqrt(c
*x^4 + b*x^2 + a))^3*B*a^2*b^2 + 40*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))
^3*A*a*b^3 - 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*A*a^2*b*c - 48*(s
qrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*B*a^3*b*sqrt(c) - 96*(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2 + a))^2*A*a^3*c^(3/2) + 30*(sqrt(c)*x^2 - sqrt(c*x^4 +
b*x^2 + a))*B*a^3*b^2 - 33*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*A*a^2*b^
3 + 24*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*B*a^4*c - 36*(sqrt(c)*x^2 -
sqrt(c*x^4 + b*x^2 + a))*A*a^3*b*c + 48*B*a^4*b*sqrt(c) - 48*A*a^3*b^2*sqrt
(c) + 32*A*a^4*c^(3/2))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^3*
a^3)
```

maple [A] time = 0.02, size = 311, normalized size = 1.76

$$-\frac{3Abc\ln\left(\frac{bx^2+2a+2\sqrt{cx^4+bx^2+a}\,\sqrt{a}}{x^2}\right)}{8a^{\frac{5}{2}}} + \frac{5Ab^3\ln\left(\frac{bx^2+2a+2\sqrt{cx^4+bx^2+a}\,\sqrt{a}}{x^2}\right)}{32a^{\frac{7}{2}}} + \frac{Bc\ln\left(\frac{bx^2+2a+2\sqrt{cx^4+bx^2+a}\,\sqrt{a}}{x^2}\right)}{4a^{\frac{3}{2}}} - \frac{3Bb^2\ln\left(\frac{bx^2+2a+2\sqrt{cx^4+bx^2+a}\,\sqrt{a}}{x^2}\right)}{16a^{\frac{5}{2}}} + \frac{\sqrt{cx^4+bx^2+a}\,Ac}{3a^2x^2} - \frac{5\sqrt{cx^4+bx^2+a}\,Ab^2}{16a^3x^2} + \frac{3\sqrt{cx^4+bx^2+a}\,Bb}{8a^2x^2} + \frac{5\sqrt{cx^4+bx^2+a}\,Ab}{24a^2x^4} - \frac{\sqrt{cx^4+bx^2+a}\,B}{4ax^4} - \frac{\sqrt{cx^4+bx^2+a}\,A}{6ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/6*A*(c*x^4+b*x^2+a)^(1/2)/a/x^6+5/24*A*b/a^2/x^4*(c*x^4+b*x^2+a)^(1/2)-5
/16*A*b^2/a^3/x^2*(c*x^4+b*x^2+a)^(1/2)+5/32*A*b^3/a^(7/2)*ln((b*x^2+2*a+2*
(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-3/8*A*b/a^(5/2)*c*ln((b*x^2+2*a+2*(c*x^
4+b*x^2+a)^(1/2)*a^(1/2))/x^2)+1/3*A/a^2*c/x^2*(c*x^4+b*x^2+a)^(1/2)-1/4*B/
a/x^4*(c*x^4+b*x^2+a)^(1/2)+3/8*B*b/a^2/x^2*(c*x^4+b*x^2+a)^(1/2)-3/16*B*b^
2/a^(5/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)+1/4*B*c/a^(3/
2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,x^2+A}{x^7\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^7*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int((A + B*x^2)/(x^7*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**7/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((A + B*x**2)/(x**7*sqrt(a + b*x**2 + c*x**4)), x)
```

3.143 $\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=98

$$-\frac{89}{48}\sqrt{x^4+5x^2+3}\,x^4-\frac{1}{384}\left(24243-3802x^2\right)\sqrt{x^4+5x^2+3}+\frac{32801}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)+\frac{3}{8}\sqrt{x^4+5x^2+$$

Rubi [A] time = 0.09, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1251, 832, 779, 621, 206}

$$\frac{3}{8}\sqrt{x^4+5x^2+3}\,x^6-\frac{89}{48}\sqrt{x^4+5x^2+3}\,x^4-\frac{1}{384}\left(24243-3802x^2\right)\sqrt{x^4+5x^2+3}+\frac{32801}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-89*x^4*Sqrt[3 + 5*x^2 + x^4])/48 + (3*x^6*Sqrt[3 + 5*x^2 + x^4])/8 - ((24
243 - 3802*x^2)*Sqrt[3 + 5*x^2 + x^4])/384 + (32801*ArcTanh[(5 + 2*x^2)/(2*
Sqrt[3 + 5*x^2 + x^4])])/256
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
```

```
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7 \left(2+3 x^2\right)}{\sqrt{3+5 x^2+x^4}} \, dx = \frac{1}{2} \text{Subst}\left(\int \frac{x^3 (2+3 x)}{\sqrt{3+5 x+x^2}} \, dx,x,x^2\right)$$
$$= \frac{3}{8} x^6 \sqrt{3+5 x^2+x^4}+\frac{1}{8} \text{Subst}\left(\int \frac{\left(-27-\frac{89 x}{2}\right) x^2}{\sqrt{3+5 x+x^2}} \, dx,x,x^2\right)$$
$$= -\frac{89}{48} x^4 \sqrt{3+5 x^2+x^4}+\frac{3}{8} x^6 \sqrt{3+5 x^2+x^4}+\frac{1}{24} \text{Subst}\left(\int \frac{x \left(267+\frac{1901 x}{4}\right)}{\sqrt{3+5 x+x^2}} \, dx,x,x^2\right)$$
$$= -\frac{89}{48} x^4 \sqrt{3+5 x^2+x^4}+\frac{3}{8} x^6 \sqrt{3+5 x^2+x^4}-\frac{1}{384} \left(24243-3802 x^2\right) \sqrt{3+5 x^2+x^4}+$$
$$= -\frac{89}{48} x^4 \sqrt{3+5 x^2+x^4}+\frac{3}{8} x^6 \sqrt{3+5 x^2+x^4}-\frac{1}{384} \left(24243-3802 x^2\right) \sqrt{3+5 x^2+x^4}+$$
$$= -\frac{89}{48} x^4 \sqrt{3+5 x^2+x^4}+\frac{3}{8} x^6 \sqrt{3+5 x^2+x^4}-\frac{1}{384} \left(24243-3802 x^2\right) \sqrt{3+5 x^2+x^4}+$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.67

$$\frac{1}{768} \left(98403 \tanh^{-1}\left(\frac{2 x^2+5}{2 \sqrt{x^4+5 x^2+3}}\right)+2 \sqrt{x^4+5 x^2+3} \left(144 x^6-712 x^4+3802 x^2-24243\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(-24243 + 3802*x^2 - 712*x^4 + 144*x^6) + 98403*Ar
cTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/768
```

IntegrateAlgebraic [A] time = 0.20, size = 64, normalized size = 0.65

$$\frac{1}{384} \sqrt{x^4+5 x^2+3} \left(144 x^6-712 x^4+3802 x^2-24243\right)-\frac{32801}{256} \log \left(-2 x^2+2 \sqrt{x^4+5 x^2+3}-5\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^7*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-24243 + 3802*x^2 - 712*x^4 + 144*x^6))/384 - (3280
1*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/256
```

fricas [A] time = 0.62, size = 56, normalized size = 0.57

$$\frac{1}{384} \left(144 x^6-712 x^4+3802 x^2-24243\right) \sqrt{x^4+5 x^2+3}-\frac{32801}{256} \log \left(-2 x^2+2 \sqrt{x^4+5 x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
[Out] 1/384*(144*x^6 - 712*x^4 + 3802*x^2 - 24243)*sqrt(x^4 + 5*x^2 + 3) - 32801/
256*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

giac [A] time = 0.37, size = 60, normalized size = 0.61

$$\frac{1}{384}\sqrt{x^4+5x^2+3}\left(2\left(4\left(18x^2-89\right)x^2+1901\right)x^2-24243\right)-\frac{32801}{256}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(18*x^2 - 89)*x^2 + 1901)*x^2 - 24243) -
32801/256*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.02, size = 87, normalized size = 0.89

$$\frac{3\sqrt{x^4+5x^2+3}\,x^6}{8}-\frac{89\sqrt{x^4+5x^2+3}\,x^4}{48}+\frac{1901\sqrt{x^4+5x^2+3}\,x^2}{192}+\frac{32801\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{256}-\frac{8081\sqrt{x^4+5x^2+3}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 3/8*(x^4+5*x^2+3)^(1/2)*x^6-89/48*(x^4+5*x^2+3)^(1/2)*x^4+1901/192*(x^4+5*x
^2+3)^(1/2)*x^2-8081/128*(x^4+5*x^2+3)^(1/2)+32801/256*ln(x^2+5/2+(x^4+5*x^
2+3)^(1/2))
```

maxima [A] time = 0.93, size = 90, normalized size = 0.92

$$\frac{3}{8}\sqrt{x^4+5x^2+3}\,x^6-\frac{89}{48}\sqrt{x^4+5x^2+3}\,x^4+\frac{1901}{192}\sqrt{x^4+5x^2+3}\,x^2-\frac{8081}{128}\sqrt{x^4+5x^2+3}+\frac{32801}{256}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(x^4 + 5*x^2 + 3)*x^6 - 89/48*sqrt(x^4 + 5*x^2 + 3)*x^4 + 1901/192*
sqrt(x^4 + 5*x^2 + 3)*x^2 - 8081/128*sqrt(x^4 + 5*x^2 + 3) + 32801/256*log(
2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7\left(3x^2+2\right)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] int((x^7*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7\left(3x^2+2\right)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**7*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

3.144 $\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=77

$$\frac{1}{2}\sqrt{x^4+5x^2+3}\,x^4+\frac{3}{16}\left(89-14x^2\right)\sqrt{x^4+5x^2+3}-\frac{1083}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1251, 832, 779, 621, 206}

$$\frac{1}{2}\sqrt{x^4+5x^2+3}\,x^4+\frac{3}{16}\left(89-14x^2\right)\sqrt{x^4+5x^2+3}-\frac{1083}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (x^4*Sqrt[3 + 5*x^2 + x^4])/2 + (3*(89 - 14*x^2)*Sqrt[3 + 5*x^2 + x^4])/16
- (1083*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^5\left(2+3x^2\right)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2(2+3x)}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)$$
$$= \frac{1}{2}x^4\sqrt{3+5x^2+x^4} + \frac{1}{6}\,\text{Subst}\left(\int \frac{\left(-18-\frac{63x}{2}\right)x}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)$$
$$= \frac{1}{2}x^4\sqrt{3+5x^2+x^4} + \frac{3}{16}\left(89-14x^2\right)\sqrt{3+5x^2+x^4} - \frac{1083}{32}\,\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx,\right.$$
$$= \frac{1}{2}x^4\sqrt{3+5x^2+x^4} + \frac{3}{16}\left(89-14x^2\right)\sqrt{3+5x^2+x^4} - \frac{1083}{16}\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{5}{\sqrt{3+}}\right.$$
$$= \frac{1}{2}x^4\sqrt{3+5x^2+x^4} + \frac{3}{16}\left(89-14x^2\right)\sqrt{3+5x^2+x^4} - \frac{1083}{32}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.79

$$\frac{1}{32}\left(2\sqrt{x^4+5x^2+3}\left(8x^4-42x^2+267\right)-1083\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(267 - 42*x^2 + 8*x^4) - 1083*ArcTanh[(5 + 2*x^2)/
(2*Sqrt[3 + 5*x^2 + x^4])])/32
```

IntegrateAlgebraic [A] time = 0.18, size = 59, normalized size = 0.77

$$\frac{1}{16}\sqrt{x^4+5x^2+3}\left(8x^4-42x^2+267\right)+\frac{1083}{32}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(267 - 42*x^2 + 8*x^4))/16 + (1083*Log[-5 - 2*x^2 +
2*Sqrt[3 + 5*x^2 + x^4]])/32
```

fricas [A] time = 0.97, size = 51, normalized size = 0.66

$$\frac{1}{16}\left(8\,x^4-42\,x^2+267\right)\sqrt{x^4+5\,x^2+3}+\frac{1083}{32}\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(8*x^4 - 42*x^2 + 267)*sqrt(x^4 + 5*x^2 + 3) + 1083/32*log(-2*x^2 + 2*
sqrt(x^4 + 5*x^2 + 3) - 5)
```

giac [A] time = 0.34, size = 53, normalized size = 0.69

$$\frac{1}{16}\sqrt{x^4+5\,x^2+3}\left(2\left(4\,x^2-21\right)x^2+267\right)+\frac{1083}{32}\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 - 21)*x^2 + 267) + 1083/32*log(2*x^2 -
 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.01, size = 70, normalized size = 0.91

$$\frac{\sqrt{x^4+5x^2+3}\,x^4}{2}-\frac{21\sqrt{x^4+5x^2+3}\,x^2}{8}-\frac{1083\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{32}+\frac{267\sqrt{x^4+5x^2+3}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 1/2*(x^4+5*x^2+3)^(1/2)*x^4-21/8*(x^4+5*x^2+3)^(1/2)*x^2+267/16*(x^4+5*x^2+
3)^(1/2)-1083/32*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))
```

maxima [A] time = 0.99, size = 73, normalized size = 0.95

$$\frac{1}{2}\sqrt{x^4+5\,x^2+3}\,x^4-\frac{21}{8}\sqrt{x^4+5\,x^2+3}\,x^2+\frac{267}{16}\sqrt{x^4+5\,x^2+3}-\frac{1083}{32}\log\left(2\,x^2+2\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^4 + 5*x^2 + 3)*x^4 - 21/8*sqrt(x^4 + 5*x^2 + 3)*x^2 + 267/16*sqr
t(x^4 + 5*x^2 + 3) - 1083/32*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\left(3\,x^2+2\right)}{\sqrt{x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] int((x^5*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(3x^2+2\right)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**5*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

3.145 $\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=56

$$\frac{149}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{1}{8}\left(37-6x^2\right)\sqrt{x^4+5x^2+3}$$

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1251, 779, 621, 206}

$$\frac{149}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{1}{8}\left(37-6x^2\right)\sqrt{x^4+5x^2+3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] -((37 - 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 + (149*ArcTanh[(5 + 2*x^2)/(2*Sqrt[
3 + 5*x^2 + x^4])])/16
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(2+3x^2\right)}{\sqrt{3+5x^2+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x(2+3x)}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\ &= -\frac{1}{8}\left(37-6x^2\right)\sqrt{3+5x^2+x^4}+\frac{149}{16}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\ &= -\frac{1}{8}\left(37-6x^2\right)\sqrt{3+5x^2+x^4}+\frac{149}{8}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)\\ &= -\frac{1}{8}\left(37-6x^2\right)\sqrt{3+5x^2+x^4}+\frac{149}{16}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 56, normalized size = 1.00

$$\frac{1}{16}\left(2\sqrt{x^4+5x^2+3}\left(6x^2-37\right)+149\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*(-37 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4] + 149*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3
+ 5*x^2 + x^4])])/16
```

IntegrateAlgebraic [A] time = 0.18, size = 54, normalized size = 0.96

$$\frac{1}{8}\left(6x^2-37\right)\sqrt{x^4+5x^2+3}-\frac{149}{16}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] ((-37 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 - (149*Log[-5 - 2*x^2 + 2*Sqrt[3 +
5*x^2 + x^4]])/16
```

fricas [A] time = 0.98, size = 46, normalized size = 0.82

$$\frac{1}{8}\sqrt{x^4+5\,x^2+3}\left(6\,x^2-37\right)-\frac{149}{16}\,\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 - 37) - 149/16*log(-2*x^2 + 2*sqrt(x^4 + 5
*x^2 + 3) - 5)
```

giac [A] time = 0.35, size = 46, normalized size = 0.82

$$\frac{1}{8}\sqrt{x^4+5\,x^2+3}\left(6\,x^2-37\right)-\frac{149}{16}\,\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 - 37) - 149/16*log(2*x^2 - 2*sqrt(x^4 + 5*
x^2 + 3) + 5)
```

maple [A] time = 0.01, size = 53, normalized size = 0.95

$$\frac{3\sqrt{x^4+5x^2+3}\,x^2}{4}+\frac{149\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{16}-\frac{37\sqrt{x^4+5x^2+3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 3/4*(x^4+5*x^2+3)^(1/2)*x^2-37/8*(x^4+5*x^2+3)^(1/2)+149/16*ln(x^2+5/2+(x^4
+5*x^2+3)^(1/2))
```

maxima [A] time = 1.04, size = 56, normalized size = 1.00

$$\frac{3}{4}\sqrt{x^4+5x^2+3}\,x^2-\frac{37}{8}\sqrt{x^4+5x^2+3}+\frac{149}{16}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(x^4 + 5*x^2 + 3)*x^2 - 37/8*sqrt(x^4 + 5*x^2 + 3) + 149/16*log(2*x
^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x^3\left(3x^2+2\right)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] int((x^3*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3\left(3x^2+2\right)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**3*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

3.146 $\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=49

$$\frac{3}{2}\sqrt{x^4+5x^2+3}-\frac{11}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1247, 640, 621, 206}

$$\frac{3}{2}\sqrt{x^4+5x^2+3}-\frac{11}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (3*Sqrt[3 + 5*x^2 + x^4])/2 - (11*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x
^4])])/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(2+3x^2\right)}{\sqrt{3+5x^2+x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{2+3x}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= \frac{3}{2}\sqrt{3+5x^2+x^4}-\frac{11}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= \frac{3}{2}\sqrt{3+5x^2+x^4}-\frac{11}{2}\operatorname{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= \frac{3}{2}\sqrt{3+5x^2+x^4}-\frac{11}{4}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$\frac{3}{2}\sqrt{x^4+5x^2+3}-\frac{11}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (3*Sqrt[3 + 5*x^2 + x^4])/2 - (11*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x
^4])])/4
```

IntegrateAlgebraic [A] time = 0.17, size = 47, normalized size = 0.96

$$\frac{3}{2}\sqrt{x^4+5x^2+3}+\frac{11}{4}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (3*Sqrt[3 + 5*x^2 + x^4])/2 + (11*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]]
)/4
```

fricas [A] time = 0.75, size = 39, normalized size = 0.80

$$\frac{3}{2}\sqrt{x^4+5x^2+3}+\frac{11}{4}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 3/2*sqrt(x^4 + 5*x^2 + 3) + 11/4*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

giac [A] time = 0.47, size = 39, normalized size = 0.80

$$\frac{3}{2}\sqrt{x^4+5x^2+3}+\frac{11}{4}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*sqrt(x^4 + 5*x^2 + 3) + 11/4*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.01, size = 36, normalized size = 0.73

$$-\frac{11\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{4}+\frac{3\sqrt{x^4+5x^2+3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 3/2*(x^4+5*x^2+3)^(1/2)-11/4*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))
```

maxima [A] time = 0.92, size = 39, normalized size = 0.80

$$\frac{3}{2}\sqrt{x^4+5x^2+3}-\frac{11}{4}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(x^4 + 5*x^2 + 3) - 11/4*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [B] time = 0.52, size = 35, normalized size = 0.71

$$\frac{3\,\sqrt{x^4+5\,x^2+3}}{2}-\frac{11\,\ln\left(\sqrt{x^4+5\,x^2+3}\,+x^2+\frac{5}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] (3*(5*x^2 + x^4 + 3)^(1/2))/2 - (11*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2
))/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(3x^2+2\right)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

3.147 $\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=69

$$\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1251, 843, 621, 206, 724}

$$\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (3*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2 - ArcTanh[(6 + 5*x^2)/
(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]/Sqrt[3]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{2+3x}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)+\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)\right)+3\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{5+2x}{\sqrt{3+5x^2}}\right.\\
&= \frac{3}{2}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)-\frac{\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{\sqrt{3}}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 69, normalized size = 1.00

$$\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (3*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2 - ArcTanh[(6 + 5*x^2)/
(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]/Sqrt[3]
```

IntegrateAlgebraic [A] time = 0.19, size = 68, normalized size = 0.99

$$\frac{2\tanh^{-1}\left(\frac{x^2}{\sqrt{3}}-\frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{3}{2}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x^2)/(x*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (2*ArcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/Sqrt[3] - (3*Log[-
5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/2
```

fricas [A] time = 0.80, size = 75, normalized size = 1.09

$$\frac{1}{3}\sqrt{3}\log\left(\frac{25x^2-2\sqrt{3}\left(5x^2+6\right)-2\sqrt{x^4+5x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)-\frac{3}{2}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(
5*sqrt(3) - 6) + 30)/x^2) - 3/2*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

giac [A] time = 0.43, size = 78, normalized size = 1.13

$$\frac{1}{3}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)-\frac{3}{2}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sq
rt(x^4 + 5*x^2 + 3))) - 3/2*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.01, size = 52, normalized size = 0.75

$$-\frac{\sqrt{3}\ \operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{3}+\frac{3\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 3/2*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))-1/3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+
5*x^2+3)^(1/2))*3^(1/2)
```

maxima [A] time = 1.93, size = 58, normalized size = 0.84

$$-\frac{1}{3}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{3}{2}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 3/2*log
(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [B] time = 1.01, size = 56, normalized size = 0.81

$$\frac{3\ln\left(\sqrt{x^4+5x^2+3}+x^2+\frac{5}{2}\right)}{2}-\frac{\sqrt{3}\left(\ln\left(\frac{1}{x^2}\right)+\ln\left(2\sqrt{3}\sqrt{x^4+5x^2+3}+5x^2+6\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x*(5*x^2 + x^4 + 3)^(1/2)),x)
```

```
[Out] (3*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/2 - (3^(1/2)*(log(1/x^2) + log
(2*3^(1/2)*(5*x^2 + x^4 + 3)^(1/2) + 5*x^2 + 6)))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{3x^2+2}{x\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x*sqrt(x**4 + 5*x**2 + 3)), x)
```

3.148 $\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{x^4+5x^2+3}}{3x^2}-\frac{2\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1251, 806, 724, 206}

$$-\frac{\sqrt{x^4+5x^2+3}}{3x^2}-\frac{2\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^3*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -Sqrt[3 + 5*x^2 + x^4]/(3*x^2) - (2*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 +
 5*x^2 + x^4])])/(3*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{2+3x}{x^2\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{3+5x^2+x^4}}{3x^2} + \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{3+5x^2+x^4}}{3x^2} - \frac{4}{3}\,\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)$$
$$= -\frac{\sqrt{3+5x^2+x^4}}{3x^2} - \frac{2\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}$$

Mathematica [A] time = 0.01, size = 62, normalized size = 1.00

$$-\frac{\sqrt{x^4+5x^2+3}}{3x^2} - \frac{2\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^3*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -1/3*Sqrt[3 + 5*x^2 + x^4]/x^2 - (2*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 +
 5*x^2 + x^4])])/(3*Sqrt[3])
```

IntegrateAlgebraic [A] time = 0.25, size = 63, normalized size = 1.02

$$\frac{4\tanh^{-1}\left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\sqrt{x^4+5x^2+3}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x^2)/(x^3*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -1/3*Sqrt[3 + 5*x^2 + x^4]/x^2 + (4*ArcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 +
x^4]/Sqrt[3]])/(3*Sqrt[3])
```

fricas [A] time = 0.78, size = 78, normalized size = 1.26

$$\frac{2\sqrt{3}\,x^2\log\left(\frac{25\,x^2-2\,\sqrt{3}\,(5\,x^2+6)-2\,\sqrt{x^4+5\,x^2+3}\,(5\,\sqrt{3}-6)+30}{x^2}\right) - 3\,x^2 - 3\,\sqrt{x^4+5\,x^2+3}}{9\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(2*sqrt(3)*x^2*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2
 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 3*x^2 - 3*sqrt(x^4 + 5*x^2 + 3))/x^2
```

giac [B] time = 0.39, size = 101, normalized size = 1.63

$$\frac{2}{9}\,\sqrt{3}\,\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right) + \frac{5\,x^2-5\,\sqrt{x^4+5\,x^2+3}+6}{3\left(\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^2-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sq
rt(x^4 + 5*x^2 + 3))) + 1/3*(5*x^2 - 5*sqrt(x^4 + 5*x^2 + 3) + 6)/((x^2 - s
qrt(x^4 + 5*x^2 + 3))^2 - 3)
```

maple [A] time = 0.02, size = 49, normalized size = 0.79

$$-\frac{2\sqrt{3}\ \operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{9}-\frac{\sqrt{x^4+5x^2+3}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -2/9*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/3*(x^4+5*
x^2+3)^(1/2)/x^2
```

maxima [A] time = 2.02, size = 51, normalized size = 0.82

$$-\frac{2}{9}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{\sqrt{x^4+5x^2+3}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) - 1/3*sqr
t(x^4 + 5*x^2 + 3)/x^2
```

mupad [B] time = 0.66, size = 83, normalized size = 1.34

$$\frac{5\sqrt{3}\ \operatorname{atanh}\left(\frac{\sqrt{3}\left(5x^2+6\right)}{6\sqrt{x^4+5x^2+3}}\right)}{18}-\frac{\sqrt{x^4+5x^2+3}}{3x^2}-\frac{\sqrt{3}\ \left(\ln\left(\frac{1}{x^2}\right)+\ln\left(2\sqrt{3}\ \sqrt{x^4+5x^2+3}\ +5x^2+6\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^3*(5*x^2 + x^4 + 3)^(1/2)),x)
```

```
[Out] (5*3^(1/2)*atanh((3^(1/2)*(5*x^2 + 6))/(6*(5*x^2 + x^4 + 3)^(1/2))))/18 - (
5*x^2 + x^4 + 3)^(1/2)/(3*x^2) - (3^(1/2)*(log(1/x^2) + log(2*3^(1/2)*(5*x^
2 + x^4 + 3)^(1/2) + 5*x^2 + 6)))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{3x^2+2}{x^3\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**3/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**3*sqrt(x**4 + 5*x**2 + 3)), x)
```

3.149 $\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=83

$$-\frac{\sqrt{x^4+5x^2+3}}{12x^2}-\frac{\sqrt{x^4+5x^2+3}}{6x^4}+\frac{1}{8}\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1251, 834, 806, 724, 206}

$$-\frac{\sqrt{x^4+5x^2+3}}{12x^2}-\frac{\sqrt{x^4+5x^2+3}}{6x^4}+\frac{1}{8}\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^5*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -Sqrt[3 + 5*x^2 + x^4]/(6*x^4) - Sqrt[3 + 5*x^2 + x^4]/(12*x^2) + (Sqrt[3]*
ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/8
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
```

```
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x^3\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{3+5x^2+x^4}}{6x^4} - \frac{1}{12}\text{Subst}\left(\int \frac{-3+2x}{x^2\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{3+5x^2+x^4}}{6x^4} - \frac{\sqrt{3+5x^2+x^4}}{12x^2} - \frac{3}{8}\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{3+5x^2+x^4}}{6x^4} - \frac{\sqrt{3+5x^2+x^4}}{12x^2} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= -\frac{\sqrt{3+5x^2+x^4}}{6x^4} - \frac{\sqrt{3+5x^2+x^4}}{12x^2} + \frac{1}{8}\sqrt{3}\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 67, normalized size = 0.81

$$\frac{1}{8}\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right) - \frac{(x^2+2)\sqrt{x^4+5x^2+3}}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^5*Sqrt[3 + 5*x^2 + x^4]),x]
[Out] -1/12*((2 + x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4 + (Sqrt[3]*ArcTanh[(6 + 5*x^2)/
(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/8
```

IntegrateAlgebraic [A] time = 0.28, size = 70, normalized size = 0.84

$$\frac{(-x^2-2)\sqrt{x^4+5x^2+3}}{12x^4} - \frac{1}{4}\sqrt{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x^2)/(x^5*Sqrt[3 + 5*x^2 + x^4]),x]
[Out] ((-2 - x^2)*Sqrt[3 + 5*x^2 + x^4])/(12*x^4) - (Sqrt[3]*ArcTanh[x^2/Sqrt[3]
- Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/4
```

fricas [A] time = 0.81, size = 83, normalized size = 1.00

$$\frac{3\sqrt{3}x^4\log\left(\frac{25x^2+2\sqrt{3}(5x^2+6)+2\sqrt{x^4+5x^2+3}(5\sqrt{3}+6)+30}{x^2}\right) - 2x^4 - 2\sqrt{x^4+5x^2+3}(x^2+2)}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
[Out] 1/24*(3*sqrt(3)*x^4*log((25*x^2 + 2*sqrt(3)*(5*x^2 + 6) + 2*sqrt(x^4 + 5*x^
2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) - 2*x^4 - 2*sqrt(x^4 + 5*x^2 + 3)*(x^2 +
2))/x^4
```

giac [B] time = 0.52, size = 145, normalized size = 1.75

$$-\frac{1}{8}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)+\frac{9\left(x^2-\sqrt{x^4+5x^2+3}\right)^3+36\left(x^2-\sqrt{x^4+5x^2+3}\right)^2+47x^2-47\sqrt{x^4+5x^2+3}+12}{12\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - s
qrt(x^4 + 5*x^2 + 3))) + 1/12*(9*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 36*(x^2
- sqrt(x^4 + 5*x^2 + 3))^2 + 47*x^2 - 47*sqrt(x^4 + 5*x^2 + 3) + 12)/((x^2
- sqrt(x^4 + 5*x^2 + 3))^2 - 3)^2
```

maple [A] time = 0.01, size = 66, normalized size = 0.80

$$\frac{\sqrt{3}\ \operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{8}-\frac{\sqrt{x^4+5x^2+3}}{12x^2}-\frac{\sqrt{x^4+5x^2+3}}{6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 1/8*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/6*(x^4+5*x
^2+3)^(1/2)/x^4-1/12*(x^4+5*x^2+3)^(1/2)/x^2
```

maxima [A] time = 2.00, size = 68, normalized size = 0.82

$$\frac{1}{8}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{\sqrt{x^4+5x^2+3}}{12x^2}-\frac{\sqrt{x^4+5x^2+3}}{6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) - 1/12*sqr
t(x^4 + 5*x^2 + 3)/x^2 - 1/6*sqrt(x^4 + 5*x^2 + 3)/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3x^2+2}{x^5\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^5*(5*x^2 + x^4 + 3)^(1/2)),x)
```

```
[Out] int((3*x^2 + 2)/(x^5*(5*x^2 + x^4 + 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+2}{x^5\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**5/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**5*sqrt(x**4 + 5*x**2 + 3)), x)
```

3.150 $\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=104

$$\frac{13\sqrt{x^4+5x^2+3}}{108x^2}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}-\frac{61\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{216\sqrt{3}}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}$$

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1251, 834, 806, 724, 206}

$$\frac{13\sqrt{x^4+5x^2+3}}{108x^2}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}-\frac{61\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{216\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^7*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -Sqrt[3 + 5*x^2 + x^4]/(9*x^6) - Sqrt[3 + 5*x^2 + x^4]/(54*x^4) + (13*Sqrt[
3 + 5*x^2 + x^4])/(108*x^2) - (61*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5
*x^2 + x^4])])/(216*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x^4\sqrt{3+5x+x^2}}\,dx,x,x^2\right)$$

$$= -\frac{\sqrt{3+5x^2+x^4}}{9x^6} - \frac{1}{18}\text{Subst}\left(\int \frac{-2+4x}{x^3\sqrt{3+5x+x^2}}\,dx,x,x^2\right)$$

$$= -\frac{\sqrt{3+5x^2+x^4}}{9x^6} - \frac{\sqrt{3+5x^2+x^4}}{54x^4} + \frac{1}{108}\text{Subst}\left(\int \frac{-39-2x}{x^2\sqrt{3+5x+x^2}}\,dx,x,x^2\right)$$

$$= -\frac{\sqrt{3+5x^2+x^4}}{9x^6} - \frac{\sqrt{3+5x^2+x^4}}{54x^4} + \frac{13\sqrt{3+5x^2+x^4}}{108x^2} + \frac{61}{216}\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+}}\right.$$

$$= -\frac{\sqrt{3+5x^2+x^4}}{9x^6} - \frac{\sqrt{3+5x^2+x^4}}{54x^4} + \frac{13\sqrt{3+5x^2+x^4}}{108x^2} - \frac{61}{108}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\right.$$

$$= -\frac{\sqrt{3+5x^2+x^4}}{9x^6} - \frac{\sqrt{3+5x^2+x^4}}{54x^4} + \frac{13\sqrt{3+5x^2+x^4}}{108x^2} - \frac{61\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{216\sqrt{3}}$$

Mathematica [A] time = 0.02, size = 77, normalized size = 0.74

$$\frac{6\sqrt{x^4+5x^2+3}\left(13x^4-2x^2-12\right)-61\sqrt{3}x^6\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{648x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^7*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (6*Sqrt[3 + 5*x^2 + x^4]*(-12 - 2*x^2 + 13*x^4) - 61*Sqrt[3]*x^6*ArcTanh[(6
 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(648*x^6)
```

IntegrateAlgebraic [A] time = 0.34, size = 75, normalized size = 0.72

$$\frac{61\tanh^{-1}\left(\frac{x^2}{\sqrt{3}}-\frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}}\right)}{108\sqrt{3}}+\frac{\sqrt{x^4+5x^2+3}\left(13x^4-2x^2-12\right)}{108x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x^2)/(x^7*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-12 - 2*x^2 + 13*x^4))/(108*x^6) + (61*ArcTanh[x^2/
Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/(108*Sqrt[3])
```

fricas [A] time = 0.86, size = 90, normalized size = 0.87

$$\frac{61\sqrt{3}x^6\log\left(\frac{25x^2-2\sqrt{3}\left(5x^2+6\right)-2\sqrt{x^4+5x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)+78x^6+6\left(13x^4-2x^2-12\right)\sqrt{x^4+5x^2+3}}{648x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/648*(61*sqrt(3)*x^6*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*
x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) + 78*x^6 + 6*(13*x^4 - 2*x^2 - 12)*sqrt
(x^4 + 5*x^2 + 3))/x^6
```

giac [B] time = 0.51, size = 167, normalized size = 1.61

$$\frac{61}{648}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)-\frac{61\left(x^2-\sqrt{x^4+5x^2+3}\right)^5-920\left(x^2-\sqrt{x^4+5x^2+3}\right)^3-2052\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-1449x^2+1449\sqrt{x^4+5x^2+3}-108}{108\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 61/648*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) -
 sqrt(x^4 + 5*x^2 + 3))) - 1/108*(61*(x^2 - sqrt(x^4 + 5*x^2 + 3))^5 - 920*
(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 - 2052*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 14
49*x^2 + 1449*sqrt(x^4 + 5*x^2 + 3) - 108)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2
 - 3)^3
```

maple [A] time = 0.02, size = 83, normalized size = 0.80

$$-\frac{61\sqrt{3}\ \operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{648}+\frac{13\sqrt{x^4+5x^2+3}}{108x^2}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -61/648*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/9*(x^4
+5*x^2+3)^(1/2)/x^6-1/54*(x^4+5*x^2+3)^(1/2)/x^4+13/108*(x^4+5*x^2+3)^(1/2)
/x^2
```

maxima [A] time = 1.98, size = 85, normalized size = 0.82

$$-\frac{61}{648}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{13\sqrt{x^4+5x^2+3}}{108x^2}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -61/648*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 13/1
08*sqrt(x^4 + 5*x^2 + 3)/x^2 - 1/54*sqrt(x^4 + 5*x^2 + 3)/x^4 - 1/9*sqrt(x^
4 + 5*x^2 + 3)/x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3x^2+2}{x^7\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^7*(5*x^2 + x^4 + 3)^(1/2)),x)
```

```
[Out] int((3*x^2 + 2)/(x^7*(5*x^2 + x^4 + 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+2}{x^7\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**7/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**7*sqrt(x**4 + 5*x**2 + 3)), x)
```

3.151 $\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{(47x^2+33)\,x^2}{13\sqrt{x^4+5x^2+3}}+\frac{133}{26}\sqrt{x^4+5x^2+3}-\frac{41}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1251, 818, 640, 621, 206}

$$-\frac{(47x^2+33)\,x^2}{13\sqrt{x^4+5x^2+3}}+\frac{133}{26}\sqrt{x^4+5x^2+3}-\frac{41}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] -(x^2*(33 + 47*x^2))/(13*Sqrt[3 + 5*x^2 + x^4]) + (133*Sqrt[3 + 5*x^2 + x^4
])/26 - (41*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
```

```
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(2+3x^2\right)}{\left(3+5x^2+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(2+3x)}{\left(3+5x+x^2\right)^{3/2}}\,dx, x, x^2\right) \\
&= -\frac{x^2\left(33+47x^2\right)}{13\sqrt{3+5x^2+x^4}} + \frac{1}{13}\operatorname{Subst}\left(\int \frac{33+\frac{133x}{2}}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right) \\
&= -\frac{x^2\left(33+47x^2\right)}{13\sqrt{3+5x^2+x^4}} + \frac{133}{26}\sqrt{3+5x^2+x^4} - \frac{41}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right) \\
&= -\frac{x^2\left(33+47x^2\right)}{13\sqrt{3+5x^2+x^4}} + \frac{133}{26}\sqrt{3+5x^2+x^4} - \frac{41}{2}\operatorname{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right) \\
&= -\frac{x^2\left(33+47x^2\right)}{13\sqrt{3+5x^2+x^4}} + \frac{133}{26}\sqrt{3+5x^2+x^4} - \frac{41}{4}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 72, normalized size = 0.94

$$\frac{78x^4+1198x^2-533\sqrt{x^4+5x^2+3}\,\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)+798}{52\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (798 + 1198*x^2 + 78*x^4 - 533*Sqrt[3 + 5*x^2 + x^4]*ArcTanh[(5 + 2*x^2)/(2
*Sqrt[3 + 5*x^2 + x^4])])/(52*Sqrt[3 + 5*x^2 + x^4])
```

IntegrateAlgebraic [A] time = 0.34, size = 59, normalized size = 0.77

$$\frac{39x^4+599x^2+399}{26\sqrt{x^4+5x^2+3}}+\frac{41}{4}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (399 + 599*x^2 + 39*x^4)/(26*Sqrt[3 + 5*x^2 + x^4]) + (41*Log[-5 - 2*x^2 +
2*Sqrt[3 + 5*x^2 + x^4]])/4
```

fricas [A] time = 1.01, size = 86, normalized size = 1.12

$$\frac{1811\,x^4+9055\,x^2+1066\left(x^4+5\,x^2+3\right)\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)+4\left(39\,x^4+599\,x^2+399\right)\sqrt{x^4+5\,x^2+3}+5433}{104\left(x^4+5\,x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/104*(1811*x^4 + 9055*x^2 + 1066*(x^4 + 5*x^2 + 3)*log(-2*x^2 + 2*sqrt(x^4
 + 5*x^2 + 3) - 5) + 4*(39*x^4 + 599*x^2 + 399)*sqrt(x^4 + 5*x^2 + 3) + 543
3)/(x^4 + 5*x^2 + 3)
```

giac [A] time = 0.38, size = 52, normalized size = 0.68

$$\frac{\left(39\,x^2+599\right)x^2+399}{26\,\sqrt{x^4+5\,x^2+3}}+\frac{41}{4}\,\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/26*((39*x^2 + 599)*x^2 + 399)/sqrt(x^4 + 5*x^2 + 3) + 41/4*log(2*x^2 - 2*
sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.02, size = 91, normalized size = 1.18

$$\frac{3x^4}{2\sqrt{x^4+5x^2+3}}+\frac{41x^2}{4\sqrt{x^4+5x^2+3}}-\frac{41\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{4}-\frac{133}{8\sqrt{x^4+5x^2+3}}+\frac{\frac{665x^2}{52}+\frac{3325}{104}}{\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 3/2*x^4/(x^4+5*x^2+3)^(1/2)+41/4*x^2/(x^4+5*x^2+3)^(1/2)-133/8/(x^4+5*x^2+3
)^(1/2)+665/104*(2*x^2+5)/(x^4+5*x^2+3)^(1/2)-41/4*ln(x^2+5/2+(x^4+5*x^2+3)
^(1/2))
```

maxima [A] time = 0.92, size = 73, normalized size = 0.95

$$\frac{3\,x^4}{2\,\sqrt{x^4+5\,x^2+3}}+\frac{599\,x^2}{26\,\sqrt{x^4+5\,x^2+3}}+\frac{399}{26\,\sqrt{x^4+5\,x^2+3}}-\frac{41}{4}\,\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/2*x^4/sqrt(x^4 + 5*x^2 + 3) + 599/26*x^2/sqrt(x^4 + 5*x^2 + 3) + 399/26/s
qrt(x^4 + 5*x^2 + 3) - 41/4*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\left(3\,x^2+2\right)}{{\left(x^4+5\,x^2+3\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] int((x^5*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(3x^2+2\right)}{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**5*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.152 $\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=56

$$\frac{-47x^2-33}{13\sqrt{x^4+5x^2+3}}+\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1251, 777, 621, 206}

$$\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{47x^2+33}{13\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] -(33 + 47*x^2)/(13*Sqrt[3 + 5*x^2 + x^4]) + (3*ArcTanh[(5 + 2*x^2)/(2*Sqrt[
3 + 5*x^2 + x^4])])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3\left(2+3x^2\right)}{\left(3+5x^2+x^4\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x(2+3x)}{\left(3+5x+x^2\right)^{3/2}}\,dx,x,x^2\right)$$
$$= -\frac{33+47x^2}{13\sqrt{3+5x^2+x^4}}+\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)$$
$$= -\frac{33+47x^2}{13\sqrt{3+5x^2+x^4}}+3\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)$$
$$= -\frac{33+47x^2}{13\sqrt{3+5x^2+x^4}}+\frac{3}{2}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

Mathematica [A] time = 0.12, size = 54, normalized size = 0.96

$$\frac{3}{2}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)-\frac{47x^2+33}{13\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] -1/13*(33 + 47*x^2)/Sqrt[3 + 5*x^2 + x^4] + (3*Log[5 + 2*x^2 + 2*Sqrt[3 + 5
*x^2 + x^4]])/2
```

IntegrateAlgebraic [A] time = 0.27, size = 54, normalized size = 0.96

$$\frac{-47x^2-33}{13\sqrt{x^4+5x^2+3}}-\frac{3}{2}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (-33 - 47*x^2)/(13*Sqrt[3 + 5*x^2 + x^4]) - (3*Log[-5 - 2*x^2 + 2*Sqrt[3 +
5*x^2 + x^4]])/2
```

fricas [A] time = 0.99, size = 81, normalized size = 1.45

$$-\frac{94\,x^4+470\,x^2+39\left(x^4+5\,x^2+3\right)\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)+2\,\sqrt{x^4+5\,x^2+3}\left(47\,x^2+33\right)+282}{26\left(x^4+5\,x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/26*(94*x^4 + 470*x^2 + 39*(x^4 + 5*x^2 + 3)*log(-2*x^2 + 2*sqrt(x^4 + 5*
x^2 + 3) - 5) + 2*sqrt(x^4 + 5*x^2 + 3)*(47*x^2 + 33) + 282)/(x^4 + 5*x^2 +
 3)
```

giac [A] time = 0.51, size = 46, normalized size = 0.82

$$-\frac{47\,x^2+33}{13\,\sqrt{x^4+5\,x^2+3}}-\frac{3}{2}\,\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] -1/13*(47*x^2 + 33)/sqrt(x^4 + 5*x^2 + 3) - 3/2*log(2*x^2 - 2*sqrt(x^4 + 5*
x^2 + 3) + 5)
```

maple [B] time = 0.01, size = 95, normalized size = 1.70

$$-\frac{3x^{2}}{2\sqrt{x^{4}+5x^{2}+3}}+\frac{3\ln\left(x^{2}+\frac{5}{2}+\sqrt{x^{4}+5x^{2}+3}\right)}{2}+\frac{15}{4\sqrt{x^{4}+5x^{2}+3}}-\frac{75\left(2x^{2}+5\right)}{52\sqrt{x^{4}+5x^{2}+3}}+\frac{\frac{10x^{2}}{13}+\frac{12}{13}}{\sqrt{x^{4}+5x^{2}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] -3/2/(x^4+5*x^2+3)^(1/2)*x^2+15/4/(x^4+5*x^2+3)^(1/2)-75/52*(2*x^2+5)/(x^4+
5*x^2+3)^(1/2)+3/2*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))+2/13/(x^4+5*x^2+3)^(1/2)
*(5*x^2+6)
```

maxima [A] time = 0.93, size = 56, normalized size = 1.00

$$-\frac{47\,x^{2}}{13\,\sqrt{x^{4}+5\,x^{2}+3}}-\frac{33}{13\,\sqrt{x^{4}+5\,x^{2}+3}}+\frac{3}{2}\,\log\left(2\,x^{2}+2\,\sqrt{x^{4}+5\,x^{2}+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] -47/13*x^2/sqrt(x^4 + 5*x^2 + 3) - 33/13/sqrt(x^4 + 5*x^2 + 3) + 3/2*log(2*
x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [B] time = 0.31, size = 52, normalized size = 0.93

$$\frac{3\,\ln\left(\sqrt{x^{4}+5\,x^{2}+3}+x^{2}+\frac{5}{2}\right)}{2}-\frac{47\,x^{2}}{13\,\sqrt{x^{4}+5\,x^{2}+3}}-\frac{33}{13\,\sqrt{x^{4}+5\,x^{2}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] (3*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/2 - (47*x^2)/(13*(5*x^2 + x^4
+ 3)^(1/2)) - 33/(13*(5*x^2 + x^4 + 3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{3}\left(3x^{2}+2\right)}{\left(x^{4}+5x^{2}+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**3*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.153 $\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1247, 636}

$$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (8 + 11*x^2)/(13*Sqrt[3 + 5*x^2 + x^4])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(2+3x^2\right)}{\left(3+5x^2+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{2+3x}{\left(3+5x+x^2\right)^{3/2}}\,dx, x, x^2\right)\\ &= \frac{8+11x^2}{13\sqrt{3+5x^2+x^4}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 25, normalized size = 1.00

$$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (8 + 11*x^2)/(13*Sqrt[3 + 5*x^2 + x^4])
```

IntegrateAlgebraic [A] time = 0.29, size = 25, normalized size = 1.00

$$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (8 + 11*x^2)/(13*Sqrt[3 + 5*x^2 + x^4])
```

fricas [B] time = 0.90, size = 46, normalized size = 1.84

$$\frac{11\,x^4+55\,x^2+\sqrt{x^4+5\,x^2+3}\left(11\,x^2+8\right)+33}{13\left(x^4+5\,x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/13*(11*x^4 + 55*x^2 + sqrt(x^4 + 5*x^2 + 3)*(11*x^2 + 8) + 33)/(x^4 + 5*x
^2 + 3)
```

giac [A] time = 0.36, size = 21, normalized size = 0.84

$$\frac{11\,x^2+8}{13\,\sqrt{x^4+5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/13*(11*x^2 + 8)/sqrt(x^4 + 5*x^2 + 3)
```

maple [A] time = 0.01, size = 22, normalized size = 0.88

$$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 1/13*(11*x^2+8)/(x^4+5*x^2+3)^(1/2)
```

maxima [A] time = 0.94, size = 32, normalized size = 1.28

$$\frac{11\,x^2}{13\,\sqrt{x^4+5\,x^2+3}}+\frac{8}{13\,\sqrt{x^4+5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 11/13*x^2/sqrt(x^4 + 5*x^2 + 3) + 8/13/sqrt(x^4 + 5*x^2 + 3)
```

mupad [B] time = 0.24, size = 21, normalized size = 0.84

$$\frac{11\,x^2+8}{13\,\sqrt{x^4+5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] (11*x^2 + 8)/(13*(5*x^2 + x^4 + 3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(3x^2+2\right)}{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.154 $\int \frac{2+3x^2}{x\left(3+5x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{-8x^2-7}{39\sqrt{x^4+5x^2+3}}-\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1251, 822, 12, 724, 206}

$$-\frac{8x^2+7}{39\sqrt{x^4+5x^2+3}}-\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] -(7 + 8*x^2)/(39*Sqrt[3 + 5*x^2 + x^4]) - ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sq
rt[3 + 5*x^2 + x^4])]/(3*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
```

```
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x\left(3+5x^2+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x\left(3+5x+x^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= -\frac{7+8x^2}{39\sqrt{3+5x^2+x^4}}-\frac{1}{39}\text{Subst}\left(\int -\frac{13}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= -\frac{7+8x^2}{39\sqrt{3+5x^2+x^4}}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= -\frac{7+8x^2}{39\sqrt{3+5x^2+x^4}}-\frac{2}{3}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= -\frac{7+8x^2}{39\sqrt{3+5x^2+x^4}}-\frac{\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 1.00

$$-\frac{8x^2+7}{39\sqrt{x^4+5x^2+3}}-\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] -1/39*(7 + 8*x^2)/Sqrt[3 + 5*x^2 + x^4] - ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sq
rt[3 + 5*x^2 + x^4])]/(3*Sqrt[3])
```

IntegrateAlgebraic [A] time = 0.36, size = 67, normalized size = 1.02

$$\frac{-8x^2-7}{39\sqrt{x^4+5x^2+3}}+\frac{2\tanh^{-1}\left(\frac{x^2}{\sqrt{3}}-\frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x^2)/(x*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-7 - 8*x^2)/(39*Sqrt[3 + 5*x^2 + x^4]) + (2*ArcTanh[x^2/Sqrt[3] - Sqrt[3 +
 5*x^2 + x^4]/Sqrt[3]])/(3*Sqrt[3])
```

fricas [B] time = 0.93, size = 107, normalized size = 1.62

$$-\frac{24\,x^4-13\,\sqrt{3}\left(x^4+5\,x^2+3\right)\log\left(\frac{25\,x^2-2\,\sqrt{3}\left(5\,x^2+6\right)-2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}-6\right)+30}{x^2}\right)+120\,x^2+3\,\sqrt{x^4+5\,x^2+3}\left(8\,x^2+7\right)+72}{117\left(x^4+5\,x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/117*(24*x^4 - 13*sqrt(3)*(x^4 + 5*x^2 + 3)*log((25*x^2 - 2*sqrt(3)*(5*x^
2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) + 120*x^2 + 3*s
qrt(x^4 + 5*x^2 + 3)*(8*x^2 + 7) + 72)/(x^4 + 5*x^2 + 3)
```

giac [A] time = 0.56, size = 78, normalized size = 1.18

$$-\frac{1}{9}\sqrt{3}\log\left(-x^2+\sqrt{3}+\sqrt{x^4+5x^2+3}\right)+\frac{1}{9}\sqrt{3}\log\left(-x^2-\sqrt{3}+\sqrt{x^4+5x^2+3}\right)-\frac{8x^2+7}{39\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*log(-x^2 + sqrt(3) + sqrt(x^4 + 5*x^2 + 3)) + 1/9*sqrt(3)*log(
-x^2 - sqrt(3) + sqrt(x^4 + 5*x^2 + 3)) - 1/39*(8*x^2 + 7)/sqrt(x^4 + 5*x^2
 + 3)
```

maple [A] time = 0.02, size = 67, normalized size = 1.02

$$-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{9}-\frac{4\left(2x^2+5\right)}{39\sqrt{x^4+5x^2+3}}+\frac{1}{3\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] -4/39*(2*x^2+5)/(x^4+5*x^2+3)^(1/2)+1/3/(x^4+5*x^2+3)^(1/2)-1/9*arctanh(1/6
*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

maxima [A] time = 2.01, size = 65, normalized size = 0.98

$$-\frac{8x^2}{39\sqrt{x^4+5x^2+3}}-\frac{1}{9}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{7}{39\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] -8/39*x^2/sqrt(x^4 + 5*x^2 + 3) - 1/9*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^
2 + 3)/x^2 + 6/x^2 + 5) - 7/39/sqrt(x^4 + 5*x^2 + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{3x^2+2}{x\left(x^4+5x^2+3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x*(5*x^2 + x^4 + 3)^(3/2)),x)
```

```
[Out] int((3*x^2 + 2)/(x*(5*x^2 + x^4 + 3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+2}{x\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

3.155 $\int \frac{2+3x^2}{x^3\left(3+5x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{-8x^2-7}{39x^2\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{39x^2}+\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1251, 822, 806, 724, 206}

$$-\frac{8x^2+7}{39x^2\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{39x^2}+\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^3*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] -(7 + 8*x^2)/(39*x^2*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[3 + 5*x^2 + x^4])/(39
*x^2) + ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]/(3*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x^3\left(3+5x^2+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{2+3x}{x^2\left(3+5x+x^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= -\frac{7+8x^2}{39x^2\sqrt{3+5x^2+x^4}}-\frac{1}{39}\operatorname{Subst}\left(\int \frac{-6+8x}{x^2\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= -\frac{7+8x^2}{39x^2\sqrt{3+5x^2+x^4}}-\frac{2\sqrt{3+5x^2+x^4}}{39x^2}-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= -\frac{7+8x^2}{39x^2\sqrt{3+5x^2+x^4}}-\frac{2\sqrt{3+5x^2+x^4}}{39x^2}+\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= -\frac{7+8x^2}{39x^2\sqrt{3+5x^2+x^4}}-\frac{2\sqrt{3+5x^2+x^4}}{39x^2}+\frac{\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 88, normalized size = 0.98

$$\frac{-6x^4-54x^2+13\sqrt{3}\sqrt{x^4+5x^2+3}\,x^2\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)-39}{117x^2\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^3*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-39 - 54*x^2 - 6*x^4 + 13*Sqrt[3]*x^2*Sqrt[3 + 5*x^2 + x^4]*ArcTanh[(6 + 5
*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(117*x^2*Sqrt[3 + 5*x^2 + x^4])
```

IntegrateAlgebraic [A] time = 0.42, size = 75, normalized size = 0.83

$$\frac{-2x^4-18x^2-13}{39x^2\sqrt{x^4+5x^2+3}}-\frac{2\tanh^{-1}\left(\frac{x^2}{\sqrt{3}}-\frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x^2)/(x^3*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-13 - 18*x^2 - 2*x^4)/(39*x^2*Sqrt[3 + 5*x^2 + x^4]) - (2*ArcTanh[x^2/Sqrt
[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/(3*Sqrt[3])
```

fricas [A] time = 1.09, size = 124, normalized size = 1.38

$$-\frac{6\,x^6+30\,x^4-13\,\sqrt{3}\left(x^6+5\,x^4+3\,x^2\right)\log\left(\frac{25\,x^2+2\,\sqrt{3}\left(5\,x^2+6\right)+2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}+6\right)+30}{x^2}\right)+18\,x^2+3\left(2\,x^4+18\,x^2+13\right)\sqrt{x^4+5\,x^2+3}}{117\left(x^6+5\,x^4+3\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/117*(6*x^6 + 30*x^4 - 13*sqrt(3)*(x^6 + 5*x^4 + 3*x^2)*log((25*x^2 + 2*s
qrt(3)*(5*x^2 + 6) + 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) + 1
8*x^2 + 3*(2*x^4 + 18*x^2 + 13)*sqrt(x^4 + 5*x^2 + 3))/(x^6 + 5*x^4 + 3*x^2
)
```

giac [A] time = 0.49, size = 122, normalized size = 1.36

$$-\frac{1}{9}\,\sqrt{3}\,\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right)+\frac{7\,x^2+11}{117\,\sqrt{x^4+5\,x^2+3}}+\frac{5\,x^2-5\,\sqrt{x^4+5\,x^2+3}+6}{9\left(\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^2-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - s
qrt(x^4 + 5*x^2 + 3))) + 1/117*(7*x^2 + 11)/sqrt(x^4 + 5*x^2 + 3) + 1/9*(5*
x^2 - 5*sqrt(x^4 + 5*x^2 + 3) + 6)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)
```

maple [A] time = 0.02, size = 84, normalized size = 0.93

$$\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{9}-\frac{1}{3\sqrt{x^4+5x^2+3}\,x^2}-\frac{1}{3\sqrt{x^4+5x^2+3}}-\frac{2x^2+5}{39\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] -1/3/(x^4+5*x^2+3)^(1/2)-1/39*(2*x^2+5)/(x^4+5*x^2+3)^(1/2)+1/9*arctanh(1/6
*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/3/x^2/(x^4+5*x^2+3)^(1/2)
```

maxima [A] time = 2.09, size = 82, normalized size = 0.91

$$-\frac{2\,x^2}{39\,\sqrt{x^4+5\,x^2+3}}+\frac{1}{9}\,\sqrt{3}\,\log\left(\frac{2\,\sqrt{3}\,\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{6}{13\,\sqrt{x^4+5\,x^2+3}}-\frac{1}{3\,\sqrt{x^4+5\,x^2+3}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/39*x^2/sqrt(x^4 + 5*x^2 + 3) + 1/9*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^
2 + 3)/x^2 + 6/x^2 + 5) - 6/13/sqrt(x^4 + 5*x^2 + 3) - 1/3/(sqrt(x^4 + 5*x^
2 + 3)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3\,x^2+2}{x^3\,\left(x^4+5\,x^2+3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^3*(5*x^2 + x^4 + 3)^(3/2)),x)
```

```
[Out] int((3*x^2 + 2)/(x^3*(5*x^2 + x^4 + 3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+2}{x^3\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**3/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**3*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

3.156 $\int (fx)^m \left(d+ex^2\right)\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=243

$$\frac{a^3d(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+7}\left(3a^2ce+3ab^2e+6abcd+b^3d\right)}{f^7(m+7)}+\frac{a^2(fx)^{m+3}(ae+3bd)}{f^3(m+3)}+\frac{3c(fx)^{m+11}\left(ace+b^2e+bcd\right)}{f^{11}(m+11)}+\frac{3a(f$$

Rubi [A] time = 0.18, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1261}

$$\frac{(fx)^{m+7}\left(3a^2ce+3ab^2e+6abcd+b^3d\right)}{f^7(m+7)}+\frac{a^2(fx)^{m+3}(ae+3bd)}{f^3(m+3)}+\frac{a^3d(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+9}\left(6abce+3ac^2d+3b^2cd+b^3e\right)}{f^9(m+9)}+\frac{3a(fx)^{m+5}\left(abe+acd+b^2d\right)}{f^5(m+5)}+\frac{3c(fx)^{m+11}\left(ace+b^2e+bcd\right)}{f^{11}(m+11)}+\frac{c^2(fx)^{m+13}(3be+cd)}{f^{13}(m+13)}+\frac{c^3e(fx)^{m+15}}{f^{15}(m+15)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*d*(f*x)^(1 + m))/(f*(1 + m)) + (a^2*(3*b*d + a*e)*(f*x)^(3 + m))/(f^3*
(3 + m)) + (3*a*(b^2*d + a*c*d + a*b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + ((b^
3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)*(f*x)^(7 + m))/(f^7*(7 + m)) + ((3
*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*(f*x)^(9 + m))/(f^9*(9 + m)) + (3
*c*(b*c*d + b^2*e + a*c*e)*(f*x)^(11 + m))/(f^11*(11 + m)) + (c^2*(c*d + 3*
b*e)*(f*x)^(13 + m))/(f^13*(13 + m)) + (c^3*e*(f*x)^(15 + m))/(f^15*(15 + m
))
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int (fx)^m\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^3\,dx=\int\left(a^3d(fx)^m+\frac{a^2(3bd+ae)(fx)^{2+m}}{f^2}+\frac{3a\left(b^2d+acd+abe\right)(fx)^{4+m}}{f^4}+\right.$$
$$=\frac{a^3d(fx)^{1+m}}{f(1+m)}+\frac{a^2(3bd+ae)(fx)^{3+m}}{f^3(3+m)}+\frac{3a\left(b^2d+acd+abe\right)(fx)^{5+m}}{f^5(5+m)}+$$

Mathematica [A] time = 0.30, size = 191, normalized size = 0.79

$$x(fx)^m\left(\frac{a^3d}{m+1}+\frac{x^6\left(3a^2ce+3ab^2e+6abcd+b^3d\right)}{m+7}+\frac{a^2x^2(ae+3bd)}{m+3}+\frac{3cx^{10}\left(ace+b^2e+bcd\right)}{m+11}+\frac{3ax^4\left(abe+acd+b^2d\right)}{m+5}+\frac{x^8\left(6abce+3ac^2d+b^3e+3b^2cd\right)}{m+9}+\frac{c^2x^{12}(3be+cd)}{m+13}+\frac{c^3ex^{14}}{m+15}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] x*(f*x)^m*((a^3*d)/(1 + m) + (a^2*(3*b*d + a*e)*x^2)/(3 + m) + (3*a*(b^2*d
+ a*c*d + a*b*e)*x^4)/(5 + m) + ((b^3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e
)*x^6)/(7 + m) + ((3*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*x^8)/(9 + m)
+ (3*c*(b*c*d + b^2*e + a*c*e)*x^10)/(11 + m) + (c^2*(c*d + 3*b*e)*x^12)/(1
3 + m) + (c^3*e*x^14)/(15 + m))
```

IntegrateAlgebraic [F] time = 1.21, size = 0, normalized size = 0.00

$$\int (fx)^m\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 0.86, size = 1357, normalized size = 5.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] ((c^3*e*m^7 + 49*c^3*e*m^6 + 973*c^3*e*m^5 + 10045*c^3*e*m^4 + 57379*c^3*e*
m^3 + 177331*c^3*e*m^2 + 264207*c^3*e*m + 135135*c^3*e)*x^15 + ((c^3*d + 3*
b*c^2*e)*m^7 + 51*(c^3*d + 3*b*c^2*e)*m^6 + 1045*(c^3*d + 3*b*c^2*e)*m^5 +
11055*(c^3*d + 3*b*c^2*e)*m^4 + 155925*c^3*d + 467775*b*c^2*e + 64339*(c^3*
d + 3*b*c^2*e)*m^3 + 201609*(c^3*d + 3*b*c^2*e)*m^2 + 303255*(c^3*d + 3*b*c
^2*e)*m)*x^13 + 3*((b*c^2*d + (b^2*c + a*c^2)*e)*m^7 + 53*(b*c^2*d + (b^2*c
 + a*c^2)*e)*m^6 + 1125*(b*c^2*d + (b^2*c + a*c^2)*e)*m^5 + 12265*(b*c^2*d
+ (b^2*c + a*c^2)*e)*m^4 + 184275*b*c^2*d + 73139*(b*c^2*d + (b^2*c + a*c^2
)*e)*m^3 + 233487*(b*c^2*d + (b^2*c + a*c^2)*e)*m^2 + 184275*(b^2*c + a*c^2
)*e + 355815*(b*c^2*d + (b^2*c + a*c^2)*e)*m)*x^11 + ((3*(b^2*c + a*c^2)*d
+ (b^3 + 6*a*b*c)*e)*m^7 + 55*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^6
 + 1213*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^5 + 13723*(3*(b^2*c + a
*c^2)*d + (b^3 + 6*a*b*c)*e)*m^4 + 84547*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*
b*c)*e)*m^3 + 277093*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^2 + 675675
*(b^2*c + a*c^2)*d + 225225*(b^3 + 6*a*b*c)*e + 430335*(3*(b^2*c + a*c^2)*d
 + (b^3 + 6*a*b*c)*e)*m)*x^9 + (((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m
^7 + 57*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^6 + 1309*((b^3 + 6*a*b*
c)*d + 3*(a*b^2 + a^2*c)*e)*m^5 + 15477*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2
*c)*e)*m^4 + 99715*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^3 + 340011*(
(b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^2 + 289575*(b^3 + 6*a*b*c)*d + 8
68725*(a*b^2 + a^2*c)*e + 544095*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*
m)*x^7 + 3*((a^2*b*e + (a*b^2 + a^2*c)*d)*m^7 + 59*(a^2*b*e + (a*b^2 + a^2*
c)*d)*m^6 + 1413*(a^2*b*e + (a*b^2 + a^2*c)*d)*m^5 + 17575*(a^2*b*e + (a*b^
2 + a^2*c)*d)*m^4 + 405405*a^2*b*e + 120179*(a^2*b*e + (a*b^2 + a^2*c)*d)*m
^3 + 437121*(a^2*b*e + (a*b^2 + a^2*c)*d)*m^2 + 405405*(a*b^2 + a^2*c)*d +
738567*(a^2*b*e + (a*b^2 + a^2*c)*d)*m)*x^5 + ((3*a^2*b*d + a^3*e)*m^7 + 61
*(3*a^2*b*d + a^3*e)*m^6 + 1525*(3*a^2*b*d + a^3*e)*m^5 + 20065*(3*a^2*b*d
+ a^3*e)*m^4 + 2027025*a^2*b*d + 675675*a^3*e + 147859*(3*a^2*b*d + a^3*e)*
m^3 + 594439*(3*a^2*b*d + a^3*e)*m^2 + 1140855*(3*a^2*b*d + a^3*e)*m)*x^3 +
 (a^3*d*m^7 + 63*a^3*d*m^6 + 1645*a^3*d*m^5 + 22995*a^3*d*m^4 + 185059*a^3*
d*m^3 + 852957*a^3*d*m^2 + 2071215*a^3*d*m + 2027025*a^3*d)*x)*(f*x)^m/(m^8
 + 64*m^7 + 1708*m^6 + 24640*m^5 + 208054*m^4 + 1038016*m^3 + 2924172*m^2 +
 4098240*m + 2027025)
```

giac [B] time = 0.75, size = 2816, normalized size = 11.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] ((f*x)^m*c^3*m^7*x^15*e + 49*(f*x)^m*c^3*m^6*x^15*e + (f*x)^m*c^3*d*m^7*x^1
3 + 3*(f*x)^m*b*c^2*m^7*x^13*e + 973*(f*x)^m*c^3*m^5*x^15*e + 51*(f*x)^m*c^
3*d*m^6*x^13 + 153*(f*x)^m*b*c^2*m^6*x^13*e + 10045*(f*x)^m*c^3*m^4*x^15*e
+ 3*(f*x)^m*b*c^2*d*m^7*x^11 + 1045*(f*x)^m*c^3*d*m^5*x^13 + 3*(f*x)^m*b^2*
c*m^7*x^11*e + 3*(f*x)^m*a*c^2*m^7*x^11*e + 3135*(f*x)^m*b*c^2*m^5*x^13*e +
 57379*(f*x)^m*c^3*m^3*x^15*e + 159*(f*x)^m*b*c^2*d*m^6*x^11 + 11055*(f*x)^
m*c^3*d*m^4*x^13 + 159*(f*x)^m*b^2*c*m^6*x^11*e + 159*(f*x)^m*a*c^2*m^6*x^1
1*e + 33165*(f*x)^m*b*c^2*m^4*x^13*e + 177331*(f*x)^m*c^3*m^2*x^15*e + 3*(f
```

```
*x)^m*b^2*c*d*m^7*x^9 + 3*(f*x)^m*a*c^2*d*m^7*x^9 + 3375*(f*x)^m*b*c^2*d*m^
5*x^11 + 64339*(f*x)^m*c^3*d*m^3*x^13 + (f*x)^m*b^3*m^7*x^9*e + 6*(f*x)^m*a
*b*c*m^7*x^9*e + 3375*(f*x)^m*b^2*c*m^5*x^11*e + 3375*(f*x)^m*a*c^2*m^5*x^1
1*e + 193017*(f*x)^m*b*c^2*m^3*x^13*e + 264207*(f*x)^m*c^3*m*x^15*e + 165*(
f*x)^m*b^2*c*d*m^6*x^9 + 165*(f*x)^m*a*c^2*d*m^6*x^9 + 36795*(f*x)^m*b*c^2*
d*m^4*x^11 + 201609*(f*x)^m*c^3*d*m^2*x^13 + 55*(f*x)^m*b^3*m^6*x^9*e + 330
*(f*x)^m*a*b*c*m^6*x^9*e + 36795*(f*x)^m*b^2*c*m^4*x^11*e + 36795*(f*x)^m*a
*c^2*m^4*x^11*e + 604827*(f*x)^m*b*c^2*m^2*x^13*e + 135135*(f*x)^m*c^3*x^15
*e + (f*x)^m*b^3*d*m^7*x^7 + 6*(f*x)^m*a*b*c*d*m^7*x^7 + 3639*(f*x)^m*b^2*c
*d*m^5*x^9 + 3639*(f*x)^m*a*c^2*d*m^5*x^9 + 219417*(f*x)^m*b*c^2*d*m^3*x^11
 + 303255*(f*x)^m*c^3*d*m*x^13 + 3*(f*x)^m*a*b^2*m^7*x^7*e + 3*(f*x)^m*a^2*
c*m^7*x^7*e + 1213*(f*x)^m*b^3*m^5*x^9*e + 7278*(f*x)^m*a*b*c*m^5*x^9*e + 2
19417*(f*x)^m*b^2*c*m^3*x^11*e + 219417*(f*x)^m*a*c^2*m^3*x^11*e + 909765*(
f*x)^m*b*c^2*m*x^13*e + 57*(f*x)^m*b^3*d*m^6*x^7 + 342*(f*x)^m*a*b*c*d*m^6*
x^7 + 41169*(f*x)^m*b^2*c*d*m^4*x^9 + 41169*(f*x)^m*a*c^2*d*m^4*x^9 + 70046
1*(f*x)^m*b*c^2*d*m^2*x^11 + 155925*(f*x)^m*c^3*d*x^13 + 171*(f*x)^m*a*b^2*
m^6*x^7*e + 171*(f*x)^m*a^2*c*m^6*x^7*e + 13723*(f*x)^m*b^3*m^4*x^9*e + 823
38*(f*x)^m*a*b*c*m^4*x^9*e + 700461*(f*x)^m*b^2*c*m^2*x^11*e + 700461*(f*x)
^m*a*c^2*m^2*x^11*e + 467775*(f*x)^m*b*c^2*x^13*e + 3*(f*x)^m*a*b^2*d*m^7*x
^5 + 3*(f*x)^m*a^2*c*d*m^7*x^5 + 1309*(f*x)^m*b^3*d*m^5*x^7 + 7854*(f*x)^m*
a*b*c*d*m^5*x^7 + 253641*(f*x)^m*b^2*c*d*m^3*x^9 + 253641*(f*x)^m*a*c^2*d*m
^3*x^9 + 1067445*(f*x)^m*b*c^2*d*m*x^11 + 3*(f*x)^m*a^2*b*m^7*x^5*e + 3927*
(f*x)^m*a*b^2*m^5*x^7*e + 3927*(f*x)^m*a^2*c*m^5*x^7*e + 84547*(f*x)^m*b^3*
m^3*x^9*e + 507282*(f*x)^m*a*b*c*m^3*x^9*e + 1067445*(f*x)^m*b^2*c*m*x^11*e
 + 1067445*(f*x)^m*a*c^2*m*x^11*e + 177*(f*x)^m*a*b^2*d*m^6*x^5 + 177*(f*x)
^m*a^2*c*d*m^6*x^5 + 15477*(f*x)^m*b^3*d*m^4*x^7 + 92862*(f*x)^m*a*b*c*d*m^
4*x^7 + 831279*(f*x)^m*b^2*c*d*m^2*x^9 + 831279*(f*x)^m*a*c^2*d*m^2*x^9 + 5
52825*(f*x)^m*b*c^2*d*x^11 + 177*(f*x)^m*a^2*b*m^6*x^5*e + 46431*(f*x)^m*a*
b^2*m^4*x^7*e + 46431*(f*x)^m*a^2*c*m^4*x^7*e + 277093*(f*x)^m*b^3*m^2*x^9*
e + 1662558*(f*x)^m*a*b*c*m^2*x^9*e + 552825*(f*x)^m*b^2*c*x^11*e + 552825*
(f*x)^m*a*c^2*x^11*e + 3*(f*x)^m*a^2*b*d*m^7*x^3 + 4239*(f*x)^m*a*b^2*d*m^5
*x^5 + 4239*(f*x)^m*a^2*c*d*m^5*x^5 + 99715*(f*x)^m*b^3*d*m^3*x^7 + 598290*
(f*x)^m*a*b*c*d*m^3*x^7 + 1291005*(f*x)^m*b^2*c*d*m*x^9 + 1291005*(f*x)^m*a
*c^2*d*m*x^9 + (f*x)^m*a^3*m^7*x^3*e + 4239*(f*x)^m*a^2*b*m^5*x^5*e + 29914
5*(f*x)^m*a*b^2*m^3*x^7*e + 299145*(f*x)^m*a^2*c*m^3*x^7*e + 430335*(f*x)^m
*b^3*m*x^9*e + 2582010*(f*x)^m*a*b*c*m*x^9*e + 183*(f*x)^m*a^2*b*d*m^6*x^3
 + 52725*(f*x)^m*a*b^2*d*m^4*x^5 + 52725*(f*x)^m*a^2*c*d*m^4*x^5 + 340011*(f
*x)^m*b^3*d*m^2*x^7 + 2040066*(f*x)^m*a*b*c*d*m^2*x^7 + 675675*(f*x)^m*b^2*
c*d*x^9 + 675675*(f*x)^m*a*c^2*d*x^9 + 61*(f*x)^m*a^3*m^6*x^3*e + 52725*(f*
x)^m*a^2*b*m^4*x^5*e + 1020033*(f*x)^m*a*b^2*m^2*x^7*e + 1020033*(f*x)^m*a^
2*c*m^2*x^7*e + 225225*(f*x)^m*b^3*x^9*e + 1351350*(f*x)^m*a*b*c*x^9*e + (f
*x)^m*a^3*d*m^7*x + 4575*(f*x)^m*a^2*b*d*m^5*x^3 + 360537*(f*x)^m*a*b^2*d*m
^3*x^5 + 360537*(f*x)^m*a^2*c*d*m^3*x^5 + 544095*(f*x)^m*b^3*d*m*x^7 + 3264
570*(f*x)^m*a*b*c*d*m*x^7 + 1525*(f*x)^m*a^3*m^5*x^3*e + 360537*(f*x)^m*a^2
*b*m^3*x^5*e + 1632285*(f*x)^m*a*b^2*m*x^7*e + 1632285*(f*x)^m*a^2*c*m*x^7*
e + 63*(f*x)^m*a^3*d*m^6*x + 60195*(f*x)^m*a^2*b*d*m^4*x^3 + 1311363*(f*x)^
m*a*b^2*d*m^2*x^5 + 1311363*(f*x)^m*a^2*c*d*m^2*x^5 + 289575*(f*x)^m*b^3*d*
x^7 + 1737450*(f*x)^m*a*b*c*d*x^7 + 20065*(f*x)^m*a^3*m^4*x^3*e + 1311363*(
f*x)^m*a^2*b*m^2*x^5*e + 868725*(f*x)^m*a*b^2*x^7*e + 868725*(f*x)^m*a^2*c*
x^7*e + 1645*(f*x)^m*a^3*d*m^5*x + 443577*(f*x)^m*a^2*b*d*m^3*x^3 + 2215701
*(f*x)^m*a*b^2*d*m*x^5 + 2215701*(f*x)^m*a^2*c*d*m*x^5 + 147859*(f*x)^m*a^3
*m^3*x^3*e + 2215701*(f*x)^m*a^2*b*m*x^5*e + 22995*(f*x)^m*a^3*d*m^4*x + 17
83317*(f*x)^m*a^2*b*d*m^2*x^3 + 1216215*(f*x)^m*a*b^2*d*x^5 + 1216215*(f*x)
^m*a^2*c*d*x^5 + 594439*(f*x)^m*a^3*m^2*x^3*e + 1216215*(f*x)^m*a^2*b*x^5*e
 + 185059*(f*x)^m*a^3*d*m^3*x + 3422565*(f*x)^m*a^2*b*d*m*x^3 + 1140855*(f*
x)^m*a^3*m*x^3*e + 852957*(f*x)^m*a^3*d*m^2*x + 2027025*(f*x)^m*a^2*b*d*x^3
 + 675675*(f*x)^m*a^3*x^3*e + 2071215*(f*x)^m*a^3*d*m*x + 2027025*(f*x)^m*a
^3*d*x)/(m^8 + 64*m^7 + 1708*m^6 + 24640*m^5 + 208054*m^4 + 1038016*m^3 + 2
924172*m^2 + 4098240*m + 2027025)
```

maple [B] time = 0.01, size = 1935, normalized size = 7.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] x*(c^3*e*m^7*x^14+49*c^3*e*m^6*x^14+3*b*c^2*e*m^7*x^12+c^3*d*m^7*x^12+973*c
^3*e*m^5*x^14+153*b*c^2*e*m^6*x^12+51*c^3*d*m^6*x^12+10045*c^3*e*m^4*x^14+3
*a*c^2*e*m^7*x^10+3*b^2*c*e*m^7*x^10+3*b*c^2*d*m^7*x^10+3135*b*c^2*e*m^5*x^
12+1045*c^3*d*m^5*x^12+57379*c^3*e*m^3*x^14+159*a*c^2*e*m^6*x^10+159*b^2*c*
e*m^6*x^10+159*b*c^2*d*m^6*x^10+33165*b*c^2*e*m^4*x^12+11055*c^3*d*m^4*x^12
+177331*c^3*e*m^2*x^14+6*a*b*c*e*m^7*x^8+3*a*c^2*d*m^7*x^8+3375*a*c^2*e*m^5
*x^10+b^3*e*m^7*x^8+3*b^2*c*d*m^7*x^8+3375*b^2*c*e*m^5*x^10+3375*b*c^2*d*m^
5*x^10+193017*b*c^2*e*m^3*x^12+64339*c^3*d*m^3*x^12+264207*c^3*e*m*x^14+330
*a*b*c*e*m^6*x^8+165*a*c^2*d*m^6*x^8+36795*a*c^2*e*m^4*x^10+55*b^3*e*m^6*x^
8+165*b^2*c*d*m^6*x^8+36795*b^2*c*e*m^4*x^10+36795*b*c^2*d*m^4*x^10+604827*
b*c^2*e*m^2*x^12+201609*c^3*d*m^2*x^12+135135*c^3*e*x^14+3*a^2*c*e*m^7*x^6+
3*a*b^2*e*m^7*x^6+6*a*b*c*d*m^7*x^6+7278*a*b*c*e*m^5*x^8+3639*a*c^2*d*m^5*x
^8+219417*a*c^2*e*m^3*x^10+b^3*d*m^7*x^6+1213*b^3*e*m^5*x^8+3639*b^2*c*d*m^
5*x^8+219417*b^2*c*e*m^3*x^10+219417*b*c^2*d*m^3*x^10+909765*b*c^2*e*m*x^12
+303255*c^3*d*m*x^12+171*a^2*c*e*m^6*x^6+171*a*b^2*e*m^6*x^6+342*a*b*c*d*m^
6*x^6+82338*a*b*c*e*m^4*x^8+41169*a*c^2*d*m^4*x^8+700461*a*c^2*e*m^2*x^10+5
7*b^3*d*m^6*x^6+13723*b^3*e*m^4*x^8+41169*b^2*c*d*m^4*x^8+700461*b^2*c*e*m^
2*x^10+700461*b*c^2*d*m^2*x^10+467775*b*c^2*e*x^12+155925*c^3*d*x^12+3*a^2*
b*e*m^7*x^4+3*a^2*c*d*m^7*x^4+3927*a^2*c*e*m^5*x^6+3*a*b^2*d*m^7*x^4+3927*a
*b^2*e*m^5*x^6+7854*a*b*c*d*m^5*x^6+507282*a*b*c*e*m^3*x^8+253641*a*c^2*d*m
^3*x^8+1067445*a*c^2*e*m*x^10+1309*b^3*d*m^5*x^6+84547*b^3*e*m^3*x^8+253641
*b^2*c*d*m^3*x^8+1067445*b^2*c*e*m*x^10+1067445*b*c^2*d*m*x^10+177*a^2*b*e*
m^6*x^4+177*a^2*c*d*m^6*x^4+46431*a^2*c*e*m^4*x^6+177*a*b^2*d*m^6*x^4+46431
*a*b^2*e*m^4*x^6+92862*a*b*c*d*m^4*x^6+1662558*a*b*c*e*m^2*x^8+831279*a*c^2
*d*m^2*x^8+552825*a*c^2*e*x^10+15477*b^3*d*m^4*x^6+277093*b^3*e*m^2*x^8+831
279*b^2*c*d*m^2*x^8+552825*b^2*c*e*x^10+552825*b*c^2*d*x^10+a^3*e*m^7*x^2+3
*a^2*b*d*m^7*x^2+4239*a^2*b*e*m^5*x^4+4239*a^2*c*d*m^5*x^4+299145*a^2*c*e*m
^3*x^6+4239*a*b^2*d*m^5*x^4+299145*a*b^2*e*m^3*x^6+598290*a*b*c*d*m^3*x^6+2
582010*a*b*c*e*m*x^8+1291005*a*c^2*d*m*x^8+99715*b^3*d*m^3*x^6+430335*b^3*e
*m*x^8+1291005*b^2*c*d*m*x^8+61*a^3*e*m^6*x^2+183*a^2*b*d*m^6*x^2+52725*a^2
*b*e*m^4*x^4+52725*a^2*c*d*m^4*x^4+1020033*a^2*c*e*m^2*x^6+52725*a*b^2*d*m^
4*x^4+1020033*a*b^2*e*m^2*x^6+2040066*a*b*c*d*m^2*x^6+1351350*a*b*c*e*x^8+6
75675*a*c^2*d*x^8+340011*b^3*d*m^2*x^6+225225*b^3*e*x^8+675675*b^2*c*d*x^8+
a^3*d*m^7+1525*a^3*e*m^5*x^2+4575*a^2*b*d*m^5*x^2+360537*a^2*b*e*m^3*x^4+36
0537*a^2*c*d*m^3*x^4+1632285*a^2*c*e*m*x^6+360537*a*b^2*d*m^3*x^4+1632285*a
*b^2*e*m*x^6+3264570*a*b*c*d*m*x^6+544095*b^3*d*m*x^6+63*a^3*d*m^6+20065*a^
3*e*m^4*x^2+60195*a^2*b*d*m^4*x^2+1311363*a^2*b*e*m^2*x^4+1311363*a^2*c*d*m
^2*x^4+868725*a^2*c*e*x^6+1311363*a*b^2*d*m^2*x^4+868725*a*b^2*e*x^6+173745
0*a*b*c*d*x^6+289575*b^3*d*x^6+1645*a^3*d*m^5+147859*a^3*e*m^3*x^2+443577*a
^2*b*d*m^3*x^2+2215701*a^2*b*e*m*x^4+2215701*a^2*c*d*m*x^4+2215701*a*b^2*d*
m*x^4+22995*a^3*d*m^4+594439*a^3*e*m^2*x^2+1783317*a^2*b*d*m^2*x^2+1216215*
a^2*b*e*x^4+1216215*a^2*c*d*x^4+1216215*a*b^2*d*x^4+185059*a^3*d*m^3+114085
5*a^3*e*m*x^2+3422565*a^2*b*d*m*x^2+852957*a^3*d*m^2+675675*a^3*e*x^2+20270
25*a^2*b*d*x^2+2071215*a^3*d*m+2027025*a^3*d)*(f*x)^m/(m+1)/(m+3)/(m+5)/(m+
7)/(m+9)/(m+11)/(m+13)/(m+15)
```

maxima [A] time = 1.39, size = 408, normalized size = 1.68

$$\frac{c^3 e f^m x^{15} x^m}{m+15} + \frac{c^3 d f^m x^{13} x^m}{m+13} + \frac{3 b c^2 e f^m x^{13} x^m}{m+13} + \frac{3 b c^2 d f^m x^{11} x^m}{m+11} + \frac{3 b^2 c e f^m x^{11} x^m}{m+11} + \frac{3 a c^2 e f^m x^{11} x^m}{m+11} + \frac{3 b^2 c d f^m x^9 x^m}{m+9} + \frac{3 a c^2 d f^m x^9 x^m}{m+9} + \frac{b^3 e f^m x^9 x^m}{m+9} + \frac{6 a b c e f^m x^9 x^m}{m+9} + \frac{b^3 d f^m x^7 x^m}{m+7} + \frac{6 a b c d f^m x^7 x^m}{m+7} + \frac{3 a b^2 e f^m x^7 x^m}{m+7} + \frac{3 a^2 c e f^m x^7 x^m}{m+7} + \frac{3 a b^2 d f^m x^5 x^m}{m+5} + \frac{3 a^2 c d f^m x^5 x^m}{m+5} + \frac{3 a^2 b e f^m x^5 x^m}{m+5} + \frac{3 a^2 b d f^m x^3 x^m}{m+3} + \frac{a^3 e f^m x^3 x^m}{m+3} + \frac{(f x)^{m+1} a^3 d}{f(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] c^3*e*f^m*x^15*x^m/(m + 15) + c^3*d*f^m*x^13*x^m/(m + 13) + 3*b*c^2*e*f^m*x
^13*x^m/(m + 13) + 3*b*c^2*d*f^m*x^11*x^m/(m + 11) + 3*b^2*c*e*f^m*x^11*x^m
/(m + 11) + 3*a*c^2*e*f^m*x^11*x^m/(m + 11) + 3*b^2*c*d*f^m*x^9*x^m/(m + 9)
 + 3*a*c^2*d*f^m*x^9*x^m/(m + 9) + b^3*e*f^m*x^9*x^m/(m + 9) + 6*a*b*c*e*f^
m*x^9*x^m/(m + 9) + b^3*d*f^m*x^7*x^m/(m + 7) + 6*a*b*c*d*f^m*x^7*x^m/(m +
7) + 3*a*b^2*e*f^m*x^7*x^m/(m + 7) + 3*a^2*c*e*f^m*x^7*x^m/(m + 7) + 3*a*b^
2*d*f^m*x^5*x^m/(m + 5) + 3*a^2*c*d*f^m*x^5*x^m/(m + 5) + 3*a^2*b*e*f^m*x^5
*x^m/(m + 5) + 3*a^2*b*d*f^m*x^3*x^m/(m + 3) + a^3*e*f^m*x^3*x^m/(m + 3) +
(f*x)^(m + 1)*a^3*d/(f*(m + 1))
```

mupad [B] time = 1.06, size = 769, normalized size = 3.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (x^7*(f*x)^m*(b^3*d + 3*a*b^2*e + 3*a^2*c*e + 6*a*b*c*d)*(544095*m + 340011
*m^2 + 99715*m^3 + 15477*m^4 + 1309*m^5 + 57*m^6 + m^7 + 289575))/(4098240*
m + 2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7
+ m^8 + 2027025) + (x^9*(f*x)^m*(b^3*e + 3*a*c^2*d + 3*b^2*c*d + 6*a*b*c*e)
*(430335*m + 277093*m^2 + 84547*m^3 + 13723*m^4 + 1213*m^5 + 55*m^6 + m^7 +
 225225))/(4098240*m + 2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 +
 1708*m^6 + 64*m^7 + m^8 + 2027025) + (a^3*d*x*(f*x)^m*(2071215*m + 852957*
m^2 + 185059*m^3 + 22995*m^4 + 1645*m^5 + 63*m^6 + m^7 + 2027025))/(4098240
*m + 2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7
 + m^8 + 2027025) + (c^3*e*x^15*(f*x)^m*(264207*m + 177331*m^2 + 57379*m^3
+ 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135))/(4098240*m + 2924172*m^2 +
1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 2027025) +
 (3*a*x^5*(f*x)^m*(b^2*d + a*b*e + a*c*d)*(738567*m + 437121*m^2 + 120179*m
^3 + 17575*m^4 + 1413*m^5 + 59*m^6 + m^7 + 405405))/(4098240*m + 2924172*m^
2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 202702
5) + (3*c*x^11*(f*x)^m*(b^2*e + a*c*e + b*c*d)*(355815*m + 233487*m^2 + 731
39*m^3 + 12265*m^4 + 1125*m^5 + 53*m^6 + m^7 + 184275))/(4098240*m + 292417
2*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 20
27025) + (a^2*x^3*(f*x)^m*(a*e + 3*b*d)*(1140855*m + 594439*m^2 + 147859*m^
3 + 20065*m^4 + 1525*m^5 + 61*m^6 + m^7 + 675675))/(4098240*m + 2924172*m^2
 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 2027025
) + (c^2*x^13*(f*x)^m*(3*b*e + c*d)*(303255*m + 201609*m^2 + 64339*m^3 + 11
055*m^4 + 1045*m^5 + 51*m^6 + m^7 + 155925))/(4098240*m + 2924172*m^2 + 103
8016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 2027025)
```

sympy [A] time = 12.38, size = 11538, normalized size = 47.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Piecewise(((-a**3*d/(14*x**14) - a**3*e/(12*x**12) - a**2*b*d/(4*x**12) - 3
*a**2*b*e/(10*x**10) - 3*a**2*c*d/(10*x**10) - 3*a**2*c*e/(8*x**8) - 3*a*b*
*2*d/(10*x**10) - 3*a*b**2*e/(8*x**8) - 3*a*b*c*d/(4*x**8) - a*b*c*e/x**6 -
 a*c**2*d/(2*x**6) - 3*a*c**2*e/(4*x**4) - b**3*d/(8*x**8) - b**3*e/(6*x**6
) - b**2*c*d/(2*x**6) - 3*b**2*c*e/(4*x**4) - 3*b*c**2*d/(4*x**4) - 3*b*c**
2*e/(2*x**2) - c**3*d/(2*x**2) + c**3*e*log(x))/f**15, Eq(m, -15)), ((-a**3
*d/(12*x**12) - a**3*e/(10*x**10) - 3*a**2*b*d/(10*x**10) - 3*a**2*b*e/(8*x
**8) - 3*a**2*c*d/(8*x**8) - a**2*c*e/(2*x**6) - 3*a*b**2*d/(8*x**8) - a*b*
*2*e/(2*x**6) - a*b*c*d/x**6 - 3*a*b*c*e/(2*x**4) - 3*a*c**2*d/(4*x**4) - 3
*a*c**2*e/(2*x**2) - b**3*d/(6*x**6) - b**3*e/(4*x**4) - 3*b**2*c*d/(4*x**4
) - 3*b**2*c*e/(2*x**2) - 3*b*c**2*d/(2*x**2) + 3*b*c**2*e*log(x) + c**3*d*
```

```
log(x) + c**3*e*x**2/2)/f**13, Eq(m, -13)), ((-a**3*d/(10*x**10) - a**3*e/(
8*x**8) - 3*a**2*b*d/(8*x**8) - a**2*b*e/(2*x**6) - a**2*c*d/(2*x**6) - 3*a
**2*c*e/(4*x**4) - a*b**2*d/(2*x**6) - 3*a*b**2*e/(4*x**4) - 3*a*b*c*d/(2*x
**4) - 3*a*b*c*e/x**2 - 3*a*c**2*d/(2*x**2) + 3*a*c**2*e*log(x) - b**3*d/(4
*x**4) - b**3*e/(2*x**2) - 3*b**2*c*d/(2*x**2) + 3*b**2*c*e*log(x) + 3*b*c*
*2*d*log(x) + 3*b*c**2*e*x**2/2 + c**3*d*x**2/2 + c**3*e*x**4/4)/f**11, Eq(
m, -11)), ((-a**3*d/(8*x**8) - a**3*e/(6*x**6) - a**2*b*d/(2*x**6) - 3*a**2
*b*e/(4*x**4) - 3*a**2*c*d/(4*x**4) - 3*a**2*c*e/(2*x**2) - 3*a*b**2*d/(4*x
**4) - 3*a*b**2*e/(2*x**2) - 3*a*b*c*d/x**2 + 6*a*b*c*e*log(x) + 3*a*c**2*d
*log(x) + 3*a*c**2*e*x**2/2 - b**3*d/(2*x**2) + b**3*e*log(x) + 3*b**2*c*d*
log(x) + 3*b**2*c*e*x**2/2 + 3*b*c**2*d*x**2/2 + 3*b*c**2*e*x**4/4 + c**3*d
*x**4/4 + c**3*e*x**6/6)/f**9, Eq(m, -9)), ((-a**3*d/(6*x**6) - a**3*e/(4*x
**4) - 3*a**2*b*d/(4*x**4) - 3*a**2*b*e/(2*x**2) - 3*a**2*c*d/(2*x**2) + 3*
a**2*c*e*log(x) - 3*a*b**2*d/(2*x**2) + 3*a*b**2*e*log(x) + 6*a*b*c*d*log(x
) + 3*a*b*c*e*x**2 + 3*a*c**2*d*x**2/2 + 3*a*c**2*e*x**4/4 + b**3*d*log(x)
+ b**3*e*x**2/2 + 3*b**2*c*d*x**2/2 + 3*b**2*c*e*x**4/4 + 3*b*c**2*d*x**4/4
 + b*c**2*e*x**6/2 + c**3*d*x**6/6 + c**3*e*x**8/8)/f**7, Eq(m, -7)), ((-a*
*3*d/(4*x**4) - a**3*e/(2*x**2) - 3*a**2*b*d/(2*x**2) + 3*a**2*b*e*log(x) +
 3*a**2*c*d*log(x) + 3*a**2*c*e*x**2/2 + 3*a*b**2*d*log(x) + 3*a*b**2*e*x**
2/2 + 3*a*b*c*d*x**2 + 3*a*b*c*e*x**4/2 + 3*a*c**2*d*x**4/4 + a*c**2*e*x**6
/2 + b**3*d*x**2/2 + b**3*e*x**4/4 + 3*b**2*c*d*x**4/4 + b**2*c*e*x**6/2 +
b*c**2*d*x**6/2 + 3*b*c**2*e*x**8/8 + c**3*d*x**8/8 + c**3*e*x**10/10)/f**5
, Eq(m, -5)), ((-a**3*d/(2*x**2) + a**3*e*log(x) + 3*a**2*b*d*log(x) + 3*a*
*2*b*e*x**2/2 + 3*a**2*c*d*x**2/2 + 3*a**2*c*e*x**4/4 + 3*a*b**2*d*x**2/2 +
 3*a*b**2*e*x**4/4 + 3*a*b*c*d*x**4/2 + a*b*c*e*x**6 + a*c**2*d*x**6/2 + 3*
a*c**2*e*x**8/8 + b**3*d*x**4/4 + b**3*e*x**6/6 + b**2*c*d*x**6/2 + 3*b**2*
c*e*x**8/8 + 3*b*c**2*d*x**8/8 + 3*b*c**2*e*x**10/10 + c**3*d*x**10/10 + c*
*3*e*x**12/12)/f**3, Eq(m, -3)), ((a**3*d*log(x) + a**3*e*x**2/2 + 3*a**2*b
*d*x**2/2 + 3*a**2*b*e*x**4/4 + 3*a**2*c*d*x**4/4 + a**2*c*e*x**6/2 + 3*a*b
**2*d*x**4/4 + a*b**2*e*x**6/2 + a*b*c*d*x**6 + 3*a*b*c*e*x**8/4 + 3*a*c**2
*d*x**8/8 + 3*a*c**2*e*x**10/10 + b**3*d*x**6/6 + b**3*e*x**8/8 + 3*b**2*c*
d*x**8/8 + 3*b**2*c*e*x**10/10 + 3*b*c**2*d*x**10/10 + b*c**2*e*x**12/4 + c
**3*d*x**12/12 + c**3*e*x**14/14)/f, Eq(m, -1)), (a**3*d*f**m*m**7*x*x**m/(
m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924
172*m**2 + 4098240*m + 2027025) + 63*a**3*d*f**m*m**6*x*x**m/(m**8 + 64*m**
7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40
98240*m + 2027025) + 1645*a**3*d*f**m*m**5*x*x**m/(m**8 + 64*m**7 + 1708*m*
*6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2
027025) + 22995*a**3*d*f**m*m**4*x*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640
*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) +
185059*a**3*d*f**m*m**3*x*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 2
08054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 852957*a*
*3*d*f**m*m**2*x*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**
4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 2071215*a**3*d*f**
m*m*x*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016
*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 2027025*a**3*d*f**m*x*x**m/(m
**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 29241
72*m**2 + 4098240*m + 2027025) + a**3*e*f**m*m**7*x**3*x**m/(m**8 + 64*m**7
 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409
8240*m + 2027025) + 61*a**3*e*f**m*m**6*x**3*x**m/(m**8 + 64*m**7 + 1708*m*
*6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2
027025) + 1525*a**3*e*f**m*m**5*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 246
40*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
+ 20065*a**3*e*f**m*m**4*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5
 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 14785
9*a**3*e*f**m*m**3*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208
054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 594439*a**3
*e*f**m*m**2*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m*
*4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1140855*a**3*e*f*
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*m*m*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103
8016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 675675*a**3*e*f**m*x**3*x
m/(m8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +
 2924172*m**2 + 4098240*m + 2027025) + 3*a**2*b*d*f**m*m**7*x**3*x**m/(m**8
 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*
m**2 + 4098240*m + 2027025) + 183*a**2*b*d*f**m*m**6*x**3*x**m/(m**8 + 64*m
7 + 1708*m6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 4575*a**2*b*d*f**m*m**5*x**3*x**m/(m**8 + 64*m**7 +
1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409824
0*m + 2027025) + 60195*a**2*b*d*f**m*m**4*x**3*x**m/(m**8 + 64*m**7 + 1708*
m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
 2027025) + 443577*a**2*b*d*f**m*m**3*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6
 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202
7025) + 1783317*a**2*b*d*f**m*m**2*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 +
24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202702
5) + 3422565*a**2*b*d*f**m*m*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*
m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 2
027025*a**2*b*d*f**m*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 2
08054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3*a**2*b*
e*f**m*m**7*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**
4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 177*a**2*b*e*f**m*
m**6*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103
8016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 4239*a**2*b*e*f**m*m**5*x
5*xm/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m
3 + 2924172*m2 + 4098240*m + 2027025) + 52725*a**2*b*e*f**m*m**4*x**5*x
m/(m8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +
 2924172*m**2 + 4098240*m + 2027025) + 360537*a**2*b*e*f**m*m**3*x**5*x**m/
(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292
4172*m**2 + 4098240*m + 2027025) + 1311363*a**2*b*e*f**m*m**2*x**5*x**m/(m*
*8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292417
2*m**2 + 4098240*m + 2027025) + 2215701*a**2*b*e*f**m*m*x**5*x**m/(m**8 + 6
4*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2
 + 4098240*m + 2027025) + 1216215*a**2*b*e*f**m*x**5*x**m/(m**8 + 64*m**7 +
 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40982
40*m + 2027025) + 3*a**2*c*d*f**m*m**7*x**5*x**m/(m**8 + 64*m**7 + 1708*m**
6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20
27025) + 177*a**2*c*d*f**m*m**6*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 246
40*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
+ 4239*a**2*c*d*f**m*m**5*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**
5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 5272
5*a**2*c*d*f**m*m**4*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 2
08054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 360537*a*
*2*c*d*f**m*m**3*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 20805
4*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1311363*a**2*
c*d*f**m*m**2*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m
4 + 1038016*m3 + 2924172*m**2 + 4098240*m + 2027025) + 2215701*a**2*c*d
*f**m*m*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 +
1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1216215*a**2*c*d*f**m*
x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*
m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3*a**2*c*e*f**m*m**7*x**7*x**m
/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 29
24172*m**2 + 4098240*m + 2027025) + 171*a**2*c*e*f**m*m**6*x**7*x**m/(m**8
+ 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m
2 + 4098240*m + 2027025) + 3927*a2*c*e*f**m*m**5*x**7*x**m/(m**8 + 64*m
7 + 1708*m6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 46431*a**2*c*e*f**m*m**4*x**7*x**m/(m**8 + 64*m**7 +
 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40982
40*m + 2027025) + 299145*a**2*c*e*f**m*m**3*x**7*x**m/(m**8 + 64*m**7 + 170
8*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m

+ 2027025) + 1020033*a**2*c*e*f**m*m**2*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1632285*a**2*c*e*f**m*m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 868725*a**2*c*e*f**m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3*a*b**2*d*f**m*m**7*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 177*a*b**2*d*f**m*m**6*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 4239*a*b**2*d*f**m*m**5*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 52725*a*b**2*d*f**m*m**4*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 360537*a*b**2*d*f**m*m**3*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1311363*a*b**2*d*f**m*m**2*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 2215701*a*b**2*d*f**m*m*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1216215*a*b**2*d*f**m*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3*a*b**2*e*f**m*m**7*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 171*a*b**2*e*f**m*m**6*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3927*a*b**2*e*f**m*m**5*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 46431*a*b**2*e*f**m*m**4*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 299145*a*b**2*e*f**m*m**3*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1020033*a*b**2*e*f**m*m**2*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1632285*a*b**2*e*f**m*m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 868725*a*b**2*e*f**m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 6*a*b*c*d*f**m*m**7*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 342*a*b*c*d*f**m*m**6*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 7854*a*b*c*d*f**m*m**5*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 92862*a*b*c*d*f**m*m**4*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 598290*a*b*c*d*f**m*m**3*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 2040066*a*b*c*d*f**m*m**2*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3264570*a*b*c*d*f**m*m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1737450*a*b*c*d*f**m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 6*a*b*c*e*f**m*m**7*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 330*a*b*c*e*f**m*m**6*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 7278*a*b*c*e*f**m*m**5*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 82338*a*b*c*e*f**m*m**4*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2

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924172*m**2 + 4098240*m + 2027025) + 507282*a*b*c*e*f**m*m**3*x**9*x**m/(m*
*8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292417
2*m**2 + 4098240*m + 2027025) + 1662558*a*b*c*e*f**m*m**2*x**9*x**m/(m**8 +
 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m*
*2 + 4098240*m + 2027025) + 2582010*a*b*c*e*f**m*m*x**9*x**m/(m**8 + 64*m**
7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40
98240*m + 2027025) + 1351350*a*b*c*e*f**m*x**9*x**m/(m**8 + 64*m**7 + 1708*
m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
 2027025) + 3*a*c**2*d*f**m*m**7*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24
640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
 + 165*a*c**2*d*f**m*m**6*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**
5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3639
*a*c**2*d*f**m*m**5*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 20
8054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 41169*a*c*
*2*d*f**m*m**4*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*
m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 253641*a*c**2*d
*f**m*m**3*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4
 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 831279*a*c**2*d*f**
m*m**2*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1
038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1291005*a*c**2*d*f**m*m
*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016
*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 675675*a*c**2*d*f**m*x**9*x**
m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2
924172*m**2 + 4098240*m + 2027025) + 3*a*c**2*e*f**m*m**7*x**11*x**m/(m**8
+ 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m
**2 + 4098240*m + 2027025) + 159*a*c**2*e*f**m*m**6*x**11*x**m/(m**8 + 64*m
**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 3375*a*c**2*e*f**m*m**5*x**11*x**m/(m**8 + 64*m**7 +
 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40982
40*m + 2027025) + 36795*a*c**2*e*f**m*m**4*x**11*x**m/(m**8 + 64*m**7 + 170
8*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m
 + 2027025) + 219417*a*c**2*e*f**m*m**3*x**11*x**m/(m**8 + 64*m**7 + 1708*m
**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
2027025) + 700461*a*c**2*e*f**m*m**2*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6
 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202
7025) + 1067445*a*c**2*e*f**m*m*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24
640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
 + 552825*a*c**2*e*f**m*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5
 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + b**3*
d*f**m*m**7*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**
4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 57*b**3*d*f**m*m**
6*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103801
6*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1309*b**3*d*f**m*m**5*x**7*x
**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +
 2924172*m**2 + 4098240*m + 2027025) + 15477*b**3*d*f**m*m**4*x**7*x**m/(m*
*8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292417
2*m**2 + 4098240*m + 2027025) + 99715*b**3*d*f**m*m**3*x**7*x**m/(m**8 + 64
*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2
+ 4098240*m + 2027025) + 340011*b**3*d*f**m*m**2*x**7*x**m/(m**8 + 64*m**7
+ 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098
240*m + 2027025) + 544095*b**3*d*f**m*m*x**7*x**m/(m**8 + 64*m**7 + 1708*m*
*6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2
027025) + 289575*b**3*d*f**m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*
m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + b
**3*e*f**m*m**7*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054
*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 55*b**3*e*f**m
*m**6*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 10
38016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1213*b**3*e*f**m*m**5*x*
*9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m*
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*3 + 2924172*m**2 + 4098240*m + 2027025) + 13723*b**3*e*f**m*m**4*x**9*x**m
/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 29
24172*m**2 + 4098240*m + 2027025) + 84547*b**3*e*f**m*m**3*x**9*x**m/(m**8
+ 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m
**2 + 4098240*m + 2027025) + 277093*b**3*e*f**m*m**2*x**9*x**m/(m**8 + 64*m
**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 430335*b**3*e*f**m*m*x**9*x**m/(m**8 + 64*m**7 + 170
8*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m
 + 2027025) + 225225*b**3*e*f**m*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24
640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
 + 3*b**2*c*d*f**m*m**7*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5
+ 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 165*b*
*2*c*d*f**m*m**6*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 20805
4*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3639*b**2*c*d
*f**m*m**5*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4
 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 41169*b**2*c*d*f**m
*m**4*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 10
38016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 253641*b**2*c*d*f**m*m**
3*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103801
6*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 831279*b**2*c*d*f**m*m**2*x*
*9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m*
*3 + 2924172*m**2 + 4098240*m + 2027025) + 1291005*b**2*c*d*f**m*m*x**9*x**
m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2
924172*m**2 + 4098240*m + 2027025) + 675675*b**2*c*d*f**m*x**9*x**m/(m**8 +
 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m*
*2 + 4098240*m + 2027025) + 3*b**2*c*e*f**m*m**7*x**11*x**m/(m**8 + 64*m**7
 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409
8240*m + 2027025) + 159*b**2*c*e*f**m*m**6*x**11*x**m/(m**8 + 64*m**7 + 170
8*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m
 + 2027025) + 3375*b**2*c*e*f**m*m**5*x**11*x**m/(m**8 + 64*m**7 + 1708*m**
6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20
27025) + 36795*b**2*c*e*f**m*m**4*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 +
24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202702
5) + 219417*b**2*c*e*f**m*m**3*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 246
40*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
+ 700461*b**2*c*e*f**m*m**2*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*
m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1
067445*b**2*c*e*f**m*m*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5
+ 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 552825
*b**2*c*e*f**m*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054
*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3*b*c**2*d*f**
m*m**7*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 +
1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 159*b*c**2*d*f**m*m**6
*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103801
6*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3375*b*c**2*d*f**m*m**5*x**1
1*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**
3 + 2924172*m**2 + 4098240*m + 2027025) + 36795*b*c**2*d*f**m*m**4*x**11*x*
*m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +
2924172*m**2 + 4098240*m + 2027025) + 219417*b*c**2*d*f**m*m**3*x**11*x**m/
(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292
4172*m**2 + 4098240*m + 2027025) + 700461*b*c**2*d*f**m*m**2*x**11*x**m/(m*
*8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292417
2*m**2 + 4098240*m + 2027025) + 1067445*b*c**2*d*f**m*m*x**11*x**m/(m**8 +
64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**
2 + 4098240*m + 2027025) + 552825*b*c**2*d*f**m*x**11*x**m/(m**8 + 64*m**7
+ 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098
240*m + 2027025) + 3*b*c**2*e*f**m*m**7*x**13*x**m/(m**8 + 64*m**7 + 1708*m
**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
2027025) + 153*b*c**2*e*f**m*m**6*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 +
```

```
24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202702
5) + 3135*b*c**2*e*f**m*m**5*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640
*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) +
33165*b*c**2*e*f**m*m**4*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**
5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1930
17*b*c**2*e*f**m*m**3*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 +
 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 604827*
b*c**2*e*f**m*m**2*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 20
8054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 909765*b*c
**2*e*f**m*m*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m
**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 467775*b*c**2*e*
f**m*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 10
38016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + c**3*d*f**m*m**7*x**13*x
**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +
 2924172*m**2 + 4098240*m + 2027025) + 51*c**3*d*f**m*m**6*x**13*x**m/(m**8
 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*
m**2 + 4098240*m + 2027025) + 1045*c**3*d*f**m*m**5*x**13*x**m/(m**8 + 64*m
**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 11055*c**3*d*f**m*m**4*x**13*x**m/(m**8 + 64*m**7 +
1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409824
0*m + 2027025) + 64339*c**3*d*f**m*m**3*x**13*x**m/(m**8 + 64*m**7 + 1708*m
**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
2027025) + 201609*c**3*d*f**m*m**2*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 +
 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20270
25) + 303255*c**3*d*f**m*m*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m
**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 15
5925*c**3*d*f**m*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 2080
54*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + c**3*e*f**m*
m**7*x**15*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 10
38016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 49*c**3*e*f**m*m**6*x**1
5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**
3 + 2924172*m**2 + 4098240*m + 2027025) + 973*c**3*e*f**m*m**5*x**15*x**m/(
m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924
172*m**2 + 4098240*m + 2027025) + 10045*c**3*e*f**m*m**4*x**15*x**m/(m**8 +
 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m*
*2 + 4098240*m + 2027025) + 57379*c**3*e*f**m*m**3*x**15*x**m/(m**8 + 64*m*
*7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4
098240*m + 2027025) + 177331*c**3*e*f**m*m**2*x**15*x**m/(m**8 + 64*m**7 +
1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409824
0*m + 2027025) + 264207*c**3*e*f**m*m*x**15*x**m/(m**8 + 64*m**7 + 1708*m**
6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20
27025) + 135135*c**3*e*f**m*x**15*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*
m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025), Tr
ue))
```

3.157 $\int (fx)^m \left(d+ex^2\right)\left(a+bx^2+cx^4\right)^2 dx$

Optimal. Leaf size=155

$$\frac{a^2d(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+7}\left(2ace+b^2e+2bcd\right)}{f^7(m+7)}+\frac{(fx)^{m+5}\left(2abe+2acd+b^2d\right)}{f^5(m+5)}+\frac{a(fx)^{m+3}(ae+2bd)}{f^3(m+3)}+\frac{c(fx)^{m+9}(2be}{f^9(m+9}$$

Rubi [A] time = 0.10, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1261}

$$\frac{a^2d(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+5}\left(2abe+2acd+b^2d\right)}{f^5(m+5)}+\frac{(fx)^{m+7}\left(2ace+b^2e+2bcd\right)}{f^7(m+7)}+\frac{a(fx)^{m+3}(ae+2bd)}{f^3(m+3)}+\frac{c(fx)^{m+9}(2be+cd)}{f^9(m+9)}+\frac{c^2e(fx)^{m+11}}{f^{11}(m+11)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*d*(f*x)^(1 + m))/(f*(1 + m)) + (a*(2*b*d + a*e)*(f*x)^(3 + m))/(f^3*(3
 + m)) + ((b^2*d + 2*a*c*d + 2*a*b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + ((2*b*
c*d + b^2*e + 2*a*c*e)*(f*x)^(7 + m))/(f^7*(7 + m)) + (c*(c*d + 2*b*e)*(f*x
)^(9 + m))/(f^9*(9 + m)) + (c^2*e*(f*x)^(11 + m))/(f^11*(11 + m))
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)\left(a+bx^2+cx^4\right)^2 dx = \int \left(a^2d(fx)^m+\frac{a(2bd+ae)(fx)^{2+m}}{f^2}+\frac{\left(b^2d+2acd+2abe\right)(fx)^{4+m}}{f^4}\right.$$
$$=\frac{a^2d(fx)^{1+m}}{f(1+m)}+\frac{a(2bd+ae)(fx)^{3+m}}{f^3(3+m)}+\frac{\left(b^2d+2acd+2abe\right)(fx)^{5+m}}{f^5(5+m)}+$$

Mathematica [A] time = 0.12, size = 117, normalized size = 0.75

$$x(fx)^m\left(\frac{a^2d}{m+1}+\frac{x^6\left(2ace+b^2e+2bcd\right)}{m+7}+\frac{x^4\left(2abe+2acd+b^2d\right)}{m+5}+\frac{ax^2(ae+2bd)}{m+3}+\frac{cx^8(2be+cd)}{m+9}+\frac{c^2ex^{10}}{m+11}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] x*(f*x)^m*((a^2*d)/(1 + m) + (a*(2*b*d + a*e)*x^2)/(3 + m) + ((b^2*d + 2*a*
c*d + 2*a*b*e)*x^4)/(5 + m) + ((2*b*c*d + b^2*e + 2*a*c*e)*x^6)/(7 + m) + (
c*(c*d + 2*b*e)*x^8)/(9 + m) + (c^2*e*x^10)/(11 + m))
```

IntegrateAlgebraic [F] time = 0.82, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+ex^2\right)\left(a+bx^2+cx^4\right)^2 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2, x]
```

fricas [B] time = 0.70, size = 573, normalized size = 3.70

$$\frac{\left(\left(c^2 e m^5 + 25 c^2 e m^4 + 230 c^2 e m^3 + 950 c^2 e m^2 + 1689 c^2 e m + 945 c^2 e\right)x^{11} + \left((c^2 d + 2bce)m^5 + 27(c^2 d + 2bce)m^4 + 262(c^2 d + 2bce)m^3 + 1155 c^2 d + 2310 bce + 1122(c^2 d + 2bce)m^2 + 2041(c^2 d + 2bce)m\right)x^9 + \left((2bcd + (b^2 + 2ac)e)m^5 + 29(2bcd + (b^2 + 2ac)e)m^4 + 302(2bcd + (b^2 + 2ac)e)m^3 + 2970 bcd + 1366(2bcd + (b^2 + 2ac)e)m^2 + 1485(b^2 + 2ac)e + 2577(2bcd + (b^2 + 2ac)e)m\right)x^7 + \left((2abe + (b^2 + 2ac)d)m^5 + 31(2abe + (b^2 + 2ac)d)m^4 + 350(2abe + (b^2 + 2ac)d)m^3 + 4158 abe + 1730(2abe + (b^2 + 2ac)d)m^2 + 2079(b^2 + 2ac)d + 3489(2abe + (b^2 + 2ac)d)m\right)x^5 + \left((2abd + a^2 e)m^5 + 33(2abd + a^2 e)m^4 + 406(2abd + a^2 e)m^3 + 6930 abd + 3465 a^2 e + 2262(2abd + a^2 e)m^2 + 5353(2abd + a^2 e)m\right)x^3 + \left(a^2 d m^5 + 35 a^2 d m^4 + 470 a^2 d m^3 + 3010 a^2 d m^2 + 9129 a^2 d m + 10395 a^2 d\right)x\right)(fx)^m}{m^6 + 36 m^5 + 505 m^4 + 3480 m^3 + 12139 m^2 + 19524 m + 10395}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] ((c^2*e*m^5 + 25*c^2*e*m^4 + 230*c^2*e*m^3 + 950*c^2*e*m^2 + 1689*c^2*e*m +
 945*c^2*e)*x^11 + ((c^2*d + 2*b*c*e)*m^5 + 27*(c^2*d + 2*b*c*e)*m^4 + 262*
(c^2*d + 2*b*c*e)*m^3 + 1155*c^2*d + 2310*b*c*e + 1122*(c^2*d + 2*b*c*e)*m^
2 + 2041*(c^2*d + 2*b*c*e)*m)*x^9 + ((2*b*c*d + (b^2 + 2*a*c)*e)*m^5 + 29*(
2*b*c*d + (b^2 + 2*a*c)*e)*m^4 + 302*(2*b*c*d + (b^2 + 2*a*c)*e)*m^3 + 2970
*b*c*d + 1366*(2*b*c*d + (b^2 + 2*a*c)*e)*m^2 + 1485*(b^2 + 2*a*c)*e + 2577
*(2*b*c*d + (b^2 + 2*a*c)*e)*m)*x^7 + ((2*a*b*e + (b^2 + 2*a*c)*d)*m^5 + 31
*(2*a*b*e + (b^2 + 2*a*c)*d)*m^4 + 350*(2*a*b*e + (b^2 + 2*a*c)*d)*m^3 + 41
58*a*b*e + 1730*(2*a*b*e + (b^2 + 2*a*c)*d)*m^2 + 2079*(b^2 + 2*a*c)*d + 34
89*(2*a*b*e + (b^2 + 2*a*c)*d)*m)*x^5 + ((2*a*b*d + a^2*e)*m^5 + 33*(2*a*b*
d + a^2*e)*m^4 + 406*(2*a*b*d + a^2*e)*m^3 + 6930*a*b*d + 3465*a^2*e + 2262
*(2*a*b*d + a^2*e)*m^2 + 5353*(2*a*b*d + a^2*e)*m)*x^3 + (a^2*d*m^5 + 35*a^
2*d*m^4 + 470*a^2*d*m^3 + 3010*a^2*d*m^2 + 9129*a^2*d*m + 10395*a^2*d)*x)*(
f*x)^m/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

giac [B] time = 0.40, size = 1178, normalized size = 7.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] ((f*x)^m*c^2*m^5*x^11*e + 25*(f*x)^m*c^2*m^4*x^11*e + (f*x)^m*c^2*d*m^5*x^9
 + 2*(f*x)^m*b*c*m^5*x^9*e + 230*(f*x)^m*c^2*m^3*x^11*e + 27*(f*x)^m*c^2*d*
m^4*x^9 + 54*(f*x)^m*b*c*m^4*x^9*e + 950*(f*x)^m*c^2*m^2*x^11*e + 2*(f*x)^m
*b*c*d*m^5*x^7 + 262*(f*x)^m*c^2*d*m^3*x^9 + (f*x)^m*b^2*m^5*x^7*e + 2*(f*x
)^m*a*c*m^5*x^7*e + 524*(f*x)^m*b*c*m^3*x^9*e + 1689*(f*x)^m*c^2*m*x^11*e +
 58*(f*x)^m*b*c*d*m^4*x^7 + 1122*(f*x)^m*c^2*d*m^2*x^9 + 29*(f*x)^m*b^2*m^4
*x^7*e + 58*(f*x)^m*a*c*m^4*x^7*e + 2244*(f*x)^m*b*c*m^2*x^9*e + 945*(f*x)^
m*c^2*x^11*e + (f*x)^m*b^2*d*m^5*x^5 + 2*(f*x)^m*a*c*d*m^5*x^5 + 604*(f*x)^
m*b*c*d*m^3*x^7 + 2041*(f*x)^m*c^2*d*m*x^9 + 2*(f*x)^m*a*b*m^5*x^5*e + 302*
(f*x)^m*b^2*m^3*x^7*e + 604*(f*x)^m*a*c*m^3*x^7*e + 4082*(f*x)^m*b*c*m*x^9*
e + 31*(f*x)^m*b^2*d*m^4*x^5 + 62*(f*x)^m*a*c*d*m^4*x^5 + 2732*(f*x)^m*b*c*
d*m^2*x^7 + 1155*(f*x)^m*c^2*d*x^9 + 62*(f*x)^m*a*b*m^4*x^5*e + 1366*(f*x)^
m*b^2*m^2*x^7*e + 2732*(f*x)^m*a*c*m^2*x^7*e + 2310*(f*x)^m*b*c*x^9*e + 2*(
f*x)^m*a*b*d*m^5*x^3 + 350*(f*x)^m*b^2*d*m^3*x^5 + 700*(f*x)^m*a*c*d*m^3*x^
5 + 5154*(f*x)^m*b*c*d*m*x^7 + (f*x)^m*a^2*m^5*x^3*e + 700*(f*x)^m*a*b*m^3*
x^5*e + 2577*(f*x)^m*b^2*m*x^7*e + 5154*(f*x)^m*a*c*m*x^7*e + 66*(f*x)^m*a*
b*d*m^4*x^3 + 1730*(f*x)^m*b^2*d*m^2*x^5 + 3460*(f*x)^m*a*c*d*m^2*x^5 + 297
0*(f*x)^m*b*c*d*x^7 + 33*(f*x)^m*a^2*m^4*x^3*e + 3460*(f*x)^m*a*b*m^2*x^5*e
 + 1485*(f*x)^m*b^2*x^7*e + 2970*(f*x)^m*a*c*x^7*e + (f*x)^m*a^2*d*m^5*x +
812*(f*x)^m*a*b*d*m^3*x^3 + 3489*(f*x)^m*b^2*d*m*x^5 + 6978*(f*x)^m*a*c*d*m
*x^5 + 406*(f*x)^m*a^2*m^3*x^3*e + 6978*(f*x)^m*a*b*m*x^5*e + 35*(f*x)^m*a^
2*d*m^4*x + 4524*(f*x)^m*a*b*d*m^2*x^3 + 2079*(f*x)^m*b^2*d*x^5 + 4158*(f*x
)^m*a*c*d*x^5 + 2262*(f*x)^m*a^2*m^2*x^3*e + 4158*(f*x)^m*a*b*x^5*e + 470*(
f*x)^m*a^2*d*m^3*x + 10706*(f*x)^m*a*b*d*m*x^3 + 5353*(f*x)^m*a^2*m*x^3*e +
 3010*(f*x)^m*a^2*d*m^2*x + 6930*(f*x)^m*a*b*d*x^3 + 3465*(f*x)^m*a^2*x^3*e
 + 9129*(f*x)^m*a^2*d*m*x + 10395*(f*x)^m*a^2*d*x)/(m^6 + 36*m^5 + 505*m^4
+ 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

maple [B]   time = 0.01, size = 783, normalized size = 5.05

$$\frac{\left(c^{2} e \,m^{5} x^{10}+25 c^{2} e \,m^{4} x^{10}+2 b c e \,m^{5} x^{8}+c^{2} d \,m^{5} x^{8}+230 c^{2} e \,m^{3} x^{10}+54 b c e \,m^{4} x^{8}+27 c^{2} d \,m^{4} x^{8}+950 c^{2} e \,m^{2} x^{10}+2 a c e \,m^{5} x^{6}+b^{2} e \,m^{5} x^{6}+2 b c d \,m^{5} x^{6}+524 b c e \,m^{3} x^{8}+262 c^{2} d \,m^{3} x^{8}+1689 c^{2} e m \,x^{10}+58 a c e \,m^{4} x^{6}+29 b^{2} e \,m^{4} x^{6}+58 b c d \,m^{4} x^{6}+2244 b c e \,m^{2} x^{8}+1122 c^{2} d \,m^{2} x^{8}+945 c^{2} e \,x^{10}+2 a b e \,m^{5} x^{4}+2 a c d \,m^{5} x^{4}+604 a c e \,m^{3} x^{6}+b^{2} d \,m^{5} x^{4}+302 b^{2} e \,m^{3} x^{6}+604 b c d \,m^{3} x^{6}+4082 b c e m \,x^{8}+2041 c^{2} d m \,x^{8}+62 a b e \,m^{4} x^{4}+62 a c d \,m^{4} x^{4}+2732 a c e \,m^{2} x^{6}+31 b^{2} d \,m^{4} x^{4}+1366 b^{2} e \,m^{2} x^{6}+2732 b c d \,m^{2} x^{6}+2310 b c e \,x^{8}+1155 c^{2} d \,x^{8}+a^{2} e \,m^{5} x^{2}+2 a b d \,m^{5} x^{2}+700 a b e \,m^{3} x^{4}+700 a c d \,m^{3} x^{4}+5154 a c e m \,x^{6}+350 b^{2} d \,m^{3} x^{4}+2577 b^{2} e m \,x^{6}+5154 b c d m \,x^{6}+33 a^{2} e \,m^{4} x^{2}+66 a b d \,m^{4} x^{2}+3460 a b e \,m^{2} x^{4}+3460 a c d \,m^{2} x^{4}+2970 a c e \,x^{6}+1730 b^{2} d \,m^{2} x^{4}+1485 b^{2} e \,x^{6}+2970 b c d \,x^{6}+a^{2} d \,m^{5}+406 a^{2} e \,m^{3} x^{2}+812 a b d \,m^{3} x^{2}+6978 a b e m \,x^{4}+6978 a c d m \,x^{4}+3489 b^{2} d m \,x^{4}+35 a^{2} d \,m^{4}+2262 a^{2} e \,m^{2} x^{2}+4524 a b d \,m^{2} x^{2}+4158 a b e \,x^{4}+4158 a c d \,x^{4}+2079 b^{2} d \,x^{4}+470 a^{2} d \,m^{3}+5353 a^{2} e m \,x^{2}+10706 a b d m \,x^{2}+3010 a^{2} d \,m^{2}+3465 a^{2} e \,x^{2}+6930 a b d \,x^{2}+9129 a^{2} d m+10395 a^{2} d \right) x \left(f x\right)^{m}}{\left(m+11\right) \left(m+9\right) \left(m+7\right) \left(m+5\right) \left(m+3\right) \left(m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] x*(c^2*e*m^5*x^10+25*c^2*e*m^4*x^10+2*b*c*e*m^5*x^8+c^2*d*m^5*x^8+230*c^2*e
*m^3*x^10+54*b*c*e*m^4*x^8+27*c^2*d*m^4*x^8+950*c^2*e*m^2*x^10+2*a*c*e*m^5*
x^6+b^2*e*m^5*x^6+2*b*c*d*m^5*x^6+524*b*c*e*m^3*x^8+262*c^2*d*m^3*x^8+1689*
c^2*e*m*x^10+58*a*c*e*m^4*x^6+29*b^2*e*m^4*x^6+58*b*c*d*m^4*x^6+2244*b*c*e*
m^2*x^8+1122*c^2*d*m^2*x^8+945*c^2*e*x^10+2*a*b*e*m^5*x^4+2*a*c*d*m^5*x^4+6
04*a*c*e*m^3*x^6+b^2*d*m^5*x^4+302*b^2*e*m^3*x^6+604*b*c*d*m^3*x^6+4082*b*c
*e*m*x^8+2041*c^2*d*m*x^8+62*a*b*e*m^4*x^4+62*a*c*d*m^4*x^4+2732*a*c*e*m^2*
x^6+31*b^2*d*m^4*x^4+1366*b^2*e*m^2*x^6+2732*b*c*d*m^2*x^6+2310*b*c*e*x^8+1
155*c^2*d*x^8+a^2*e*m^5*x^2+2*a*b*d*m^5*x^2+700*a*b*e*m^3*x^4+700*a*c*d*m^3
*x^4+5154*a*c*e*m*x^6+350*b^2*d*m^3*x^4+2577*b^2*e*m*x^6+5154*b*c*d*m*x^6+3
3*a^2*e*m^4*x^2+66*a*b*d*m^4*x^2+3460*a*b*e*m^2*x^4+3460*a*c*d*m^2*x^4+2970
*a*c*e*x^6+1730*b^2*d*m^2*x^4+1485*b^2*e*x^6+2970*b*c*d*x^6+a^2*d*m^5+406*a
^2*e*m^3*x^2+812*a*b*d*m^3*x^2+6978*a*b*e*m*x^4+6978*a*c*d*m*x^4+3489*b^2*d
*m*x^4+35*a^2*d*m^4+2262*a^2*e*m^2*x^2+4524*a*b*d*m^2*x^2+4158*a*b*e*x^4+41
58*a*c*d*x^4+2079*b^2*d*x^4+470*a^2*d*m^3+5353*a^2*e*m*x^2+10706*a*b*d*m*x^
2+3010*a^2*d*m^2+3465*a^2*e*x^2+6930*a*b*d*x^2+9129*a^2*d*m+10395*a^2*d)*(f
*x)^m/(m+11)/(m+9)/(m+7)/(m+5)/(m+3)/(m+1)
```

maxima [A]   time = 1.22, size = 230, normalized size = 1.48

$$\frac{c^{2} e f^{m} x^{11} x^{m}}{m+11}+\frac{c^{2} d f^{m} x^{9} x^{m}}{m+9}+\frac{2 \, b c e f^{m} x^{9} x^{m}}{m+9}+\frac{2 \, b c d f^{m} x^{7} x^{m}}{m+7}+\frac{b^{2} e f^{m} x^{7} x^{m}}{m+7}+\frac{2 \, a c e f^{m} x^{7} x^{m}}{m+7}+\frac{b^{2} d f^{m} x^{5} x^{m}}{m+5}+\frac{2 \, a c d f^{m} x^{5} x^{m}}{m+5}+\frac{2 \, a b e f^{m} x^{5} x^{m}}{m+5}+\frac{2 \, a b d f^{m} x^{3} x^{m}}{m+3}+\frac{a^{2} e f^{m} x^{3} x^{m}}{m+3}+\frac{\left(f x\right)^{m+1} a^{2} d}{f (m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] c^2*e*f^m*x^11*x^m/(m + 11) + c^2*d*f^m*x^9*x^m/(m + 9) + 2*b*c*e*f^m*x^9*x
^m/(m + 9) + 2*b*c*d*f^m*x^7*x^m/(m + 7) + b^2*e*f^m*x^7*x^m/(m + 7) + 2*a*
c*e*f^m*x^7*x^m/(m + 7) + b^2*d*f^m*x^5*x^m/(m + 5) + 2*a*c*d*f^m*x^5*x^m/(
m + 5) + 2*a*b*e*f^m*x^5*x^m/(m + 5) + 2*a*b*d*f^m*x^3*x^m/(m + 3) + a^2*e*
f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a^2*d/(f*(m + 1))
```

mupad [B]   time = 0.60, size = 429, normalized size = 2.77

$$\frac{x^5 \left(f\,x\right)^m\,\left(d\,b^2+2\,a\,e\,b+2\,a\,c\,d\right)\,\left(m^5+31\,m^4+350\,m^3+1730\,m^2+3489\,m+2079\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{x^7 \left(f\,x\right)^m\,\left(e\,b^2+2\,c\,d\,b+2\,a\,c\,e\right)\,\left(m^5+29\,m^4+302\,m^3+1366\,m^2+2577\,m+1485\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{a^2\,d\,x\,\left(f\,x\right)^m\,\left(m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{a\,x^3 \left(f\,x\right)^m\,\left(a\,e+2\,b\,d\right)\,\left(m^5+33\,m^4+406\,m^3+2262\,m^2+5353\,m+3465\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{c\,x^9 \left(f\,x\right)^m\,\left(2\,b\,e+c\,d\right)\,\left(m^5+27\,m^4+262\,m^3+1122\,m^2+2041\,m+1155\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{c^2\,e\,x^{11} \left(f\,x\right)^m\,\left(m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] (x^5*(f*x)^m*(b^2*d + 2*a*b*e + 2*a*c*d)*(3489*m + 1730*m^2 + 350*m^3 + 31*
m^4 + m^5 + 2079))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6
 + 10395) + (x^7*(f*x)^m*(b^2*e + 2*a*c*e + 2*b*c*d)*(2577*m + 1366*m^2 + 3
02*m^3 + 29*m^4 + m^5 + 1485))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 +
36*m^5 + m^6 + 10395) + (a^2*d*x*(f*x)^m*(9129*m + 3010*m^2 + 470*m^3 + 35*
m^4 + m^5 + 10395))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^
6 + 10395) + (a*x^3*(f*x)^m*(a*e + 2*b*d)*(5353*m + 2262*m^2 + 406*m^3 + 33
*m^4 + m^5 + 3465))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^
6 + 10395) + (c*x^9*(f*x)^m*(2*b*e + c*d)*(2041*m + 1122*m^2 + 262*m^3 + 27
*m^4 + m^5 + 1155))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^
6 + 10395) + (c^2*e*x^11*(f*x)^m*(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5
 + 945))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395)
```

sympy [A]   time = 5.44, size = 4190, normalized size = 27.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Piecewise(((-a**2*d/(10*x**10) - a**2*e/(8*x**8) - a*b*d/(4*x**8) - a*b*e/(
3*x**6) - a*c*d/(3*x**6) - a*c*e/(2*x**4) - b**2*d/(6*x**6) - b**2*e/(4*x**
4) - b*c*d/(2*x**4) - b*c*e/x**2 - c**2*d/(2*x**2) + c**2*e*log(x))/f**11,
Eq(m, -11)), ((-a**2*d/(8*x**8) - a**2*e/(6*x**6) - a*b*d/(3*x**6) - a*b*e/
(2*x**4) - a*c*d/(2*x**4) - a*c*e/x**2 - b**2*d/(4*x**4) - b**2*e/(2*x**2)
- b*c*d/x**2 + 2*b*c*e*log(x) + c**2*d*log(x) + c**2*e*x**2/2)/f**9, Eq(m,
-9)), ((-a**2*d/(6*x**6) - a**2*e/(4*x**4) - a*b*d/(2*x**4) - a*b*e/x**2 -
a*c*d/x**2 + 2*a*c*e*log(x) - b**2*d/(2*x**2) + b**2*e*log(x) + 2*b*c*d*log
(x) + b*c*e*x**2 + c**2*d*x**2/2 + c**2*e*x**4/4)/f**7, Eq(m, -7)), ((-a**2
*d/(4*x**4) - a**2*e/(2*x**2) - a*b*d/x**2 + 2*a*b*e*log(x) + 2*a*c*d*log(x
) + a*c*e*x**2 + b**2*d*log(x) + b**2*e*x**2/2 + b*c*d*x**2 + b*c*e*x**4/2
+ c**2*d*x**4/4 + c**2*e*x**6/6)/f**5, Eq(m, -5)), ((-a**2*d/(2*x**2) + a**
2*e*log(x) + 2*a*b*d*log(x) + a*b*e*x**2 + a*c*d*x**2 + a*c*e*x**4/2 + b**2
*d*x**2/2 + b**2*e*x**4/4 + b*c*d*x**4/2 + b*c*e*x**6/3 + c**2*d*x**6/6 + c
**2*e*x**8/8)/f**3, Eq(m, -3)), ((a**2*d*log(x) + a**2*e*x**2/2 + a*b*d*x**
2 + a*b*e*x**4/2 + a*c*d*x**4/2 + a*c*e*x**6/3 + b**2*d*x**4/4 + b**2*e*x**
6/6 + b*c*d*x**6/3 + b*c*e*x**8/4 + c**2*d*x**8/8 + c**2*e*x**10/10)/f, Eq(
m, -1)), (a**2*d*f**m*m**5*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 +
12139*m**2 + 19524*m + 10395) + 35*a**2*d*f**m*m**4*x*x**m/(m**6 + 36*m**5
+ 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 470*a**2*d*f**m*m*
*3*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1
0395) + 3010*a**2*d*f**m*m**2*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3
 + 12139*m**2 + 19524*m + 10395) + 9129*a**2*d*f**m*m*x*x**m/(m**6 + 36*m**
5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 10395*a**2*d*f**
m*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10
395) + a**2*e*f**m*m**5*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 +
12139*m**2 + 19524*m + 10395) + 33*a**2*e*f**m*m**4*x**3*x**m/(m**6 + 36*m*
*5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 406*a**2*e*f**m
*m**3*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524
*m + 10395) + 2262*a**2*e*f**m*m**2*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 +
3480*m**3 + 12139*m**2 + 19524*m + 10395) + 5353*a**2*e*f**m*m*x**3*x**m/(m
**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3465
*a**2*e*f**m*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2
+ 19524*m + 10395) + 2*a*b*d*f**m*m**5*x**3*x**m/(m**6 + 36*m**5 + 505*m**4
 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 66*a*b*d*f**m*m**4*x**3*x**m
/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 8
12*a*b*d*f**m*m**3*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139
*m**2 + 19524*m + 10395) + 4524*a*b*d*f**m*m**2*x**3*x**m/(m**6 + 36*m**5 +
 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 10706*a*b*d*f**m*m*
x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1
0395) + 6930*a*b*d*f**m*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 +
12139*m**2 + 19524*m + 10395) + 2*a*b*e*f**m*m**5*x**5*x**m/(m**6 + 36*m**5
 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 62*a*b*e*f**m*m**
4*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m +
 10395) + 700*a*b*e*f**m*m**3*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m
**3 + 12139*m**2 + 19524*m + 10395) + 3460*a*b*e*f**m*m**2*x**5*x**m/(m**6
+ 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 6978*a*b
*e*f**m*m*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 1
9524*m + 10395) + 4158*a*b*e*f**m*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 34
80*m**3 + 12139*m**2 + 19524*m + 10395) + 2*a*c*d*f**m*m**5*x**5*x**m/(m**6
 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 62*a*c*
d*f**m*m**4*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 +
 19524*m + 10395) + 700*a*c*d*f**m*m**3*x**5*x**m/(m**6 + 36*m**5 + 505*m**
4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3460*a*c*d*f**m*m**2*x**5*x
```

```
**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
 + 6978*a*c*d*f**m*m*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1213
9*m**2 + 19524*m + 10395) + 4158*a*c*d*f**m*x**5*x**m/(m**6 + 36*m**5 + 505
*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2*a*c*e*f**m*m**5*x**7*
x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
 + 58*a*c*e*f**m*m**4*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12
139*m**2 + 19524*m + 10395) + 604*a*c*e*f**m*m**3*x**7*x**m/(m**6 + 36*m**5
 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2732*a*c*e*f**m*m
**2*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
 + 10395) + 5154*a*c*e*f**m*m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m
**3 + 12139*m**2 + 19524*m + 10395) + 2970*a*c*e*f**m*x**7*x**m/(m**6 + 36*
m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + b**2*d*f**m*m
**5*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
 + 10395) + 31*b**2*d*f**m*m**4*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480
*m**3 + 12139*m**2 + 19524*m + 10395) + 350*b**2*d*f**m*m**3*x**5*x**m/(m**
6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1730*b
**2*d*f**m*m**2*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m*
*2 + 19524*m + 10395) + 3489*b**2*d*f**m*m*x**5*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2079*b**2*d*f**m*x**5*x*
*m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
 b**2*e*f**m*m**5*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*
m**2 + 19524*m + 10395) + 29*b**2*e*f**m*m**4*x**7*x**m/(m**6 + 36*m**5 + 5
05*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 302*b**2*e*f**m*m**3*
x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1
0395) + 1366*b**2*e*f**m*m**2*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m
**3 + 12139*m**2 + 19524*m + 10395) + 2577*b**2*e*f**m*m*x**7*x**m/(m**6 +
36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1485*b**2*
e*f**m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 1952
4*m + 10395) + 2*b*c*d*f**m*m**5*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 348
0*m**3 + 12139*m**2 + 19524*m + 10395) + 58*b*c*d*f**m*m**4*x**7*x**m/(m**6
 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 604*b*c
*d*f**m*m**3*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2
+ 19524*m + 10395) + 2732*b*c*d*f**m*m**2*x**7*x**m/(m**6 + 36*m**5 + 505*m
**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 5154*b*c*d*f**m*m*x**7*x*
*m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
 2970*b*c*d*f**m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m
**2 + 19524*m + 10395) + 2*b*c*e*f**m*m**5*x**9*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 54*b*c*e*f**m*m**4*x**9*
x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
 + 524*b*c*e*f**m*m**3*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1
2139*m**2 + 19524*m + 10395) + 2244*b*c*e*f**m*m**2*x**9*x**m/(m**6 + 36*m*
*5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 4082*b*c*e*f**m
*m*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
+ 10395) + 2310*b*c*e*f**m*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3
 + 12139*m**2 + 19524*m + 10395) + c**2*d*f**m*m**5*x**9*x**m/(m**6 + 36*m*
*5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 27*c**2*d*f**m*
m**4*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*
m + 10395) + 262*c**2*d*f**m*m**3*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 34
80*m**3 + 12139*m**2 + 19524*m + 10395) + 1122*c**2*d*f**m*m**2*x**9*x**m/(
m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 204
1*c**2*d*f**m*m*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m*
*2 + 19524*m + 10395) + 1155*c**2*d*f**m*x**9*x**m/(m**6 + 36*m**5 + 505*m*
*4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + c**2*e*f**m*m**5*x**11*x**
m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
25*c**2*e*f**m*m**4*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 121
39*m**2 + 19524*m + 10395) + 230*c**2*e*f**m*m**3*x**11*x**m/(m**6 + 36*m**
5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 950*c**2*e*f**m*
m**2*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524
*m + 10395) + 1689*c**2*e*f**m*m*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 34
```

```
80*m**3 + 12139*m**2 + 19524*m + 10395) + 945*c**2*e*f**m*x**11*x**m/(m**6
+ 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395), True))
```

3.158 $\int (fx)^m \left(d+ex^2\right)\left(a+bx^2+cx^4\right) dx$

Optimal. Leaf size=83

$$\frac{(fx)^{m+3}(ae+bd)}{f^3(m+3)}+\frac{ad(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+5}(be+cd)}{f^5(m+5)}+\frac{ce(fx)^{m+7}}{f^7(m+7)}$$

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1261}

$$\frac{(fx)^{m+3}(ae+bd)}{f^3(m+3)}+\frac{ad(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+5}(be+cd)}{f^5(m+5)}+\frac{ce(fx)^{m+7}}{f^7(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d*(f*x)^(1 + m))/(f*(1 + m)) + ((b*d + a*e)*(f*x)^(3 + m))/(f^3*(3 + m))
 + ((c*d + b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + (c*e*(f*x)^(7 + m))/(f^7*(7
+ m))
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int (fx)^m\left(d+ex^2\right)\left(a+bx^2+cx^4\right)dx&=\int\left(ad(fx)^m+\frac{(bd+ae)(fx)^{2+m}}{f^2}+\frac{(cd+be)(fx)^{4+m}}{f^4}+\frac{ce(fx)^{6+m}}{f^6}\right)dx\\&=\frac{ad(fx)^{1+m}}{f(1+m)}+\frac{(bd+ae)(fx)^{3+m}}{f^3(3+m)}+\frac{(cd+be)(fx)^{5+m}}{f^5(5+m)}+\frac{ce(fx)^{7+m}}{f^7(7+m)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.71

$$x(fx)^m\left(\frac{x^2(ae+bd)}{m+3}+\frac{ad}{m+1}+\frac{x^4(be+cd)}{m+5}+\frac{cex^6}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] x*(f*x)^m*((a*d)/(1 + m) + ((b*d + a*e)*x^2)/(3 + m) + ((c*d + b*e)*x^4)/(5
 + m) + (c*e*x^6)/(7 + m))
```

IntegrateAlgebraic [F] time = 0.12, size = 0, normalized size = 0.00

$$\int (fx)^m\left(d+ex^2\right)\left(a+bx^2+cx^4\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] Defer[IntegrateAlgebraic][(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4), x]
```

fricas [B] time = 0.72, size = 171, normalized size = 2.06

$$\frac{\left(\left(c e m^{3} + 9 \, c e m^{2} + 23 \, c e m + 15 \, c e\right)} x^{7} + \left({\left(c d + b e\right)} m^{3} + 11 \, {\left(c d + b e\right)} m^{2} + 21 \, c d + 21 \, b e + 31 \, {\left(c d + b e\right)} m\right)} x^{5} + \left({\left(b d + a e\right)} m^{3} + 13 \, {\left(b d + a e\right)} m^{2} + 35 \, b d + 35 \, a e + 47 \, {\left(b d + a e\right)} m\right)} x^{3} + {\left(a d m^{3} + 15 \, a d m^{2} + 71 \, a d m + 105 \, a d\right)} x\right)} \left(f x\right)^{m}}{m^{4} + 16 \, m^{3} + 86 \, m^{2} + 176 \, m + 105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] ((c*e*m^3 + 9*c*e*m^2 + 23*c*e*m + 15*c*e)*x^7 + ((c*d + b*e)*m^3 + 11*(c*d
 + b*e)*m^2 + 21*c*d + 21*b*e + 31*(c*d + b*e)*m)*x^5 + ((b*d + a*e)*m^3 +
13*(b*d + a*e)*m^2 + 35*b*d + 35*a*e + 47*(b*d + a*e)*m)*x^3 + (a*d*m^3 + 1
5*a*d*m^2 + 71*a*d*m + 105*a*d)*x)*(f*x)^m/(m^4 + 16*m^3 + 86*m^2 + 176*m +
 105)
```

giac [B] time = 0.42, size = 350, normalized size = 4.22

$$\frac{\left(f x\right)^{m} c m^{3} x^{7} e + 9 \, \left(f x\right)^{m} c m^{2} x^{7} e + \left(f x\right)^{m} c d m^{3} x^{5} + \left(f x\right)^{m} b m^{3} x^{5} e + 23 \, \left(f x\right)^{m} c m x^{7} e + 11 \, \left(f x\right)^{m} c d m^{2} x^{5} + 11 \, \left(f x\right)^{m} b m^{2} x^{5} e + 15 \, \left(f x\right)^{m} c x^{7} e + \left(f x\right)^{m} b d m^{3} x^{3} + 31 \, \left(f x\right)^{m} c d m x^{5} + \left(f x\right)^{m} a m^{3} x^{3} e + 31 \, \left(f x\right)^{m} b m x^{5} e + 13 \, \left(f x\right)^{m} b d m^{2} x^{3} + 21 \, \left(f x\right)^{m} c d x^{5} + 13 \, \left(f x\right)^{m} a m^{2} x^{3} e + 21 \, \left(f x\right)^{m} b x^{5} e + \left(f x\right)^{m} a d m^{3} x + 47 \, \left(f x\right)^{m} b d m x^{3} + 47 \, \left(f x\right)^{m} a m x^{3} e + 15 \, \left(f x\right)^{m} a d m^{2} x + 35 \, \left(f x\right)^{m} b d x^{3} + 35 \, \left(f x\right)^{m} a x^{3} e + 71 \, \left(f x\right)^{m} a d m x + 105 \, \left(f x\right)^{m} a d x}{m^{4} + 16 \, m^{3} + 86 \, m^{2} + 176 \, m + 105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] ((f*x)^m*c*m^3*x^7*e + 9*(f*x)^m*c*m^2*x^7*e + (f*x)^m*c*d*m^3*x^5 + (f*x)^
m*b*m^3*x^5*e + 23*(f*x)^m*c*m*x^7*e + 11*(f*x)^m*c*d*m^2*x^5 + 11*(f*x)^m*
b*m^2*x^5*e + 15*(f*x)^m*c*x^7*e + (f*x)^m*b*d*m^3*x^3 + 31*(f*x)^m*c*d*m*x
^5 + (f*x)^m*a*m^3*x^3*e + 31*(f*x)^m*b*m*x^5*e + 13*(f*x)^m*b*d*m^2*x^3 +
21*(f*x)^m*c*d*x^5 + 13*(f*x)^m*a*m^2*x^3*e + 21*(f*x)^m*b*x^5*e + (f*x)^m*
a*d*m^3*x + 47*(f*x)^m*b*d*m*x^3 + 47*(f*x)^m*a*m*x^3*e + 15*(f*x)^m*a*d*m^
2*x + 35*(f*x)^m*b*d*x^3 + 35*(f*x)^m*a*x^3*e + 71*(f*x)^m*a*d*m*x + 105*(f
*x)^m*a*d*x)/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

maple [B] time = 0.00, size = 221, normalized size = 2.66

$$\frac{\left(c e \,m^{3} x^{6}+9 c e \,m^{2} x^{6}+b e \,m^{3} x^{4}+c d \,m^{3} x^{4}+23 c e m \,x^{6}+11 b e \,m^{2} x^{4}+11 c d \,m^{2} x^{4}+15 c e \,x^{6}+a e \,m^{3} x^{2}+b d \,m^{3} x^{2}+31 b e m \,x^{4}+31 c d m \,x^{4}+13 a e \,m^{2} x^{2}+13 b d \,m^{2} x^{2}+21 b e \,x^{4}+21 c d \,x^{4}+a d \,m^{3}+47 a e m \,x^{2}+47 b d m \,x^{2}+15 a d \,m^{2}+35 a e \,x^{2}+35 b d \,x^{2}+71 a d m +105 a d \right) x \left(f x \right)^{m}}{\left(m +7\right) \left(m +5\right) \left(m +3\right) \left(m +1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x)
```

```
[Out] x*(c*e*m^3*x^6+9*c*e*m^2*x^6+b*e*m^3*x^4+c*d*m^3*x^4+23*c*e*m*x^6+11*b*e*m^
2*x^4+11*c*d*m^2*x^4+15*c*e*x^6+a*e*m^3*x^2+b*d*m^3*x^2+31*b*e*m*x^4+31*c*d
*m*x^4+13*a*e*m^2*x^2+13*b*d*m^2*x^2+21*b*e*x^4+21*c*d*x^4+a*d*m^3+47*a*e*m
*x^2+47*b*d*m*x^2+15*a*d*m^2+35*a*e*x^2+35*b*d*x^2+71*a*d*m+105*a*d)*(f*x)^
m/(m+7)/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.06, size = 104, normalized size = 1.25

$$\frac{c e f^{m} x^{7} x^{m}}{m + 7} + \frac{c d f^{m} x^{5} x^{m}}{m + 5} + \frac{b e f^{m} x^{5} x^{m}}{m + 5} + \frac{b d f^{m} x^{3} x^{m}}{m + 3} + \frac{a e f^{m} x^{3} x^{m}}{m + 3} + \frac{\left(f x\right)^{m + 1} a d}{f {\left(m + 1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] c*e*f^m*x^7*x^m/(m + 7) + c*d*f^m*x^5*x^m/(m + 5) + b*e*f^m*x^5*x^m/(m + 5)
 + b*d*f^m*x^3*x^m/(m + 3) + a*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d/(f
*(m + 1))
```

mupad [B] time = 0.34, size = 171, normalized size = 2.06

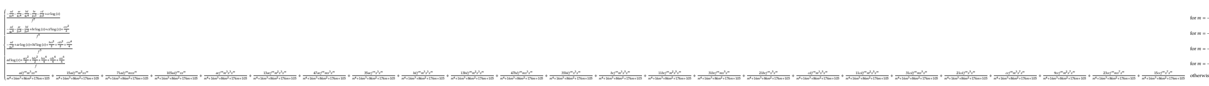
$${\left(f\,x\right)}^m\,\left(\frac{x^3\,\left(a\,e+b\,d\right)\,\left(m^3+13\,m^2+47\,m+35\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{x^5\,\left(b\,e+c\,d\right)\,\left(m^3+11\,m^2+31\,m+21\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{a\,d\,x\,\left(m^3+15\,m^2+71\,m+105\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{c\,e\,x^7\,\left(m^3+9\,m^2+23\,m+15\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] (f*x)^m*((x^3*(a*e + b*d)*(47*m + 13*m^2 + m^3 + 35))/(176*m + 86*m^2 + 16*
m^3 + m^4 + 105) + (x^5*(b*e + c*d)*(31*m + 11*m^2 + m^3 + 21))/(176*m + 86
*m^2 + 16*m^3 + m^4 + 105) + (a*d*x*(71*m + 15*m^2 + m^3 + 105))/(176*m + 8
6*m^2 + 16*m^3 + m^4 + 105) + (c*e*x^7*(23*m + 9*m^2 + m^3 + 15))/(176*m +
86*m^2 + 16*m^3 + m^4 + 105))
```

sympy [A] time = 1.86, size = 1056, normalized size = 12.72



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] Piecewise(((-a*d/(6*x**6) - a*e/(4*x**4) - b*d/(4*x**4) - b*e/(2*x**2) - c*
d/(2*x**2) + c*e*log(x))/f**7, Eq(m, -7)), ((-a*d/(4*x**4) - a*e/(2*x**2) -
 b*d/(2*x**2) + b*e*log(x) + c*d*log(x) + c*e*x**2/2)/f**5, Eq(m, -5)), ((-
a*d/(2*x**2) + a*e*log(x) + b*d*log(x) + b*e*x**2/2 + c*d*x**2/2 + c*e*x**4
/4)/f**3, Eq(m, -3)), ((a*d*log(x) + a*e*x**2/2 + b*d*x**2/2 + b*e*x**4/4 +
 c*d*x**4/4 + c*e*x**6/6)/f, Eq(m, -1)), (a*d*f**m*m**3*x*x**m/(m**4 + 16*m
**3 + 86*m**2 + 176*m + 105) + 15*a*d*f**m*m**2*x*x**m/(m**4 + 16*m**3 + 86
*m**2 + 176*m + 105) + 71*a*d*f**m*m*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176
*m + 105) + 105*a*d*f**m*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) +
a*e*f**m*m**3*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 13*a*e*f
**m*m**2*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 47*a*e*f**m*m
*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 35*a*e*f**m*x**3*x**m
/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + b*d*f**m*m**3*x**3*x**m/(m**4 +
 16*m**3 + 86*m**2 + 176*m + 105) + 13*b*d*f**m*m**2*x**3*x**m/(m**4 + 16*m
**3 + 86*m**2 + 176*m + 105) + 47*b*d*f**m*m*x**3*x**m/(m**4 + 16*m**3 + 86
*m**2 + 176*m + 105) + 35*b*d*f**m*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 17
6*m + 105) + b*e*f**m*m**3*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 10
5) + 11*b*e*f**m*m**2*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) +
31*b*e*f**m*m*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 21*b*e*f
**m*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + c*d*f**m*m**3*x**5
*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 11*c*d*f**m*m**2*x**5*x**m
/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 31*c*d*f**m*m*x**5*x**m/(m**4 +
 16*m**3 + 86*m**2 + 176*m + 105) + 21*c*d*f**m*x**5*x**m/(m**4 + 16*m**3 +
 86*m**2 + 176*m + 105) + c*e*f**m*m**3*x**7*x**m/(m**4 + 16*m**3 + 86*m**2
 + 176*m + 105) + 9*c*e*f**m*m**2*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176
*m + 105) + 23*c*e*f**m*m*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105
) + 15*c*e*f**m*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105), True))
```

3.159 $\int \frac{x^9}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx$

Optimal. Leaf size=134

$$\frac{a^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(ae^2+cd^2\right)}-\frac{a^2e\log\left(a+cx^4\right)}{4c^2\left(ae^2+cd^2\right)}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(ae^2+cd^2\right)}-\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}$$

Rubi [A] time = 0.18, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1252, 1629, 635, 205, 260}

$$-\frac{a^2e\log\left(a+cx^4\right)}{4c^2\left(ae^2+cd^2\right)}+\frac{a^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(ae^2+cd^2\right)}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(ae^2+cd^2\right)}-\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -(d*x^2)/(2*c*e^2) + x^4/(4*c*e) + (a^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]
)/(2*c^(3/2)*(c*d^2 + a*e^2)) + (d^4*Log[d + e*x^2])/(2*e^3*(c*d^2 + a*e^2)
) - (a^2*e*Log[a + c*x^4])/(4*c^2*(c*d^2 + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^9}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{(d+ex)\left(a+cx^2\right)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int\left(-\frac{d}{ce^2}+\frac{x}{ce}+\frac{d^4}{e^2\left(cd^2+ae^2\right)(d+ex)}+\frac{a^2(d-ex)}{c\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right)dx, x,$$
$$= -\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(cd^2+ae^2\right)}+\frac{a^2\,\text{Subst}\left(\int\frac{d-ex}{a+cx^2}\,dx, x, x^2\right)}{2c\left(cd^2+ae^2\right)}$$
$$= -\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(cd^2+ae^2\right)}+\frac{\left(a^2d\right)\text{Subst}\left(\int\frac{1}{a+cx^2}\,dx, x, x^2\right)}{2c\left(cd^2+ae^2\right)}-\frac{\left(a^2e\right)\text{Subst}\left(\right.}{2c\left(c\right.}$$
$$= -\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}+\frac{a^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(cd^2+ae^2\right)}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(cd^2+ae^2\right)}-\frac{a^2e\log\left(a+cx^4\right)}{4c^2\left(cd^2+ae^2\right)}$$

Mathematica [A] time = 0.06, size = 134, normalized size = 1.00

$$\frac{a^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(ae^2+cd^2\right)}-\frac{a^2e\log\left(a+cx^4\right)}{4c^2\left(ae^2+cd^2\right)}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(ae^2+cd^2\right)}-\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/2*(d*x^2)/(c*e^2) + x^4/(4*c*e) + (a^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a
]])/(2*c^(3/2)*(c*d^2 + a*e^2)) + (d^4*Log[d + e*x^2])/(2*e^3*(c*d^2 + a*e^
2)) - (a^2*e*Log[a + c*x^4])/(4*c^2*(c*d^2 + a*e^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^9/((d + e*x^2)*(a + c*x^4)), x]
```

fricas [A] time = 11.00, size = 277, normalized size = 2.07

$$\left[\frac{acde^3\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4+2cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)-a^2e^4\log\left(cx^4+a\right)+2c^2d^4\log\left(ex^2+d\right)+\left(c^2d^2e^2+ace^4\right)x^4-2\left(c^2d^3e+acde^3\right)x^2}{4\left(c^3d^2e^3+ac^2e^5\right)}, \frac{2acde^3\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right)-a^2e^4\log\left(cx^4+a\right)+2c^2d^4\log\left(ex^2+d\right)+\left(c^2d^2e^2+ace^4\right)x^4-2\left(c^2d^3e+acde^3\right)x^2}{4\left(c^3d^2e^3+ac^2e^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(a*c*d*e^3*sqrt(-a/c)*log((c*x^4 + 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)
) - a^2*e^4*log(c*x^4 + a) + 2*c^2*d^4*log(e*x^2 + d) + (c^2*d^2*e^2 + a*c*
e^4)*x^4 - 2*(c^2*d^3*e + a*c*d*e^3)*x^2)/(c^3*d^2*e^3 + a*c^2*e^5), 1/4*(2
*a*c*d*e^3*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) - a^2*e^4*log(c*x^4 + a) + 2
*c^2*d^4*log(e*x^2 + d) + (c^2*d^2*e^2 + a*c*e^4)*x^4 - 2*(c^2*d^3*e + a*c*
d*e^3)*x^2)/(c^3*d^2*e^3 + a*c^2*e^5)]
```

giac [A] time = 0.31, size = 121, normalized size = 0.90

$$\frac{d^4\log\left(\left|x^2e+d\right|\right)}{2\left(cd^2e^3+ae^5\right)}-\frac{a^2e\log\left(cx^4+a\right)}{4\left(c^3d^2+ac^2e^2\right)}+\frac{a^2d\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(c^2d^2+ace^2\right)\sqrt{ac}}+\frac{\left(cx^4e-2\,cdx^2\right)e^{(-2)}}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*d^4*log(abs(x^2*e + d))/(c*d^2*e^3 + a*e^5) - 1/4*a^2*e*log(c*x^4 + a)/
(c^3*d^2 + a*c^2*e^2) + 1/2*a^2*d*arctan(c*x^2/sqrt(a*c))/((c^2*d^2 + a*c*e
^2)*sqrt(a*c)) + 1/4*(c*x^4*e - 2*c*d*x^2)*e^(-2)/c^2
```

maple [A] time = 0.01, size = 122, normalized size = 0.91

$$\frac{a^2d\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{2\left(a\,e^2+c\,d^2\right)\sqrt{ac}\;c}-\frac{a^2e\ln\left(c\,x^4+a\right)}{4\left(a\,e^2+c\,d^2\right)c^2}+\frac{x^4}{4ce}+\frac{d^4\ln\left(e\,x^2+d\right)}{2\left(a\,e^2+c\,d^2\right)e^3}-\frac{d\,x^2}{2c\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/4*x^4/c/e-1/2*d*x^2/c/e^2-1/4*a^2*e*ln(c*x^4+a)/c^2/(a*e^2+c*d^2)+1/2*a^2
/(a*e^2+c*d^2)/c*d/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))+1/2*d^4*ln(e*x^2+d
)/e^3/(a*e^2+c*d^2)
```

maxima [A] time = 2.00, size = 120, normalized size = 0.90

$$\frac{d^4\log\left(ex^2+d\right)}{2\left(cd^2e^3+ae^5\right)}-\frac{a^2e\log\left(cx^4+a\right)}{4\left(c^3d^2+ac^2e^2\right)}+\frac{a^2d\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(c^2d^2+ace^2\right)\sqrt{ac}}+\frac{ex^4-2\,dx^2}{4\,ce^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*d^4*log(e*x^2 + d)/(c*d^2*e^3 + a*e^5) - 1/4*a^2*e*log(c*x^4 + a)/(c^3*
d^2 + a*c^2*e^2) + 1/2*a^2*d*arctan(c*x^2/sqrt(a*c))/((c^2*d^2 + a*c*e^2)*s
qrt(a*c)) + 1/4*(e*x^4 - 2*d*x^2)/(c*e^2)
```

mupad [B] time = 0.87, size = 181, normalized size = 1.35

$$\frac{\ln\left(\sqrt{-a^3\,c^5}+a\,c^3\,x^2\right)\left(d\,\sqrt{-a^3\,c^5}-a^2\,c^2\,e\right)}{4\,c^5\,d^2+4\,a\,c^4\,e^2}-\frac{\ln\left(\sqrt{-a^3\,c^5}-a\,c^3\,x^2\right)\left(d\,\sqrt{-a^3\,c^5}+a^2\,c^2\,e\right)}{4\,\left(c^5\,d^2+a\,c^4\,e^2\right)}+\frac{d^4\,\ln\left(e\,x^2+d\right)}{2\,c\,d^2\,e^3+2\,a\,e^5}+\frac{x^4}{4\,c\,e}-\frac{d\,x^2}{2\,c\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (log((-a^3*c^5)^(1/2) + a*c^3*x^2)*(d*(-a^3*c^5)^(1/2) - a^2*c^2*e))/(4*c^5
*d^2 + 4*a*c^4*e^2) - (log((-a^3*c^5)^(1/2) - a*c^3*x^2)*(d*(-a^3*c^5)^(1/2
) + a^2*c^2*e))/(4*(c^5*d^2 + a*c^4*e^2)) + (d^4*log(d + e*x^2))/(2*a*e^5 +
 2*c*d^2*e^3) + x^4/(4*c*e) - (d*x^2)/(2*c*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.160 $\int \frac{x^7}{\left(d+ex^2\right)\left(a+cx^4\right)} \, dx$

Optimal. Leaf size=118

$$-\frac{a^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(ae^2+cd^2\right)}-\frac{ad\log\left(a+cx^4\right)}{4c\left(ae^2+cd^2\right)}-\frac{d^3\log\left(d+ex^2\right)}{2e^2\left(ae^2+cd^2\right)}+\frac{x^2}{2ce}$$

Rubi [A] time = 0.15, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1252, 1629, 635, 205, 260}

$$-\frac{a^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(ae^2+cd^2\right)}-\frac{d^3\log\left(d+ex^2\right)}{2e^2\left(ae^2+cd^2\right)}-\frac{ad\log\left(a+cx^4\right)}{4c\left(ae^2+cd^2\right)}+\frac{x^2}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] x^2/(2*c*e) - (a^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*c^(3/2)*(c*d^2 +
 a*e^2)) - (d^3*Log[d + e*x^2])/(2*e^2*(c*d^2 + a*e^2)) - (a*d*Log[a + c*x^
4])/(4*c*(c*d^2 + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{(d+ex)\left(a+cx^2\right)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{ce}-\frac{d^3}{e\left(cd^2+ae^2\right)(d+ex)}-\frac{a(ae+cdx)}{c\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right)dx,x,x^2\right)\\
&= \frac{x^2}{2ce}-\frac{d^3\log\left(d+ex^2\right)}{2e^2\left(cd^2+ae^2\right)}-\frac{a\,\text{Subst}\left(\int \frac{ae+cdx}{a+cx^2}\,dx,x,x^2\right)}{2c\left(cd^2+ae^2\right)}\\
&= \frac{x^2}{2ce}-\frac{d^3\log\left(d+ex^2\right)}{2e^2\left(cd^2+ae^2\right)}-\frac{(ad)\,\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^2+ae^2\right)}-\frac{\left(a^2e\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x\right.}{2c\left(cd^2+ae^2\right)}\\
&= \frac{x^2}{2ce}-\frac{a^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(cd^2+ae^2\right)}-\frac{d^3\log\left(d+ex^2\right)}{2e^2\left(cd^2+ae^2\right)}-\frac{ad\log\left(a+cx^4\right)}{4c\left(cd^2+ae^2\right)}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 99, normalized size = 0.84

$$
\frac{-\frac{2a^{3/2}e^3\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{c^{3/2}}+\frac{e\left(2x^2\left(ae^2+cd^2\right)-ade\log\left(a+cx^4\right)\right)}{c}-2d^3\log\left(d+ex^2\right)}{4e^2\left(ae^2+cd^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] ((-2*a^(3/2)*e^3*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/c^(3/2) - 2*d^3*Log[d + e*x
^2] + (e*(2*(c*d^2 + a*e^2)*x^2 - a*d*e*Log[a + c*x^4]))/c)/(4*e^2*(c*d^2 +
 a*e^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^7}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^7/((d + e*x^2)*(a + c*x^4)), x]
```

fricas [A] time = 5.18, size = 212, normalized size = 1.80

$$
\left[\frac{ae^3\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4-2cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)-ade^2\log\left(cx^4+a\right)-2cd^3\log\left(ex^2+d\right)+2\left(cd^2e+ae^3\right)x^2}{4\left(c^2d^2e^2+ace^4\right)},\ -\frac{2ae^3\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right)+ade^2\log\left(cx^4+a\right)+2cd^3\log\left(ex^2+d\right)-2\left(cd^2e+ae^3\right)x^2}{4\left(c^2d^2e^2+ace^4\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(a*e^3*sqrt(-a/c)*log((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) -
a*d*e^2*log(c*x^4 + a) - 2*c*d^3*log(e*x^2 + d) + 2*(c*d^2*e + a*e^3)*x^2)/
(c^2*d^2*e^2 + a*c*e^4), -1/4*(2*a*e^3*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a)
+ a*d*e^2*log(c*x^4 + a) + 2*c*d^3*log(e*x^2 + d) - 2*(c*d^2*e + a*e^3)*x^2
)/(c^2*d^2*e^2 + a*c*e^4)]
```

giac [A] time = 0.31, size = 105, normalized size = 0.89

$$-\frac{d^3\log\left(\left|x^2e+d\right|\right)}{2\left(cd^2e^2+ae^4\right)}-\frac{a^2\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)e}{2\left(c^2d^2+ace^2\right)\sqrt{ac}}+\frac{x^2e^{(-1)}}{2c}-\frac{ad\log\left(cx^4+a\right)}{4\left(c^2d^2+ace^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*d^3*log(abs(x^2*e + d))/(c*d^2*e^2 + a*e^4) - 1/2*a^2*arctan(c*x^2/sqr
t(a*c))*e/((c^2*d^2 + a*c*e^2)*sqrt(a*c)) + 1/2*x^2*e^(-1)/c - 1/4*a*d*log(
c*x^4 + a)/(c^2*d^2 + a*c*e^2)
```

maple [A] time = 0.01, size = 108, normalized size = 0.92

$$-\frac{a^2e\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(ae^2+cd^2\right)\sqrt{ac}\,c}-\frac{ad\ln\left(cx^4+a\right)}{4\left(ae^2+cd^2\right)c}-\frac{d^3\ln\left(ex^2+d\right)}{2\left(ae^2+cd^2\right)e^2}+\frac{x^2}{2ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/2*x^2/c/e-1/4*a*d*ln(c*x^4+a)/c/(a*e^2+c*d^2)-1/2*a^2/(a*e^2+c*d^2)/c*e/(
a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)-1/2*d^3*ln(e*x^2+d)/e^2/(a*e^2+c*d^2
)
```

maxima [A] time = 2.02, size = 107, normalized size = 0.91

$$-\frac{d^3\log\left(ex^2+d\right)}{2\left(cd^2e^2+ae^4\right)}-\frac{a^2e\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(c^2d^2+ace^2\right)\sqrt{ac}}-\frac{ad\log\left(cx^4+a\right)}{4\left(c^2d^2+ace^2\right)}+\frac{x^2}{2\,ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/2*d^3*log(e*x^2 + d)/(c*d^2*e^2 + a*e^4) - 1/2*a^2*e*arctan(c*x^2/sqrt(a
*c))/((c^2*d^2 + a*c*e^2)*sqrt(a*c)) - 1/4*a*d*log(c*x^4 + a)/(c^2*d^2 + a*
c*e^2) + 1/2*x^2/(c*e)
```

mupad [B] time = 0.72, size = 166, normalized size = 1.41

$$\frac{x^2}{2\,c\,e}-\frac{d^3\ln\left(e\,x^2+d\right)}{2\,c\,d^2\,e^2+2\,a\,e^4}-\frac{\ln\left(\sqrt{-a^3\,c^3}+a\,c^2\,x^2\right)\,\left(e\,\sqrt{-a^3\,c^3}+a\,c^2\,d\right)}{4\,\left(c^4\,d^2+a\,c^3\,e^2\right)}+\frac{\ln\left(\sqrt{-a^3\,c^3}-a\,c^2\,x^2\right)\,\left(e\,\sqrt{-a^3\,c^3}-a\,c^2\,d\right)}{4\,c^4\,d^2+4\,a\,c^3\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] x^2/(2*c*e) - (d^3*log(d + e*x^2))/(2*a*e^4 + 2*c*d^2*e^2) - (log((-a^3*c^3
)^(1/2) + a*c^2*x^2)*(e*(-a^3*c^3)^(1/2) + a*c^2*d))/(4*(c^4*d^2 + a*c^3*e^
2)) + (log((-a^3*c^3)^(1/2) - a*c^2*x^2)*(e*(-a^3*c^3)^(1/2) - a*c^2*d))/(4
*c^4*d^2 + 4*a*c^3*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.161 $\int \frac{x^5}{(d+ex^2)(a+cx^4)} \, dx$

Optimal. Leaf size=105

$$\frac{ae\log\left(a+cx^4\right)}{4c\left(ae^2+cd^2\right)}+\frac{d^2\log\left(d+ex^2\right)}{2e\left(ae^2+cd^2\right)}-\frac{\sqrt{a}\,d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{c}\,\left(ae^2+cd^2\right)}$$

Rubi [A] time = 0.14, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1252, 1629, 635, 205, 260}

$$\frac{d^2\log\left(d+ex^2\right)}{2e\left(ae^2+cd^2\right)}+\frac{ae\log\left(a+cx^4\right)}{4c\left(ae^2+cd^2\right)}-\frac{\sqrt{a}\,d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{c}\,\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -(Sqrt[a]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[c]*(c*d^2 + a*e^2)) + (d
^2*Log[d + e*x^2])/(2*e*(c*d^2 + a*e^2)) + (a*e*Log[a + c*x^4])/(4*c*(c*d^2
 + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{(d+ex^2)(a+cx^4)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d^2}{(cd^2+ae^2)(d+ex)} - \frac{a(d-ex)}{(cd^2+ae^2)(a+cx^2)}\right)dx, x, x^2\right)\\
&= \frac{d^2\log(d+ex^2)}{2e(cd^2+ae^2)} - \frac{a\,\text{Subst}\left(\int \frac{d-ex}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)}\\
&= \frac{d^2\log(d+ex^2)}{2e(cd^2+ae^2)} - \frac{(ad)\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)} + \frac{(ae)\,\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)}\\
&= -\frac{\sqrt{a}\,d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{c}\,(cd^2+ae^2)} + \frac{d^2\log(d+ex^2)}{2e(cd^2+ae^2)} + \frac{ae\log(a+cx^4)}{4c(cd^2+ae^2)}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.73

$$
\frac{-2\sqrt{a}\sqrt{c}\,de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right) + ae^2\log(a+cx^4) + 2cd^2\log(d+ex^2)}{4ace^3+4c^2d^2e}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-2*Sqrt[a]*Sqrt[c]*d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]] + 2*c*d^2*Log[d + e*x
^2] + a*e^2*Log[a + c*x^4])/(4*c^2*d^2*e + 4*a*c*e^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^5}{(d+ex^2)(a+cx^4)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^5/((d + e*x^2)*(a + c*x^4)), x]
```

fricas [A] time = 2.92, size = 170, normalized size = 1.62

$$
\left[\frac{cde\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4-2cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right) + ae^2\log(cx^4+a) + 2cd^2\log(ex^2+d)}{4(c^2d^2e+ace^3)}, -\frac{2cde\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right) - ae^2\log(cx^4+a) - 2cd^2\log(ex^2+d)}{4(c^2d^2e+ace^3)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(c*d*e*sqrt(-a/c)*log((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) +
a*e^2*log(c*x^4 + a) + 2*c*d^2*log(e*x^2 + d))/(c^2*d^2*e + a*c*e^3), -1/4*
(2*c*d*e*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) - a*e^2*log(c*x^4 + a) - 2*c*d
^2*log(e*x^2 + d))/(c^2*d^2*e + a*c*e^3)]
```

giac [A] time = 0.36, size = 90, normalized size = 0.86

$$
\frac{ae\log(cx^4+a)}{4(c^2d^2+ace^2)} + \frac{d^2\log(|x^2e+d|)}{2(cd^2e+ae^3)} - \frac{ad\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(cd^2+ae^2)\sqrt{ac}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*a*e*log(c*x^4 + a)/(c^2*d^2 + a*c*e^2) + 1/2*d^2*log(abs(x^2*e + d))/(c
*d^2*e + a*e^3) - 1/2*a*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c
))
```

maple [A] time = 0.01, size = 92, normalized size = 0.88

$$-\frac{ad\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{2\left(a\,e^2+c\,d^2\right)\sqrt{ac}}+\frac{ae\ln\left(c\,x^4+a\right)}{4\left(a\,e^2+c\,d^2\right)c}+\frac{d^2\ln\left(e\,x^2+d\right)}{2\left(a\,e^2+c\,d^2\right)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/4*a*e*ln(c*x^4+a)/c/(a*e^2+c*d^2)-1/2*a/(a*e^2+c*d^2)*d/(a*c)^(1/2)*arcta
n(1/(a*c)^(1/2)*c*x^2)+1/2*d^2*ln(e*x^2+d)/e/(a*e^2+c*d^2)
```

maxima [A] time = 2.00, size = 89, normalized size = 0.85

$$\frac{ae\log\left(cx^4+a\right)}{4\left(c^2d^2+ace^2\right)}+\frac{d^2\log\left(ex^2+d\right)}{2\left(cd^2e+ae^3\right)}-\frac{ad\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(cd^2+ae^2\right)\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*a*e*log(c*x^4 + a)/(c^2*d^2 + a*c*e^2) + 1/2*d^2*log(e*x^2 + d)/(c*d^2*
e + a*e^3) - 1/2*a*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c))
```

mupad [B] time = 0.99, size = 138, normalized size = 1.31

$$\frac{d^2\,\ln\left(e\,x^2+d\right)}{2\,c\,d^2\,e+2\,a\,e^3}-\frac{\ln\left(\sqrt{-a\,c^3}+c^2\,x^2\right)\,\left(d\,\sqrt{-a\,c^3}-a\,c\,e\right)}{4\,\left(c^3\,d^2+a\,c^2\,e^2\right)}+\frac{\ln\left(\sqrt{-a\,c^3}-c^2\,x^2\right)\,\left(d\,\sqrt{-a\,c^3}+a\,c\,e\right)}{4\,c^3\,d^2+4\,a\,c^2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (d^2*log(d + e*x^2))/(2*a*e^3 + 2*c*d^2*e) - (log((-a*c^3)^(1/2) + c^2*x^2)
*(d*(-a*c^3)^(1/2) - a*c*e))/(4*(c^3*d^2 + a*c^2*e^2)) + (log((-a*c^3)^(1/2
) - c^2*x^2)*(d*(-a*c^3)^(1/2) + a*c*e))/(4*c^3*d^2 + 4*a*c^2*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.162 $\int \frac{x^3}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=96

$$\frac{d\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)}-\frac{d\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)}+\frac{\sqrt{a}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{c}\,\left(ae^2+cd^2\right)}$$

Rubi [A] time = 0.09, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1252, 801, 635, 205, 260}

$$-\frac{d\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)}+\frac{d\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)}+\frac{\sqrt{a}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{c}\,\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (Sqrt[a]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[c]*(c*d^2 + a*e^2)) - (d*
Log[d + e*x^2])/(2*(c*d^2 + a*e^2)) + (d*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)
)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{de}{(cd^2+ae^2)(d+ex)} + \frac{ae+cdx}{(cd^2+ae^2)(a+cx^2)}\right) dx, x, x^2\right)$$
$$= -\frac{d\log(d+ex^2)}{2(cd^2+ae^2)} + \frac{\operatorname{Subst}\left(\int \frac{ae+cdx}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)}$$
$$= -\frac{d\log(d+ex^2)}{2(cd^2+ae^2)} + \frac{(cd)\operatorname{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)} + \frac{(ae)\operatorname{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)}$$
$$= \frac{\sqrt{a}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{c}\,(cd^2+ae^2)} - \frac{d\log(d+ex^2)}{2(cd^2+ae^2)} + \frac{d\log(a+cx^4)}{4(cd^2+ae^2)}$$

Mathematica [A] time = 0.04, size = 66, normalized size = 0.69

$$\frac{d\log(a+cx^4) + \frac{2\sqrt{a}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{c}} - 2d\log(d+ex^2)}{4ae^2+4cd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] ((2*Sqrt[a]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/Sqrt[c] - 2*d*Log[d + e*x^2] +
 d*Log[a + c*x^4])/(4*c*d^2 + 4*a*e^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^3/((d + e*x^2)*(a + c*x^4)), x]
```

fricas [A] time = 1.33, size = 145, normalized size = 1.51

$$\left[\frac{e\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4+2cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right) + d\log(cx^4+a) - 2d\log(ex^2+d)}{4(cd^2+ae^2)}, \frac{2e\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right) + d\log(cx^4+a) - 2d\log(ex^2+d)}{4(cd^2+ae^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(e*sqrt(-a/c)*log((c*x^4 + 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) + d*lo
g(c*x^4 + a) - 2*d*log(e*x^2 + d))/(c*d^2 + a*e^2), 1/4*(2*e*sqrt(a/c)*arct
an(c*x^2*sqrt(a/c)/a) + d*log(c*x^4 + a) - 2*d*log(e*x^2 + d))/(c*d^2 + a*e
^2)]
```

giac [A] time = 0.39, size = 86, normalized size = 0.90

$$-\frac{de\log\left(\left|x^2e+d\right|\right)}{2\left(cd^2e+ae^3\right)}+\frac{a\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)e}{2\left(cd^2+ae^2\right)\sqrt{ac}}+\frac{d\log\left(cx^4+a\right)}{4\left(cd^2+ae^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*d*e*log(abs(x^2*e + d))/(c*d^2*e + a*e^3) + 1/2*a*arctan(c*x^2/sqrt(a*
c))*e/((c*d^2 + a*e^2)*sqrt(a*c)) + 1/4*d*log(c*x^4 + a)/(c*d^2 + a*e^2)
```

maple [A] time = 0.01, size = 83, normalized size = 0.86

$$\frac{ae\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(ae^2+cd^2\right)\sqrt{ac}}+\frac{d\ln\left(cx^4+a\right)}{4ae^2+4cd^2}-\frac{d\ln\left(ex^2+d\right)}{2\left(ae^2+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/4*d*ln(c*x^4+a)/(a*e^2+c*d^2)+1/2/(a*e^2+c*d^2)*a*e/(a*c)^(1/2)*arctan(1/
(a*c)^(1/2)*c*x^2)-1/2*d*ln(e*x^2+d)/(a*e^2+c*d^2)
```

maxima [A] time = 1.99, size = 82, normalized size = 0.85

$$\frac{ae\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(cd^2+ae^2\right)\sqrt{ac}}+\frac{d\log\left(cx^4+a\right)}{4\left(cd^2+ae^2\right)}-\frac{d\log\left(ex^2+d\right)}{2\left(cd^2+ae^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*a*e*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) + 1/4*d*log(c*x
^4 + a)/(c*d^2 + a*e^2) - 1/2*d*log(e*x^2 + d)/(c*d^2 + a*e^2)
```

mupad [B] time = 1.94, size = 944, normalized size = 9.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (c*d*log(a^4*e^6 - 9*a*c^3*d^6 - 39*a^3*c*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/2
) - 9*c^3*d^6*x^2*(-a*c)^(1/2) + 79*a^2*c^2*d^4*e^2 - 42*c*d^5*e*(-a*c)^(3/
2) + 76*a*d^3*e^3*(-a*c)^(3/2) + 10*a^3*d*e^5*(-a*c)^(1/2) + 76*a^2*c^2*d^3
*e^3*x^2 - 42*a*c^3*d^5*e*x^2 - 10*a^3*c*d*e^5*x^2 + 39*a*d^2*e^4*x^2*(-a*c
)^(3/2) - 79*c*d^4*e^2*x^2*(-a*c)^(3/2)))/(4*c^2*d^2 + 4*a*c*e^2) - (d*log(
d + e*x^2))/(2*(a*e^2 + c*d^2)) + (c*d*log(9*a*c^3*d^6 - a^4*e^6 + 39*a^3*c
*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/2) - 9*c^3*d^6*x^2*(-a*c)^(1/2) - 79*a^2*c
^2*d^4*e^2 + 10*a^3*d*e^5*(-a*c)^(1/2) + 42*a*c^2*d^5*e*(-a*c)^(1/2) - 76*a
^2*c^2*d^3*e^3*x^2 + 42*a*c^3*d^5*e*x^2 + 10*a^3*c*d*e^5*x^2 - 76*a^2*c*d^3
*e^3*(-a*c)^(1/2) + 79*a*c^2*d^4*e^2*x^2*(-a*c)^(1/2) - 39*a^2*c*d^2*e^4*x^
2*(-a*c)^(1/2)))/(4*c^2*d^2 + 4*a*c*e^2) - (e*log(a^4*e^6 - 9*a*c^3*d^6 - 3
9*a^3*c*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/2) - 9*c^3*d^6*x^2*(-a*c)^(1/2) + 7
9*a^2*c^2*d^4*e^2 - 42*c*d^5*e*(-a*c)^(3/2) + 76*a*d^3*e^3*(-a*c)^(3/2) + 1
0*a^3*d*e^5*(-a*c)^(1/2) + 76*a^2*c^2*d^3*e^3*x^2 - 42*a*c^3*d^5*e*x^2 - 10
*a^3*c*d*e^5*x^2 + 39*a*d^2*e^4*x^2*(-a*c)^(3/2) - 79*c*d^4*e^2*x^2*(-a*c)^
```

```
(3/2))*(-a*c)^(1/2))/(4*c^2*d^2 + 4*a*c*e^2) + (e*log(9*a*c^3*d^6 - a^4*e^6
 + 39*a^3*c*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/2) - 9*c^3*d^6*x^2*(-a*c)^(1/2)
 - 79*a^2*c^2*d^4*e^2 + 10*a^3*d*e^5*(-a*c)^(1/2) + 42*a*c^2*d^5*e*(-a*c)^(
1/2) - 76*a^2*c^2*d^3*e^3*x^2 + 42*a*c^3*d^5*e*x^2 + 10*a^3*c*d*e^5*x^2 - 7
6*a^2*c*d^3*e^3*(-a*c)^(1/2) + 79*a*c^2*d^4*e^2*x^2*(-a*c)^(1/2) - 39*a^2*c
*d^2*e^4*x^2*(-a*c)^(1/2))*(-a*c)^(1/2))/(4*c^2*d^2 + 4*a*c*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+a),x)

[Out] Timed out
```

3.163 $\int \frac{x}{(d+ex^2)(a+cx^4)} \, dx$

Optimal. Leaf size=96

$$-\frac{e \log \left(a+c x^4\right)}{4 \left(a e^2+c d^2\right)}+\frac{e \log \left(d+e x^2\right)}{2 \left(a e^2+c d^2\right)}+\frac{\sqrt{c} d \tan ^{-1}\left(\frac{\sqrt{c} x^2}{\sqrt{a}}\right)}{2 \sqrt{a} \left(a e^2+c d^2\right)}$$

Rubi [A] time = 0.06, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1248, 706, 31, 635, 205, 260}

$$\frac{e \log \left(d+e x^2\right)}{2 \left(a e^2+c d^2\right)}-\frac{e \log \left(a+c x^4\right)}{4 \left(a e^2+c d^2\right)}+\frac{\sqrt{c} d \tan ^{-1}\left(\frac{\sqrt{c} x^2}{\sqrt{a}}\right)}{2 \sqrt{a} \left(a e^2+c d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (Sqrt[c]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^2 + a*e^2)) + (e*
Log[d + e*x^2])/(2*(c*d^2 + a*e^2)) - (e*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(d+ex)\left(a+cx^2\right)}\,dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{cd-cex}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)} + \frac{e^2\,\text{Subst}\left(\int \frac{1}{d+ex}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)}$$
$$= \frac{e\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)} + \frac{(cd)\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)} - \frac{(ce)\,\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)}$$
$$= \frac{\sqrt{c}\,d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\left(cd^2+ae^2\right)} + \frac{e\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)} - \frac{e\log\left(a+cx^4\right)}{4\left(cd^2+ae^2\right)}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 0.70

$$\frac{\frac{2\sqrt{c}\,d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}} - e\log\left(a+cx^4\right) + 2e\log\left(d+ex^2\right)}{4ae^2+4cd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] ((2*Sqrt[c]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/Sqrt[a] + 2*e*Log[d + e*x^2] -
 e*Log[a + c*x^4])/(4*c*d^2 + 4*a*e^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x/((d + e*x^2)*(a + c*x^4)), x]
```

fricas [A] time = 1.50, size = 146, normalized size = 1.52

$$\left[\frac{d\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4+2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right) - e\log\left(cx^4+a\right) + 2\,e\log\left(ex^2+d\right)}{4\left(cd^2+ae^2\right)}, -\frac{2\,d\sqrt{\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^2}\right) + e\log\left(cx^4+a\right) - 2\,e\log\left(ex^2+d\right)}{4\left(cd^2+ae^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(d*sqrt(-c/a)*log((c*x^4 + 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) - e*lo
g(c*x^4 + a) + 2*e*log(e*x^2 + d))/(c*d^2 + a*e^2), -1/4*(2*d*sqrt(c/a)*arc
tan(a*sqrt(c/a)/(c*x^2)) + e*log(c*x^4 + a) - 2*e*log(e*x^2 + d))/(c*d^2 +
a*e^2)]
```

giac [A] time = 0.35, size = 85, normalized size = 0.89

$$\frac{cd\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(cd^2+ae^2\right)\sqrt{ac}} - \frac{e\log\left(cx^4+a\right)}{4\left(cd^2+ae^2\right)} + \frac{e^2\log\left(\left|x^2e+d\right|\right)}{2\left(cd^2e+ae^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*c*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) - 1/4*e*log(c*x
^4 + a)/(c*d^2 + a*e^2) + 1/2*e^2*log(abs(x^2*e + d))/(c*d^2*e + a*e^3)
```

maple [A] time = 0.01, size = 83, normalized size = 0.86

$$\frac{cd\arctan\left(\frac{c\,x^{2}}{\sqrt{ac}}\right)}{2\left(a\,e^{2}+c\,d^{2}\right)\sqrt{ac}}-\frac{e\ln\left(c\,x^{4}+a\right)}{4\left(a\,e^{2}+c\,d^{2}\right)}+\frac{e\ln\left(e\,x^{2}+d\right)}{2a\,e^{2}+2c\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] -1/4*e*ln(c*x^4+a)/(a*e^2+c*d^2)+1/2*c/(a*e^2+c*d^2)*d/(a*c)^(1/2)*arctan(1
/(a*c)^(1/2)*c*x^2)+1/2*e*ln(e*x^2+d)/(a*e^2+c*d^2)
```

maxima [A] time = 1.98, size = 82, normalized size = 0.85

$$\frac{cd\arctan\left(\frac{cx^{2}}{\sqrt{ac}}\right)}{2\left(cd^{2}+ae^{2}\right)\sqrt{ac}}-\frac{e\log\left(cx^{4}+a\right)}{4\left(cd^{2}+ae^{2}\right)}+\frac{e\log\left(ex^{2}+d\right)}{2\left(cd^{2}+ae^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*c*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) - 1/4*e*log(c*x
^4 + a)/(c*d^2 + a*e^2) + 1/2*e*log(e*x^2 + d)/(c*d^2 + a*e^2)
```

mupad [B] time = 1.02, size = 328, normalized size = 3.42

$$\frac{e\,\ln\left(e\,x^2+d\right)}{2\,c\,d^2+2\,a\,e^2}-\frac{\ln\left(a\,c^5\,d^6\,x^2-c^3\,d^6\,{\left(-a\,c\right)}^{3/2}-9\,a^3\,e^6\,{\left(-a\,c\right)}^{3/2}+9\,a^4\,c^2\,e^6\,x^2+19\,a\,d^2\,e^4\,{\left(-a\,c\right)}^{5/2}+11\,c\,d^4\,e^2\,{\left(-a\,c\right)}^{5/2}+11\,a^2\,c^4\,d^4\,e^2\,x^2+19\,a^3\,c^3\,d^2\,e^4\,x^2\right)\,\left(a\,e-d\,\sqrt{-a\,c}\right)}{4\,\left(a^2\,e^2+c\,a\,d^2\right)}-\frac{\ln\left(9\,a^3\,e^6\,{\left(-a\,c\right)}^{3/2}+c^3\,d^6\,{\left(-a\,c\right)}^{3/2}+a\,c^5\,d^6\,x^2+9\,a^4\,c^2\,e^6\,x^2-19\,a\,d^2\,e^4\,{\left(-a\,c\right)}^{5/2}-11\,c\,d^4\,e^2\,{\left(-a\,c\right)}^{5/2}+11\,a^2\,c^4\,d^4\,e^2\,x^2+19\,a^3\,c^3\,d^2\,e^4\,x^2\right)\,\left(a\,e+d\,\sqrt{-a\,c}\right)}{4\,\left(a^2\,e^2+c\,a\,d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (e*log(d + e*x^2))/(2*a*e^2 + 2*c*d^2) - (log(a*c^5*d^6*x^2 - c^3*d^6*(-a*c
)^(3/2) - 9*a^3*e^6*(-a*c)^(3/2) + 9*a^4*c^2*e^6*x^2 + 19*a*d^2*e^4*(-a*c)^
(5/2) + 11*c*d^4*e^2*(-a*c)^(5/2) + 11*a^2*c^4*d^4*e^2*x^2 + 19*a^3*c^3*d^2
*e^4*x^2)*(a*e - d*(-a*c)^(1/2)))/(4*(a^2*e^2 + a*c*d^2)) - (log(9*a^3*e^6*
(-a*c)^(3/2) + c^3*d^6*(-a*c)^(3/2) + a*c^5*d^6*x^2 + 9*a^4*c^2*e^6*x^2 - 1
9*a*d^2*e^4*(-a*c)^(5/2) - 11*c*d^4*e^2*(-a*c)^(5/2) + 11*a^2*c^4*d^4*e^2*x
^2 + 19*a^3*c^3*d^2*e^4*x^2)*(a*e + d*(-a*c)^(1/2)))/(4*(a^2*e^2 + a*c*d^2)
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.164 $\int \frac{1}{x(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=114

$$-\frac{cd\log\left(a+cx^4\right)}{4a\left(ae^2+cd^2\right)}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(ae^2+cd^2\right)}-\frac{\sqrt{c}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^2+cd^2\right)}+\frac{\log(x)}{ad}$$

Rubi [A] time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1252, 894, 635, 205, 260}

$$-\frac{e^2\log\left(d+ex^2\right)}{2d\left(ae^2+cd^2\right)}-\frac{cd\log\left(a+cx^4\right)}{4a\left(ae^2+cd^2\right)}-\frac{\sqrt{c}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^2+cd^2\right)}+\frac{\log(x)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -(Sqrt[c]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^2 + a*e^2)) + Lo
g[x]/(a*d) - (e^2*Log[d + e*x^2])/(2*d*(c*d^2 + a*e^2)) - (c*d*Log[a + c*x^
4])/(4*a*(c*d^2 + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(d+ex^2\right)\left(a+cx^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(d+ex)\left(a+cx^2\right)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{adx}-\frac{e^3}{d\left(cd^2+ae^2\right)(d+ex)}-\frac{c(ae+cdx)}{a\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right)dx, x, x^2\right)\\
&= \frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2+ae^2\right)}-\frac{c\,\text{Subst}\left(\int \frac{ae+cdx}{a+cx^2}\,dx, x, x^2\right)}{2a\left(cd^2+ae^2\right)}\\
&= \frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2+ae^2\right)}-\frac{\left(c^2d\right)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2a\left(cd^2+ae^2\right)}-\frac{(ce)\,\text{Subst}\left(\int \frac{1}{a+cx^2}\right.}{2\left(cd^2+ae\right.}\\
&= -\frac{\sqrt{c}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^2+ae^2\right)}+\frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2+ae^2\right)}-\frac{cd\log\left(a+cx^4\right)}{4a\left(cd^2+ae^2\right)}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 134, normalized size = 1.18

$$
\frac{-cd^2\log\left(a+cx^4\right)+2\sqrt{a}\sqrt{c}\,de\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+2\sqrt{a}\sqrt{c}\,de\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)-2ae^2\log\left(d+ex^2\right)+4ae^2\log(x)+4cd^2\log(x)}{4a^2de^2+4acd^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[c]*d*e*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[a]*
Sqrt[c]*d*e*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 4*c*d^2*Log[x] + 4*a*
e^2*Log[x] - 2*a*e^2*Log[d + e*x^2] - c*d^2*Log[a + c*x^4])/(4*a*c*d^3 + 4*
a^2*d*e^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x\left(d+ex^2\right)\left(a+cx^4\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(d + e*x^2)*(a + c*x^4)), x]
```

fricas [A] time = 16.72, size = 201, normalized size = 1.76

$$
\left[\frac{ade\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4-2ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)-cd^2\log\left(cx^4+a\right)-2ae^2\log\left(ex^2+d\right)+4\left(cd^2+ae^2\right)\log(x)}{4\left(acd^3+a^2de^2\right)}, \frac{2ade\sqrt{\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^2}\right)-cd^2\log\left(cx^4+a\right)-2ae^2\log\left(ex^2+d\right)+4\left(cd^2+ae^2\right)\log(x)}{4\left(acd^3+a^2de^2\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(a*d*e*sqrt(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) -
c*d^2*log(c*x^4 + a) - 2*a*e^2*log(e*x^2 + d) + 4*(c*d^2 + a*e^2)*log(x))/(
a*c*d^3 + a^2*d*e^2), 1/4*(2*a*d*e*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) -
c*d^2*log(c*x^4 + a) - 2*a*e^2*log(e*x^2 + d) + 4*(c*d^2 + a*e^2)*log(x))/(
a*c*d^3 + a^2*d*e^2)]
```

giac [A] time = 0.29, size = 102, normalized size = 0.89

$$-\frac{cd\log\left(cx^4+a\right)}{4\left(acd^2+a^2e^2\right)}-\frac{c\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)e}{2\left(cd^2+ae^2\right)\sqrt{ac}}-\frac{e^3\log\left(\left|x^2e+d\right|\right)}{2\left(cd^3e+ade^3\right)}+\frac{\log\left(x^2\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*c*d*log(c*x^4 + a)/(a*c*d^2 + a^2*e^2) - 1/2*c*arctan(c*x^2/sqrt(a*c))
*e/((c*d^2 + a*e^2)*sqrt(a*c)) - 1/2*e^3*log(abs(x^2*e + d))/(c*d^3*e + a*d
*e^3) + 1/2*log(x^2)/(a*d)
```

maple [A] time = 0.01, size = 101, normalized size = 0.89

$$-\frac{ce\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(ae^2+cd^2\right)\sqrt{ac}}-\frac{cd\ln\left(cx^4+a\right)}{4\left(ae^2+cd^2\right)a}-\frac{e^2\ln\left(ex^2+d\right)}{2\left(ae^2+cd^2\right)d}+\frac{\ln(x)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] ln(x)/a/d-1/4*c*d*ln(c*x^4+a)/a/(a*e^2+c*d^2)-1/2*c/(a*e^2+c*d^2)*e/(a*c)^(
1/2)*arctan(1/(a*c)^(1/2)*c*x^2)-1/2*e^2*ln(e*x^2+d)/d/(a*e^2+c*d^2)
```

maxima [A] time = 1.97, size = 101, normalized size = 0.89

$$-\frac{cd\log\left(cx^4+a\right)}{4\left(acd^2+a^2e^2\right)}-\frac{e^2\log\left(ex^2+d\right)}{2\left(cd^3+ade^2\right)}-\frac{ce\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(cd^2+ae^2\right)\sqrt{ac}}+\frac{\log\left(x^2\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/4*c*d*log(c*x^4 + a)/(a*c*d^2 + a^2*e^2) - 1/2*e^2*log(e*x^2 + d)/(c*d^3
 + a*d*e^2) - 1/2*c*e*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) +
 1/2*log(x^2)/(a*d)
```

mupad [B] time = 0.96, size = 527, normalized size = 4.62

$$\frac{\ln\left(64a^7ce^{10}x^2-64a^6e^{10}\sqrt{-a^3c}-25ac^5d^{10}\sqrt{-a^3c}+25a^2c^6d^{10}x^2+180a^2d^2e^8\left(-a^3c\right)^{3/2}-41c^2d^6e^4\left(-a^3c\right)^{3/2}-9a^3c^5d^8e^2x^2-41a^4c^4d^6e^4x^2+109a^5c^3d^4e^6x^2+180a^6c^2d^2e^8x^2+9a^2c^4d^8e^2\sqrt{-a^3c}+109acd^4e^6\left(-a^3c\right)^{3/2}\right)\left(e\sqrt{-a^3c}-acd\right)}{4a^3e^2+4ca^2d^2}-\frac{\ln\left(64a^6e^{10}\sqrt{-a^3c}+64a^7ce^{10}x^2+25ac^5d^{10}\sqrt{-a^3c}+25a^2c^6d^{10}x^2-180a^2d^2e^8\left(-a^3c\right)^{3/2}+41c^2d^6e^4\left(-a^3c\right)^{3/2}-9a^3c^5d^8e^2x^2-41a^4c^4d^6e^4x^2+109a^5c^3d^4e^6x^2+180a^6c^2d^2e^8x^2-9a^2c^4d^8e^2\sqrt{-a^3c}-109acd^4e^6\left(-a^3c\right)^{3/2}\right)\left(e\sqrt{-a^3c}+acd\right)}{4\left(a^3e^2+ca^2d^2\right)}-\frac{e^2\ln\left(ex^2+d\right)}{2cd^3+2ade^2}+\frac{\ln(x)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (log(64*a^7*c*e^10*x^2 - 64*a^6*e^10*(-a^3*c)^(1/2) - 25*a*c^5*d^10*(-a^3*c
)^(1/2) + 25*a^2*c^6*d^10*x^2 + 180*a^2*d^2*e^8*(-a^3*c)^(3/2) - 41*c^2*d^6
*e^4*(-a^3*c)^(3/2) - 9*a^3*c^5*d^8*e^2*x^2 - 41*a^4*c^4*d^6*e^4*x^2 + 109*
a^5*c^3*d^4*e^6*x^2 + 180*a^6*c^2*d^2*e^8*x^2 + 9*a^2*c^4*d^8*e^2*(-a^3*c)^
(1/2) + 109*a*c*d^4*e^6*(-a^3*c)^(3/2))*(e*(-a^3*c)^(1/2) - a*c*d))/(4*a^3*
e^2 + 4*a^2*c*d^2) - (log(64*a^6*e^10*(-a^3*c)^(1/2) + 64*a^7*c*e^10*x^2 +
25*a*c^5*d^10*(-a^3*c)^(1/2) + 25*a^2*c^6*d^10*x^2 - 180*a^2*d^2*e^8*(-a^3*
c)^(3/2) + 41*c^2*d^6*e^4*(-a^3*c)^(3/2) - 9*a^3*c^5*d^8*e^2*x^2 - 41*a^4*c
^4*d^6*e^4*x^2 + 109*a^5*c^3*d^4*e^6*x^2 + 180*a^6*c^2*d^2*e^8*x^2 - 9*a^2*
c^4*d^8*e^2*(-a^3*c)^(1/2) - 109*a*c*d^4*e^6*(-a^3*c)^(3/2))*(e*(-a^3*c)^(1
/2) + a*c*d))/(4*(a^3*e^2 + a^2*c*d^2)) - (e^2*log(d + e*x^2))/(2*c*d^3 + 2
*a*d*e^2) + log(x)/(a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.165 $\int \frac{1}{x^3(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=129

$$-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)}+\frac{ce\log\left(a+cx^4\right)}{4a\left(ae^2+cd^2\right)}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(ae^2+cd^2\right)}-\frac{e\log(x)}{ad^2}-\frac{1}{2adx^2}$$

Rubi [A] time = 0.15, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1252, 894, 635, 205, 260}

$$-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(ae^2+cd^2\right)}+\frac{ce\log\left(a+cx^4\right)}{4a\left(ae^2+cd^2\right)}-\frac{e\log(x)}{ad^2}-\frac{1}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/(2*a*d*x^2) - (c^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^
2 + a*e^2)) - (e*Log[x])/(a*d^2) + (e^3*Log[d + e*x^2])/(2*d^2*(c*d^2 + a*e
^2)) + (c*e*Log[a + c*x^4])/(4*a*(c*d^2 + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(d+ex^2\right)\left(a+cx^4\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{adx^2}-\frac{e}{ad^2x}+\frac{e^4}{d^2\left(cd^2+ae^2\right)(d+ex)}-\frac{c^2(d-ex)}{a\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right)\right.\\
&= -\frac{1}{2adx^2}-\frac{e\log(x)}{ad^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2+ae^2\right)}-\frac{c^2\,\text{Subst}\left(\int \frac{d-ex}{a+cx^2}\,dx,x,x^2\right)}{2a\left(cd^2+ae^2\right)}\\
&= -\frac{1}{2adx^2}-\frac{e\log(x)}{ad^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2+ae^2\right)}-\frac{\left(c^2d\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)}{2a\left(cd^2+ae^2\right)}+\frac{\left(c^2e\right)}{}\\
&= -\frac{1}{2adx^2}-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(cd^2+ae^2\right)}-\frac{e\log(x)}{ad^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2+ae^2\right)}+\frac{ce\log\left(a+cx^4\right)}{4a\left(cd^2+ae^2\right)}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 169, normalized size = 1.31

$$
\frac{2c^{3/2}d^3x^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2c^{3/2}d^3x^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)+\sqrt{a}\left(-4ex^2\log(x)\left(ae^2+cd^2\right)+cd^2ex^2\log\left(a+cx^4\right)+2ae^3x^2\log\left(d+ex^2\right)-2ade^2-2cd^3\right)}{4a^{3/2}d^2x^2\left(ae^2+cd^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (2*c^(3/2)*d^3*x^2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*c^(3/2)*d^3*
x^2*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[a]*(-2*c*d^3 - 2*a*d*e^2
 - 4*e*(c*d^2 + a*e^2)*x^2*Log[x] + 2*a*e^3*x^2*Log[d + e*x^2] + c*d^2*e*x^
2*Log[a + c*x^4]))/(4*a^(3/2)*d^2*(c*d^2 + a*e^2)*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x^3\left(d+ex^2\right)\left(a+cx^4\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(d + e*x^2)*(a + c*x^4)), x]
```

fricas [A] time = 101.42, size = 265, normalized size = 2.05

$$
\left[\frac{cd^3x^2\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4-2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)+cd^2ex^2\log\left(cx^4+a\right)+2\,ae^3x^2\log\left(ex^2+d\right)-2\,cd^3-2\,ade^2-4\left(cd^2e+ae^3\right)x^2\log(x)}{4\left(acd^4+a^2d^2e^2\right)x^2},\frac{2\,cd^3x^2\sqrt{\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^2}\right)+cd^2ex^2\log\left(cx^4+a\right)+2\,ae^3x^2\log\left(ex^2+d\right)-2\,cd^3-2\,ade^2-4\left(cd^2e+ae^3\right)x^2\log(x)}{4\left(acd^4+a^2d^2e^2\right)x^2}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(c*d^3*x^2*sqrt(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)
) + c*d^2*e*x^2*log(c*x^4 + a) + 2*a*e^3*x^2*log(e*x^2 + d) - 2*c*d^3 - 2*a
*d*e^2 - 4*(c*d^2*e + a*e^3)*x^2*log(x))/((a*c*d^4 + a^2*d^2*e^2)*x^2), 1/4
*(2*c*d^3*x^2*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) + c*d^2*e*x^2*log(c*x^4
 + a) + 2*a*e^3*x^2*log(e*x^2 + d) - 2*c*d^3 - 2*a*d*e^2 - 4*(c*d^2*e + a*e
^3)*x^2*log(x))/((a*c*d^4 + a^2*d^2*e^2)*x^2)]
```

giac [A] time = 0.30, size = 132, normalized size = 1.02

$$-\frac{c^2 d \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(acd^2 + a^2e^2\right)\sqrt{ac}} + \frac{ce\log\left(cx^4 + a\right)}{4\left(acd^2 + a^2e^2\right)} + \frac{e^4\log\left(\left|x^2e + d\right|\right)}{2\left(cd^4e + ad^2e^3\right)} - \frac{e\log\left(x^2\right)}{2\,ad^2} + \frac{x^2e - d}{2\,ad^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*c^2*d*arctan(c*x^2/sqrt(a*c))/((a*c*d^2 + a^2*e^2)*sqrt(a*c)) + 1/4*c*
e*log(c*x^4 + a)/(a*c*d^2 + a^2*e^2) + 1/2*e^4*log(abs(x^2*e + d))/(c*d^4*e
 + a*d^2*e^3) - 1/2*e*log(x^2)/(a*d^2) + 1/2*(x^2*e - d)/(a*d^2*x^2)
```

maple [A] time = 0.01, size = 119, normalized size = 0.92

$$-\frac{c^2 d \arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{2\left(a\,e^2 + c\,d^2\right)\sqrt{ac}\;a} + \frac{ce\ln\left(c\,x^4 + a\right)}{4\left(a\,e^2 + c\,d^2\right)a} + \frac{e^3\ln\left(e\,x^2 + d\right)}{2\left(a\,e^2 + c\,d^2\right)d^2} - \frac{e\ln(x)}{a\,d^2} - \frac{1}{2ad\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] -1/2/a/d/x^2-e*ln(x)/a/d^2+1/4*c*e*ln(c*x^4+a)/a/(a*e^2+c*d^2)-1/2*c^2/(a*e
^2+c*d^2)/a*d/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)+1/2*e^3*ln(e*x^2+d)/d
^2/(a*e^2+c*d^2)
```

maxima [A] time = 2.00, size = 120, normalized size = 0.93

$$\frac{e^3\log\left(ex^2 + d\right)}{2\left(cd^4 + ad^2e^2\right)} - \frac{c^2 d \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(acd^2 + a^2e^2\right)\sqrt{ac}} + \frac{ce\log\left(cx^4 + a\right)}{4\left(acd^2 + a^2e^2\right)} - \frac{e\log\left(x^2\right)}{2\,ad^2} - \frac{1}{2\,adx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*e^3*log(e*x^2 + d)/(c*d^4 + a*d^2*e^2) - 1/2*c^2*d*arctan(c*x^2/sqrt(a*
c))/((a*c*d^2 + a^2*e^2)*sqrt(a*c)) + 1/4*c*e*log(c*x^4 + a)/(a*c*d^2 + a^2
*e^2) - 1/2*e*log(x^2)/(a*d^2) - 1/2/(a*d*x^2)
```

mupad [B] time = 1.38, size = 820, normalized size = 6.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (log(a^6*c^12*d^16*x^2 + 64*a^14*c^4*e^16*x^2 + a^2*c^7*d^16*(-a^3*c^3)^(3/
2) - 64*a^13*c^2*e^16*(-a^3*c^3)^(1/2) + 63*a^3*d^8*e^8*(-a^3*c^3)^(5/2) +
224*a^9*d^2*e^14*(-a^3*c^3)^(3/2) - 28*c^3*d^14*e^2*(-a^3*c^3)^(5/2) + 28*a
^7*c^11*d^14*e^2*x^2 + 114*a^8*c^10*d^12*e^4*x^2 + 108*a^9*c^9*d^10*e^6*x^2
 - 63*a^10*c^8*d^8*e^8*x^2 - 32*a^11*c^7*d^6*e^10*x^2 + 212*a^12*c^6*d^4*e^
12*x^2 + 224*a^13*c^5*d^2*e^14*x^2 - 114*a*c^2*d^12*e^4*(-a^3*c^3)^(5/2) -
108*a^2*c*d^10*e^6*(-a^3*c^3)^(5/2) + 212*a^8*c*d^4*e^12*(-a^3*c^3)^(3/2) -
 32*a^7*c^2*d^6*e^10*(-a^3*c^3)^(3/2))*(d*(-a^3*c^3)^(1/2) + a^2*c*e))/(4*a
^4*e^2 + 4*a^3*c*d^2) - (log(a^6*c^12*d^16*x^2 + 64*a^14*c^4*e^16*x^2 - a^2
*c^7*d^16*(-a^3*c^3)^(3/2) + 64*a^13*c^2*e^16*(-a^3*c^3)^(1/2) - 63*a^3*d^8
*e^8*(-a^3*c^3)^(5/2) - 224*a^9*d^2*e^14*(-a^3*c^3)^(3/2) + 28*c^3*d^14*e^2
*(-a^3*c^3)^(5/2) + 28*a^7*c^11*d^14*e^2*x^2 + 114*a^8*c^10*d^12*e^4*x^2 +
```

```
108*a^9*c^9*d^10*e^6*x^2 - 63*a^10*c^8*d^8*e^8*x^2 - 32*a^11*c^7*d^6*e^10*x
^2 + 212*a^12*c^6*d^4*e^12*x^2 + 224*a^13*c^5*d^2*e^14*x^2 + 114*a*c^2*d^12
*e^4*(-a^3*c^3)^(5/2) + 108*a^2*c*d^10*e^6*(-a^3*c^3)^(5/2) - 212*a^8*c*d^4
*e^12*(-a^3*c^3)^(3/2) + 32*a^7*c^2*d^6*e^10*(-a^3*c^3)^(3/2))*(d*(-a^3*c^3
)^(1/2) - a^2*c*e))/(4*(a^4*e^2 + a^3*c*d^2)) + (e^3*log(d + e*x^2))/(2*c*d
^4 + 2*a*d^2*e^2) - 1/(2*a*d*x^2) - (e*log(x))/(a*d^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.166 $\int \frac{1}{x^5(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=156

$$\frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)}+\frac{c^2d\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)}-\frac{\log(x)\left(cd^2-ae^2\right)}{a^2d^3}-\frac{e^4\log\left(d+ex^2\right)}{2d^3\left(ae^2+cd^2\right)}+\frac{e}{2ad^2x^2}-\frac{1}{4adx^4}$$

Rubi [A] time = 0.18, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1252, 894, 635, 205, 260}

$$\frac{c^2d\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)}+\frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)}-\frac{\log(x)\left(cd^2-ae^2\right)}{a^2d^3}-\frac{e^4\log\left(d+ex^2\right)}{2d^3\left(ae^2+cd^2\right)}+\frac{e}{2ad^2x^2}-\frac{1}{4adx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/(4*a*d*x^4) + e/(2*a*d^2*x^2) + (c^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]
)/(2*a^(3/2)*(c*d^2 + a*e^2)) - ((c*d^2 - a*e^2)*Log[x])/(a^2*d^3) - (e^4*L
og[d + e*x^2])/(2*d^3*(c*d^2 + a*e^2)) + (c^2*d*Log[a + c*x^4])/(4*a^2*(c*d
^2 + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{1}{x^5\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{adx^3} - \frac{e}{ad^2x^2} + \frac{-cd^2+ae^2}{a^2d^3x} - \frac{e^5}{d^3\left(cd^2+ae^2\right)(d+ex)} + \frac{c^2(}{a^2\left(cd^2+\right.}\right.\right.$$
$$= -\frac{1}{4adx^4} + \frac{e}{2ad^2x^2} - \frac{\left(cd^2-ae^2\right)\log(x)}{a^2d^3} - \frac{e^4\log\left(d+ex^2\right)}{2d^3\left(cd^2+ae^2\right)} + \frac{c^2\,\text{Subst}\left(\int \frac{ae+cdx}{a+cx^2}\right.}{2a^2\left(cd^2+a\right.}$$
$$= -\frac{1}{4adx^4} + \frac{e}{2ad^2x^2} - \frac{\left(cd^2-ae^2\right)\log(x)}{a^2d^3} - \frac{e^4\log\left(d+ex^2\right)}{2d^3\left(cd^2+ae^2\right)} + \frac{\left(c^3d\right)\text{Subst}\left(\int \frac{x}{a+c}\right.}{2a^2\left(cd^2+\right.}$$
$$= -\frac{1}{4adx^4} + \frac{e}{2ad^2x^2} + \frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(cd^2+ae^2\right)} - \frac{\left(cd^2-ae^2\right)\log(x)}{a^2d^3} - \frac{e^4\log\left(d+ex^2\right)}{2d^3\left(cd^2+ae^2\right)}$$

Mathematica [A] time = 0.09, size = 209, normalized size = 1.34

$$-\frac{a^2d^2e^2+2a^2e^4x^4\log\left(d+ex^2\right)-2a^2de^3x^2-4a^2e^4x^4\log(x)+2\sqrt{a}\,c^{3/2}d^3ex^4\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+2\sqrt{a}\,c^{3/2}d^3ex^4\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)-c^2d^4x^4\log\left(a+cx^4\right)+acd^4-2acd^3ex^2+4c^2d^4x^4\log(x)}{4a^2d^3x^4\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/4*(a*c*d^4 + a^2*d^2*e^2 - 2*a*c*d^3*e*x^2 - 2*a^2*d*e^3*x^2 + 2*Sqrt[a]
*c^(3/2)*d^3*e*x^4*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[a]*c^(3
/2)*d^3*e*x^4*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 4*c^2*d^4*x^4*Log[x
] - 4*a^2*e^4*x^4*Log[x] + 2*a^2*e^4*x^4*Log[d + e*x^2] - c^2*d^4*x^4*Log[a
 + c*x^4])/(a^2*d^3*(c*d^2 + a*e^2)*x^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(d+ex^2\right)\left(a+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(d + e*x^2)*(a + c*x^4)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.34, size = 168, normalized size = 1.08

$$\frac{c^2d\log\left(cx^4+a\right)}{4\left(a^2cd^2+a^3e^2\right)} + \frac{c^2\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)e}{2\left(acd^2+a^2e^2\right)\sqrt{ac}} - \frac{e^5\log\left(\left|x^2e+d\right|\right)}{2\left(cd^5e+ad^3e^3\right)} - \frac{\left(cd^2-ae^2\right)\log\left(x^2\right)}{2\,a^2d^3} + \frac{3\,cd^2x^4-3\,ax^4e^2+2\,adx^2e-ad^2}{4\,a^2d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*c^2*d*log(c*x^4 + a)/(a^2*c*d^2 + a^3*e^2) + 1/2*c^2*arctan(c*x^2/sqrt(
a*c))*e/((a*c*d^2 + a^2*e^2)*sqrt(a*c)) - 1/2*e^5*log(abs(x^2*e + d))/(c*d^
5*e + a*d^3*e^3) - 1/2*(c*d^2 - a*e^2)*log(x^2)/(a^2*d^3) + 1/4*(3*c*d^2*x^
4 - 3*a*x^4*e^2 + 2*a*d*x^2*e - a*d^2)/(a^2*d^3*x^4)
```

maple [A] time = 0.02, size = 145, normalized size = 0.93

$$\frac{c^2 e \arctan\left(\frac{c x^2}{\sqrt{ac}}\right)}{2\left(a e^2+c d^2\right)\sqrt{ac}\, a}+\frac{c^2 d \ln\left(c x^4+a\right)}{4\left(a e^2+c d^2\right)a^2}-\frac{e^4 \ln\left(e x^2+d\right)}{2\left(a e^2+c d^2\right)d^3}+\frac{e^2 \ln(x)}{a d^3}-\frac{c \ln(x)}{a^2 d}+\frac{e}{2a d^2 x^2}-\frac{1}{4ad x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] -1/4/a/d/x^4+1/d^3/a*ln(x)*e^2-1/d/a^2*ln(x)*c+1/2*e/a/d^2/x^2+1/4*c^2*d*ln
(c*x^4+a)/a^2/(a*e^2+c*d^2)+1/2*c^2/(a*e^2+c*d^2)/a*e/(a*c)^(1/2)*arctan(1/
(a*c)^(1/2)*c*x^2)-1/2*e^4*ln(e*x^2+d)/d^3/(a*e^2+c*d^2)
```

maxima [A] time = 2.05, size = 145, normalized size = 0.93

$$-\frac{e^4 \log\left(ex^2+d\right)}{2\left(cd^5+ad^3e^2\right)}+\frac{c^2 d \log\left(cx^4+a\right)}{4\left(a^2cd^2+a^3e^2\right)}+\frac{c^2 e \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(acd^2+a^2e^2\right)\sqrt{ac}}-\frac{\left(cd^2-ae^2\right)\log\left(x^2\right)}{2\,a^2d^3}+\frac{2\,ex^2-d}{4\,ad^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/2*e^4*log(e*x^2 + d)/(c*d^5 + a*d^3*e^2) + 1/4*c^2*d*log(c*x^4 + a)/(a^2
*c*d^2 + a^3*e^2) + 1/2*c^2*e*arctan(c*x^2/sqrt(a*c))/((a*c*d^2 + a^2*e^2)*
sqrt(a*c)) - 1/2*(c*d^2 - a*e^2)*log(x^2)/(a^2*d^3) + 1/4*(2*e*x^2 - d)/(a*
d^2*x^4)
```

mupad [B] time = 1.87, size = 1017, normalized size = 6.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (log(25*a^2*c^9*d^20*(-a^5*c^3)^(3/2) - 64*a^19*c^4*e^20*x^2 - 25*a^9*c^14*
d^20*x^2 - 64*a^17*c^2*e^20*(-a^5*c^3)^(1/2) + 100*a^3*d^8*e^12*(-a^5*c^3)^
(5/2) + 128*a^11*d^2*e^18*(-a^5*c^3)^(3/2) - 112*c^3*d^14*e^6*(-a^5*c^3)^(5
/2) - 76*a^10*c^13*d^18*e^2*x^2 - 138*a^11*c^12*d^16*e^4*x^2 - 112*a^12*c^1
1*d^14*e^6*x^2 + 55*a^13*c^10*d^12*e^8*x^2 + 104*a^14*c^9*d^10*e^10*x^2 + 1
00*a^15*c^8*d^8*e^12*x^2 + 172*a^16*c^7*d^6*e^14*x^2 + 32*a^17*c^6*d^4*e^16
*x^2 - 128*a^18*c^5*d^2*e^18*x^2 + 55*a*c^2*d^12*e^8*(-a^5*c^3)^(5/2) + 104
*a^2*c*d^10*e^10*(-a^5*c^3)^(5/2) - 32*a^10*c*d^4*e^16*(-a^5*c^3)^(3/2) + 7
6*a^3*c^8*d^18*e^2*(-a^5*c^3)^(3/2) + 138*a^4*c^7*d^16*e^4*(-a^5*c^3)^(3/2)
 - 172*a^9*c^2*d^6*e^14*(-a^5*c^3)^(3/2))*(e*(-a^5*c^3)^(1/2) + a^2*c^2*d))
/(4*a^5*e^2 + 4*a^4*c*d^2) - (e^4*log(d + e*x^2))/(2*(c*d^5 + a*d^3*e^2)) -
 (log(25*a^9*c^14*d^20*x^2 + 64*a^19*c^4*e^20*x^2 + 25*a^2*c^9*d^20*(-a^5*c
^3)^(3/2) - 64*a^17*c^2*e^20*(-a^5*c^3)^(1/2) + 100*a^3*d^8*e^12*(-a^5*c^3)
^(5/2) + 128*a^11*d^2*e^18*(-a^5*c^3)^(3/2) - 112*c^3*d^14*e^6*(-a^5*c^3)^(
5/2) + 76*a^10*c^13*d^18*e^2*x^2 + 138*a^11*c^12*d^16*e^4*x^2 + 112*a^12*c^
11*d^14*e^6*x^2 - 55*a^13*c^10*d^12*e^8*x^2 - 104*a^14*c^9*d^10*e^10*x^2 -
100*a^15*c^8*d^8*e^12*x^2 - 172*a^16*c^7*d^6*e^14*x^2 - 32*a^17*c^6*d^4*e^1
6*x^2 + 128*a^18*c^5*d^2*e^18*x^2 + 55*a*c^2*d^12*e^8*(-a^5*c^3)^(5/2) + 10
```

```
4*a^2*c*d^10*e^10*(-a^5*c^3)^(5/2) - 32*a^10*c*d^4*e^16*(-a^5*c^3)^(3/2) +
76*a^3*c^8*d^18*e^2*(-a^5*c^3)^(3/2) + 138*a^4*c^7*d^16*e^4*(-a^5*c^3)^(3/2
) - 172*a^9*c^2*d^6*e^14*(-a^5*c^3)^(3/2))*(e*(-a^5*c^3)^(1/2) - a^2*c^2*d)
)/(4*(a^5*e^2 + a^4*c*d^2)) - (1/(4*a*d) - (e*x^2)/(2*a*d^2))/x^4 + (log(x)
*(a*e^2 - c*d^2))/(a^2*d^3)
```

sympy [**F(-1**)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.167 $\int \frac{x^8}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=359

$$-\frac{a^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{7/4}\left(ae^2+cd^2\right)}+\frac{a^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{7/4}\left(ae^2+cd^2\right)}-\frac{a^{5/4}}{}$$

Rubi [A] time = 0.35, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{a^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{7/4}\left(ae^2+cd^2\right)}+\frac{a^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{7/4}\left(ae^2+cd^2\right)}-\frac{a^{5/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,c^{7/4}\left(ae^2+cd^2\right)}+\frac{a^{5/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,c^{7/4}\left(ae^2+cd^2\right)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{5/2}\left(ae^2+cd^2\right)}-\frac{dx}{ce^2}+\frac{x^3}{3ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -((d*x)/(c*e^2)) + x^3/(3*c*e) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(
5/2)*(c*d^2 + a*e^2)) - (a^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) + (a^(5/4)*(Sqrt
[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(7
/4)*(c*d^2 + a*e^2)) - (a^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) +
(a^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^8}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \int \left(-\frac{d}{ce^2}+\frac{x^2}{ce}+\frac{d^4}{e^2\left(cd^2+ae^2\right)\left(d+ex^2\right)}+\frac{a^2\left(d-ex^2\right)}{c\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx$$

$$= -\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{a^2\int \frac{d-ex^2}{a+cx^4}\,dx}{c\left(cd^2+ae^2\right)}+\frac{d^4\int \frac{1}{d+ex^2}\,dx}{e^2\left(cd^2+ae^2\right)}$$

$$= -\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{5/2}\left(cd^2+ae^2\right)}+\frac{\left(a^2\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c^2\left(cd^2+ae^2\right)}+\frac{\left(a^2\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)\right)\cdots}{2c^2\left(cd^2\cdots\right.}$$

$$= -\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{5/2}\left(cd^2+ae^2\right)}+\frac{\left(a^2\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4c^2\left(cd^2+ae^2\right)}+\frac{\left(a^2\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-\cdots\right.\right.}{4\cdots}$$

$$= -\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{5/2}\left(cd^2+ae^2\right)}-\frac{a^{5/4}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\cdots\right.}{4\sqrt{2}\,c^{7/4}\left(cd^2+ae^2\right)}\cdots$$

$$= -\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{5/2}\left(cd^2+ae^2\right)}-\frac{a^{7/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,c^{7/4}\left(cd^2+ae^2\right)}+\frac{a^{7/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-\cdots\right.}{2\sqrt{2}\cdots}$$

Mathematica [A] time = 0.36, size = 344, normalized size = 0.96

$$\frac{-3\sqrt{2}\,ae^{5/2}\left(a^{3/4}e+\sqrt[4]{a}\,\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)+3\sqrt{2}\,ae^{5/2}\left(a^{3/4}e+\sqrt[4]{a}\,\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)+6\sqrt{2}\,a^{5/4}e^{5/2}\left(\sqrt{a}\,e-\sqrt{c}\,d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)-6\sqrt{2}\,a^{5/4}e^{5/2}\left(\sqrt{a}\,e-\sqrt{c}\,d\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)-24c^{3/4}d\sqrt{e}\,x\left(ae^2+cd^2\right)+8c^{3/4}e^{3/2}x^3\left(ae^2+cd^2\right)+24c^{7/4}d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{24c^{7/4}e^{5/2}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-24*c^(3/4)*d*Sqrt[e]*(c*d^2 + a*e^2)*x + 8*c^(3/4)*e^(3/2)*(c*d^2 + a*e^2
)*x^3 + 24*c^(7/4)*d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + 6*Sqrt[2]*a^(5/4)*
e^(5/2)*(-(Sqrt[c]*d) + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]
- 6*Sqrt[2]*a^(5/4)*e^(5/2)*(-(Sqrt[c]*d) + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*
c^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*a*e^(5/2)*(a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*L
```

```
og[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*Sqrt[2]*a*e^(5/2)
*(a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(24*c^(7/4)*e^(5/2)*(c*d^2 + a*e^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^8/((d + e*x^2)*(a + c*x^4)), x]
```

fricas [B] time = 32.47, size = 4414, normalized size = 12.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/12*(6*c*d^3*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) +
 4*(c*d^2*e + a*e^3)*x^3 - 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e + (c^5
*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2
+ a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2
*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^3*
c*d^2 - a^4*e^2)*x + (a^2*c^3*d^3 - a^3*c^2*d*e^2 + (c^7*d^4*e + 2*a*c^6*d^
2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*
d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^
8)))*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5
*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*
c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2
 + a^2*c^3*e^4))) + 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e + (c^5*d^4 +
2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e
^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 +
a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^3*c*d^2 -
 a^4*e^2)*x - (a^2*c^3*d^3 - a^3*c^2*d*e^2 + (c^7*d^4*e + 2*a*c^6*d^2*e^3 +
 a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4
*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))*sq
rt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^
4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4
*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*
c^3*e^4))) - 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4
*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^
11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7
*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^3*c*d^2 - a^4*e^
2)*x + (a^2*c^3*d^3 - a^3*c^2*d*e^2 - (c^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*c^
5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10
*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))*sqrt((2*a
^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a
^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 +
4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4
))) + 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^
2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8
+ 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))
/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^3*c*d^2 - a^4*e^2)*x -
(a^2*c^3*d^3 - a^3*c^2*d*e^2 - (c^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*c^5*e^5)*
sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^
2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))*sqrt((2*a^3*d*e
- (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^
```

```
2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c
^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))) - 1
2*(c*d^3 + a*d*e^2)*x)/(c^2*d^2*e^2 + a*c*e^4), 1/12*(12*c*d^3*sqrt(d/e)*ar
ctan(e*x*sqrt(d/e)/d) + 4*(c*d^2*e + a*e^3)*x^3 - 3*(c^2*d^2*e^2 + a*c*e^4)
*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2
*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*
d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a
^2*c^3*e^4))*log(-(a^3*c*d^2 - a^4*e^2)*x + (a^2*c^3*d^3 - a^3*c^2*d*e^2 +
(c^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^
2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c
^8*d^2*e^6 + a^4*c^7*e^8)))*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 +
a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*
a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^
5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))) + 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((
2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 -
2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4
 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*
e^4))*log(-(a^3*c*d^2 - a^4*e^2)*x - (a^2*c^3*d^3 - a^3*c^2*d*e^2 + (c^7*d^
4*e + 2*a*c^6*d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 +
 a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*
e^6 + a^4*c^7*e^8)))*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3
*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*
d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 +
 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))) - 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d
*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c
*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^
3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*l
og(-(a^3*c*d^2 - a^4*e^2)*x + (a^2*c^3*d^3 - a^3*c^2*d*e^2 - (c^7*d^4*e + 2
*a*c^6*d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^
4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a
^4*c^7*e^8)))*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*s
qrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2
 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^
4*d^2*e^2 + a^2*c^3*e^4))) + 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e - (c
^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^
2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d
^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^
3*c*d^2 - a^4*e^2)*x - (a^2*c^3*d^3 - a^3*c^2*d*e^2 - (c^7*d^4*e + 2*a*c^6*
d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^1
1*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*
e^8)))*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a
^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^
2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e
^2 + a^2*c^3*e^4))) - 12*(c*d^3 + a*d*e^2)*x)/(c^2*d^2*e^2 + a*c*e^4)]
```

giac [A] time = 0.54, size = 363, normalized size = 1.01

$$\frac{d^{\frac{7}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{cd^2e^2+ae^4}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ac^2d-\left(ac^3\right)^{\frac{3}{4}}ae\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^5d^2+\sqrt{2}ac^4e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ac^2d-\left(ac^3\right)^{\frac{3}{4}}ae\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^5d^2+\sqrt{2}ac^4e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ac^2d+\left(ac^3\right)^{\frac{3}{4}}ae\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^5d^2+\sqrt{2}ac^4e^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ac^2d+\left(ac^3\right)^{\frac{3}{4}}ae\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^5d^2+\sqrt{2}ac^4e^2\right)}+\frac{\left(c^2x^3e^2-3c^2dxe\right)e^{(-3)}}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] d^(7/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2*e^2 + a*e^4) + 1/2*((a*c^
3)^(1/4)*a*c^2*d - (a*c^3)^(3/4)*a*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/
c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^5*d^2 + sqrt(2)*a*c^4*e^2) + 1/2*((a*c^3)
^(1/4)*a*c^2*d - (a*c^3)^(3/4)*a*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)
^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^5*d^2 + sqrt(2)*a*c^4*e^2) + 1/4*((a*c^3)^(
1/4)*a*c^2*d + (a*c^3)^(3/4)*a*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/
c))/(sqrt(2)*c^5*d^2 + sqrt(2)*a*c^4*e^2) - 1/4*((a*c^3)^(1/4)*a*c^2*d + (a
```

```
*c^3)^(3/4)*a*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^5*
d^2 + sqrt(2)*a*c^4*e^2) + 1/3*(c^2*x^3*e^2 - 3*c^2*d*x*e)*e^(-3)/c^3
```

maple [A] time = 0.02, size = 405, normalized size = 1.13

$$\frac{d^4\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(a e^2+c d^2\right)\sqrt{de}\, e^2}-\frac{\sqrt{2}\, a^2 e \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(a e^2+c d^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}} c^2}-\frac{\sqrt{2}\, a^2 e \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(a e^2+c d^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}} c^2}-\frac{\sqrt{2}\, a^2 e \ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\, x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\, x+\sqrt{\frac{a}{c}}}\right)}{8\left(a e^2+c d^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}} c^2}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\, a d \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(a e^2+c d^2\right) c}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\, a d \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(a e^2+c d^2\right) c}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\, a d \ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\, x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\, x+\sqrt{\frac{a}{c}}}\right)}{8\left(a e^2+c d^2\right) c}+\frac{x^3}{3ce}-\frac{dx}{c e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/3*x^3/c/e-d*x/c/e^2+1/8*a/(a*e^2+c*d^2)/c*d*(a/c)^(1/4)*2^(1/2)*ln((x^2+(
a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+
1/4*a/(a*e^2+c*d^2)/c*d*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)
+1/4*a/(a*e^2+c*d^2)/c*d*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1
)-1/8*a^2/(a*e^2+c*d^2)/c^2*e/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(
1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))-1/4*a^2/(a*e^2+c
*d^2)/c^2*e/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)-1/4*a^2/(a*
e^2+c*d^2)/c^2*e/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/e^2*
d^4/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

maxima [A] time = 2.05, size = 294, normalized size = 0.82

$$\frac{d^4\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(cd^2e^2+ae^4\right)\sqrt{de}}+\frac{a^2\left(\frac{2\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{8\left(c^2d^2+ace^2\right)}+\frac{ex^3-3\,dx}{3\,ce^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] d^4*arctan(e*x/sqrt(d*e))/((c*d^2*e^2 + a*e^4)*sqrt(d*e)) + 1/8*a^2*(2*sqrt
(2)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/
4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c))
+ 2*sqrt(2)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(
2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*s
qrt(c)) + sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)
*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*l
og(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(c
^2*d^2 + a*c*e^2) + 1/3*(e*x^3 - 3*d*x)/(c*e^2)
```

mupad [B] time = 2.06, size = 6097, normalized size = 16.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (log(a^7*d^4*e^26 + 16*c^7*d^18*e^12 - 16*c^7*x*(-d^7*e^5)^(5/2) + 2*a^6*c*
d^6*e^24 + 16*a^3*c^4*d^12*e^18 + a^5*c^2*d^8*e^22 - a^7*e^24*x*(-d^7*e^5)^
(1/2) - a^5*c^2*d^4*e^20*x*(-d^7*e^5)^(1/2) + 16*a^3*c^4*d*e^11*x*(-d^7*e^5
)^(3/2) - 2*a^6*c*d^2*e^22*x*(-d^7*e^5)^(1/2))*(-d^7*e^5)^(1/2))/(2*a*e^7 +
 2*c*d^2*e^5) - atan(((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*
a^5*c^6*d^2*e^9)/(c^3*e^3) - (2*x*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^
7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(
1/2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6 - 256*a^3*c^9*d^4*e^8 + 256*a
^4*c^8*d^2*e^10))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1
/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)
+ (2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a^4*c^6*d^5*e^5 - 16*a^5*c^
5*d^3*e^7))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) +
```

```
2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (16*
a^3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4
*a^6*c^3*d^3*e^6)/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1
/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)
- (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e
^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d
^2*e^2)))^(1/2)*1i - (((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a
^5*c^6*d^2*e^9)/(c^3*e^3) + (2*x*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7
)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1
/2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6 - 256*a^3*c^9*d^4*e^8 + 256*a^
4*c^8*d^2*e^10))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/
2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) -
 (2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a^4*c^6*d^5*e^5 - 16*a^5*c^5
*d^3*e^7))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) + 2
*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (16*a
^3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*
a^6*c^3*d^3*e^6)/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/
2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) +
 (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^
2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^
2*e^2)))^(1/2)*1i)/((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^
5*c^6*d^2*e^9)/(c^3*e^3) - (2*x*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)
^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/
2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6 - 256*a^3*c^9*d^4*e^8 + 256*a^4
*c^8*d^2*e^10))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2
) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) +
(2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a^4*c^6*d^5*e^5 - 16*a^5*c^5*
d^3*e^7))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) + 2*
a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (16*a^
3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a
^6*c^3*d^3*e^6)/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2
) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) -
(2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2
*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2
*e^2)))^(1/2) + (((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^
6*d^2*e^9)/(c^3*e^3) + (2*x*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/
2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*(
256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6 - 256*a^3*c^9*d^4*e^8 + 256*a^4*c^8
*d^2*e^10))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) +
2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (2*x
*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a^4*c^6*d^5*e^5 - 16*a^5*c^5*d^3*
e^7))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) + 2*a^3*
c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (16*a^3*c^
6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c
^3*d^3*e^6)/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) +
2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) + (2*x
*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a
^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2
)))^(1/2) + (2*(a^7*d^4*e^3 - a^6*c*d^6*e))/(c^3*e^3)))*((c*d^2*(-a^5*c^7)^
(1/2) - a*e^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4
+ 2*a*c^8*d^2*e^2)))^(1/2)*2i - (log(a^7*d^4*e^26 + 16*c^7*d^18*e^12 + 16*c
^7*x*(-d^7*e^5)^(5/2) + 2*a^6*c*d^6*e^24 + 16*a^3*c^4*d^12*e^18 + a^5*c^2*d
^8*e^22 + a^7*e^24*x*(-d^7*e^5)^(1/2) + a^5*c^2*d^4*e^20*x*(-d^7*e^5)^(1/2)
 - 16*a^3*c^4*d*e^11*x*(-d^7*e^5)^(3/2) + 2*a^6*c*d^2*e^22*x*(-d^7*e^5)^(1/
2))*(-d^7*e^5)^(1/2))/(2*(a*e^7 + c*d^2*e^5)) - atan((((((192*a^3*c^8*d^6*
e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9)/(c^3*e^3) - (2*x*((a*e^2*(
-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^
2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e
^6 - 256*a^3*c^9*d^4*e^8 + 256*a^4*c^8*d^2*e^10))/(c^3*e^3))*((a*e^2*(-a^5*
```

```
c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7
*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) + (2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9
 - 8*a^4*c^6*d^5*e^5 - 16*a^5*c^5*d^3*e^7))/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(
1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 +
 2*a*c^8*d^2*e^2)))^(1/2) - (16*a^3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*
d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6)/(c^3*e^3))*((a*e^2*(-a^5*
c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7
*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3)
)*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c
^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*1i - (((((192*a^3*c^8*d^6*e
^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9)/(c^3*e^3) + (2*x*((a*e^2*(-
a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2
*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^
6 - 256*a^3*c^9*d^4*e^8 + 256*a^4*c^8*d^2*e^10))/(c^3*e^3))*((a*e^2*(-a^5*c
^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*
e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9
 - 8*a^4*c^6*d^5*e^5 - 16*a^5*c^5*d^3*e^7))/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(1
/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 +
2*a*c^8*d^2*e^2)))^(1/2) - (16*a^3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d
^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6)/(c^3*e^3))*((a*e^2*(-a^5*c
^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*
e^4 + 2*a*c^8*d^2*e^2)))^(1/2) + (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))
*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^
9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*1i)/((((((192*a^3*c^8*d^6*e^
5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9)/(c^3*e^3) - (2*x*((a*e^2*(-a
^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*
c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6
 - 256*a^3*c^9*d^4*e^8 + 256*a^4*c^8*d^2*e^10))/(c^3*e^3))*((a*e^2*(-a^5*c^
7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e
^4 + 2*a*c^8*d^2*e^2)))^(1/2) + (2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 -
 8*a^4*c^6*d^5*e^5 - 16*a^5*c^5*d^3*e^7))/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(1/
2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2
*a*c^8*d^2*e^2)))^(1/2) - (16*a^3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^
7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6)/(c^3*e^3))*((a*e^2*(-a^5*c^
7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e
^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*
((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9
*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) + (((((192*a^3*c^8*d^6*e^5 +
384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9)/(c^3*e^3) + (2*x*((a*e^2*(-a^5*c
^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*
e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6 - 2
56*a^3*c^9*d^4*e^8 + 256*a^4*c^8*d^2*e^10))/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(
1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 +
 2*a*c^8*d^2*e^2)))^(1/2) - (2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a
^4*c^6*d^5*e^5 - 16*a^5*c^5*d^3*e^7))/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(1/2) -
 c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c
^8*d^2*e^2)))^(1/2) - (16*a^3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^
2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6)/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(
1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 +
 2*a*c^8*d^2*e^2)))^(1/2) + (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*((a*
e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4
 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) + (2*(a^7*d^4*e^3 - a^6*c*d^6*e))
/(c^3*e^3)))*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*
d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*2i + x^3/(3*c*e)
 - (d*x)/(c*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.168 $\int \frac{x^6}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=345

$$-\frac{a^{3/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{5/4}\left(ae^2+cd^2\right)}+\frac{a^{3/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{5/4}\left(ae^2+cd^2\right)}+\frac{a^{3/4}\,(\cdots}{\cdots}$$

Rubi [A] time = 0.30, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{a^{3/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{5/4}\left(ae^2+cd^2\right)}+\frac{a^{3/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{5/4}\left(ae^2+cd^2\right)}+\frac{a^{3/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,c^{5/4}\left(ae^2+cd^2\right)}-\frac{a^{3/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,c^{5/4}\left(ae^2+cd^2\right)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}\left(ae^2+cd^2\right)}+\frac{x}{ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] x/(c*e) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 + a*e^2)) +
 (a^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/
(2*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) - (a^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) -
(a^(3/4)*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(4*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) + (a^(3/4)*(Sqrt[c]*d - S
qrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2
]*c^(5/4)*(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^6}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \int \left(\frac{1}{ce}-\frac{d^3}{e\left(cd^2+ae^2\right)\left(d+ex^2\right)}-\frac{a\left(ae+cdx^2\right)}{c\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx$$

$$=\frac{x}{ce}-\frac{a\int\frac{ae+cdx^2}{a+cx^4}\,dx}{c\left(cd^2+ae^2\right)}-\frac{d^3\int\frac{1}{d+ex^2}\,dx}{e\left(cd^2+ae^2\right)}$$

$$=\frac{x}{ce}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\left(cd^2+ae^2\right)}+\frac{\left(a\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c\left(cd^2+ae^2\right)}-\frac{\left(a\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}}{2c\left(cd^2+ae^2\right)}$$

$$=\frac{x}{ce}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\left(cd^2+ae^2\right)}-\frac{\left(a^{3/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)}-\frac{\left(a^{3/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int}{4\sqrt{2}c^{3/4}\left(\right.}$$

$$=\frac{x}{ce}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\left(cd^2+ae^2\right)}-\frac{a^{3/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)}+\frac{a^{3/4}\left(d\right.}{}$$

$$=\frac{x}{ce}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\left(cd^2+ae^2\right)}+\frac{a^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{5/4}\left(cd^2+ae^2\right)}-\frac{a^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right.}{2\sqrt{2}c^{5/4}}$$

Mathematica [A] time = 0.24, size = 373, normalized size = 1.08

$$-\frac{\left(a^{3/4}cd-a^{5/4}\sqrt{c}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}+\frac{\left(a^{3/4}cd-a^{5/4}\sqrt{c}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}-\frac{\left(a^{3/4}cd+a^{5/4}\sqrt{c}e\right)\tan^{-1}\left(\frac{2\sqrt[4]{c}x-\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}-\frac{\left(a^{3/4}cd+a^{5/4}\sqrt{c}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{c}x}{\sqrt{2}\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\left(ae^2+cd^2\right)}+\frac{x}{ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] x/(c*e) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 + a*e^2)) -
 ((a^(3/4)*c*d + a^(5/4)*Sqrt[c]*e)*ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*c^(1/4)*
x)/(Sqrt[2]*a^(1/4))])/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - ((a^(3/4)*c*d
```

```
+ a^(5/4)*Sqrt[c]*e)*ArcTan[(Sqrt[2]*a^(1/4) + 2*c^(1/4)*x)/(Sqrt[2]*a^(1/4
))])/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - ((a^(3/4)*c*d - a^(5/4)*Sqrt[c]*
e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/
4)*(c*d^2 + a*e^2)) + ((a^(3/4)*c*d - a^(5/4)*Sqrt[c]*e)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^6/((d + e*x^2)*(a + c*x^4)), x]
```

fricas [B] time = 4.86, size = 4354, normalized size = 12.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*c*d^2*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) +
(c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c
^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*
d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 +
 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x + (a^2*c^2*d^
2*e - a^3*c*e^3 - (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^
2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d
^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2
*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^
4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4
*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) - (c^2*d^2*e + a*c*
e^3)*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^
3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c
^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2
+ a^2*c^2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 -
 (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d
^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^
6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 +
a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a
*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*
d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) + (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2
*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4
*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^
3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*l
og(-(a^2*c*d^2 - a^3*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3 + (c^6*d^5 + 2*a*c
^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)
/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c
^5*e^8)))*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt
(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*
a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2
*e^2 + a^2*c^2*e^4))) - (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e - (c^4*d^4 +
 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*
e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a
^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2 -
a^3*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 + (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*
c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c
^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(
```

```
2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 -
2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 +
 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^
4))) + 4*(c*d^2 + a*e^2)*x)/(c^2*d^2*e + a*c*e^3), -1/4*(4*c*d^2*sqrt(d/e)*
arctan(e*x*sqrt(d/e)/d) - (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e + (c^4*d^4
 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^
5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 +
 a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2
- a^3*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3 - (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^
2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a
*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(
-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4
- 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4
 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*
e^4))) + (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^
2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 +
 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(
c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x - (a
^2*c^2*d^2*e - a^3*c*e^3 - (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt
(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*
a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e + (c^
4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2
 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*
e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) - (c^2*d^2
*e + a*c*e^3)*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*
sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2
+ 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3
*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x + (a^2*c^2*d^2*e - a^
3*c*e^3 + (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 -
2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 +
 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d
^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*
d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8
)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) + (c^2*d^2*e + a*c*e^3)*sqr
t(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^
4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e
^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^
2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 + (c^6*d^
5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 +
 a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^
6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*
e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6
*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*
a*c^3*d^2*e^2 + a^2*c^2*e^4))) - 4*(c*d^2 + a*e^2)*x)/(c^2*d^2*e + a*c*e^3)
]
```

giac [A] time = 0.44, size = 333, normalized size = 0.97

$$-\frac{d^{\frac{5}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{cd^2e+ae^3}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}+\frac{xe^{(-1)}}{c}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace-\left(ac^3\right)^{\frac{3}{4}}d\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace-\left(ac^3\right)^{\frac{3}{4}}d\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -d^(5/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2*e + a*e^3) - 1/2*((a*c^3
)^(1/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1
/4))/(a/c)^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) - 1/2*((a*c^3)^(1/4
)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) + x*e^(-1)/c - 1/4*((a*c^
3)^(1/4)*a*c*e - (a*c^3)^(3/4)*d)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/
```

```
c))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) + 1/4*((a*c^3)^(1/4)*a*c*e - (a*c
^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^4*d^2
+ sqrt(2)*a*c^3*e^2)
```

maple [A]   time = 0.01, size = 387, normalized size = 1.12

$$-\frac{d^{3} \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(a e^{2}+c d^{2}\right) \sqrt{de}\, e}-\frac{\sqrt{2}\, ad \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(a e^{2}+c d^{2}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}} c}-\frac{\sqrt{2}\, ad \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(a e^{2}+c d^{2}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}} c}-\frac{\sqrt{2}\, ad \ln\left(\frac{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{c}}}{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{c}}}\right)}{8\left(a e^{2}+c d^{2}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}} c}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}\, ae \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(a e^{2}+c d^{2}\right) c}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}\, ae \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(a e^{2}+c d^{2}\right) c}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}\, ae \ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{c}}}\right)}{8\left(a e^{2}+c d^{2}\right) c}+\frac{x}{ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] x/c/e-1/4*a/(a*e^2+c*d^2)/c*e*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4
)*x-1)-1/8*a/(a*e^2+c*d^2)/c*e*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1
/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))-1/4*a/(a*e^2+c*
d^2)/c*e*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)-1/8*a/(a*e^2+c
*d^2)/c*d/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x
^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))-1/4*a/(a*e^2+c*d^2)/c*d/(a/c)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)-1/4*a/(a*e^2+c*d^2)/c*d/(a/c)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)-1/e*d^3/(a*e^2+c*d^2)/(d*e)^(1/2)*
arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A]   time = 2.09, size = 289, normalized size = 0.84

$$-\frac{d^{3} \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(cd^{2} e+ae^{3}\right) \sqrt{de}}-\frac{a\left(\frac{2 \sqrt{2}\left(\sqrt{a}\, cd+a \sqrt{c}\, e\right) \arctan\left(\frac{\sqrt{2}\left(2 \sqrt{c}\, x+\sqrt{2}\, a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{2 \sqrt{\sqrt{a} \sqrt{c}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{c}} \sqrt{c}}+\frac{2 \sqrt{2}\left(\sqrt{a}\, cd+a \sqrt{c}\, e\right) \arctan\left(\frac{\sqrt{2}\left(2 \sqrt{c}\, x-\sqrt{2}\, a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{2 \sqrt{\sqrt{a} \sqrt{c}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{c}} \sqrt{c}}-\frac{\sqrt{2}\left(\sqrt{a}\, cd-a \sqrt{c}\, e\right) \log\left(\sqrt{c}\, x^{2}+\sqrt{2}\, a^{\frac{1}{4}} c^{\frac{1}{4}} x+\sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{3}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}\, cd-a \sqrt{c}\, e\right) \log\left(\sqrt{c}\, x^{2}-\sqrt{2}\, a^{\frac{1}{4}} c^{\frac{1}{4}} x+\sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{3}{4}}}\right)}{8\left(c^{2} d^{2}+ace^{2}\right)}+\frac{x}{ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -d^3*arctan(e*x/sqrt(d*e))/((c*d^2*e + a*e^3)*sqrt(d*e)) - 1/8*a*(2*sqrt(2)
*(sqrt(a)*c*d + a*sqrt(c)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1
/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c))
 + 2*sqrt(2)*(sqrt(a)*c*d + a*sqrt(c)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x -
sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
c))*sqrt(c)) - sqrt(2)*(sqrt(a)*c*d - a*sqrt(c)*e)*log(sqrt(c)*x^2 + sqrt(2
)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + sqrt(2)*(sqrt(a)*c*d - a
*sqrt(c)*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)
*c^(3/4)))/(c^2*d^2 + a*c*e^2) + x/(c*e)
```

mupad [B]   time = 1.83, size = 5908, normalized size = 17.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] atan((((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*
e) - (2*x*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*
e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*(256*a^5*c^5*e^10
- 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*e))*
(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^
7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (2*x*(64*a^2*c^6*d^7*e - 5
6*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e))*(-(a*e^2*
(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a
^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (48*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*
e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5
)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1
```

```
/2) - (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*
d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*
d^2*e^2)))^(1/2)*1i - (((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*
c^5*d^3*e^6)/(c*e) + (2*x*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2
) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*(2
56*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d
^2*e^8))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*
c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*x*(64*a
^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/
(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/
(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (48*a^3*c^4*d^6*e -
 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) -
 c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c
^6*d^2*e^2)))^(1/2) + (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e))*(-(a*e^2*(-a^3
*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^
5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*1i)/((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^
5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) - (2*x*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2
*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2
*e^2)))^(1/2)*(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6
 + 256*a^4*c^6*d^2*e^8))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5
)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1
/2) + (2*x*(64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^
4*c^4*d^3*e^5))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) +
 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (48
*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(a*e^2*(-
a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2
*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e)
)*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(
c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (((((64*a^5*c^4*d*e^8 +
64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) + (2*x*(-(a*e^2*(-a^3*c^5)^
(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4
+ 2*a*c^6*d^2*e^2)))^(1/2)*(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^
3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*
d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*
d^2*e^2)))^(1/2) - (2*x*(64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^
5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3
*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))
)^(1/2) - (48*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e)
)*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(
c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (2*x*(a^6*e^6 - 2*a^3*c^
3*d^6))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c
^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*(a^4*c*d
^5 - a^5*d^3*e^2))/(c*e)))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/
2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*2
i + atan((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)
/(c*e) - (2*x*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^
3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*(256*a^5*c^5*e
^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*
e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16
*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (2*x*(64*a^2*c^6*d^7*e
 - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e))*(-(c*
d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4
 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (48*a^3*c^4*d^6*e - 60*a^4*c^3*
d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3
*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))
)^(1/2) - (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2)
- a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*
c^6*d^2*e^2)))^(1/2)*1i - (((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*
a^4*c^5*d^3*e^6)/(c*e) + (2*x*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^
```

```
(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2
)*(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c
^6*d^2*e^8))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*
a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*x*(
64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^
5))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d
*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (48*a^3*c^4*d^6
*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/
2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2
*a*c^6*d^2*e^2)))^(1/2) + (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e))*(-(c*d^2*(
-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^
2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*1i)/((((((64*a^5*c^4*d*e^8 + 64*a^3*c^
6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) - (2*x*(-(c*d^2*(-a^3*c^5)^(1/2) - a
*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6
*d^2*e^2)))^(1/2)*(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4
*e^6 + 256*a^4*c^6*d^2*e^8))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3
*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))
)^(1/2) + (2*x*(64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 1
6*a^4*c^4*d^3*e^5))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/
2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) -
 (48*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(c*d^
2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 +
 a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(
c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(
16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (((((64*a^5*c^4*d*e^
8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) + (2*x*(-(c*d^2*(-a^3*c
^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*
e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 25
6*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2)
- a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*
c^6*d^2*e^2)))^(1/2) - (2*x*(64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^
5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(
-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e
^2)))^(1/2) - (48*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(
c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(
16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (2*x*(a^6*e^6 - 2*a^
3*c^3*d^6))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a
^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*(a^4
*c*d^5 - a^5*d^3*e^2))/(c*e)))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)
^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/
2)*2i + x/(c*e) - (log(16*c^5*x*(-d^5*e^3)^(5/2) - 16*c^5*d^13*e^7 - a^5*d^
3*e^17 - 2*a^4*c*d^5*e^15 + 16*a^2*c^3*d^9*e^11 - a^3*c^2*d^7*e^13 + a^5*e^
16*x*(-d^5*e^3)^(1/2) + a^3*c^2*d^4*e^12*x*(-d^5*e^3)^(1/2) + 16*a^2*c^3*d*
e^7*x*(-d^5*e^3)^(3/2) + 2*a^4*c*d^2*e^14*x*(-d^5*e^3)^(1/2))*(-d^5*e^3)^(1
/2))/(2*(a*e^5 + c*d^2*e^3)) + (log(a^5*d^3*e^17 + 16*c^5*d^13*e^7 + 16*c^5
*x*(-d^5*e^3)^(5/2) + 2*a^4*c*d^5*e^15 - 16*a^2*c^3*d^9*e^11 + a^3*c^2*d^7*
e^13 + a^5*e^16*x*(-d^5*e^3)^(1/2) + a^3*c^2*d^4*e^12*x*(-d^5*e^3)^(1/2) +
16*a^2*c^3*d*e^7*x*(-d^5*e^3)^(3/2) + 2*a^4*c*d^2*e^14*x*(-d^5*e^3)^(1/2))*
(-d^5*e^3)^(1/2))/(2*a*e^5 + 2*c*d^2*e^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.169 $\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=336

$$\frac{\sqrt[4]{a}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{a}}{}$$

Rubi [A] time = 0.27, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{a}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,c^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,c^{3/4}\left(ae^2+cd^2\right)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 + a*e^2)) + (a^(1/4)*
(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*c^(3/4)*(c*d^2 + a*e^2)) - (a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)) + (a^(1/4)*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)) - (a^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(3/4)*
(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \int \left(\frac{d^2}{\left(cd^2+ae^2\right)\left(d+ex^2\right)} - \frac{a\left(d-ex^2\right)}{\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx$$

$$= -\frac{a\int \frac{d-ex^2}{a+cx^4}\,dx}{cd^2+ae^2} + \frac{d^2\int \frac{1}{d+ex^2}\,dx}{cd^2+ae^2}$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)} - \frac{\left(a\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c\left(cd^2+ae^2\right)} - \frac{\left(a\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c\left(cd^2+ae^2\right)}$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)} - \frac{\left(a\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4c\left(cd^2+ae^2\right)} - \frac{\left(a\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}}{4c\left(cd^2+ae^2\right)}$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)} + \frac{\sqrt[4]{a}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{3/4}\left(cd^2+ae^2\right)} - \sqrt[4]{a}\left(\sqrt{c}\,d+\right.$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)} + \frac{a^{3/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,c^{3/4}\left(cd^2+ae^2\right)} - \frac{a^{3/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\tan^{-1}\left(1+\frac{\sqrt{2}}{\ }\right.}{2\sqrt{2}\,c^{3/4}\left(cd^2+ae^2\right)}$$

Mathematica [A] time = 0.15, size = 233, normalized size = 0.69

$$\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt{e}\left(\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\left(\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)-\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)\right)+2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+\left(2\sqrt{a}\,e-2\sqrt{c}\,d\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\right)+8c^{3/4}d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8c^{3/4}\sqrt{e}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (8*c^(3/4)*d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*a^(1/4)*Sqrt[e]*(2
*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + (-2*Sqrt
[c]*d + 2*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + (Sqrt[c]*d +
 Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[S
```

```
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])))/(8*c^(3/4)*Sqrt[e]*(c*
d^2 + a*e^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^4/((d + e*x^2)*(a + c*x^4)), x]
```

fricas [B] time = 1.76, size = 4040, normalized size = 12.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*((c*d^2 + a*e^2)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^
4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2
 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^
2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2 - a*e^2)*x + (c^2*d^3 - a*c*d*e^2 + (c^
4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2
 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*
e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4
)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2
+ 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2
*d^2*e^2 + a^2*c*e^4))) - (c*d^2 + a*e^2)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^
2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d
^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)
))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2 - a*e^2)*x - (c^2*d
^3 - a*c*d*e^2 + (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d
^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*
e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2
*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^
8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))
)/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))) + (c*d^2 + a*e^2)*sqrt((2*a*d*e
 - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e
^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^
2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2
 - a*e^2)*x + (c^2*d^3 - a*c*d*e^2 - (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2
*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*
e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e
- (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^
2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2
*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))) - (c*d^2 +
a*e^2)*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2
*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^
4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2
*c*e^4))*log(-(c*d^2 - a*e^2)*x - (c^2*d^3 - a*c*d*e^2 - (c^4*d^4*e + 2*a*c
^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^
7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e
^8)))*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*
d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4
*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*
c*e^4))) + 2*d*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)))/
(c*d^2 + a*e^2), 1/4*(4*d*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + (c*d^2 + a*e^
2)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4
```

```
 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^
4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e
^4))*log(-(c*d^2 - a*e^2)*x + (c^2*d^3 - a*c*d*e^2 + (c^4*d^4*e + 2*a*c^3*d
^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^
8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))
)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4
- 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4
 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^
4))) - (c*d^2 + a*e^2)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e
^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^
2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c
^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2 - a*e^2)*x - (c^2*d^3 - a*c*d*e^2 + (c
^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^
2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2
*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^
4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2
 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^
2*d^2*e^2 + a^2*c*e^4))) + (c*d^2 + a*e^2)*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c
^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*
d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8
)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2 - a*e^2)*x + (c^2*
d^3 - a*c*d*e^2 - (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*
d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4
*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c^
2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d
^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)
))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))) - (c*d^2 + a*e^2)*sqrt((2*a*d*
e - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*
e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d
^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^
2 - a*e^2)*x - (c^2*d^3 - a*c*d*e^2 - (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^
2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6
*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e
 - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e
^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^
2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))))/(c*d^2 +
a*e^2)]
```

giac [A] time = 0.43, size = 327, normalized size = 0.97

$$\frac{d^{\frac{3}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{cd^2+ae^2}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d+\left(ac^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d+\left(ac^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] d^(3/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2 + a*e^2) - 1/2*((a*c^3)^(
1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4)
)/(a/c)^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) - 1/2*((a*c^3)^(1/4)*c
^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c
)^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) - 1/4*((a*c^3)^(1/4)*c^2*d +
 (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^4
*d^2 + sqrt(2)*a*c^3*e^2) + 1/4*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*log
(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*
e^2)
```

maple [A] time = 0.01, size = 363, normalized size = 1.08

$$\frac{d^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(a\,e^2+c\,d^2\right)\sqrt{de}}+\frac{\sqrt{2}\,ae\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(a\,e^2+c\,d^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}c}+\frac{\sqrt{2}\,ae\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(a\,e^2+c\,d^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}c}+\frac{\sqrt{2}\,ae\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8\left(a\,e^2+c\,d^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}c}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(a\,e^2+c\,d^2\right)}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(a\,e^2+c\,d^2\right)}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8\left(a\,e^2+c\,d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] -1/8/(a*e^2+c*d^2)*d*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c
)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))-1/4/(a*e^2+c*d^2)*d*(a/c)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)-1/4/(a*e^2+c*d^2)*d*(a/c)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/8*a/(a*e^2+c*d^2)*e/c/(a/c)^(
1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^
(1/2)*x+(a/c)^(1/2)))+1/4*a/(a*e^2+c*d^2)*e/c/(a/c)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/c)^(1/4)*x+1)+1/4*a/(a*e^2+c*d^2)*e/c/(a/c)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/c)^(1/4)*x-1)+d^2/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*
e*x)
```

maxima [A] time = 2.63, size = 268, normalized size = 0.80

$$\frac{d^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(cd^2+ae^2\right)\sqrt{de}} - \frac{a\left(\frac{2\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{8\left(cd^2+ae^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] d^2*arctan(e*x/sqrt(d*e))/((c*d^2 + a*e^2)*sqrt(d*e)) - 1/8*a*(2*sqrt(2)*(s
qrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(
1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sq
rt(2)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(
1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)
) + sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/
4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*log(sqr
t(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(c*d^2 +
 a*e^2)
```

mupad [B] time = 2.20, size = 5111, normalized size = 15.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] atan(-((((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c
^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 11
2*a^2*c^5*d^5*e^2 - 32*a^3*c^4*d^3*e^4) + ((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-
a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))
^(1/2)*(64*a^2*c^6*d^6*e^2 - x*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2
) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512
*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*
e^7) + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((a*e^2*(-a*c^3)^(1/2) -
c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^
2*e^2)))^(1/2) + 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3) -
 x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)
^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)
*1i + (((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^
5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112
*a^2*c^5*d^5*e^2 - 32*a^3*c^4*d^3*e^4) - ((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a
*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^
(1/2)*(x*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(
c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2
*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 64*a^2*c^6*d^6*
```

```
e^2 + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((a*e^2*(-a*c^3)^(1/2) - c
*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2
*e^2)))^(1/2) - 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3) -
x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^
(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*
1i)/((((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5
*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112*
a^2*c^5*d^5*e^2 - 32*a^3*c^4*d^3*e^4) + ((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*
c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(
1/2)*(64*a^2*c^6*d^6*e^2 - x*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2)
+ 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a
^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^
7) + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((a*e^2*(-a*c^3)^(1/2) - c*
d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*
e^2)))^(1/2) + 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3) - x
*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(
1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2) -
 (((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4
 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112*a^2*
c^5*d^5*e^2 - 32*a^3*c^4*d^3*e^4) - ((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)
^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)
*(x*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d
^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*
d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 64*a^2*c^6*d^6*e^2 +
 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*
(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)
))^(1/2) - 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3) - x*(2*
a^4*c*e^5 + 4*a^2*c^3*d^4*e))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2)
 + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2) + 2*a
^3*c*d^2*e^2))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)
/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*2i + atan(-((((c*d^2
*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^
3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112*a^2*c^5*d^5*e
^2 - 32*a^3*c^4*d^3*e^4) + ((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) +
2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(64*a^2*
c^6*d^6*e^2 - x*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e
)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 -
512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 128*a^3*
c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^
(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)
+ 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3) - x*(2*a^4*c*e^5
 + 4*a^2*c^3*d^4*e))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^
2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*1i + (((c*d^2*
(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3
*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112*a^2*c^5*d^5*e^
2 - 32*a^3*c^4*d^3*e^4) - ((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2
*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(x*((c*d^
2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c
^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 -
512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 64*a^2*c^6*d^6*e^2 + 128*a^3*c
^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(
1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2) -
 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3) - x*(2*a^4*c*e^5
+ 4*a^2*c^3*d^4*e))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2
*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*1i)/((((c*d^2*(
-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*
e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112*a^2*c^5*d^5*e^2
 - 32*a^3*c^4*d^3*e^4) + ((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*
a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(64*a^2*c^
```

```
6*d^6*e^2 - x*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/
(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 51
2*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 128*a^3*c^
5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1
/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2) +
16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3) - x*(2*a^4*c*e^5 +
 4*a^2*c^3*d^4*e))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*
d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2) - (((c*d^2*(-a*c
^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4
+ 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112*a^2*c^5*d^5*e^2 - 3
2*a^3*c^4*d^3*e^4) - ((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^
2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(x*((c*d^2*(-a
*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^
4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a
^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 64*a^2*c^6*d^6*e^2 + 128*a^3*c^5*d^
4*e^4 + 64*a^4*c^4*d^2*e^6))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2)
+ 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2) - 16*a
^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3) - x*(2*a^4*c*e^5 + 4*a
^2*c^3*d^4*e))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)
/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2) + 2*a^3*c*d^2*e^2))*
((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 +
 a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*2i - (log(16*c^3*x*(-d^3*e)^(5/2) +
 a^3*d^2*e^8 + 16*c^3*d^8*e^2 + 17*a*c^2*d^6*e^4 + 2*a^2*c*d^4*e^6 + a^3*e^
8*x*(-d^3*e)^(1/2) - 17*a*c^2*d*e^3*x*(-d^3*e)^(3/2) + 2*a^2*c*d^2*e^6*x*(-
d^3*e)^(1/2))*(-d^3*e)^(1/2))/(2*(a*e^3 + c*d^2*e)) + (log(a^3*d^2*e^8 - 16
*c^3*x*(-d^3*e)^(5/2) + 16*c^3*d^8*e^2 + 17*a*c^2*d^6*e^4 + 2*a^2*c*d^4*e^6
 - a^3*e^8*x*(-d^3*e)^(1/2) + 17*a*c^2*d*e^3*x*(-d^3*e)^(3/2) - 2*a^2*c*d^2
*e^6*x*(-d^3*e)^(1/2))*(-d^3*e)^(1/2))/(2*a*e^3 + 2*c*d^2*e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.170 $\int \frac{x^2}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=337

$$\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,\left(ae^2+cd^2\right)}-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,\left(ae^2+cd^2\right)}-\frac{\sqrt{d}\,\sqrt{e}\,\tan^{-1}\left(\right.}{ae^2+cd^2}$$

Rubi [A] time = 0.27, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,\left(ae^2+cd^2\right)}-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,\left(ae^2+cd^2\right)}-\frac{\sqrt{d}\,\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{ae^2+cd^2}-\frac{\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -((Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)) - ((Sqrt[c
]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4
)*c^(1/4)*(c*d^2 + a*e^2)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)) + ((Sqrt[c]
*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*
Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)) - ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)
*(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \int \left(-\frac{de}{\left(cd^2+ae^2\right)\left(d+ex^2\right)}+\frac{ae+cdx^2}{\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx$$

$$=\frac{\int \frac{ae+cdx^2}{a+cx^4}\,dx}{cd^2+ae^2}-\frac{(de)\int \frac{1}{d+ex^2}\,dx}{cd^2+ae^2}$$

$$=-\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2+ae^2}-\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)}+\frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)}$$

$$=-\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2+ae^2}+\frac{\left(\sqrt[4]{c}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}\left(cd^2+ae^2\right)}+\frac{\left(\sqrt[4]{c}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int \cdots}{4\sqrt{2}\sqrt[4]{a}\left(\cdots\right.}$$

$$=-\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2+ae^2}+\frac{\sqrt[4]{c}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\left(cd^2+ae^2\right)}-\frac{\sqrt[4]{c}\left(d-\cdots\right.}{\cdots}$$

$$=-\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2+ae^2}-\frac{\sqrt[4]{c}\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\left(cd^2+ae^2\right)}+\frac{\sqrt[4]{c}\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(\cdots\right.}{2\sqrt{2}\sqrt[4]{a}\left(cd^2+\cdots\right.}$$

Mathematica [A] time = 0.13, size = 232, normalized size = 0.69

$$\frac{\sqrt{2}\left(\left(\sqrt{c}d-\sqrt{a}e\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)\right)-2\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\right)-8\sqrt[4]{a}\sqrt[4]{c}\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-8*a^(1/4)*c^(1/4)*Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*(
-2*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sqr
t[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + (Sqrt[c]*d -
Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[Sq
```

```
rt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])))/(8*a^(1/4)*c^(1/4)*(c*d
^2 + a*e^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^2/((d + e*x^2)*(a + c*x^4)), x]
```

fricas [B] time = 1.78, size = 3892, normalized size = 11.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((c*d^2 + a*e^2)*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*s
qrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6
*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^
2 + a^2*e^4))*log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 +
2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/
(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^
5*c*e^8)))*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^
4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4
*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)
)) - (c*d^2 + a*e^2)*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqr
t(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a
^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2
+ a^2*e^4))*log(-(c*d^2 - a*e^2)*x - (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 + 2*
a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a
*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*
c*e^8)))*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4
- 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e
^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)))
 + (c*d^2 + a*e^2)*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(
-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3
*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 +
a^2*e^4))*log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^
2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c
^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*
e^8)))*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 -
2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4
 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))) -
 (c*d^2 + a*e^2)*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(
c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c
^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^
2*e^4))*log(-(c*d^2 - a*e^2)*x - (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^2*
c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5
*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^
8)))*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*
a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 +
 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))) - 2
*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(c*d^2 + a*e^2),
 -1/4*((c*d^2 + a*e^2)*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*s
qrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6
*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^
```

```
2 + a^2*e^4))*log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 +
2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/
(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^
5*c*e^8)))*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^
4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4
*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)
)) - (c*d^2 + a*e^2)*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqr
t(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a
^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2
+ a^2*e^4))*log(-(c*d^2 - a*e^2)*x - (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 + 2*
a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a
*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*
c*e^8)))*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4
- 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e
^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)))
 + (c*d^2 + a*e^2)*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(
-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3
*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 +
a^2*e^4))*log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^
2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c
^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*
e^8)))*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 -
2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4
 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))) -
 (c*d^2 + a*e^2)*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(
c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c
^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^
2*e^4))*log(-(c*d^2 - a*e^2)*x - (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^2*
c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5
*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^
8)))*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*
a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 +
 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))) + 4
*sqrt(d*e)*arctan(sqrt(d*e)*x/d))/(c*d^2 + a*e^2)]
```

giac [A] time = 0.38, size = 336, normalized size = 1.00

$$-\frac{\sqrt{d}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{cd^2+ae^2}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace-\left(ac^3\right)^{\frac{3}{4}}d\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace-\left(ac^3\right)^{\frac{3}{4}}d\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/(c*d^2 + a*e^2) + 1/2*((a*c^3)^(
1/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4)
)/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/2*((a*c^3)^(1/
4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/
(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/4*((a*c^3)^(1/4)
*a*c*e - (a*c^3)^(3/4)*d)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqr
t(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) - 1/4*((a*c^3)^(1/4)*a*c*e - (a*c^3)^
(3/4)*d)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^2 +
sqrt(2)*a^2*c^2*e^2)
```

maple [A] time = 0.01, size = 351, normalized size = 1.04

$$-\frac{de\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(ae^2+cd^2\right)\sqrt{de}}+\frac{\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(ae^2+cd^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(ae^2+cd^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,d\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8\left(ae^2+cd^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4ae^2+4cd^2}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4ae^2+4cd^2}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,e\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8ae^2+8cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/4/(a*e^2+c*d^2)*e*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/8
/(a*e^2+c*d^2)*e*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1
/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4/(a*e^2+c*d^2)*e*(a/c)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/8/(a*e^2+c*d^2)*d/(a/c)^(1/4)*
2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)
*x+(a/c)^(1/2)))+1/4/(a*e^2+c*d^2)*d/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
c)^(1/4)*x+1)+1/4/(a*e^2+c*d^2)*d/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^
(1/4)*x-1)-d*e/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 1.43, size = 275, normalized size = 0.82

$$-\frac{de\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(cd^2+ae^2\right)\sqrt{de}}+\frac{\frac{2\sqrt{2}\left(\sqrt{a}cd+a\sqrt{c}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(\sqrt{a}cd+a\sqrt{c}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\left(\sqrt{a}cd-a\sqrt{c}e\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}cd-a\sqrt{c}e\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}}{8\left(cd^2+ae^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -d*e*arctan(e*x/sqrt(d*e))/((c*d^2 + a*e^2)*sqrt(d*e)) + 1/8*(2*sqrt(2)*(sq
rt(a)*c*d + a*sqrt(c)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*
c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2
*sqrt(2)*(sqrt(a)*c*d + a*sqrt(c)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt
(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*
sqrt(c)) - sqrt(2)*(sqrt(a)*c*d - a*sqrt(c)*e)*log(sqrt(c)*x^2 + sqrt(2)*a^
(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + sqrt(2)*(sqrt(a)*c*d - a*sqr
t(c)*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(
3/4)))/(c*d^2 + a*e^2)
```

mupad [B] time = 1.59, size = 4720, normalized size = 14.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (log(a^2*d*e^7 + c^2*d^5*e^3 - c^2*d*x*(-d*e)^(7/2) + 2*a*c*d^3*e^5 + a^2*e
^7*x*(-d*e)^(1/2) + 2*a*c*e^3*x*(-d*e)^(5/2))*(-d*e)^(1/2))/(2*a*e^2 + 2*c*
d^2) - atan((((-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(
a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(((-(c*d^2*(-a*c)^(1/2)
- a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^
2*e^2)))^(1/2)*(x*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(
16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 5
12*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^4*c
^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5) - x*(16*a*c^6*d^5*e^2
 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*d^3*e^4))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*
(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))
^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4) + x*(2*a^2*c^3*e^5 - 4*a*c^4
*d^2*e^3))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c
^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*1i - ((-(c*d^2*(-a*c)^(1/2)
 - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d
^2*e^2)))^(1/2)*(((-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(
16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(192*a^4*c^4*d*e^7 -
 x*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 +
 a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*
e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^2*c^6*d^5*e^3 + 38
4*a^3*c^5*d^3*e^5) + x*(16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*
d^3*e^4))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^
3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c
^4*d^2*e^4) - x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3))*(-(c*d^2*(-a*c)^(1/2) -
```

```
a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*
e^2)))^(1/2)*1i)/(((-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/
(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(((-(c*d^2*(-a*c)^(
1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c
^2*d^2*e^2)))^(1/2)*(x*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d
*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^
9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*
a^4*c^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5) - x*(16*a*c^6*d^
5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*d^3*e^4))*(-(c*d^2*(-a*c)^(1/2) - a
*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e
^2)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4) + x*(2*a^2*c^3*e^5 - 4*
a*c^4*d^2*e^3))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16
*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2) + ((-(c*d^2*(-a*c)^(1/
2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2
*d^2*e^2)))^(1/2)*(((-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)
/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(192*a^4*c^4*d*e^7
 - x*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4
 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^
6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^2*c^6*d^5*e^3 +
384*a^3*c^5*d^3*e^5) + x*(16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^
5*d^3*e^4))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*
c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2
*c^4*d^2*e^4) - x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3))*(-(c*d^2*(-a*c)^(1/2)
- a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^
2*e^2)))^(1/2) + 2*a*c^3*d*e^3))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2)
 + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*2i -
(log(a^2*d*e^7 + c^2*d^5*e^3 + c^2*d*x*(-d*e)^(7/2) + 2*a*c*d^3*e^5 - a^2*e
^7*x*(-d*e)^(1/2) - 2*a*c*e^3*x*(-d*e)^(5/2))*(-d*e)^(1/2))/(2*(a*e^2 + c*d
^2)) - atan((((-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(
a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(((-(a*e^2*(-a*c)^(1/2)
- c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^
2*e^2)))^(1/2)*(x*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(
16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 5
12*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^4*c
^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5) - x*(16*a*c^6*d^5*e^2
 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*d^3*e^4))*(-(a*e^2*(-a*c)^(1/2) - c*d^2*
(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))
^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4) + x*(2*a^2*c^3*e^5 - 4*a*c^4
*d^2*e^3))*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c
^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*1i - ((-(a*e^2*(-a*c)^(1/2)
 - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d
^2*e^2)))^(1/2)*(((-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(
16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(192*a^4*c^4*d*e^7 -
 x*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 +
 a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*
e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^2*c^6*d^5*e^3 + 38
4*a^3*c^5*d^3*e^5) + x*(16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*
d^3*e^4))*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^
3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c
^4*d^2*e^4) - x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3))*(-(a*e^2*(-a*c)^(1/2) -
c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*
e^2)))^(1/2)*1i)/(((-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/
(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(((-(a*e^2*(-a*c)^(
1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c
^2*d^2*e^2)))^(1/2)*(x*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d
*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^
9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*
a^4*c^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5) - x*(16*a*c^6*d^
5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*d^3*e^4))*(-(a*e^2*(-a*c)^(1/2) - c
```

```
*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e
^2)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4) + x*(2*a^2*c^3*e^5 - 4*
a*c^4*d^2*e^3))*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16
*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2) + ((-(a*e^2*(-a*c)^(1/
2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2
*d^2*e^2)))^(1/2)*(((-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)
/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(192*a^4*c^4*d*e^7
 - x*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4
 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^
6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^2*c^6*d^5*e^3 +
384*a^3*c^5*d^3*e^5) + x*(16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^
5*d^3*e^4))*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*
c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2
*c^4*d^2*e^4) - x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3))*(-(a*e^2*(-a*c)^(1/2)
- c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^
2*e^2)))^(1/2) + 2*a*c^3*d*e^3))*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2)
 + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x**2+d)/(c*x**4+a),x)
[Out] Timed out
```

3.171 $\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=336

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)}-\ldots$$

Rubi [A] time = 0.27, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1171, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)) - (c^(1/4)*
(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*
(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \int \left(\frac{e^2}{\left(cd^2+ae^2\right)\left(d+ex^2\right)}+\frac{c\left(d-ex^2\right)}{\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx$$

$$=\frac{c\int \frac{d-ex^2}{a+cx^4}\,dx}{cd^2+ae^2}+\frac{e^2\int \frac{1}{d+ex^2}\,dx}{cd^2+ae^2}$$

$$=\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)}+\frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)}+\frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)}$$

$$=\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)}+\frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4\left(cd^2+ae^2\right)}+\frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4\left(cd^2+ae^2\right)}-$$

$$=\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(cd^2+ae^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{c}\,d+\right.}{}$$

$$=\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\left(cd^2+ae^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(\right.}{2\sqrt{2}\,a^{3/4}\left(cd^2+a\right.}$$

Mathematica [A] time = 0.15, size = 234, normalized size = 0.70

$$\frac{8a^{3/4}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)+\sqrt{2}\,\sqrt[4]{c}\,\sqrt{d}\left(-\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\left(\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)-\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)\right)+\left(2\sqrt{a}\,e-2\sqrt{c}\,d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\right)}{8a^{3/4}\sqrt{d}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (8*a^(3/4)*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*c^(1/4)*Sqrt[d]*((
-2*Sqrt[c]*d + 2*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sq
rt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - (Sqrt[c]*d +
 Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]))/(8*a^(3/4)*Sqrt[d]*(c*
d^2 + a*e^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x^2)*(a + c*x^4)),x]
[Out] IntegrateAlgebraic[1/((d + e*x^2)*(a + c*x^4)), x]
```

fricas [B] time = 3.20, size = 4084, normalized size = 12.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
[Out] [-1/4*((c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e
^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*
d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a
^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*
e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*
d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 +
 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2
+ a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a
^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^
4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d
^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^
4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 +
 a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^
2)*x - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^
5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d
^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (
a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a
^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^
2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e
^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4
- 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2
*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*
e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*
e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4
)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 +
a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c
^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*
a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2
 + a^3*e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^
2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4
*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*
d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x - (a*c^2*d^3 -
 a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 -
 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*
d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c
*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*
d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/
(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - 2*e*sqrt(-e/d)*log((e*x^2 + 2*d
*x*sqrt(-e/d) - d)/(e*x^2 + d)))/(c*d^2 + a*e^2), 1/4*(4*e*sqrt(e/d)*arctan
(x*sqrt(e/d)) - (c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^
2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4
*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*
d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 -
```

```
a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 -
2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*
d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c
*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*
d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/
(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e^2)*sqrt((2*c*d*e +
(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 +
a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*
d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2
- a*c*e^2)*x - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3
+ a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*
a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*
c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^
2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4
*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*
d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-
(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 +
6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e
^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 - (a^3
*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 +
a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d
^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)
*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6
*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*
c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2
*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^
4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))
)/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x - (a*
c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(
c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6
*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4
+ 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/
(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^
7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))))/(c*d^2 + a*e^2)]
```

giac [A] time = 0.42, size = 339, normalized size = 1.01

$$\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d-\left(ac^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^{3}d^{2}+\sqrt{2}a^{2}c^{2}e^{2}\right)}+\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d-\left(ac^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^{3}d^{2}+\sqrt{2}a^{2}c^{2}e^{2}\right)}+\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d+\left(ac^{3}\right)^{\frac{3}{4}}e\right)\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^{3}d^{2}+\sqrt{2}a^{2}c^{2}e^{2}\right)}-\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d+\left(ac^{3}\right)^{\frac{3}{4}}e\right)\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^{3}d^{2}+\sqrt{2}a^{2}c^{2}e^{2}\right)}+\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{3}{2}}}{\left(cd^{2}+ae^{2}\right)\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/
2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)
*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/4*
((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + s
qrt(a/c))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) - 1/4*((a*c^3)^(1/4)*c^
2*d + (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2
)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/((c*
d^2 + a*e^2)*sqrt(d))
```

maple [A] time = 0.01, size = 363, normalized size = 1.08

$$\frac{e^{2}\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(ae^{2}+cd^{2}\right)\sqrt{de}}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,cd\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(ae^{2}+cd^{2}\right)a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,cd\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(ae^{2}+cd^{2}\right)a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,cd\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8\left(ae^{2}+cd^{2}\right)a}-\frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(ae^{2}+cd^{2}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(ae^{2}+cd^{2}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\,e\ln\left(\frac{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8\left(ae^{2}+cd^{2}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/8*c/(a*e^2+c*d^2)*d*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(
a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4*c/(a*e^2+c*d^2)*d*
(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/4*c/(a*e^2+c*d^2)*d
*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)-1/8/(a*e^2+c*d^2)*e/
(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(
1/4)*2^(1/2)*x+(a/c)^(1/2)))-1/4/(a*e^2+c*d^2)*e/(a/c)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/c)^(1/4)*x+1)-1/4/(a*e^2+c*d^2)*e/(a/c)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/c)^(1/4)*x-1)+e^2/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e
*x)
```

maxima [A] time = 1.09, size = 268, normalized size = 0.80

$$\frac{e^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(cd^2+ae^2\right)\sqrt{de}}+\frac{c\left(\frac{2\sqrt{2}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{8\left(cd^2+ae^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] e^2*arctan(e*x/sqrt(d*e))/((c*d^2 + a*e^2)*sqrt(d*e)) + 1/8*c*(2*sqrt(2)*(s
qrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(
1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sq
rt(2)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(
1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)
) + sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/
4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*log(sqr
t(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(c*d^2 +
 a*e^2)
```

mupad [B] time = 1.67, size = 4802, normalized size = 14.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] atan(((((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^
5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-
a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d
^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*((a*e^2*(-a^3*c)^(1/2) -
 c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d
^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^
5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3
*c^5*d^2*e^6) + x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*
((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 +
 a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((
a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a
^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*1i - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*
(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)
))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) +
2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4
*c^4*e^8 - x*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(
16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512
*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^
6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32
*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c
)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2
```

) + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*1i)/((((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) + x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 - x*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)))*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*2i + atan(((((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) + x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*1i - (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 - x*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*1i)/((((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) + x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))

```
^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^
3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(
1/2) + (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a
^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((c*d^2*(
-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*
d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 - x*((c*d^2*(-a^3*c)^(1/2)
- a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*
d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e
^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^
3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))
*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4
+ a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(
(c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 +
a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)))*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a
^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^
(1/2)*2i - (log(16*a^2*e^2*(-d*e^3)^(3/2) + c^2*d^5*e^3*x - c^2*d^5*e*(-d*e
^3)^(1/2) + 16*a^2*d*e^7*x + a*c*d^2*(-d*e^3)^(3/2) + a*c*d^3*e^5*x)*(-d*e^
3)^(1/2))/(2*(c*d^3 + a*d*e^2)) + (log(c^2*d^5*e^3*x - 16*a^2*e^2*(-d*e^3)^
(3/2) + c^2*d^5*e*(-d*e^3)^(1/2) + 16*a^2*d*e^7*x + 4*a*c*d^2*(-d*e^3)^(3/2
) + a*c*d^3*e^5*x + 5*a*c*d^3*e^3*(-d*e^3)^(1/2))*(-d*e^3)^(1/2))/(2*c*d^3
+ 2*a*d*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.172 $\int \frac{1}{x^2(d+ex^2)(a+cx^4)} \, dx$

Optimal. Leaf size=348

$$-\frac{c^{3/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{5/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{5/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(\right.}{}$$

Rubi [A] time = 0.30, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{c^{3/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{5/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{5/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{5/4}\left(ae^2+cd^2\right)}-\frac{c^{3/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{5/4}\left(ae^2+cd^2\right)}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(ae^2+cd^2\right)}-\frac{1}{adx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -(1/(a*d*x)) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 + a*e^
2)) + (c^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/
4)])/(2*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2)) - (c^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)
*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2
)) - (c^(3/4)*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2)) + (c^(3/4)*(Sqrt[c]*
d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*S
qrt[2]*a^(5/4)*(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^2\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \int \left(\frac{1}{adx^2} - \frac{e^3}{d\left(cd^2+ae^2\right)\left(d+ex^2\right)} - \frac{c\left(ae+cdx^2\right)}{a\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx$$

$$= -\frac{1}{adx} - \frac{c\int \frac{ae+cdx^2}{a+cx^4}\,dx}{a\left(cd^2+ae^2\right)} - \frac{e^3\int \frac{1}{d+ex^2}\,dx}{d\left(cd^2+ae^2\right)}$$

$$= -\frac{1}{adx} - \frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)} + \frac{\left(c\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{2a\left(cd^2+ae^2\right)} - \frac{\left(c\left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\right)\int \frac{\sqrt{a}\cdots}{a\cdots}}{2a\left(cd^2+ae^2\cdots\right.}$$

$$= -\frac{1}{adx} - \frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)} - \frac{\left(c^{5/4}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\,a^{5/4}\left(cd^2+ae^2\right)} - \frac{\left(c^{5/4}\left(d-\frac{\sqrt{a}}{\sqrt{c}}\cdots\right.\right.}{4\sqrt{2}\cdots}$$

$$= -\frac{1}{adx} - \frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)} - \frac{c^{5/4}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{5/4}\left(cd^2+ae^2\right)} + \frac{c^{5}\cdots}{\cdots}$$

$$= -\frac{1}{adx} - \frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)} + \frac{c^{3/4}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{5/4}\left(cd^2+ae^2\right)} - \frac{c^{3/4}\left(\sqrt{c}\,d+\cdots\right.}{2\sqrt{\cdots}}$$

Mathematica [A] time = 0.25, size = 389, normalized size = 1.12

$$\frac{-\sqrt{d}\left(8a^{5/4}e^2+\sqrt{2}c^{5/4}d^2x\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-\sqrt{2}c^{5/4}d^2x\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-\sqrt{2}\sqrt{a}c^{3/4}dex\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+\sqrt{2}\sqrt{a}c^{3/4}dex\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-2\sqrt{2}c^{3/4}dx\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\sqrt{2}c^{3/4}dx\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)+8\sqrt[4]{a}cd^2\right)-8a^{5/4}e^{5/2}x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8a^{5/4}d^{3/2}x\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-8*a^(5/4)*e^(5/2)*x*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - Sqrt[d]*(8*a^(1/4)*c*d^
2 + 8*a^(5/4)*e^2 - 2*Sqrt[2]*c^(3/4)*d*(Sqrt[c]*d + Sqrt[a]*e)*x*ArcTan[1
- (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*c^(3/4)*d*(Sqrt[c]*d + Sqrt[a]*e
)*x*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[2]*c^(5/4)*d^2*x*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Sqrt[2]*Sqrt[a]*c^(3/4)*d
```

```
*e*x*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Sqrt[2]*c^(5/
4)*d^2*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*S
qrt[a]*c^(3/4)*d*e*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
))/(8*a^(5/4)*d^(3/2)*(c*d^2 + a*e^2)*x)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(d + ex^2\right) \left(a + cx^4\right)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(d + e*x^2)*(a + c*x^4)), x]
```

fricas [B] time = 9.08, size = 4362, normalized size = 12.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*e^2*x*sqrt(-e/d)*log((e*x^2 - 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d))
+ (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2
 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 +
4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c
^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x + (a^2*c^
2*d^2*e - a^3*c*e^3 - (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^
5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6
*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e + (a^2*c^2
*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^
3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e
^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) - (a*c*d^3 + a^
2*d*e^2)*x*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqr
t(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e
^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*
c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x - (a^2*c^2*d^2*e - a^3*c
*e^3 - (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4
*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^
4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*
d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4
*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))
/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) + (a*c*d^3 + a^2*d*e^2)*x*sqrt
(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2
*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*
d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4
*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3 + (a^4*c^2
*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*
c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2
*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e
^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^
3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 +
 2*a^3*c*d^2*e^2 + a^4*e^4))) - (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e -
(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2
+ a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8
*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^
3*d^2 - a*c^2*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 + (a^4*c^2*d^5 + 2*a^5*c*
d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c
^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)
))*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*
```

```
d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a
^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^
2 + a^4*e^4))) - 4*c*d^2 - 4*a*e^2)/((a*c*d^3 + a^2*d*e^2)*x), -1/4*(4*a*e^
2*x*sqrt(e/d)*arctan(x*sqrt(e/d)) - (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*
e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*
e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4
*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(
-(c^3*d^2 - a*c^2*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3 - (a^4*c^2*d^5 + 2*a^
5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a
^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*
e^8)))*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(
c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 +
 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^
2*e^2 + a^4*e^4))) + (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e + (a^2*c^2*d^
4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e
^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6
+ a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c
^2*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 - (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a
^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*
a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2
*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c
^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*
e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4
))) - (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2
*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^
8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a
^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x + (a^
2*c^2*d^2*e - a^3*c*e^3 + (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(
-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2
 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e - (a^2
*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^
2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d
^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) + (a*c*d^3
+ a^2*d*e^2)*x*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)
*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d
^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*
a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x - (a^2*c^2*d^2*e - a
^3*c*e^3 + (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a
*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^
4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^
3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5
*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^
8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) + 4*c*d^2 + 4*a*e^2)/((a*c
*d^3 + a^2*d*e^2)*x)]
```

giac [A] time = 0.40, size = 348, normalized size = 1.00

$$-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace-\left(ac^3\right)^{\frac{3}{4}}d\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace-\left(ac^3\right)^{\frac{3}{4}}d\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)}-\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{5}{2}}}{\left(cd^3+ade^2\right)\sqrt{d}}-\frac{1}{adx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*((a*c^3)^(1/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x + sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1
/2*((a*c^3)^(1/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2
)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1/4
*((a*c^3)^(1/4)*a*c*e - (a*c^3)^(3/4)*d)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) +
sqrt(a/c))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) + 1/4*((a*c^3)^(1/4)*a
```

```
*c*e - (a*c^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(
2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - arctan(x*e^(1/2)/sqrt(d))*e^(5/2)/((c
*d^3 + a*d*e^2)*sqrt(d)) - 1/(a*d*x)
```

maple [A] time = 0.01, size = 390, normalized size = 1.12

$$-\frac{e^{3}\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(a\,e^{2}+c\,d^{2}\right)\sqrt{de}\,d}-\frac{\sqrt{2}\,cd\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(a\,e^{2}+c\,d^{2}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}a}-\frac{\sqrt{2}\,cd\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(a\,e^{2}+c\,d^{2}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}a}-\frac{\sqrt{2}\,cd\ln\left(\frac{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8\left(a\,e^{2}+c\,d^{2}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}a}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,ce\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(a\,e^{2}+c\,d^{2}\right)a}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,ce\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(a\,e^{2}+c\,d^{2}\right)a}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,ce\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8\left(a\,e^{2}+c\,d^{2}\right)a}-\frac{1}{adx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] -1/a/d/x-1/4*c/(a*e^2+c*d^2)/a*e*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(
1/4)*x-1)-1/8*c/(a*e^2+c*d^2)/a*e*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2
^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))-1/4*c/(a*e^2
+c*d^2)/a*e*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)-1/8*c/(a*e^
2+c*d^2)/a*d/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))
/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))-1/4*c/(a*e^2+c*d^2)/a*d/(a/c)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)-1/4*c/(a*e^2+c*d^2)/a*d/(a/c)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)-1/d*e^3/(a*e^2+c*d^2)/(d*e)^(1/
2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 1.27, size = 292, normalized size = 0.84

$$-\frac{e^{3}\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(cd^{3}+ade^{2}\right)\sqrt{de}}-\frac{c\left(\frac{2\,\sqrt{2}\left(\sqrt{a}\,cd+a\sqrt{c}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\left(\sqrt{a}\,cd+a\sqrt{c}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\left(\sqrt{a}\,cd-a\sqrt{c}\,e\right)\log\left(\sqrt{c}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}\,cd-a\sqrt{c}\,e\right)\log\left(\sqrt{c}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{8\left(acd^{2}+a^{2}e^{2}\right)}-\frac{1}{adx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -e^3*arctan(e*x/sqrt(d*e))/((c*d^3 + a*d*e^2)*sqrt(d*e)) - 1/8*c*(2*sqrt(2)
*(sqrt(a)*c*d + a*sqrt(c)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1
/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c))
 + 2*sqrt(2)*(sqrt(a)*c*d + a*sqrt(c)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x -
sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
c))*sqrt(c)) - sqrt(2)*(sqrt(a)*c*d - a*sqrt(c)*e)*log(sqrt(c)*x^2 + sqrt(2
)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + sqrt(2)*(sqrt(a)*c*d - a
*sqrt(c)*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)
*c^(3/4)))/(a*c*d^2 + a^2*e^2) - 1/(a*d*x)
```

mupad [B] time = 2.00, size = 5761, normalized size = 16.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] atan(((x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7) - (-(a*e^2*(-a^5*c^3)^(1/2
) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*
a^6*c*d^2*e^2)))^(1/2)*(((-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2)
 + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*
(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^
7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512
*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9) - 192*a^
10*c^7*d^14*e^3 - 128*a^11*c^6*d^12*e^5 + 320*a^12*c^5*d^10*e^7 + 256*a^13*
c^4*d^8*e^9) + x*(16*a^8*c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 112*a^10*c^6*
d^10*e^6 + 128*a^11*c^5*d^8*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c
^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^
```

```
(1/2) - 4*a^7*c^8*d^13*e^2 - 4*a^8*c^7*d^11*e^4 + 16*a^10*c^5*d^7*e^8))*(-(
a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e
^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*1i + (x*(2*a^7*c^7*d^9*e^5 - 4*
a^8*c^6*d^7*e^7) - (-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a
^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(((-(a*e^
2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 +
 a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^
2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^
2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^
5*d^11*e^7 - 512*a^14*c^4*d^9*e^9) + 192*a^10*c^7*d^14*e^3 + 128*a^11*c^6*d
^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9) + x*(16*a^8*c^8*d^1
4*e^2 + 32*a^9*c^7*d^12*e^4 - 112*a^10*c^6*d^10*e^6 + 128*a^11*c^5*d^8*e^8)
)*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(
a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a
^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(
-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e
^2)))^(1/2)*1i)/((x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7) - (-(a*e^2*(-a^
5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c
^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(((-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5
*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2))
)^(1/2)*(x*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d
*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^1
5*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^
9) - 192*a^10*c^7*d^14*e^3 - 128*a^11*c^6*d^12*e^5 + 320*a^12*c^5*d^10*e^7
+ 256*a^13*c^4*d^8*e^9) + x*(16*a^8*c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 11
2*a^10*c^6*d^10*e^6 + 128*a^11*c^5*d^8*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*
d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*
d^2*e^2)))^(1/2) - 4*a^7*c^8*d^13*e^2 - 4*a^8*c^7*d^11*e^4 + 16*a^10*c^5*d^
7*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)
/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2) - (x*(2*a^7*c^7*d^9*
e^5 - 4*a^8*c^6*d^7*e^7) - (-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/
2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(
((-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a
^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*(-(a*e^2*(-a^5*c^3)^(1/2
) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*
a^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512
*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9) + 192*a^10*c^7*d^14*e^3 + 128*a^
11*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9) + x*(16*a^8
*c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 112*a^10*c^6*d^10*e^6 + 128*a^11*c^5*
d^8*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*
e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e
^2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) -
 c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6
*c*d^2*e^2)))^(1/2)))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) +
2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*2i + a
tan(((x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7) - (-(c*d^2*(-a^5*c^3)^(1/2)
 - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a
^6*c*d^2*e^2)))^(1/2)*(((-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2)
+ 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*(
-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7
*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*
a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9) - 192*a^1
0*c^7*d^14*e^3 - 128*a^11*c^6*d^12*e^5 + 320*a^12*c^5*d^10*e^7 + 256*a^13*c
^4*d^8*e^9) + x*(16*a^8*c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 112*a^10*c^6*d
^10*e^6 + 128*a^11*c^5*d^8*e^8))*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^
3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(
1/2) - 4*a^7*c^8*d^13*e^2 - 4*a^8*c^7*d^11*e^4 + 16*a^10*c^5*d^7*e^8))*(-(c
*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^
4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*1i + (x*(2*a^7*c^7*d^9*e^5 - 4*a
```

```
^8*c^6*d^7*e^7) - (-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^
3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(((-(c*d^2
*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 +
a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2
*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2
*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5
*d^11*e^7 - 512*a^14*c^4*d^9*e^9) + 192*a^10*c^7*d^14*e^3 + 128*a^11*c^6*d^
12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9) + x*(16*a^8*c^8*d^14
*e^2 + 32*a^9*c^7*d^12*e^4 - 112*a^10*c^6*d^10*e^6 + 128*a^11*c^5*d^8*e^8))
*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a
^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^
8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8))*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-
a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^
2)))^(1/2)*1i)/((x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7) - (-(c*d^2*(-a^5
*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^
2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(((-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*
c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))
^(1/2)*(x*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*
e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15
*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9
) - 192*a^10*c^7*d^14*e^3 - 128*a^11*c^6*d^12*e^5 + 320*a^12*c^5*d^10*e^7 +
 256*a^13*c^4*d^8*e^9) + x*(16*a^8*c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 112
*a^10*c^6*d^10*e^6 + 128*a^11*c^5*d^8*e^8))*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e
^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d
^2*e^2)))^(1/2) - 4*a^7*c^8*d^13*e^2 - 4*a^8*c^7*d^11*e^4 + 16*a^10*c^5*d^7
*e^8))*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/
(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2) - (x*(2*a^7*c^7*d^9*e
^5 - 4*a^8*c^6*d^7*e^7) - (-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2
) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*((
(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^
7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*(-(c*d^2*(-a^5*c^3)^(1/2)
 - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a
^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*
a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9) + 192*a^10*c^7*d^14*e^3 + 128*a^1
1*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9) + x*(16*a^8*
c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 112*a^10*c^6*d^10*e^6 + 128*a^11*c^5*d
^8*e^8))*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e
)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^
2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8))*(-(c*d^2*(-a^5*c^3)^(1/2) -
a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*
c*d^2*e^2)))^(1/2)))*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2
*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*2i - 1/
(a*d*x) - (log(c^4*d^11*(-d^3*e^5)^(1/2) - 16*a^4*e^3*(-d^3*e^5)^(3/2) + 16
*a^4*d^4*e^11*x + c^4*d^12*e^3*x + a*c^3*d^9*e^2*(-d^3*e^5)^(1/2) + a*c^3*d
^10*e^5*x - 16*a^3*c*d^6*e^9*x + 16*a^3*c*d^2*e*(-d^3*e^5)^(3/2))*(-d^3*e^5
)^(1/2))/(2*(c*d^5 + a*d^3*e^2)) + (log(16*a^4*e^3*(-d^3*e^5)^(3/2) - c^4*d
^11*(-d^3*e^5)^(1/2) + 16*a^4*d^4*e^11*x + c^4*d^12*e^3*x - a*c^3*d^9*e^2*(
-d^3*e^5)^(1/2) + a*c^3*d^10*e^5*x - 16*a^3*c*d^6*e^9*x - 16*a^3*c*d^2*e*(-
d^3*e^5)^(3/2))*(-d^3*e^5)^(1/2))/(2*c*d^5 + 2*a*d^3*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.173 $\int \frac{1}{x^4(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=360

$$\frac{c^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{7/4}\left(ae^2+cd^2\right)}-\frac{c^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{7/4}\left(ae^2+cd^2\right)}+\frac{c^{5/4}}{}$$

Rubi [A] time = 0.30, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{c^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{7/4}\left(ae^2+cd^2\right)}-\frac{c^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{7/4}\left(ae^2+cd^2\right)}+\frac{c^{5/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{7/4}\left(ae^2+cd^2\right)}-\frac{c^{5/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{7/4}\left(ae^2+cd^2\right)}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}\left(ae^2+cd^2\right)}+\frac{e}{ad^2x}-\frac{1}{3adx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/(3*a*d*x^3) + e/(a*d^2*x) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/
2)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*
c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(5/4)*(Sqrt[c
]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(7/4
)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c
^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sq
rt[c]*x^2])/(4*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^4\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \int \left(\frac{1}{adx^4}-\frac{e}{ad^2x^2}+\frac{e^4}{d^2\left(cd^2+ae^2\right)\left(d+ex^2\right)}-\frac{c^2\left(d-ex^2\right)}{a\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx$$

$$= -\frac{1}{3adx^3}+\frac{e}{ad^2x}-\frac{c^2\int \frac{d-ex^2}{a+cx^4}\,dx}{a\left(cd^2+ae^2\right)}+\frac{e^4\int \frac{1}{d+ex^2}\,dx}{d^2\left(cd^2+ae^2\right)}$$

$$= -\frac{1}{3adx^3}+\frac{e}{ad^2x}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)}-\frac{\left(c\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{2a\left(cd^2+ae^2\right)}-\frac{\left(c\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)\right.}{2a\left(c\right.}$$

$$= -\frac{1}{3adx^3}+\frac{e}{ad^2x}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)}-\frac{\left(c\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4a\left(cd^2+ae^2\right)}-\frac{\left(c\left(\frac{\sqrt{c}\,d}{\sqrt{a}}\right.\right.}{}$$

$$= -\frac{1}{3adx^3}+\frac{e}{ad^2x}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)}+\frac{c^{5/4}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\right.}{4\sqrt{2}\,a^{7/4}\left(cd^2+ae^2\right)}$$

$$= -\frac{1}{3adx^3}+\frac{e}{ad^2x}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)}+\frac{c^{5/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{7/4}\left(cd^2+ae^2\right)}-\frac{c^{5/4}}{}$$

Mathematica [A]   time = 0.40, size = 367, normalized size = 1.02

$$\frac{3\sqrt{2}\,c^{5/4}d^{5/2}x^3\left(a^{3/4}e+\sqrt[4]{a}\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)-3\sqrt{2}\,c^{5/4}d^{5/2}x^3\left(a^{3/4}e+\sqrt[4]{a}\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)+24a^2e^{7/2}x^3\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)+6\sqrt{2}\,\sqrt[4]{a}\,c^{5/4}d^{5/2}x^3\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+6\sqrt{2}\,\sqrt[4]{a}\,c^{5/4}d^{5/2}x^3\left(\sqrt{a}\,e-\sqrt{c}\,d\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)+24a\sqrt{d}\,ex^2\left(ae^2+cd^2\right)-8ad^{3/2}\left(ae^2+cd^2\right)}{24a^2d^{5/2}x^3\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-8*a*d^(3/2)*(c*d^2 + a*e^2) + 24*a*Sqrt[d]*e*(c*d^2 + a*e^2)*x^2 + 24*a^2
*e^(7/2)*x^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + 6*Sqrt[2]*a^(1/4)*c^(5/4)*d^(5/2
)*(Sqrt[c]*d - Sqrt[a]*e)*x^3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*S
qrt[2]*a^(1/4)*c^(5/4)*d^(5/2)*(-(Sqrt[c]*d) + Sqrt[a]*e)*x^3*ArcTan[1 + (S
qrt[2]*c^(1/4)*x)/a^(1/4)] + 3*Sqrt[2]*c^(5/4)*d^(5/2)*(a^(1/4)*Sqrt[c]*d +
```

```
a^(3/4)*e)*x^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - 3*
Sqrt[2]*c^(5/4)*d^(5/2)*(a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*x^3*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(24*a^2*d^(5/2)*(c*d^2 + a*e^2)*x^
3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(d+ex^2\right) \left(a+cx^4\right)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(d + e*x^2)*(a + c*x^4)), x]
```

fricas [B] time = 26.63, size = 4442, normalized size = 12.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/12*(6*a*e^3*x^3*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d
)) + 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c
*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^
4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8
)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x
 + (a^2*c^4*d^3 - a^3*c^3*d*e^2 + (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^
5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3
*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d
*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2
*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 +
4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)))
- 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^
2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d
^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))
/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x -
(a^2*c^4*d^3 - a^3*c^3*d*e^2 + (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*
sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^
6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e
+ (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^
2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a
^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) + 3
*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e
^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8
+ 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a
^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x + (a^
2*c^4*d^3 - a^3*c^3*d*e^2 - (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqr
t(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e
^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e - (
a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 +
 a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10
*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 3*(a
*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2
+ a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4
*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*
c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x - (a^2*c
^4*d^3 - a^3*c^3*d*e^2 - (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-
(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2
+ 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e - (a^3
```

```
*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^
2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*
d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 4*c*d^3
 - 4*a*d*e^2 + 12*(c*d^2*e + a*e^3)*x^2)/((a*c*d^4 + a^2*d^2*e^2)*x^3), 1/1
2*(12*a*e^3*x^3*sqrt(e/d)*arctan(x*sqrt(e/d)) + 3*(a*c*d^4 + a^2*d^2*e^2)*x
^3*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d
^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^
9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e
^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x + (a^2*c^4*d^3 - a^3*c^3*d*e^2
+ (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*
e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4
*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*
e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8
 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(
a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*
sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4
- 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c
^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2
+ a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x - (a^2*c^4*d^3 - a^3*c^3*d*e^2 + (
a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2
 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^
10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2
 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 +
4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3
*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) + 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqr
t((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2
*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*
d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a
^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x + (a^2*c^4*d^3 - a^3*c^3*d*e^2 - (a^6
*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 +
a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*
c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 +
a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a
^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^
2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((
2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*
c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4
*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*
e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x - (a^2*c^4*d^3 - a^3*c^3*d*e^2 - (a^6*c^
2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2
*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d
^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5
*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*
c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d
^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 4*c*d^3 - 4*a*d*e^2 + 12*(c*d^2*e + a*e
^3)*x^2)/((a*c*d^4 + a^2*d^2*e^2)*x^3)]
```

giac [A] time = 0.40, size = 364, normalized size = 1.01

$$-\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d-\left(ac^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^{2}c^{2}d^{2}+\sqrt{2}a^{3}ce^{2}\right)}-\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d-\left(ac^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^{2}c^{2}d^{2}+\sqrt{2}a^{3}ce^{2}\right)}-\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d+\left(ac^{3}\right)^{\frac{3}{4}}e\right)\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}a^{2}c^{2}d^{2}+\sqrt{2}a^{3}ce^{2}\right)}+\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d+\left(ac^{3}\right)^{\frac{3}{4}}e\right)\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}a^{2}c^{2}d^{2}+\sqrt{2}a^{3}ce^{2}\right)}+\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{7}{2}}}{\left(cd^{4}+ad^{2}e^{2}\right)\sqrt{d}}+\frac{3x^{2}e-d}{3ad^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1
/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2
)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1/4
*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) +
```

```
sqrt(a/c))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) + 1/4*((a*c^3)^(1/4)*c
^2*d + (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(
2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) + arctan(x*e^(1/2)/sqrt(d))*e^(7/2)/((c
*d^4 + a*d^2*e^2)*sqrt(d)) + 1/3*(3*x^2*e - d)/(a*d^2*x^3)
```

maple [A] time = 0.01, size = 406, normalized size = 1.13

$$\frac{e^{4}\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(a\,e^{2}+c\,d^{2}\right)\sqrt{de}\;d^{2}}+\frac{\sqrt{2}\;ce\arctan\left(\frac{\sqrt{2}\;x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(a\,e^{2}+c\,d^{2}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}a}+\frac{\sqrt{2}\;ce\arctan\left(\frac{\sqrt{2}\;x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(a\,e^{2}+c\,d^{2}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}a}+\frac{\sqrt{2}\;ce\ln\left(\frac{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\;x+\sqrt{\frac{a}{c}}}{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\;x+\sqrt{\frac{a}{c}}}\right)}{8\left(a\,e^{2}+c\,d^{2}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}a}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\;c^{2}d\arctan\left(\frac{\sqrt{2}\;x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(a\,e^{2}+c\,d^{2}\right)a^{2}}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\;c^{2}d\arctan\left(\frac{\sqrt{2}\;x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(a\,e^{2}+c\,d^{2}\right)a^{2}}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\;c^{2}d\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\;x+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\;x+\sqrt{\frac{a}{c}}}\right)}{8\left(a\,e^{2}+c\,d^{2}\right)a^{2}}+\frac{e}{a\,d^{2}x}-\frac{1}{3ad\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] -1/3/a/d/x^3+e/a/d^2/x-1/8*c^2/(a*e^2+c*d^2)/a^2*d*(a/c)^(1/4)*2^(1/2)*ln((
x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/
2)))-1/4*c^2/(a*e^2+c*d^2)/a^2*d*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(
1/4)*x+1)-1/4*c^2/(a*e^2+c*d^2)/a^2*d*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/c)^(1/4)*x-1)+1/8*c/(a*e^2+c*d^2)/a*e/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1
/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4*c/(
a*e^2+c*d^2)/a*e/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/4*c/
(a*e^2+c*d^2)/a*e/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/d^2
*e^4/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.12, size = 297, normalized size = 0.82

$$\frac{e^{4}\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(cd^{4}+ad^{2}e^{2}\right)\sqrt{de}}-\frac{c^{2}\left(\frac{2\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{c}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{c}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{8\left(acd^{2}+a^{2}e^{2}\right)}+\frac{3\,ex^{2}-d}{3\,ad^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] e^4*arctan(e*x/sqrt(d*e))/((c*d^4 + a*d^2*e^2)*sqrt(d*e)) - 1/8*c^2*(2*sqrt
(2)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/
4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c))
+ 2*sqrt(2)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(
2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*s
qrt(c)) + sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)
*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*l
og(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(a
*c*d^2 + a^2*e^2) + 1/3*(3*e*x^2 - d)/(a*d^2*x^3)
```

mupad [B] time = 2.26, size = 5972, normalized size = 16.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] atan(((x*(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((a*e^2*(-a^7*c^5)^(1/
2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2
*a^8*c*d^2*e^2)))^(1/2)*((((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2)
 + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(x*
((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9
*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^24*e^3 + 512*
a^12*c^6*d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) - 64*a^9
*c^8*d^24*e^2 + 128*a^10*c^7*d^22*e^4 + 192*a^11*c^6*d^20*e^6 - 256*a^12*c^
5*d^18*e^8 - 256*a^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32*a^8*c^8*
d^21*e^4 - 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10))*((a*e^2*(-a^7*c^
```

```
5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d
^4 + 2*a^8*c*d^2*e^2)))^(1/2) - 4*a^6*c^9*d^21*e^3 - 4*a^7*c^8*d^19*e^5 + 4
8*a^9*c^6*d^15*e^9))*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*
a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*1i + (x*
(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((a*e^2*(-a^7*c^5)^(1/2) - c*d^
2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^
2*e^2)))^(1/2)*((((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*
c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(x*((a*e^2*(
-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^
7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^24*e^3 + 512*a^12*c^6*
d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) + 64*a^9*c^8*d^24
*e^2 - 128*a^10*c^7*d^22*e^4 - 192*a^11*c^6*d^20*e^6 + 256*a^12*c^5*d^18*e^
8 + 256*a^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32*a^8*c^8*d^21*e^4
- 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10))*((a*e^2*(-a^7*c^5)^(1/2)
- c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^
8*c*d^2*e^2)))^(1/2) + 4*a^6*c^9*d^21*e^3 + 4*a^7*c^8*d^19*e^5 - 48*a^9*c^6
*d^15*e^9))*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d
*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*1i)/((x*(2*a^5*c^
9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c
^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^
(1/2)*((((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/
(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(x*((a*e^2*(-a^7*c^5)
^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4
 + 2*a^8*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^24*e^3 + 512*a^12*c^6*d^22*e^5
- 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) - 64*a^9*c^8*d^24*e^2 + 12
8*a^10*c^7*d^22*e^4 + 192*a^11*c^6*d^20*e^6 - 256*a^12*c^5*d^18*e^8 - 256*a
^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32*a^8*c^8*d^21*e^4 - 112*a^9
*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10))*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(
-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e
^2)))^(1/2) - 4*a^6*c^9*d^21*e^3 - 4*a^7*c^8*d^19*e^5 + 48*a^9*c^6*d^15*e^9
))*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(
a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) - (x*(2*a^5*c^9*d^18*e^5 +
 4*a^7*c^7*d^14*e^9) - ((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) +
2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*((((a*
e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4
 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(x*((a*e^2*(-a^7*c^5)^(1/2) - c*d
^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d
^2*e^2)))^(1/2)*(512*a^11*c^7*d^24*e^3 + 512*a^12*c^6*d^22*e^5 - 512*a^13*c
^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) + 64*a^9*c^8*d^24*e^2 - 128*a^10*c^7*d
^22*e^4 - 192*a^11*c^6*d^20*e^6 + 256*a^12*c^5*d^18*e^8 + 256*a^13*c^4*d^16
*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32*a^8*c^8*d^21*e^4 - 112*a^9*c^7*d^19*e^
6 - 128*a^11*c^5*d^15*e^10))*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1
/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)
+ 4*a^6*c^9*d^21*e^3 + 4*a^7*c^8*d^19*e^5 - 48*a^9*c^6*d^15*e^9))*((a*e^2*(
-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^
7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) + 2*a^5*c^8*d^14*e^8))*((a*e^2*(-a^7*c
^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*
d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*2i + atan(((x*(2*a^5*c^9*d^18*e^5 + 4*a^7*c^
7*d^14*e^9) - ((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3
*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*((((c*d^2*(-a^7
*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^
2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(x*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*
c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))
^(1/2)*(512*a^11*c^7*d^24*e^3 + 512*a^12*c^6*d^22*e^5 - 512*a^13*c^5*d^20*e
^7 - 512*a^14*c^4*d^18*e^9) - 64*a^9*c^8*d^24*e^2 + 128*a^10*c^7*d^22*e^4 +
 192*a^11*c^6*d^20*e^6 - 256*a^12*c^5*d^18*e^8 - 256*a^13*c^4*d^16*e^10) -
x*(16*a^7*c^9*d^23*e^2 + 32*a^8*c^8*d^21*e^4 - 112*a^9*c^7*d^19*e^6 - 128*a
^11*c^5*d^15*e^10))*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a
^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) - 4*a^6*c
```

```
^9*d^21*e^3 - 4*a^7*c^8*d^19*e^5 + 48*a^9*c^6*d^15*e^9))*((c*d^2*(-a^7*c^5)
^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4
 + 2*a^8*c*d^2*e^2)))^(1/2)*1i + (x*(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^
9) - ((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16
*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*((((c*d^2*(-a^7*c^5)^(1/
2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2
*a^8*c*d^2*e^2)))^(1/2)*(x*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2
) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(5
12*a^11*c^7*d^24*e^3 + 512*a^12*c^6*d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*
a^14*c^4*d^18*e^9) + 64*a^9*c^8*d^24*e^2 - 128*a^10*c^7*d^22*e^4 - 192*a^11
*c^6*d^20*e^6 + 256*a^12*c^5*d^18*e^8 + 256*a^13*c^4*d^16*e^10) - x*(16*a^7
*c^9*d^23*e^2 + 32*a^8*c^8*d^21*e^4 - 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d
^15*e^10))*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*
e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) + 4*a^6*c^9*d^21*e
^3 + 4*a^7*c^8*d^19*e^5 - 48*a^9*c^6*d^15*e^9))*((c*d^2*(-a^7*c^5)^(1/2) -
a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*
c*d^2*e^2)))^(1/2)*1i)/((x*(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((c*
d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4
 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*((((c*d^2*(-a^7*c^5)^(1/2) - a*e^
2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^
2*e^2)))^(1/2)*(x*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4
*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(512*a^11*c
^7*d^24*e^3 + 512*a^12*c^6*d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*
d^18*e^9) - 64*a^9*c^8*d^24*e^2 + 128*a^10*c^7*d^22*e^4 + 192*a^11*c^6*d^20
*e^6 - 256*a^12*c^5*d^18*e^8 - 256*a^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23
*e^2 + 32*a^8*c^8*d^21*e^4 - 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10)
)*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a
^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) - 4*a^6*c^9*d^21*e^3 - 4*a^
7*c^8*d^19*e^5 + 48*a^9*c^6*d^15*e^9))*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a
^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2
)))^(1/2) - (x*(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((c*d^2*(-a^7*c^
5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d
^4 + 2*a^8*c*d^2*e^2)))^(1/2)*((((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)
^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/
2)*(x*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(1
6*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^24*e^3
+ 512*a^12*c^6*d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) +
64*a^9*c^8*d^24*e^2 - 128*a^10*c^7*d^22*e^4 - 192*a^11*c^6*d^20*e^6 + 256*a
^12*c^5*d^18*e^8 + 256*a^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32*a^
8*c^8*d^21*e^4 - 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10))*((c*d^2*(-
a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7
*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) + 4*a^6*c^9*d^21*e^3 + 4*a^7*c^8*d^19*e
^5 - 48*a^9*c^6*d^15*e^9))*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2
) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) +
2*a^5*c^8*d^14*e^8))*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*
a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*2i - (1/
(3*a*d) - (e*x^2)/(a*d^2))/x^3 - (log(16*a^7*d^13*e^20 + c^7*d^27*e^6 + 2*a
*c^6*d^25*e^8 + a^2*c^5*d^23*e^10 + 16*a^4*c^3*d^19*e^14 + 16*a^7*e^3*x*(-d
^5*e^7)^(5/2) - a^2*c^5*d^15*x*(-d^5*e^7)^(3/2) + c^7*d^24*e^3*x*(-d^5*e^7)
^(1/2) - 16*a^4*c^3*d^11*e^4*x*(-d^5*e^7)^(3/2) + 2*a*c^6*d^22*e^5*x*(-d^5*
e^7)^(1/2))*(-d^5*e^7)^(1/2))/(2*(c*d^7 + a*d^5*e^2)) + (log(16*a^7*d^13*e^
20 + c^7*d^27*e^6 + 2*a*c^6*d^25*e^8 + a^2*c^5*d^23*e^10 + 16*a^4*c^3*d^19*
e^14 - 16*a^7*e^3*x*(-d^5*e^7)^(5/2) + a^2*c^5*d^15*x*(-d^5*e^7)^(3/2) - c^
7*d^24*e^3*x*(-d^5*e^7)^(1/2) + 16*a^4*c^3*d^11*e^4*x*(-d^5*e^7)^(3/2) - 2*
a*c^6*d^22*e^5*x*(-d^5*e^7)^(1/2))*(-d^5*e^7)^(1/2))/(2*c*d^7 + 2*a*d^5*e^2
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.174 $\int \frac{x^9}{(d+ex^2)(a+cx^4)^2} dx$

Optimal. Leaf size=169

$$-\frac{\sqrt{a}\, d\left(ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4c^{3/2}\left(ae^2+cd^2\right)^2}+\frac{ae\left(ae^2+2cd^2\right)\log\left(a+cx^4\right)}{4c^2\left(ae^2+cd^2\right)^2}+\frac{a\left(ae+cdx^2\right)}{4c^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{d^4\log\left(d+ex^2\right)}{2e\left(ae^2+cd^2\right)^2}$$

Rubi [A] time = 0.37, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1252, 1647, 1629, 635, 205, 260}

$$\frac{a\left(ae+cdx^2\right)}{4c^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{ae\left(ae^2+2cd^2\right)\log\left(a+cx^4\right)}{4c^2\left(ae^2+cd^2\right)^2}-\frac{\sqrt{a}\, d\left(ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4c^{3/2}\left(ae^2+cd^2\right)^2}+\frac{d^4\log\left(d+ex^2\right)}{2e\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*(a*e + c*d*x^2))/(4*c^2*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[a]*d*(3*c*d
^2 + a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*c^(3/2)*(c*d^2 + a*e^2)^2) +
(d^4*Log[d + e*x^2])/(2*e*(c*d^2 + a*e^2)^2) + (a*e*(2*c*d^2 + a*e^2)*Log[a
 + c*x^4])/(4*c^2*(c*d^2 + a*e^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
```

```
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{1}{2}\mathrm{Subst}\left(\int \frac{x^4}{(d+ex)(a+cx^2)^2}\,dx, x, x^2\right)$$

$$= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} - \frac{\mathrm{Subst}\left(\int \frac{\frac{a^2d^2}{cd^2+ae^2}-\frac{a^2dex}{cd^2+ae^2}-2ax^2}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)}{4ac}$$

$$= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} - \frac{\mathrm{Subst}\left(\int \left(-\frac{2acd^4}{(cd^2+ae^2)^2(d+ex)} + \frac{a^2(d(3cd^2+ae^2)-2e(2cd^2+ae^2)x)}{(cd^2+ae^2)^2(a+cx^2)}\right)\right.}{4ac}$$

$$= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} + \frac{d^4\log(d+ex^2)}{2e(cd^2+ae^2)^2} - \frac{a\,\mathrm{Subst}\left(\int \frac{d(3cd^2+ae^2)-2e(2cd^2+ae^2)x}{a+cx^2}\,dx,\right.}{4c(cd^2+ae^2)^2}$$

$$= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} + \frac{d^4\log(d+ex^2)}{2e(cd^2+ae^2)^2} + \frac{(ae(2cd^2+ae^2))\,\mathrm{Subst}\left(\int \frac{x}{a+cx^2}\,dx,\right.}{2c(cd^2+ae^2)^2}$$

$$= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{a}\,d(3cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4c^{3/2}(cd^2+ae^2)^2} + \frac{d^4\log(d+ex^2)}{2e(cd^2+ae^2)^2} + \frac{ae}{}$$

Mathematica [A] time = 0.21, size = 135, normalized size = 0.80

$$\frac{-\frac{\sqrt{a}\,d(ae^2+3cd^2)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{c^{3/2}} + \frac{ae(ae^2+2cd^2)\log(a+cx^4)}{c^2} + \frac{a(ae^2+cd^2)(ae+cdx^2)}{c^2(a+cx^4)} + \frac{2d^4\log(d+ex^2)}{e}}{4(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((a*(c*d^2 + a*e^2)*(a*e + c*d*x^2))/(c^2*(a + c*x^4)) - (Sqrt[a]*d*(3*c*d^
2 + a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/c^(3/2) + (2*d^4*Log[d + e*x^2])/
e + (a*e*(2*c*d^2 + a*e^2)*Log[a + c*x^4])/c^2)/(4*(c*d^2 + a*e^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[x^9/((d + e*x^2)*(a + c*x^4)^2), x]
```

fricas [A] time = 35.28, size = 555, normalized size = 3.28

$$\left[\frac{2\,a^{2}cd^{2}e^{2}+2\,a^{3}e^{4}+2\left(ac^{2}d^{3}e+a^{2}cde^{3}\right)x^{2}+\left(3\,ac^{2}d^{3}e+a^{2}cde^{3}+\left(3\,c^{3}d^{3}e+ac^{2}de^{3}\right)x^{4}\right)\sqrt{-\frac{a}{c}}\log\left(\frac{cx^{4}-2\,cx^{2}\sqrt{-\frac{a}{c}}-a}{cx^{4}+a}\right)+2\left(2\,a^{2}cd^{2}e^{2}+a^{3}e^{4}+\left(2\,ac^{2}d^{2}e^{2}+a^{2}ce^{4}\right)x^{4}\right)\log\left(cx^{4}+a\right)+4\left(c^{3}d^{4}x^{4}+ac^{2}d^{4}\right)\log\left(ex^{2}+d\right)}{8\left(ac^{4}d^{4}e+2\,a^{2}c^{3}d^{2}e^{3}+a^{3}c^{2}e^{5}+\left(c^{5}d^{4}e+2\,ac^{4}d^{2}e^{3}+a^{2}c^{3}e^{5}\right)x^{4}\right)},\ \frac{a^{2}cd^{2}e^{2}+a^{3}e^{4}+\left(ac^{2}d^{3}e+a^{2}cde^{3}\right)x^{2}-\left(3\,ac^{2}d^{3}e+a^{2}cde^{3}+\left(3\,c^{3}d^{3}e+ac^{2}de^{3}\right)x^{4}\right)\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^{2}\sqrt{\frac{a}{c}}}{a}\right)+\left(2\,a^{2}cd^{2}e^{2}+a^{3}e^{4}+\left(2\,ac^{2}d^{2}e^{2}+a^{2}ce^{4}\right)x^{4}\right)\log\left(cx^{4}+a\right)+2\left(c^{3}d^{4}x^{4}+ac^{2}d^{4}\right)\log\left(ex^{2}+d\right)}{4\left(ac^{4}d^{4}e+2\,a^{2}c^{3}d^{2}e^{3}+a^{3}c^{2}e^{5}+\left(c^{5}d^{4}e+2\,ac^{4}d^{2}e^{3}+a^{2}c^{3}e^{5}\right)x^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(2*a^2*c*d^2*e^2 + 2*a^3*e^4 + 2*(a*c^2*d^3*e + a^2*c*d*e^3)*x^2 + (3*
a*c^2*d^3*e + a^2*c*d*e^3 + (3*c^3*d^3*e + a*c^2*d*e^3)*x^4)*sqrt(-a/c)*log
((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) + 2*(2*a^2*c*d^2*e^2 + a^3*e
^4 + (2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*log(c*x^4 + a) + 4*(c^3*d^4*x^4 + a
*c^2*d^4)*log(e*x^2 + d))/(a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e^5 +
(c^5*d^4*e + 2*a*c^4*d^2*e^3 + a^2*c^3*e^5)*x^4), 1/4*(a^2*c*d^2*e^2 + a^3*
e^4 + (a*c^2*d^3*e + a^2*c*d*e^3)*x^2 - (3*a*c^2*d^3*e + a^2*c*d*e^3 + (3*c
^3*d^3*e + a*c^2*d*e^3)*x^4)*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) + (2*a^2*c
*d^2*e^2 + a^3*e^4 + (2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*log(c*x^4 + a) + 2*
(c^3*d^4*x^4 + a*c^2*d^4)*log(e*x^2 + d))/(a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3
+ a^3*c^2*e^5 + (c^5*d^4*e + 2*a*c^4*d^2*e^3 + a^2*c^3*e^5)*x^4)]
```

giac [A] time = 0.36, size = 251, normalized size = 1.49

$$\frac{d^{4}\log\left(\left|x^{2}e+d\right|\right)}{2\left(c^{2}d^{4}e+2\,acd^{2}e^{3}+a^{2}e^{5}\right)}+\frac{\left(2\,acd^{2}e+a^{2}e^{3}\right)\log\left(cx^{4}+a\right)}{4\left(c^{4}d^{4}+2\,ac^{3}d^{2}e^{2}+a^{2}c^{2}e^{4}\right)}-\frac{\left(3\,acd^{3}+a^{2}de^{2}\right)\arctan\left(\frac{cx^{2}}{\sqrt{ac}}\right)}{4\left(c^{3}d^{4}+2\,ac^{2}d^{2}e^{2}+a^{2}ce^{4}\right)\sqrt{ac}}-\frac{2\,acd^{2}x^{4}e-acd^{3}x^{2}+a^{2}x^{4}e^{3}-a^{2}dx^{2}e^{2}+a^{2}d^{2}e}{4\left(c^{3}d^{4}+2\,ac^{2}d^{2}e^{2}+a^{2}ce^{4}\right)\left(cx^{4}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*d^4*log(abs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*(2*
a*c*d^2*e + a^2*e^3)*log(c*x^4 + a)/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^
4) - 1/4*(3*a*c*d^3 + a^2*d*e^2)*arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a*c^
2*d^2*e^2 + a^2*c*e^4)*sqrt(a*c)) - 1/4*(2*a*c*d^2*x^4*e - a*c*d^3*x^2 + a^
2*x^4*e^3 - a^2*d*x^2*e^2 + a^2*d^2*e)/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*
e^4)*(c*x^4 + a))
```

maple [A] time = 0.02, size = 305, normalized size = 1.80

$$\frac{a^{2}d\,e^{2}x^{2}}{4\left(a\,e^{2}+c\,d^{2}\right)^{2}\left(c\,x^{4}+a\right)c}+\frac{a\,d^{3}x^{2}}{4\left(a\,e^{2}+c\,d^{2}\right)^{2}\left(c\,x^{4}+a\right)}-\frac{a^{2}d\,e^{2}\arctan\left(\frac{c\,x^{2}}{\sqrt{a c}}\right)}{4\left(a\,e^{2}+c\,d^{2}\right)^{2}\sqrt{a c}\,c}-\frac{3 a\,d^{3}\arctan\left(\frac{c\,x^{2}}{\sqrt{a c}}\right)}{4\left(a\,e^{2}+c\,d^{2}\right)^{2}\sqrt{a c}}+\frac{a^{3}e^{3}}{4\left(a\,e^{2}+c\,d^{2}\right)^{2}\left(c\,x^{4}+a\right)c^{2}}+\frac{a^{2}d^{2}e}{4\left(a\,e^{2}+c\,d^{2}\right)^{2}\left(c\,x^{4}+a\right)c}+\frac{a^{2}e^{3}\ln\left(c\,x^{4}+a\right)}{4\left(a\,e^{2}+c\,d^{2}\right)^{2}c^{2}}+\frac{a\,d^{2}e\ln\left(c\,x^{4}+a\right)}{2\left(a\,e^{2}+c\,d^{2}\right)^{2}c}+\frac{d^{4}\ln\left(e\,x^{2}+d\right)}{2\left(a\,e^{2}+c\,d^{2}\right)^{2}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4*a^2/(a*e^2+c*d^2)^2/(c*x^4+a)*d/c*x^2*e^2+1/4*a/(a*e^2+c*d^2)^2/(c*x^4+
a)*x^2*d^3+1/4*a^3/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3/c^2+1/4*a^2/(a*e^2+c*d^2)^
2/(c*x^4+a)*e/c*d^2+1/4*a^2/(a*e^2+c*d^2)^2/c^2*ln(c*x^4+a)*e^3+1/2*a/(a*e^
2+c*d^2)^2/c*ln(c*x^4+a)*d^2*e-1/4*a^2/(a*e^2+c*d^2)^2/c/(a*c)^(1/2)*arctan
(1/(a*c)^(1/2)*c*x^2)*d*e^2-3/4*a/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(1/(a*c
)^(1/2)*c*x^2)*d^3+1/2*d^4*ln(e*x^2+d)/e/(a*e^2+c*d^2)^2
```

maxima [A] time = 2.08, size = 220, normalized size = 1.30

$$\frac{d^{4}\log\left(ex^{2}+d\right)}{2\left(c^{2}d^{4}e+2\,acd^{2}e^{3}+a^{2}e^{5}\right)}+\frac{\left(2\,acd^{2}e+a^{2}e^{3}\right)\log\left(cx^{4}+a\right)}{4\left(c^{4}d^{4}+2\,ac^{3}d^{2}e^{2}+a^{2}c^{2}e^{4}\right)}-\frac{\left(3\,acd^{3}+a^{2}de^{2}\right)\arctan\left(\frac{cx^{2}}{\sqrt{ac}}\right)}{4\left(c^{3}d^{4}+2\,ac^{2}d^{2}e^{2}+a^{2}ce^{4}\right)\sqrt{ac}}+\frac{acdx^{2}+a^{2}e}{4\left(ac^{3}d^{2}+a^{2}c^{2}e^{2}+\left(c^{4}d^{2}+ac^{3}e^{2}\right)x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*d^4*log(e*x^2 + d)/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*(2*a*c*d
^2*e + a^2*e^3)*log(c*x^4 + a)/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4) -
```

```
1/4*(3*a*c*d^3 + a^2*d*e^2)*arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a*c^2*d^2
*e^2 + a^2*c*e^4)*sqrt(a*c)) + 1/4*(a*c*d*x^2 + a^2*e)/(a*c^3*d^2 + a^2*c^2
*e^2 + (c^4*d^2 + a*c^3*e^2)*x^4)
```

mupad [B] time = 1.30, size = 305, normalized size = 1.80

$$\frac{\frac{a^2\,e}{4\,c^2\,\left(c\,d^2+a\,e^2\right)}+\frac{a\,d\,x^2}{4\,c\,\left(c\,d^2+a\,e^2\right)}}{c\,x^4+a}-\frac{\ln\left(\sqrt{-a\,c^5}+c^3\,x^2\right)\,\left(3\,c\,d^3\,\sqrt{-a\,c^5}-2\,a^2\,c^2\,e^3-4\,a\,c^3\,d^2\,e+a\,d\,e^2\,\sqrt{-a\,c^5}\right)}{8\,\left(a^2\,c^4\,e^4+2\,a\,c^5\,d^2\,e^2+c^6\,d^4\right)}+\frac{\ln\left(\sqrt{-a\,c^5}-c^3\,x^2\right)\,\left(3\,c\,d^3\,\sqrt{-a\,c^5}+2\,a^2\,c^2\,e^3+4\,a\,c^3\,d^2\,e+a\,d\,e^2\,\sqrt{-a\,c^5}\right)}{8\,\left(a^2\,c^4\,e^4+2\,a\,c^5\,d^2\,e^2+c^6\,d^4\right)}+\frac{d^4\,\ln\left(e\,x^2+d\right)}{2\,a^2\,e^5+4\,a\,c\,d^2\,e^3+2\,c^2\,d^4\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((a^2*e)/(4*c^2*(a*e^2 + c*d^2)) + (a*d*x^2)/(4*c*(a*e^2 + c*d^2)))/(a + c*
x^4) - (log((-a*c^5)^(1/2) + c^3*x^2)*(3*c*d^3*(-a*c^5)^(1/2) - 2*a^2*c^2*e
^3 - 4*a*c^3*d^2*e + a*d*e^2*(-a*c^5)^(1/2)))/(8*(c^6*d^4 + a^2*c^4*e^4 + 2
*a*c^5*d^2*e^2)) + (log((-a*c^5)^(1/2) - c^3*x^2)*(3*c*d^3*(-a*c^5)^(1/2) +
 2*a^2*c^2*e^3 + 4*a*c^3*d^2*e + a*d*e^2*(-a*c^5)^(1/2)))/(8*(c^6*d^4 + a^2
*c^4*e^4 + 2*a*c^5*d^2*e^2)) + (d^4*log(d + e*x^2))/(2*a^2*e^5 + 2*c^2*d^4*
e + 4*a*c*d^2*e^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.175 $\int \frac{x^7}{(d+ex^2)(a+cx^4)^2} \, dx$

Optimal. Leaf size=150

$$\frac{\sqrt{a}\,e\left(ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4c^{3/2}\left(ae^2+cd^2\right)^2}+\frac{a\left(d-ex^2\right)}{4c\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{d^3\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}-\frac{d^3\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}$$

Rubi [A] time = 0.25, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1252, 1647, 801, 635, 205, 260}

$$\frac{\sqrt{a}\,e\left(ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4c^{3/2}\left(ae^2+cd^2\right)^2}+\frac{a\left(d-ex^2\right)}{4c\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{d^3\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}+\frac{d^3\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*(d - e*x^2))/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) + (Sqrt[a]*e*(3*c*d^2 + a
*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*c^(3/2)*(c*d^2 + a*e^2)^2) - (d^3*L
og[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) + (d^3*Log[a + c*x^4])/(4*(c*d^2 + a*e
^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
```

```
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(d+ex)(a+cx^2)^2}\,dx, x, x^2\right)$$

$$= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \frac{-\frac{a^2de}{cd^2+ae^2}-\frac{a(2cd^2+ae^2)x}{cd^2+ae^2}}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)}{4ac}$$

$$= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \left(\frac{2acd^3e}{(cd^2+ae^2)^2(d+ex)} - \frac{a(3acd^2e+a^2e^3+2c^2d^3x)}{(cd^2+ae^2)^2(a+cx^2)}\right) dx, x, x^2\right)}{4ac}$$

$$= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^3\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{\text{Subst}\left(\int \frac{3acd^2e+a^2e^3+2c^2d^3x}{a+cx^2}\,dx, x, x^2\right)}{4c(cd^2+ae^2)^2}$$

$$= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^3\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{(cd^3)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)^2} + \frac{(ae(\ldots}{\ldots}$$

$$= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} + \frac{\sqrt{a}\,e(3cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{3/2}(cd^2+ae^2)^2} - \frac{d^3\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{d^3\,l\ldots}{4(\ldots}$$

Mathematica [A] time = 0.11, size = 142, normalized size = 0.95

$$\frac{\sqrt{a}\,e(a+cx^4)(ae^2+3cd^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)+\sqrt{c}\left(-2cd^3(a+cx^4)\log(d+ex^2)+cd^3(a+cx^4)\log(a+cx^4)+a(d-ex^2)(ae^2+cd^2)\right)}{4c^{3/2}(a+cx^4)(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (Sqrt[a]*e*(3*c*d^2 + a*e^2)*(a + c*x^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]] + Sq
rt[c]*(a*(c*d^2 + a*e^2)*(d - e*x^2) - 2*c*d^3*(a + c*x^4)*Log[d + e*x^2] +
 c*d^3*(a + c*x^4)*Log[a + c*x^4]))/(4*c^(3/2)*(c*d^2 + a*e^2)^2*(a + c*x^4
))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[x^7/((d + e*x^2)*(a + c*x^4)^2), x]
```

fricas [A] time = 17.04, size = 457, normalized size = 3.05

$$\left[\frac{2\,acd^3+2\,a^2de^2-2\,\left(acd^2e+a^2e^3\right)x^2+\left(3\,acd^2e+a^2e^3+\left(3\,c^2d^2e+ace^3\right)x^4\right)\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4+2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)+2\,\left(c^2d^3x^4+acd^3\right)\log\left(cx^4+a\right)-4\,\left(c^2d^3x^4+acd^3\right)\log\left(ex^2+d\right)}{8\,\left(ac^3d^4+2\,a^2c^2d^2e^2+a^3ce^4+\left(c^4d^4+2\,ac^3d^2e^2+a^2c^2e^4\right)x^4\right)},\ \frac{acd^3+a^2de^2-\left(acd^2e+a^2e^3\right)x^2+\left(3\,acd^2e+a^2e^3+\left(3\,c^2d^2e+ace^3\right)x^4\right)\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right)+\left(c^2d^3x^4+acd^3\right)\log\left(cx^4+a\right)-2\,\left(c^2d^3x^4+acd^3\right)\log\left(ex^2+d\right)}{4\,\left(ac^3d^4+2\,a^2c^2d^2e^2+a^3ce^4+\left(c^4d^4+2\,ac^3d^2e^2+a^2c^2e^4\right)x^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*c*d^3 + 2*a^2*d*e^2 - 2*(a*c*d^2*e + a^2*e^3)*x^2 + (3*a*c*d^2*e
+ a^2*e^3 + (3*c^2*d^2*e + a*c*e^3)*x^4)*sqrt(-a/c)*log((c*x^4 + 2*c*x^2*sq
rt(-a/c) - a)/(c*x^4 + a)) + 2*(c^2*d^3*x^4 + a*c*d^3)*log(c*x^4 + a) - 4*(
c^2*d^3*x^4 + a*c*d^3)*log(e*x^2 + d))/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3
*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4), 1/4*(a*c*d^3 + a^2
*d*e^2 - (a*c*d^2*e + a^2*e^3)*x^2 + (3*a*c*d^2*e + a^2*e^3 + (3*c^2*d^2*e
+ a*c*e^3)*x^4)*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) + (c^2*d^3*x^4 + a*c*d^
3)*log(c*x^4 + a) - 2*(c^2*d^3*x^4 + a*c*d^3)*log(e*x^2 + d))/(a*c^3*d^4 +
2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x
^4)]
```

giac [A] time = 0.35, size = 223, normalized size = 1.49

$$-\frac{d^3e\log\left(\left|x^2e+d\right|\right)}{2\,\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)}+\frac{d^3\log\left(cx^4+a\right)}{4\,\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}+\frac{\left(3\,acd^2e+a^2e^3\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right)\sqrt{ac}}-\frac{c^2d^3x^4+acd^2x^2e+a^2x^2e^3-a^2de^2}{4\,\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right)\left(cx^4+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*d^3*e*log(abs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*
d^3*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(3*a*c*d^2*e +
 a^2*e^3)*arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*
sqrt(a*c)) - 1/4*(c^2*d^3*x^4 + a*c*d^2*x^2*e + a^2*x^2*e^3 - a^2*d*e^2)/((
c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*(c*x^4 + a))
```

maple [A] time = 0.02, size = 260, normalized size = 1.73

$$-\frac{a^2e^3x^2}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)c}-\frac{ad^2ex^2}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)}+\frac{a^2e^3\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(ae^2+cd^2\right)^2\sqrt{ac}\,c}+\frac{3ad^2e\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(ae^2+cd^2\right)^2\sqrt{ac}}+\frac{a^2de^2}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)c}+\frac{ad^3}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)}+\frac{d^3\ln\left(cx^4+a\right)}{4\left(ae^2+cd^2\right)^2}-\frac{d^3\ln\left(ex^2+d\right)}{2\left(ae^2+cd^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a^2*e^3/c*x^2-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*
a*e*x^2*d^2+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a^2*d/c*e^2+1/4/(a*e^2+c*d^2)^2/(
c*x^4+a)*a*d^3+1/4*d^3*ln(c*x^4+a)/(a*e^2+c*d^2)^2+1/4/(a*e^2+c*d^2)^2/c/(a
*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)*a^2*e^3+3/4/(a*e^2+c*d^2)^2/(a*c)^(1/
2)*arctan(1/(a*c)^(1/2)*c*x^2)*a*d^2*e-1/2*d^3*ln(e*x^2+d)/(a*e^2+c*d^2)^2
```

maxima [A] time = 2.05, size = 197, normalized size = 1.31

$$\frac{d^3\log\left(cx^4+a\right)}{4\,\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}-\frac{d^3\log\left(ex^2+d\right)}{2\,\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}+\frac{\left(3\,acd^2e+a^2e^3\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right)\sqrt{ac}}-\frac{aex^2-ad}{4\,\left(ac^2d^2+a^2ce^2+\left(c^3d^2+ac^2e^2\right)x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*d^3*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/2*d^3*log(e*
x^2 + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(3*a*c*d^2*e + a^2*e^3)*
arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(a*c))
 - 1/4*(a*e*x^2 - a*d)/(a*c^2*d^2 + a^2*c*e^2 + (c^3*d^2 + a*c^2*e^2)*x^4)
```

mupad [B] time = 1.49, size = 647, normalized size = 4.31

$$\frac{\frac{a\,d}{4\,c\,\left(c\,d^2+a\,e^2\right)}-\frac{a\,e\,x^2}{4\,c\,\left(c\,d^2+a\,e^2\right)}}{c\,x^4+a}-\frac{d^3\,\ln\left(e\,x^2+d\right)}{2\,\left(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}+\frac{\ln\left(36\,c^8\,d^{10}\,x^2+36\,c^6\,d^{10}\,\sqrt{-a\,c^3}+a^5\,c\,e^{10}\,\sqrt{-a\,c^3}+a^5\,c^3\,e^{10}\,x^2-22\,a^2\,d^4\,e^6\,{\left(-a\,c^3\right)}^{3/2}-81\,c^2\,d^8\,e^2\,{\left(-a\,c^3\right)}^{3/2}+60\,a^2\,c^6\,d^6\,e^4\,x^2+22\,a^3\,c^5\,d^4\,e^6\,x^2+8\,a^4\,c^4\,d^2\,e^8\,x^2+8\,a^4\,c^2\,d^2\,e^8\,\sqrt{-a\,c^3}-60\,a\,c\,d^6\,e^4\,{\left(-a\,c^3\right)}^{3/2}+81\,a\,c^7\,d^8\,e^2\,x^2\right)\,\left(2\,c^3\,d^3+a\,e^3\,\sqrt{-a\,c^3}+3\,c\,d^2\,e\,\sqrt{-a\,c^3}\right)}{8\,\left(a^2\,c^3\,e^4+2\,a\,c^4\,d^2\,e^2+c^5\,d^4\right)}-\frac{\ln\left(36\,c^8\,d^{10}\,x^2-36\,c^6\,d^{10}\,\sqrt{-a\,c^3}-a^5\,c\,e^{10}\,\sqrt{-a\,c^3}+a^5\,c^3\,e^{10}\,x^2+22\,a^2\,d^4\,e^6\,{\left(-a\,c^3\right)}^{3/2}+81\,c^2\,d^8\,e^2\,{\left(-a\,c^3\right)}^{3/2}+60\,a^2\,c^6\,d^6\,e^4\,x^2+22\,a^3\,c^5\,d^4\,e^6\,x^2+8\,a^4\,c^4\,d^2\,e^8\,x^2-8\,a^4\,c^2\,d^2\,e^8\,\sqrt{-a\,c^3}+60\,a\,c\,d^6\,e^4\,{\left(-a\,c^3\right)}^{3/2}+81\,a\,c^7\,d^8\,e^2\,x^2\right)\,\left(a\,e^3\,\sqrt{-a\,c^3}-2\,c^3\,d^3+3\,c\,d^2\,e\,\sqrt{-a\,c^3}\right)}{8\,\left(a^2\,c^3\,e^4+2\,a\,c^4\,d^2\,e^2+c^5\,d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((a*d)/(4*c*(a*e^2 + c*d^2)) - (a*e*x^2)/(4*c*(a*e^2 + c*d^2)))/(a + c*x^4)
 - (d^3*log(d + e*x^2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (log(36*c
^8*d^10*x^2 + 36*c^6*d^10*(-a*c^3)^(1/2) + a^5*c*e^10*(-a*c^3)^(1/2) + a^5*
c^3*e^10*x^2 - 22*a^2*d^4*e^6*(-a*c^3)^(3/2) - 81*c^2*d^8*e^2*(-a*c^3)^(3/2
) + 60*a^2*c^6*d^6*e^4*x^2 + 22*a^3*c^5*d^4*e^6*x^2 + 8*a^4*c^4*d^2*e^8*x^2
 + 8*a^4*c^2*d^2*e^8*(-a*c^3)^(1/2) - 60*a*c*d^6*e^4*(-a*c^3)^(3/2) + 81*a*
c^7*d^8*e^2*x^2)*(2*c^3*d^3 + a*e^3*(-a*c^3)^(1/2) + 3*c*d^2*e*(-a*c^3)^(1/
2)))/(8*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)) - (log(36*c^8*d^10*x^2 -
 36*c^6*d^10*(-a*c^3)^(1/2) - a^5*c*e^10*(-a*c^3)^(1/2) + a^5*c^3*e^10*x^2
+ 22*a^2*d^4*e^6*(-a*c^3)^(3/2) + 81*c^2*d^8*e^2*(-a*c^3)^(3/2) + 60*a^2*c^
6*d^6*e^4*x^2 + 22*a^3*c^5*d^4*e^6*x^2 + 8*a^4*c^4*d^2*e^8*x^2 - 8*a^4*c^2*
d^2*e^8*(-a*c^3)^(1/2) + 60*a*c*d^6*e^4*(-a*c^3)^(3/2) + 81*a*c^7*d^8*e^2*x
^2)*(a*e^3*(-a*c^3)^(1/2) - 2*c^3*d^3 + 3*c*d^2*e*(-a*c^3)^(1/2)))/(8*(c^5*
d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.176 $\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=155

$$-\frac{d^2e\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}+\frac{d^2e\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}+\frac{d\left(cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}\left(ae^2+cd^2\right)^2}+\frac{-ae-cdx^2}{4c\left(a+cx^4\right)\left(ae^2+cd^2\right)}$$

Rubi [A] time = 0.25, antiderivative size = 153, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1252, 1647, 801, 635, 205, 260}

$$-\frac{ae+cdx^2}{4c\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{d^2e\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}-\frac{d^2e\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}+\frac{d\left(cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -(a*e + c*d*x^2)/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) + (d*(c*d^2 - a*e^2)*Arc
Tan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2)^2) + (d^2*e*
Log[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) - (d^2*e*Log[a + c*x^4])/(4*(c*d^2 +
a*e^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
```

```
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(d+ex)(a+cx^2)^2}\,dx, x, x^2\right)$$

$$= -\frac{ae+cdx^2}{4c(cd^2+ae^2)(a+cx^4)} - \frac{\operatorname{Subst}\left(\int \frac{-\frac{acd^2}{cd^2+ae^2}+\frac{acdex}{cd^2+ae^2}}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)}{4ac}$$

$$= -\frac{ae+cdx^2}{4c(cd^2+ae^2)(a+cx^4)} - \frac{\operatorname{Subst}\left(\int \left(-\frac{2acd^2e^2}{(cd^2+ae^2)^2(d+ex)} + \frac{acd(-cd^2+ae^2+2cdex)}{(cd^2+ae^2)^2(a+cx^2)}\right)dx, x, x^2\right)}{4ac}$$

$$= -\frac{ae+cdx^2}{4c(cd^2+ae^2)(a+cx^4)} + \frac{d^2e\log(d+ex^2)}{2(cd^2+ae^2)^2} - \frac{d\operatorname{Subst}\left(\int \frac{-cd^2+ae^2+2cdex}{a+cx^2}\,dx, x, x^2\right)}{4(cd^2+ae^2)^2}$$

$$= -\frac{ae+cdx^2}{4c(cd^2+ae^2)(a+cx^4)} + \frac{d^2e\log(d+ex^2)}{2(cd^2+ae^2)^2} - \frac{(cd^2e)\operatorname{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)^2} + \cdots$$

$$= -\frac{ae+cdx^2}{4c(cd^2+ae^2)(a+cx^4)} + \frac{d(cd^2-ae^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}(cd^2+ae^2)^2} + \frac{d^2e\log(d+ex^2)}{2(cd^2+ae^2)^2} - \frac{d^2e\log\cdots}{4(c\cdots}$$

Mathematica [A] time = 0.15, size = 120, normalized size = 0.77

$$\frac{\frac{d(cd^2-ae^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}} - \frac{(ae^2+cd^2)(ae+cdx^2)}{c(a+cx^4)} - d^2e\log(a+cx^4) + 2d^2e\log(d+ex^2)}{4(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-(((c*d^2 + a*e^2)*(a*e + c*d*x^2))/(c*(a + c*x^4))) + (d*(c*d^2 - a*e^2)*
ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) + 2*d^2*e*Log[d + e*x^2] -
 d^2*e*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[x^5/((d + e*x^2)*(a + c*x^4)^2), x]
```

fricas [A] time = 6.20, size = 487, normalized size = 3.14

$$\left[-\frac{2\,a^{2}cd^{2}e + 2\,a^{3}e^{3} + 2\,\left(ac^{2}d^{3} + a^{2}cde^{2}\right)x^{2} - \left(acd^{3} - a^{2}de^{2} + \left(c^{2}d^{3} - acde^{2}\right)x^{4}\right)\sqrt{-ac}\,\log\left(\frac{cx^{4}+2\,\sqrt{-ac}\,x^{2}-a}{cx^{4}+a}\right) + 2\,\left(ac^{2}d^{2}ex^{4} + a^{2}cd^{2}e\right)\log\left(cx^{4} + a\right) - 4\,\left(ac^{2}d^{2}ex^{4} + a^{2}cd^{2}e\right)\log\left(ex^{2} + d\right)}{8\,\left(a^{2}c^{3}d^{4} + 2\,a^{3}c^{2}d^{2}e^{2} + a^{4}ce^{4} + \left(ac^{4}d^{4} + 2\,a^{2}c^{3}d^{2}e^{2} + a^{3}c^{2}e^{4}\right)x^{4}\right)}, -\frac{a^{2}cd^{2}e + a^{3}e^{3} + \left(ac^{2}d^{3} + a^{2}cde^{2}\right)x^{2} + \left(acd^{3} - a^{2}de^{2} + \left(c^{2}d^{3} - acde^{2}\right)x^{4}\right)\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}}{cx^{2}}\right) + \left(ac^{2}d^{2}ex^{4} + a^{2}cd^{2}e\right)\log\left(cx^{4} + a\right) - 2\,\left(ac^{2}d^{2}ex^{4} + a^{2}cd^{2}e\right)\log\left(ex^{2} + d\right)}{4\,\left(a^{2}c^{3}d^{4} + 2\,a^{3}c^{2}d^{2}e^{2} + a^{4}ce^{4} + \left(ac^{4}d^{4} + 2\,a^{2}c^{3}d^{2}e^{2} + a^{3}c^{2}e^{4}\right)x^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*a^2*c*d^2*e + 2*a^3*e^3 + 2*(a*c^2*d^3 + a^2*c*d*e^2)*x^2 - (a*c*d
^3 - a^2*d*e^2 + (c^2*d^3 - a*c*d*e^2)*x^4)*sqrt(-a*c)*log((c*x^4 + 2*sqrt(
-a*c)*x^2 - a)/(c*x^4 + a)) + 2*(a*c^2*d^2*e*x^4 + a^2*c*d^2*e)*log(c*x^4 +
 a) - 4*(a*c^2*d^2*e*x^4 + a^2*c*d^2*e)*log(e*x^2 + d))/(a^2*c^3*d^4 + 2*a^
3*c^2*d^2*e^2 + a^4*c*e^4 + (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*x
^4), -1/4*(a^2*c*d^2*e + a^3*e^3 + (a*c^2*d^3 + a^2*c*d*e^2)*x^2 + (a*c*d^3
 - a^2*d*e^2 + (c^2*d^3 - a*c*d*e^2)*x^4)*sqrt(a*c)*arctan(sqrt(a*c)/(c*x^2
)) + (a*c^2*d^2*e*x^4 + a^2*c*d^2*e)*log(c*x^4 + a) - 2*(a*c^2*d^2*e*x^4 +
a^2*c*d^2*e)*log(e*x^2 + d))/(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4 +
 (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*x^4)]
```

giac [A] time = 0.33, size = 220, normalized size = 1.42

$$-\frac{d^{2}e\log\left(cx^{4} + a\right)}{4\,\left(c^{2}d^{4} + 2\,acd^{2}e^{2} + a^{2}e^{4}\right)} + \frac{d^{2}e^{2}\log\left(\left|x^{2}e + d\right|\right)}{2\,\left(c^{2}d^{4}e + 2\,acd^{2}e^{3} + a^{2}e^{5}\right)} + \frac{\left(cd^{3} - ade^{2}\right)\arctan\left(\frac{cx^{2}}{\sqrt{ac}}\right)}{4\,\left(c^{2}d^{4} + 2\,acd^{2}e^{2} + a^{2}e^{4}\right)\sqrt{ac}} + \frac{c^{2}d^{2}x^{4}e - c^{2}d^{3}x^{2} - acdx^{2}e^{2} - a^{2}e^{3}}{4\,\left(c^{3}d^{4} + 2\,ac^{2}d^{2}e^{2} + a^{2}ce^{4}\right)\left(cx^{4} + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*d^2*e*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*d^2*e^2
*log(abs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*(c*d^3 - a
*d*e^2)*arctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a
*c)) + 1/4*(c^2*d^2*x^4*e - c^2*d^3*x^2 - a*c*d*x^2*e^2 - a^2*e^3)/((c^3*d^
4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*(c*x^4 + a))
```

maple [A] time = 0.02, size = 252, normalized size = 1.63

$$-\frac{a\,d\,e^{2}x^{2}}{4\left(a\,e^{2} + c\,d^{2}\right)^{2}\left(c\,x^{4} + a\right)} - \frac{c\,d^{3}x^{2}}{4\left(a\,e^{2} + c\,d^{2}\right)^{2}\left(c\,x^{4} + a\right)} - \frac{a\,d\,e^{2}\arctan\left(\frac{c\,x^{2}}{\sqrt{ac}}\right)}{4\left(a\,e^{2} + c\,d^{2}\right)^{2}\sqrt{ac}} + \frac{c\,d^{3}\arctan\left(\frac{c\,x^{2}}{\sqrt{ac}}\right)}{4\left(a\,e^{2} + c\,d^{2}\right)^{2}\sqrt{ac}} - \frac{a^{2}e^{3}}{4\left(a\,e^{2} + c\,d^{2}\right)^{2}\left(c\,x^{4} + a\right)c} - \frac{a\,d^{2}e}{4\left(a\,e^{2} + c\,d^{2}\right)^{2}\left(c\,x^{4} + a\right)} - \frac{d^{2}e\ln\left(c\,x^{4} + a\right)}{4\left(a\,e^{2} + c\,d^{2}\right)^{2}} + \frac{d^{2}e\ln\left(e\,x^{2} + d\right)}{2\left(a\,e^{2} + c\,d^{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^2*a*d*e^2-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^
2*c*d^3-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a^2*e^3/c-1/4/(a*e^2+c*d^2)^2/(c*x^4+
a)*a*e*d^2-1/4*d^2*e*ln(c*x^4+a)/(a*e^2+c*d^2)^2-1/4/(a*e^2+c*d^2)^2*d/(a*c
)^(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)*a*e^2+1/4/(a*e^2+c*d^2)^2/(a*c)^(1/2)*a
rctan(1/(a*c)^(1/2)*c*x^2)*c*d^3+1/2*d^2*e*ln(e*x^2+d)/(a*e^2+c*d^2)^2
```

maxima [A] time = 2.08, size = 192, normalized size = 1.24

$$-\frac{d^{2}e\log\left(cx^{4} + a\right)}{4\,\left(c^{2}d^{4} + 2\,acd^{2}e^{2} + a^{2}e^{4}\right)} + \frac{d^{2}e\log\left(ex^{2} + d\right)}{2\,\left(c^{2}d^{4} + 2\,acd^{2}e^{2} + a^{2}e^{4}\right)} + \frac{\left(cd^{3} - ade^{2}\right)\arctan\left(\frac{cx^{2}}{\sqrt{ac}}\right)}{4\,\left(c^{2}d^{4} + 2\,acd^{2}e^{2} + a^{2}e^{4}\right)\sqrt{ac}} - \frac{cdx^{2} + ae}{4\,\left(ac^{2}d^{2} + a^{2}ce^{2} + \left(c^{3}d^{2} + ac^{2}e^{2}\right)x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*d^2*e*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*d^2*e*l
og(e*x^2 + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(c*d^3 - a*d*e^2)*a
rctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)) - 1/
4*(c*d*x^2 + a*e)/(a*c^2*d^2 + a^2*c*e^2 + (c^3*d^2 + a*c^2*e^2)*x^4)
```

mupad [B] time = 1.52, size = 528, normalized size = 3.41

$$\frac{\ln\left(a^4\,e^8\,\sqrt{-a\,c}+c^4\,d^8\,\sqrt{-a\,c}+70\,d^4\,e^4\,{\left(-a\,c\right)}^{5/2}+c^5\,d^8\,x^2+a^4\,c\,e^8\,x^2-36\,a^2\,d^2\,e^6\,{\left(-a\,c\right)}^{3/2}-36\,c^2\,d^6\,e^2\,{\left(-a\,c\right)}^{3/2}+70\,a^2\,c^3\,d^4\,e^4\,x^2+36\,a^3\,c^2\,d^2\,e^6\,x^2+36\,a\,c^4\,d^6\,e^2\,x^2\right)\,\left(c\,\left(\frac{d^3\,\sqrt{-a\,c}}{8}-\frac{a\,d^2\,e}{4}\right)-\frac{a\,d\,e^2\,\sqrt{-a\,c}}{8}\right)}{a^3\,c\,e^4+2\,a^2\,c^2\,d^2\,e^2+a\,c^3\,d^4}-\frac{\frac{d\,x^2}{4\,\left(c\,d^2+a\,e^2\right)}+\frac{a\,e}{4\,c\,\left(c\,d^2+a\,e^2\right)}}{c\,x^4+a}-\frac{\ln\left(c^5\,d^8\,x^2-c^4\,d^8\,\sqrt{-a\,c}-70\,d^4\,e^4\,{\left(-a\,c\right)}^{5/2}-a^4\,e^8\,\sqrt{-a\,c}+a^4\,c\,e^8\,x^2+36\,a^2\,d^2\,e^6\,{\left(-a\,c\right)}^{3/2}+36\,c^2\,d^6\,e^2\,{\left(-a\,c\right)}^{3/2}+70\,a^2\,c^3\,d^4\,e^4\,x^2+36\,a^3\,c^2\,d^2\,e^6\,x^2+36\,a\,c^4\,d^6\,e^2\,x^2\right)\,\left(c\,\left(\frac{d^3\,\sqrt{-a\,c}}{8}+\frac{a\,d^2\,e}{4}\right)-\frac{a\,d\,e^2\,\sqrt{-a\,c}}{8}\right)}{a^3\,c\,e^4+2\,a^2\,c^2\,d^2\,e^2+a\,c^3\,d^4}+\frac{d^2\,e\,\ln\left(e\,x^2+d\right)}{2\,\left(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] (log(a^4*e^8*(-a*c)^(1/2) + c^4*d^8*(-a*c)^(1/2) + 70*d^4*e^4*(-a*c)^(5/2)
+ c^5*d^8*x^2 + a^4*c*e^8*x^2 - 36*a^2*d^2*e^6*(-a*c)^(3/2) - 36*c^2*d^6*e^
2*(-a*c)^(3/2) + 70*a^2*c^3*d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*c^4
*d^6*e^2*x^2)*(c*((d^3*(-a*c)^(1/2))/8 - (a*d^2*e)/4) - (a*d*e^2*(-a*c)^(1/
2))/8))/(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2) - ((d*x^2)/(4*(a*e^2 +
c*d^2)) + (a*e)/(4*c*(a*e^2 + c*d^2)))/(a + c*x^4) - (log(c^5*d^8*x^2 - c^4
*d^8*(-a*c)^(1/2) - 70*d^4*e^4*(-a*c)^(5/2) - a^4*e^8*(-a*c)^(1/2) + a^4*c*
e^8*x^2 + 36*a^2*d^2*e^6*(-a*c)^(3/2) + 36*c^2*d^6*e^2*(-a*c)^(3/2) + 70*a^
2*c^3*d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*c^4*d^6*e^2*x^2)*(c*((d^3
*(-a*c)^(1/2))/8 + (a*d^2*e)/4) - (a*d*e^2*(-a*c)^(1/2))/8))/(a*c^3*d^4 + a
^3*c*e^4 + 2*a^2*c^2*d^2*e^2) + (d^2*e*log(d + e*x^2))/(2*(a^2*e^4 + c^2*d^
4 + 2*a*c*d^2*e^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.177 $\int \frac{x^3}{(d+ex^2)(a+cx^4)^2} \, dx$

Optimal. Leaf size=149

$$\frac{de^2 \log \left(a + cx^4\right)}{4 \left(ae^2 + cd^2\right)^2} - \frac{de^2 \log \left(d + ex^2\right)}{2 \left(ae^2 + cd^2\right)^2} - \frac{e \left(cd^2 - ae^2\right) \tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\sqrt{a}\,\sqrt{c}\,\left(ae^2 + cd^2\right)^2} + \frac{ex^2 - d}{4 \left(a + cx^4\right) \left(ae^2 + cd^2\right)}$$

Rubi [A] time = 0.19, antiderivative size = 148, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1252, 823, 801, 635, 205, 260}

$$-\frac{d - ex^2}{4 \left(a + cx^4\right) \left(ae^2 + cd^2\right)} - \frac{de^2 \log \left(d + ex^2\right)}{2 \left(ae^2 + cd^2\right)^2} + \frac{de^2 \log \left(a + cx^4\right)}{4 \left(ae^2 + cd^2\right)^2} - \frac{e \left(cd^2 - ae^2\right) \tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\sqrt{a}\,\sqrt{c}\,\left(ae^2 + cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -(d - e*x^2)/(4*(c*d^2 + a*e^2)*(a + c*x^4)) - (e*(c*d^2 - a*e^2)*ArcTan[(S
qrt[c]*x^2)/Sqrt[a]])/(4*Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2)^2) - (d*e^2*Log[d
+ e*x^2])/(2*(c*d^2 + a*e^2)^2) + (d*e^2*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)
^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{x^3}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \frac{1}{2}\mathrm{Subst}\left(\int \frac{x}{(d+ex)\left(a+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{d-ex^2}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\mathrm{Subst}\left(\int \frac{acde-ace^2x}{(d+ex)\left(a+cx^2\right)}\,dx, x, x^2\right)}{4ac\left(cd^2+ae^2\right)}$$

$$= -\frac{d-ex^2}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\mathrm{Subst}\left(\int \left(\frac{2acde^3}{\left(cd^2+ae^2\right)(d+ex)} - \frac{ace\left(-cd^2+ae^2+2cdex\right)}{\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right)dx, x, x^2\right)}{4ac\left(cd^2+ae^2\right)}$$

$$= -\frac{d-ex^2}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{de^2\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)^2} + \frac{e\,\mathrm{Subst}\left(\int \frac{-cd^2+ae^2+2cdex}{a+cx^2}\,dx, x, x^2\right)}{4\left(cd^2+ae^2\right)^2}$$

$$= -\frac{d-ex^2}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{de^2\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)^2} + \frac{\left(cde^2\right)\mathrm{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)^2} - \frac{(e}{}$$

$$= -\frac{d-ex^2}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{e\left(cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}\left(cd^2+ae^2\right)^2} - \frac{de^2\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)^2} + \frac{de^2\log}{4\left(cd^2\right.}$$

Mathematica [A] time = 0.14, size = 114, normalized size = 0.77

$$\frac{\frac{e\left(ae^2-cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}} + \frac{\left(ex^2-d\right)\left(ae^2+cd^2\right)}{a+cx^4} + de^2\log\left(a+cx^4\right) - 2de^2\log\left(d+ex^2\right)}{4\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (((c*d^2 + a*e^2)*(-d + e*x^2))/(a + c*x^4) + (e*(-(c*d^2) + a*e^2)*ArcTan[
(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - 2*d*e^2*Log[d + e*x^2] + d*e^2*
Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[x^3/((d + e*x^2)*(a + c*x^4)^2), x]
```

fricas [A] time = 7.09, size = 492, normalized size = 3.30

$$\left[-\frac{2\,ac^2d^3+2\,a^2cde^2-2\,\left(ac^2d^2e+a^2ce^3\right)x^2-\left(acd^2e-a^2e^3+\left(c^2d^2e-ace^3\right)x^4\right)\sqrt{-ac}\,\log\left(\frac{cx^4-2\,\sqrt{-ac}\,x^2-a}{cx^4+a}\right)-2\,\left(ac^2de^2x^4+a^2cde^2\right)\log\left(cx^4+a\right)+4\,\left(ac^2de^2x^4+a^2cde^2\right)\log\left(ex^2+d\right)}{8\,\left(a^2c^3d^4+2\,a^3c^2d^2e^2+a^4ce^4+\left(ac^4d^4+2\,a^2c^3d^2e^2+a^3c^2e^4\right)x^4\right)},\ -\frac{ac^2d^3+a^2cde^2-\left(ac^2d^2e+a^2ce^3\right)x^2-\left(acd^2e-a^2e^3+\left(c^2d^2e-ace^3\right)x^4\right)\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}}{cx^2}\right)-\left(ac^2de^2x^4+a^2cde^2\right)\log\left(cx^4+a\right)+2\,\left(ac^2de^2x^4+a^2cde^2\right)\log\left(ex^2+d\right)}{4\,\left(a^2c^3d^4+2\,a^3c^2d^2e^2+a^4ce^4+\left(ac^4d^4+2\,a^2c^3d^2e^2+a^3c^2e^4\right)x^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*a*c^2*d^3 + 2*a^2*c*d*e^2 - 2*(a*c^2*d^2*e + a^2*c*e^3)*x^2 - (a*c
*d^2*e - a^2*e^3 + (c^2*d^2*e - a*c*e^3)*x^4)*sqrt(-a*c)*log((c*x^4 - 2*sqr
t(-a*c)*x^2 - a)/(c*x^4 + a)) - 2*(a*c^2*d*e^2*x^4 + a^2*c*d*e^2)*log(c*x^4
 + a) + 4*(a*c^2*d*e^2*x^4 + a^2*c*d*e^2)*log(e*x^2 + d))/(a^2*c^3*d^4 + 2*
a^3*c^2*d^2*e^2 + a^4*c*e^4 + (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)
*x^4), -1/4*(a*c^2*d^3 + a^2*c*d*e^2 - (a*c^2*d^2*e + a^2*c*e^3)*x^2 - (a*c
*d^2*e - a^2*e^3 + (c^2*d^2*e - a*c*e^3)*x^4)*sqrt(a*c)*arctan(sqrt(a*c)/(c
*x^2)) - (a*c^2*d*e^2*x^4 + a^2*c*d*e^2)*log(c*x^4 + a) + 2*(a*c^2*d*e^2*x^
4 + a^2*c*d*e^2)*log(e*x^2 + d))/(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e
^4 + (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*x^4)]
```

giac [A] time = 0.28, size = 188, normalized size = 1.26

$$\frac{de^2\log\left(cx^4+a\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}-\frac{de^3\log\left(\left|x^2e+d\right|\right)}{2\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)}-\frac{\left(cd^2e-ae^3\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\sqrt{ac}}-\frac{cd^3-\left(cd^2e+ae^3\right)x^2+ade^2}{4\left(cx^4+a\right)\left(cd^2+ae^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*d*e^2*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/2*d*e^3*lo
g(abs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) - 1/4*(c*d^2*e - a*
e^3)*arctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)
) - 1/4*(c*d^3 - (c*d^2*e + a*e^3)*x^2 + a*d*e^2)/((c*x^4 + a)*(c*d^2 + a*e
^2)^2)
```

maple [A] time = 0.02, size = 247, normalized size = 1.66

$$\frac{a\,e^3x^2}{4\left(a\,e^2+c\,d^2\right)^2\left(c\,x^4+a\right)}+\frac{c\,d^2e\,x^2}{4\left(a\,e^2+c\,d^2\right)^2\left(c\,x^4+a\right)}+\frac{a\,e^3\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{4\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}}-\frac{c\,d^2e\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{4\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}}-\frac{a\,d\,e^2}{4\left(a\,e^2+c\,d^2\right)^2\left(c\,x^4+a\right)}-\frac{c\,d^3}{4\left(a\,e^2+c\,d^2\right)^2\left(c\,x^4+a\right)}+\frac{d\,e^2\ln\left(c\,x^4+a\right)}{4\left(a\,e^2+c\,d^2\right)^2}-\frac{d\,e^2\ln\left(e\,x^2+d\right)}{2\left(a\,e^2+c\,d^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^2*e^3*a+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^2*e
*c*d^2-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a*d*e^2-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*
c*d^3+1/4*d*e^2*ln(c*x^4+a)/(a*e^2+c*d^2)^2+1/4/(a*e^2+c*d^2)^2/(a*c)^(1/2)
*arctan(1/(a*c)^(1/2)*c*x^2)*a*e^3-1/4/(a*e^2+c*d^2)^2*e/(a*c)^(1/2)*arctan
(1/(a*c)^(1/2)*c*x^2)*c*d^2-1/2*d*e^2*ln(e*x^2+d)/(a*e^2+c*d^2)^2
```

maxima [A] time = 2.03, size = 186, normalized size = 1.25

$$\frac{de^2\log\left(cx^4+a\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}-\frac{de^2\log\left(ex^2+d\right)}{2\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}-\frac{\left(cd^2e-ae^3\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\sqrt{ac}}+\frac{ex^2-d}{4\left(\left(c^2d^2+ace^2\right)x^4+acd^2+a^2e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*d*e^2*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/2*d*e^2*lo
g(e*x^2 + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/4*(c*d^2*e - a*e^3)*ar
ctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)) + 1/4
*(e*x^2 - d)/((c^2*d^2 + a*c*e^2)*x^4 + a*c*d^2 + a^2*e^2)
```

mupad [B] time = 1.41, size = 527, normalized size = 3.54

$$\frac{\ln\left(a^4\,e^8\,\sqrt{-a\,c}+c^4\,d^8\,\sqrt{-a\,c}+70\,d^4\,e^4\,{\left(-a\,c\right)}^{5/2}+c^5\,d^8\,x^2+a^4\,c\,e^8\,x^2-36\,a^2\,d^2\,e^6\,{\left(-a\,c\right)}^{3/2}-36\,c^2\,d^6\,e^2\,{\left(-a\,c\right)}^{3/2}+70\,a^2\,c^3\,d^4\,e^4\,x^2+36\,a^3\,c^2\,d^2\,e^6\,x^2+36\,a\,c^4\,d^6\,e^2\,x^2\right)\,\left(a\,\left(\frac{e^3\,\sqrt{-a\,c}}{8}+\frac{c\,d\,e^2}{4}\right)-\frac{c\,d^2\,e\,\sqrt{-a\,c}}{8}\right)}{a^3\,c\,e^4+2\,a^2\,c^2\,d^2\,e^2+a\,c^3\,d^4}-\frac{\frac{d}{4\,\left(c\,d^2+a\,e^2\right)}-\frac{e\,x^2}{4\,\left(c\,d^2+a\,e^2\right)}}{c\,x^4+a}-\frac{\ln\left(c^5\,d^8\,x^2-c^4\,d^8\,\sqrt{-a\,c}-70\,d^4\,e^4\,{\left(-a\,c\right)}^{5/2}-a^4\,e^8\,\sqrt{-a\,c}+a^4\,c\,e^8\,x^2+36\,a^2\,d^2\,e^6\,{\left(-a\,c\right)}^{3/2}+36\,c^2\,d^6\,e^2\,{\left(-a\,c\right)}^{3/2}+70\,a^2\,c^3\,d^4\,e^4\,x^2+36\,a^3\,c^2\,d^2\,e^6\,x^2+36\,a\,c^4\,d^6\,e^2\,x^2\right)\,\left(a\,\left(\frac{e^3\,\sqrt{-a\,c}}{8}-\frac{c\,d\,e^2}{4}\right)-\frac{c\,d^2\,e\,\sqrt{-a\,c}}{8}\right)}{a^3\,c\,e^4+2\,a^2\,c^2\,d^2\,e^2+a\,c^3\,d^4}-\frac{d\,e^2\,\ln\left(e\,x^2+d\right)}{2\,\left(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] (log(a^4*e^8*(-a*c)^(1/2) + c^4*d^8*(-a*c)^(1/2) + 70*d^4*e^4*(-a*c)^(5/2)
+ c^5*d^8*x^2 + a^4*c*e^8*x^2 - 36*a^2*d^2*e^6*(-a*c)^(3/2) - 36*c^2*d^6*e^
2*(-a*c)^(3/2) + 70*a^2*c^3*d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*c^4
*d^6*e^2*x^2)*(a*((e^3*(-a*c)^(1/2))/8 + (c*d*e^2)/4) - (c*d^2*e*(-a*c)^(1/
2))/8))/(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2) - (d/(4*(a*e^2 + c*d^2)
) - (e*x^2)/(4*(a*e^2 + c*d^2)))/(a + c*x^4) - (log(c^5*d^8*x^2 - c^4*d^8*(
-a*c)^(1/2) - 70*d^4*e^4*(-a*c)^(5/2) - a^4*e^8*(-a*c)^(1/2) + a^4*c*e^8*x^
2 + 36*a^2*d^2*e^6*(-a*c)^(3/2) + 36*c^2*d^6*e^2*(-a*c)^(3/2) + 70*a^2*c^3*
d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*c^4*d^6*e^2*x^2)*(a*((e^3*(-a*c
)^(1/2))/8 - (c*d*e^2)/4) - (c*d^2*e*(-a*c)^(1/2))/8))/(a*c^3*d^4 + a^3*c*e
^4 + 2*a^2*c^2*d^2*e^2) - (d*e^2*log(d + e*x^2))/(2*(a^2*e^4 + c^2*d^4 + 2*
a*c*d^2*e^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.178 $\int \frac{x}{(d+ex^2)(a+cx^4)^2} \, dx$

Optimal. Leaf size=151

$$\frac{\sqrt{c}\,d\left(3ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(ae^2+cd^2\right)^2}+\frac{ae+cdx^2}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{e^3\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}+\frac{e^3\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}$$

Rubi [A] time = 0.18, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1248, 741, 801, 635, 205, 260}

$$\frac{\sqrt{c}\,d\left(3ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(ae^2+cd^2\right)^2}+\frac{ae+cdx^2}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{e^3\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}-\frac{e^3\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*e + c*d*x^2)/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (Sqrt[c]*d*(c*d^2 + 3*a
*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^2 + a*e^2)^2) + (e^3*L
og[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) - (e^3*Log[a + c*x^4])/(4*(c*d^2 + a*e
^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(d+ex)\left(a+cx^2\right)^2}\,dx, x, x^2\right)\\
&= \frac{ae+cdx^2}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-cd^2-2ae^2-cdex}{(d+ex)\left(a+cx^2\right)}\,dx, x, x^2\right)}{4a\left(cd^2+ae^2\right)}\\
&= \frac{ae+cdx^2}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\text{Subst}\left(\int \left(-\frac{2ae^4}{\left(cd^2+ae^2\right)(d+ex)} - \frac{c\left(cd^3+3ade^2-2ae^3x\right)}{\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right) dx, x, x^2\right)}{4a\left(cd^2+ae^2\right)}\\
&= \frac{ae+cdx^2}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^3\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)^2} + \frac{c\,\text{Subst}\left(\int \frac{cd^3+3ade^2-2ae^3x}{a+cx^2}\,dx, x, x^2\right)}{4a\left(cd^2+ae^2\right)^2}\\
&= \frac{ae+cdx^2}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^3\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)^2} - \frac{\left(ce^3\right)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)^2} + \frac{(cd\,(\ldots}{\ldots}\\
&= \frac{ae+cdx^2}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{\sqrt{c}\,d\left(cd^2+3ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(cd^2+ae^2\right)^2} + \frac{e^3\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)^2} - \frac{e^3\,\text{l}\ldots}{4(\ldots}
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 117, normalized size = 0.77

$$
\frac{\frac{\sqrt{c}\,d\left(3ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\left(ae^2+cd^2\right)\left(ae+cdx^2\right)}{a\left(a+cx^4\right)} - e^3\log\left(a+cx^4\right) + 2e^3\log\left(d+ex^2\right)}{4\left(ae^2+cd^2\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (((c*d^2 + a*e^2)*(a*e + c*d*x^2))/(a*(a + c*x^4)) + (Sqrt[c]*d*(c*d^2 + 3*
a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/a^(3/2) + 2*e^3*Log[d + e*x^2] - e^3*
Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[x/((d + e*x^2)*(a + c*x^4)^2), x]
```

fricas [A] time = 16.31, size = 458, normalized size = 3.03

$$
\left[\frac{2\,acd^2e + 2\,a^2e^3 + 2\left(c^2d^3 + acde^2\right)x^2 + \left(acd^3 + 3\,a^2de^2 + \left(c^2d^3 + 3\,acde^2\right)x^4\right)\sqrt{-\frac{c}{a}}\,\log\left(\frac{cx^4 + 2\,ax^2\sqrt{-\frac{c}{a}} - a}{cx^4 + a}\right) - 2\left(ace^3x^4 + a^2e^3\right)\log\left(cx^4 + a\right) + 4\left(ace^3x^4 + a^2e^3\right)\log\left(ex^2 + d\right)}{8\left(a^2c^2d^4 + 2\,a^3cd^2e^2 + a^4e^4 + \left(ac^3d^4 + 2\,a^2c^2d^2e^2 + a^3ce^4\right)x^4\right)},\ \frac{acd^2e + a^2e^3 + \left(c^2d^3 + acde^2\right)x^2 - \left(acd^3 + 3\,a^2de^2 + \left(c^2d^3 + 3\,acde^2\right)x^4\right)\sqrt{\frac{c}{a}}\,\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^2}\right) - \left(ace^3x^4 + a^2e^3\right)\log\left(cx^4 + a\right) + 2\left(ace^3x^4 + a^2e^3\right)\log\left(ex^2 + d\right)}{4\left(a^2c^2d^4 + 2\,a^3cd^2e^2 + a^4e^4 + \left(ac^3d^4 + 2\,a^2c^2d^2e^2 + a^3ce^4\right)x^4\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*c*d^2*e + 2*a^2*e^3 + 2*(c^2*d^3 + a*c*d*e^2)*x^2 + (a*c*d^3 + 3*
a^2*d*e^2 + (c^2*d^3 + 3*a*c*d*e^2)*x^4)*sqrt(-c/a)*log((c*x^4 + 2*a*x^2*sq
rt(-c/a) - a)/(c*x^4 + a)) - 2*(a*c*e^3*x^4 + a^2*e^3)*log(c*x^4 + a) + 4*(
a*c*e^3*x^4 + a^2*e^3)*log(e*x^2 + d))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4
*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4), 1/4*(a*c*d^2*e + a
^2*e^3 + (c^2*d^3 + a*c*d*e^2)*x^2 - (a*c*d^3 + 3*a^2*d*e^2 + (c^2*d^3 + 3*
a*c*d*e^2)*x^4)*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) - (a*c*e^3*x^4 + a^2*
e^3)*log(c*x^4 + a) + 2*(a*c*e^3*x^4 + a^2*e^3)*log(e*x^2 + d))/(a^2*c^2*d^
4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)
*x^4)]
```

giac [A] time = 0.38, size = 199, normalized size = 1.32

$$-\frac{e^{3}\log\left(cx^{4}+a\right)}{4\left(c^{2}d^{4}+2\,acd^{2}e^{2}+a^{2}e^{4}\right)}+\frac{e^{4}\log\left(\left|x^{2}e+d\right|\right)}{2\left(c^{2}d^{4}e+2\,acd^{2}e^{3}+a^{2}e^{5}\right)}+\frac{\left(c^{2}d^{3}+3\,acde^{2}\right)\arctan\left(\frac{cx^{2}}{\sqrt{ac}}\right)}{4\left(ac^{2}d^{4}+2\,a^{2}cd^{2}e^{2}+a^{3}e^{4}\right)\sqrt{ac}}+\frac{acd^{2}e+\left(c^{2}d^{3}+acde^{2}\right)x^{2}+a^{2}e^{3}}{4\left(cx^{4}+a\right)\left(cd^{2}+ae^{2}\right)^{2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*e^3*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*e^4*log(a
bs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*(c^2*d^3 + 3*a*c
*d*e^2)*arctan(c*x^2/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sq
rt(a*c)) + 1/4*(a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x^2 + a^2*e^3)/((c*x^4 +
a)*(c*d^2 + a*e^2)^2*a)
```

maple [A] time = 0.02, size = 255, normalized size = 1.69

$$\frac{c^{2}d^{3}x^{2}}{4\left(a e^{2}+c d^{2}\right)^{2}\left(c x^{4}+a\right)a}+\frac{c d e^{2}x^{2}}{4\left(a e^{2}+c d^{2}\right)^{2}\left(c x^{4}+a\right)}+\frac{c^{2}d^{3}\arctan\left(\frac{c x^{2}}{\sqrt{ac}}\right)}{4\left(a e^{2}+c d^{2}\right)^{2}\sqrt{ac}\,a}+\frac{3cd\,e^{2}\arctan\left(\frac{c x^{2}}{\sqrt{ac}}\right)}{4\left(a e^{2}+c d^{2}\right)^{2}\sqrt{ac}}+\frac{a e^{3}}{4\left(a e^{2}+c d^{2}\right)^{2}\left(c x^{4}+a\right)}+\frac{c d^{2}e}{4\left(a e^{2}+c d^{2}\right)^{2}\left(c x^{4}+a\right)}-\frac{e^{3}\ln\left(c x^{4}+a\right)}{4\left(a e^{2}+c d^{2}\right)^{2}}+\frac{e^{3}\ln\left(e x^{2}+d\right)}{2\left(a e^{2}+c d^{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*d*x^2*e^2+1/4*c^2/(a*e^2+c*d^2)^2/(c*x^4+a)
*d^3/a*x^2+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3*a+1/4*c/(a*e^2+c*d^2)^2/(c*x^4
+a)*e*d^2-1/4*e^3*ln(c*x^4+a)/(a*e^2+c*d^2)^2+3/4*c/(a*e^2+c*d^2)^2/(a*c)^(
1/2)*arctan(1/(a*c)^(1/2)*c*x^2)*d*e^2+1/4*c^2/(a*e^2+c*d^2)^2/a/(a*c)^(1/2
)*arctan(1/(a*c)^(1/2)*c*x^2)*d^3+1/2*e^3*ln(e*x^2+d)/(a*e^2+c*d^2)^2
```

maxima [A] time = 2.03, size = 196, normalized size = 1.30

$$-\frac{e^{3}\log\left(cx^{4}+a\right)}{4\left(c^{2}d^{4}+2\,acd^{2}e^{2}+a^{2}e^{4}\right)}+\frac{e^{3}\log\left(ex^{2}+d\right)}{2\left(c^{2}d^{4}+2\,acd^{2}e^{2}+a^{2}e^{4}\right)}+\frac{\left(c^{2}d^{3}+3\,acde^{2}\right)\arctan\left(\frac{cx^{2}}{\sqrt{ac}}\right)}{4\left(ac^{2}d^{4}+2\,a^{2}cd^{2}e^{2}+a^{3}e^{4}\right)\sqrt{ac}}+\frac{cdx^{2}+ae}{4\left(a^{2}cd^{2}+a^{3}e^{2}+\left(ac^{2}d^{2}+a^{2}ce^{2}\right)x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*e^3*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*e^3*log(e
*x^2 + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(c^2*d^3 + 3*a*c*d*e^2)
*arctan(c*x^2/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(a*c)
) + 1/4*(c*d*x^2 + a*e)/(a^2*c*d^2 + a^3*e^2 + (a*c^2*d^2 + a^2*c*e^2)*x^4)
```

mupad [B] time = 1.49, size = 649, normalized size = 4.30

$$\frac{\frac{e}{4\left(c\,d^{2}+a\,e^{2}\right)}+\frac{c\,d\,x^{2}}{4\,a\left(c\,d^{2}+a\,e^{2}\right)}}{c\,x^{4}+a}+\frac{e^{3}\ln\left(e\,x^{2}+d\right)}{2\left(a^{2}e^{4}+2\,a\,c\,d^{2}e^{2}+c^{2}d^{4}\right)}+\frac{\ln\left(36\,a^{6}e^{10}\sqrt{-a^{3}c}+36\,a^{7}c\,e^{10}x^{2}+a\,c^{5}d^{10}\sqrt{-a^{3}c}+a^{2}c^{6}d^{10}x^{2}-81\,a^{2}d^{2}e^{8}\left(-a^{3}c\right)^{3/2}-22\,c^{2}d^{6}e^{4}\left(-a^{3}c\right)^{3/2}+8\,a^{3}c^{5}d^{8}e^{2}x^{2}+22\,a^{4}c^{4}d^{6}e^{4}x^{2}+60\,a^{5}c^{3}d^{4}e^{6}x^{2}+81\,a^{6}c^{2}d^{2}e^{8}x^{2}+8\,a^{2}c^{4}d^{8}e^{2}\sqrt{-a^{3}c}-60\,a\,c\,d^{4}e^{6}\left(-a^{3}c\right)^{3/2}\right)\left(c\,d^{3}\sqrt{-a^{3}c}-2\,a^{3}e^{3}+3\,a\,d\,e^{2}\sqrt{-a^{3}c}\right)}{8\left(a^{5}e^{4}+2\,a^{4}c\,d^{2}e^{2}+a^{3}c^{2}d^{4}\right)}-\frac{\ln\left(36\,a^{7}c\,e^{10}x^{2}-36\,a^{6}e^{10}\sqrt{-a^{3}c}-a\,c^{5}d^{10}\sqrt{-a^{3}c}+a^{2}c^{6}d^{10}x^{2}+81\,a^{2}d^{2}e^{8}\left(-a^{3}c\right)^{3/2}+22\,c^{2}d^{6}e^{4}\left(-a^{3}c\right)^{3/2}+8\,a^{3}c^{5}d^{8}e^{2}x^{2}+22\,a^{4}c^{4}d^{6}e^{4}x^{2}+60\,a^{5}c^{3}d^{4}e^{6}x^{2}+81\,a^{6}c^{2}d^{2}e^{8}x^{2}-8\,a^{2}c^{4}d^{8}e^{2}\sqrt{-a^{3}c}+60\,a\,c\,d^{4}e^{6}\left(-a^{3}c\right)^{3/2}\right)\left(2\,a^{3}e^{3}+c\,d^{3}\sqrt{-a^{3}c}+3\,a\,d\,e^{2}\sqrt{-a^{3}c}\right)}{8\left(a^{5}e^{4}+2\,a^{4}c\,d^{2}e^{2}+a^{3}c^{2}d^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] (e/(4*(a*e^2 + c*d^2)) + (c*d*x^2)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^4) + (e^
3*log(d + e*x^2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (log(36*a^6*e^1
0*(-a^3*c)^(1/2) + 36*a^7*c*e^10*x^2 + a*c^5*d^10*(-a^3*c)^(1/2) + a^2*c^6*
d^10*x^2 - 81*a^2*d^2*e^8*(-a^3*c)^(3/2) - 22*c^2*d^6*e^4*(-a^3*c)^(3/2) +
8*a^3*c^5*d^8*e^2*x^2 + 22*a^4*c^4*d^6*e^4*x^2 + 60*a^5*c^3*d^4*e^6*x^2 + 8
1*a^6*c^2*d^2*e^8*x^2 + 8*a^2*c^4*d^8*e^2*(-a^3*c)^(1/2) - 60*a*c*d^4*e^6*(
-a^3*c)^(3/2))*(c*d^3*(-a^3*c)^(1/2) - 2*a^3*e^3 + 3*a*d*e^2*(-a^3*c)^(1/2)
))/(8*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)) - (log(36*a^7*c*e^10*x^2 -
 36*a^6*e^10*(-a^3*c)^(1/2) - a*c^5*d^10*(-a^3*c)^(1/2) + a^2*c^6*d^10*x^2
+ 81*a^2*d^2*e^8*(-a^3*c)^(3/2) + 22*c^2*d^6*e^4*(-a^3*c)^(3/2) + 8*a^3*c^5
*d^8*e^2*x^2 + 22*a^4*c^4*d^6*e^4*x^2 + 60*a^5*c^3*d^4*e^6*x^2 + 81*a^6*c^2
*d^2*e^8*x^2 - 8*a^2*c^4*d^8*e^2*(-a^3*c)^(1/2) + 60*a*c*d^4*e^6*(-a^3*c)^(
3/2))*(2*a^3*e^3 + c*d^3*(-a^3*c)^(1/2) + 3*a*d*e^2*(-a^3*c)^(1/2)))/(8*(a^
5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.179 $\int \frac{1}{x(d+ex^2)(a+cx^4)^2} \, dx$

Optimal. Leaf size=209

$$-\frac{\sqrt{c}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(ae^2+cd^2\right)}-\frac{cd\left(2ae^2+cd^2\right)\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)^2}+\frac{\log(x)}{a^2d}+\frac{c\left(d-ex^2\right)}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{e^4\log\left(d+ex^2\right)}{2d\left(ae^2+cd^2\right)^2}-\frac{\sqrt{c}\,e^3}{2\sqrt{a}}$$

Rubi [A] time = 0.24, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1252, 894, 639, 205, 635, 260}

$$-\frac{cd\left(2ae^2+cd^2\right)\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)^2}-\frac{\sqrt{c}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(ae^2+cd^2\right)}+\frac{\log(x)}{a^2d}+\frac{c\left(d-ex^2\right)}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{e^4\log\left(d+ex^2\right)}{2d\left(ae^2+cd^2\right)^2}-\frac{\sqrt{c}\,e^3\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (c*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[c]*e^3*ArcTan[(Sq
rt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^2 + a*e^2)^2) - (Sqrt[c]*e*ArcTan[(Sqr
t[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^2 + a*e^2)) + Log[x]/(a^2*d) - (e^4*Log
[d + e*x^2])/(2*d*(c*d^2 + a*e^2)^2) - (c*d*(c*d^2 + 2*a*e^2)*Log[a + c*x^4
])/(4*a^2*(c*d^2 + a*e^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(d+ex)\left(a+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int\left(\frac{1}{a^2dx} - \frac{e^5}{d\left(cd^2+ae^2\right)^2(d+ex)} - \frac{c(ae+cdx)}{a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{c\left(-a^2e^3 \ldots}{a^2\left(cd \ldots}\right.\right.$$

$$= \frac{\log(x)}{a^2d} - \frac{e^4\log\left(d+ex^2\right)}{2d\left(cd^2+ae^2\right)^2} + \frac{c\,\text{Subst}\left(\int \frac{-a^2e^3-cd\left(cd^2+2ae^2\right)x}{a+cx^2}\,dx, x, x^2\right)}{2a^2\left(cd^2+ae^2\right)^2} - \frac{c\,\text{Subst}\left(\int \ldots\right.}{2a\left(c \ldots\right.}$$

$$= \frac{c\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{\log(x)}{a^2d} - \frac{e^4\log\left(d+ex^2\right)}{2d\left(cd^2+ae^2\right)^2} - \frac{\left(ce^3\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, \ldots\right.}{2\left(cd^2+ae^2\right)^2}$$

$$= \frac{c\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\sqrt{c}\,e^3\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^2+ae^2\right)^2} - \frac{\sqrt{c}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(cd^2+ae^2\right)} + \frac{\log(x)}{a^2d} - \frac{e^4 \ldots}{2d \ldots}$$

Mathematica [A] time = 0.18, size = 241, normalized size = 1.15

$$\frac{-2a^2e^4\left(a+cx^4\right)\log\left(d+ex^2\right)+4\log(x)\left(a+cx^4\right)\left(ae^2+cd^2\right)^2-cd^2\left(a+cx^4\right)\left(2ae^2+cd^2\right)\log\left(a+cx^4\right)+\sqrt{a}\sqrt{c}\,de\left(a+cx^4\right)\left(3ae^2+cd^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+\sqrt{a}\sqrt{c}\,de\left(a+cx^4\right)\left(3ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)+acd\left(d-ex^2\right)\left(ae^2+cd^2\right)}{4a^2d\left(a+cx^4\right)\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*c*d*(c*d^2 + a*e^2)*(d - e*x^2) + Sqrt[a]*Sqrt[c]*d*e*(c*d^2 + 3*a*e^2)*
(a + c*x^4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[a]*Sqrt[c]*d*e*(
c*d^2 + 3*a*e^2)*(a + c*x^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 4*(c
*d^2 + a*e^2)^2*(a + c*x^4)*Log[x] - 2*a^2*e^4*(a + c*x^4)*Log[d + e*x^2] -
 c*d^2*(c*d^2 + 2*a*e^2)*(a + c*x^4)*Log[a + c*x^4])/(4*a^2*d*(c*d^2 + a*e^
2)^2*(a + c*x^4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(d + e*x^2)*(a + c*x^4)^2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

[Out] Timed out

giac [A] time = 0.37, size = 279, normalized size = 1.33

$$-\frac{\left(c^{2} d^{3}+2 \, a c d e^{2}\right) \log \left(c x^{4}+a\right)}{4 \left(a^{2} c^{2} d^{4}+2 \, a^{3} c d^{2} e^{2}+a^{4} e^{4}\right)}-\frac{e^{5} \log \left(\left|x^{2} e+d\right|\right)}{2 \left(c^{2} d^{5} e+2 \, a c d^{3} e^{3}+a^{2} d e^{5}\right)}-\frac{\left(c^{2} d^{2} e+3 \, a c e^{3}\right) \arctan \left(\frac{c x^{2}}{\sqrt{a c}}\right)}{4 \left(a c^{2} d^{4}+2 \, a^{2} c d^{2} e^{2}+a^{3} e^{4}\right) \sqrt{a c}}+\frac{c^{3} d^{3} x^{4}+2 \, a c^{2} d x^{4} e^{2}-a c^{2} d^{2} x^{2} e+2 \, a c^{2} d^{3}-a^{2} c x^{2} e^{3}+3 \, a^{2} c d e^{2}}{4 \left(a^{2} c^{2} d^{4}+2 \, a^{3} c d^{2} e^{2}+a^{4} e^{4}\right) \left(c x^{4}+a\right)}+\frac{\log \left(x^{2}\right)}{2 \, a^{2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(c^2*d^3 + 2*a*c*d*e^2)*log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2
+ a^4*e^4) - 1/2*e^5*log(abs(x^2*e + d))/(c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d
*e^5) - 1/4*(c^2*d^2*e + 3*a*c*e^3)*arctan(c*x^2/sqrt(a*c))/((a*c^2*d^4 + 2
*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(a*c)) + 1/4*(c^3*d^3*x^4 + 2*a*c^2*d*x^4*e^2
 - a*c^2*d^2*x^2*e + 2*a*c^2*d^3 - a^2*c*x^2*e^3 + 3*a^2*c*d*e^2)/((a^2*c^2
*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*(c*x^4 + a)) + 1/2*log(x^2)/(a^2*d)
```

maple [A] time = 0.02, size = 309, normalized size = 1.48

$$-\frac{c^{2} d^{2} e \,x^{2}}{4 \left(a \,e^{2}+c \,d^{2}\right)^{2} \left(c \,x^{4}+a \right) a}-\frac{c \,e^{3} x^{2}}{4 \left(a \,e^{2}+c \,d^{2}\right)^{2} \left(c \,x^{4}+a \right)}-\frac{c^{2} d^{2} e \arctan \left(\frac{c \,x^{2}}{\sqrt{a c}}\right)}{4 \left(a \,e^{2}+c \,d^{2}\right)^{2} \sqrt{a c}\, a}-\frac{3 c \,e^{3} \arctan \left(\frac{c \,x^{2}}{\sqrt{a c}}\right)}{4 \left(a \,e^{2}+c \,d^{2}\right)^{2} \sqrt{a c}}+\frac{c^{2} d^{3}}{4 \left(a \,e^{2}+c \,d^{2}\right)^{2} \left(c \,x^{4}+a \right) a}-\frac{c d \,e^{2} \ln \left(c \,x^{4}+a \right)}{2 \left(a \,e^{2}+c \,d^{2}\right)^{2} a}-\frac{c^{2} d^{3} \ln \left(c \,x^{4}+a \right)}{4 \left(a \,e^{2}+c \,d^{2}\right)^{2} a^{2}}+\frac{c d \,e^{2}}{4 \left(a \,e^{2}+c \,d^{2}\right)^{2} \left(c \,x^{4}+a \right)}-\frac{e^{4} \ln \left(e \,x^{2}+d \right)}{2 \left(a \,e^{2}+c \,d^{2}\right)^{2} d}+\frac{\ln \left(x \right)}{a^{2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] ln(x)/a^2/d-1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3*x^2-1/4*c^2/(a*e^2+c*d^2)^2
/a/(c*x^4+a)*x^2*e*d^2+1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*d*e^2+1/4*c^2/(a*e^2
+c*d^2)^2/a/(c*x^4+a)*d^3-1/2*c/(a*e^2+c*d^2)^2/a*ln(c*x^4+a)*d*e^2-1/4*c^2
/(a*e^2+c*d^2)^2/a^2*ln(c*x^4+a)*d^3-3/4*c/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arct
an(1/(a*c)^(1/2)*c*x^2)*e^3-1/4*c^2/(a*e^2+c*d^2)^2/a/(a*c)^(1/2)*arctan(1/
(a*c)^(1/2)*c*x^2)*e*d^2-1/2*e^4*ln(e*x^2+d)/d/(a*e^2+c*d^2)^2
```

maxima [A] time = 2.05, size = 228, normalized size = 1.09

$$-\frac{e^{4} \log \left(e x^{2}+d\right)}{2 \left(c^{2} d^{5}+2 \, a c d^{3} e^{2}+a^{2} d e^{4}\right)}-\frac{\left(c^{2} d^{3}+2 \, a c d e^{2}\right) \log \left(c x^{4}+a\right)}{4 \left(a^{2} c^{2} d^{4}+2 \, a^{3} c d^{2} e^{2}+a^{4} e^{4}\right)}-\frac{\left(c^{2} d^{2} e+3 \, a c e^{3}\right) \arctan \left(\frac{c x^{2}}{\sqrt{a c}}\right)}{4 \left(a c^{2} d^{4}+2 \, a^{2} c d^{2} e^{2}+a^{3} e^{4}\right) \sqrt{a c}}-\frac{c e x^{2}-c d}{4 \left(a^{2} c d^{2}+a^{3} e^{2}+\left(a c^{2} d^{2}+a^{2} c e^{2}\right) x^{4}\right)}+\frac{\log \left(x^{2}\right)}{2 \, a^{2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*e^4*log(e*x^2 + d)/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4) - 1/4*(c^2*d^
3 + 2*a*c*d*e^2)*log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4) -
 1/4*(c^2*d^2*e + 3*a*c*e^3)*arctan(c*x^2/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*
d^2*e^2 + a^3*e^4)*sqrt(a*c)) - 1/4*(c*e*x^2 - c*d)/(a^2*c*d^2 + a^3*e^2 +
(a*c^2*d^2 + a^2*c*e^2)*x^4) + 1/2*log(x^2)/(a^2*d)
```

mupad [B] time = 2.58, size = 1082, normalized size = 5.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((c*d)/(4*a*(a*e^2 + c*d^2)) - (c*e*x^2)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^4)
 - (log(400*a^9*c^12*d^20*x^2 - 10481*d^4*e^16*(-a^5*c)^(7/2) - 1024*a^12*e
^20*(-a^5*c)^(3/2) + 1024*a^19*c^2*e^20*x^2 - 400*a^2*c^10*d^20*(-a^5*c)^(3
/2) + 5840*a^6*d^2*e^18*(-a^5*c)^(5/2) + 33710*c^6*d^14*e^6*(-a^5*c)^(5/2)
+ 4104*a^10*c^11*d^18*e^2*x^2 + 16689*a^11*c^10*d^16*e^4*x^2 + 33710*a^12*c
^9*d^14*e^6*x^2 + 33391*a^13*c^8*d^12*e^8*x^2 + 10748*a^14*c^7*d^10*e^10*x^
2 - 3585*a^15*c^6*d^8*e^12*x^2 + 3998*a^16*c^5*d^6*e^14*x^2 + 10481*a^17*c^
```

```
4*d^4*e^16*x^2 + 5840*a^18*c^3*d^2*e^18*x^2 + 10748*a^2*c^4*d^10*e^10*(-a^5
*c)^(5/2) - 3585*a^3*c^3*d^8*e^12*(-a^5*c)^(5/2) + 3998*a^4*c^2*d^6*e^14*(-
a^5*c)^(5/2) - 4104*a^3*c^9*d^18*e^2*(-a^5*c)^(3/2) - 16689*a^4*c^8*d^16*e^
4*(-a^5*c)^(3/2) + 33391*a*c^5*d^12*e^8*(-a^5*c)^(5/2))*(3*a*e^3*(-a^5*c)^(
1/2) + 2*a^2*c^2*d^3 + 4*a^3*c*d*e^2 + c*d^2*e*(-a^5*c)^(1/2)))/(8*(a^6*e^4
 + a^4*c^2*d^4 + 2*a^5*c*d^2*e^2)) + (log(1024*a^12*e^20*(-a^5*c)^(3/2) + 1
0481*d^4*e^16*(-a^5*c)^(7/2) + 400*a^9*c^12*d^20*x^2 + 1024*a^19*c^2*e^20*x
^2 + 400*a^2*c^10*d^20*(-a^5*c)^(3/2) - 5840*a^6*d^2*e^18*(-a^5*c)^(5/2) -
33710*c^6*d^14*e^6*(-a^5*c)^(5/2) + 4104*a^10*c^11*d^18*e^2*x^2 + 16689*a^1
1*c^10*d^16*e^4*x^2 + 33710*a^12*c^9*d^14*e^6*x^2 + 33391*a^13*c^8*d^12*e^8
*x^2 + 10748*a^14*c^7*d^10*e^10*x^2 - 3585*a^15*c^6*d^8*e^12*x^2 + 3998*a^1
6*c^5*d^6*e^14*x^2 + 10481*a^17*c^4*d^4*e^16*x^2 + 5840*a^18*c^3*d^2*e^18*x
^2 - 10748*a^2*c^4*d^10*e^10*(-a^5*c)^(5/2) + 3585*a^3*c^3*d^8*e^12*(-a^5*c
)^(5/2) - 3998*a^4*c^2*d^6*e^14*(-a^5*c)^(5/2) + 4104*a^3*c^9*d^18*e^2*(-a^
5*c)^(3/2) + 16689*a^4*c^8*d^16*e^4*(-a^5*c)^(3/2) - 33391*a*c^5*d^12*e^8*(
-a^5*c)^(5/2))*(3*a*e^3*(-a^5*c)^(1/2) - 2*a^2*c^2*d^3 - 4*a^3*c*d*e^2 + c*
d^2*e*(-a^5*c)^(1/2)))/(8*(a^6*e^4 + a^4*c^2*d^4 + 2*a^5*c*d^2*e^2)) - (e^4
*log(d + e*x^2))/(2*c^2*d^5 + 2*a^2*d*e^4 + 4*a*c*d^3*e^2) + log(x)/(a^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.180 $\int \frac{1}{x^3(d+ex^2)(a+cx^4)^2} dx$

Optimal. Leaf size=236

$$-\frac{c^{3/2}d\left(2ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}\left(ae^2+cd^2\right)^2}-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(ae^2+cd^2\right)}+\frac{ce\left(2ae^2+cd^2\right)\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)^2}-\frac{c\left(ae+cdx^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}$$

Rubi [A] time = 0.26, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1252, 894, 639, 205, 635, 260}

$$-\frac{c^{3/2}d\left(2ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}\left(ae^2+cd^2\right)^2}-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(ae^2+cd^2\right)}-\frac{c\left(ae+cdx^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{ce\left(2ae^2+cd^2\right)\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)^2}-\frac{e\log(x)}{a^2d^2}-\frac{1}{2a^2dx^2}+\frac{e^5\log\left(d+ex^2\right)}{2d^2\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/(2*a^2*d*x^2) - (c*(a*e + c*d*x^2))/(4*a^2*(c*d^2 + a*e^2)*(a + c*x^4))
- (c^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(5/2)*(c*d^2 + a*e^2)) - (
c^(3/2)*d*(c*d^2 + 2*a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(5/2)*(c*d^
2 + a*e^2)^2) - (e*Log[x])/(a^2*d^2) + (e^5*Log[d + e*x^2])/(2*d^2*(c*d^2 +
 a*e^2)^2) + (c*e*(c*d^2 + 2*a*e^2)*Log[a + c*x^4])/(4*a^2*(c*d^2 + a*e^2)^
2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2dx^2} - \frac{e}{a^2d^2x} + \frac{e^6}{d^2\left(cd^2+ae^2\right)^2(d+ex)} - \frac{c^2(d-ex)}{a\left(cd^2+ae^2\right)\left(a+cx^2\right)^{2}}\right.\right.$$

$$= -\frac{1}{2a^2dx^2} - \frac{e\log(x)}{a^2d^2} + \frac{e^5\log\left(d+ex^2\right)}{2d^2\left(cd^2+ae^2\right)^2} - \frac{c^2\,\text{Subst}\left(\int \frac{d-ex}{\left(a+cx^2\right)^2}\,dx, x, x^2\right)}{2a\left(cd^2+ae^2\right)} - \frac{\left(c^2\left(cd\right.\right.}{}$$

$$= -\frac{1}{2a^2dx^2} - \frac{c\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{e\log(x)}{a^2d^2} + \frac{e^5\log\left(d+ex^2\right)}{2d^2\left(cd^2+ae^2\right)^2} - \frac{\left(c^2d\right)\text{Subs}}{4a^2}$$

$$= -\frac{1}{2a^2dx^2} - \frac{c\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(cd^2+ae^2\right)} - \frac{c^{3/2}d\left(cd^2+2ae^2\right)\text{ta}}{2a^{5/2}\left(cd^2+a\right.}$$

Mathematica [A] time = 0.43, size = 248, normalized size = 1.05

$$\frac{1}{4}\left(\frac{c^{3/2}d\left(5ae^2+3cd^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{5/2}\left(ae^2+cd^2\right)^2} + \frac{c^{3/2}d\left(5ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{a^{5/2}\left(ae^2+cd^2\right)^2} - \frac{c\left(ae+cdx^2\right)}{a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)} + \frac{c\left(2ae^3+cd^2e\right)\log\left(a+cx^4\right)}{a^2\left(ae^2+cd^2\right)^2} - \frac{4e\log(x)}{a^2d^2} - \frac{2}{a^2dx^2} + \frac{2e^5\log\left(d+ex^2\right)}{\left(ade^2+cd^3\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-2/(a^2*d*x^2) - (c*(a*e + c*d*x^2))/(a^2*(c*d^2 + a*e^2)*(a + c*x^4)) + (
c^(3/2)*d*(3*c*d^2 + 5*a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(
5/2)*(c*d^2 + a*e^2)^2) + (c^(3/2)*d*(3*c*d^2 + 5*a*e^2)*ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(a^(5/2)*(c*d^2 + a*e^2)^2) - (4*e*Log[x])/(a^2*d^2)
 + (2*e^5*Log[d + e*x^2])/(c*d^3 + a*d*e^2)^2 + (c*(c*d^2*e + 2*a*e^3)*Log[
a + c*x^4])/(a^2*(c*d^2 + a*e^2)^2))/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(d + e*x^2)*(a + c*x^4)^2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.35, size = 344, normalized size = 1.46

$$\frac{\left(c^{2} d^{2} e + 2 \, a c e^{3}\right) \log\left(c x^{4} + a\right)}{4 \, {\left(a^{2} c^{2} d^{4} + 2 \, a^{3} c d^{2} e^{2} + a^{4} e^{4}\right)}} + \frac{e^{6} \log\left({\left| x^{2} e + d \right|}\right)}{2 \, {\left(c^{2} d^{6} e + 2 \, a c d^{4} e^{3} + a^{2} d^{2} e^{5}\right)}} - \frac{{\left(3 \, c^{3} d^{3} + 5 \, a c^{2} d e^{2}\right)} \arctan\left(\frac{c x^{2}}{\sqrt{a c}}\right)}{4 \, {\left(a^{2} c^{2} d^{4} + 2 \, a^{3} c d^{2} e^{2} + a^{4} e^{4}\right)} \sqrt{a c}} - \frac{9 \, c^{3} d^{5} x^{4} + 15 \, a c^{2} d^{3} x^{4} e^{2} - 2 \, a^{2} c x^{6} e^{5} + 3 \, a c^{2} d^{4} x^{2} e + 6 \, a^{2} c d x^{4} e^{4} + 6 \, a c^{2} d^{5} + 3 \, a^{2} c d^{2} x^{2} e^{3} + 12 \, a^{2} c d^{3} e^{2} - 2 \, a^{3} x^{2} e^{5} + 6 \, a^{3} d e^{4}}{12 \, {\left(a^{2} c^{2} d^{6} + 2 \, a^{3} c d^{4} e^{2} + a^{4} d^{2} e^{4}\right)} {\left(c x^{6} + a x^{2}\right)}} - \frac{e \log\left(x^{2}\right)}{2 \, a^{2} d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out] 1/4*(c^2*d^2*e + 2*a*c*e^3)*log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4) + 1/2*e^6*log(abs(x^2*e + d))/(c^2*d^6*e + 2*a*c*d^4*e^3 + a^2*d^2*e^5) - 1/4*(3*c^3*d^3 + 5*a*c^2*d*e^2)*arctan(c*x^2/sqrt(a*c))/((a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(a*c)) - 1/12*(9*c^3*d^5*x^4 + 15*a*c^2*d^3*x^4*e^2 - 2*a^2*c*x^6*e^5 + 3*a*c^2*d^4*x^2*e + 6*a^2*c*d*x^4*e^4 + 6*a*c^2*d^5 + 3*a^2*c*d^2*x^2*e^3 + 12*a^2*c*d^3*e^2 - 2*a^3*x^2*e^5 + 6*a^3*d*e^4)/((a^2*c^2*d^6 + 2*a^3*c*d^4*e^2 + a^4*d^2*e^4)*(c*x^6 + a*x^2)) - 1/2*e*log(x^2)/(a^2*d^2)

maple [A] time = 0.02, size = 332, normalized size = 1.41

$$-\frac{c^{2} d \,e^{2} x^{2}}{4 \left(a \,e^{2}+c \,d^{2}\right)^{2} \left(c \,x^{4}+a \right) a}-\frac{c^{3} d^{3} x^{2}}{4 \left(a \,e^{2}+c \,d^{2}\right)^{2} \left(c \,x^{4}+a \right) a^{2}}-\frac{5 c^{2} d \,e^{2} \arctan \left(\frac{c \,x^{2}}{\sqrt{a c}}\right)}{4 \left(a \,e^{2}+c \,d^{2}\right)^{2} \sqrt{a c}\, a}-\frac{3 c^{3} d^{3} \arctan \left(\frac{c \,x^{2}}{\sqrt{a c}}\right)}{4 \left(a \,e^{2}+c \,d^{2}\right)^{2} \sqrt{a c}\, a^{2}}-\frac{c^{2} d^{2} e}{4 \left(a \,e^{2}+c \,d^{2}\right)^{2} \left(c \,x^{4}+a \right) a}+\frac{c \,e^{3} \ln \left(c \,x^{4}+a \right)}{2 \left(a \,e^{2}+c \,d^{2}\right)^{2} a}+\frac{c^{2} d^{2} e \ln \left(c \,x^{4}+a \right)}{4 \left(a \,e^{2}+c \,d^{2}\right)^{2} a^{2}}-\frac{c \,e^{3}}{4 \left(a \,e^{2}+c \,d^{2}\right)^{2} \left(c \,x^{4}+a \right)}+\frac{e^{5} \ln \left(e \,x^{2}+d \right)}{2 \left(a \,e^{2}+c \,d^{2}\right)^{2} d^{2}}-\frac{e \ln \left(x \right)}{a^{2} d^{2}}-\frac{1}{2 a^{2} d \,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x)

[Out] -1/2/a^2/d/x^2-e*ln(x)/a^2/d^2-1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*x^2*d*e^2-1/4*c^3/(a*e^2+c*d^2)^2/a^2/(c*x^4+a)*x^2*d^3-1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3-1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*e*d^2+1/2*c/(a*e^2+c*d^2)^2/a*ln(c*x^4+a)*e^3+1/4*c^2/(a*e^2+c*d^2)^2/a^2*ln(c*x^4+a)*e*d^2-5/4*c^2/(a*e^2+c*d^2)^2/a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)*d*e^2-3/4*c^3/(a*e^2+c*d^2)^2/a^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)*d^3+1/2*e^5*ln(e*x^2+d)/d^2/(a*e^2+c*d^2)^2

maxima [A] time = 2.01, size = 278, normalized size = 1.18

$$\frac{e^{5} \log\left(e x^{2} + d\right)}{2 \, {\left(c^{2} d^{6} + 2 \, a c d^{4} e^{2} + a^{2} d^{2} e^{4}\right)}} + \frac{{\left(c^{2} d^{2} e + 2 \, a c e^{3}\right)} \log\left(c x^{4} + a\right)}{4 \, {\left(a^{2} c^{2} d^{4} + 2 \, a^{3} c d^{2} e^{2} + a^{4} e^{4}\right)}} - \frac{{\left(3 \, c^{3} d^{3} + 5 \, a c^{2} d e^{2}\right)} \arctan\left(\frac{c x^{2}}{\sqrt{a c}}\right)}{4 \, {\left(a^{2} c^{2} d^{4} + 2 \, a^{3} c d^{2} e^{2} + a^{4} e^{4}\right)} \sqrt{a c}} - \frac{a c d e x^{2} + {\left(3 \, c^{2} d^{2} + 2 \, a c e^{2}\right)} x^{4} + 2 \, a c d^{2} + 2 \, a^{2} e^{2}}{4 \, {\left({\left(a^{2} c^{2} d^{3} + a^{3} c d e^{2}\right)} x^{6} + {\left(a^{3} c d^{3} + a^{4} d e^{2}\right)} x^{2}\right)}} - \frac{e \log\left(x^{2}\right)}{2 \, a^{2} d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")

[Out] 1/2*e^5*log(e*x^2 + d)/(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4) + 1/4*(c^2*d^2*e + 2*a*c*e^3)*log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4) - 1/4*(3*c^3*d^3 + 5*a*c^2*d*e^2)*arctan(c*x^2/sqrt(a*c))/((a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(a*c)) - 1/4*(a*c*d*e*x^2 + (3*c^2*d^2 + 2*a*c*e^2)*x^4 + 2*a*c*d^2 + 2*a^2*e^2)/((a^2*c^2*d^3 + a^3*c*d*e^2)*x^6 + (a^3*c*d^3 + a^4*d*e^2)*x^2) - 1/2*e*log(x^2)/(a^2*d^2)

mupad [B] time = 2.94, size = 1337, normalized size = 5.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a + c*x^4)^2*(d + e*x^2)),x)

[Out] (log(81*a^10*c^16*d^24*x^2 + 1024*a^22*c^4*e^24*x^2 - 81*a^3*c^11*d^24*(-a^5*c^3)^(3/2) + 1024*a^20*c^2*e^24*(-a^5*c^3)^(1/2) - 14496*a^6*d^8*e^16*(-a^5*c^3)^(5/2) - 5120*a^14*d^2*e^22*(-a^5*c^3)^(3/2) + 11647*c^6*d^20*e^4*(-

```
a^5*c^3)^(5/2) + 1638*a^11*c^15*d^22*e^2*x^2 + 11647*a^12*c^14*d^20*e^4*x^2
 + 43524*a^13*c^13*d^18*e^6*x^2 + 97311*a^14*c^12*d^16*e^8*x^2 + 133334*a^1
5*c^11*d^14*e^10*x^2 + 103633*a^16*c^10*d^12*e^12*x^2 + 29456*a^17*c^9*d^10
*e^14*x^2 - 14496*a^18*c^8*d^8*e^16*x^2 - 7984*a^19*c^7*d^6*e^18*x^2 + 5888
*a^20*c^6*d^4*e^20*x^2 + 5120*a^21*c^5*d^2*e^22*x^2 + 43524*a*c^5*d^18*e^6*
(-a^5*c^3)^(5/2) + 29456*a^5*c*d^10*e^14*(-a^5*c^3)^(5/2) - 5888*a^13*c*d^4
*e^20*(-a^5*c^3)^(3/2) + 97311*a^2*c^4*d^16*e^8*(-a^5*c^3)^(5/2) + 133334*a
^3*c^3*d^14*e^10*(-a^5*c^3)^(5/2) + 103633*a^4*c^2*d^12*e^12*(-a^5*c^3)^(5/
2) - 1638*a^4*c^10*d^22*e^2*(-a^5*c^3)^(3/2) + 7984*a^12*c^2*d^6*e^18*(-a^5
*c^3)^(3/2))*(4*a^4*c*e^3 - 3*c*d^3*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d^2*e - 5*
a*d*e^2*(-a^5*c^3)^(1/2)))/(8*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)) -
(1/(2*a*d) + (c*e*x^2)/(4*a*(a*e^2 + c*d^2)) + (c*x^4*(2*a*e^2 + 3*c*d^2))/
(4*a^2*d*(a*e^2 + c*d^2)))/(a*x^2 + c*x^6) + (log(81*a^10*c^16*d^24*x^2 + 1
024*a^22*c^4*e^24*x^2 + 81*a^3*c^11*d^24*(-a^5*c^3)^(3/2) - 1024*a^20*c^2*e
^24*(-a^5*c^3)^(1/2) + 14496*a^6*d^8*e^16*(-a^5*c^3)^(5/2) + 5120*a^14*d^2*
e^22*(-a^5*c^3)^(3/2) - 11647*c^6*d^20*e^4*(-a^5*c^3)^(5/2) + 1638*a^11*c^1
5*d^22*e^2*x^2 + 11647*a^12*c^14*d^20*e^4*x^2 + 43524*a^13*c^13*d^18*e^6*x^
2 + 97311*a^14*c^12*d^16*e^8*x^2 + 133334*a^15*c^11*d^14*e^10*x^2 + 103633*
a^16*c^10*d^12*e^12*x^2 + 29456*a^17*c^9*d^10*e^14*x^2 - 14496*a^18*c^8*d^8
*e^16*x^2 - 7984*a^19*c^7*d^6*e^18*x^2 + 5888*a^20*c^6*d^4*e^20*x^2 + 5120*
a^21*c^5*d^2*e^22*x^2 - 43524*a*c^5*d^18*e^6*(-a^5*c^3)^(5/2) - 29456*a^5*c
*d^10*e^14*(-a^5*c^3)^(5/2) + 5888*a^13*c*d^4*e^20*(-a^5*c^3)^(3/2) - 97311
*a^2*c^4*d^16*e^8*(-a^5*c^3)^(5/2) - 133334*a^3*c^3*d^14*e^10*(-a^5*c^3)^(5
/2) - 103633*a^4*c^2*d^12*e^12*(-a^5*c^3)^(5/2) + 1638*a^4*c^10*d^22*e^2*(-
a^5*c^3)^(3/2) - 7984*a^12*c^2*d^6*e^18*(-a^5*c^3)^(3/2))*(4*a^4*c*e^3 + 3*
c*d^3*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d^2*e + 5*a*d*e^2*(-a^5*c^3)^(1/2)))/(8*
(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)) + (e^5*log(d + e*x^2))/(2*c^2*d^
6 + 2*a^2*d^2*e^4 + 4*a*c*d^4*e^2) - (e*log(x))/(a^2*d^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.181 $\int \frac{1}{x^5(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=265

$$\frac{c^{3/2}e\left(2ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{5/2}\left(ae^2+cd^2\right)^2}+\frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(ae^2+cd^2\right)}+\frac{c^2d\left(3ae^2+2cd^2\right)\log\left(a+cx^4\right)}{4a^3\left(ae^2+cd^2\right)^2}-\frac{\log(x)\left(2cd^2-ae^2\right)}{a^3d^3}-\frac{}{4a}$$

Rubi [A] time = 0.33, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1252, 894, 639, 205, 635, 260}

$$-\frac{c^2\left(d-ex^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{c^2d\left(3ae^2+2cd^2\right)\log\left(a+cx^4\right)}{4a^3\left(ae^2+cd^2\right)^2}+\frac{c^{3/2}e\left(2ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{5/2}\left(ae^2+cd^2\right)^2}+\frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(ae^2+cd^2\right)}-\frac{\log(x)\left(2cd^2-ae^2\right)}{a^3d^3}+\frac{e}{2a^2d^2x^2}-\frac{1}{4a^2dx^4}-\frac{e^6\log\left(d+ex^2\right)}{2d^3\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/(4*a^2*d*x^4) + e/(2*a^2*d^2*x^2) - (c^2*(d - e*x^2))/(4*a^2*(c*d^2 + a*
e^2)*(a + c*x^4)) + (c^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(5/2)*(c
*d^2 + a*e^2)) + (c^(3/2)*e*(c*d^2 + 2*a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]
)/(2*a^(5/2)*(c*d^2 + a*e^2)^2) - ((2*c*d^2 - a*e^2)*Log[x])/(a^3*d^3) - (e
^6*Log[d + e*x^2])/(2*d^3*(c*d^2 + a*e^2)^2) + (c^2*d*(2*c*d^2 + 3*a*e^2)*L
og[a + c*x^4])/(4*a^3*(c*d^2 + a*e^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{1}{x^5 \left(d+ex^2\right) \left(a+cx^4\right)^2} \, dx = \frac{1}{2} \text{Subst}\left(\int \frac{1}{x^3 (d+ex) \left(a+cx^2\right)^2} \, dx, x, x^2\right)$$

$$= \frac{1}{2} \text{Subst}\left(\int \left(\frac{1}{a^2 d x^3} - \frac{e}{a^2 d^2 x^2} + \frac{-2cd^2+ae^2}{a^3 d^3 x} - \frac{e^7}{d^3 \left(cd^2+ae^2\right)^2 (d+ex)} + \frac{c}{a^2 \left(cd^2 \cdots\right.} \cdots\right.\right.$$

$$= -\frac{1}{4a^2 d x^4} + \frac{e}{2a^2 d^2 x^2} - \frac{\left(2cd^2-ae^2\right) \log(x)}{a^3 d^3} - \frac{e^6 \log\left(d+ex^2\right)}{2d^3 \left(cd^2+ae^2\right)^2} + \frac{c^2 \text{Subst}\left(\int \frac{ae(c\cdots}{\cdots}\right.}{2\cdots}$$

$$= -\frac{1}{4a^2 d x^4} + \frac{e}{2a^2 d^2 x^2} - \frac{c^2 \left(d-ex^2\right)}{4a^2 \left(cd^2+ae^2\right) \left(a+cx^4\right)} - \frac{\left(2cd^2-ae^2\right) \log(x)}{a^3 d^3} - \frac{e^6 \log\left(\cdots\right.}{2d^3 \left(cd^2 \cdots\right.}$$

$$= -\frac{1}{4a^2 d x^4} + \frac{e}{2a^2 d^2 x^2} - \frac{c^2 \left(d-ex^2\right)}{4a^2 \left(cd^2+ae^2\right) \left(a+cx^4\right)} + \frac{c^{3/2} e \tan^{-1}\left(\frac{\sqrt{c} x^2}{\sqrt{a}}\right)}{4a^{5/2} \left(cd^2+ae^2\right)} + \frac{c^{3/2} e \left(cd^2 \cdots\right.}{2a \cdots}$$

Mathematica [A] time = 0.39, size = 278, normalized size = 1.05

$$\frac{1}{4}\left(-\frac{c^{3/2} e \left(5ae^2+3cd^2\right) \tan^{-1}\left(1-\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{a^{5/2} \left(ae^2+cd^2\right)^2} - \frac{c^{3/2} e \left(5ae^2+3cd^2\right) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}+1\right)}{a^{5/2} \left(ae^2+cd^2\right)^2} + \frac{c^2 \left(3ade^2+2cd^3\right) \log\left(a+cx^4\right)}{a^3 \left(ae^2+cd^2\right)^2} + \frac{4 \log(x) \left(ae^2-2cd^2\right)}{a^3 d^3} + \frac{c^2 \left(ex^2-d\right)}{a^2 \left(a+cx^4\right) \left(ae^2+cd^2\right)} + \frac{2e}{a^2 d^2 x^2} - \frac{1}{a^2 d x^4} - \frac{2e^6 \log\left(d+ex^2\right)}{d^3 \left(ae^2+cd^2\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-(1/(a^2*d*x^4)) + (2*e)/(a^2*d^2*x^2) + (c^2*(-d + e*x^2))/(a^2*(c*d^2 +
a*e^2)*(a + c*x^4)) - (c^(3/2)*e*(3*c*d^2 + 5*a*e^2)*ArcTan[1 - (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(a^(5/2)*(c*d^2 + a*e^2)^2) - (c^(3/2)*e*(3*c*d^2 + 5*a*
e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(5/2)*(c*d^2 + a*e^2)^2) +
 (4*(-2*c*d^2 + a*e^2)*Log[x])/(a^3*d^3) - (2*e^6*Log[d + e*x^2])/(d^3*(c*d
^2 + a*e^2)^2) + (c^2*(2*c*d^3 + 3*a*d*e^2)*Log[a + c*x^4])/(a^3*(c*d^2 + a
*e^2)^2))/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(d+ex^2\right) \left(a+cx^4\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(d + e*x^2)*(a + c*x^4)^2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.37, size = 350, normalized size = 1.32

$$\frac{\left(2 c^{3} d^{3}+3 a c^{2} d e^{2}\right) \log \left(c x^{4}+a\right)}{4\left(a^{3} c^{2} d^{4}+2 a^{4} c d^{2} e^{2}+a^{5} e^{4}\right)}-\frac{e^{7} \log \left(\left|x^{2} e+d\right|\right)}{2\left(c^{2} d^{7} e+2 a c d^{5} e^{3}+a^{2} d^{3} e^{5}\right)}+\frac{\left(3 c^{3} d^{2} e+5 a c^{2} e^{3}\right) \arctan \left(\frac{c x^{2}}{\sqrt{a c}}\right)}{4\left(a^{2} c^{2} d^{4}+2 a^{3} c d^{2} e^{2}+a^{4} e^{4}\right) \sqrt{a c}}-\frac{2 c^{4} d^{3} x^{4}+3 a c^{3} d x^{4} e^{2}-a c^{3} d^{2} x^{2} e+3 a c^{3} d^{3}-a^{2} c^{2} x^{2} e^{3}+4 a^{2} c^{2} d e^{2}}{4\left(a^{3} c^{2} d^{4}+2 a^{4} c d^{2} e^{2}+a^{5} e^{4}\right)\left(c x^{4}+a\right)}-\frac{\left(2 c d^{2}-a e^{2}\right) \log \left(x^{2}\right)}{2 a^{3} d^{3}}+\frac{6 c d^{2} x^{4}-3 a x^{4} e^{2}+2 a d x^{2} e-a d^{2}}{4 a^{3} d^{3} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*c^3*d^3 + 3*a*c^2*d*e^2)*log(c*x^4 + a)/(a^3*c^2*d^4 + 2*a^4*c*d^2*e
^2 + a^5*e^4) - 1/2*e^7*log(abs(x^2*e + d))/(c^2*d^7*e + 2*a*c*d^5*e^3 + a^
2*d^3*e^5) + 1/4*(3*c^3*d^2*e + 5*a*c^2*e^3)*arctan(c*x^2/sqrt(a*c))/((a^2*
c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(a*c)) - 1/4*(2*c^4*d^3*x^4 + 3*a*
c^3*d*x^4*e^2 - a*c^3*d^2*x^2*e + 3*a*c^3*d^3 - a^2*c^2*x^2*e^3 + 4*a^2*c^2
*d*e^2)/((a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*(c*x^4 + a)) - 1/2*(2*c*
d^2 - a*e^2)*log(x^2)/(a^3*d^3) + 1/4*(6*c*d^2*x^4 - 3*a*x^4*e^2 + 2*a*d*x^
2*e - a*d^2)/(a^3*d^3*x^4)
```

maple [A] time = 0.02, size = 363, normalized size = 1.37

$$\frac{c^{2} e^{3} x^{2}}{4\left(a e^{2}+c d^{2}\right)^{2}\left(c x^{4}+a\right) a}+\frac{c^{3} d^{2} e x^{2}}{4\left(a e^{2}+c d^{2}\right)^{2}\left(c x^{4}+a\right) a^{2}}+\frac{5 c^{2} e^{3} \arctan \left(\frac{c x^{2}}{\sqrt{a c}}\right)}{4\left(a e^{2}+c d^{2}\right)^{2} \sqrt{a c}\, a}+\frac{3 c^{3} d^{2} e \arctan \left(\frac{c x^{2}}{\sqrt{a c}}\right)}{4\left(a e^{2}+c d^{2}\right)^{2} \sqrt{a c}\, a^{2}}-\frac{c^{2} d e^{2}}{4\left(a e^{2}+c d^{2}\right)^{2}\left(c x^{4}+a\right) a}-\frac{c^{3} d^{3}}{4\left(a e^{2}+c d^{2}\right)^{2}\left(c x^{4}+a\right) a^{2}}+\frac{3 c^{2} d e^{2} \ln \left(c x^{4}+a\right)}{4\left(a e^{2}+c d^{2}\right)^{2} a^{2}}+\frac{c^{3} d^{3} \ln \left(c x^{4}+a\right)}{2\left(a e^{2}+c d^{2}\right)^{2} a^{3}}-\frac{e^{6} \ln \left(e x^{2}+d\right)}{2\left(a e^{2}+c d^{2}\right)^{2} d^{3}}+\frac{e^{2} \ln (x)}{a^{2} d^{3}}-\frac{2 c \ln (x)}{a^{3} d}+\frac{e}{2 a^{2} d^{2} x^{2}}-\frac{1}{4 a^{2} d x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4/a^2/d/x^4+1/a^2/d^3*ln(x)*e^2-2/a^3/d*ln(x)*c+1/2*e/a^2/d^2/x^2+1/4*c^
2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*e^3*x^2+1/4*c^3/(a*e^2+c*d^2)^2/a^2/(c*x^4+a)
*x^2*e*d^2-1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*d*e^2-1/4*c^3/(a*e^2+c*d^2)^
2/a^2/(c*x^4+a)*d^3+3/4*c^2/(a*e^2+c*d^2)^2/a^2*ln(c*x^4+a)*d*e^2+1/2*c^3/(
a*e^2+c*d^2)^2/a^3*ln(c*x^4+a)*d^3+5/4*c^2/(a*e^2+c*d^2)^2/a/(a*c)^(1/2)*ar
ctan(1/(a*c)^(1/2)*c*x^2)*e^3+3/4*c^3/(a*e^2+c*d^2)^2/a^2/(a*c)^(1/2)*arcta
n(1/(a*c)^(1/2)*c*x^2)*e*d^2-1/2*e^6*ln(e*x^2+d)/d^3/(a*e^2+c*d^2)^2
```

maxima [A] time = 2.08, size = 332, normalized size = 1.25

$$-\frac{e^{6} \log \left(e x^{2}+d\right)}{2\left(c^{2} d^{7}+2 a c d^{5} e^{2}+a^{2} d^{3} e^{4}\right)}+\frac{\left(2 c^{3} d^{3}+3 a c^{2} d e^{2}\right) \log \left(c x^{4}+a\right)}{4\left(a^{3} c^{2} d^{4}+2 a^{4} c d^{2} e^{2}+a^{5} e^{4}\right)}+\frac{\left(3 c^{3} d^{2} e+5 a c^{2} e^{3}\right) \arctan \left(\frac{c x^{2}}{\sqrt{a c}}\right)}{4\left(a^{2} c^{2} d^{4}+2 a^{3} c d^{2} e^{2}+a^{4} e^{4}\right) \sqrt{a c}}+\frac{\left(3 c^{2} d^{2} e+2 a c e^{3}\right) x^{6}-a c d^{3}-a^{2} d e^{2}-\left(2 c^{2} d^{3}+a c d e^{2}\right) x^{4}+2\left(a c d^{2} e+a^{2} e^{3}\right) x^{2}}{4\left(\left(a^{2} c^{2} d^{4}+a^{3} c d^{2} e^{2}\right) x^{8}+\left(a^{3} c d^{4}+a^{4} d^{2} e^{2}\right) x^{4}\right)}-\frac{\left(2 c d^{2}-a e^{2}\right) \log \left(x^{2}\right)}{2 a^{3} d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*e^6*log(e*x^2 + d)/(c^2*d^7 + 2*a*c*d^5*e^2 + a^2*d^3*e^4) + 1/4*(2*c^
3*d^3 + 3*a*c^2*d*e^2)*log(c*x^4 + a)/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*
e^4) + 1/4*(3*c^3*d^2*e + 5*a*c^2*e^3)*arctan(c*x^2/sqrt(a*c))/((a^2*c^2*d^
4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(a*c)) + 1/4*((3*c^2*d^2*e + 2*a*c*e^3)*
x^6 - a*c*d^3 - a^2*d*e^2 - (2*c^2*d^3 + a*c*d*e^2)*x^4 + 2*(a*c*d^2*e + a^
2*e^3)*x^2)/((a^2*c^2*d^4 + a^3*c*d^2*e^2)*x^8 + (a^3*c*d^4 + a^4*d^2*e^2)*
x^4) - 1/2*(2*c*d^2 - a*e^2)*log(x^2)/(a^3*d^3)
```

mupad [B] time = 3.48, size = 1545, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] (log(6400*a^13*c^18*d^28*x^2 + 1024*a^27*c^4*e^28*x^2 - 6400*a^3*c^13*d^28*
(-a^7*c^3)^(3/2) + 1024*a^24*c^2*e^28*(-a^7*c^3)^(1/2) - 10688*a^6*d^8*e^20
*(-a^7*c^3)^(5/2) - 2048*a^16*d^2*e^26*(-a^7*c^3)^(3/2) + 536959*c^6*d^20*e
```

```
^8*(-a^7*c^3)^(5/2) + 54944*a^14*c^17*d^26*e^2*x^2 + 200881*a^15*c^16*d^24*
e^4*x^2 + 413414*a^16*c^15*d^22*e^6*x^2 + 536959*a^17*c^14*d^20*e^8*x^2 + 4
65092*a^18*c^13*d^18*e^10*x^2 + 256991*a^19*c^12*d^16*e^12*x^2 + 52822*a^20
*c^11*d^14*e^14*x^2 - 37423*a^21*c^10*d^12*e^16*x^2 - 27472*a^22*c^9*d^10*e
^18*x^2 - 10688*a^23*c^8*d^8*e^20*x^2 - 10288*a^24*c^7*d^6*e^22*x^2 - 3584*
a^25*c^6*d^4*e^24*x^2 + 2048*a^26*c^5*d^2*e^26*x^2 + 465092*a*c^5*d^18*e^10
*(-a^7*c^3)^(5/2) - 27472*a^5*c*d^10*e^18*(-a^7*c^3)^(5/2) + 3584*a^15*c*d^
4*e^24*(-a^7*c^3)^(3/2) + 256991*a^2*c^4*d^16*e^12*(-a^7*c^3)^(5/2) + 52822
*a^3*c^3*d^14*e^14*(-a^7*c^3)^(5/2) - 37423*a^4*c^2*d^12*e^16*(-a^7*c^3)^(5
/2) - 54944*a^4*c^12*d^26*e^2*(-a^7*c^3)^(3/2) - 200881*a^5*c^11*d^24*e^4*(
-a^7*c^3)^(3/2) - 413414*a^6*c^10*d^22*e^6*(-a^7*c^3)^(3/2) + 10288*a^14*c^
2*d^6*e^22*(-a^7*c^3)^(3/2))*(4*a^3*c^3*d^3 + 5*a*e^3*(-a^7*c^3)^(1/2) + 6*
a^4*c^2*d*e^2 + 3*c*d^2*e*(-a^7*c^3)^(1/2)))/(8*(a^8*e^4 + a^6*c^2*d^4 + 2*
a^7*c*d^2*e^2)) - (e^6*log(d + e*x^2))/(2*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^
5*e^2)) - (1/(4*a*d) - (e*x^2)/(2*a*d^2) + (x^4*(2*c^2*d^2 + a*c*e^2))/(4*a
^2*d*(a*e^2 + c*d^2)) - (c*e*x^6*(2*a*e^2 + 3*c*d^2))/(4*a^2*d^2*(a*e^2 + c
*d^2)))/(a*x^4 + c*x^8) + (log(6400*a^13*c^18*d^28*x^2 + 1024*a^27*c^4*e^28
*x^2 + 6400*a^3*c^13*d^28*(-a^7*c^3)^(3/2) - 1024*a^24*c^2*e^28*(-a^7*c^3)^
(1/2) + 10688*a^6*d^8*e^20*(-a^7*c^3)^(5/2) + 2048*a^16*d^2*e^26*(-a^7*c^3)
^(3/2) - 536959*c^6*d^20*e^8*(-a^7*c^3)^(5/2) + 54944*a^14*c^17*d^26*e^2*x^
2 + 200881*a^15*c^16*d^24*e^4*x^2 + 413414*a^16*c^15*d^22*e^6*x^2 + 536959*
a^17*c^14*d^20*e^8*x^2 + 465092*a^18*c^13*d^18*e^10*x^2 + 256991*a^19*c^12*
d^16*e^12*x^2 + 52822*a^20*c^11*d^14*e^14*x^2 - 37423*a^21*c^10*d^12*e^16*x
^2 - 27472*a^22*c^9*d^10*e^18*x^2 - 10688*a^23*c^8*d^8*e^20*x^2 - 10288*a^2
4*c^7*d^6*e^22*x^2 - 3584*a^25*c^6*d^4*e^24*x^2 + 2048*a^26*c^5*d^2*e^26*x^
2 - 465092*a*c^5*d^18*e^10*(-a^7*c^3)^(5/2) + 27472*a^5*c*d^10*e^18*(-a^7*c
^3)^(5/2) - 3584*a^15*c*d^4*e^24*(-a^7*c^3)^(3/2) - 256991*a^2*c^4*d^16*e^1
2*(-a^7*c^3)^(5/2) - 52822*a^3*c^3*d^14*e^14*(-a^7*c^3)^(5/2) + 37423*a^4*c
^2*d^12*e^16*(-a^7*c^3)^(5/2) + 54944*a^4*c^12*d^26*e^2*(-a^7*c^3)^(3/2) +
200881*a^5*c^11*d^24*e^4*(-a^7*c^3)^(3/2) + 413414*a^6*c^10*d^22*e^6*(-a^7*
c^3)^(3/2) - 10288*a^14*c^2*d^6*e^22*(-a^7*c^3)^(3/2))*(4*a^3*c^3*d^3 - 5*a
*e^3*(-a^7*c^3)^(1/2) + 6*a^4*c^2*d*e^2 - 3*c*d^2*e*(-a^7*c^3)^(1/2)))/(8*(
a^8*e^4 + a^6*c^2*d^4 + 2*a^7*c*d^2*e^2)) + (log(x)*(a*e^2 - 2*c*d^2))/(a^3
*d^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.182 $\int \frac{x^8}{(d+ex^2)(a+cx^4)^2} \, dx$

Optimal. Leaf size=712

$$\frac{\sqrt[4]{a} d^2 \left(\sqrt{a} e + \sqrt{c} d\right) \log \left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{4\sqrt{2} c^{3/4} \left(ae^2 + cd^2\right)^2} - \frac{\sqrt[4]{a} d^2 \left(\sqrt{a} e + \sqrt{c} d\right) \log \left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{4\sqrt{2} c^{3/4} \left(ae^2 + cd^2\right)^2} -$$

Rubi [A] time = 0.67, antiderivative size = 712, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1314, 1276, 1280, 1168, 1162, 617, 204, 1165, 628, 1288, 205}

$$\frac{\sqrt[4]{a} d^2 \left(\sqrt{a} e + \sqrt{c} d\right) \log \left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{4\sqrt{2} c^{3/4} \left(ae^2 + cd^2\right)^2} - \frac{\sqrt[4]{a} d^2 \left(\sqrt{a} e + \sqrt{c} d\right) \log \left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{4\sqrt{2} c^{3/4} \left(ae^2 + cd^2\right)^2} + \frac{\sqrt[4]{a} \left(3\sqrt{a} e + \sqrt{c} d\right) \log \left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{16\sqrt{2} c^{7/4} \left(ae^2 + cd^2\right)} - \frac{\sqrt[4]{a} \left(3\sqrt{a} e + \sqrt{c} d\right) \log \left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{16\sqrt{2} c^{7/4} \left(ae^2 + cd^2\right)} + \frac{\sqrt[4]{a} d^2 \left(\sqrt{c} d - \sqrt{a} e\right) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} c^{3/4} \left(ae^2 + cd^2\right)^2} - \frac{\sqrt[4]{a} d^2 \left(\sqrt{c} d - \sqrt{a} e\right) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2} c^{3/4} \left(ae^2 + cd^2\right)^2} + \frac{\sqrt[4]{a} \left(\sqrt{c} d - 3\sqrt{a} e\right) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{8\sqrt{2} c^{7/4} \left(ae^2 + cd^2\right)} - \frac{\sqrt[4]{a} \left(\sqrt{c} d - 3\sqrt{a} e\right) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}} + 1\right)}{8\sqrt{2} c^{7/4} \left(ae^2 + cd^2\right)} - \frac{x^3 \left(ae + cdx^2\right)}{4c \left(a + cx^4\right) \left(ae^2 + cd^2\right)} + \frac{dx}{4c \left(ae^2 + cd^2\right)} + \frac{d^{7/2} \tan^{-1}\left(\frac{\sqrt{e} x}{\sqrt{d}}\right)}{\sqrt{e} \left(ae^2 + cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (d*x)/(4*c*(c*d^2 + a*e^2)) - (x^3*(a*e + c*d*x^2))/(4*c*(c*d^2 + a*e^2)*(a
 + c*x^4)) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 + a*e^2)
^2) + (a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) + (a^(1/4)*(Sqrt[c]*d - 3*Sq
rt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*c^(7/4)*(c*d^2
 + a*e^2)) - (a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) - (a^(1/4)*(Sqrt[c]*d
 - 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*c^(7/4)
*(c*d^2 + a*e^2)) + (a^(1/4)*d^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2)
 + (a^(1/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - (a^(1/4)*d^2*(Sqr
t[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) - (a^(1/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*c^(7/4
)*(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1276

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1)*(a*e - c*d*x^2))/(4*a
*c*(p + 1)), x] - Dist[f^2/(4*a*c*(p + 1)), Int[(f*x)^(m - 2)*(a + c*x^4)^(
p + 1)*(a*e*(m - 1) - c*d*(4*p + 4 + m + 1)*x^2), x], x] /; FreeQ[{a, c, d,
 e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || I
ntegerQ[m])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rule 1314

```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> -Dist[(a*f^4)/(c*d^2 + a*e^2), Int[(f*x)^(m - 4)*(d - e*x^2)*
(a + c*x^4)^p, x], x] + Dist[(d^2*f^4)/(c*d^2 + a*e^2), Int[((f*x)^(m - 4)*
(a + c*x^4)^(p + 1))/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && Lt
Q[p, -1] && GtQ[m, 2]
```

Rubi steps

$$\int \frac{x^8}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = -\frac{a\int \frac{x^4\left(d-ex^2\right)}{\left(a+cx^4\right)^2}\,dx}{cd^2+ae^2} + \frac{d^2\int \frac{x^4}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx}{cd^2+ae^2}$$

$$= -\frac{x^3\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\int \frac{x^2\left(-3ae-cdx^2\right)}{a+cx^4}\,dx}{4c\left(cd^2+ae^2\right)} + \frac{d^2\int \left(\frac{d^2}{\left(cd^2+ae^2\right)\left(d+ex^2\right)} - \frac{a\left(d-ex^2\right.}{\left(cd^2+ae^2\right)\left(a\right.}\right.}{cd^2+ae^2}$$

$$= \frac{dx}{4c\left(cd^2+ae^2\right)} - \frac{x^3\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\left(ad^2\right)\int \frac{d-ex^2}{a+cx^4}\,dx}{\left(cd^2+ae^2\right)^2} + \frac{d^4\int \frac{1}{d+ex^2}\,dx}{\left(cd^2+ae^2\right)^2} + \frac{\int}{4}$$

$$= \frac{dx}{4c\left(cd^2+ae^2\right)} - \frac{x^3\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)^2} - \frac{\left(ad^2\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\right)\int}{2c\left(cd^2+\right.}$$

$$= \frac{dx}{4c\left(cd^2+ae^2\right)} - \frac{x^3\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)^2} - \frac{\left(ad^2\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\right)\int}{4c\left(cd^2\right.}$$

$$= \frac{dx}{4c\left(cd^2+ae^2\right)} - \frac{x^3\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)^2} + \frac{\sqrt[4]{a}\,d^2\left(\sqrt{c}\,d+\sqrt{a}\right.}{4}$$

$$= \frac{dx}{4c\left(cd^2+ae^2\right)} - \frac{x^3\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)^2} + \frac{a^{3/4}d^2\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\mathrm{t}}{2\sqrt{2}\,c^{3/4}\left(\right.}$$

Mathematica [A] time = 0.30, size = 431, normalized size = 0.61

$$\frac{\frac{\sqrt{2}\sqrt[4]{a}\left(3a^{3/2}e^3+7\sqrt{a}\,cd^2e+a\sqrt{c}\,de^2+5c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{c^{7/4}} - \frac{\sqrt{2}\sqrt[4]{a}\left(3a^{3/2}e^3+7\sqrt{a}\,cd^2e+a\sqrt{c}\,de^2+5c^{3/2}d^3\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{c^{7/4}} - \frac{2\sqrt{2}\sqrt[4]{a}\left(3a^{3/2}e^3+7\sqrt{a}\,cd^2e-a\sqrt{c}\,de^2-5c^{3/2}d^3\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{c^{7/4}} + \frac{2\sqrt{2}\sqrt[4]{a}\left(3a^{3/2}e^3+7\sqrt{a}\,cd^2e-a\sqrt{c}\,de^2-5c^{3/2}d^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{c^{7/4}} + \frac{8ax\left(d-ex^2\right)\left(ae^2+cd^2\right)}{c\left(a+cx^4\right)} + \frac{32d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}}{32\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*a*(c*d^2 + a*e^2)*x*(d - e*x^2))/(c*(a + c*x^4)) + (32*d^(7/2)*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (2*Sqrt[2]*a^(1/4)*(-5*c^(3/2)*d^3 + 7*Sqrt[
a]*c*d^2*e - a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/c^(7/4) + (2*Sqrt[2]*a^(1/4)*(-5*c^(3/2)*d^3 + 7*Sqrt[a]*c*d^2*
e - a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/c^(7/4) + (Sqrt[2]*a^(1/4)*(5*c^(3/2)*d^3 + 7*Sqrt[a]*c*d^2*e + a*Sqrt[c
]*d*e^2 + 3*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/c^(7/4) - (Sqrt[2]*a^(1/4)*(5*c^(3/2)*d^3 + 7*Sqrt[a]*c*d^2*e + a*Sqr
t[c]*d*e^2 + 3*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2])/c^(7/4))/(32*(c*d^2 + a*e^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/((d + e*x^2)*(a + c*x^4)^2),x]
```

[Out] IntegrateAlgebraic[x^8/((d + e*x^2)*(a + c*x^4)^2), x]

fricas [B] time = 43.13, size = 9856, normalized size = 13.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(a*c*d^2*e + a^2*e^3)*x^3 - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*
c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt((70*a*c^2*d^5*e
 + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*
d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2
*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d
^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 +
 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5
*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))
/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c
^3*e^8))*log(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594
*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2
*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 + (7*c^10*d^10*e + 31*a
*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9
 + 3*a^5*c^5*e^11)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*
c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e
^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 +
56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*
c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))*sqrt((70*a*c^2*d^5*e +
44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4
*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^
5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*
e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28
*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^
10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c
^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*
e^8))) + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^
2*e^2 + a^2*c^2*e^4)*x^4)*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d
*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 +
 a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d
^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 +
 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^
3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d
^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 +
 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*log(-(625*c^4*d^8 -
750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*
x - (125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3
*e^6 - 9*a^4*c^2*d*e^8 + (7*c^10*d^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6
*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*sqrt(-(625
*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^
6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16
 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4
*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*
e^14 + a^8*c^7*e^16)))*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^
5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^
4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*
e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81
*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c
^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*
e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*
a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))) - (a*c^3*d^4 + 2*a^2*c
^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqr
```

```
t((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6
*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^
6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6
 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*
a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11
*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14
+ a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^
4*d^2*e^6 + a^4*c^3*e^8))*log(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*
c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (125*c^6*d^9 - 170*a*c^5*
d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 - (7*c
^10*d^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 1
9*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^
10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8
+ 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2
*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d
^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))*sqrt((
70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^
2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d
^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 +
2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c
^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^
8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a
^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d
^2*e^6 + a^4*c^3*e^8))) + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4
*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt((70*a*c^2*d^5*e + 44*a^2*c*
d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*
a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^
2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738
*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13
*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^
10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 +
4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*log
(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e
^6 - 81*a^4*e^8)*x - (125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4
 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 - (7*c^10*d^10*e + 31*a*c^9*d^8*e^3
 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5
*e^11)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4
+ 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7
*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*
d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12
 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3
*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3
*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 -
 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^
6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^
12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10
+ 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a
*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))) - 8*(
c^2*d^3*x^4 + a*c*d^3)*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2
 + d)) - 4*(a*c*d^3 + a^2*d*e^2)*x)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*
e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4), -1/16*(4*(a*c*d^2*e +
 a^2*e^3)*x^3 - 16*(c^2*d^3*x^4 + a*c*d^3)*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d
) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2
 + a^2*c^2*e^4)*x^4)*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5
+ (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*
c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^
4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a
^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^1
2*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^
```

```
12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^
2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*log(-(625*c^4*d^8 - 750*a
*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (
125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6
- 9*a^4*c^2*d*e^8 + (7*c^10*d^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5
+ 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*sqrt(-(625*a*c^
6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6
 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*
a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11
*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14
+ a^8*c^7*e^16)))*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (
c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3
*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 +
 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*
e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d
^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12
+ 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c
^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))) + (a*c^3*d^4 + 2*a^2*c^2*d^
2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt((70
*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2
+ 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^1
2 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 23
83*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^1
4*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*
e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8
*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2
*e^6 + a^4*c^3*e^8))*log(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d
^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*x - (125*c^6*d^9 - 170*a*c^5*d^7*e
^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 + (7*c^10*d
^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4
*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^
2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738
*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13
*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^
10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))*sqrt((70*a*
c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6
*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 -
 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*
a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d
^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8
 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^
7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^
6 + a^4*c^3*e^8))) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4
+ 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e
^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c
^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 5
29*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*
c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12
*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 +
28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c
^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*log(-(62
5*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 -
81*a^4*e^8)*x + (125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86
*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 - (7*c^10*d^10*e + 31*a*c^9*d^8*e^3 + 54
*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11
)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 274
8*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12
)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*
e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*
```

```
a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3
+ 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*
d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*
a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d
^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^
4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*
a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*
d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))) + (a*c^3*d
^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e
^4)*x^4)*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 +
 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqr
t(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4
*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^
15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 +
 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c
^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^
4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*log(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2
 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*x - (125*c^6*d^9
- 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*
d*e^8 - (7*c^10*d^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7
*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*sqrt(-(625*a*c^6*d^12 - 195
0*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*
c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*
e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 5
6*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^
16)))*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*
a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-
(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^
3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*
d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70
*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*
d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 +
 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))) - 4*(a*c*d^3 + a^2*d*e^2)*x)/(a*c^3*d^4
 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4
)*x^4)]
```

giac [A] time = 0.57, size = 581, normalized size = 0.82

$$\frac{d^{\frac{7}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{c^2d^4+2\,acd^2e^2+a^2e^4}-\frac{\left(5\,\left(ac^3\right)^{\frac{1}{4}}c^3d^3+\left(ac^3\right)^{\frac{1}{4}}ac^2de^2-7\,\left(ac^3\right)^{\frac{3}{4}}cd^2e-3\,\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,c^6d^4+2\,\sqrt{2}\,ac^5d^2e^2+\sqrt{2}\,a^2c^4e^4\right)}-\frac{\left(5\,\left(ac^3\right)^{\frac{1}{4}}c^3d^3+\left(ac^3\right)^{\frac{1}{4}}ac^2de^2-7\,\left(ac^3\right)^{\frac{3}{4}}cd^2e-3\,\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,c^6d^4+2\,\sqrt{2}\,ac^5d^2e^2+\sqrt{2}\,a^2c^4e^4\right)}-\frac{\left(5\,\left(ac^3\right)^{\frac{1}{4}}c^3d^3+\left(ac^3\right)^{\frac{1}{4}}ac^2de^2+7\,\left(ac^3\right)^{\frac{3}{4}}cd^2e+3\,\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}\,c^6d^4+2\,\sqrt{2}\,ac^5d^2e^2+\sqrt{2}\,a^2c^4e^4\right)}+\frac{\left(5\,\left(ac^3\right)^{\frac{1}{4}}c^3d^3+\left(ac^3\right)^{\frac{1}{4}}ac^2de^2+7\,\left(ac^3\right)^{\frac{3}{4}}cd^2e+3\,\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}\,c^6d^4+2\,\sqrt{2}\,ac^5d^2e^2+\sqrt{2}\,a^2c^4e^4\right)}-\frac{ax^3e-adx}{4\left(cx^4+a\right)\left(c^2d^2+ace^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] d^(7/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e
^4) - 1/8*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 - 7*(a*c^3)^
(3/4)*c*d^2*e - 3*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a
/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2*e^2 + sqrt(2
)*a^2*c^4*e^4) - 1/8*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 -
 7*(a*c^3)^(3/4)*c*d^2*e - 3*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x -
 sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2*e
^2 + sqrt(2)*a^2*c^4*e^4) - 1/16*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a
*c^2*d*e^2 + 7*(a*c^3)^(3/4)*c*d^2*e + 3*(a*c^3)^(3/4)*a*e^3)*log(x^2 + sqr
t(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2*e^2
+ sqrt(2)*a^2*c^4*e^4) + 1/16*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^
2*d*e^2 + 7*(a*c^3)^(3/4)*c*d^2*e + 3*(a*c^3)^(3/4)*a*e^3)*log(x^2 - sqrt(2
)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2*e^2 + s
qrt(2)*a^2*c^4*e^4) - 1/4*(a*x^3*e - a*d*x)/((c*x^4 + a)*(c^2*d^2 + a*c*e^2
))
```

maple [A] time = 0.02, size = 873, normalized size = 1.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4*a^2/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3/c*x^3-1/4*a/(a*e^2+c*d^2)^2/(c*x^4+a
)*e*x^3*d^2+1/4*a^2/(a*e^2+c*d^2)^2/(c*x^4+a)*d/c*x*e^2+1/4*a/(a*e^2+c*d^2)
^2/(c*x^4+a)*d^3*x-1/16*a/(a*e^2+c*d^2)^2/c*(a/c)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/c)^(1/4)*x-1)*d*e^2-5/16/(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/c)^(1/4)*x-1)*d^3-1/32*a/(a*e^2+c*d^2)^2/c*(a/c)^(1/4)*2^(1/2)*l
n((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^
(1/2)))*d*e^2-5/32/(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*
2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^3-1/16*a/
(a*e^2+c*d^2)^2/c*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^2
-5/16/(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d
^3+3/32*a^2/(a*e^2+c*d^2)^2/c^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(
1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*e^3+7/32*a/(a*
e^2+c*d^2)^2/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2
))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^2*e+3/16*a^2/(a*e^2+c*d^2)^2/
c^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^3+7/16*a/(a*e^2+c
*d^2)^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2*e+3/16*a^
2/(a*e^2+c*d^2)^2/c^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e
^3+7/16*a/(a*e^2+c*d^2)^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*
x+1)*d^2*e+d^4/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.06, size = 504, normalized size = 0.71

$$\frac{d^4 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(c^2d^4 + 2\,acd^2e^2 + a^2e^4\right)\sqrt{de}} - \frac{a\left(\frac{2\sqrt{2}\left(5c^{\frac{3}{2}}d^3 - 7\sqrt{a}cd^2e + a\sqrt{c}de^2 - 3a^{\frac{3}{2}}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\left(5c^{\frac{3}{2}}d^3 - 7\sqrt{a}cd^2e + a\sqrt{c}de^2 - 3a^{\frac{3}{2}}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{\sqrt{2}\left(5c^{\frac{3}{2}}d^3 + 7\sqrt{a}cd^2e + a\sqrt{c}de^2 + 3a^{\frac{3}{2}}e^3\right)\log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}\left(5c^{\frac{3}{2}}d^3 + 7\sqrt{a}cd^2e + a\sqrt{c}de^2 + 3a^{\frac{3}{2}}e^3\right)\log\left(\sqrt{c}x^2 - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{32\left(c^3d^4 + 2\,ac^2d^2e^2 + a^2ce^4\right)} - \frac{\left(acd^2e + a^2e^3\right)x^3 - \left(acd^3 + a^2de^2\right)x}{4\left(ac^3d^4 + 2\,a^2c^2d^2e^2 + a^3ce^4 + \left(c^4d^4 + 2\,ac^3d^2e^2 + a^2c^2e^4\right)x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] d^4*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e)) -
 1/32*a*(2*sqrt(2)*(5*c^(3/2)*d^3 - 7*sqrt(a)*c*d^2*e + a*sqrt(c)*d*e^2 - 3
*a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sq
rt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(5
*c^(3/2)*d^3 - 7*sqrt(a)*c*d^2*e + a*sqrt(c)*d*e^2 - 3*a^(3/2)*e^3)*arctan(
1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/
(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(5*c^(3/2)*d^3 + 7*sqrt(a
)*c*d^2*e + a*sqrt(c)*d*e^2 + 3*a^(3/2)*e^3)*log(sqrt(c)*x^2 + sqrt(2)*a^(1
/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(5*c^(3/2)*d^3 + 7*sqr
t(a)*c*d^2*e + a*sqrt(c)*d*e^2 + 3*a^(3/2)*e^3)*log(sqrt(c)*x^2 - sqrt(2)*a
^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 +
 a^2*c*e^4) - 1/4*((a*c*d^2*e + a^2*e^3)*x^3 - (a*c*d^3 + a^2*d*e^2)*x)/(a*
c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*
c^2*e^4)*x^4)
```

mupad [B] time = 2.86, size = 18343, normalized size = 25.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((a*d*x)/(4*c*(a*e^2 + c*d^2)) - (a*e*x^3)/(4*c*(a*e^2 + c*d^2)))/(a + c*x^
4) + atan(((((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d*e^13 - 17232*a^3*c^7*d^11
*e^3 - 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e^7 + 4320*a^6*c^4*d^5*e^9
 + 2928*a^7*c^3*d^3*e^11)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6
*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((81920*a^5*c^9*d^8*e^8 - 73728*a
```

^3*c^11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480*a^2*c^12*d^14*e^2 + 1843
20*a^6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 28672*a^8*c^6*d^2*e^14)/(25
6*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*
d^2*e^6)) - (x*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a
^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c
^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 +
4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*(65536*a
^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a
^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589
824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(128*(c^7*d^8 + a^4*c^3*e^
8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6
*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d
^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^
4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c
^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(1920*a^8*c^4*d*e^14 + 13184*a
^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a
^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5*d^3*e^12))/(128*(c^
7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e
^6)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*
e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2)
- 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10
*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2))*((25*c^3*d^6*(-a
*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e
^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-
a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d
^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e
+ 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^
5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^
6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^
(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 7
0*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)
^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4
+ 4*a^3*c^8*d^2*e^6)))^(1/2)*1i - (((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d*e
^13 - 17232*a^3*c^7*d^11*e^3 - 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e^
7 + 4320*a^6*c^4*d^5*e^9 + 2928*a^7*c^3*d^3*e^11)/(256*(c^7*d^8 + a^4*c^3*e
^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((81920*a
^5*c^9*d^8*e^8 - 73728*a^3*c^11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480*
a^2*c^12*d^14*e^2 + 184320*a^6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 286
72*a^8*c^6*d^2*e^14)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*
c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) + (x*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*
e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e
- 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(
c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d
^2*e^6)))^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*
c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680
*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(1
28*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4
*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*
c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)
^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*
a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) - (x*(1920*
a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a
^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7
*c^5*d^3*e^12))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d
^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c
^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^
2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8
+ a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))
^(1/2))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*

d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/
2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^
10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) - (x*(81*a^8*e^
13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a
^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(128*(c^7*d^
8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6))
)*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5
+ 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 4
1*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6
*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*1i)/((81*a^6*d^4*e^8
+ 450*a^5*c*d^6*e^6 + 300*a^3*c^3*d^10*e^2 + 733*a^4*c^2*d^8*e^4)/(128*(c^7
*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^
6)) + (((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d*e^13 - 17232*a^3*c^7*d^11*e^3
- 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e^7 + 4320*a^6*c^4*d^5*e^9 + 29
28*a^7*c^3*d^3*e^11)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*
c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((81920*a^5*c^9*d^8*e^8 - 73728*a^3*c^
11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480*a^2*c^12*d^14*e^2 + 184320*a^
6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 28672*a^8*c^6*d^2*e^14)/(256*(c^
7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e
^6)) - (x*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^
4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(
1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*
c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*(65536*a^9*c^
7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^
12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a
^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4
*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*
c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^
3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a
c^7)^(1/2))/(256(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^
4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(1920*a^8*c^4*d*e^14 + 13184*a^2*c^
10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*c^
7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5*d^3*e^12))/(128*(c^7*d^8
 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))
*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 +
 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41
*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*
e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2))*((25*c^3*d^6*(-a*c^7)
^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 +
70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7
)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^
4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e + 612
*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^5*c^3
*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6
*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^(1/2)
 - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c
^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2
))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*
a^3*c^8*d^2*e^6)))^(1/2) + (((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d*e^13 - 17
232*a^3*c^7*d^11*e^3 - 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e^7 + 4320
*a^6*c^4*d^5*e^9 + 2928*a^7*c^3*d^3*e^11)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a
*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((81920*a^5*c^9*d
^8*e^8 - 73728*a^3*c^11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480*a^2*c^12
*d^14*e^2 + 184320*a^6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 28672*a^8*c
^6*d^2*e^14)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*
e^4 + 4*a^3*c^4*d^2*e^6)) + (x*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*
c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c
^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8
 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6))

)^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^1
2*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^1
0*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(128*(c^7*
d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6
)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^
5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) -
41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d
^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) - (x*(1920*a^8*c^4*
d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c^8*d
^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5*d^3
e^12))/(128(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 +
4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2
) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^
2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^
7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2))*
((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 +
44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*
a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e
^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) - (x*(81*a^8*e^13 + 800
*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d
^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(128*(c^7*d^8 + a^4*
c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c
^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2
*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*
d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6
*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)))*((25*c^3*d^6*(-a*c^7)^(1/2)
- 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c
^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2
))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*
a^3*c^8*d^2*e^6)))^(1/2)*2i + atan(((((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d*
e^13 - 17232*a^3*c^7*d^11*e^3 - 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e
^7 + 4320*a^6*c^4*d^5*e^9 + 2928*a^7*c^3*d^3*e^11)/(256*(c^7*d^8 + a^4*c^3*
e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((81920*
a^5*c^9*d^8*e^8 - 73728*a^3*c^11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480
*a^2*c^12*d^14*e^2 + 184320*a^6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 28
672*a^8*c^6*d^2*e^14)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2
*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (x*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3
d^6(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e
+ 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*
(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*
d^2*e^6)))^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3
*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 32768
0*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(
128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^
4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3
*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7
)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4
*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(1920
*a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*
a^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^
7*c^5*d^3*e^12))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*
d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*
c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c
^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8
+ a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6))
)^(1/2))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4
*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1
/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c
^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(81*a^8*e

^13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*
a^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(128*(c^7*d
^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)
))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5
 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) +
41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^
6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*1i - (((5120*a^2*c^8
*d^13*e + 432*a^8*c^2*d*e^13 - 17232*a^3*c^7*d^11*e^3 - 37776*a^4*c^6*d^9*e
^5 - 13600*a^5*c^5*d^7*e^7 + 4320*a^6*c^4*d^5*e^9 + 2928*a^7*c^3*d^3*e^11)/
(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c
^4*d^2*e^6)) - (((81920*a^5*c^9*d^8*e^8 - 73728*a^3*c^11*d^12*e^4 - 61440*a
^4*c^10*d^10*e^6 - 20480*a^2*c^12*d^14*e^2 + 184320*a^6*c^8*d^6*e^10 + 1228
80*a^7*c^7*d^4*e^12 + 28672*a^8*c^6*d^2*e^14)/(256*(c^7*d^8 + a^4*c^3*e^8 +
 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) + (x*((9*a^3*e^6
*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*
d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e
^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*
c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^
14*d^14*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*
a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327
680*a^8*c^8*d^2*e^15))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^
2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^
6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e +
39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^
11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2
e^6)))^(1/2) - (x(1920*a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a
^3*c^9*d^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6
*c^6*d^5*e^10 + 20736*a^7*c^5*d^3*e^12))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*
c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)
^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 +
 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^
7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e
^4 + 4*a^3*c^8*d^2*e^6)))^(1/2))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-
a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a
*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d
^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6
)))^(1/2) - (x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832
*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c
^2*d^4*e^9))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*
e^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)
^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d
^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a
^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1
/2)*1i)/((81*a^6*d^4*e^8 + 450*a^5*c*d^6*e^6 + 300*a^3*c^3*d^10*e^2 + 733*a
^4*c^2*d^8*e^4)/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d
^4*e^4 + 4*a^3*c^4*d^2*e^6)) + (((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d*e^13
- 17232*a^3*c^7*d^11*e^3 - 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e^7 +
4320*a^6*c^4*d^5*e^9 + 2928*a^7*c^3*d^3*e^11)/(256*(c^7*d^8 + a^4*c^3*e^8 +
 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((81920*a^5*c
^9*d^8*e^8 - 73728*a^3*c^11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480*a^2*
c^12*d^14*e^2 + 184320*a^6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 28672*a
^8*c^6*d^2*e^14)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*
d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (x*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*
(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39
*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11
*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e
^6)))^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13
*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6
*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(128*(

c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2
e^6)))((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*
d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/
2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^
10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(1920*a^8*
c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c
^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5
*d^3*e^12))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e
^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^
(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^
4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^
4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/
2))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^
5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) +
 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d
^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(81*a^8*e^13 +
 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c
^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(128*(c^7*d^8 +
a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((
9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44
*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^
2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2
 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (((5120*a^2*c^8*d^13*e
+ 432*a^8*c^2*d*e^13 - 17232*a^3*c^7*d^11*e^3 - 37776*a^4*c^6*d^9*e^5 - 136
00*a^5*c^5*d^7*e^7 + 4320*a^6*c^4*d^5*e^9 + 2928*a^7*c^3*d^3*e^11)/(256*(c^
7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e
^6)) - (((81920*a^5*c^9*d^8*e^8 - 73728*a^3*c^11*d^12*e^4 - 61440*a^4*c^10*
d^10*e^6 - 20480*a^2*c^12*d^14*e^2 + 184320*a^6*c^8*d^6*e^10 + 122880*a^7*c
^7*d^4*e^12 + 28672*a^8*c^6*d^2*e^14)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6
*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) + (x*((9*a^3*e^6*(-a*c^7
)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3
+ 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c
^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*
e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*
e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11
*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*
c^8*d^2*e^15))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^
4*e^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^
7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2
*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 +
 a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^
(1/2) - (x*(1920*a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d
^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5
*e^10 + 20736*a^7*c^5*d^3*e^12))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*
e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) -
 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^
6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2)
)/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a
^3*c^8*d^2*e^6)))^(1/2))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(
1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4
e^2(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4
*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2
) - (x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5
*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e
^9))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*
a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) +
 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(
-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e
^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)))*((

9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*2i + (atan(((((x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) + (((20*a^2*c^8*d^13*e + (27*a^8*c^2*d*e^13)/16 - (1077*a^3*c^7*d^11*e^3)/16 - (2361*a^4*c^6*d^9*e^5)/16 - (425*a^5*c^5*d^7*e^7)/8 + (135*a^6*c^4*d^5*e^9)/8 + (183*a^7*c^3*d^3*e^11)/16)/(2*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - ((-d^7*e)^(1/2)*(((-d^7*e)^(1/2)*((320*a^5*c^9*d^8*e^8 - 288*a^3*c^11*d^12*e^4 - 240*a^4*c^10*d^10*e^6 - 80*a^2*c^12*d^14*e^2 + 720*a^6*c^8*d^6*e^10 + 480*a^7*c^7*d^4*e^12 + 112*a^8*c^6*d^2*e^14)/(2*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (x*(-d^7*e)^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(512*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6))))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)) + (x*(1920*a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5*d^3*e^12))/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6))))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)))*(-d^7*e)^(1/2))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)))*(-d^7*e)^(1/2)*1i)/(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3) + (((x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((20*a^2*c^8*d^13*e + (27*a^8*c^2*d*e^13)/16 - (1077*a^3*c^7*d^11*e^3)/16 - (2361*a^4*c^6*d^9*e^5)/16 - (425*a^5*c^5*d^7*e^7)/8 + (135*a^6*c^4*d^5*e^9)/8 + (183*a^7*c^3*d^3*e^11)/16)/(2*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - ((-d^7*e)^(1/2)*(((-d^7*e)^(1/2)*((320*a^5*c^9*d^8*e^8 - 288*a^3*c^11*d^12*e^4 - 240*a^4*c^10*d^10*e^6 - 80*a^2*c^12*d^14*e^2 + 720*a^6*c^8*d^6*e^10 + 480*a^7*c^7*d^4*e^12 + 112*a^8*c^6*d^2*e^14)/(2*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) + (x*(-d^7*e)^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(512*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6))))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)) - (x*(1920*a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5*d^3*e^12))/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6))))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)))*(-d^7*e)^(1/2))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)))*(-d^7*e)^(1/2)*1i)/(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3))/(((81*a^6*d^4*e^8)/128 + (225*a^5*c*d^6*e^6)/64 + (75*a^3*c^3*d^10*e^2)/32 + (733*a^4*c^2*d^8*e^4)/128)/(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6) + (((x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) + (((20*a^2*c^8*d^13*e + (27*a^8*c^2*d*e^13)/16 - (1077*a^3*c^7*d^11*e^3)/16 - (2361*a^4*c^6*d^9*e^5)/16 - (425*a^5*c^5*d^7*e^7)/8 + (135*a^6*c^4*d^5*e^9)/8 + (183*a^7*c^3*d^3*e^11)/16)/(2*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - ((-d^7*e)^(1/2)*(((-d^7*e)^(1/2)*((320*a^5*c^9*d^8*e^8 - 288*a^3*c^11*d^12*e^4 - 240*a^4*c^10*d^10*e^6 - 80*a^2*c^12*d^14*e^2 + 720*a^6*c^8*d^6*e^10 + 480*a^7*c^7*d^4*e^12 + 112*a^8*c^6*d^2*e^

```
14)/(2*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3
*c^4*d^2*e^6)) - (x*(-d^7*e)^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^1
4*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^
11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^
8*c^8*d^2*e^15))/(512*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)*(c^7*d^8 + a^4*
c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6))))/(2*(a
^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)) + (x*(1920*a^8*c^4*d*e^14 + 13184*a^2*
c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*
c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5*d^3*e^12))/(256*(c^7*d
^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)
)))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)))*(-d^7*e)^(1/2))/(2*(a^2*e^5
+ c^2*d^4*e + 2*a*c*d^2*e^3)))*(-d^7*e)^(1/2))/(a^2*e^5 + c^2*d^4*e + 2*a*c
*d^2*e^3) - (((x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 8
32*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6
*c^2*d^4*e^9))/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^
4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((20*a^2*c^8*d^13*e + (27*a^8*c^2*d*e^13)/16
 - (1077*a^3*c^7*d^11*e^3)/16 - (2361*a^4*c^6*d^9*e^5)/16 - (425*a^5*c^5*d^
7*e^7)/8 + (135*a^6*c^4*d^5*e^9)/8 + (183*a^7*c^3*d^3*e^11)/16)/(2*(c^7*d^8
 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6))
- ((-d^7*e)^(1/2)*(((-d^7*e)^(1/2)*((320*a^5*c^9*d^8*e^8 - 288*a^3*c^11*d^1
2*e^4 - 240*a^4*c^10*d^10*e^6 - 80*a^2*c^12*d^14*e^2 + 720*a^6*c^8*d^6*e^10
 + 480*a^7*c^7*d^4*e^12 + 112*a^8*c^6*d^2*e^14)/(2*(c^7*d^8 + a^4*c^3*e^8 +
 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) + (x*(-d^7*e)^(1
/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^12*e^
5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^
6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(512*(a^2*e^5
+ c^2*d^4*e + 2*a*c*d^2*e^3)*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a
^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6))))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*
e^3)) - (x*(1920*a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d
^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5
*e^10 + 20736*a^7*c^5*d^3*e^12))/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*
e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6))))/(2*(a^2*e^5 + c^2*d^4*e + 2
*a*c*d^2*e^3)))*(-d^7*e)^(1/2))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)))*
(-d^7*e)^(1/2))/(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)))*(-d^7*e)^(1/2)*1i)/
(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.183 $\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=687

$$-\frac{\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\sqrt[4]{a}\,c^{5/4}\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\sqrt[4]{a}\,c^{5/4}\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right.}{8\sqrt{2}\sqrt[4]{a}}$$

Rubi [A] time = 0.60, antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1314, 1276, 1168, 1162, 617, 204, 1165, 628, 1288, 205}

$$-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}\sqrt[4]{a}c^{5/4}\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}\sqrt[4]{a}c^{5/4}\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\sqrt[4]{a}c^{5/4}\left(ae^2+cd^2\right)}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\sqrt[4]{a}c^{5/4}\left(ae^2+cd^2\right)}-\frac{x\left(ae+cdx^2\right)}{4c\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{d^2\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)^2}-\frac{d^2\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)^2}-\frac{d^{5/2}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(ae^2+cd^2\right)^2}-\frac{d^2\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)^2}+\frac{d^2\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -(x*(a*e + c*d*x^2))/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) - (d^(5/2)*Sqrt[e]*A
rcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 - (d^2*(Sqrt[c]*d + Sqrt[a]*e
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^
2 + a*e^2)^2) + ((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(8*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2)) + (d^2*(Sqrt[c]*d + Sqrt
[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*
(c*d^2 + a*e^2)^2) - ((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/(8*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2)) + (d^2*(Sqrt[c]*d -
 Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt
[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)^2) - ((Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(1/4)*c^(5/4)*
(c*d^2 + a*e^2)) - (d^2*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)^2)
+ ((Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c
]*x^2])/(16*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1276

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1)*(a*e - c*d*x^2))/(4*a
*c*(p + 1)), x] - Dist[f^2/(4*a*c*(p + 1)), Int[(f*x)^(m - 2)*(a + c*x^4)^(
p + 1)*(a*e*(m - 1) - c*d*(4*p + 4 + m + 1)*x^2), x], x] /; FreeQ[{a, c, d,
 e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || I
ntegerQ[m])
```

Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rule 1314

```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> -Dist[(a*f^4)/(c*d^2 + a*e^2), Int[(f*x)^(m - 4)*(d - e*x^2)*
(a + c*x^4)^p, x], x] + Dist[(d^2*f^4)/(c*d^2 + a*e^2), Int[((f*x)^(m - 4)*
(a + c*x^4)^(p + 1))/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && Lt
Q[p, -1] && GtQ[m, 2]
```

Rubi steps

$$\int \frac{x^6}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = -\frac{a\int \frac{x^2\left(d-ex^2\right)}{\left(a+cx^4\right)^2}\,dx}{cd^2+ae^2} + \frac{d^2\int \frac{x^2}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx}{cd^2+ae^2}$$

$$= -\frac{x\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\int \frac{-ae+cdx^2}{a+cx^4}\,dx}{4c\left(cd^2+ae^2\right)} + \frac{d^2\int \left(-\frac{de}{\left(cd^2+ae^2\right)\left(d+ex^2\right)} + \frac{ae+cdx^2}{\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right.}{cd^2+ae^2}$$

$$= -\frac{x\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^2\int \frac{ae+cdx^2}{a+cx^4}\,dx}{\left(cd^2+ae^2\right)^2} - \frac{\left(d^3e\right)\int \frac{1}{d+ex^2}\,dx}{\left(cd^2+ae^2\right)^2} - \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{\sqrt{a}\,\sqrt{c}+}{a+cx}}{8c\left(cd^2+ae^2\right.}$$

$$= -\frac{x\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{d^{5/2}\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} - \frac{\left(d^2\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)^2} +$$

$$= -\frac{x\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{d^{5/2}\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} - \frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\right.}{16\sqrt{2}\,\sqrt[4]{a}\,c^{5/4}\left(cd^2+a\right.}$$

$$= -\frac{x\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{d^{5/2}\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} + \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,\sqrt[4]{a}\,c^{3/4}\left(cd^2+ae^2\right)} -$$

$$= -\frac{x\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{d^{5/2}\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}\,d^2\left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}}{\sqrt[4]{a}}\right.}{2\sqrt{2}\,\sqrt[4]{a}\,\left(cd^2+ae^2\right)^2}$$

Mathematica [A] time = 0.37, size = 428, normalized size = 0.62

$$-\frac{\frac{\sqrt{2}\left(a^{3/2}e^3+5\sqrt{a}\,cd^2e+a\sqrt{c}\,de^2-3c^{3/2}d^3\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{\sqrt[4]{a}\,c^{5/4}} - \frac{\sqrt{2}\left(a^{3/2}e^3+5\sqrt{a}\,cd^2e+a\sqrt{c}\,de^2-3c^{3/2}d^3\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{\sqrt[4]{a}\,c^{5/4}} + \frac{2\sqrt{2}\left(a^{3/2}e^3+5\sqrt{a}\,cd^2e-a\sqrt{c}\,de^2+3c^{3/2}d^3\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,c^{5/4}} - \frac{2\sqrt{2}\left(a^{3/2}e^3+5\sqrt{a}\,cd^2e-a\sqrt{c}\,de^2+3c^{3/2}d^3\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{a}\,c^{5/4}} + \frac{8\left(ae^2+cd^2\right)\left(aex+cdx^3\right)}{c\left(a+cx^4\right)} + 32d^{5/2}\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{32\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/32*((8*(c*d^2 + a*e^2)*(a*e*x + c*d*x^3))/(c*(a + c*x^4)) + 32*d^(5/2)*S
qrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + (2*Sqrt[2]*(3*c^(3/2)*d^3 + 5*Sqrt[a]*
c*d^2*e - a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(a^(1/4)*c^(5/4)) - (2*Sqrt[2]*(3*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e -
a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^
(1/4)*c^(5/4)) + (Sqrt[2]*(-3*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + a*Sqrt[c]*d
*e^2 + a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(a^(1/4)*c^(5/4)) - (Sqrt[2]*(-3*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + a*Sqrt[
c]*d*e^2 + a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/(a^(1/4)*c^(5/4)))/(c*d^2 + a*e^2)^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[x^6/((d + e*x^2)*(a + c*x^4)^2), x]
```

fricas [B] time = 27.28, size = 9822, normalized size = 14.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(c^2*d^3 + a*c*d*e^2)*x^3 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*
c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e
- 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*
e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*
e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*
a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11
*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10
 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*
a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-
(81*c^4*d^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 -
a^4*e^8)*x + (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 1
4*a^4*c^2*d^2*e^7 - a^5*c*e^9 - (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3
*c^7*d^7*e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-
(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e
^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a
^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9
*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 +
 a^9*c^5*e^16)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d
^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)
*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^
3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^1
6 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*
a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2
*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*
a^3*c^3*d^2*e^6 + a^4*c^2*e^8))) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e
^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4
*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4
 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2
 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5
*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^
12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 +
28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c
^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81
*c^4*d^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4
*e^8)*x - (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a
^4*c^2*d^2*e^7 - a^5*c*e^9 - (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^
7*d^7*e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81
*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6
+ 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*
c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^
8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^
9*c^5*e^16)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8
+ 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sq
rt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d
^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 +
 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5
*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^
14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3
*c^3*d^2*e^6 + a^4*c^2*e^8))) + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4
+ (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4*a*
```

```
c*d^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 +
4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 +
799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*
d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*
e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*
a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*
d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81*c^
4*d^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^
8)*x + (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*
c^2*d^2*e^7 - a^5*c*e^9 + (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d
^7*e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^
6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 1
43*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^1
2*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e
^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c
^5*e^16)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4
*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(
-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*
e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*
a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^
9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14
 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^
3*d^2*e^6 + a^4*c^2*e^8))) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (
c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4*a*c*d
^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a
^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799
*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2
*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4
 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7
*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6
*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81*c^4*d
^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*
x - (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2
*d^2*e^7 - a^5*c*e^9 + (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*
e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^6*d
^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*
a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d
^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8
+ 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*
e^16)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4*a*
c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(8
1*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6
 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2
*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d
^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a
^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d
^2*e^6 + a^4*c^2*e^8))) - 8*(c^2*d^2*x^4 + a*c*d^2)*sqrt(-d*e)*log((e*x^2 -
 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 4*(a*c*d^2*e + a^2*e^3)*x)/(a*c^3*d^4 +
 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*
x^4), -1/16*(4*(c^2*d^3 + a*c*d*e^2)*x^3 + 16*(c^2*d^2*x^4 + a*c*d^2)*sqrt(
d*e)*arctan(sqrt(d*e)*x/d) + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (
c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4*a*c*d
^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a
^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799
*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2
*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4
 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7
*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6
*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81*c^4*d
```

^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*
x + (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2
*d^2*e^7 - a^5*c*e^9 - (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*
e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^6*d
^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*
a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d
^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8
+ 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*
e^16)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*
c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(8
1*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6
 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2
*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d
^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a
^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d
^2*e^6 + a^4*c^2*e^8))) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4
*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*
e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*
c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^
2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^
10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 +
56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^
7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^
2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81*c^4*d^8
- 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*x -
 (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2*d^
2*e^7 - a^5*c*e^9 - (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*e^4
 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^6*d^12
 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4
*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14
*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 5
6*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^1
6)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5
*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c
^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 +
143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^
12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*
e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*
c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*
e^6 + a^4*c^2*e^8))) + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^
4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3
 - 2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3
*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c
^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10
+ a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*
a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d
^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 +
 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81*c^4*d^8 - 2
70*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*x + (4
5*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2*d^2*e
^7 - a^5*c*e^9 + (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*e^4 +
6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^6*d^12 -
558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^
2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^
2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a
^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16))
)*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^5*d^
6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*
d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143

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*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*
d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8
 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5
*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6
 + a^4*c^2*e^8))) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 +
 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 -
2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^
2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*
d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a
^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4
*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*
e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*
a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81*c^4*d^8 - 270*
a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*x - (45*a
*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2*d^2*e^7
- a^5*c*e^9 + (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*e^4 + 6*a
^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^6*d^12 - 558
*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d
^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 +
 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*
c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))*s
qrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^5*d^6*e
^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^1
2 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^
4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^1
4*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 +
56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^
16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 +
a^4*c^2*e^8))) + 4*(a*c*d^2*e + a^2*e^3)*x)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2
+ a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)]
```

giac [A] time = 0.59, size = 595, normalized size = 0.87

$$-\frac{d^{\frac{5}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{c^2d^4+2\,acd^2e^2+a^2e^4}+\frac{\left(5\,(ac^3)^{\frac{1}{4}}ac^2d^2e+3\,(ac^3)^{\frac{3}{4}}cd^3+(ac^3)^{\frac{1}{4}}a^2ce^3-(ac^3)^{\frac{3}{4}}ade^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,ac^5d^4+2\,\sqrt{2}\,a^2c^4d^2e^2+\sqrt{2}\,a^3c^3e^4\right)}+\frac{\left(5\,(ac^3)^{\frac{1}{4}}ac^2d^2e+3\,(ac^3)^{\frac{3}{4}}cd^3+(ac^3)^{\frac{1}{4}}a^2ce^3-(ac^3)^{\frac{3}{4}}ade^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,ac^5d^4+2\,\sqrt{2}\,a^2c^4d^2e^2+\sqrt{2}\,a^3c^3e^4\right)}+\frac{\left(5\,(ac^3)^{\frac{1}{4}}ac^2d^2e-3\,(ac^3)^{\frac{3}{4}}cd^3+(ac^3)^{\frac{1}{4}}a^2ce^3+(ac^3)^{\frac{3}{4}}ade^2\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}\,ac^5d^4+2\,\sqrt{2}\,a^2c^4d^2e^2+\sqrt{2}\,a^3c^3e^4\right)}-\frac{\left(5\,(ac^3)^{\frac{1}{4}}ac^2d^2e-3\,(ac^3)^{\frac{3}{4}}cd^3+(ac^3)^{\frac{1}{4}}a^2ce^3+(ac^3)^{\frac{3}{4}}ade^2\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}\,ac^5d^4+2\,\sqrt{2}\,a^2c^4d^2e^2+\sqrt{2}\,a^3c^3e^4\right)}-\frac{cdx^3+axe}{4\,(cx^4+a)\left(c^2d^2+ace^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -d^(5/2)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e
^4) + 1/8*(5*(a*c^3)^(1/4)*a*c^2*d^2*e + 3*(a*c^3)^(3/4)*c*d^3 + (a*c^3)^(1
/4)*a^2*c*e^3 - (a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a
/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2*e^2 + sq
rt(2)*a^3*c^3*e^4) + 1/8*(5*(a*c^3)^(1/4)*a*c^2*d^2*e + 3*(a*c^3)^(3/4)*c*d
^3 + (a*c^3)^(1/4)*a^2*c*e^3 - (a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c
^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) + 1/16*(5*(a*c^3)^(1/4)*a*c^2*d^2*e - 3*(
a*c^3)^(3/4)*c*d^3 + (a*c^3)^(1/4)*a^2*c*e^3 + (a*c^3)^(3/4)*a*d*e^2)*log(x
^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*
c^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) - 1/16*(5*(a*c^3)^(1/4)*a*c^2*d^2*e - 3*
(a*c^3)^(3/4)*c*d^3 + (a*c^3)^(1/4)*a^2*c*e^3 + (a*c^3)^(3/4)*a*d*e^2)*log(
x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2
*c^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) - 1/4*(c*d*x^3 + a*x*e)/((c*x^4 + a)*(c
^2*d^2 + a*c*e^2))
```

maple [A] time = 0.02, size = 852, normalized size = 1.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^3*a*d*e^2-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^
3*c*d^3-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a^2*e^3/c*x-1/4/(a*e^2+c*d^2)^2/(c*x^
4+a)*e*a*d^2*x+1/16/(a*e^2+c*d^2)^2/c*(a/c)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/
(a/c)^(1/4)*x-1)*e^3+5/16/(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(a/c)^(1/4)*x-1)*d^2*e+1/32/(a*e^2+c*d^2)^2/c*(a/c)^(1/4)*a*2^(1/2)*ln((x
^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2
)))*e^3+5/32/(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2
)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^2*e+1/16/(a*e^2
+c*d^2)^2/c*(a/c)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e^3+5/16/
(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^2*e-1
/32/(a*e^2+c*d^2)^2/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/
c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*a*d*e^2+3/32/(a*e^2+c*d^
2)^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a
/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^3-1/16/(a*e^2+c*d^2)^2/c/(a/c)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*a*d*e^2+3/16/(a*e^2+c*d^2)^2/(a/c)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^3-1/16/(a*e^2+c*d^2)^2/c/(a/c
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*a*d*e^2+3/16/(a*e^2+c*d^2)^
2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^3-d^3*e/(a*e^2+c*d^
2)^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.08, size = 476, normalized size = 0.69

$$-\frac{d^3 e \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\sqrt{de}}-\frac{cdx^3+aex}{4\left(ac^2d^2+a^2ce^2+\left(c^3d^2+ac^2e^2\right)x^4\right)}+\frac{\frac{2\sqrt{2}\left(3\sqrt{a}c^2d^3+5\,ac^{\frac{3}{2}}d^2e-a^{\frac{3}{2}}cde^2+a^2\sqrt{c}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(3\sqrt{a}c^2d^3+5\,ac^{\frac{3}{2}}d^2e-a^{\frac{3}{2}}cde^2+a^2\sqrt{c}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\left(3\sqrt{a}c^2d^3-5\,ac^{\frac{3}{2}}d^2e-a^{\frac{3}{2}}cde^2-a^2\sqrt{c}e^3\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\left(3\sqrt{a}c^2d^3-5\,ac^{\frac{3}{2}}d^2e-a^{\frac{3}{2}}cde^2-a^2\sqrt{c}e^3\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}}{32\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -d^3*e*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e)
) - 1/4*(c*d*x^3 + a*e*x)/(a*c^2*d^2 + a^2*c*e^2 + (c^3*d^2 + a*c^2*e^2)*x^
4) + 1/32*(2*sqrt(2)*(3*sqrt(a)*c^2*d^3 + 5*a*c^(3/2)*d^2*e - a^(3/2)*c*d*e
^2 + a^2*sqrt(c)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(
1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sq
rt(2)*(3*sqrt(a)*c^2*d^3 + 5*a*c^(3/2)*d^2*e - a^(3/2)*c*d*e^2 + a^2*sqrt(c
)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt
(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) - sqrt(2)*(3*sqrt(a)*
c^2*d^3 - 5*a*c^(3/2)*d^2*e - a^(3/2)*c*d*e^2 - a^2*sqrt(c)*e^3)*log(sqrt(c
)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + sqrt(2)*(3
*sqrt(a)*c^2*d^3 - 5*a*c^(3/2)*d^2*e - a^(3/2)*c*d*e^2 - a^2*sqrt(c)*e^3)*l
og(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(c
^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)
```

mupad [B] time = 2.82, size = 17909, normalized size = 26.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] atan(((((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d^10*e^4 + 12000*a^3*c^5*d^8*e^
6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^10 + 48*a^6*c^2*d^2*e^12)/(256
*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2
*e^6)) + (((45056*a^2*c^10*d^13*e^3 - 4096*a^8*c^4*d*e^15 + 221184*a^3*c^9*
d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6
*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6
*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) - (x*(-(a^3*e^6*(-a*c^5)^(1/
2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*
c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2
))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 +
4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 -
327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^
```

9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*
e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*
a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2
*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*
c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8
+ 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (x*(
1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*
a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c
^4*d^3*e^12))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e
^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1
/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e
^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c
^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)
)*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 -
4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^
2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e
^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) - (x*(a^6*e^13 - 288*a*
c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7
 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^
2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*
(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*
a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*
d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e
^6)))^(1/2)*1i - (((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d^10*e^4 + 12000*a^3
*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^10 + 48*a^6*c^2*d^2
e^12)/(256(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*
a^3*c^2*d^2*e^6)) + (((45056*a^2*c^10*d^13*e^3 - 4096*a^8*c^4*d*e^15 + 2211
84*a^3*c^9*d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184
320*a^6*c^6*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(256*(c^5*d^8 + a^4*c*e^8 +
4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (x*(-(a^3*e^6*(
-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*
e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-
a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7
*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*
d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5
*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a
^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d
^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5
)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d
^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a
^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(
1/2) - (x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e
^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 +
 4864*a^6*c^4*d^3*e^12))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^
2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*
(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*
a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*
d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e
^6)))^(1/2))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c
^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(
1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^
2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (x*(a^6*e^
13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*
c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6
*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) -
 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*
d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(
256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^
4*c^6*d^2*e^6)))^(1/2)*1i)/((((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d^10*e^4

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+ 12000*a^3*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^10 + 48*
a^6*c^2*d^2*e^12)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d
^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((45056*a^2*c^10*d^13*e^3 - 4096*a^8*c^4*d*
e^15 + 221184*a^3*c^9*d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^
7*e^9 + 184320*a^6*c^6*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(256*(c^5*d^8 + a
^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) - (x*(
-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a
^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c
*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2
+ 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c^5*e^17 - 6553
6*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 -
 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13
 + 327680*a^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 +
6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*
d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e +
 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*
c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d
^2*e^6)))^(1/2) + (x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2
*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5
*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6
*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) -
 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*
d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(
256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^
4*c^6*d^2*e^6)))^(1/2))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2
) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2
*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5
*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) -
 (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5
 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d^8 + a^4*c*e^8 +
4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c
^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3
+ 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^
5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4
*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d
^10*e^4 + 12000*a^3*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^
10 + 48*a^6*c^2*d^2*e^12)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a
^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((45056*a^2*c^10*d^13*e^3 - 4096*a^
8*c^4*d*e^15 + 221184*a^3*c^9*d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^
5*c^7*d^7*e^9 + 184320*a^6*c^6*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(256*(c^5
*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)
) + (x*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e
^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) +
 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*
d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c^5*e^1
7 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^
10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*
d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^
6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2)
- 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5
*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/
(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a
^4*c^6*d^2*e^6)))^(1/2) - (x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 2
1248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296
*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a
*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)
^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 3
0*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^
(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^
```

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4 + 4*a^4*c^6*d^2*e^6)))^(1/2))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c
^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2
*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 +
 a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))
^(1/2) + (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4
*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d^8 + a^4*
c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e
^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*
d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^
4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3
*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (a^4*d^3*e^9 + 108*a*c^3*d^9*e^
3 + 18*a^3*c*d^5*e^7 + 93*a^2*c^2*d^7*e^5)/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*
c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6))))*(-(a^3*e^6*(-a*c^5)
^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 3
0*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^
(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^
4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*2i - ((d*x^3)/(4*(a*e^2 + c*d^2)) + (a*e*x)/
(4*c*(a*e^2 + c*d^2)))/(a + c*x^4) + atan(((((432*a*c^7*d^12*e^2 + 13040*a^
2*c^6*d^10*e^4 + 12000*a^3*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3
*d^4*e^10 + 48*a^6*c^2*d^2*e^12)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^
2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((45056*a^2*c^10*d^13*e^3 -
4096*a^8*c^4*d*e^15 + 221184*a^3*c^9*d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 40
9600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(2
56*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d
^2*e^6)) - (x*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c
^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(
1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^
2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c
^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c
^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^
7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*
c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^(
1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*
a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1
/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4
+ 4*a^4*c^6*d^2*e^6)))^(1/2) + (x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^1
4 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 -
 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(128*(c^5*d^8 + a^4*c*e^8
+ 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*
c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3
 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c
^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^
4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-
a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*
c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^
8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6
)))^(1/2) - (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*
c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d^8 + a
^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3
*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^
4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*
e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a
^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*1i - (((432*a*c^7*d^12*e^2 + 13
040*a^2*c^6*d^10*e^4 + 12000*a^3*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a
^5*c^3*d^4*e^10 + 48*a^6*c^2*d^2*e^12)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*
d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((45056*a^2*c^10*d^13*
e^3 - 4096*a^8*c^4*d*e^15 + 221184*a^3*c^9*d^11*e^5 + 430080*a^4*c^8*d^9*e^
7 + 409600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6*d^5*e^11 + 24576*a^7*c^5*d^3*e^
13)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3
```

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*c^2*d^2*e^6)) + (x*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2
*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*
c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8
+ 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*(65536
*a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824
*a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589
824*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a^4*c*e^8
+ 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*
c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3
 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c
^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^
4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) - (x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3
*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7
*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(128*(c^5*d^8 + a^4*
c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^
6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d
^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4
*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*
c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*
d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e +
 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*
c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d
^2*e^6)))^(1/2) + (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 1
7*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d
^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))
*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*
a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*
c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2
 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*1i)/((((432*a*c^7*d^12*e^
2 + 13040*a^2*c^6*d^10*e^4 + 12000*a^3*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 -
 400*a^5*c^3*d^4*e^10 + 48*a^6*c^2*d^2*e^12)/(256*(c^5*d^8 + a^4*c*e^8 + 4*
a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((45056*a^2*c^10
*d^13*e^3 - 4096*a^8*c^4*d*e^15 + 221184*a^3*c^9*d^11*e^5 + 430080*a^4*c^8*
d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6*d^5*e^11 + 24576*a^7*c^5*
d^3*e^13)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 +
 4*a^3*c^2*d^2*e^6)) - (x*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/
2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^
2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^
5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*
(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 -
589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11
 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a^4*
c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^
6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d
^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4
*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*
c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (x*(1152*a*c^9*d^13*e^2 + 1152*a
^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c
^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(128*(c^5*d^8
+ a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((
a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2
*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d
^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 +
6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2))*((a^3*e^6*(-a*c^5)^(1/2) -
9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d
^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(2
56*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4
*c^6*d^2*e^6)))^(1/2) - (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^
11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(128*
```

(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d^10*e^4 + 12000*a^3*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^10 + 48*a^6*c^2*d^2*e^12)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((45056*a^2*c^10*d^13*e^3 - 4096*a^8*c^4*d*e^15 + 221184*a^3*c^9*d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (x*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) - (x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (a^4*d^3*e^9 + 108*a*c^3*d^9*e^3 + 18*a^3*c*d^5*e^7 + 93*a^2*c^2*d^7*e^5)/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6))))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*2i - (atan(-(((-d^5*e)^(1/2)*(((-d^5*e)^(1/2)*(((27*a*c^7*d^12*e^2)/16 + (815*a^2*c^6*d^10*e^4)/16 + (375*a^3*c^5*d^8*e^6)/8 - (33*a^4*c^4*d^6*e^8)/8 - (25*a^5*c^3*d^4*e^10)/16 + (3*a^6*c^2*d^2*e^12)/16)/(2*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((176*a^2*c^10*d^13*e^3 - 16*a^8*c^4*d*e^15 + 864*a^3*c^9*d^11*e^5 + 1680*a^4*c^8*d^9*e^7 + 1600*a^5*c^7*d^7*e^9 + 720*a^6*c^6*d^5*e^11 + 96*a^7*c^5*d^3*e^13)/(2*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) - (x*(-d^5*e)^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c

^6*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(c^5*d^8 + a^4*c*e^8
 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-d^5*e)^(1/2
))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^5*e)^(1/2))/(2*(a^2*e^4 + c
^2*d^4 + 2*a*c*d^2*e^2))))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (x*(a^
6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*
a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4
*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*1i)/(a^2*e^4 + c^2*d^4
 + 2*a*c*d^2*e^2) - ((-d^5*e)^(1/2)*(((-d^5*e)^(1/2)*(((27*a*c^7*d^12*e^2)/1
6 + (815*a^2*c^6*d^10*e^4)/16 + (375*a^3*c^5*d^8*e^6)/8 - (33*a^4*c^4*d^6*e
^8)/8 - (25*a^5*c^3*d^4*e^10)/16 + (3*a^6*c^2*d^2*e^12)/16)/(2*(c^5*d^8 + a
^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) - (((x
*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 2547
2*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6
*c^4*d^3*e^12))/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4
*e^4 + 4*a^3*c^2*d^2*e^6)) - (((176*a^2*c^10*d^13*e^3 - 16*a^8*c^4*d*e^15 +
 864*a^3*c^9*d^11*e^5 + 1680*a^4*c^8*d^9*e^7 + 1600*a^5*c^7*d^7*e^9 + 720*a
^6*c^6*d^5*e^11 + 96*a^7*c^5*d^3*e^13)/(2*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^
6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (x*(-d^5*e)^(1/2)*(65536*
a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*
a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 5898
24*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2
*a*c*d^2*e^2)*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 +
4*a^3*c^2*d^2*e^6)))*(-d^5*e)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)
))*(-d^5*e)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))))/(2*(a^2*e^4 +
c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^
2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(
256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*
d^2*e^6)))*1i)/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))/(((a^4*d^3*e^9)/128 + (
27*a*c^3*d^9*e^3)/32 + (9*a^3*c*d^5*e^7)/64 + (93*a^2*c^2*d^7*e^5)/128)/(c^
5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6
) + ((-d^5*e)^(1/2)*(((-d^5*e)^(1/2)*(((27*a*c^7*d^12*e^2)/16 + (815*a^2*c^
6*d^10*e^4)/16 + (375*a^3*c^5*d^8*e^6)/8 - (33*a^4*c^4*d^6*e^8)/8 - (25*a^5
*c^3*d^4*e^10)/16 + (3*a^6*c^2*d^2*e^12)/16)/(2*(c^5*d^8 + a^4*c*e^8 + 4*a*
c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((x*(1152*a*c^9*d^
13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e
^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))
/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^
2*d^2*e^6)) + (((176*a^2*c^10*d^13*e^3 - 16*a^8*c^4*d*e^15 + 864*a^3*c^9*d^
11*e^5 + 1680*a^4*c^8*d^9*e^7 + 1600*a^5*c^7*d^7*e^9 + 720*a^6*c^6*d^5*e^11
 + 96*a^7*c^5*d^3*e^13)/(2*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c
^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) - (x*(-d^5*e)^(1/2)*(65536*a^9*c^5*e^17 -
65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e
^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*
e^13 + 327680*a^8*c^6*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(
c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e
^6)))*(-d^5*e)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^5*e)^(1/
2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c
*d^2*e^2)) - (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2
*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(256*(c^5*d^8 +
a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6))))/(a^
2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2) + ((-d^5*e)^(1/2)*(((-d^5*e)^(1/2)*(((27*a
*c^7*d^12*e^2)/16 + (815*a^2*c^6*d^10*e^4)/16 + (375*a^3*c^5*d^8*e^6)/8 - (
33*a^4*c^4*d^6*e^8)/8 - (25*a^5*c^3*d^4*e^10)/16 + (3*a^6*c^2*d^2*e^12)/16)
/(2*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*
d^2*e^6)) - (((x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8
*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5
*e^10 + 4864*a^6*c^4*d^3*e^12))/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2
 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) - (((176*a^2*c^10*d^13*e^3 - 16*
a^8*c^4*d*e^15 + 864*a^3*c^9*d^11*e^5 + 1680*a^4*c^8*d^9*e^7 + 1600*a^5*c^7

```
*d^7*e^9 + 720*a^6*c^6*d^5*e^11 + 96*a^7*c^5*d^3*e^13)/(2*(c^5*d^8 + a^4*c*
e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (x*(-d^5*
e)^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^
12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8
*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(512*(a^2*e
^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^
2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-d^5*e)^(1/2))/(2*(a^2*e^4 + c^2*d^4
+ 2*a*c*d^2*e^2)))*(-d^5*e)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))
)/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(a^6*e^13 - 288*a*c^5*d^10*e
^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4
*c^2*d^4*e^9))/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*
e^4 + 4*a^3*c^2*d^2*e^6))))/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^5*e)^
(1/2)*1i)/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.184 $\int \frac{x^4}{(d+ex^2)(a+cx^4)^2} dx$

Optimal. Leaf size=685

$$\frac{\left(\sqrt{a}\,e+3\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{3/4}c^{3/4}\left(ae^2+cd^2\right)}-\frac{\left(\sqrt{a}\,e+3\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{3/4}c^{3/4}\left(ae^2+cd^2\right)}+\frac{\left(3\sqrt{c}\,d-\right.}{8\sqrt{}}$$

Rubi [A] time = 0.61, antiderivative size = 685, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1314, 1179, 1168, 1162, 617, 204, 1165, 628, 1171, 205}

$$\frac{\left(\sqrt{a}\,e+3\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{3/4}c^{3/4}\left(ae^2+cd^2\right)}-\frac{\left(\sqrt{a}\,e+3\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{3/4}c^{3/4}\left(ae^2+cd^2\right)}+\frac{\left(3\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{3/4}c^{3/4}\left(ae^2+cd^2\right)}-\frac{\left(3\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\,a^{3/4}c^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{c}\,d^2\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}\,d^2\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}\,d^2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}\,d^2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{x\left(d-ex^2\right)}{4\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -(x*(d - e*x^2))/(4*(c*d^2 + a*e^2)*(a + c*x^4)) + (d^(3/2)*e^(3/2)*ArcTan[
(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 - (c^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]
*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*
e^2)^2) + ((3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/(8*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*d^2*(Sqrt[c]*d -
Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d
^2 + a*e^2)^2) - ((3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(8*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*d^2*(Sqrt[
c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + ((3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(3/4)*c^(3/4)*
(c*d^2 + a*e^2)) + (c^(1/4)*d^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2)
- ((3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(16*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1314

```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> -Dist[(a*f^4)/(c*d^2 + a*e^2), Int[(f*x)^(m - 4)*(d - e*x^2)*
(a + c*x^4)^p, x], x] + Dist[(d^2*f^4)/(c*d^2 + a*e^2), Int[((f*x)^(m - 4)*
(a + c*x^4)^(p + 1))/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && Lt
Q[p, -1] && GtQ[m, 2]
```

Rubi steps

$$\int \frac{x^4}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = -\frac{a\int \frac{d-ex^2}{\left(a+cx^4\right)^2}dx}{cd^2+ae^2} + \frac{d^2\int \frac{1}{\left(d+ex^2\right)\left(a+cx^4\right)}dx}{cd^2+ae^2}$$

$$= -\frac{x\left(d-ex^2\right)}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{\int \frac{-3d+ex^2}{a+cx^4}dx}{4\left(cd^2+ae^2\right)} + \frac{d^2\int \left(\frac{e^2}{\left(cd^2+ae^2\right)\left(d+ex^2\right)} + \frac{c\left(d-ex^2\right)}{\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right.}{cd^2+ae^2}$$

$$= -\frac{x\left(d-ex^2\right)}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{\left(cd^2\right)\int \frac{d-ex^2}{a+cx^4}dx}{\left(cd^2+ae^2\right)^2} + \frac{\left(d^2e^2\right)\int \frac{1}{d+ex^2}dx}{\left(cd^2+ae^2\right)^2} - \frac{\left(\frac{3\sqrt{c}\,d}{\sqrt{a}}-e\right)\int}{8c\left(cd^2\right.}$$

$$= -\frac{x\left(d-ex^2\right)}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} + \frac{\left(d^2\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}dx}{2\left(cd^2+ae^2\right)^2}$$

$$= -\frac{x\left(d-ex^2\right)}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} + \frac{\left(3\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\right.}{16\sqrt{2}\,a^{3/4}c^{3/4}\left(cd^2\right.}$$

$$= -\frac{x\left(d-ex^2\right)}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} + \frac{\left(3\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{8\sqrt{2}\,a^{3/4}c^{3/4}\left(cd^2+ae^2\right)}$$

$$= -\frac{x\left(d-ex^2\right)}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}\,d^2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\right.}{2\sqrt{2}\,a^{3/4}\left(cd^2+ae^2\right)^2}$$

Mathematica [A] time = 0.27, size = 423, normalized size = 0.62

$$\frac{\frac{\sqrt{2}\left(a^{3/2}e^3-3\sqrt{a}\,cd^2e+3a\sqrt{c}\,de^2-c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{a^{3/4}c^{3/4}} + \frac{\sqrt{2}\left(-a^{3/2}e^3+3\sqrt{a}\,cd^2e-3a\sqrt{c}\,de^2+c^{3/2}d^3\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{a^{3/4}c^{3/4}} - \frac{2\sqrt{2}\left(a^{3/2}e^3-3\sqrt{a}\,cd^2e-3a\sqrt{c}\,de^2+c^{3/2}d^3\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{3/4}c^{3/4}} + \frac{2\sqrt{2}\left(a^{3/2}e^3-3\sqrt{a}\,cd^2e-3a\sqrt{c}\,de^2+c^{3/2}d^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{a^{3/4}c^{3/4}} + \frac{8\left(ex^3-dx\right)\left(ae^2+cd^2\right)}{a+cx^4} + 32d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{32\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*(c*d^2 + a*e^2)*(-(d*x) + e*x^3))/(a + c*x^4) + 32*d^(3/2)*e^(3/2)*ArcT
an[(Sqrt[e]*x)/Sqrt[d]] - (2*Sqrt[2]*(c^(3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a
*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(
3/4)*c^(3/4)) + (2*Sqrt[2]*(c^(3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d
*e^2 + a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(3/4)*c^(3/
4)) + (Sqrt[2]*(-(c^(3/2)*d^3) - 3*Sqrt[a]*c*d^2*e + 3*a*Sqrt[c]*d*e^2 + a^
(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(3/4)
*c^(3/4)) + (Sqrt[2]*(c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 -
 a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(3
/4)*c^(3/4)))/(32*(c*d^2 + a*e^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[x^4/((d + e*x^2)*(a + c*x^4)^2), x]
```

fricas [B] time = 24.28, size = 9678, normalized size = 14.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(c*d^2*e + a*e^3)*x^3 - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (
c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*
e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*
a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*
c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10
 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 5
6*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*
d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^
6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))*log(-(c^4*d^8 -
 14*a*c^3*d^6*e^2 + 14*a^3*c*d^2*e^6 - a^4*e^8)*x + (a*c^5*d^9 - 18*a^2*c^4
*d^7*e^2 + 60*a^3*c^3*d^5*e^4 - 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 + (3*a^3
*c^7*d^10*e + 11*a^4*c^6*d^8*e^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 -
 a^7*c^3*d^2*e^9 - a^8*c^2*e^11)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*
a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*
e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4
 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*
c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))*sqrt((6*c^2*d^5*e - 2
0*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^
4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2
+ 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*
c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^
12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 2
8*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*
a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) + (a
*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e
^4)*x^4)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*
a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-
(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 +
 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4
*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8
*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^
11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c
^2*d^2*e^6 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a*c^3*d^6*e^2 + 14*a^3*c*d^2*e^
6 - a^4*e^8)*x - (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 + 60*a^3*c^3*d^5*e^4 - 46*
a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 + (3*a^3*c^7*d^10*e + 11*a^4*c^6*d^8*e^3 +
14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*c^3*d^2*e^9 - a^8*c^2*e^11)*sq
rt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e
^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8
*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7
*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14
+ a^11*c^3*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^
5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e
^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3
*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^
16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a
^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2
*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4
 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^
4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*sqrt((6*c^2*d^5*e - 20*a*c
*d^3*e^3 + 6*a^2*d*e^5 - (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4
```

```
 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255
*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2
*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^
4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9
*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c
^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))*log(-(c^4*
d^8 - 14*a*c^3*d^6*e^2 + 14*a^3*c*d^2*e^6 - a^4*e^8)*x + (a*c^5*d^9 - 18*a^
2*c^4*d^7*e^2 + 60*a^3*c^3*d^5*e^4 - 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 - (
3*a^3*c^7*d^10*e + 11*a^4*c^6*d^8*e^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*
e^7 - a^7*c^3*d^2*e^9 - a^8*c^2*e^11)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 +
 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c
*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^1
2*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28
*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))*sqrt((6*c^2*d^5*
e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c
^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10
*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30
*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c
^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^1
0 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8
 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))
 + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^
2*c*e^4)*x^4)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - (a*c^5*d^8
 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*s
qrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*
e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 +
8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^
7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14
 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*
a^4*c^2*d^2*e^6 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a*c^3*d^6*e^2 + 14*a^3*c*d
^2*e^6 - a^4*e^8)*x - (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 + 60*a^3*c^3*d^5*e^4
- 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 - (3*a^3*c^7*d^10*e + 11*a^4*c^6*d^8*e
^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*c^3*d^2*e^9 - a^8*c^2*e^1
1)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*
d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^1
6 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^
7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*
e^14 + a^11*c^3*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 -
(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^
5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^
3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^
11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 +
 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^
4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^
4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) + 8*(c*d*e*x^4 + a*d*e)*sqrt(-d*e)
*log((e*x^2 + 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 4*(c*d^3 + a*d*e^2)*x)/(a*
c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^
4)*x^4), 1/16*(4*(c*d^2*e + a*e^3)*x^3 + 16*(c*d*e*x^4 + a*d*e)*sqrt(d*e)*a
rctan(sqrt(d*e)*x/d) - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 +
2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^
2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^
2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^
4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^1
2)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*
d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 +
 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*
a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a*c^3*
d^6*e^2 + 14*a^3*c*d^2*e^6 - a^4*e^8)*x + (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 +
 60*a^3*c^3*d^5*e^4 - 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 + (3*a^3*c^7*d^10*
```

e + 11*a^4*c^6*d^8*e^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*c^3*d
^2*e^9 - a^8*c^2*e^11)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^
8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6
*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*
c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^
12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c*d^3*
e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*
a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*
c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10
 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 5
6*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*
d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^
6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) + (a*c^2*d^4 +
 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*s
qrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^4*d^
6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12
- 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c
^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14
*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*
a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^1
6)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6
 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a*c^3*d^6*e^2 + 14*a^3*c*d^2*e^6 - a^4*e^
8)*x - (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 + 60*a^3*c^3*d^5*e^4 - 46*a^4*c^2*d^
3*e^6 + 3*a^5*c*d*e^8 + (3*a^3*c^7*d^10*e + 11*a^4*c^6*d^8*e^3 + 14*a^5*c^5
*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*c^3*d^2*e^9 - a^8*c^2*e^11)*sqrt(-(c^6*d
^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a
^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*
d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 +
 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3
*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*
a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-
(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 +
 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4
*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8
*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^
11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c
^2*d^2*e^6 + a^5*c*e^8))) - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d
^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 +
 6*a^2*d*e^5 - (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c
^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d
^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^
6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6
*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e
^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2
 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a
*c^3*d^6*e^2 + 14*a^3*c*d^2*e^6 - a^4*e^8)*x + (a*c^5*d^9 - 18*a^2*c^4*d^7*
e^2 + 60*a^3*c^3*d^5*e^4 - 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 - (3*a^3*c^7*
d^10*e + 11*a^4*c^6*d^8*e^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*
c^3*d^2*e^9 - a^8*c^2*e^11)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c
^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10
+ a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56
*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d
^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c
*d^3*e^3 + 6*a^2*d*e^5 - (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4
 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255
*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2
*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^
4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9
*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c

```
^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) + (a*c^2*
d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x
^4)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - (a*c^5*d^8 + 4*a^2*c
^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*
d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*
a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10
*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8
+ 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^
3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^
2*e^6 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a*c^3*d^6*e^2 + 14*a^3*c*d^2*e^6 - a
^4*e^8)*x - (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 + 60*a^3*c^3*d^5*e^4 - 46*a^4*c
^2*d^3*e^6 + 3*a^5*c*d*e^8 - (3*a^3*c^7*d^10*e + 11*a^4*c^6*d^8*e^3 + 14*a^
5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*c^3*d^2*e^9 - a^8*c^2*e^11)*sqrt(-(
c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 +
255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*
c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*
e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^1
1*c^3*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - (a*c^5*d^8
 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*s
qrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*
e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 +
8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^
7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14
 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*
a^4*c^2*d^2*e^6 + a^5*c*e^8))) - 4*(c*d^3 + a*d*e^2)*x)/(a*c^2*d^4 + 2*a^2*
c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)]
```

giac [A] time = 0.47, size = 586, normalized size = 0.86

$$\frac{d^{\frac{3}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{3}{2}}}{c^2d^4+2\,acd^2e^2+a^2e^4}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^3d^3-3\left(ac^3\right)^{\frac{1}{4}}ac^2de^2-3\left(ac^3\right)^{\frac{3}{4}}cd^2e+\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,ac^5d^4+2\sqrt{2}\,a^2c^4d^2e^2+\sqrt{2}\,a^3c^3e^4\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^3d^3-3\left(ac^3\right)^{\frac{1}{4}}ac^2de^2-3\left(ac^3\right)^{\frac{3}{4}}cd^2e+\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,ac^5d^4+2\sqrt{2}\,a^2c^4d^2e^2+\sqrt{2}\,a^3c^3e^4\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^3d^3-3\left(ac^3\right)^{\frac{1}{4}}ac^2de^2+3\left(ac^3\right)^{\frac{3}{4}}cd^2e-\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}\,ac^5d^4+2\sqrt{2}\,a^2c^4d^2e^2+\sqrt{2}\,a^3c^3e^4\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^3d^3-3\left(ac^3\right)^{\frac{1}{4}}ac^2de^2+3\left(ac^3\right)^{\frac{3}{4}}cd^2e-\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}\,ac^5d^4+2\sqrt{2}\,a^2c^4d^2e^2+\sqrt{2}\,a^3c^3e^4\right)}+\frac{x^3e-dx}{4\left(cx^4+a\right)\left(cd^2+ae^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] d^(3/2)*arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^
4) + 1/8*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 - 3*(a*c^3)^(
3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)
^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2*e^2 + sqrt(
2)*a^3*c^3*e^4) + 1/8*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2
- 3*(a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x -
sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^
2*e^2 + sqrt(2)*a^3*c^3*e^4) + 1/16*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4
)*a*c^2*d*e^2 + 3*(a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*log(x^2 + sq
rt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2
*e^2 + sqrt(2)*a^3*c^3*e^4) - 1/16*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)
*a*c^2*d*e^2 + 3*(a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*log(x^2 - sqr
t(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2*
e^2 + sqrt(2)*a^3*c^3*e^4) + 1/4*(x^3*e - d*x)/((c*x^4 + a)*(c*d^2 + a*e^2)
)
```

maple [A] time = 0.02, size = 848, normalized size = 1.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^3*e^3*a+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^3*e
*c*d^2-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x*a*d*e^2-1/4/(a*e^2+c*d^2)^2/(c*x^4+a
)*x*c*d^3-3/16/(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/
```

```
4)*x-1)*d*e^2+1/16/(a*e^2+c*d^2)^2*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/
c)^(1/4)*x-1)*c*d^3-3/32/(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^
(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d*e^2
+1/32/(a*e^2+c*d^2)^2*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(
a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*c*d^3-3/16/(a*e^2+c*d^
2)^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^2+1/16/(a*e^2+
c*d^2)^2*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*c*d^3+1/32/(
a*e^2+c*d^2)^2/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1
/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*a*e^3-3/32/(a*e^2+c*d^2)^2/(a
/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/
4)*2^(1/2)*x+(a/c)^(1/2)))*d^2*e+1/16/(a*e^2+c*d^2)^2/c/(a/c)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*a*e^3-3/16/(a*e^2+c*d^2)^2/(a/c)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2*e+1/16/(a*e^2+c*d^2)^2/c/(a/c)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*a*e^3-3/16/(a*e^2+c*d^2)^2/(a/c)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^2*e+e^2/(a*e^2+c*d^2)^2*d^
2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.09, size = 490, normalized size = 0.72

$$\frac{d^2e^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\sqrt{de}}+\frac{\left(cd^2e+ae^3\right)x^3-\left(cd^3+ade^2\right)x}{4\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4+\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right)x^4\right)}+\frac{\frac{2\sqrt{2}\left(c^{\frac{3}{2}}d^3-3\sqrt{a}cd^2e-3\,a\sqrt{c}de^2+a^{\frac{3}{2}}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(c^{\frac{3}{2}}d^3-3\sqrt{a}cd^2e-3\,a\sqrt{c}de^2+a^{\frac{3}{2}}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(c^{\frac{3}{2}}d^3+3\sqrt{a}cd^2e-3\,a\sqrt{c}de^2-a^{\frac{3}{2}}e^3\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(c^{\frac{3}{2}}d^3+3\sqrt{a}cd^2e-3\,a\sqrt{c}de^2-a^{\frac{3}{2}}e^3\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}}{32\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] d^2*e^2*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e
)) + 1/4*((c*d^2*e + a*e^3)*x^3 - (c*d^3 + a*d*e^2)*x)/(a*c^2*d^4 + 2*a^2*c
*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4) + 1/32*(2
*sqrt(2)*(c^(3/2)*d^3 - 3*sqrt(a)*c*d^2*e - 3*a*sqrt(c)*d*e^2 + a^(3/2)*e^3
)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*s
qrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(c^(3/2)*d^3 -
 3*sqrt(a)*c*d^2*e - 3*a*sqrt(c)*d*e^2 + a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2
*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(c^(3/2)*d^3 + 3*sqrt(a)*c*d^2*e - 3*a*
sqrt(c)*d*e^2 - a^(3/2)*e^3)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x +
sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(c^(3/2)*d^3 + 3*sqrt(a)*c*d^2*e - 3*a
*sqrt(c)*d*e^2 - a^(3/2)*e^3)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x +
 sqrt(a))/(a^(3/4)*c^(3/4)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)
```

mupad [B] time = 4.87, size = 17180, normalized size = 25.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] - atan((((((28672*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*
d^8*e^6 + 253952*a^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*
d^2*e^12)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (
x*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e +
 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) -
 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4
*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(65536*a^9*c^
4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^
9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*
c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*
d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2)
 - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^
3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)
^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4
*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e
```

^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 +
4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4
*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3
)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20
*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a
^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*
c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*
e^5 + 16*a^4*c^2*d*e^11 + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256
*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((a^3*e^6*(-a^3*
c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 -
20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*
(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a
^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) - (x*(a^4*c*e^13 + 33*c^5*d^8*e
^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^
4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*
((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6
*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 1
5*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c
^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*1i - (((((28672
*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*d^8*e^6 + 253952*a
^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*d^2*e^12)/(256*(a^
3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x*((a^3*e^6*(-a^3*
c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 -
20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*
(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a
^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(65536*a^9*c^4*e^17 - 65536*a^2
*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 32768
0*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327
680*a^8*c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*
d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c
^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c
^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*
c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d
^2*e^6)))^(1/2) - (x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*
d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e
^10 - 3840*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4
*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*
(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 +
15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(25
6*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^
6*c^4*d^2*e^6)))^(1/2) + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*e^5 + 16*a^4*c^2*d
*e^11 + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256*(a^3*e^6 + c^3*d^
6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d
^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^
3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/
(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4
*a^6*c^4*d^2*e^6)))^(1/2) + (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^6
*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^4*e^8 + c^4*d^8 +
4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^
3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 2
0*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-
a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5
*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*1i)/((5*c^2*d^4*e^6 + a*c*d^2*e^8
)/(128*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (((((2867
2*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*d^8*e^6 + 253952*
a^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*d^2*e^12)/(256*(a
^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (x*((a^3*e^6*(-a^3
*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5
- 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4

*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*
a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(65536*a^9*c^4*e^17 - 65536*a^
2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 3276
80*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 32
7680*a^8*c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c
*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*
c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*
c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3
*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*
d^2*e^6)))^(1/2) + (x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3
*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*
e^10 - 3840*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 +
4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6
*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3
+ 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(2
56*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a
^6*c^4*d^2*e^6)))^(1/2) + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*e^5 + 16*a^4*c^2*
d*e^11 + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256*(a^3*e^6 + c^3*d
^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*
d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e
^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))
/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 +
4*a^6*c^4*d^2*e^6)))^(1/2) - (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^
6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^4*e^8 + c^4*d^8 +
4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c
^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 -
20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(
-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^
5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (((((28672*a^2*c^8*d^10*e^4 -
4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*d^8*e^6 + 253952*a^4*c^6*d^6*e^8 + 176
128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*d^2*e^12)/(256*(a^3*e^6 + c^3*d^6 + 3*
a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6
*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3
+ 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(2
56*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a
^6*c^4*d^2*e^6)))^(1/2)*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327
680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 +
327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15
))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*
d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^
4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3
)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^
8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) - (x
*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c
^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d
^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a
^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6
*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-
a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7
*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/
2) + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*e^5 + 16*a^4*c^2*d*e^11 + 8288*a^2*c^4
*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2
+ 3*a^2*c*d^2*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2)
+ 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2
*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 +
a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^
(1/2) + (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^
4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*
a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(

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-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 +
15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256
*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6
*c^4*d^2*e^6)))^(1/2)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/
2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*
e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8
 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)
))^(1/2)*2i - atan((((((28672*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155
648*a^3*c^7*d^8*e^6 + 253952*a^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45
056*a^6*c^4*d^2*e^12)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d
^2*e^4)) - (x*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2
*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*
c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3
*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(
65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 5
89824*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 +
 589824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^
8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^
3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5
 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^
4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6
*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (x*(256*a*c^8*d^11*e^4 - 12
8*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^
6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 +
 c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d
^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^
2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c
*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*
e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (16*c^6*d^9*e^3 - 96
0*a*c^5*d^7*e^5 + 16*a^4*c^2*d*e^11 + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d
^3*e^9)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((c^
3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4
*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^
2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d
^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) - (x*(a^4*c*e^13 +
33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^1
1))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2
*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c
^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^
3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e
^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*1i
- (((((28672*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*d^8*e^
6 + 253952*a^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*d^2*e^
12)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x*((c^
3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4
*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^
2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d
^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(65536*a^9*c^4*e^17
 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10
*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^
4*e^13 + 327680*a^8*c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^
2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3
*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*
e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2)
)/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 +
 4*a^6*c^4*d^2*e^6)))^(1/2) - (x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2
944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a
^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3
*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2
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) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c
^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3
)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^
4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*e^5 +
16*a^4*c^2*d*e^11 + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256*(a^3*
e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((c^3*d^6*(-a^3*c^3)^(
1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^
3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*
c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5
*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 1
88*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^4*e^8
+ c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*
d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c
^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*
c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6
*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*1i)/((5*c^2*d^4*e^6 +
a*c*d^2*e^8)/(128*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4))
+ (((((28672*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*d^8*e
^6 + 253952*a^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*d^2*e
^12)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (x*((c
^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^
4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a
^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*
d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(65536*a^9*c^4*e^1
7 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^1
0*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d
^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e
^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^
3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3
*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2
))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4
+ 4*a^6*c^4*d^2*e^6)))^(1/2) + (x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 +
2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*
a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^
3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/
2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*
c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^
3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d
^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*e^5 +
16*a^4*c^2*d*e^11 + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256*(a^3
*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((c^3*d^6*(-a^3*c^3)^
(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a
^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3
c^3)^(1/2))/(256(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^
5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) - (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 -
188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^4*e^8
+ c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3
d^6(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*
c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2
*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^
6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (((((28672*a^2*c^8
*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*d^8*e^6 + 253952*a^4*c^6*d
^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*d^2*e^12)/(256*(a^3*e^6 +
c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x*((c^3*d^6*(-a^3*c^3)^(1/
2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*
c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^
3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d
^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^
14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^

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8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*
c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6
+ 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2
) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e
^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8
+ a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6))
)^(1/2) - (x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 +
 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 38
40*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d
^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^
3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^
2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c
^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^
2*e^6)))^(1/2) + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*e^5 + 16*a^4*c^2*d*e^11 +
8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256*(a^3*e^6 + c^3*d^6 + 3*a*
c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3
*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a
*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^
3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4
*d^2*e^6)))^(1/2) + (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 3
8*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*
d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2)
 - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^
3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)
^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4
*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-
a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 1
5*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*
(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*
c^4*d^2*e^6)))^(1/2)*2i - ((d*x)/(4*(a*e^2 + c*d^2)) - (e*x^3)/(4*(a*e^2 +
c*d^2)))/(a + c*x^4) - (atan((((((((c^6*d^9*e^3)/16 - (15*a*c^5*d^7*e^5)/4
+ (a^4*c^2*d*e^11)/16 + (259*a^2*c^4*d^5*e^7)/8 - (47*a^3*c^3*d^3*e^9)/4)/(
2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (((x*(256*a*c^
8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6
 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/
(256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4
*e^4)) + (((112*a^2*c^8*d^10*e^4 - 16*a*c^9*d^12*e^2 + 608*a^3*c^7*d^8*e^6
+ 992*a^4*c^6*d^6*e^8 + 688*a^5*c^5*d^4*e^10 + 176*a^6*c^4*d^2*e^12)/(2*(a^
3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (x*(-d^3*e^3)^(1/2)
*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 -
 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11
 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(512*(a^2*e^4 + c^2*
d^4 + 2*a*c*d^2*e^2)*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6
 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2
*e^2)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^
3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (x*(a^4*c*e^13 + 33*c^5
*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(2
56*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e
^4)))*(-d^3*e^3)^(1/2)*1i)/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2) - ((((((c^6*
d^9*e^3)/16 - (15*a*c^5*d^7*e^5)/4 + (a^4*c^2*d*e^11)/16 + (259*a^2*c^4*d^5
*e^7)/8 - (47*a^3*c^3*d^3*e^9)/4)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 +
 3*a^2*c*d^2*e^4)) - (((x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6
*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*
d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^
2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) - (((112*a^2*c^8*d^10*e^4 - 16*a*
c^9*d^12*e^2 + 608*a^3*c^7*d^8*e^6 + 992*a^4*c^6*d^6*e^8 + 688*a^5*c^5*d^4*
e^10 + 176*a^6*c^4*d^2*e^12)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^
2*c*d^2*e^4)) + (x*(-d^3*e^3)^(1/2)*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^
14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^
```

```
8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*
c^5*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a^4*e^8 + c^4*d^8
+ 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^(1/2)
)/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 +
c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*
d^2*e^2)) + (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^
3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2
+ 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^(1/2)*1i)/(a^2*e^4 + c^
2*d^4 + 2*a*c*d^2*e^2))/(((5*c^2*d^4*e^6)/128 + (a*c*d^2*e^8)/128)/(a^3*e^6
 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4) + ((((((c^6*d^9*e^3)/16 - (
15*a*c^5*d^7*e^5)/4 + (a^4*c^2*d*e^11)/16 + (259*a^2*c^4*d^5*e^7)/8 - (47*a
^3*c^3*d^3*e^9)/4)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^
4)) + (((x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 2
1632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840
*a^5*c^4*d^3*e^12))/(256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2
*e^6 + 6*a^2*c^2*d^4*e^4)) + (((112*a^2*c^8*d^10*e^4 - 16*a*c^9*d^12*e^2 +
608*a^3*c^7*d^8*e^6 + 992*a^4*c^6*d^6*e^8 + 688*a^5*c^5*d^4*e^10 + 176*a^6*
c^4*d^2*e^12)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) -
 (x*(-d^3*e^3)^(1/2)*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680
*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327
680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/
(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e
^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 +
 c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c
*d^2*e^2)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (x*
(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a
^3*c^2*d^2*e^11))/(256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e
^6 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^(1/2))/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*
e^2) + ((((((c^6*d^9*e^3)/16 - (15*a*c^5*d^7*e^5)/4 + (a^4*c^2*d*e^11)/16 +
 (259*a^2*c^4*d^5*e^7)/8 - (47*a^3*c^3*d^3*e^9)/4)/(2*(a^3*e^6 + c^3*d^6 +
3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (((x*(256*a*c^8*d^11*e^4 - 128*c^9*d^
13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^
8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(256*(a^4*e^8 + c^4*d^8
 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) - (((112*a^2*c^8
*d^10*e^4 - 16*a*c^9*d^12*e^2 + 608*a^3*c^7*d^8*e^6 + 992*a^4*c^6*d^6*e^8 +
 688*a^5*c^5*d^4*e^10 + 176*a^6*c^4*d^2*e^12)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c
^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x*(-d^3*e^3)^(1/2)*(65536*a^9*c^4*e^17 -
65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^
7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e
^13 + 327680*a^8*c^5*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a
^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))
*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^(1/2
))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 +
 c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^
6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(256*(a^4*e^8 + c^4*d^8 +
 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^(1/2))
/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^(1/2)*1i)/(a^2*e^4 + c^2*
d^4 + 2*a*c*d^2*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x**2+d)/(c*x**4+a)**2,x)
[Out] Timed out
```

3.185 $\int \frac{x^2}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=685

$$\frac{\sqrt[4]{c}\,de\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}\,de\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}-$$

Rubi [A] time = 0.56, antiderivative size = 685, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {1316, 1179, 1168, 1162, 617, 204, 1165, 628, 1171, 205}

$$\frac{\sqrt[4]{c}\,de\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}\,de\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\left(\sqrt{c}\,d-3\sqrt{a}\,e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{5/4}\sqrt[4]{c}\left(ae^2+cd^2\right)}-\frac{\left(\sqrt{c}\,d-3\sqrt{a}\,e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{5/4}\sqrt[4]{c}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\,de\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}\,de\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\left(3\sqrt{a}\,e+\sqrt{c}\,d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{5/4}\sqrt[4]{c}\left(ae^2+cd^2\right)}+\frac{\left(3\sqrt{a}\,e+\sqrt{c}\,d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\,a^{5/4}\sqrt[4]{c}\left(ae^2+cd^2\right)}+\frac{x\left(ae+cdx^2\right)}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{\sqrt{d}\,e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (x*(a*e + c*d*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[d]*e^(5/2)*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 + (c^(1/4)*d*e*(Sqrt[c]*d - Sq
rt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2
 + a*e^2)^2) - ((Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^
(1/4)])/(8*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2)) - (c^(1/4)*d*e*(Sqrt[c]
*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)
*(c*d^2 + a*e^2)^2) + ((Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/(8*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2)) + (c^(1/4)*d*e*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + ((Sqrt[c]*d - 3*Sqrt[a]*e)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(5/4)*c^(
1/4)*(c*d^2 + a*e^2)) - (c^(1/4)*d*e*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2
)^2) - ((Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
 Sqrt[c]*x^2])/(16*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1316

```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[f^2/(c*d^2 + a*e^2), Int[(f*x)^(m - 2)*(a*e + c*d*x^2)*(
a + c*x^4)^p, x], x] - Dist[(d*e*f^2)/(c*d^2 + a*e^2), Int[((f*x)^(m - 2)*(
a + c*x^4)^(p + 1))/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && LtQ
[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \frac{\int \frac{ae+cdx^2}{\left(a+cx^4\right)^2}\,dx}{cd^2+ae^2} - \frac{(de)\int \frac{1}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx}{cd^2+ae^2}$$

$$= \frac{x\left(ae+cdx^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\int \frac{-3ae-cdx^2}{a+cx^4}\,dx}{4a\left(cd^2+ae^2\right)} - \frac{(de)\int \left(\frac{e^2}{\left(cd^2+ae^2\right)\left(d+ex^2\right)} + \frac{c\left(d-ex^2\right)}{\left(cd^2+ae^2\right)\left(a+cx\right.}\right.}{cd^2+ae^2}$$

$$= \frac{x\left(ae+cdx^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{(cde)\int \frac{d-ex^2}{a+cx^4}\,dx}{\left(cd^2+ae^2\right)^2} - \frac{\left(de^3\right)\int \frac{1}{d+ex^2}\,dx}{\left(cd^2+ae^2\right)^2} - \frac{\left(d-\frac{3\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{\sqrt{a}}{}}{8a\left(cd^2+\right.}$$

$$= \frac{x\left(ae+cdx^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\sqrt{d}\,e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} - \frac{\left(d\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)e\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)^2}$$

$$= \frac{x\left(ae+cdx^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\sqrt{d}\,e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} + \frac{\sqrt[4]{c}\left(d-\frac{3\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\,\right.}{16\sqrt{2}\,a^{5/4}\left(cd^2+\right.}$$

$$= \frac{x\left(ae+cdx^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\sqrt{d}\,e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}\left(d+\frac{3\sqrt{a}\,e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}}{\sqrt[4]{a}}\right.}{8\sqrt{2}\,a^{5/4}\left(cd^2+ae^2\right)}$$

$$= \frac{x\left(ae+cdx^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\sqrt{d}\,e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} + \frac{\sqrt[4]{c}\,de\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\right.}{2\sqrt{2}\,a^{3/4}\left(cd^2+ae^2\right)^2}$$

Mathematica [A] time = 0.28, size = 428, normalized size = 0.62

$$\frac{\frac{\sqrt{2}\left(-3a^{3/2}e^3+\sqrt{a}\,cd^2e+5a\sqrt{c}\,de^2+c^{3/2}d^3\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{a^{5/4}\sqrt[4]{c}} - \frac{\sqrt{2}\left(-3a^{3/2}e^3+\sqrt{a}\,cd^2e+5a\sqrt{c}\,de^2+c^{3/2}d^3\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{a^{5/4}\sqrt[4]{c}} - \frac{2\sqrt{2}\left(3a^{3/2}e^3-\sqrt{a}\,cd^2e+5a\sqrt{c}\,de^2+c^{3/2}d^3\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{5/4}\sqrt[4]{c}} + \frac{2\sqrt{2}\left(3a^{3/2}e^3-\sqrt{a}\,cd^2e+5a\sqrt{c}\,de^2+c^{3/2}d^3\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{a^{5/4}\sqrt[4]{c}} + \frac{8\left(ae^2+cd^2\right)\left(aex+cdx^3\right)}{a\left(a+cx^4\right)} - 32\sqrt{d}\,e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{32\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*(c*d^2 + a*e^2)*(a*e*x + c*d*x^3))/(a*(a + c*x^4)) - 32*Sqrt[d]*e^(5/2)
*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - (2*Sqrt[2]*(c^(3/2)*d^3 - Sqrt[a]*c*d^2*e +
5*a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(a^(5/4)*c^(1/4)) + (2*Sqrt[2]*(c^(3/2)*d^3 - Sqrt[a]*c*d^2*e + 5*a*Sqrt[c
]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(5/4)*
c^(1/4)) + (Sqrt[2]*(c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 5*a*Sqrt[c]*d*e^2 - 3*
a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(5/
4)*c^(1/4)) - (Sqrt[2]*(c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 5*a*Sqrt[c]*d*e^2 -
 3*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^
(5/4)*c^(1/4)))/(32*(c*d^2 + a*e^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[x^2/((d + e*x^2)*(a + c*x^4)^2), x]
```

fricas [B] time = 26.34, size = 9774, normalized size = 14.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(c^2*d^3 + a*c*d*e^2)*x^3 + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e
^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4
*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*
d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 +
143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c
*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^
12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 +
28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 +
4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c
^4*d^8 + 18*a*c^3*d^6*e^2 + 112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^
4*e^8)*x + (a^2*c^4*d^8*e + 6*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5
*c*d^2*e^7 + 27*a^6*e^9 - (a^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^
7*e^4 + 34*a^7*c^3*d^5*e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^
6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 79
9*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6
*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*
e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a
^13*c*e^16)))*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d
^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqr
t(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^
6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 +
 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^
5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^
14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 +
4*a^5*c*d^2*e^6 + a^6*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (
a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d
^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^
4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^
2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e
^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4
 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^1
1*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*
c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8
 + 18*a*c^3*d^6*e^2 + 112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)
*x - (a^2*c^4*d^8*e + 6*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2
*e^7 + 27*a^6*e^9 - (a^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4
+ 34*a^7*c^3*d^5*e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12
 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*
c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d
^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 +
56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*
e^16)))*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4
*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^
6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 79
9*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6
*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*
e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a
^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*
c*d^2*e^6 + a^6*e^8))) + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*
d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3
```

```
 - 30*a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*
a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*
d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 +
81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*
a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*
d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^
6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8 + 18*
a*c^3*d^6*e^2 + 112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (
a^2*c^4*d^8*e + 6*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 +
 27*a^6*e^9 + (a^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a
^7*c^3*d^5*e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*
a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^
4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^
2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^1
0*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16))
)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c
^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12
 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*
c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d
^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 +
56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*
e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*
e^6 + a^6*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 +
2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*
a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*
d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^
4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6
*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^
6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^
12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2
+ 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8 + 18*a*c^3*
d^6*e^2 + 112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x - (a^2*c^
4*d^8*e + 6*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^
6*e^9 + (a^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3
*d^5*e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*a*c^5*
d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8
- 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28
*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*
d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt
((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c^3*d^6
*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*
a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^
4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^
2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^1
0*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16))
)/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 +
a^6*e^8))) + 8*(a*c*e^2*x^4 + a^2*e^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)
*x - d)/(e*x^2 + d)) + 4*(a*c*d^2*e + a^2*e^3)*x)/(a^2*c^2*d^4 + 2*a^3*c*d^
2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4), 1/16*(4
*(c^2*d^3 + a*c*d*e^2)*x^3 - 16*(a*c*e^2*x^4 + a^2*e^2)*sqrt(d*e)*arctan(sq
rt(d*e)*x/d) + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^
2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*
d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*
e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 +
540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^1
2)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^
10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 +
 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*
a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8 + 18*a*c^3*d^6*
```

e^2 + 112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (a^2*c^4*d^
8*e + 6*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^6*e^
9 - (a^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3*d^5
*e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10
*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 55
8*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7
*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*
e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt((2*
c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2
 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^
5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^
8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 +
28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^
4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a
^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*
e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*
d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5
+ (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 +
a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^
3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^
5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6
 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^1
2*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^
2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8 + 18*a*c^3*d^6*e^2 +
112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x - (a^2*c^4*d^8*e +
6*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^6*e^9 - (a
^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3*d^5*e^6 +
 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 +
 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*
c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d
^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 +
 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt((2*c^2*d^
5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a
^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10
*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 55
8*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7
*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*
e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4
*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)))
 + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^
2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 - (a^2
*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^
8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*
d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*
d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*
a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*
d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*
e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8 + 18*a*c^3*d^6*e^2 + 112*a^
2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (a^2*c^4*d^8*e + 6*a^3*
c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^6*e^9 + (a^4*c^6
*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3*d^5*e^6 + 21*a^
8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a
^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*
e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^
4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^
11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt((2*c^2*d^5*e +
4*a*c*d^3*e^3 - 30*a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2
*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 +
 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*

```
c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d
^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 +
 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 +
 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))) - (a^
2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^
3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 - (a^2*c^4*d
^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqr
t(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^
6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 +
 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^
5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^
14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 +
4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8 + 18*a*c^3*d^6*e^2 + 112*a^2*c^2*
d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x - (a^2*c^4*d^8*e + 6*a^3*c^3*d^
6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^6*e^9 + (a^4*c^6*d^11
+ 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3*d^5*e^6 + 21*a^8*c^2*
d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4
*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 +
 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56
*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3
*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt((2*c^2*d^5*e + 4*a*c*
d^3*e^3 - 30*a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e
^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a
^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*
e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^
4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^
11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3
*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))) + 4*(a*c*d^
2*e + a^2*e^3)*x)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2
*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)]
```

giac [A] time = 0.50, size = 603, normalized size = 0.88

$$-\frac{\sqrt{d}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{5}{2}}}{c^2d^4+2\,acd^2e^2+a^2e^4}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e-\left(ac^3\right)^{\frac{3}{4}}cd^3-3\left(ac^3\right)^{\frac{1}{4}}a^2ce^3-5\left(ac^3\right)^{\frac{3}{4}}ade^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2c^4d^4+2\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e-\left(ac^3\right)^{\frac{3}{4}}cd^3-3\left(ac^3\right)^{\frac{1}{4}}a^2ce^3-5\left(ac^3\right)^{\frac{3}{4}}ade^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2c^4d^4+2\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e+\left(ac^3\right)^{\frac{3}{4}}cd^3-3\left(ac^3\right)^{\frac{1}{4}}a^2ce^3+5\left(ac^3\right)^{\frac{3}{4}}ade^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^2c^4d^4+2\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e+\left(ac^3\right)^{\frac{3}{4}}cd^3-3\left(ac^3\right)^{\frac{1}{4}}a^2ce^3+5\left(ac^3\right)^{\frac{3}{4}}ade^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^2c^4d^4+2\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)}+\frac{cdx^3+axe}{4\left(cx^4+a\right)\left(acd^2+a^2e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(5/2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e
^4) - 1/8*((a*c^3)^(1/4)*a*c^2*d^2*e - (a*c^3)^(3/4)*c*d^3 - 3*(a*c^3)^(1/4
)*a^2*c*e^3 - 5*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a
/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 +
sqrt(2)*a^4*c^2*e^4) - 1/8*((a*c^3)^(1/4)*a*c^2*d^2*e - (a*c^3)^(3/4)*c*d^3
 - 3*(a*c^3)^(1/4)*a^2*c*e^3 - 5*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*
(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a
^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/16*((a*c^3)^(1/4)*a*c^2*d^2*e + (
a*c^3)^(3/4)*c*d^3 - 3*(a*c^3)^(1/4)*a^2*c*e^3 + 5*(a*c^3)^(3/4)*a*d*e^2)*l
og(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2
)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/16*((a*c^3)^(1/4)*a*c^2*d^2*e
+ (a*c^3)^(3/4)*c*d^3 - 3*(a*c^3)^(1/4)*a^2*c*e^3 + 5*(a*c^3)^(3/4)*a*d*e^2
)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqr
t(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/4*(c*d*x^3 + a*x*e)/((c*x^4
 + a)*(a*c*d^2 + a^2*e^2))
```

maple [A] time = 0.02, size = 852, normalized size = 1.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*c*d*x^3*e^2+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*c^2
*d^3/a*x^3+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x*e^3*a+1/4/(a*e^2+c*d^2)^2/(c*x^4
+a)*x*e*c*d^2+3/16/(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)
^(1/4)*x-1)*e^3-1/16/(a*e^2+c*d^2)^2/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
a/c)^(1/4)*x-1)*c*d^2*e+3/32/(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a
/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*e
^3-1/32/(a*e^2+c*d^2)^2/a*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x
+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*c*d^2*e+3/16/(a*e^2+
c*d^2)^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e^3-1/16/(a*e^
2+c*d^2)^2/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*c*d^2*e+5/
32/(a*e^2+c*d^2)^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^
(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d*e^2+1/32/(a*e^2+c*d^2)^2/
a*c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/
c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^3+5/16/(a*e^2+c*d^2)^2/(a/c)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d*e^2+1/16/(a*e^2+c*d^2)^2/a*c/(a/c)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^3+5/16/(a*e^2+c*d^2)^2/(a/c)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^2+1/16/(a*e^2+c*d^2)^2/a*c
/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^3-d*e^3/(a*e^2+c*d^2
)^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.15, size = 472, normalized size = 0.69

$$-\frac{de^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\sqrt{de}}+\frac{cdx^3+aex}{4\left(a^2cd^2+a^3e^2+\left(ac^2d^2+a^2ce^2\right)x^4\right)}+\frac{\frac{2\sqrt{2}\left(\sqrt{a}c^2d^3-ac^{\frac{3}{2}}d^2e+5\,a^{\frac{3}{2}}cde^2+3\,a^2\sqrt{c}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(\sqrt{a}c^2d^3-ac^{\frac{3}{2}}d^2e+5\,a^{\frac{3}{2}}cde^2+3\,a^2\sqrt{c}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\left(\sqrt{a}c^2d^3+ac^{\frac{3}{2}}d^2e+5\,a^{\frac{3}{2}}cde^2-3\,a^2\sqrt{c}e^3\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}c^2d^3+ac^{\frac{3}{2}}d^2e+5\,a^{\frac{3}{2}}cde^2-3\,a^2\sqrt{c}e^3\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}}{32\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -d*e^3*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e)
) + 1/4*(c*d*x^3 + a*e*x)/(a^2*c*d^2 + a^3*e^2 + (a*c^2*d^2 + a^2*c*e^2)*x^
4) + 1/32*(2*sqrt(2)*(sqrt(a)*c^2*d^3 - a*c^(3/2)*d^2*e + 5*a^(3/2)*c*d*e^2
 + 3*a^2*sqrt(c)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(
1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sq
rt(2)*(sqrt(a)*c^2*d^3 - a*c^(3/2)*d^2*e + 5*a^(3/2)*c*d*e^2 + 3*a^2*sqrt(c
)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt
(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) - sqrt(2)*(sqrt(a)*c^
2*d^3 + a*c^(3/2)*d^2*e + 5*a^(3/2)*c*d*e^2 - 3*a^2*sqrt(c)*e^3)*log(sqrt(c
)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + sqrt(2)*(s
qrt(a)*c^2*d^3 + a*c^(3/2)*d^2*e + 5*a^(3/2)*c*d*e^2 - 3*a^2*sqrt(c)*e^3)*l
og(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(a
*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)
```

mupad [B] time = 2.87, size = 17812, normalized size = 26.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((e*x)/(4*(a*e^2 + c*d^2)) + (c*d*x^3)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^4) +
 atan((((((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d
^11*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*
d^5*e^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d
^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - (x*(-(c^3*d^6*(-a^5*c)^(
1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*
a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1
/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4
+ 4*a^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3
 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*
e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d
^2*e^15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2
```

```
 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2
) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*
(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5
*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) +
 (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 39
68*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*
a^6*c^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^
3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^
5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2
*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8
+ a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6))
)^(1/2) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928
*a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a
^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*
e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5
*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) +
31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*
d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(81*a^4*c^3*e
^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2
*e^11))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 +
 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2)
- 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-
a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d
^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*1i
- (((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d^11*e
^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*d^5*e
^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^
6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + (x*(-(c^3*d^6*(-a^5*c)^(1/2)
- 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c
*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/
(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a
^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 32
7680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 +
 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^
15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*
a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2
*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5
*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8
+ 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(
128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^
3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c
^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6
*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^
(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*
e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5
*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/
2) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928*a^3*
c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a^6*e^
8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4))
)*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e -
4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^
2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e
^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (x*(81*a^4*c^3*e^13 +
 c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11
))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^
4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a
^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c
)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 +
4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*1i)/((((
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((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d^11*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*d^5*e^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - (x*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928*a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d^11*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*d^5*e^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + (x*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928*a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a^6*e^8 + a^2*c^4*d

^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (5*c^5*d^5*e^7 + 54*a*c^4*d^3*e^9 + 81*a^2*c^3*d*e^11)/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4))))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*2i + atan((((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d^11*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*d^5*e^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - (x*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928*a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*1i - (((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d^11*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*d^5*e^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + (x*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 6553

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6*a^4*c^11*d^14*e^3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 -
327680*a^7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13
+ 327680*a^10*c^5*d^2*e^15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6
+ 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3
*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5
+ 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a
^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*
d^2*e^6)))^(1/2) - (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2
*c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^
6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*
d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2)
 - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*
c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))
/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*
a^8*c^2*d^2*e^6)))^(1/2) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*
c^7*d^8*e^6 + 928*a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*
d^2*e^12)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2
 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2)
 + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(
-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*
d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) +
(x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 -
 108*a^3*c^4*d^2*e^11))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a
^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*
(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a
*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*
e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e
^6)))^(1/2)*1i)/((((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728
*a^4*c^9*d^11*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 55296
0*a^7*c^6*d^5*e^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 +
 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - (x*((c^3*d^6*(
-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*
e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-
a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3
*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11
*d^14*e^3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7
*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a
^10*c^5*d^2*e^15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^
3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5
*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*
d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 +
 a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))
^(1/2) + (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*
e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10
 - 7424*a^6*c^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 +
4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e
^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 +
9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9
*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^
2*e^6)))^(1/2) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^
6 + 928*a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/
(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c
^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c
^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1
/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^
6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(81*a^4
*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c
^4*d^2*e^11))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6
*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(
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1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e
^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*
c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2
) + (((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d^11
*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*d^5
*e^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*
e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + (x*((c^3*d^6*(-a^5*c)^(1/2)
 - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*
c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))
/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*
a^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 3
27680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9
+ 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e
^15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6
*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2
*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5
*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8
+ 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(
128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^
3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c
^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6
*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(
1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e
^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*
c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2
) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928*a^3*c
^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a^6*e^8
 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))
*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*
a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*
c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2
 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (x*(81*a^4*c^3*e^13 + c
^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))
/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*
c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*
c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(
1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a
^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (5*c^5*d^
5*e^7 + 54*a*c^4*d^3*e^9 + 81*a^2*c^3*d*e^11)/(128*(a^6*e^8 + a^2*c^4*d^8 +
 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4))))*((c^3*d^6*(-a^
5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3
 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5
*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^
4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*2i - (atan((((-d*e^5)^(1/2)*((x*(81*a^4*
c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^
4*d^2*e^11))/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*
e^2 + 6*a^4*c^2*d^4*e^4)) - ((((c^9*d^12*e^2)/16 + (13*a*c^8*d^10*e^4)/16 +
 (21*a^2*c^7*d^8*e^6)/8 + (29*a^3*c^6*d^6*e^8)/8 + (805*a^4*c^5*d^4*e^10)/1
6 + (777*a^5*c^4*d^2*e^12)/16)/(2*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6
+ 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + ((((-d*e^5)^(1/2)*((208*a^9*c^4
*d*e^15 + 16*a^3*c^10*d^13*e^3 + 288*a^4*c^9*d^11*e^5 + 1200*a^5*c^8*d^9*e^
7 + 2240*a^6*c^7*d^7*e^9 + 2160*a^7*c^6*d^5*e^11 + 1056*a^8*c^5*d^3*e^13)/(
2*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*
d^4*e^4)) - (x*(-d*e^5)^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^
3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8
*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*
d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a^6*e^8 + a^2*c^4*d^8
+ 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4))))/(2*(a^2*e^4 +
 c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14
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+ 768*a^2*c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30
592*a^5*c^6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/(256*(a^6*e^8 + a^2*c^4*d^8
+ 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-d*e^5)^(1/2)
)/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d*e^5)^(1/2))/(2*(a^2*e^4 + c^
2*d^4 + 2*a*c*d^2*e^2)))*1i)/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2) + ((-d*e^5
)^(1/2)*((x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*
d^4*e^9 - 108*a^3*c^4*d^2*e^11))/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*
e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + ((((c^9*d^12*e^2)/16 + (13*
a*c^8*d^10*e^4)/16 + (21*a^2*c^7*d^8*e^6)/8 + (29*a^3*c^6*d^6*e^8)/8 + (805
*a^4*c^5*d^4*e^10)/16 + (777*a^5*c^4*d^2*e^12)/16)/(2*(a^6*e^8 + a^2*c^4*d^
8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + ((((-d*e^5)
^(1/2)*((208*a^9*c^4*d*e^15 + 16*a^3*c^10*d^13*e^3 + 288*a^4*c^9*d^11*e^5 +
1200*a^5*c^8*d^9*e^7 + 2240*a^6*c^7*d^7*e^9 + 2160*a^7*c^6*d^5*e^11 + 1056
*a^8*c^5*d^3*e^13)/(2*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*
d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + (x*(-d*e^5)^(1/2)*(65536*a^11*c^4*e^17 - 65
536*a^4*c^11*d^14*e^3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7
- 327680*a^7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^1
3 + 327680*a^10*c^5*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a^
6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e
^4))))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (x*(128*a*c^10*d^13*e^2 -
14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*
a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/(256*(a^
6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e
^4)))*(-d*e^5)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d*e^5)^(1/
2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*1i)/(a^2*e^4 + c^2*d^4 + 2*a*c
*d^2*e^2))/(((5*c^5*d^5*e^7)/128 + (27*a*c^4*d^3*e^9)/64 + (81*a^2*c^3*d*e^
11)/128)/(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a
^4*c^2*d^4*e^4) - ((-d*e^5)^(1/2)*((x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a
*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))/(256*(a^6*e^8 +
a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - (
(((c^9*d^12*e^2)/16 + (13*a*c^8*d^10*e^4)/16 + (21*a^2*c^7*d^8*e^6)/8 + (29
*a^3*c^6*d^6*e^8)/8 + (805*a^4*c^5*d^4*e^10)/16 + (777*a^5*c^4*d^2*e^12)/16
)/(2*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c
^2*d^4*e^4)) + ((((-d*e^5)^(1/2)*((208*a^9*c^4*d*e^15 + 16*a^3*c^10*d^13*e^
3 + 288*a^4*c^9*d^11*e^5 + 1200*a^5*c^8*d^9*e^7 + 2240*a^6*c^7*d^7*e^9 + 21
60*a^7*c^6*d^5*e^11 + 1056*a^8*c^5*d^3*e^13)/(2*(a^6*e^8 + a^2*c^4*d^8 + 4*
a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - (x*(-d*e^5)^(1/2)
*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 327680*a^5*c^10*d^12*e^5
- 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^1
1 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^15))/(512*(a^2*e^4 + c^
2*d^4 + 2*a*c*d^2*e^2)*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3
*d^6*e^2 + 6*a^4*c^2*d^4*e^4))))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) +
(x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 396
8*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a
^6*c^5*d^3*e^12))/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3
*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-d*e^5)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a
*c*d^2*e^2)))*(-d*e^5)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))))/(a^
2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2) + ((-d*e^5)^(1/2)*((x*(81*a^4*c^3*e^13 + c
^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))
/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*
c^2*d^4*e^4)) + ((((c^9*d^12*e^2)/16 + (13*a*c^8*d^10*e^4)/16 + (21*a^2*c^7
*d^8*e^6)/8 + (29*a^3*c^6*d^6*e^8)/8 + (805*a^4*c^5*d^4*e^10)/16 + (777*a^5
*c^4*d^2*e^12)/16)/(2*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*
d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + ((((-d*e^5)^(1/2)*((208*a^9*c^4*d*e^15 + 16
*a^3*c^10*d^13*e^3 + 288*a^4*c^9*d^11*e^5 + 1200*a^5*c^8*d^9*e^7 + 2240*a^6
*c^7*d^7*e^9 + 2160*a^7*c^6*d^5*e^11 + 1056*a^8*c^5*d^3*e^13)/(2*(a^6*e^8 +
a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) +
(x*(-d*e^5)^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 327680*a
^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 + 32768

```
0*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^15))/(
512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^
2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4))))/(2*(a^2*e^4 + c^2*d^4 + 2
*a*c*d^2*e^2)) - (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c
^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*
d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^
2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-d*e^5)^(1/2))/(2*(a^2*e^
4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d*e^5)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*
c*d^2*e^2))))/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d*e^5)^(1/2)*1i)/(a^2
*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.186 $\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=689

$$-\frac{\sqrt[4]{c}\,e^2\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}\,e^2\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}-$$

Rubi [A] time = 0.60, antiderivative size = 689, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1239, 205, 1179, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{c}\,e^2\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}\,e^2\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}\left(\sqrt{a}\,e+3\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{a}\,e+3\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{c}\,e^2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}\,e^2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}\left(3\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(3\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\,a^{7/4}\left(ae^2+cd^2\right)}+\frac{cx\left(d-ex^2\right)}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (c*x*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (e^(7/2)*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)^2) - (c^(1/4)*e^2*(Sqrt[c]*d - Sq
rt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2
 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]
*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)
*(c*d^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(1/4)*e^2*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]
*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(
7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2
)^2) + (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1239

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx=\int \left(\frac{e^4}{\left(cd^2+ae^2\right)^2\left(d+ex^2\right)}+\frac{c\left(d-ex^2\right)}{\left(cd^2+ae^2\right)\left(a+cx^4\right)^2}-\frac{ce^2\left(-d+ex^2\right)}{\left(cd^2+ae^2\right)^2\left(a+cx^4\right)}\right)dx$$

$$=-\frac{\left(ce^2\right)\int \frac{-d+ex^2}{a+cx^4}\,dx}{\left(cd^2+ae^2\right)^2}+\frac{e^4\int \frac{1}{d+ex^2}\,dx}{\left(cd^2+ae^2\right)^2}+\frac{c\int \frac{d-ex^2}{\left(a+cx^4\right)^2}\,dx}{cd^2+ae^2}$$

$$=\frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2}+\frac{\left(\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)e^2\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)^2}+\frac{\left(e^2\left(\right.\right.}{}$$

$$=\frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2}+\frac{\left(\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4\left(cd^2+ae^2\right)^2}+$$

$$=\frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2}-\frac{\sqrt[4]{c}\,e^2\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\right.}{4\sqrt{2}\,a^{3/4}\left(cd^2+ae^2\right)^2}$$

$$=\frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2}-\frac{\sqrt[4]{c}\,e^2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right.}{2\sqrt{2}\,a^{3/4}\left(cd^2+ae^2\right)^2}$$

$$=\frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2}-\frac{\sqrt[4]{c}\,e^2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right.}{2\sqrt{2}\,a^{3/4}\left(cd^2+ae^2\right)^2}$$

Mathematica [A] time = 0.29, size = 429, normalized size = 0.62

$$\frac{-\frac{\sqrt{2}\sqrt[4]{c}\left(5a^{3/2}e^3+\sqrt{a}\,cd^2e+7a\sqrt{c}\,de^2+3c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{a^{7/4}}+\frac{\sqrt{2}\sqrt[4]{c}\left(5a^{3/2}e^3+\sqrt{a}\,cd^2e+7a\sqrt{c}\,de^2+3c^{3/2}d^3\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{a^{7/4}}+\frac{2\sqrt{2}\sqrt[4]{c}\left(5a^{3/2}e^3+\sqrt{a}\,cd^2e-7a\sqrt{c}\,de^2-3c^{3/2}d^3\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{7/4}}-\frac{2\sqrt{2}\sqrt[4]{c}\left(5a^{3/2}e^3+\sqrt{a}\,cd^2e-7a\sqrt{c}\,de^2-3c^{3/2}d^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{a^{7/4}}+\frac{8cx\left(d-ex^2\right)\left(ae^2+cd^2\right)}{a\left(a+cx^4\right)}+\frac{32e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}}}{32\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*c*(c*d^2 + a*e^2)*x*(d - e*x^2))/(a*(a + c*x^4)) + (32*e^(7/2)*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]
*c*d^2*e - 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/a^(7/4) - (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e
- 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/a^(7/4) - (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*Sqrt[c
]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*Sqr
t[c]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2])/a^(7/4))/(32*(c*d^2 + a*e^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((d + e*x^2)*(a + c*x^4)^2), x]
```

fricas [B] time = 41.28, size = 9892, normalized size = 14.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(c^2*d^2*e + a*c*e^3)*x^3 + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*
e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e +
44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^
5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d
^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8
 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e
^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*
a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))
/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a
^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*
a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x + (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2
+ 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^
10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^1
0*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a
^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*
d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*
d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^1
0 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5
*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 +
 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*
c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^
4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d
^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8
+ 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^
16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^
6 + a^7*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*
a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70
*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6
*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^
5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e
^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*
e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 2
8*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4
*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5
*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625
*a^4*c*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4
 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*
e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*
e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748
*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*
e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^
5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e
^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3
 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 +
4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a
^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*
d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*
d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^1
0 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 +
4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) + (a^2
*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3
```

*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x + (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 - (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 - (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) - 8*(a*c*e^3*x^4 + a^2*e^3)*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)) - 4*(c^2*d^3 + a*c*d*e^2)*x)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4), -1/16*(4*(c^2*d^2*e + a*c*e^3)*x^3 - 16*(a*c*e^3*x^4 + a^2*e^3)*sqrt(e/d)*arctan(x*sqrt(e/d)) + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c

```
^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*
c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8
))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^
2*d^2*e^6 - 625*a^4*c*e^8)*x + (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*
a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e +
 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^
2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5
*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^
10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e
^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28
*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 4
4*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5
*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^
10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8
- 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^
2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a
^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/
(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^
7*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^
2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c
*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2
*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*
e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 +
625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 +
56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13
*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d
^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 +
 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c
*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198
*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 +
26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*
sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c
^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/
(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10
*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 +
8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*
a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*
c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5
*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^
10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e
^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28
*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*
c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) + (a^2*c^2*d
^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4
)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8
 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(
-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^
6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*
c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6
+ 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^1
4*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^
4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 +
1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x + (27*a^2*c^5
*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 17
5*a^6*c*d*e^8 - (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 +
34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 +
738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*
c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8
*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^
```

```
8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a
^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^
4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*
sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c
^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/
(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10
*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 +
8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c
^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2
+ a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^
5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2
+ 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a
*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d
^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*
d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8
 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e
^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e
^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4
+ 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*c^4*d^
7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 - (a^6*
c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 +
21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 +
2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^
5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^
9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d
^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c
^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6
*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 +
738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*
c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8
*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^
8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a
^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*
d^2*e^6 + a^7*e^8))) - 4*(c^2*d^3 + a*c*d*e^2)*x)/(a^2*c^2*d^4 + 2*a^3*c*d^
2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)]
```

giac [A] time = 0.47, size = 603, normalized size = 0.88

$$\frac{\left(3\left(ac^3\right)^{\frac{1}{4}}c^3d^3+7\left(ac^3\right)^{\frac{1}{4}}ac^2de^2-\left(ac^3\right)^{\frac{3}{4}}cd^2e-5\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2c^4d^4+2\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)}+\frac{\left(3\left(ac^3\right)^{\frac{1}{4}}c^3d^3+7\left(ac^3\right)^{\frac{1}{4}}ac^2de^2-\left(ac^3\right)^{\frac{3}{4}}cd^2e-5\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2c^4d^4+2\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)}+\frac{\left(3\left(ac^3\right)^{\frac{1}{4}}c^3d^3+7\left(ac^3\right)^{\frac{1}{4}}ac^2de^2+\left(ac^3\right)^{\frac{3}{4}}cd^2e+5\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^2c^4d^4+2\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)}-\frac{\left(3\left(ac^3\right)^{\frac{1}{4}}c^3d^3+7\left(ac^3\right)^{\frac{1}{4}}ac^2de^2+\left(ac^3\right)^{\frac{3}{4}}cd^2e+5\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^2c^4d^4+2\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)}+\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{7}{2}}}{\left(c^2d^4+2acd^2e^2+a^2e^4\right)\sqrt{d}}-\frac{cx^3e-cdx}{4\left(cx^4+a\right)\left(acd^2+a^2e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c^3)^(3/4)*
c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1
/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + sqrt(2
)*a^4*c^2*e^4) + 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2
 - (a*c^3)^(3/4)*c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x -
 sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3
*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/16*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)
^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)*a*e^3)*log(x^2
 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*
c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/16*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c
^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)*a*e^3)*log(
x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a
^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + arctan(x*e^(1/2)/sqrt(d))*e^(7/2)/(
(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d)) - 1/4*(c*x^3*e - c*d*x)/((c*x^
4 + a)*(a*c*d^2 + a^2*e^2))
```

maple [A] time = 0.02, size = 873, normalized size = 1.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3*x^3-1/4*c^2/(a*e^2+c*d^2)^2/(c*x^4+a)*
e/a*x^3*d^2+1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*d*x*e^2+1/4*c^2/(a*e^2+c*d^2)^2
/(c*x^4+a)*d^3/a*x+7/16*c/(a*e^2+c*d^2)^2/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/c)^(1/4)*x-1)*d*e^2+3/16*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^3+7/32*c/(a*e^2+c*d^2)^2/a*(a/c)^(1/4)*2
^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*
x+(a/c)^(1/2)))*d*e^2+3/32*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^(1/4)*2^(1/2)*ln((
x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/
2)))*d^3+7/16*c/(a*e^2+c*d^2)^2/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^
(1/4)*x+1)*d*e^2+3/16*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/c)^(1/4)*x+1)*d^3-5/32/(a*e^2+c*d^2)^2/(a/c)^(1/4)*2^(1/2)*ln((x^2
-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))
)*e^3-1/32*c/(a*e^2+c*d^2)^2/a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1
/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^2*e-5/16/(a*e
^2+c*d^2)^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^3-1/16*c/
(a*e^2+c*d^2)^2/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2*e
-5/16/(a*e^2+c*d^2)^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e
^3-1/16*c/(a*e^2+c*d^2)^2/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*
x+1)*d^2*e+e^4/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.06, size = 506, normalized size = 0.73

$$\frac{e^4 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(c^2d^4 + 2\,acd^2e^2 + a^2e^4\right)\sqrt{de}} + \frac{c\left(\frac{2\,\sqrt{2}\left(3\,c^{\frac{3}{2}}d^3 - \sqrt{a}\,cd^2e + 7\,a\sqrt{c}\,de^2 - 5\,a^{\frac{3}{2}}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}\,x + \sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{c}}\,\sqrt{c}} + \frac{2\,\sqrt{2}\left(3\,c^{\frac{3}{2}}d^3 - \sqrt{a}\,cd^2e + 7\,a\sqrt{c}\,de^2 - 5\,a^{\frac{3}{2}}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}\,x - \sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{c}}\,\sqrt{c}} + \frac{\sqrt{2}\left(3\,c^{\frac{3}{2}}d^3 + \sqrt{a}\,cd^2e + 7\,a\sqrt{c}\,de^2 + 5\,a^{\frac{3}{2}}e^3\right)\log\left(\sqrt{c}\,x^2 + \sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}\left(3\,c^{\frac{3}{2}}d^3 + \sqrt{a}\,cd^2e + 7\,a\sqrt{c}\,de^2 + 5\,a^{\frac{3}{2}}e^3\right)\log\left(\sqrt{c}\,x^2 - \sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{32\left(ac^2d^4 + 2\,a^2cd^2e^2 + a^3e^4\right)} - \frac{\left(c^2d^2e + ace^3\right)x^3 - \left(c^2d^3 + acde^2\right)x}{4\left(a^2c^2d^4 + 2\,a^3cd^2e^2 + a^4e^4 + \left(ac^3d^4 + 2\,a^2c^2d^2e^2 + a^3ce^4\right)x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] e^4*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e)) +
 1/32*c*(2*sqrt(2)*(3*c^(3/2)*d^3 - sqrt(a)*c*d^2*e + 7*a*sqrt(c)*d*e^2 - 5
*a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sq
rt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(3
*c^(3/2)*d^3 - sqrt(a)*c*d^2*e + 7*a*sqrt(c)*d*e^2 - 5*a^(3/2)*e^3)*arctan(
1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/
(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(3*c^(3/2)*d^3 + sqrt(a)*
c*d^2*e + 7*a*sqrt(c)*d*e^2 + 5*a^(3/2)*e^3)*log(sqrt(c)*x^2 + sqrt(2)*a^(1
/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*c^(3/2)*d^3 + sqrt(
a)*c*d^2*e + 7*a*sqrt(c)*d*e^2 + 5*a^(3/2)*e^3)*log(sqrt(c)*x^2 - sqrt(2)*a
^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2
 + a^3*e^4) - 1/4*((c^2*d^2*e + a*c*e^3)*x^3 - (c^2*d^3 + a*c*d*e^2)*x)/(a^
2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^
3*c*e^4)*x^4)
```

mupad [B] time = 2.73, size = 17945, normalized size = 26.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((c*d*x)/(4*a*(a*e^2 + c*d^2)) - (c*e*x^3)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^
4) - atan((((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c
^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8
*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a
^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*
```

```
e^4)) - (x*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c
^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^
(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a
^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*
c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*
c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a
^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 +
 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a
^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*
e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(
-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*
d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a
^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c
^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e
^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)
))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e
 + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) +
39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^
2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e
^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 +
21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8
 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(
-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^
3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4
*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^
3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8
*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(
128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^
2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4
*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c
)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4
*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*1i - (((((
65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 3
6864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 +
663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4
*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*((9*c
^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^
5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c
*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 +
4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536
*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 3
27680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13
 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6
 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25
*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d
*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(
256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*
c^2*d^4*e^4)))^(1/2) + (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 +
7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 666
88*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8
 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(
-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^
3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4
*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^
3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c
^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^
5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e
^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) -
25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c
```

*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*1i)/((125*a^2*c^5*e^12 + 81*c^7*d^4*e^8 + 270*a*c^6*d^2*e^10)/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^

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5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2)
 + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c
*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (x*(1152*a^2*c^
11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9
*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d
^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2
 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1
/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*
e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*
c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)
 - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 123
20*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(
a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4
*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*
d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/
2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10
*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (x*(1425*a^4*
c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532
*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c
^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(
-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*
a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*
e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^
4)))^(1/2)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4
*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c
)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4
*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*2i - atan(
((((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e
^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e
^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a
^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x
*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e +
 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39
*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*
e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 -
 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e
^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^
4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d
^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/
2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a
^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1
/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 +
6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e
^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8
 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c
^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^
3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*
c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d
^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*
a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432
*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*
c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c
*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(
1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70
*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^
(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2
+ 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612
*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e
^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)
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))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e
 + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) -
39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^
2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*1i - (((((65536*a^11
*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c
^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 + 663552*a^9
*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a
^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*((25*a^3*e^6*(-
a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*
e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(
-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*
d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*
d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*
c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*
a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c
^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(
-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*
a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*
e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^
4)))^(1/2) + (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c
^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7
*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c
*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(
1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70
*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^
(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2
+ 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13
+ 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33
296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5
*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6
*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 4
1*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^1
1*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*
e^4)))^(1/2) + (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 +
 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8
 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*
(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^
3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4
*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^
3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*1i)/((125*a^2*c^5*e^12 + 81*c^7*d^4*
e^8 + 270*a*c^6*d^2*e^10)/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4
*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (((((65536*a^11*c^4*e^16 - 12288*a
^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 24576
0*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331
776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^
5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^
3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^
5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256
*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2
*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a
^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 32768
0*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))
/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*
c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a
^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7
*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 +
 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(11
52*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 2035
2*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*
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a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*2i + (atan(-(((((((45*a*c^10*d^11*e^3)/16 + (1277*a^6*c^5*d*e^13)/16 + (305*a^2*c^9*d^9*e^5)/16

+ (385*a^3*c^8*d^7*e^7)/8 + (657*a^4*c^7*d^5*e^9)/8 + (2081*a^5*c^6*d^3*e^11)/16)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (((((256*a^11*c^4*e^16 - 48*a^4*c^11*d^14*e^2 - 224*a^5*c^10*d^12*e^4 - 144*a^6*c^9*d^10*e^6 + 960*a^7*c^8*d^8*e^8 + 2480*a^8*c^7*d^6*e^10 + 2592*a^9*c^6*d^4*e^12 + 1296*a^10*c^5*d^2*e^14)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x*(-d*e^7)^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(512*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) + (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2)*1i)/(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2) - ((((((45*a*c^10*d^11*e^3)/16 + (1277*a^6*c^5*d*e^13)/16 + (305*a^2*c^9*d^9*e^5)/16 + (385*a^3*c^8*d^7*e^7)/8 + (657*a^4*c^7*d^5*e^9)/8 + (2081*a^5*c^6*d^3*e^11)/16)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (((((256*a^11*c^4*e^16 - 48*a^4*c^11*d^14*e^2 - 224*a^5*c^10*d^12*e^4 - 144*a^6*c^9*d^10*e^6 + 960*a^7*c^8*d^8*e^8 + 2480*a^8*c^7*d^6*e^10 + 2592*a^9*c^6*d^4*e^12 + 1296*a^10*c^5*d^2*e^14)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*(-d*e^7)^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(512*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) + (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) - (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2)*1i)/(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2))/(((((((45*a*c^10*d^11*e^3)/16 + (1277*a^6*c^5*d*e^13)/16 + (305*a^2*c^9*d^9*e^5)/16 + (385*a^3*c^8*d^7*e^7)/8 + (657*a^4*c^7*d^5*e^9)/8 + (2081*a^5*c^6*d^3*e^11)/16)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (((((256*a^11*c^4*e^16 - 48*a^4*c^11*d^14*e^2 - 224*a^5*c^10*d^12*e^4 - 144*a^6*c^9*d^10*e^6 + 960*a^7*c^8*d^8*e^8 + 2480*a^8*c^7*d^6*e^10 + 2592*a^9*c^6*d^4*e^12 + 1296*a^10*c^5*d^2*e^14)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x*(-d*e^7)^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(512*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) + (x*(1425*a^4*c^5*

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e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3
*c^6*d^2*e^11))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d
^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(c^2*d^5 + a^2*d*e^4 + 2*a*c*
d^3*e^2) - ((125*a^2*c^5*e^12)/128 + (81*c^7*d^4*e^8)/128 + (135*a*c^6*d^2*
e^10)/64)/(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*
a^6*c^2*d^4*e^4) + ((((((45*a*c^10*d^11*e^3)/16 + (1277*a^6*c^5*d*e^13)/16
+ (305*a^2*c^9*d^9*e^5)/16 + (385*a^3*c^8*d^7*e^7)/8 + (657*a^4*c^7*d^5*e^9
)/8 + (2081*a^5*c^6*d^3*e^11)/16)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e
^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (((((256*a^11*c^4*e^16 - 48*
a^4*c^11*d^14*e^2 - 224*a^5*c^10*d^12*e^4 - 144*a^6*c^9*d^10*e^6 + 960*a^7*
c^8*d^8*e^8 + 2480*a^8*c^7*d^6*e^10 + 2592*a^9*c^6*d^4*e^12 + 1296*a^10*c^5
*d^2*e^14)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2
+ 6*a^6*c^2*d^4*e^4)) + (x*(-d*e^7)^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*
c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680
*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 32
7680*a^12*c^5*d^2*e^15))/(512*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)*(a^8*e^
8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4))
)*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) + (x*(1152*a^2*
c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c
^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6
*d^3*e^12))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e
^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d
^3*e^2)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) - (x*(1
425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^
9 + 2532*a^3*c^6*d^2*e^11))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 +
 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(c^2*d^5 + a^2*d*
e^4 + 2*a*c*d^3*e^2)))*(-d*e^7)^(1/2)*1i)/(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*
e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.187 $\int \frac{1}{x^2(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=745

$$-\frac{c^{3/4}\left(\sqrt{c}\,d-3\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{9/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(\sqrt{c}\,d-3\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{9/4}\left(ae^2+cd^2\right)}+$$

Rubi [A] time = 0.77, antiderivative size = 745, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1336, 205, 1179, 1168, 1162, 617, 204, 1165, 628}

$$\frac{c^{3/4}\left(a^{3/2}e^3-\sqrt{c}\,d\left(2ae^2+cd^2\right)\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{9/4}\left(ae^2+cd^2\right)^2}-\frac{c^{3/4}\left(\sqrt{c}\,d-3\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{9/4}\left(ae^2+cd^2\right)}-\frac{c^{3/4}\left(a^{3/2}e^3-\sqrt{c}\,d\left(2ae^2+cd^2\right)\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{9/4}\left(ae^2+cd^2\right)^2}+\frac{c^{3/4}\left(\sqrt{c}\,d-3\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{9/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(a^{3/2}e^3+\sqrt{c}\,d\left(2ae^2+cd^2\right)\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{9/4}\left(ae^2+cd^2\right)^2}+\frac{c^{3/4}\left(3\sqrt{a}\,e+\sqrt{c}\,d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{9/4}\left(ae^2+cd^2\right)}-\frac{c^{3/4}\left(a^{3/2}e^3+\sqrt{c}\,d\left(2ae^2+cd^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{9/4}\left(ae^2+cd^2\right)^2}-\frac{c^{3/4}\left(3\sqrt{a}\,e+\sqrt{c}\,d\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\,a^{9/4}\left(ae^2+cd^2\right)}-\frac{cx\left(ae+cdx^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{1}{a^2dx}-\frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -(1/(a^2*d*x)) - (c*x*(a*e + c*d*x^2))/(4*a^2*(c*d^2 + a*e^2)*(a + c*x^4))
- (e^(9/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 + a*e^2)^2) + (c^(3
/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*S
qrt[2]*a^(9/4)*(c*d^2 + a*e^2)) + (c^(3/4)*(a^(3/2)*e^3 + Sqrt[c]*d*(c*d^2
+ 2*a*e^2))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*(c*
d^2 + a*e^2)^2) - (c^(3/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)) - (c^(3/4)*(a^(3/2)*
e^3 + Sqrt[c]*d*(c*d^2 + 2*a*e^2))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)*(Sqrt[c]*d - 3*Sqrt[a]*e)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(9/4
)*(c*d^2 + a*e^2)) + (c^(3/4)*(a^(3/2)*e^3 - Sqrt[c]*d*(c*d^2 + 2*a*e^2))*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(9/4)*(
c*d^2 + a*e^2)^2) + (c^(3/4)*(Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)) -
(c^(3/4)*(a^(3/2)*e^3 - Sqrt[c]*d*(c*d^2 + 2*a*e^2))*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1336

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rubi steps

$$\int \frac{1}{x^2\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \int \left(\frac{1}{a^2dx^2} - \frac{e^5}{d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} - \frac{c\left(ae+cdx^2\right)}{a\left(cd^2+ae^2\right)\left(a+cx^4\right)^2} + \frac{c\left(-a^2e^3 - c\right.}{a^2\left(cd^2+\right.}\right.$$

$$= -\frac{1}{a^2dx} + \frac{c\int \frac{-a^2e^3-cd\left(cd^2+2ae^2\right)x^2}{a+cx^4}\,dx}{a^2\left(cd^2+ae^2\right)^2} - \frac{e^5\int \frac{1}{d+ex^2}\,dx}{d\left(cd^2+ae^2\right)^2} - \frac{c\int \frac{ae+cdx^2}{\left(a+cx^4\right)^2}\,dx}{a\left(cd^2+ae^2\right)}$$

$$= -\frac{1}{a^2dx} - \frac{cx\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)^2} + \frac{c\int \frac{-3ae-cdx^2}{a+cx^4}\,dx}{4a^2\left(cd^2+ae^2\right)} + \frac{(c}{}$$

$$= -\frac{1}{a^2dx} - \frac{cx\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)^2} + \frac{\left(c\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\right)\int \frac{\sqrt{a}\,\sqrt{}}{a+}}{8a^2\left(cd^2+ae^2\right.}$$

$$= -\frac{1}{a^2dx} - \frac{cx\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)^2} - \frac{c^{5/4}\left(cd^3+2ade^2-\frac{a^3}{}\right.}{4\sqrt{}}$$

$$= -\frac{1}{a^2dx} - \frac{cx\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)^2} + \frac{c^{5/4}\left(cd^3+2ade^2+\frac{a}{}\right.}{2\sqrt{2}\,a^{9/4}}$$

$$= -\frac{1}{a^2dx} - \frac{cx\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)^2} + \frac{c^{3/4}\left(\sqrt{c}\,d+3\sqrt{a}\,e\right)\text{ta}}{8\sqrt{2}\,a^{9/4}\left(cd\right.}$$

Mathematica [A] time = 0.37, size = 499, normalized size = 0.67

$$\frac{1}{32}\left(\frac{\sqrt{2}c^{3/4}\left(7a^{3/2}e^3+3\sqrt{a}cd^2e-9a\sqrt{c}de^2-5c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{9/4}\left(ae^2+cd^2\right)^2} + \frac{\sqrt{2}c^{3/4}\left(-7a^{3/2}e^3-3\sqrt{a}cd^2e+9a\sqrt{c}de^2+5c^{3/2}d^3\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{9/4}\left(ae^2+cd^2\right)^2} + \frac{2\sqrt{2}c^{3/4}\left(7a^{3/2}e^3+3\sqrt{a}cd^2e+9a\sqrt{c}de^2+5c^{3/2}d^3\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{9/4}\left(ae^2+cd^2\right)^2} - \frac{2\sqrt{2}c^{3/4}\left(7a^{3/2}e^3+3\sqrt{a}cd^2e+9a\sqrt{c}de^2+5c^{3/2}d^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{a^{9/4}\left(ae^2+cd^2\right)^2} - \frac{8cx\left(ae+cdx^2\right)}{a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)} - \frac{32}{a^2dx} - \frac{32e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(ae^2+cd^2\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-32/(a^2*d*x) - (8*c*x*(a*e + c*d*x^2))/(a^2*(c*d^2 + a*e^2)*(a + c*x^4))
- (32*e^(9/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 + a*e^2)^2) + (2
*Sqrt[2]*c^(3/4)*(5*c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e + 9*a*Sqrt[c]*d*e^2 + 7
*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(9/4)*(c*d^2 + a*
e^2)^2) - (2*Sqrt[2]*c^(3/4)*(5*c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e + 9*a*Sqrt[
c]*d*e^2 + 7*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(9/4)
*(c*d^2 + a*e^2)^2) + (Sqrt[2]*c^(3/4)*(-5*c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e
- 9*a*Sqrt[c]*d*e^2 + 7*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*
x + Sqrt[c]*x^2])/(a^(9/4)*(c*d^2 + a*e^2)^2) + (Sqrt[2]*c^(3/4)*(5*c^(3/2)
*d^3 - 3*Sqrt[a]*c*d^2*e + 9*a*Sqrt[c]*d*e^2 - 7*a^(3/2)*e^3)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(9/4)*(c*d^2 + a*e^2)^2))/32
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(d + e*x^2)*(a + c*x^4)^2), x]
```

fricas [B] time = 111.35, size = 10188, normalized size = 13.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*a*c^2*d^4 + 32*a^2*c*d^2*e^2 + 16*a^3*e^4 + 4*(5*c^3*d^4 + 9*a*c
^2*d^2*e^2 + 4*a^2*c*e^4)*x^4 + 4*(a*c^2*d^3*e + a^2*c*d*e^3)*x^2 - ((a^2*c
^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*
e^2 + a^5*d*e^4)*x)*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d
*e^5 + (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e
^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*
e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 +
 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*
e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 2
8*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5
*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))*log(-(625*c^
6*d^8 + 3250*a*c^5*d^6*e^2 + 4944*a^2*c^4*d^4*e^4 + 686*a^3*c^3*d^2*e^6 - 2
401*a^4*c^2*e^8)*x + (75*a^3*c^5*d^8*e + 418*a^4*c^4*d^6*e^3 + 684*a^5*c^3*
d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a^7*c*e^9 - (5*a^7*c^5*d^11 + 29*a^8*c^
4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + 74*a^10*c^2*d^5*e^6 + 41*a^11*c*d^3*e^8 +
9*a^12*d*e^10)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8
*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10
+ 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12
*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 +
28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))*sqrt(-(30*c^4*d^5*e
 + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 + (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2
 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 405
0*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c
^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a
^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^
4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14
 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^
7*c*d^2*e^6 + a^8*e^8))) + ((a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)
*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4)*x)*sqrt(-(30*c^4*d^5*e +
 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 + (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 +
 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*
a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5
*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^1
0*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*
d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 +
 a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*
c*d^2*e^6 + a^8*e^8))*log(-(625*c^6*d^8 + 3250*a*c^5*d^6*e^2 + 4944*a^2*c^4
*d^4*e^4 + 686*a^3*c^3*d^2*e^6 - 2401*a^4*c^2*e^8)*x - (75*a^3*c^5*d^8*e +
418*a^4*c^4*d^6*e^3 + 684*a^5*c^3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a^7*c
*e^9 - (5*a^7*c^5*d^11 + 29*a^8*c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + 74*a^10*
c^2*d^5*e^6 + 41*a^11*c*d^3*e^8 + 9*a^12*d*e^10)*sqrt(-(625*c^9*d^12 + 4050
*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^
5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^
10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4
*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14
+ a^17*e^16)))*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5
+ (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 +
a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 +
 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401
*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 +
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 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^1
5*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*
d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))) - ((a^2*c^3*d^5
+ 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a
^5*d*e^4)*x)*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 -
(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^
8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3
868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a
^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 5
6*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*
c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^
6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))*log(-(625*c^6*d^8 +
 3250*a*c^5*d^6*e^2 + 4944*a^2*c^4*d^4*e^4 + 686*a^3*c^3*d^2*e^6 - 2401*a^4
*c^2*e^8)*x + (75*a^3*c^5*d^8*e + 418*a^4*c^4*d^6*e^3 + 684*a^5*c^3*d^4*e^5
 + 126*a^6*c^2*d^2*e^7 - 343*a^7*c*e^9 + (5*a^7*c^5*d^11 + 29*a^8*c^4*d^9*e
^2 + 66*a^9*c^3*d^7*e^4 + 74*a^10*c^2*d^5*e^6 + 41*a^11*c*d^3*e^8 + 9*a^12*
d*e^10)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 +
3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*
a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 +
56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15
*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))*sqrt(-(30*c^4*d^5*e + 124*
a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^
6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8
*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*
e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7
*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e
^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17
*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2
*e^6 + a^8*e^8))) + ((a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 +
(a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4)*x)*sqrt(-(30*c^4*d^5*e + 124*a*
c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*
c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d
^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^
8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d
^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8
 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e
^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e
^6 + a^8*e^8))*log(-(625*c^6*d^8 + 3250*a*c^5*d^6*e^2 + 4944*a^2*c^4*d^4*e^
4 + 686*a^3*c^3*d^2*e^6 - 2401*a^4*c^2*e^8)*x - (75*a^3*c^5*d^8*e + 418*a^4
*c^4*d^6*e^3 + 684*a^5*c^3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a^7*c*e^9 +
(5*a^7*c^5*d^11 + 29*a^8*c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + 74*a^10*c^2*d^5
*e^6 + 41*a^11*c*d^3*e^8 + 9*a^12*d*e^10)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*
d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e
^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*
d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^
8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*
e^16)))*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 - (a^4*
c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8
)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a
^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^
3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^1
2*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d
^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2
 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))) - 8*(a^2*c*e^4*x^5 + a^
3*e^4*x)*sqrt(-e/d)*log((e*x^2 - 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)))/((a^2*
c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3
*e^2 + a^5*d*e^4)*x), -1/16*(16*a*c^2*d^4 + 32*a^2*c*d^2*e^2 + 16*a^3*e^4 +
 4*(5*c^3*d^4 + 9*a*c^2*d^2*e^2 + 4*a^2*c*e^4)*x^4 + 4*(a*c^2*d^3*e + a^2*c
*d*e^3)*x^2 + 16*(a^2*c*e^4*x^5 + a^3*e^4*x)*sqrt(e/d)*arctan(x*sqrt(e/d))
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- ((a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a
^4*c*d^3*e^2 + a^5*d*e^4)*x)*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*
a^2*c^2*d*e^5 + (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^
7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2
*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d
^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*
c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6
*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^
8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))*log
(-(625*c^6*d^8 + 3250*a*c^5*d^6*e^2 + 4944*a^2*c^4*d^4*e^4 + 686*a^3*c^3*d^
2*e^6 - 2401*a^4*c^2*e^8)*x + (75*a^3*c^5*d^8*e + 418*a^4*c^4*d^6*e^3 + 684
*a^5*c^3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a^7*c*e^9 - (5*a^7*c^5*d^11 +
29*a^8*c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + 74*a^10*c^2*d^5*e^6 + 41*a^11*c*d
^3*e^8 + 9*a^12*d*e^10)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^
2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*
d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11
*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^
6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))*sqrt(-(30*
c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 + (a^4*c^4*d^8 + 4*a^5*c^
3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d
^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6
417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d
^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 7
0*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c
*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e
^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))) + ((a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4
*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4)*x)*sqrt(-(30*c^
4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 + (a^4*c^4*d^8 + 4*a^5*c^3*
d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^1
2 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 641
7*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^1
6 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*
a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d
^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4
 + 4*a^7*c*d^2*e^6 + a^8*e^8))*log(-(625*c^6*d^8 + 3250*a*c^5*d^6*e^2 + 494
4*a^2*c^4*d^4*e^4 + 686*a^3*c^3*d^2*e^6 - 2401*a^4*c^2*e^8)*x - (75*a^3*c^5
*d^8*e + 418*a^4*c^4*d^6*e^3 + 684*a^5*c^3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 -
343*a^7*c*e^9 - (5*a^7*c^5*d^11 + 29*a^8*c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 +
 74*a^10*c^2*d^5*e^6 + 41*a^11*c*d^3*e^8 + 9*a^12*d*e^10)*sqrt(-(625*c^9*d^
12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 64
17*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^
16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70
*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*
d^2*e^14 + a^17*e^16)))*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c
^2*d*e^5 + (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d
^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*
d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^
10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d
^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10
 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4
*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))) - ((a^2
*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^
3*e^2 + a^5*d*e^4)*x)*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2
*d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2
*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^
8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10
 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^1
2*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 +
 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a
```

```
^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))*log(-(625*
c^6*d^8 + 3250*a*c^5*d^6*e^2 + 4944*a^2*c^4*d^4*e^4 + 686*a^3*c^3*d^2*e^6 -
 2401*a^4*c^2*e^8)*x + (75*a^3*c^5*d^8*e + 418*a^4*c^4*d^6*e^3 + 684*a^5*c^
3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a^7*c*e^9 + (5*a^7*c^5*d^11 + 29*a^8*
c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + 74*a^10*c^2*d^5*e^6 + 41*a^11*c*d^3*e^8
+ 9*a^12*d*e^10)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d
^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^1
0 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^
12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10
+ 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))*sqrt(-(30*c^4*d^5
*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e
^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4
050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4
*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8
*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*
c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^
14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*
a^7*c*d^2*e^6 + a^8*e^8))) + ((a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^
4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4)*x)*sqrt(-(30*c^4*d^5*e
 + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2
 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 405
0*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c
^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a
^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^
4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14
 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^
7*c*d^2*e^6 + a^8*e^8))*log(-(625*c^6*d^8 + 3250*a*c^5*d^6*e^2 + 4944*a^2*c
^4*d^4*e^4 + 686*a^3*c^3*d^2*e^6 - 2401*a^4*c^2*e^8)*x - (75*a^3*c^5*d^8*e
+ 418*a^4*c^4*d^6*e^3 + 684*a^5*c^3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a^7
*c*e^9 + (5*a^7*c^5*d^11 + 29*a^8*c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + 74*a^1
0*c^2*d^5*e^6 + 41*a^11*c*d^3*e^8 + 9*a^12*d*e^10)*sqrt(-(625*c^9*d^12 + 40
50*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*
c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*
a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c
^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^1
4 + a^17*e^16)))*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^
5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6
+ a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4
 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 24
01*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4
 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a
^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^
3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))))/((a^2*c^3*d^5
 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 +
a^5*d*e^4)*x)]
```

giac [A] time = 0.45, size = 639, normalized size = 0.86

$$-\frac{\left(3\left(ac^{3}\right)^{\frac{1}{4}}ac^{2}d^{2}e+5\left(ac^{3}\right)^{\frac{3}{4}}cd^{3}+7\left(ac^{3}\right)^{\frac{1}{4}}a^{2}ce^{3}+9\left(ac^{3}\right)^{\frac{3}{4}}ade^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^{3}c^{3}d^{4}+2\sqrt{2}a^{4}c^{2}d^{2}e^{2}+\sqrt{2}a^{5}ce^{4}\right)}-\frac{\left(3\left(ac^{3}\right)^{\frac{1}{4}}ac^{2}d^{2}e+5\left(ac^{3}\right)^{\frac{3}{4}}cd^{3}+7\left(ac^{3}\right)^{\frac{1}{4}}a^{2}ce^{3}+9\left(ac^{3}\right)^{\frac{3}{4}}ade^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^{3}c^{3}d^{4}+2\sqrt{2}a^{4}c^{2}d^{2}e^{2}+\sqrt{2}a^{5}ce^{4}\right)}-\frac{\left(3\left(ac^{3}\right)^{\frac{1}{4}}ac^{2}d^{2}e-5\left(ac^{3}\right)^{\frac{3}{4}}cd^{3}+7\left(ac^{3}\right)^{\frac{1}{4}}a^{2}ce^{3}-9\left(ac^{3}\right)^{\frac{3}{4}}ade^{2}\right)\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^{3}c^{3}d^{4}+2\sqrt{2}a^{4}c^{2}d^{2}e^{2}+\sqrt{2}a^{5}ce^{4}\right)}+\frac{\left(3\left(ac^{3}\right)^{\frac{1}{4}}ac^{2}d^{2}e-5\left(ac^{3}\right)^{\frac{3}{4}}cd^{3}+7\left(ac^{3}\right)^{\frac{1}{4}}a^{2}ce^{3}-9\left(ac^{3}\right)^{\frac{3}{4}}ade^{2}\right)\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^{3}c^{3}d^{4}+2\sqrt{2}a^{4}c^{2}d^{2}e^{2}+\sqrt{2}a^{5}ce^{4}\right)}-\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{9}{2}}}{\left(c^{2}d^{5}+2acd^{3}e^{2}+a^{2}de^{4}\right)\sqrt{d}}-\frac{5c^{2}d^{2}x^{4}+4acx^{4}e^{2}+acdx^{2}e+4acd^{2}+4a^{2}e^{2}}{4\left(a^{2}cd^{3}+a^{3}de^{2}\right)\left(cx^{5}+ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*(3*(a*c^3)^(1/4)*a*c^2*d^2*e + 5*(a*c^3)^(3/4)*c*d^3 + 7*(a*c^3)^(1/4)
*a^2*c*e^3 + 9*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/
c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^4*c^2*d^2*e^2 + s
qrt(2)*a^5*c*e^4) - 1/8*(3*(a*c^3)^(1/4)*a*c^2*d^2*e + 5*(a*c^3)^(3/4)*c*d^
3 + 7*(a*c^3)^(1/4)*a^2*c*e^3 + 9*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*
```

```
a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - 1/16*(3*(a*c^3)^(1/4)*a*c^2*d^2*e -
5*(a*c^3)^(3/4)*c*d^3 + 7*(a*c^3)^(1/4)*a^2*c*e^3 - 9*(a*c^3)^(3/4)*a*d*e^2
)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^3*d^4 + 2*sqr
t(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) + 1/16*(3*(a*c^3)^(1/4)*a*c^2*d^2
*e - 5*(a*c^3)^(3/4)*c*d^3 + 7*(a*c^3)^(1/4)*a^2*c*e^3 - 9*(a*c^3)^(3/4)*a*
d*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^3*d^4 +
2*sqrt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - arctan(x*e^(1/2)/sqrt(d))*
e^(9/2)/((c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4)*sqrt(d)) - 1/4*(5*c^2*d^2*x^
4 + 4*a*c*x^4*e^2 + a*c*d*x^2*e + 4*a*c*d^2 + 4*a^2*e^2)/((a^2*c*d^3 + a^3*
d*e^2)*(c*x^5 + a*x))
```

maple [A] time = 0.02, size = 911, normalized size = 1.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/a^2/d/x-1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*x^3*d*e^2-1/4*c^3/(a*e^2+c*d
^2)^2/a^2/(c*x^4+a)*x^3*d^3-1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*x*e^3-1/4*c^2/(
a*e^2+c*d^2)^2/a/(c*x^4+a)*x*e*d^2-7/32*c/(a*e^2+c*d^2)^2/a*(a/c)^(1/4)*2^(
1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+
(a/c)^(1/2)))*e^3-3/32*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+
(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))
*d^2*e-7/16*c/(a*e^2+c*d^2)^2/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1
/4)*x-1)*e^3-3/16*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(a/c)^(1/4)*x-1)*d^2*e-7/16*c/(a*e^2+c*d^2)^2/a*(a/c)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(a/c)^(1/4)*x+1)*e^3-3/16*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^2*e-9/32*c/(a*e^2+c*d^2)^2/a/(a/c)^(
1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^
(1/2)*x+(a/c)^(1/2)))*d*e^2-5/32*c^2/(a*e^2+c*d^2)^2/a^2/(a/c)^(1/4)*2^(1/2
)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/
c)^(1/2)))*d^3-9/16*c/(a*e^2+c*d^2)^2/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a/c)^(1/4)*x-1)*d*e^2-5/16*c^2/(a*e^2+c*d^2)^2/a^2/(a/c)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(a/c)^(1/4)*x-1)*d^3-9/16*c/(a*e^2+c*d^2)^2/a/(a/c)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^2-5/16*c^2/(a*e^2+c*d^2)^2/a^2/(a/c)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^3-1/d*e^5/(a*e^2+c*d^2)^2/
(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.11, size = 521, normalized size = 0.70

$$-\frac{e^{5} \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(c^{2} d^{5} + 2\,acd^{3} e^{2} + a^{2} d e^{4}\right)\sqrt{de}} - \frac{c\left(\frac{2\sqrt{2}\left(5\sqrt{a}c^{2}d^{3}+3\,ac^{\frac{3}{2}}d^{2}e+9\,a^{\frac{3}{2}}cde^{2}+7\,a^{2}\sqrt{c}e^{3}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\left(5\sqrt{a}c^{2}d^{3}+3\,ac^{\frac{3}{2}}d^{2}e+9\,a^{\frac{3}{2}}cde^{2}+7\,a^{2}\sqrt{c}e^{3}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\left(5\sqrt{a}c^{2}d^{3}-3\,ac^{\frac{3}{2}}d^{2}e+9\,a^{\frac{3}{2}}cde^{2}-7\,a^{2}\sqrt{c}e^{3}\right)\log\left(\sqrt{c}x^{2}+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\left(5\sqrt{a}c^{2}d^{3}-3\,ac^{\frac{3}{2}}d^{2}e+9\,a^{\frac{3}{2}}cde^{2}-7\,a^{2}\sqrt{c}e^{3}\right)\log\left(\sqrt{c}x^{2}-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{32\left(a^{2}c^{2}d^{4}+2\,a^{3}cd^{2}e^{2}+a^{4}e^{4}\right)} - \frac{acdex^{2}+\left(5\,c^{2}d^{2}+4\,ace^{2}\right)x^{4}+4\,acd^{2}+4\,a^{2}e^{2}}{4\left(\left(a^{2}c^{2}d^{3}+a^{3}cde^{2}\right)x^{5}+\left(a^{3}cd^{3}+a^{4}de^{2}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -e^5*arctan(e*x/sqrt(d*e))/((c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4)*sqrt(d*e)
) - 1/32*c*(2*sqrt(2)*(5*sqrt(a)*c^2*d^3 + 3*a*c^(3/2)*d^2*e + 9*a^(3/2)*c*
d*e^2 + 7*a^2*sqrt(c)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4
)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) +
 2*sqrt(2)*(5*sqrt(a)*c^2*d^3 + 3*a*c^(3/2)*d^2*e + 9*a^(3/2)*c*d*e^2 + 7*a
^2*sqrt(c)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/
sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) - sqrt(2)*(5
*sqrt(a)*c^2*d^3 - 3*a*c^(3/2)*d^2*e + 9*a^(3/2)*c*d*e^2 - 7*a^2*sqrt(c)*e^
3)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4))
 + sqrt(2)*(5*sqrt(a)*c^2*d^3 - 3*a*c^(3/2)*d^2*e + 9*a^(3/2)*c*d*e^2 - 7*a
^2*sqrt(c)*e^3)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(
3/4)*c^(3/4)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4) - 1/4*(a*c*d*e*x^2
```

```
 + (5*c^2*d^2 + 4*a*c*e^2)*x^4 + 4*a*c*d^2 + 4*a^2*e^2)/((a^2*c^2*d^3 + a^3
*c*d*e^2)*x^5 + (a^3*c*d^3 + a^4*d*e^2)*x)
```

mupad [B] time = 5.16, size = 24015, normalized size = 32.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] - (1/(a*d) + (c*e*x^2)/(4*a*(a*e^2 + c*d^2)) + (c*x^4*(4*a*e^2 + 5*c*d^2))/
(4*a^2*d*(a*e^2 + c*d^2)))/(a*x + c*x^5) - atan(((11875*a^5*c^10*d^15*e - a
^9*c^3*(72128*a^3*d*e^15 + 265655*c^3*d^7*e^9 - 76440*a*c^2*d^5*e^11 - 1785
85*a^2*c*d^3*e^13) + 68800*a^6*c^9*d^13*e^3 + 89403*a^7*c^8*d^11*e^5 - 1264
88*a^8*c^7*d^9*e^7)*(a^25*d^2*e^19*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^
3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3
*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^
(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*
a^11*c^2*d^4*e^4))^(5/2)*2i - a^15*c^2*e^17*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2
) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 12
4*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-
a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6
*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*3136i - a^11*c^10*d^19*x*(-(49*a^3*e^6*(-
a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c
^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2
*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4
*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*25i - a^16*c^9*d^20*e*x*(-(4
9*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e
 + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1
/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c
*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*2i + a^24*c*d^4*
e^17*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^
5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-
a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8
 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*14i +
 a^8*c^9*d^14*e^3*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^
(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c
^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8
+ a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4)
)^(1/2)*1250i + a^9*c^8*d^12*e^5*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*
d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d
^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1
/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^
11*c^2*d^4*e^4))^(1/2)*9900i + a^10*c^7*d^10*e^7*x*(-(49*a^3*e^6*(-a^9*c^3)
^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5
 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e
^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^
3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*31902i + a^11*c^6*d^8*e^9*x*(-(49*a^
3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 1
26*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2)
- 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2
*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*52008i + a^12*c^5*d^
6*e^11*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*
a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*
(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d
^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*422
38i + a^13*c^4*d^4*e^13*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9
*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 -
81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^1
3*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^
```

4*e^4))^(1/2)*10924i - a^14*c^3*d^2*e^15*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*5694i - a^12*c^9*d^17*e^2*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*216i - a^13*c^8*d^15*e^4*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*700i - a^14*c^7*d^13*e^6*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*808i + a^15*c^6*d^11*e^8*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*778i + a^16*c^5*d^9*e^10*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*3224i + a^17*c^4*d^7*e^12*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*3460i + a^18*c^3*d^5*e^14*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*1384i - a^19*c^2*d^3*e^16*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*57i - a^17*c^8*d^18*e^3*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*14i - a^18*c^7*d^16*e^5*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*40i - a^19*c^6*d^14*e^7*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*56i - a^20*c^5*d^12*e^9*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*28i + a^21*c^4*d^10*e^11*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)

*28i + a^22*c^3*d^8*e^13*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^
9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 -
 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^
13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d
^4*e^4))^(5/2)*56i + a^23*c^2*d^6*e^15*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 2
5*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6
*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c
^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2
+ 6*a^11*c^2*d^4*e^4))^(5/2)*40i - a^20*c*d*e^18*x*(-(49*a^3*e^6*(-a^9*c^3)
^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5
 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e
^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^
3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*128i) + (-a^9*c^3)^(1/2)*(3125*c^9*d
^16 + 21952*a^8*c*e^16 + 3000*a*c^8*d^14*e^2 - 77435*a^2*c^7*d^12*e^4 - 242
104*a^3*c^6*d^10*e^6 - 127665*a^4*c^5*d^8*e^8 + 240064*a^5*c^4*d^6*e^10 + 1
18199*a^6*c^3*d^4*e^12 - 130368*a^7*c^2*d^2*e^14)*(a^25*d^2*e^19*x*(-(49*a^
3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 1
26*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2)
- 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2
*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*2i - a^15*c^2*e^17*x
*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*
d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^
3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a
^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*3136i - a^1
1*c^10*d^19*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2)
+ 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4
e^2(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*
c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2
)*25i - a^16*c^9*d^20*e*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9
*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 -
81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^1
3*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^
4*e^4))^(5/2)*2i + a^24*c*d^4*e^17*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^
3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3
*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^
(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*
a^11*c^2*d^4*e^4))^(5/2)*14i + a^8*c^9*d^14*e^3*x*(-(49*a^3*e^6*(-a^9*c^3)^
(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5
+ 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^
4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3
*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*1250i + a^9*c^8*d^12*e^5*x*(-(49*a^3*
e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126
*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) -
39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e
^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*9900i + a^10*c^7*d^10*
e^7*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5
*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a
^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8
+ 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*31902i
 + a^11*c^6*d^8*e^9*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3
)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a
*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^
8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^
4))^(1/2)*52008i + a^12*c^5*d^6*e^11*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*
c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c
^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3
)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 +
6*a^11*c^2*d^4*e^4))^(1/2)*42238i + a^13*c^4*d^4*e^13*x*(-(49*a^3*e^6*(-a^9
*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*

d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*
d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^
10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*10924i - a^14*c^3*d^2*e^15*x*(-
(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5
*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^
(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12
*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*5694i - a^12*c
^9*d^17*e^2*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2)
+ 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4
e^2(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*
c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2
)*216i - a^13*c^8*d^15*e^4*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-
a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3
 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(
a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2
*d^4*e^4))^(3/2)*700i - a^14*c^7*d^13*e^6*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2)
- 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*
a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^
9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e
^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*808i + a^15*c^6*d^11*e^8*x*(-(49*a^3*e^6*(-
a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c
^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2
*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4
*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*778i + a^16*c^5*d^9*e^10*x*(
-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^
5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)
^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^1
2*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*3224i + a^17*
c^4*d^7*e^12*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2)
 + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^
4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9
*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/
2)*3460i + a^18*c^3*d^5*e^14*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*
(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e
^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))
/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c
^2*d^4*e^4))^(3/2)*1384i - a^19*c^2*d^3*e^16*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/
2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 1
24*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(
-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^
6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*57i - a^17*c^8*d^18*e^3*x*(-(49*a^3*e^6*
(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7
*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a
^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 +
 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*14i - a^18*c^7*d^16*e^5*x*
(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d
^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3
)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^
12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*40i - a^19*c
^6*d^14*e^7*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2)
+ 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4
e^2(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*
c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2
)*56i - a^20*c^5*d^12*e^9*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a
^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3
- 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a
^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*
d^4*e^4))^(5/2)*28i + a^21*c^4*d^10*e^11*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) -
 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a

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^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9
*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^
2 + 6*a^11*c^2*d^4*e^4))^(5/2)*28i + a^22*c^3*d^8*e^13*x*(-(49*a^3*e^6*(-a^
9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2
*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c
*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a
^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*56i + a^23*c^2*d^6*e^15*x*(-(4
9*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e
 + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1
/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c
*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*40i - a^20*c*d*e
^18*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5
*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a
^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8
+ 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*128i))
/(9765625*a^9*c^21*d^32 + 481890304*a^25*c^5*e^32 + 159765625*a^10*c^20*d^3
0*e^2 + 1159031250*a^11*c^19*d^28*e^4 + 4879001250*a^12*c^18*d^26*e^6 + 130
43411775*a^13*c^17*d^24*e^8 + 22507897839*a^14*c^16*d^22*e^10 + 23209461788
*a^15*c^15*d^20*e^12 + 7790140604*a^16*c^14*d^18*e^14 - 15160518297*a^17*c^
13*d^16*e^16 - 24964288057*a^18*c^12*d^14*e^18 - 11511478798*a^19*c^11*d^12
*e^20 + 8613907074*a^20*c^10*d^10*e^22 + 11397074817*a^21*c^9*d^8*e^24 + 58
6708977*a^22*c^8*d^6*e^26 - 3576733440*a^23*c^7*d^4*e^28 - 521228288*a^24*c
^6*d^2*e^30))*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2)
+ 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4
*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(256*(a^13*e^8 +
 a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))
)^(1/2)*2i - atan(((11875*a^5*c^10*d^15*e - a^9*c^3*(72128*a^3*d*e^15 + 265
655*c^3*d^7*e^9 - 76440*a*c^2*d^5*e^11 - 178585*a^2*c*d^3*e^13) + 68800*a^6
*c^9*d^13*e^3 + 89403*a^7*c^8*d^11*e^5 - 126488*a^8*c^7*d^9*e^7)*(a^25*d^2*
e^19*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^
5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-
a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8
 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*2i -
a^15*c^2*e^17*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2
) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d
^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^
9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1
/2)*3136i - a^11*c^10*d^19*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-
a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3
 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(
a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2
*d^4*e^4))^(3/2)*25i - a^16*c^9*d^20*e*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 4
9*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6
*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c
^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2
+ 6*a^11*c^2*d^4*e^4))^(5/2)*2i + a^24*c*d^4*e^17*x*(-(25*c^3*d^6*(-a^9*c^3
)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^
5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*
e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c
^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*14i + a^8*c^9*d^14*e^3*x*(-(25*c^3*
d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126
*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) +
39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e
^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*1250i + a^9*c^8*d^12*e
^5*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*
c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^
9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 +
 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*9900i +
 a^10*c^7*d^10*e^7*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)
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^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*31902i + a^11*c^6*d^8*e^9*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*52008i + a^12*c^5*d^6*e^11*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*42238i + a^13*c^4*d^4*e^13*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*10924i - a^14*c^3*d^2*e^15*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*5694i - a^12*c^9*d^17*e^2*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*216i - a^13*c^8*d^15*e^4*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*700i - a^14*c^7*d^13*e^6*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*808i + a^15*c^6*d^11*e^8*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*778i + a^16*c^5*d^9*e^10*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*3224i + a^17*c^4*d^7*e^12*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*3460i + a^18*c^3*d^5*e^14*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*1384i - a^19*c^2*d^3*e^16*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*57i - a^17*c^8*d^18*e^3*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*14i - a^18*c^7*d^16*e^5*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2)

+ 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*40i - a^19*c^6*d^14*e^7*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*56i - a^20*c^5*d^12*e^9*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*28i + a^21*c^4*d^10*e^11*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*28i + a^22*c^3*d^8*e^13*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*56i + a^23*c^2*d^6*e^15*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*40i - a^20*c*d*e^18*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*128i) - (-a^9*c^3)^(1/2)*(3125*c^9*d^16 + 21952*a^8*c*e^16 + 3000*a*c^8*d^14*e^2 - 77435*a^2*c^7*d^12*e^4 - 242104*a^3*c^6*d^10*e^6 - 127665*a^4*c^5*d^8*e^8 + 240064*a^5*c^4*d^6*e^10 + 118199*a^6*c^3*d^4*e^12 - 130368*a^7*c^2*d^2*e^14)*(a^25*d^2*e^19*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*2i - a^15*c^2*e^17*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*3136i - a^11*c^10*d^19*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*25i - a^16*c^9*d^20*e*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*2i + a^24*c*d^4*e^17*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*14i + a^8*c^9*d^14*e^3*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*1250i + a^9*c^8*d^12*e^5*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1

/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*9900i + a^10*c^7*d^10*e^7*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*31902i + a^11*c^6*d^8*e^9*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*52008i + a^12*c^5*d^6*e^11*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*42238i + a^13*c^4*d^4*e^13*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*10924i - a^14*c^3*d^2*e^15*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*5694i - a^12*c^9*d^17*e^2*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*216i - a^13*c^8*d^15*e^4*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*700i - a^14*c^7*d^13*e^6*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*808i + a^15*c^6*d^11*e^8*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*778i + a^16*c^5*d^9*e^10*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*3224i + a^17*c^4*d^7*e^12*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*3460i + a^18*c^3*d^5*e^14*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*1384i - a^19*c^2*d^3*e^16*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*57i - a^17*c^8*d^18*e^3*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))

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/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c
^2*d^4*e^4))^(5/2)*14i - a^18*c^7*d^16*e^5*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2)
 - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124
*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a
^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*
e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*40i - a^19*c^6*d^14*e^7*x*(-(25*c^3*d^6*(-
a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c
^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2
*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4
*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*56i - a^20*c^5*d^12*e^9*x*(-
(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5
*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^
(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12
*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*28i + a^21*c^4
*d^10*e^11*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) +
 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*
e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c
^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)
*28i + a^22*c^3*d^8*e^13*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^
9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 +
 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^
13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d
^4*e^4))^(5/2)*56i + a^23*c^2*d^6*e^15*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 4
9*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6
*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c
^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2
+ 6*a^11*c^2*d^4*e^4))^(5/2)*40i - a^20*c*d*e^18*x*(-(25*c^3*d^6*(-a^9*c^3)
^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5
 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e
^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^
3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*128i))/(9765625*a^9*c^21*d^32 + 4818
90304*a^25*c^5*e^32 + 159765625*a^10*c^20*d^30*e^2 + 1159031250*a^11*c^19*d
^28*e^4 + 4879001250*a^12*c^18*d^26*e^6 + 13043411775*a^13*c^17*d^24*e^8 +
22507897839*a^14*c^16*d^22*e^10 + 23209461788*a^15*c^15*d^20*e^12 + 7790140
604*a^16*c^14*d^18*e^14 - 15160518297*a^17*c^13*d^16*e^16 - 24964288057*a^1
8*c^12*d^14*e^18 - 11511478798*a^19*c^11*d^12*e^20 + 8613907074*a^20*c^10*d
^10*e^22 + 11397074817*a^21*c^9*d^8*e^24 + 586708977*a^22*c^8*d^6*e^26 - 35
76733440*a^23*c^7*d^4*e^28 - 521228288*a^24*c^6*d^2*e^30))*(-(25*c^3*d^6*(-
a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c
^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2
*c*d^2*e^4*(-a^9*c^3)^(1/2))/(256*(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^
6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4)))^(1/2)*2i - (atan((a^9*e^3*x*
(-d^3*e^9)^(5/2)*4096i - a^3*c^6*d^15*x*(-d^3*e^9)^(3/2)*26804i + c^9*d^24*
e^3*x*(-d^3*e^9)^(1/2)*625i - a^4*c^5*d^13*e^2*x*(-d^3*e^9)^(3/2)*24831i -
a^5*c^4*d^11*e^4*x*(-d^3*e^9)^(3/2)*8214i + a^6*c^3*d^9*e^6*x*(-d^3*e^9)^(3
/2)*13471i + a^7*c^2*d^7*e^8*x*(-d^3*e^9)^(3/2)*16128i + a^2*c^7*d^20*e^7*x
*(-d^3*e^9)^(1/2)*15951i + a*c^8*d^22*e^5*x*(-d^3*e^9)^(1/2)*4950i)/(4096*a
^9*d^8*e^25 + 625*c^9*d^26*e^7 + 4950*a*c^8*d^24*e^9 + 15951*a^2*c^7*d^22*e
^11 + 26804*a^3*c^6*d^20*e^13 + 24831*a^4*c^5*d^18*e^15 + 8214*a^5*c^4*d^16
*e^17 - 13471*a^6*c^3*d^14*e^19 - 16128*a^7*c^2*d^12*e^21))*(-d^3*e^9)^(1/2
)*1i)/(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.188 $\int \frac{1}{x^4(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=751

$$\frac{c^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\left(2ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{11/4}\left(ae^2+cd^2\right)^2}+\frac{c^{5/4}\left(\sqrt{a}\,e+3\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\right.}{16\sqrt{2}\,a^{11/4}\left(ae^2+cd^2\right)}$$

Rubi [A] time = 0.69, antiderivative size = 751, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1336, 205, 1179, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{c^2x\left(d-ex^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{c^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\left(2ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{11/4}\left(ae^2+cd^2\right)^2}+\frac{c^{5/4}\left(\sqrt{a}\,e+3\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{11/4}\left(ae^2+cd^2\right)}-\frac{c^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\left(2ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{11/4}\left(ae^2+cd^2\right)^2}-\frac{c^{5/4}\left(\sqrt{a}\,e+3\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{11/4}\left(ae^2+cd^2\right)}+\frac{c^{5/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\left(2ae^2+cd^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{11/4}\left(ae^2+cd^2\right)^2}+\frac{c^{5/4}\left(3\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{11/4}\left(ae^2+cd^2\right)}-\frac{c^{5/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\left(2ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{11/4}\left(ae^2+cd^2\right)^2}-\frac{c^{5/4}\left(3\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\,a^{11/4}\left(ae^2+cd^2\right)}+\frac{e}{a^2d^2x}-\frac{1}{3a^2dx^3}+\frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/(3*a^2*d*x^3) + e/(a^2*d^2*x) - (c^2*x*(d - e*x^2))/(4*a^2*(c*d^2 + a*e^
2)*(a + c*x^4)) + (e^(11/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/2)*(c*d^2 +
a*e^2)^2) + (c^(5/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*
x)/a^(1/4)])/(8*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d - S
qrt[a]*e)*(c*d^2 + 2*a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqr
t[2]*a^(11/4)*(c*d^2 + a*e^2)^2) - (c^(5/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTa
n[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)) -
(c^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + 2*a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)^2) + (c^(5/4)*(3*Sqrt[
c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
16*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*(c*
d^2 + 2*a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*S
qrt[2]*a^(11/4)*(c*d^2 + a*e^2)^2) - (c^(5/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(11/4)*(
c*d^2 + a*e^2)) - (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*(c*d^2 + 2*a*e^2)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(11/4)*(c*d^
2 + a*e^2)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1336

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rubi steps

$$\int \frac{1}{x^4\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \int\left(\frac{1}{a^2dx^4} - \frac{e}{a^2d^2x^2} + \frac{e^6}{d^2\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} - \frac{c^2\left(d-ex^2\right)}{a\left(cd^2+ae^2\right)\left(a+cx^4\right)^2} - \frac{c^2\left(\cdots\right.}{a^2\left(\cdots\right.}\right.$$

$$= -\frac{1}{3a^2dx^3} + \frac{e}{a^2d^2x} + \frac{e^6\int\frac{1}{d+ex^2}\,dx}{d^2\left(cd^2+ae^2\right)^2} - \frac{c^2\int\frac{d-ex^2}{\left(a+cx^4\right)^2}\,dx}{a\left(cd^2+ae^2\right)} - \frac{\left(c^2\left(cd^2+2ae^2\right)\right)\int\frac{d-ex^2}{a+cx^4}\cdots}{a^2\left(cd^2+ae^2\right)^2}$$

$$= -\frac{1}{3a^2dx^3} + \frac{e}{a^2d^2x} - \frac{c^2x\left(d-ex^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)^2} + \frac{c^2\int\frac{-3d+ex^2}{a+cx^4}\cdots}{4a^2\left(cd^2+a\cdots\right.}$$

$$= -\frac{1}{3a^2dx^3} + \frac{e}{a^2d^2x} - \frac{c^2x\left(d-ex^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)^2} - \frac{\left(c\left(\frac{3\sqrt{c}\,d}{\sqrt{a}} - e\cdots\right.\right.}{8a^2\left(\cdots\right.}$$

$$= -\frac{1}{3a^2dx^3} + \frac{e}{a^2d^2x} - \frac{c^2x\left(d-ex^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)^2} + \frac{c^{5/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}} + \cdots\right.}{\cdots}$$

$$= -\frac{1}{3a^2dx^3} + \frac{e}{a^2d^2x} - \frac{c^2x\left(d-ex^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)^2} + \frac{c^{5/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}} - \cdots\right.}{\cdots}$$

$$= -\frac{1}{3a^2dx^3} + \frac{e}{a^2d^2x} - \frac{c^2x\left(d-ex^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)^2} + \frac{c^{5/4}\left(\frac{3\sqrt{c}\,d}{\sqrt{a}} - \cdots\right.}{8\sqrt{2}\cdots}$$

Mathematica [A] time = 0.39, size = 513, normalized size = 0.68

$$\frac{1}{96}\left(\frac{3\sqrt{2}c^{5/4}\left(9a^{3/2}e^3+5\sqrt{a}cd^2e+11a\sqrt{c}de^2+7c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{11/4}\left(ae^2+cd^2\right)^2} - \frac{3\sqrt{2}c^{5/4}\left(9a^{3/2}e^3+5\sqrt{a}cd^2e+11a\sqrt{c}de^2+7c^{3/2}d^3\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{11/4}\left(ae^2+cd^2\right)^2} + \frac{6\sqrt{2}c^{5/4}\left(-9a^{3/2}e^3-5\sqrt{a}cd^2e+11a\sqrt{c}de^2+7c^{3/2}d^3\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{11/4}\left(ae^2+cd^2\right)^2} + \frac{6\sqrt{2}c^{5/4}\left(9a^{3/2}e^3+5\sqrt{a}cd^2e-11a\sqrt{c}de^2-7c^{3/2}d^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{a^{11/4}\left(ae^2+cd^2\right)^2} - \frac{24c^2x\left(d-ex^2\right)}{a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)} + \frac{96e}{a^2d^2x} - \frac{32}{a^2dx^3} + \frac{96e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(ae^2+cd^2\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-32/(a^2*d*x^3) + (96*e)/(a^2*d^2*x) - (24*c^2*x*(d - e*x^2))/(a^2*(c*d^2
+ a*e^2)*(a + c*x^4)) + (96*e^(11/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/2)*
(c*d^2 + a*e^2)^2) + (6*Sqrt[2]*c^(5/4)*(7*c^(3/2)*d^3 - 5*Sqrt[a]*c*d^2*e
+ 11*a*Sqrt[c]*d*e^2 - 9*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4
)])/(a^(11/4)*(c*d^2 + a*e^2)^2) + (6*Sqrt[2]*c^(5/4)*(-7*c^(3/2)*d^3 + 5*S
qrt[a]*c*d^2*e - 11*a*Sqrt[c]*d*e^2 + 9*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(a^(11/4)*(c*d^2 + a*e^2)^2) + (3*Sqrt[2]*c^(5/4)*(7*c^(
3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + 11*a*Sqrt[c]*d*e^2 + 9*a^(3/2)*e^3)*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(11/4)*(c*d^2 + a*e^2)^2
) - (3*Sqrt[2]*c^(5/4)*(7*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + 11*a*Sqrt[c]*d*
e^2 + 9*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
)/(a^(11/4)*(c*d^2 + a*e^2)^2))/96
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(d + e*x^2)*(a + c*x^4)^2), x]
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**A**] time = 0.50, size = 628, normalized size = 0.84

$$-\frac{\left(7\left(ac^{3}\right)^{\frac{1}{4}}c^{3}d^{3}+11\left(ac^{3}\right)^{\frac{1}{4}}ac^{2}de^{2}-5\left(ac^{3}\right)^{\frac{3}{4}}cd^{2}e-9\left(ac^{3}\right)^{\frac{3}{4}}ae^{3}\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^{3}c^{3}d^{4}+2\sqrt{2}a^{4}c^{2}d^{2}e^{2}+\sqrt{2}a^{5}ce^{4}\right)}-\frac{\left(7\left(ac^{3}\right)^{\frac{1}{4}}c^{3}d^{3}+11\left(ac^{3}\right)^{\frac{1}{4}}ac^{2}de^{2}-5\left(ac^{3}\right)^{\frac{3}{4}}cd^{2}e-9\left(ac^{3}\right)^{\frac{3}{4}}ae^{3}\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^{3}c^{3}d^{4}+2\sqrt{2}a^{4}c^{2}d^{2}e^{2}+\sqrt{2}a^{5}ce^{4}\right)}-\frac{\left(7\left(ac^{3}\right)^{\frac{1}{4}}c^{3}d^{3}+11\left(ac^{3}\right)^{\frac{1}{4}}ac^{2}de^{2}+5\left(ac^{3}\right)^{\frac{3}{4}}cd^{2}e+9\left(ac^{3}\right)^{\frac{3}{4}}ae^{3}\right)\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^{3}c^{3}d^{4}+2\sqrt{2}a^{4}c^{2}d^{2}e^{2}+\sqrt{2}a^{5}ce^{4}\right)}+\frac{\left(7\left(ac^{3}\right)^{\frac{1}{4}}c^{3}d^{3}+11\left(ac^{3}\right)^{\frac{1}{4}}ac^{2}de^{2}+5\left(ac^{3}\right)^{\frac{3}{4}}cd^{2}e+9\left(ac^{3}\right)^{\frac{3}{4}}ae^{3}\right)\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^{3}c^{3}d^{4}+2\sqrt{2}a^{4}c^{2}d^{2}e^{2}+\sqrt{2}a^{5}ce^{4}\right)}+\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{11}{2}}}{\left(c^{2}d^{6}+2acd^{4}e^{2}+a^{2}d^{2}e^{4}\right)\sqrt{d}}+\frac{c^{2}x^{3}e-c^{2}dx}{4\left(a^{2}cd^{2}+a^{3}e^{2}\right)\left(cx^{4}+a\right)}+\frac{3x^{2}e-d}{3a^{2}d^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*(7*(a*c^3)^(1/4)*c^3*d^3 + 11*(a*c^3)^(1/4)*a*c^2*d*e^2 - 5*(a*c^3)^(3
/4)*c*d^2*e - 9*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c
)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^4*c^2*d^2*e^2 + sq
rt(2)*a^5*c*e^4) - 1/8*(7*(a*c^3)^(1/4)*c^3*d^3 + 11*(a*c^3)^(1/4)*a*c^2*d*
e^2 - 5*(a*c^3)^(3/4)*c*d^2*e - 9*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^
4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - 1/16*(7*(a*c^3)^(1/4)*c^3*d^3 + 11*(a*
c^3)^(1/4)*a*c^2*d*e^2 + 5*(a*c^3)^(3/4)*c*d^2*e + 9*(a*c^3)^(3/4)*a*e^3)*l
og(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2
)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) + 1/16*(7*(a*c^3)^(1/4)*c^3*d^3 + 11
*(a*c^3)^(1/4)*a*c^2*d*e^2 + 5*(a*c^3)^(3/4)*c*d^2*e + 9*(a*c^3)^(3/4)*a*e^
3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^3*d^4 + 2*sq
rt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) + arctan(x*e^(1/2)/sqrt(d))*e^(1
1/2)/((c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4)*sqrt(d)) + 1/4*(c^2*x^3*e - c
^2*d*x)/((a^2*c*d^2 + a^3*e^2)*(c*x^4 + a)) + 1/3*(3*x^2*e - d)/(a^2*d^2*x^
3)
```

maple [**A**] time = 0.02, size = 932, normalized size = 1.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/3/a^2/d/x^3+e/a^2/d^2/x+1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*x^3*e^3+1/4*
c^3/(a*e^2+c*d^2)^2/a^2/(c*x^4+a)*x^3*e*d^2-1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^
4+a)*x*d*e^2-1/4*c^3/(a*e^2+c*d^2)^2/a^2/(c*x^4+a)*x*d^3-11/16*c^2/(a*e^2+c
*d^2)^2/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d*e^2-7/16*
c^3/(a*e^2+c*d^2)^2/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
*d^3-11/32*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*
2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d*e^2-7/32*
c^3/(a*e^2+c*d^2)^2/a^3*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(
a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^3-11/16*c^2/(a*e^2+c
*d^2)^2/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^2-7/16*
c^3/(a*e^2+c*d^2)^2/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)
*d^3+9/32*c/(a*e^2+c*d^2)^2/a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/
2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*e^3+5/32*c^2/(a*
e^2+c*d^2)^2/a^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1
/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^2*e+9/16*c/(a*e^2+c*d^2)^2/
a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^3+5/16*c^2/(a*e^2+c
*d^2)^2/a^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2*e+9/16*
```

```
c/(a*e^2+c*d^2)^2/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e^3
+5/16*c^2/(a*e^2+c*d^2)^2/a^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4
)*x+1)*d^2*e+1/d^2*e^6/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x
)
```

maxima [A] time = 2.11, size = 543, normalized size = 0.72

$$\frac{e^6\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(c^2d^6+2\,acd^4e^2+a^2d^2e^4\right)\sqrt{de}}-\frac{c^2\left(\frac{2\sqrt{2}\left(7c^{\frac{3}{2}}d^3-5\sqrt{a}cd^2e+11a\sqrt{c}de^2-9a^{\frac{3}{2}}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(7c^{\frac{3}{2}}d^3-5\sqrt{a}cd^2e+11a\sqrt{c}de^2-9a^{\frac{3}{2}}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(7c^{\frac{3}{2}}d^3+5\sqrt{a}cd^2e+11a\sqrt{c}de^2+9a^{\frac{3}{2}}e^3\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(7c^{\frac{3}{2}}d^3+5\sqrt{a}cd^2e+11a\sqrt{c}de^2+9a^{\frac{3}{2}}e^3\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{32\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4\right)}+\frac{3\left(5c^2d^2e+4\,ace^3\right)x^6-4\,acd^3-4\,a^2de^2-\left(7c^2d^3+4\,acde^2\right)x^4+12\left(acd^2e+a^2e^3\right)x^2}{12\left(\left(a^2c^2d^4+a^3cd^2e^2\right)x^7+\left(a^3cd^4+a^4d^2e^2\right)x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] e^6*arctan(e*x/sqrt(d*e))/((c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4)*sqrt(d*e
)) - 1/32*c^2*(2*sqrt(2)*(7*c^(3/2)*d^3 - 5*sqrt(a)*c*d^2*e + 11*a*sqrt(c)*
d*e^2 - 9*a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^
(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*s
qrt(2)*(7*c^(3/2)*d^3 - 5*sqrt(a)*c*d^2*e + 11*a*sqrt(c)*d*e^2 - 9*a^(3/2)*
e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a
)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(7*c^(3/2)*d^
3 + 5*sqrt(a)*c*d^2*e + 11*a*sqrt(c)*d*e^2 + 9*a^(3/2)*e^3)*log(sqrt(c)*x^2
 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(7*c^(3
/2)*d^3 + 5*sqrt(a)*c*d^2*e + 11*a*sqrt(c)*d*e^2 + 9*a^(3/2)*e^3)*log(sqrt(
c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(a^2*c^2*d
^4 + 2*a^3*c*d^2*e^2 + a^4*e^4) + 1/12*(3*(5*c^2*d^2*e + 4*a*c*e^3)*x^6 - 4
*a*c*d^3 - 4*a^2*d*e^2 - (7*c^2*d^3 + 4*a*c*d*e^2)*x^4 + 12*(a*c*d^2*e + a^
2*e^3)*x^2)/((a^2*c^2*d^4 + a^3*c*d^2*e^2)*x^7 + (a^3*c*d^4 + a^4*d^2*e^2)*
x^3)
```

mupad [B] time = 5.22, size = 20828, normalized size = 27.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] atan(((x*(4917248*a^10*c^18*d^36*e^5 + 50677760*a^11*c^17*d^34*e^7 + 230498
304*a^12*c^16*d^32*e^9 + 607559680*a^13*c^15*d^30*e^11 + 1026486272*a^14*c^
14*d^28*e^13 + 1166602240*a^15*c^13*d^26*e^15 + 923508736*a^16*c^12*d^24*e^
17 + 539500544*a^17*c^11*d^22*e^19 + 259409920*a^18*c^10*d^20*e^21 + 109709
312*a^19*c^9*d^18*e^23 + 34537472*a^20*c^8*d^16*e^25 + 5308416*a^21*c^7*d^1
4*e^27) - ((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 7
0*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e
^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 +
 a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)
))^(1/2)*((x*(1787297792*a^19*c^13*d^31*e^12 - 147587072*a^15*c^17*d^39*e^4
 - 698089472*a^16*c^16*d^37*e^6 - 1660157952*a^17*c^15*d^35*e^8 - 158806835
2*a^18*c^14*d^33*e^10 - 12845056*a^14*c^18*d^41*e^2 + 7839678464*a^20*c^12*
d^29*e^14 + 11879841792*a^21*c^11*d^27*e^16 + 10631249920*a^22*c^10*d^25*e^
18 + 6274940928*a^23*c^9*d^23*e^20 + 2652110848*a^24*c^8*d^21*e^22 + 891027
456*a^25*c^7*d^19*e^24 + 234881024*a^26*c^6*d^17*e^26 + 33554432*a^27*c^5*d
^15*e^28) + ((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) +
 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4
*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8
 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^
4)))^(1/2)*(x*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2)
 + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d
^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e
^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*
e^4)))^(1/2)*(134217728*a^20*c^16*d^42*e^3 + 1342177280*a^21*c^15*d^40*e^5
```

+ 5905580032*a^22*c^14*d^38*e^7 + 14763950080*a^23*c^13*d^36*e^9 + 22145925
120*a^24*c^12*d^34*e^11 + 17716740096*a^25*c^11*d^32*e^13 - 17716740096*a^2
7*c^9*d^28*e^17 - 22145925120*a^28*c^8*d^26*e^19 - 14763950080*a^29*c^7*d^2
4*e^21 - 5905580032*a^30*c^6*d^22*e^23 - 1342177280*a^31*c^5*d^20*e^25 - 13
4217728*a^32*c^4*d^18*e^27) + 29360128*a^17*c^17*d^42*e^2 + 239075328*a^18*
c^16*d^40*e^4 + 708837376*a^19*c^15*d^38*e^6 + 465567744*a^20*c^14*d^36*e^8
 - 2726297600*a^21*c^13*d^34*e^10 - 9084862464*a^22*c^12*d^32*e^12 - 136147
10784*a^23*c^11*d^30*e^14 - 10745806848*a^24*c^10*d^28*e^16 - 2403336192*a^
25*c^9*d^26*e^18 + 3879731200*a^26*c^8*d^24*e^20 + 4517265408*a^27*c^7*d^22
*e^22 + 2294284288*a^28*c^6*d^20*e^24 + 603979776*a^29*c^5*d^18*e^26 + 6710
8864*a^30*c^4*d^16*e^28))*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^1
1*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 -
 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/
(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a
^13*c^2*d^4*e^4)))^(1/2) + 7225344*a^12*c^18*d^39*e^3 + 76972032*a^13*c^17*
d^37*e^5 + 367607808*a^14*c^16*d^35*e^7 + 1036910592*a^15*c^15*d^33*e^9 + 1
876983808*a^16*c^14*d^31*e^11 + 2115436544*a^17*c^13*d^29*e^13 + 1052803072
*a^18*c^12*d^27*e^15 - 848429056*a^19*c^11*d^25*e^17 - 2105458688*a^20*c^10
*d^23*e^19 - 1909030912*a^21*c^9*d^21*e^21 - 959037440*a^22*c^8*d^19*e^23 -
 262144000*a^23*c^7*d^17*e^25 - 30408704*a^24*c^6*d^15*e^27))*((81*a^3*e^6*
(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a
^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) -
31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*
c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*1i + (x*(49172
48*a^10*c^18*d^36*e^5 + 50677760*a^11*c^17*d^34*e^7 + 230498304*a^12*c^16*d
^32*e^9 + 607559680*a^13*c^15*d^30*e^11 + 1026486272*a^14*c^14*d^28*e^13 +
1166602240*a^15*c^13*d^26*e^15 + 923508736*a^16*c^12*d^24*e^17 + 539500544*
a^17*c^11*d^22*e^19 + 259409920*a^18*c^10*d^20*e^21 + 109709312*a^19*c^9*d^
18*e^23 + 34537472*a^20*c^8*d^16*e^25 + 5308416*a^21*c^7*d^14*e^27) - ((81*
a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e
 + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^
(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 +
 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*((x*(1
787297792*a^19*c^13*d^31*e^12 - 147587072*a^15*c^17*d^39*e^4 - 698089472*a^
16*c^16*d^37*e^6 - 1660157952*a^17*c^15*d^35*e^8 - 1588068352*a^18*c^14*d^3
3*e^10 - 12845056*a^14*c^18*d^41*e^2 + 7839678464*a^20*c^12*d^29*e^14 + 118
79841792*a^21*c^11*d^27*e^16 + 10631249920*a^22*c^10*d^25*e^18 + 6274940928
*a^23*c^9*d^23*e^20 + 2652110848*a^24*c^8*d^21*e^22 + 891027456*a^25*c^7*d^
19*e^24 + 234881024*a^26*c^6*d^17*e^26 + 33554432*a^27*c^5*d^15*e^28) - ((8
1*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5
*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5
)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8
 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*(293
60128*a^17*c^17*d^42*e^2 - x*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-
a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^
3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2
))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 +
6*a^13*c^2*d^4*e^4)))^(1/2)*(134217728*a^20*c^16*d^42*e^3 + 1342177280*a^21
*c^15*d^40*e^5 + 5905580032*a^22*c^14*d^38*e^7 + 14763950080*a^23*c^13*d^36
*e^9 + 22145925120*a^24*c^12*d^34*e^11 + 17716740096*a^25*c^11*d^32*e^13 -
17716740096*a^27*c^9*d^28*e^17 - 22145925120*a^28*c^8*d^26*e^19 - 147639500
80*a^29*c^7*d^24*e^21 - 5905580032*a^30*c^6*d^22*e^23 - 1342177280*a^31*c^5
*d^20*e^25 - 134217728*a^32*c^4*d^18*e^27) + 239075328*a^18*c^16*d^40*e^4 +
 708837376*a^19*c^15*d^38*e^6 + 465567744*a^20*c^14*d^36*e^8 - 2726297600*a
^21*c^13*d^34*e^10 - 9084862464*a^22*c^12*d^32*e^12 - 13614710784*a^23*c^11
*d^30*e^14 - 10745806848*a^24*c^10*d^28*e^16 - 2403336192*a^25*c^9*d^26*e^1
8 + 3879731200*a^26*c^8*d^24*e^20 + 4517265408*a^27*c^7*d^22*e^22 + 2294284
288*a^28*c^6*d^20*e^24 + 603979776*a^29*c^5*d^18*e^26 + 67108864*a^30*c^4*d
^16*e^28))*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) +

70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2) - 7225344*a^12*c^18*d^39*e^3 - 76972032*a^13*c^17*d^37*e^5 - 367607808*a^14*c^16*d^35*e^7 - 1036910592*a^15*c^15*d^33*e^9 - 1876983808*a^16*c^14*d^31*e^11 - 2115436544*a^17*c^13*d^29*e^13 - 1052803072*a^18*c^12*d^27*e^15 + 848429056*a^19*c^11*d^25*e^17 + 2105458688*a^20*c^10*d^23*e^19 + 1909030912*a^21*c^9*d^21*e^21 + 959037440*a^22*c^8*d^19*e^23 + 262144000*a^23*c^7*d^17*e^25 + 30408704*a^24*c^6*d^15*e^27))*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*1i)/((x*(4917248*a^10*c^18*d^36*e^5 + 50677760*a^11*c^17*d^34*e^7 + 230498304*a^12*c^16*d^32*e^9 + 607559680*a^13*c^15*d^30*e^11 + 1026486272*a^14*c^14*d^28*e^13 + 1166602240*a^15*c^13*d^26*e^15 + 923508736*a^16*c^12*d^24*e^17 + 539500544*a^17*c^11*d^22*e^19 + 259409920*a^18*c^10*d^20*e^21 + 109709312*a^19*c^9*d^18*e^23 + 34537472*a^20*c^8*d^16*e^25 + 5308416*a^21*c^7*d^14*e^27) - ((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*((x*(1787297792*a^19*c^13*d^31*e^12 - 147587072*a^15*c^17*d^39*e^4 - 698089472*a^16*c^16*d^37*e^6 - 1660157952*a^17*c^15*d^35*e^8 - 1588068352*a^18*c^14*d^33*e^10 - 12845056*a^14*c^18*d^41*e^2 + 7839678464*a^20*c^12*d^29*e^14 + 11879841792*a^21*c^11*d^27*e^16 + 10631249920*a^22*c^10*d^25*e^18 + 6274940928*a^23*c^9*d^23*e^20 + 2652110848*a^24*c^8*d^21*e^22 + 891027456*a^25*c^7*d^19*e^24 + 234881024*a^26*c^6*d^17*e^26 + 33554432*a^27*c^5*d^15*e^28) - ((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*(29360128*a^17*c^17*d^42*e^2 - x*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*(134217728*a^20*c^16*d^42*e^3 + 1342177280*a^21*c^15*d^40*e^5 + 5905580032*a^22*c^14*d^38*e^7 + 14763950080*a^23*c^13*d^36*e^9 + 22145925120*a^24*c^12*d^34*e^11 + 17716740096*a^25*c^11*d^32*e^13 - 17716740096*a^27*c^9*d^28*e^17 - 22145925120*a^28*c^8*d^26*e^19 - 14763950080*a^29*c^7*d^24*e^21 - 5905580032*a^30*c^6*d^22*e^23 - 1342177280*a^31*c^5*d^20*e^25 - 134217728*a^32*c^4*d^18*e^27) + 239075328*a^18*c^16*d^40*e^4 + 708837376*a^19*c^15*d^38*e^6 + 465567744*a^20*c^14*d^36*e^8 - 2726297600*a^21*c^13*d^34*e^10 - 9084862464*a^22*c^12*d^32*e^12 - 13614710784*a^23*c^11*d^30*e^14 - 10745806848*a^24*c^10*d^28*e^16 - 2403336192*a^25*c^9*d^26*e^18 + 3879731200*a^26*c^8*d^24*e^20 + 4517265408*a^27*c^7*d^22*e^22 + 2294284288*a^28*c^6*d^20*e^24 + 603979776*a^29*c^5*d^18*e^26 + 67108864*a^30*c^4*d^16*e^28))*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2) - 7225344*a^12*c^18*d^39*e^3 - 76972032*a^13*c^17*d^37*e^5 - 367607808*a^14*c^16*d^35*e^7 - 1036910592*a^15*c^15*d^33*e^9 - 1876983808*a^16*c^14*d^31*e^11 - 2115436544*a^17*c^13*d^29*e^13 - 1052803072*a^18*c^12*d^27*e^15 + 848429056*a^19*c^11*d^25*e^17 + 2105458688*a^20*c^10*d^23*e^19 + 1909030912*a^21*c^9*d^21*e^21 + 959037440*a^22*c^8*d^19*e^23 + 262144000*a^23*c^7*d^17*e^25 + 30408704*a^24*c^6*d^15*e^27))*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(

1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2) - (x*(4917248*a^10*c^18*d^36*e^5 + 50677760*a^11*c^17*d^34*e^7 + 230498304*a^12*c^16*d^32*e^9 + 607559680*a^13*c^15*d^30*e^11 + 1026486272*a^14*c^14*d^28*e^13 + 1166602240*a^15*c^13*d^26*e^15 + 923508736*a^16*c^12*d^24*e^17 + 539500544*a^17*c^11*d^22*e^19 + 259409920*a^18*c^10*d^20*e^21 + 109709312*a^19*c^9*d^18*e^23 + 34537472*a^20*c^8*d^16*e^25 + 5308416*a^21*c^7*d^14*e^27) - ((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*((x*(1787297792*a^19*c^13*d^31*e^12 - 147587072*a^15*c^17*d^39*e^4 - 698089472*a^16*c^16*d^37*e^6 - 1660157952*a^17*c^15*d^35*e^8 - 1588068352*a^18*c^14*d^33*e^10 - 12845056*a^14*c^18*d^41*e^2 + 7839678464*a^20*c^12*d^29*e^14 + 11879841792*a^21*c^11*d^27*e^16 + 10631249920*a^22*c^10*d^25*e^18 + 6274940928*a^23*c^9*d^23*e^20 + 2652110848*a^24*c^8*d^21*e^22 + 891027456*a^25*c^7*d^19*e^24 + 234881024*a^26*c^6*d^17*e^26 + 33554432*a^27*c^5*d^15*e^28) + ((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*(x*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*(134217728*a^20*c^16*d^42*e^3 + 1342177280*a^21*c^15*d^40*e^5 + 5905580032*a^22*c^14*d^38*e^7 + 14763950080*a^23*c^13*d^36*e^9 + 22145925120*a^24*c^12*d^34*e^11 + 17716740096*a^25*c^11*d^32*e^13 - 17716740096*a^27*c^9*d^28*e^17 - 22145925120*a^28*c^8*d^26*e^19 - 14763950080*a^29*c^7*d^24*e^21 - 5905580032*a^30*c^6*d^22*e^23 - 1342177280*a^31*c^5*d^20*e^25 - 134217728*a^32*c^4*d^18*e^27) + 29360128*a^17*c^17*d^42*e^2 + 239075328*a^18*c^16*d^40*e^4 + 708837376*a^19*c^15*d^38*e^6 + 465567744*a^20*c^14*d^36*e^8 - 2726297600*a^21*c^13*d^34*e^10 - 9084862464*a^22*c^12*d^32*e^12 - 13614710784*a^23*c^11*d^30*e^14 - 10745806848*a^24*c^10*d^28*e^16 - 2403336192*a^25*c^9*d^26*e^18 + 3879731200*a^26*c^8*d^24*e^20 + 4517265408*a^27*c^7*d^22*e^22 + 2294284288*a^28*c^6*d^20*e^24 + 603979776*a^29*c^5*d^18*e^26 + 67108864*a^30*c^4*d^16*e^28))*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2) + 7225344*a^12*c^18*d^39*e^3 + 76972032*a^13*c^17*d^37*e^5 + 367607808*a^14*c^16*d^35*e^7 + 1036910592*a^15*c^15*d^33*e^9 + 1876983808*a^16*c^14*d^31*e^11 + 2115436544*a^17*c^13*d^29*e^13 + 1052803072*a^18*c^12*d^27*e^15 - 848429056*a^19*c^11*d^25*e^17 - 2105458688*a^20*c^10*d^23*e^19 - 1909030912*a^21*c^9*d^21*e^21 - 959037440*a^22*c^8*d^19*e^23 - 262144000*a^23*c^7*d^17*e^25 - 30408704*a^24*c^6*d^15*e^27))*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2) + 4917248*a^10*c^16*d^30*e^10 + 40843264*a^11*c^15*d^28*e^12 + 147507200*a^12*c^14*d^26*e^14 + 302962688*a^13*c^13*d^24*e^16 + 387512320*a^14*c^12*d^22*e^18 + 316418048*a^15*c^11*d^20*e^20 + 161224704*a^16*c^10*d^18*e^22 + 46909440*a^17*c^9*d^16*e^24 + 5971968*a^18*c^8*d^14*e^26))*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*2i - (1/(3*a*d) - (e*x^2)/(a*d^2) + (x^4*(7*c^2*d^2 + 4*a*c*e^2))/(12*a^2*d*(a*e^2 + c*d^2)) - (c*x^6*(4*a*e^3 + 5*c*d^2*e))/(4*a^2*d^2*(a*e^2 + c*d^2)))/(a *x^3 + c*x^7) + atan(((a^11*c^5*(156627*c^2*d^6*e^12 - 245952*a^2*d^2*e^16 + 324032*a*c*d^4*e^14) - 16807*a^5*c^13*d^18 + 46656*a^14*c^4*e^18 + 24696*a^6*c^12*d^16*e^2 + 455609*a^7*c^11*d^14*e^4 + 856936*a^8*c^10*d^12*e^6 - 27429*a^9*c^9*d^10*e^8 - 805344*a^10*c^8*d^8*e^10)*(a^13*c^11*d^21*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*49i + a^17*c^3*e^19*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*5184i + a^28*d^4*e^19*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*2i - a^19*c^9*d^22*e*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*2i + a^27*c*d^6*e^17*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*14i + a^9*c^11*d^16*e^3*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*4802i + a^10*c^10*d^14*e^5*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*35084i + a^11*c^9*d^12*e^7*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*105438i + a^12*c^8*d^10*e^9*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*166952i + a^13*c^7*d^8*e^11*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*150174i + a^14*c^6*d^6*e^13*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*82444i + a^15*c^5*d^4*e^15*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*37058i + a^16*c^4*d^2*e^17*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)

*18176i + a^14*c^10*d^19*e^2*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*
(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*
e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1
/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a
^13*c^2*d^4*e^4))^(3/2)*416i + a^15*c^9*d^17*e^4*x*((49*c^3*d^6*(-a^11*c^5)
^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^
5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^
2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^
12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*1268i + a^16*c^8*d^15*e^6*x*((4
9*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5
*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5
)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*
a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*1232i - a^
17*c^7*d^13*e^8*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(
1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c
^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^
8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e
^4))^(3/2)*1858i - a^18*c^6*d^11*e^10*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81
*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7
*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^1
1*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*
e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*6208i - a^19*c^5*d^9*e^12*x*((49*c^3*d^6*(
-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^
8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 3
1*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*
e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*6940i - a^20*c^4*d^7*
e^14*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a
^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*
(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^
4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*
4016i - a^21*c^3*d^5*e^16*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a
^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3
 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2)
)/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13
*c^2*d^4*e^4))^(3/2)*1479i - a^22*c^2*d^3*e^18*x*((49*c^3*d^6*(-a^11*c^5)^(
1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5
+ 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*
e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12
*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*512i - a^20*c^8*d^20*e^3*x*((49*c
^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e
+ 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(
1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^1
4*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*14i - a^21*c^
7*d^18*e^5*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2)
+ 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^
4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a
^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^
(5/2)*40i - a^22*c^6*d^16*e^7*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6
*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3
*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(
1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*
a^13*c^2*d^4*e^4))^(5/2)*56i - a^23*c^5*d^14*e^9*x*((49*c^3*d^6*(-a^11*c^5)
^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^
5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^
2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^
12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*28i + a^24*c^4*d^12*e^11*x*((49
*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*
e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)
^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a

^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*28i + a^25*c^3*d^10*e^13*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*56i + a^26*c^2*d^8*e^15*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*40i - a^23*c*d*e^20*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*128i) - (-a^11*c^5)^(1/2)*(69629*c^10*d^17*e + 286944*a*c^9*d^15*e^3 + 150336*a^8*c^2*d*e^17 + 110645*a^2*c^8*d^13*e^5 - 770024*a^3*c^7*d^11*e^7 - 606089*a^4*c^6*d^9*e^9 + 566984*a^5*c^5*d^7*e^11 + 157207*a^6*c^4*d^5*e^13 - 327104*a^7*c^3*d^3*e^15)*(a^13*c^11*d^21*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*49i + a^17*c^3*e^19*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*5184i + a^28*d^4*e^19*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*2i - a^19*c^9*d^22*e*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*2i + a^27*c*d^6*e^17*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*14i + a^9*c^11*d^16*e^3*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*4802i + a^10*c^10*d^14*e^5*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*35084i + a^11*c^9*d^12*e^7*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*105438i + a^12*c^8*d^10*e^9*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*166952i + a^13*c^7*d^8*e^11*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6

*a^13*c^2*d^4*e^4))^(1/2)*150174i + a^14*c^6*d^6*e^13*x*((49*c^3*d^6*(-a^11
*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3
*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2
*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 +
 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*82444i + a^15*c^5*d^4*e^15
x((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c
^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^
11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^
8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*3705
8i + a^16*c^4*d^2*e^17*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11
*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 +
129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(
a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^
2*d^4*e^4))^(1/2)*18176i + a^14*c^10*d^19*e^2*x*((49*c^3*d^6*(-a^11*c^5)^(1
/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 +
 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e
^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*
c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*416i + a^15*c^9*d^17*e^4*x*((49*c^
3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e +
 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1
/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14
*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*1268i + a^16*c
^8*d^15*e^6*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2)
 + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d
^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 +
a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))
^(3/2)*1232i - a^17*c^7*d^13*e^8*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*
e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*
d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5
)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 +
 6*a^13*c^2*d^4*e^4))^(3/2)*1858i - a^18*c^6*d^11*e^10*x*((49*c^3*d^6*(-a^1
1*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^
3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^
2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6
+ 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*6208i - a^19*c^5*d^9*e^12
x((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c
^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^
11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^
8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*6940
i - a^20*c^4*d^7*e^14*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*
c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 1
29*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a
^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2
*d^4*e^4))^(3/2)*4016i - a^21*c^3*d^5*e^16*x*((49*c^3*d^6*(-a^11*c^5)^(1/2)
 - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 23
6*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*
(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3
*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*1479i - a^22*c^2*d^3*e^18*x*((49*c^3*
d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 1
98*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2
) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c
*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*512i - a^20*c^8*
d^20*e^3*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) +
70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*
e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^1
1*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5
/2)*14i - a^21*c^7*d^18*e^5*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(
-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e
^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/

```
2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^
13*c^2*d^4*e^4))^(5/2)*40i - a^22*c^6*d^16*e^7*x*((49*c^3*d^6*(-a^11*c^5)^(
1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5
+ 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*
e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12
*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*56i - a^23*c^5*d^14*e^9*x*((49*c^
3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e +
 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1
/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14
*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*28i + a^24*c^4
*d^12*e^11*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2)
+ 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^
4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a
^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^
(5/2)*28i + a^25*c^3*d^10*e^13*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^
6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^
3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^
(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6
*a^13*c^2*d^4*e^4))^(5/2)*56i + a^26*c^2*d^8*e^15*x*((49*c^3*d^6*(-a^11*c^5
)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e
^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d
^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a
^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*40i - a^23*c*d*e^20*x*((49*c^3
*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e +
198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/
2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*
c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*128i))/(2824752
49*a^10*c^26*d^36 + 2176782336*a^28*c^8*e^36 + 4018066297*a^11*c^25*d^34*e^
2 + 25254299042*a^12*c^24*d^32*e^4 + 91443453570*a^13*c^23*d^30*e^6 + 20709
3177767*a^14*c^22*d^28*e^8 + 292503608847*a^15*c^21*d^26*e^10 + 22503434162
8*a^16*c^20*d^24*e^12 + 22083537020*a^17*c^19*d^22*e^14 - 108969417553*a^18
*c^18*d^20*e^16 - 43670306041*a^19*c^17*d^18*e^18 + 58023955010*a^20*c^16*d
^16*e^20 + 18862267874*a^21*c^15*d^14*e^22 - 60676266279*a^22*c^14*d^12*e^2
4 - 33348619375*a^23*c^13*d^10*e^26 + 20433166080*a^24*c^12*d^8*e^28 + 9487
311616*a^25*c^11*d^6*e^30 - 7622553600*a^26*c^10*d^4*e^32 - 349360128*a^27*
c^9*d^2*e^34))*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2
) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*
d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*
e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4
*e^4)))^(1/2)*2i + (atan((a^11*e^3*x*(-d^5*e^11)^(5/2)*4096i - a^4*c^7*d^19
*x*(-d^5*e^11)^(3/2)*73519i + c^11*d^32*e^3*x*(-d^5*e^11)^(1/2)*2401i - a^5
*c^6*d^17*e^2*x*(-d^5*e^11)^(3/2)*34182i - a^6*c^5*d^15*e^4*x*(-d^5*e^11)^(
3/2)*15521i - a^7*c^4*d^13*e^6*x*(-d^5*e^11)^(3/2)*30208i - a^8*c^3*d^11*e^
8*x*(-d^5*e^11)^(3/2)*25344i + a^2*c^9*d^28*e^7*x*(-d^5*e^11)^(1/2)*52719i
+ a^3*c^8*d^26*e^9*x*(-d^5*e^11)^(1/2)*83476i + a*c^10*d^30*e^5*x*(-d^5*e^1
1)^(1/2)*17542i)/(4096*a^11*d^13*e^30 + 2401*c^11*d^35*e^8 + 17542*a*c^10*d
^33*e^10 + 52719*a^2*c^9*d^31*e^12 + 83476*a^3*c^8*d^29*e^14 + 73519*a^4*c^
7*d^27*e^16 + 34182*a^5*c^6*d^25*e^18 + 15521*a^6*c^5*d^23*e^20 + 30208*a^7
*c^4*d^21*e^22 + 25344*a^8*c^3*d^19*e^24))*(-d^5*e^11)^(1/2)*1i)/(c^2*d^9 +
 a^2*d^5*e^4 + 2*a*c*d^7*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.189 $\int x^2\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=243

$$\frac{c^2\sqrt{a^2+2abx^2+b^2x^4}\,(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16d^{5/2}\left(a+bx^2\right)}-\frac{cx\sqrt{a^2+2abx^2+b^2x^4}\,\sqrt{c+dx^2}\,(bc-2ad)}{16d^2\left(a+bx^2\right)}+\frac{bx^3\sqrt{a^2+2ab}}{6}$$

Rubi [A] time = 0.13, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1250, 459, 279, 321, 217, 206}

$$\frac{c^2\sqrt{a^2+2abx^2+b^2x^4}\,(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16d^{5/2}\left(a+bx^2\right)}-\frac{cx\sqrt{a^2+2abx^2+b^2x^4}\,\sqrt{c+dx^2}\,(bc-2ad)}{16d^2\left(a+bx^2\right)}+\frac{bx^3\sqrt{a^2+2abx^2+b^2x^4}\,\left(c+dx^2\right)^{3/2}}{6d\left(a+bx^2\right)}-\frac{x^3\sqrt{a^2+2abx^2+b^2x^4}\,\sqrt{c+dx^2}\,(bc-2ad)}{8d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -(c*(b*c - 2*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*d^
2*(a + b*x^2)) - ((b*c - 2*a*d)*x^3*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(8*d*(a + b*x^2)) + (b*x^3*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(6*d*(a + b*x^2)) + (c^2*(b*c - 2*a*d)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*d^(5/2)*(a + b*x^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\int x^2\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^2\left(ab+b^2x^2\right)\sqrt{c+dx^2}\,dx}{ab+b^2x^2}$$

$$= \frac{bx^3\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{6d\left(a+bx^2\right)} - \frac{\left(b(bc-2ad)\sqrt{a^2+2abx^2+b^2x^4}\right.}{2d\left(ab+b^2x^2\right.}$$

$$= -\frac{(bc-2ad)x^3\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{8d\left(a+bx^2\right)} + \frac{bx^3\left(c+dx^2\right)^{3/2}\sqrt{a^2+2}}{6d\left(a+bx^2\right)}$$

$$= -\frac{c(bc-2ad)x\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{16d^2\left(a+bx^2\right)} - \frac{(bc-2ad)x^3\sqrt{c+dx^2}\,\sqrt{a}}{8d\left(a+b\right.}$$

$$= -\frac{c(bc-2ad)x\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{16d^2\left(a+bx^2\right)} - \frac{(bc-2ad)x^3\sqrt{c+dx^2}\,\sqrt{a}}{8d\left(a+b\right.}$$

$$= -\frac{c(bc-2ad)x\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{16d^2\left(a+bx^2\right)} - \frac{(bc-2ad)x^3\sqrt{c+dx^2}\,\sqrt{a}}{8d\left(a+b\right.}$$

Mathematica [A] time = 0.18, size = 142, normalized size = 0.58

$$\frac{\sqrt{\left(a+bx^2\right)^2}\sqrt{c+dx^2}\left(3c^{3/2}(bc-2ad)\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)+\sqrt{d}x\sqrt{\frac{dx^2}{c}+1}\left(6ad\left(c+2dx^2\right)+b\left(-3c^2+2cdx^2+8d^2x^4\right)\right)\right)}{48d^{5/2}\left(a+bx^2\right)\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*Sqrt[c + d*x^2]*(Sqrt[d]*x*Sqrt[1 + (d*x^2)/c]*(6*a*d*
(c + 2*d*x^2) + b*(-3*c^2 + 2*c*d*x^2 + 8*d^2*x^4)) + 3*c^(3/2)*(b*c - 2*a*
d)*ArcSinh[(Sqrt[d]*x)/Sqrt[c]]))/(48*d^(5/2)*(a + b*x^2)*Sqrt[1 + (d*x^2)/
c])
```

IntegrateAlgebraic [A] time = 0.20, size = 126, normalized size = 0.52

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(\frac{\sqrt{c+dx^2}\left(6acdx+12ad^2x^3-3bc^2x+2bcdx^3+8bd^2x^5\right)}{48d^2}+\frac{\left(2ac^2d-bc^3\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{16d^{5/2}}\right)}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*((Sqrt[c + d*x^2]*(-3*b*c^2*x + 6*a*c*d*x + 2*b*c*d*x^
3 + 12*a*d^2*x^3 + 8*b*d^2*x^5))/(48*d^2) + ((-(b*c^3) + 2*a*c^2*d)*Log[-(S
qrt[d]*x) + Sqrt[c + d*x^2]])/(16*d^(5/2))))/(a + b*x^2)
```

fricas [A] time = 0.85, size = 206, normalized size = 0.85

$$\left[-\frac{3\left(bc^3-2\,ac^2d\right)\sqrt{d}\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\sqrt{d}\,x-c\right)-2\left(8\,bd^3x^5+2\left(bcd^2+6\,ad^3\right)x^3-3\left(bc^2d-2\,acd^2\right)x\right)\sqrt{dx^2+c}}{96\,d^3},-\frac{3\left(bc^3-2\,ac^2d\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-\left(8\,bd^3x^5+2\left(bcd^2+6\,ad^3\right)x^3-3\left(bc^2d-2\,acd^2\right)x\right)\sqrt{dx^2+c}}{48\,d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b*c^3 - 2*a*c^2*d)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt
(d)*x - c) - 2*(8*b*d^3*x^5 + 2*(b*c*d^2 + 6*a*d^3)*x^3 - 3*(b*c^2*d - 2*a*
c*d^2)*x)*sqrt(d*x^2 + c))/d^3, -1/48*(3*(b*c^3 - 2*a*c^2*d)*sqrt(-d)*arcta
n(sqrt(-d)*x/sqrt(d*x^2 + c)) - (8*b*d^3*x^5 + 2*(b*c*d^2 + 6*a*d^3)*x^3 -
3*(b*c^2*d - 2*a*c*d^2)*x)*sqrt(d*x^2 + c))/d^3]
```

giac [A] time = 0.38, size = 156, normalized size = 0.64

$$\frac{1}{48}\left(2\left(4\,bx^2\mathrm{sgn}\left(bx^2+a\right)+\frac{bcd^3\mathrm{sgn}\left(bx^2+a\right)+6\,ad^4\mathrm{sgn}\left(bx^2+a\right)}{d^4}\right)x^2-\frac{3\left(bc^2d^2\mathrm{sgn}\left(bx^2+a\right)-2\,acd^3\mathrm{sgn}\left(bx^2+a\right)\right)}{d^4}\right)\sqrt{dx^2+c}\,x-\frac{\left(bc^3\mathrm{sgn}\left(bx^2+a\right)-2\,ac^2d\mathrm{sgn}\left(bx^2+a\right)\right)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)}{16\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b*x^2*sgn(b*x^2 + a) + (b*c*d^3*sgn(b*x^2 + a) + 6*a*d^4*sgn(b*x
^2 + a))/d^4)*x^2 - 3*(b*c^2*d^2*sgn(b*x^2 + a) - 2*a*c*d^3*sgn(b*x^2 + a))
/d^4)*sqrt(d*x^2 + c)*x - 1/16*(b*c^3*sgn(b*x^2 + a) - 2*a*c^2*d*sgn(b*x^2
+ a))*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(5/2)
```

maple [A] time = 0.01, size = 159, normalized size = 0.65

$$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\left(8\left(d\,x^2+c\right)^{\frac{3}{2}}b\,d^{\frac{3}{2}}x^3-6a\,c^2d\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)+3b\,c^3\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)-6\sqrt{d\,x^2+c}\,ac\,d^{\frac{3}{2}}x+3\sqrt{d\,x^2+c}\,b\,c^2\sqrt{d}\,x+12\left(d\,x^2+c\right)^{\frac{3}{2}}a\,d^{\frac{3}{2}}x-6\left(d\,x^2+c\right)^{\frac{3}{2}}bc\sqrt{d}\,x\right)}{48\left(b\,x^2+a\right)d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/48*((b*x^2+a)^2)^(1/2)*(8*d^(3/2)*(d*x^2+c)^(3/2)*x^3*b+12*d^(3/2)*(d*x^2
+c)^(3/2)*x*a-6*d^(1/2)*(d*x^2+c)^(3/2)*x*b*c-6*d^(3/2)*(d*x^2+c)^(1/2)*x*a
*c+3*d^(1/2)*(d*x^2+c)^(1/2)*x*b*c^2-6*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*a*c^2*
d+3*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*b*c^3)/(b*x^2+a)/d^(5/2)
```

maxima [A] time = 0.92, size = 124, normalized size = 0.51

$$\frac{\left(dx^2+c\right)^{\frac{3}{2}}bx^3}{6\,d}-\frac{\left(dx^2+c\right)^{\frac{3}{2}}bcx}{8\,d^2}+\frac{\sqrt{dx^2+c}\,bc^2x}{16\,d^2}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}ax}{4\,d}-\frac{\sqrt{dx^2+c}\,acx}{8\,d}+\frac{bc^3\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\,d^{\frac{5}{2}}}-\frac{ac^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(d*x^2 + c)^(3/2)*b*x^3/d - 1/8*(d*x^2 + c)^(3/2)*b*c*x/d^2 + 1/16*sqrt
(d*x^2 + c)*b*c^2*x/d^2 + 1/4*(d*x^2 + c)^(3/2)*a*x/d - 1/8*sqrt(d*x^2 + c)
*a*c*x/d + 1/16*b*c^3*arcsinh(d*x/sqrt(c*d))/d^(5/2) - 1/8*a*c^2*arcsinh(d*
x/sqrt(c*d))/d^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{d\,x^2+c}\,\sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2),x)
[Out] int(x^2*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
[Out] Timed out
```

3.190 $\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=108

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{5/2}}{5d^2\left(a+bx^2\right)}-\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}(bc-ad)}{3d^2\left(a+bx^2\right)}$$

Rubi [A] time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1247, 646, 43}

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{5/2}}{5d^2\left(a+bx^2\right)}-\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}(bc-ad)}{3d^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -((b*c - a*d)*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^2*(a
+ b*x^2)) + (b*(c + d*x^2)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^2*(a
 + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int\sqrt{c+dx}\sqrt{a^2+2abx+b^2x^2}\,dx,x,x^2\right)\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int\left(ab+b^2x\right)\sqrt{c+dx}\,dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int\left(-\frac{b(bc-ad)\sqrt{c+dx}}{d}+\frac{b^2(c+dx)^{3/2}}{d}\right)dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}\\
&= -\frac{(bc-ad)\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^2\left(a+bx^2\right)}+\frac{b\left(c+dx^2\right)^{5/2}\sqrt{a^2+2a}}{5d^2\left(a+bx^2\right.}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.52

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(c+dx^2\right)^{3/2}\left(5ad-2bc+3bdx^2\right)}{15d^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(c + d*x^2)^(3/2)*(-2*b*c + 5*a*d + 3*b*d*x^2))/(15*d^
2*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 0.09, size = 56, normalized size = 0.52

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(c+dx^2\right)^{3/2}\left(5ad-2bc+3bdx^2\right)}{15d^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(c + d*x^2)^(3/2)*(-2*b*c + 5*a*d + 3*b*d*x^2))/(15*d^
2*(a + b*x^2))
```

fricas [A] time = 0.60, size = 50, normalized size = 0.46

$$\frac{\left(3\,bd^2x^4-2\,bc^2+5\,acd+\left(bcd+5\,ad^2\right)x^2\right)\sqrt{dx^2+c}}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*d^2*x^4 - 2*b*c^2 + 5*a*c*d + (b*c*d + 5*a*d^2)*x^2)*sqrt(d*x^2 +
 c)/d^2
```

giac [A] time = 0.34, size = 68, normalized size = 0.63

$$\frac{3\left(dx^2+c\right)^{\frac{5}{2}}b\mathrm{sgn}\left(bx^2+a\right)-5\left(dx^2+c\right)^{\frac{3}{2}}bc\mathrm{sgn}\left(bx^2+a\right)+5\left(dx^2+c\right)^{\frac{3}{2}}ad\mathrm{sgn}\left(bx^2+a\right)}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(3*(d*x^2 + c)^(5/2)*b*sgn(b*x^2 + a) - 5*(d*x^2 + c)^(3/2)*b*c*sgn(b*
x^2 + a) + 5*(d*x^2 + c)^(3/2)*a*d*sgn(b*x^2 + a))/d^2
```

maple [A] time = 0.00, size = 51, normalized size = 0.47

$$\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}\left(3bd\,x^2+5ad-2bc\right)\sqrt{\left(b\,x^2+a\right)^2}}{15\left(b\,x^2+a\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/15*(d*x^2+c)^(3/2)*(3*b*d*x^2+5*a*d-2*b*c)*((b*x^2+a)^2)^(1/2)/d^2/(b*x^2
+a)
```

maxima [A] time = 0.90, size = 50, normalized size = 0.46

$$\frac{\left(dx^2+c\right)^{\frac{3}{2}}bx^2}{5\,d}-\frac{2\left(dx^2+c\right)^{\frac{3}{2}}bc}{15\,d^2}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}a}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(d*x^2 + c)^(3/2)*b*x^2/d - 2/15*(d*x^2 + c)^(3/2)*b*c/d^2 + 1/3*(d*x^2
 + c)^(3/2)*a/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{d\,x^2+c}\,\sqrt{\left(b\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(x*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.191 $\int \sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=178

$$-\frac{c\sqrt{a^2+2abx^2+b^2x^4}\,(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}\left(a+bx^2\right)}+\frac{bx\sqrt{a^2+2abx^2+b^2x^4}\,\left(c+dx^2\right)^{3/2}}{4d\left(a+bx^2\right)}-\frac{x\sqrt{a^2+2abx^2+b^2x^4}\,\sqrt{}}{8d\left(a+b\right.}$$

Rubi [A] time = 0.08, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1148, 388, 195, 217, 206}

$$-\frac{c\sqrt{a^2+2abx^2+b^2x^4}\,(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}\left(a+bx^2\right)}+\frac{bx\sqrt{a^2+2abx^2+b^2x^4}\,\left(c+dx^2\right)^{3/2}}{4d\left(a+bx^2\right)}-\frac{x\sqrt{a^2+2abx^2+b^2x^4}\,\sqrt{c+dx^2}\,(bc-4ad)}{8d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -((b*c - 4*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*d*(a
+ b*x^2)) + (b*x*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*d*(a
 + b*x^2)) - (c*(b*c - 4*a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(Sqrt
[d]*x)/Sqrt[c + d*x^2]])/(8*d^(3/2)*(a + b*x^2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1148

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_),
x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^
2)^(2*FracPart[p])), Int[(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ
[{a, b, c, d, e, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(ab+b^2x^2\right)\sqrt{c+dx^2}\,dx}{ab+b^2x^2}$$

$$= \frac{bx\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{4d\left(a+bx^2\right)} - \frac{\left(b(bc-4ad)\sqrt{a^2+2abx^2+b^2x^4}\right.}{4d\left(ab+b^2x^2\right)}$$

$$= -\frac{(bc-4ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d\left(a+bx^2\right)} + \frac{bx\left(c+dx^2\right)^{3/2}\sqrt{a^2+2a}}{4d\left(a+bx^2\right)}$$

$$= -\frac{(bc-4ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d\left(a+bx^2\right)} + \frac{bx\left(c+dx^2\right)^{3/2}\sqrt{a^2+2a}}{4d\left(a+bx^2\right)}$$

$$= -\frac{(bc-4ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d\left(a+bx^2\right)} + \frac{bx\left(c+dx^2\right)^{3/2}\sqrt{a^2+2a}}{4d\left(a+bx^2\right)}$$

Mathematica [A] time = 0.11, size = 121, normalized size = 0.68

$$\frac{\sqrt{\left(a+bx^2\right)^2}\sqrt{c+dx^2}\left(\sqrt{d}\,x\sqrt{\frac{dx^2}{c}+1}\left(4ad+b\left(c+2dx^2\right)\right)-\sqrt{c}\,(bc-4ad)\sinh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\right)}{8d^{3/2}\left(a+bx^2\right)\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*Sqrt[c + d*x^2]*(Sqrt[d]*x*Sqrt[1 + (d*x^2)/c]*(4*a*d
+ b*(c + 2*d*x^2)) - Sqrt[c]*(b*c - 4*a*d)*ArcSinh[(Sqrt[d]*x)/Sqrt[c]]))/(
8*d^(3/2)*(a + b*x^2)*Sqrt[1 + (d*x^2)/c])
```

IntegrateAlgebraic [A] time = 0.16, size = 100, normalized size = 0.56

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(\frac{\left(bc^2-4acd\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{8d^{3/2}}+\frac{\sqrt{c+dx^2}\left(4adx+bcx+2bdx^3\right)}{8d}\right)}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*((Sqrt[c + d*x^2]*(b*c*x + 4*a*d*x + 2*b*d*x^3))/(8*d)
 + ((b*c^2 - 4*a*c*d)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(8*d^(3/2))))/(a
 + b*x^2)
```

fricas [A] time = 0.94, size = 155, normalized size = 0.87

$$\left[-\frac{\left(bc^2-4\,acd\right)\sqrt{d}\,\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)-2\left(2\,bd^2x^3+\left(bcd+4\,ad^2\right)x\right)\sqrt{dx^2+c}}{16\,d^2},\ \frac{\left(bc^2-4\,acd\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+\left(2\,bd^2x^3+\left(bcd+4\,ad^2\right)x\right)\sqrt{dx^2+c}}{8\,d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((b*c^2 - 4*a*c*d)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*
x - c) - 2*(2*b*d^2*x^3 + (b*c*d + 4*a*d^2)*x)*sqrt(d*x^2 + c))/d^2, 1/8*((
b*c^2 - 4*a*c*d)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (2*b*d^2*x^3
 + (b*c*d + 4*a*d^2)*x)*sqrt(d*x^2 + c))/d^2]
```

giac [A] time = 0.46, size = 109, normalized size = 0.61

$$\frac{1}{8}\left(2\,bx^2\mathrm{sgn}\left(bx^2+a\right)+\frac{bcd\mathrm{sgn}\left(bx^2+a\right)+4\,ad^2\mathrm{sgn}\left(bx^2+a\right)}{d^2}\right)\sqrt{dx^2+c}\,x+\frac{\left(bc^2\mathrm{sgn}\left(bx^2+a\right)-4\,acd\mathrm{sgn}\left(bx^2+a\right)\right)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)}{8\,d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*x^2*sgn(b*x^2 + a) + (b*c*d*sgn(b*x^2 + a) + 4*a*d^2*sgn(b*x^2 + a
))/d^2)*sqrt(d*x^2 + c)*x + 1/8*(b*c^2*sgn(b*x^2 + a) - 4*a*c*d*sgn(b*x^2 +
 a))*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(3/2)
```

maple [A] time = 0.01, size = 119, normalized size = 0.67

$$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\left(4acd\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)-b\,c^2\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)+4\sqrt{d\,x^2+c}\,a\,d^{\frac{3}{2}}x-\sqrt{d\,x^2+c}\,bc\sqrt{d}\,x+2\left(d\,x^2+c\right)^{\frac{3}{2}}b\sqrt{d}\,x\right)}{8\left(b\,x^2+a\right)d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/8*((b*x^2+a)^2)^(1/2)*(2*d^(1/2)*(d*x^2+c)^(3/2)*x*b+4*d^(3/2)*(d*x^2+c)^
(1/2)*x*a-d^(1/2)*(d*x^2+c)^(1/2)*x*b*c+4*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*a*c
*d-ln(d^(1/2)*x+(d*x^2+c)^(1/2))*b*c^2)/(b*x^2+a)/d^(3/2)
```

maxima [A] time = 1.08, size = 81, normalized size = 0.46

$$\frac{1}{2}\sqrt{dx^2+c}\,ax+\frac{\left(dx^2+c\right)^{\frac{3}{2}}bx}{4\,d}-\frac{\sqrt{dx^2+c}\,bcx}{8\,d}-\frac{bc^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{3}{2}}}+\frac{ac\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*a*x + 1/4*(d*x^2 + c)^(3/2)*b*x/d - 1/8*sqrt(d*x^2 + c)
*b*c*x/d - 1/8*b*c^2*arcsinh(d*x/sqrt(c*d))/d^(3/2) + 1/2*a*c*arcsinh(d*x/s
qrt(c*d))/sqrt(d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d\,x^2+c}\,\sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx^2}\,\sqrt{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x**2)**2), x)
```

3.192 $\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx$

Optimal. Leaf size=152

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{3d\left(a+bx^2\right)}+\frac{a\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}}{a+bx^2}-\frac{a\sqrt{c}\sqrt{a^2+2abx^2+b^2x^4}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx^2}$$

Rubi [A] time = 0.09, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1250, 446, 80, 50, 63, 208}

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{3d\left(a+bx^2\right)}+\frac{a\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}}{a+bx^2}-\frac{a\sqrt{c}\sqrt{a^2+2abx^2+b^2x^4}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]
```

```
[Out] (a*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (b*(c + d
*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) - (a*Sqrt[c]
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(a + b*x
^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)\sqrt{c+dx^2}}{x}\,dx}{ab+b^2x^2}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{(ab+b^2x)\sqrt{c+dx}}{x}\,dx, x, x^2\right)}{2\left(ab+b^2x^2\right)}$$

$$= \frac{b\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)} + \frac{\left(ab\sqrt{a^2+2abx^2+b^2x^4}\right)\text{Subst}\left(\int \cdots\right.}{2\left(ab+b^2x^2\right)}$$

$$= \frac{a\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)} + \cdots$$

$$= \frac{a\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)} + \cdots$$

$$= \frac{a\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)} - \cdots$$

Mathematica [A] time = 0.05, size = 83, normalized size = 0.55

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(\sqrt{c+dx^2}\left(3ad+b\left(c+dx^2\right)\right)-3a\sqrt{c}\,d\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)}{3d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(Sqrt[c + d*x^2]*(3*a*d + b*(c + d*x^2)) - 3*a*Sqrt[c]
*d*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/(3*d*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 0.11, size = 82, normalized size = 0.54

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(\frac{\sqrt{c+dx^2}\left(3ad+bc+bdx^2\right)}{3d}-a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*((Sqrt[c + d*x^2]*(b*c + 3*a*d + b*d*x^2))/(3*d) - a*S
qrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/(a + b*x^2)
```

fricas [A] time = 0.95, size = 123, normalized size = 0.81

$$\left[\frac{3\,a\sqrt{c}\,d\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)+2\,\left(bdx^2+bc+3\,ad\right)\sqrt{dx^2+c}}{6\,d},\ \frac{3\,a\sqrt{-c}\,d\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\left(bdx^2+bc+3\,ad\right)\sqrt{dx^2+c}}{3\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*sqrt(c)*d*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2
*(b*d*x^2 + b*c + 3*a*d)*sqrt(d*x^2 + c))/d, 1/3*(3*a*sqrt(-c)*d*arctan(sqr
t(-c)/sqrt(d*x^2 + c)) + (b*d*x^2 + b*c + 3*a*d)*sqrt(d*x^2 + c))/d]
```

giac [A] time = 0.45, size = 84, normalized size = 0.55

$$\frac{ac\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{-c}}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}bd^2\mathrm{sgn}\left(bx^2+a\right)+3\,\sqrt{dx^2+c}\,ad^3\mathrm{sgn}\left(bx^2+a\right)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] a*c*arctan(sqrt(d*x^2 + c)/sqrt(-c))*sgn(b*x^2 + a)/sqrt(-c) + 1/3*((d*x^2
+ c)^(3/2)*b*d^2*sgn(b*x^2 + a) + 3*sqrt(d*x^2 + c)*a*d^3*sgn(b*x^2 + a))/d
^3
```

maple [A] time = 0.01, size = 80, normalized size = 0.53

$$-\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\left(3a\sqrt{c}\,d\ln\left(\frac{2c+2\sqrt{d\,x^2+c}\,\sqrt{c}}{x}\right)-3\sqrt{d\,x^2+c}\,ad-\left(d\,x^2+c\right)^{\frac{3}{2}}b\right)}{3\left(b\,x^2+a\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x)
```

```
[Out] -1/3*((b*x^2+a)^2)^(1/2)*(3*ln(2*(c^(1/2)*(d*x^2+c)^(1/2)+c)/x)*c^(1/2)*a*d
-b*(d*x^2+c)^(3/2)-3*(d*x^2+c)^(1/2)*a*d)/(b*x^2+a)/d
```

maxima [A] time = 1.45, size = 45, normalized size = 0.30

$$-a\sqrt{c}\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)+\sqrt{dx^2+c}\,a+\frac{\left(dx^2+c\right)^{\frac{3}{2}}b}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -a*sqrt(c)*arcsinh(c/(sqrt(c*d)*abs(x))) + sqrt(d*x^2 + c)*a + 1/3*(d*x^2 +
 c)^(3/2)*b/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}\,\sqrt{\left(b\,x^2+a\right)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x,x)
```

```
[Out] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2}\,\sqrt{\left(a + bx^2\right)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x**2)**2)/x, x)
```

3.193 $\int \frac{\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx$

Optimal. Leaf size=177

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\,\left(c+dx^2\right)^{3/2}}{cx\left(a+bx^2\right)}+\frac{x\sqrt{a^2+2abx^2+b^2x^4}\,\sqrt{c+dx^2}\,(2ad+bc)}{2c\left(a+bx^2\right)}+\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(2ad+b}{2\sqrt{d}\,\left(a+bx\right.}$$

Rubi [A] time = 0.09, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1250, 453, 195, 217, 206}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\,\left(c+dx^2\right)^{3/2}}{cx\left(a+bx^2\right)}+\frac{x\sqrt{a^2+2abx^2+b^2x^4}\,\sqrt{c+dx^2}\,(2ad+bc)}{2c\left(a+bx^2\right)}+\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}\,\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2,x]
```

```
[Out] ((b*c + 2*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*c*(a +
 b*x^2)) - (a*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(c*x*(a +
b*x^2)) + ((b*c + 2*a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(Sqrt[d]*x
)/Sqrt[c + d*x^2]])/(2*Sqrt[d]*(a + b*x^2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)\sqrt{c+dx^2}}{x^2}\,dx}{ab+b^2x^2}$$
$$= -\frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{cx\left(a+bx^2\right)} + -\frac{\left(\left(-b^2c-2abd\right)\sqrt{a^2+2abx^2+b^2\right.}{c\left(ab+b^2x^2\right)}$$
$$= \frac{(bc+2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c\left(a+bx^2\right)} - \frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+}}{cx\left(a+bx^2\right)}$$
$$= \frac{(bc+2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c\left(a+bx^2\right)} - \frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+}}{cx\left(a+bx^2\right)}$$
$$= \frac{(bc+2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c\left(a+bx^2\right)} - \frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+}}{cx\left(a+bx^2\right)}$$

Mathematica [A] time = 0.11, size = 122, normalized size = 0.69

$$\frac{\sqrt{\left(a+bx^2\right)^2}\sqrt{c+dx^2}\left(\sqrt{c}\sqrt{d}\left(bx^2-2a\right)\sqrt{\frac{dx^2}{c}+1}+x(2ad+bc)\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right)}{2\sqrt{c}\sqrt{d}x\left(a+bx^2\right)\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*Sqrt[c + d*x^2]*(Sqrt[c]*Sqrt[d]*(-2*a + b*x^2)*Sqrt[1
 + (d*x^2)/c] + (b*c + 2*a*d)*x*ArcSinh[(Sqrt[d]*x)/Sqrt[c]]))/(2*Sqrt[c]*S
qrt[d]*x*(a + b*x^2)*Sqrt[1 + (d*x^2)/c])
```

IntegrateAlgebraic [A] time = 0.20, size = 90, normalized size = 0.51

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(\frac{\left(bx^2-2a\right)\sqrt{c+dx^2}}{2x}+\frac{(-2ad-bc)\log\left(\sqrt{c+dx^2}-\sqrt{d}x\right)}{2\sqrt{d}}\right)}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(((-2*a + b*x^2)*Sqrt[c + d*x^2])/(2*x) + ((-(b*c) - 2
*a*d)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*Sqrt[d])))/(a + b*x^2)
```

fricas [A] time = 0.88, size = 134, normalized size = 0.76

$$\left[\frac{(bc+2ad)\sqrt{d}x\log\left(-2dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\left(bdx^2-2ad\right)\sqrt{dx^2+c}}{4dx}, -\frac{(bc+2ad)\sqrt{-d}x\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-\left(bdx^2-2ad\right)\sqrt{dx^2+c}}{2dx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*((b*c + 2*a*d)*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x -
c) + 2*(b*d*x^2 - 2*a*d)*sqrt(d*x^2 + c))/(d*x), -1/2*((b*c + 2*a*d)*sqrt(-
```

```
d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (b*d*x^2 - 2*a*d)*sqrt(d*x^2 + c)
)/(d*x)]
```

giac [A] time = 0.47, size = 116, normalized size = 0.66

$$\frac{1}{2}\sqrt{dx^2+c}\,bx\mathrm{sgn}\left(bx^2+a\right)+\frac{2\,ac\sqrt{d}\,\mathrm{sgn}\left(bx^2+a\right)}{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c}-\frac{\left(bc\sqrt{d}\,\mathrm{sgn}\left(bx^2+a\right)+2\,ad^{\frac{3}{2}}\mathrm{sgn}\left(bx^2+a\right)\right)\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b*x*sgn(b*x^2 + a) + 2*a*c*sqrt(d)*sgn(b*x^2 + a)/((sqr
t(d)*x - sqrt(d*x^2 + c))^2 - c) - 1/4*(b*c*sqrt(d)*sgn(b*x^2 + a) + 2*a*d^
(3/2)*sgn(b*x^2 + a))*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d
```

maple [A] time = 0.01, size = 128, normalized size = 0.72

$$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\left(2acdx\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)+b\,c^2x\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)+2\sqrt{d\,x^2+c}\,a\,d^{\frac{3}{2}}x^2+\sqrt{d\,x^2+c}\,bc\sqrt{d}\,x^2-2\left(d\,x^2+c\right)^{\frac{3}{2}}a\sqrt{d}\right)}{2\left(b\,x^2+a\right)c\sqrt{d}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x)
```

```
[Out] 1/2*((b*x^2+a)^2)^(1/2)*(2*d^(3/2)*(d*x^2+c)^(1/2)*x^2*a+d^(1/2)*(d*x^2+c)^
(1/2)*x^2*b*c-2*d^(1/2)*(d*x^2+c)^(3/2)*a+2*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*x
*a*c*d+ln(d^(1/2)*x+(d*x^2+c)^(1/2))*x*b*c^2)/(b*x^2+a)/c/x/d^(1/2)
```

maxima [A] time = 1.08, size = 59, normalized size = 0.33

$$\frac{1}{2}\sqrt{dx^2+c}\,bx+\frac{bc\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{d}}+a\sqrt{d}\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)-\frac{\sqrt{dx^2+c}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b*x + 1/2*b*c*arcsinh(d*x/sqrt(c*d))/sqrt(d) + a*sqrt(d
)*arcsinh(d*x/sqrt(c*d)) - sqrt(d*x^2 + c)*a/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}\,\sqrt{\left(b\,x^2+a\right)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x^2,x)
```

```
[Out] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}\,\sqrt{\left(a+bx^2\right)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x**2)**2)/x**2, x)
```

3.194 $\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx$

Optimal. Leaf size=177

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{2cx^2\left(a+bx^2\right)}+\frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}\,(ad+2bc)}{2c\left(a+bx^2\right)}-\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(ad+2bc)\,\text{ta}}{2\sqrt{c}\left(a+bx^2\right)}$$

Rubi [A] time = 0.12, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1250, 446, 78, 50, 63, 208}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{2cx^2\left(a+bx^2\right)}+\frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}\,(ad+2bc)}{2c\left(a+bx^2\right)}-\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]
```

```
[Out] ((2*b*c + a*d)*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*c*(a + b
*x^2)) - (a*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*c*x^2*(a
+ b*x^2)) - ((2*b*c + a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[Sqrt[c +
 d*x^2]/Sqrt[c]])/(2*Sqrt[c]*(a + b*x^2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)\sqrt{c+dx^2}}{x^3}\,dx}{ab+b^2x^2}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)\sqrt{c+dx}}{x^2}\,dx, x, x^2\right)}{2\left(ab+b^2x^2\right)}$$

$$= -\frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{2cx^2\left(a+bx^2\right)} + \frac{\left(\left(b^2c+\frac{abd}{2}\right)\sqrt{a^2+2abx^2+b^2x\right.}{2c\left(ab+\right.}$$

$$= \frac{(2bc+ad)\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c\left(a+bx^2\right)} - \frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2}}{2cx^2\left(a+bx^2\right)}$$

$$= \frac{(2bc+ad)\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c\left(a+bx^2\right)} - \frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2}}{2cx^2\left(a+bx^2\right)}$$

$$= \frac{(2bc+ad)\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c\left(a+bx^2\right)} - \frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2}}{2cx^2\left(a+bx^2\right)}$$

Mathematica [A] time = 0.05, size = 90, normalized size = 0.51

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(\sqrt{c}\left(a-2bx^2\right)\sqrt{c+dx^2}+x^2(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)}{2\sqrt{c}\,x^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]
```

```
[Out] -1/2*(Sqrt[(a + b*x^2)^2]*(Sqrt[c]*(a - 2*b*x^2)*Sqrt[c + d*x^2] + (2*b*c +
 a*d)*x^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/(Sqrt[c]*x^2*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 0.15, size = 88, normalized size = 0.50

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(\frac{\left(2bx^2-a\right)\sqrt{c+dx^2}}{2x^2}+\frac{(-ad-2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(((-a + 2*b*x^2)*Sqrt[c + d*x^2])/(2*x^2) + ((-2*b*c -
 a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*Sqrt[c])))/(a + b*x^2)
```

fricas [A] time = 0.87, size = 141, normalized size = 0.80

$$\left[\frac{(2\,bc+ad)\sqrt{c}\,x^{2}\log\left(-\frac{dx^{2}-2\,\sqrt{dx^{2}+c}\,\sqrt{c}+2\,c}{x^{2}}\right)+2\left(2\,bcx^{2}-ac\right)\sqrt{dx^{2}+c}}{4\,cx^{2}},\;\frac{(2\,bc+ad)\sqrt{-c}\,x^{2}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)+\left(2\,bcx^{2}-ac\right)\sqrt{dx^{2}+c}}{2\,cx^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*((2*b*c + a*d)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2
*c)/x^2) + 2*(2*b*c*x^2 - a*c)*sqrt(d*x^2 + c))/(c*x^2), 1/2*((2*b*c + a*d)
*sqrt(-c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (2*b*c*x^2 - a*c)*sqrt(d*x
^2 + c))/(c*x^2)]
```

giac [A] time = 0.44, size = 100, normalized size = 0.56

$$\frac{2\,\sqrt{dx^{2}+c}\,bd\mathrm{sgn}\left(bx^{2}+a\right)+\frac{\left(2\,bcd\mathrm{sgn}\left(bx^{2}+a\right)+ad^{2}\mathrm{sgn}\left(bx^{2}+a\right)\right)\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{dx^{2}+c}\,ad\mathrm{sgn}\left(bx^{2}+a\right)}{x^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(d*x^2 + c)*b*d*sgn(b*x^2 + a) + (2*b*c*d*sgn(b*x^2 + a) + a*d^2
*sgn(b*x^2 + a))*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) - sqrt(d*x^2 + c
)*a*d*sgn(b*x^2 + a)/x^2)/d
```

maple [A] time = 0.01, size = 133, normalized size = 0.75

$$-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(a\sqrt{c}\,d\,x^{2}\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)+2b\,c^{\frac{3}{2}}x^{2}\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)-\sqrt{d\,x^{2}+c}\,ad\,x^{2}-2\sqrt{d\,x^{2}+c}\,bc\,x^{2}+\left(d\,x^{2}+c\right)^{\frac{3}{2}}a\right)}{2\left(b\,x^{2}+a\right)c\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x)
```

```
[Out] -1/2*((b*x^2+a)^2)^(1/2)*(c^(1/2)*ln(2*(c+(d*x^2+c)^(1/2)*c^(1/2))/x)*x^2*a
*d+2*c^(3/2)*ln(2*(c+(d*x^2+c)^(1/2)*c^(1/2))/x)*x^2*b-(d*x^2+c)^(1/2)*x^2*
a*d-2*(d*x^2+c)^(1/2)*x^2*b*c+(d*x^2+c)^(3/2)*a)/(b*x^2+a)/c/x^2
```

maxima [A] time = 1.23, size = 83, normalized size = 0.47

$$-b\sqrt{c}\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)-\frac{ad\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{2\,\sqrt{c}}+\sqrt{dx^{2}+c}\,b+\frac{\sqrt{dx^{2}+c}\,ad}{2\,c}-\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}a}{2\,cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -b*sqrt(c)*arcsinh(c/(sqrt(c*d)*abs(x))) - 1/2*a*d*arcsinh(c/(sqrt(c*d)*abs
(x)))/sqrt(c) + sqrt(d*x^2 + c)*b + 1/2*sqrt(d*x^2 + c)*a*d/c - 1/2*(d*x^2
+ c)^(3/2)*a/(c*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}\,\sqrt{{\left(b\,x^2+a\right)}^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x^3,x)
[Out] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**3,x)
[Out] Timed out
```

3.195 $\int x^3 \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) dx$

Optimal. Leaf size=78

$$\frac{1}{8}x^8\left(e(ae+2bd)+cd^2\right)+\frac{1}{6}dx^6(2ae+bd)+\frac{1}{4}ad^2x^4+\frac{1}{10}ex^{10}(be+2cd)+\frac{1}{12}ce^2x^{12}$$

Rubi [A] time = 0.13, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1251, 771}

$$\frac{1}{8}x^8\left(e(ae+2bd)+cd^2\right)+\frac{1}{6}dx^6(2ae+bd)+\frac{1}{4}ad^2x^4+\frac{1}{10}ex^{10}(be+2cd)+\frac{1}{12}ce^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d^2*x^4)/4 + (d*(b*d + 2*a*e)*x^6)/6 + ((c*d^2 + e*(2*b*d + a*e))*x^8)/8
 + (e*(2*c*d + b*e)*x^10)/10 + (c*e^2*x^12)/12
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^3\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)dx&=\frac{1}{2}\text{Subst}\left(\int x(d+ex)^2\left(a+bx+cx^2\right)dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(ad^2x+d(bd+2ae)x^2+\left(cd^2+e(2bd+ae)\right)x^3+e(2cd+be)x\right.\right.\\&=\frac{1}{4}ad^2x^4+\frac{1}{6}d(bd+2ae)x^6+\frac{1}{8}\left(cd^2+e(2bd+ae)\right)x^8+\frac{1}{10}e(2cd+be)x^{10}+\end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.92

$$\frac{1}{120}x^4\left(15x^4\left(e(ae+2bd)+cd^2\right)+20dx^2(2ae+bd)+30ad^2+12ex^6(be+2cd)+10ce^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (x^4*(30*a*d^2 + 20*d*(b*d + 2*a*e)*x^2 + 15*(c*d^2 + e*(2*b*d + a*e))*x^4
 + 12*e*(2*c*d + b*e)*x^6 + 10*c*e^2*x^8))/120
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4), x]
```

fricas [A] time = 0.83, size = 79, normalized size = 1.01

$$\frac{1}{12}x^{12}e^2c+\frac{1}{5}x^{10}edc+\frac{1}{10}x^{10}e^2b+\frac{1}{8}x^8d^2c+\frac{1}{4}x^8edb+\frac{1}{8}x^8e^2a+\frac{1}{6}x^6d^2b+\frac{1}{3}x^6eda+\frac{1}{4}x^4d^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*x^12*e^2*c + 1/5*x^10*e*d*c + 1/10*x^10*e^2*b + 1/8*x^8*d^2*c + 1/4*x^
8*e*d*b + 1/8*x^8*e^2*a + 1/6*x^6*d^2*b + 1/3*x^6*e*d*a + 1/4*x^4*d^2*a
```

giac [A] time = 0.27, size = 79, normalized size = 1.01

$$\frac{1}{12}cx^{12}e^2+\frac{1}{5}cdx^{10}e+\frac{1}{10}bx^{10}e^2+\frac{1}{8}cd^2x^8+\frac{1}{4}bdx^8e+\frac{1}{8}ax^8e^2+\frac{1}{6}bd^2x^6+\frac{1}{3}adx^6e+\frac{1}{4}ad^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/12*c*x^12*e^2 + 1/5*c*d*x^10*e + 1/10*b*x^10*e^2 + 1/8*c*d^2*x^8 + 1/4*b*
d*x^8*e + 1/8*a*x^8*e^2 + 1/6*b*d^2*x^6 + 1/3*a*d*x^6*e + 1/4*a*d^2*x^4
```

maple [A] time = 0.00, size = 73, normalized size = 0.94

$$\frac{c\,e^2x^{12}}{12}+\frac{\left(e^2b+2dec\right)x^{10}}{10}+\frac{\left(a\,e^2+2deb+c\,d^2\right)x^8}{8}+\frac{a\,d^2x^4}{4}+\frac{\left(2dea+d^2b\right)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/12*c*e^2*x^12+1/10*(b*e^2+2*c*d*e)*x^10+1/8*(a*e^2+2*b*d*e+c*d^2)*x^8+1/6
*(2*a*d*e+b*d^2)*x^6+1/4*a*d^2*x^4
```

maxima [A] time = 1.11, size = 72, normalized size = 0.92

$$\frac{1}{12}ce^2x^{12}+\frac{1}{10}\left(2\,cde+be^2\right)x^{10}+\frac{1}{8}\left(cd^2+2\,bde+ae^2\right)x^8+\frac{1}{4}ad^2x^4+\frac{1}{6}\left(bd^2+2\,ade\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/12*c*e^2*x^12 + 1/10*(2*c*d*e + b*e^2)*x^10 + 1/8*(c*d^2 + 2*b*d*e + a*e^
2)*x^8 + 1/4*a*d^2*x^4 + 1/6*(b*d^2 + 2*a*d*e)*x^6
```

mupad [B] time = 0.04, size = 73, normalized size = 0.94

$$x^8\left(\frac{c\,d^2}{8}+\frac{b\,d\,e}{4}+\frac{a\,e^2}{8}\right)+x^6\left(\frac{b\,d^2}{6}+\frac{a\,e\,d}{3}\right)+x^{10}\left(\frac{b\,e^2}{10}+\frac{c\,d\,e}{5}\right)+\frac{a\,d^2\,x^4}{4}+\frac{c\,e^2\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^8*((a*e^2)/8 + (c*d^2)/8 + (b*d*e)/4) + x^6*((b*d^2)/6 + (a*d*e)/3) + x^1
0*((b*e^2)/10 + (c*d*e)/5) + (a*d^2*x^4)/4 + (c*e^2*x^12)/12
```

sympy [A] time = 0.08, size = 76, normalized size = 0.97

$$\frac{ad^2x^4}{4}+\frac{ce^2x^{12}}{12}+x^{10}\left(\frac{be^2}{10}+\frac{cde}{5}\right)+x^8\left(\frac{ae^2}{8}+\frac{bde}{4}+\frac{cd^2}{8}\right)+x^6\left(\frac{ade}{3}+\frac{bd^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x**4/4 + c*e**2*x**12/12 + x**10*(b*e**2/10 + c*d*e/5) + x**8*(a*e**
2/8 + b*d*e/4 + c*d**2/8) + x**6*(a*d*e/3 + b*d**2/6)
```

3.196 $\int x^2 \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) dx$

Optimal. Leaf size=78

$$\frac{1}{7}x^7\left(e(ae+2bd)+cd^2\right)+\frac{1}{5}dx^5(2ae+bd)+\frac{1}{3}ad^2x^3+\frac{1}{9}ex^9(be+2cd)+\frac{1}{11}ce^2x^{11}$$

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1261}

$$\frac{1}{7}x^7\left(e(ae+2bd)+cd^2\right)+\frac{1}{5}dx^5(2ae+bd)+\frac{1}{3}ad^2x^3+\frac{1}{9}ex^9(be+2cd)+\frac{1}{11}ce^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d^2*x^3)/3 + (d*(b*d + 2*a*e)*x^5)/5 + ((c*d^2 + e*(2*b*d + a*e))*x^7)/7
 + (e*(2*c*d + b*e)*x^9)/9 + (c*e^2*x^11)/11
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int x^2\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)dx=\int\left(ad^2x^2+d(bd+2ae)x^4+\left(cd^2+e(2bd+ae)\right)x^6+e(2cd+be)x^8+ce\right.$$
$$=\frac{1}{3}ad^2x^3+\frac{1}{5}d(bd+2ae)x^5+\frac{1}{7}\left(cd^2+e(2bd+ae)\right)x^7+\frac{1}{9}e(2cd+be)x^9$$

Mathematica [A] time = 0.02, size = 78, normalized size = 1.00

$$\frac{1}{7}x^7\left(ae^2+2bde+cd^2\right)+\frac{1}{5}dx^5(2ae+bd)+\frac{1}{3}ad^2x^3+\frac{1}{9}ex^9(be+2cd)+\frac{1}{11}ce^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d^2*x^3)/3 + (d*(b*d + 2*a*e)*x^5)/5 + ((c*d^2 + 2*b*d*e + a*e^2)*x^7)/7
 + (e*(2*c*d + b*e)*x^9)/9 + (c*e^2*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4), x]
```

fricas [A] time = 0.79, size = 79, normalized size = 1.01

$$\frac{1}{11}x^{11}e^2c+\frac{2}{9}x^9edc+\frac{1}{9}x^9e^2b+\frac{1}{7}x^7d^2c+\frac{2}{7}x^7edb+\frac{1}{7}x^7e^2a+\frac{1}{5}x^5d^2b+\frac{2}{5}x^5eda+\frac{1}{3}x^3d^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e^2*c + 2/9*x^9*e*d*c + 1/9*x^9*e^2*b + 1/7*x^7*d^2*c + 2/7*x^7*e
*d*b + 1/7*x^7*e^2*a + 1/5*x^5*d^2*b + 2/5*x^5*e*d*a + 1/3*x^3*d^2*a
```

giac [A] time = 0.39, size = 79, normalized size = 1.01

$$\frac{1}{11}cx^{11}e^2+\frac{2}{9}cdx^9e+\frac{1}{9}bx^9e^2+\frac{1}{7}cd^2x^7+\frac{2}{7}bdx^7e+\frac{1}{7}ax^7e^2+\frac{1}{5}bd^2x^5+\frac{2}{5}adx^5e+\frac{1}{3}ad^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/11*c*x^11*e^2 + 2/9*c*d*x^9*e + 1/9*b*x^9*e^2 + 1/7*c*d^2*x^7 + 2/7*b*d*x
^7*e + 1/7*a*x^7*e^2 + 1/5*b*d^2*x^5 + 2/5*a*d*x^5*e + 1/3*a*d^2*x^3
```

maple [A] time = 0.00, size = 73, normalized size = 0.94

$$\frac{c\,e^2x^{11}}{11}+\frac{\left(e^2b+2dec\right)x^9}{9}+\frac{\left(a\,e^2+2deb+c\,d^2\right)x^7}{7}+\frac{a\,d^2x^3}{3}+\frac{\left(2dea+d^2b\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/11*c*e^2*x^11+1/9*(b*e^2+2*c*d*e)*x^9+1/7*(a*e^2+2*b*d*e+c*d^2)*x^7+1/5*(
2*a*d*e+b*d^2)*x^5+1/3*a*d^2*x^3
```

maxima [A] time = 1.19, size = 72, normalized size = 0.92

$$\frac{1}{11}ce^2x^{11}+\frac{1}{9}\left(2\,cde+be^2\right)x^9+\frac{1}{7}\left(cd^2+2\,bde+ae^2\right)x^7+\frac{1}{3}ad^2x^3+\frac{1}{5}\left(bd^2+2\,ade\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/11*c*e^2*x^11 + 1/9*(2*c*d*e + b*e^2)*x^9 + 1/7*(c*d^2 + 2*b*d*e + a*e^2)
*x^7 + 1/3*a*d^2*x^3 + 1/5*(b*d^2 + 2*a*d*e)*x^5
```

mupad [B] time = 0.03, size = 73, normalized size = 0.94

$$x^7\left(\frac{c\,d^2}{7}+\frac{2\,b\,d\,e}{7}+\frac{a\,e^2}{7}\right)+x^5\left(\frac{b\,d^2}{5}+\frac{2\,a\,e\,d}{5}\right)+x^9\left(\frac{b\,e^2}{9}+\frac{2\,c\,d\,e}{9}\right)+\frac{a\,d^2\,x^3}{3}+\frac{c\,e^2\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^7*((a*e^2)/7 + (c*d^2)/7 + (2*b*d*e)/7) + x^5*((b*d^2)/5 + (2*a*d*e)/5) +
 x^9*((b*e^2)/9 + (2*c*d*e)/9) + (a*d^2*x^3)/3 + (c*e^2*x^11)/11
```

sympy [A] time = 0.08, size = 82, normalized size = 1.05

$$\frac{ad^2x^3}{3}+\frac{ce^2x^{11}}{11}+x^9\left(\frac{be^2}{9}+\frac{2cde}{9}\right)+x^7\left(\frac{ae^2}{7}+\frac{2bde}{7}+\frac{cd^2}{7}\right)+x^5\left(\frac{2ade}{5}+\frac{bd^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x**3/3 + c*e**2*x**11/11 + x**9*(b*e**2/9 + 2*c*d*e/9) + x**7*(a*e**
2/7 + 2*b*d*e/7 + c*d**2/7) + x**5*(2*a*d*e/5 + b*d**2/5)
```

3.197 $\int x\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=75

$$\frac{\left(d+ex^2\right)^3\left(ae^2-bde+cd^2\right)}{6e^3}-\frac{\left(d+ex^2\right)^4(2cd-be)}{8e^3}+\frac{c\left(d+ex^2\right)^5}{10e^3}$$

Rubi [A] time = 0.13, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1247, 698}

$$\frac{\left(d+ex^2\right)^3\left(ae^2-bde+cd^2\right)}{6e^3}-\frac{\left(d+ex^2\right)^4(2cd-be)}{8e^3}+\frac{c\left(d+ex^2\right)^5}{10e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(d + e*x^2)^3)/(6*e^3) - ((2*c*d - b*e)*(d + e*x^2
)^4)/(8*e^3) + (c*(d + e*x^2)^5)/(10*e^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int (d+ex)^2\left(a+bx+cx^2\right)\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{\left(cd^2-bde+ae^2\right)(d+ex)^2}{e^2}+\frac{(-2cd+be)(d+ex)^3}{e^2}+\frac{c(d+\right.\right.\\ &= \frac{\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)^3}{6e^3}-\frac{(2cd-be)\left(d+ex^2\right)^4}{8e^3}+\frac{c\left(d+ex^2\right)^5}{10e^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 72, normalized size = 0.96

$$\frac{1}{120}x^2\left(20x^4\left(e(ae+2bd)+cd^2\right)+30dx^2(2ae+bd)+60ad^2+15ex^6(be+2cd)+12ce^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (x^2*(60*a*d^2 + 30*d*(b*d + 2*a*e)*x^2 + 20*(c*d^2 + e*(2*b*d + a*e))*x^4
+ 15*e*(2*c*d + b*e)*x^6 + 12*c*e^2*x^8))/120
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[x*(d + e*x^2)^2*(a + b*x^2 + c*x^4), x]
```

fricas [A] time = 0.81, size = 79, normalized size = 1.05

$$\frac{1}{10}x^{10}e^2c + \frac{1}{4}x^8edc + \frac{1}{8}x^8e^2b + \frac{1}{6}x^6d^2c + \frac{1}{3}x^6edb + \frac{1}{6}x^6e^2a + \frac{1}{4}x^4d^2b + \frac{1}{2}x^4eda + \frac{1}{2}x^2d^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e^2*c + 1/4*x^8*e*d*c + 1/8*x^8*e^2*b + 1/6*x^6*d^2*c + 1/3*x^6*e
*d*b + 1/6*x^6*e^2*a + 1/4*x^4*d^2*b + 1/2*x^4*e*d*a + 1/2*x^2*d^2*a
```

giac [A] time = 0.27, size = 79, normalized size = 1.05

$$\frac{1}{10}cx^{10}e^2 + \frac{1}{4}cdx^8e + \frac{1}{8}bx^8e^2 + \frac{1}{6}cd^2x^6 + \frac{1}{3}bdx^6e + \frac{1}{6}ax^6e^2 + \frac{1}{4}bd^2x^4 + \frac{1}{2}adx^4e + \frac{1}{2}ad^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/10*c*x^10*e^2 + 1/4*c*d*x^8*e + 1/8*b*x^8*e^2 + 1/6*c*d^2*x^6 + 1/3*b*d*x
^6*e + 1/6*a*x^6*e^2 + 1/4*b*d^2*x^4 + 1/2*a*d*x^4*e + 1/2*a*d^2*x^2
```

maple [A] time = 0.00, size = 73, normalized size = 0.97

$$\frac{c e^2 x^{10}}{10} + \frac{\left(e^2b+2dec\right)x^8}{8} + \frac{\left(a e^2+2deb+c d^2\right)x^6}{6} + \frac{a d^2x^2}{2} + \frac{\left(2dea+d^2b\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/10*c*e^2*x^10+1/8*(b*e^2+2*c*d*e)*x^8+1/6*(a*e^2+2*b*d*e+c*d^2)*x^6+1/4*(
2*a*d*e+b*d^2)*x^4+1/2*a*d^2*x^2
```

maxima [A] time = 1.21, size = 72, normalized size = 0.96

$$\frac{1}{10}ce^2x^{10} + \frac{1}{8}\left(2\,cde+be^2\right)x^8 + \frac{1}{6}\left(cd^2+2\,bde+ae^2\right)x^6 + \frac{1}{2}ad^2x^2 + \frac{1}{4}\left(bd^2+2\,ade\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/10*c*e^2*x^10 + 1/8*(2*c*d*e + b*e^2)*x^8 + 1/6*(c*d^2 + 2*b*d*e + a*e^2)
*x^6 + 1/2*a*d^2*x^2 + 1/4*(b*d^2 + 2*a*d*e)*x^4
```

mupad [B] time = 0.03, size = 73, normalized size = 0.97

$$x^6\left(\frac{c\,d^2}{6}+\frac{b\,d\,e}{3}+\frac{a\,e^2}{6}\right)+x^4\left(\frac{b\,d^2}{4}+\frac{a\,e\,d}{2}\right)+x^8\left(\frac{b\,e^2}{8}+\frac{c\,d\,e}{4}\right)+\frac{a\,d^2\,x^2}{2}+\frac{c\,e^2\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^6*((a*e^2)/6 + (c*d^2)/6 + (b*d*e)/3) + x^4*((b*d^2)/4 + (a*d*e)/2) + x^8
*((b*e^2)/8 + (c*d*e)/4) + (a*d^2*x^2)/2 + (c*e^2*x^10)/10
```

sympy [A] time = 0.08, size = 76, normalized size = 1.01

$$\frac{ad^2x^2}{2} + \frac{ce^2x^{10}}{10} + x^8\left(\frac{be^2}{8} + \frac{cde}{4}\right) + x^6\left(\frac{ae^2}{6} + \frac{bde}{3} + \frac{cd^2}{6}\right) + x^4\left(\frac{ade}{2} + \frac{bd^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x**2/2 + c*e**2*x**10/10 + x**8*(b*e**2/8 + c*d*e/4) + x**6*(a*e**2/
6 + b*d*e/3 + c*d**2/6) + x**4*(a*d*e/2 + b*d**2/4)
```

3.198 $\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) dx$

Optimal. Leaf size=73

$$\frac{1}{5}x^5 \left(e(ae+2bd)+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1153}

$$\frac{1}{5}x^5 \left(e(ae+2bd)+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + e*(2*b*d + a*e))*x^5)/5 + (e*
(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) dx &= \int \left(ad^2+d(bd+2ae)x^2+\left(cd^2+e(2bd+ae)\right)x^4+e(2cd+be)x^6+ce^2x^8\right) dx\\ &= ad^2x+\frac{1}{3}d(bd+2ae)x^3+\frac{1}{5}\left(cd^2+e(2bd+ae)\right)x^5+\frac{1}{7}e(2cd+be)x^7+\frac{1}{9}ce^2x^9\end{aligned}$$

Mathematica [A] time = 0.02, size = 73, normalized size = 1.00

$$\frac{1}{5}x^5 \left(ae^2+2bde+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + 2*b*d*e + a*e^2)*x^5)/5 + (e*
(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^2)^2*(a + b*x^2 + c*x^4), x]
```

fricas [A] time = 0.52, size = 76, normalized size = 1.04

$$\frac{1}{9}x^9e^2c+\frac{2}{7}x^7edc+\frac{1}{7}x^7e^2b+\frac{1}{5}x^5d^2c+\frac{2}{5}x^5edb+\frac{1}{5}x^5e^2a+\frac{1}{3}x^3d^2b+\frac{2}{3}x^3eda+xd^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e^2*c + 2/7*x^7*e*d*c + 1/7*x^7*e^2*b + 1/5*x^5*d^2*c + 2/5*x^5*e*d
*b + 1/5*x^5*e^2*a + 1/3*x^3*d^2*b + 2/3*x^3*e*d*a + x*d^2*a
```

giac [A] time = 0.35, size = 76, normalized size = 1.04

$$\frac{1}{9}\,cx^9e^2 + \frac{2}{7}\,cdx^7e + \frac{1}{7}\,bx^7e^2 + \frac{1}{5}\,cd^2x^5 + \frac{2}{5}\,bdx^5e + \frac{1}{5}\,ax^5e^2 + \frac{1}{3}\,bd^2x^3 + \frac{2}{3}\,adx^3e + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/9*c*x^9*e^2 + 2/7*c*d*x^7*e + 1/7*b*x^7*e^2 + 1/5*c*d^2*x^5 + 2/5*b*d*x^5
*e + 1/5*a*x^5*e^2 + 1/3*b*d^2*x^3 + 2/3*a*d*x^3*e + a*d^2*x
```

maple [A] time = 0.00, size = 70, normalized size = 0.96

$$\frac{c\,e^2x^9}{9} + \frac{\left(e^2b + 2dec\right)x^7}{7} + \frac{\left(a\,e^2 + 2deb + c\,d^2\right)x^5}{5} + a\,d^2x + \frac{\left(2dea + d^2b\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/9*c*e^2*x^9+1/7*(b*e^2+2*c*d*e)*x^7+1/5*(a*e^2+2*b*d*e+c*d^2)*x^5+1/3*(2*
a*d*e+b*d^2)*x^3+a*d^2*x
```

maxima [A] time = 1.17, size = 69, normalized size = 0.95

$$\frac{1}{9}\,ce^2x^9 + \frac{1}{7}\,\left(2\,cde + be^2\right)x^7 + \frac{1}{5}\,\left(cd^2 + 2\,bde + ae^2\right)x^5 + ad^2x + \frac{1}{3}\,\left(bd^2 + 2\,ade\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/9*c*e^2*x^9 + 1/7*(2*c*d*e + b*e^2)*x^7 + 1/5*(c*d^2 + 2*b*d*e + a*e^2)*x
^5 + a*d^2*x + 1/3*(b*d^2 + 2*a*d*e)*x^3
```

mupad [B] time = 0.03, size = 70, normalized size = 0.96

$$x^5\,\left(\frac{c\,d^2}{5}+\frac{2\,b\,d\,e}{5}+\frac{a\,e^2}{5}\right)+x^3\,\left(\frac{b\,d^2}{3}+\frac{2\,a\,e\,d}{3}\right)+x^7\,\left(\frac{b\,e^2}{7}+\frac{2\,c\,d\,e}{7}\right)+\frac{c\,e^2\,x^9}{9}+a\,d^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^5*((a*e^2)/5 + (c*d^2)/5 + (2*b*d*e)/5) + x^3*((b*d^2)/3 + (2*a*d*e)/3) +
 x^7*((b*e^2)/7 + (2*c*d*e)/7) + (c*e^2*x^9)/9 + a*d^2*x
```

sympy [A] time = 0.08, size = 78, normalized size = 1.07

$$ad^2x + \frac{ce^2x^9}{9} + x^7\left(\frac{be^2}{7} + \frac{2cde}{7}\right) + x^5\left(\frac{ae^2}{5} + \frac{2bde}{5} + \frac{cd^2}{5}\right) + x^3\left(\frac{2ade}{3} + \frac{bd^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x + c*e**2*x**9/9 + x**7*(b*e**2/7 + 2*c*d*e/7) + x**5*(a*e**2/5 + 2
*b*d*e/5 + c*d**2/5) + x**3*(2*a*d*e/3 + b*d**2/3)
```

3.199 $\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x} \, dx$

Optimal. Leaf size=74

$$\frac{1}{4}x^4\left(e(ae+2bd)+cd^2\right)+\frac{1}{2}dx^2(2ae+bd)+ad^2\log(x)+\frac{1}{6}ex^6(be+2cd)+\frac{1}{8}ce^2x^8$$

Rubi [A] time = 0.09, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1251, 893}

$$\frac{1}{4}x^4\left(e(ae+2bd)+cd^2\right)+\frac{1}{2}dx^2(2ae+bd)+ad^2\log(x)+\frac{1}{6}ex^6(be+2cd)+\frac{1}{8}ce^2x^8$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x]
```

```
[Out] (d*(b*d + 2*a*e)*x^2)/2 + ((c*d^2 + e*(2*b*d + a*e))*x^4)/4 + (e*(2*c*d + b
*e)*x^6)/6 + (c*e^2*x^8)/8 + a*d^2*Log[x]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)^2\left(a+bx+cx^2\right)}{x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(d(bd+2ae)+\frac{ad^2}{x}+\left(cd^2+e(2bd+ae)\right)x+e(2cd+be)x^2+ce^2\right.\right.\\ &= \frac{1}{2}d(bd+2ae)x^2+\frac{1}{4}\left(cd^2+e(2bd+ae)\right)x^4+\frac{1}{6}e(2cd+be)x^6+\frac{1}{8}ce^2x^8+ad^2 1\end{aligned}$$

Mathematica [A] time = 0.02, size = 74, normalized size = 1.00

$$\frac{1}{4}x^4\left(ae^2+2bde+cd^2\right)+\frac{1}{2}dx^2(2ae+bd)+ad^2\log(x)+\frac{1}{6}ex^6(be+2cd)+\frac{1}{8}ce^2x^8$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x]
```

```
[Out] (d*(b*d + 2*a*e)*x^2)/2 + ((c*d^2 + 2*b*d*e + a*e^2)*x^4)/4 + (e*(2*c*d + b
*e)*x^6)/6 + (c*e^2*x^8)/8 + a*d^2*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^2 (a+bx^2+cx^4)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x]
```

```
[Out] IntegrateAlgebraic[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x, x]
```

fricas [A] time = 0.90, size = 70, normalized size = 0.95

$$\frac{1}{8}ce^2x^8 + \frac{1}{6}\left(2\,cde + be^2\right)x^6 + \frac{1}{4}\left(cd^2 + 2\,bde + ae^2\right)x^4 + ad^2\log(x) + \frac{1}{2}\left(bd^2 + 2\,ade\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x, algorithm="fricas")
```

```
[Out] 1/8*c*e^2*x^8 + 1/6*(2*c*d*e + b*e^2)*x^6 + 1/4*(c*d^2 + 2*b*d*e + a*e^2)*x
^4 + a*d^2*log(x) + 1/2*(b*d^2 + 2*a*d*e)*x^2
```

giac [A] time = 0.26, size = 79, normalized size = 1.07

$$\frac{1}{8}cx^8e^2 + \frac{1}{3}cdx^6e + \frac{1}{6}bx^6e^2 + \frac{1}{4}cd^2x^4 + \frac{1}{2}bdx^4e + \frac{1}{4}ax^4e^2 + \frac{1}{2}bd^2x^2 + adx^2e + \frac{1}{2}ad^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x, algorithm="giac")
```

```
[Out] 1/8*c*x^8*e^2 + 1/3*c*d*x^6*e + 1/6*b*x^6*e^2 + 1/4*c*d^2*x^4 + 1/2*b*d*x^4
*e + 1/4*a*x^4*e^2 + 1/2*b*d^2*x^2 + a*d*x^2*e + 1/2*a*d^2*log(x^2)
```

maple [A] time = 0.00, size = 77, normalized size = 1.04

$$\frac{c\,e^2x^8}{8} + \frac{b\,e^2x^6}{6} + \frac{cde\,x^6}{3} + \frac{a\,e^2x^4}{4} + \frac{bde\,x^4}{2} + \frac{c\,d^2x^4}{4} + ade\,x^2 + \frac{b\,d^2x^2}{2} + a\,d^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x)
```

```
[Out] 1/8*c*e^2*x^8+1/6*x^6*b*e^2+1/3*x^6*c*d*e+1/4*x^4*a*e^2+1/2*x^4*b*d*e+1/4*x
^4*c*d^2+x^2*a*d*e+1/2*x^2*b*d^2+a*d^2*ln(x)
```

maxima [A] time = 1.12, size = 73, normalized size = 0.99

$$\frac{1}{8}ce^2x^8 + \frac{1}{6}\left(2\,cde + be^2\right)x^6 + \frac{1}{4}\left(cd^2 + 2\,bde + ae^2\right)x^4 + \frac{1}{2}ad^2\log\left(x^2\right) + \frac{1}{2}\left(bd^2 + 2\,ade\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x, algorithm="maxima")
```

```
[Out] 1/8*c*e^2*x^8 + 1/6*(2*c*d*e + b*e^2)*x^6 + 1/4*(c*d^2 + 2*b*d*e + a*e^2)*x
^4 + 1/2*a*d^2*log(x^2) + 1/2*(b*d^2 + 2*a*d*e)*x^2
```

mupad [B] time = 0.03, size = 70, normalized size = 0.95

$$x^4\left(\frac{c\,d^2}{4} + \frac{b\,d\,e}{2} + \frac{a\,e^2}{4}\right) + x^2\left(\frac{b\,d^2}{2} + a\,e\,d\right) + x^6\left(\frac{b\,e^2}{6} + \frac{c\,d\,e}{3}\right) + \frac{c\,e^2\,x^8}{8} + a\,d^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x)
```

```
[Out] x^4*((a*e^2)/4 + (c*d^2)/4 + (b*d*e)/2) + x^2*((b*d^2)/2 + a*d*e) + x^6*((b
*e^2)/6 + (c*d*e)/3) + (c*e^2*x^8)/8 + a*d^2*log(x)
```

sympy [A] time = 0.17, size = 73, normalized size = 0.99

$$ad^2\log{(x)} + \frac{ce^2x^8}{8} + x^6\left(\frac{be^2}{6} + \frac{cde}{3}\right) + x^4\left(\frac{ae^2}{4} + \frac{bde}{2} + \frac{cd^2}{4}\right) + x^2\left(ade + \frac{bd^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x,x)
```

```
[Out] a*d**2*log(x) + c*e**2*x**8/8 + x**6*(b*e**2/6 + c*d*e/3) + x**4*(a*e**2/4
+ b*d*e/2 + c*d**2/4) + x**2*(a*d*e + b*d**2/2)
```

3.200 $\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^2} dx$

Optimal. Leaf size=71

$$\frac{1}{3}x^3\left(e(ae+2bd)+cd^2\right)+dx(2ae+bd)-\frac{ad^2}{x}+\frac{1}{5}ex^5(be+2cd)+\frac{1}{7}ce^2x^7$$

Rubi [A] time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1261}

$$\frac{1}{3}x^3\left(e(ae+2bd)+cd^2\right)+dx(2ae+bd)-\frac{ad^2}{x}+\frac{1}{5}ex^5(be+2cd)+\frac{1}{7}ce^2x^7$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2,x]
```

```
[Out] -((a*d^2)/x) + d*(b*d + 2*a*e)*x + ((c*d^2 + e*(2*b*d + a*e))*x^3)/3 + (e*(
2*c*d + b*e)*x^5)/5 + (c*e^2*x^7)/7
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^2}\,dx &= \int \left(d(bd+2ae)+\frac{ad^2}{x^2}+\left(cd^2+e(2bd+ae)\right)x^2+e(2cd+be)x^4+ce^2x^6\right) dx\\ &= -\frac{ad^2}{x}+d(bd+2ae)x+\frac{1}{3}\left(cd^2+e(2bd+ae)\right)x^3+\frac{1}{5}e(2cd+be)x^5+\frac{1}{7}ce^2x^7\end{aligned}$$

Mathematica [A] time = 0.03, size = 71, normalized size = 1.00

$$\frac{1}{3}x^3\left(ae^2+2bde+cd^2\right)+dx(2ae+bd)-\frac{ad^2}{x}+\frac{1}{5}ex^5(be+2cd)+\frac{1}{7}ce^2x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2,x]
```

```
[Out] -((a*d^2)/x) + d*(b*d + 2*a*e)*x + ((c*d^2 + 2*b*d*e + a*e^2)*x^3)/3 + (e*(
2*c*d + b*e)*x^5)/5 + (c*e^2*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2,x]
```

```
[Out] IntegrateAlgebraic[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2, x]
```

fricas [A] time = 0.84, size = 74, normalized size = 1.04

$$\frac{15\,ce^{2}x^{8}+21\,\left(2\,cde+be^{2}\right)x^{6}+35\,\left(cd^{2}+2\,bde+ae^{2}\right)x^{4}-105\,ad^{2}+105\,\left(bd^{2}+2\,ade\right)x^{2}}{105\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*c*e^2*x^8 + 21*(2*c*d*e + b*e^2)*x^6 + 35*(c*d^2 + 2*b*d*e + a*e^
2)*x^4 - 105*a*d^2 + 105*(b*d^2 + 2*a*d*e)*x^2)/x
```

giac [A] time = 0.36, size = 74, normalized size = 1.04

$$\frac{1}{7}\,cx^{7}e^{2}+\frac{2}{5}\,cdx^{5}e+\frac{1}{5}\,bx^{5}e^{2}+\frac{1}{3}\,cd^{2}x^{3}+\frac{2}{3}\,bdx^{3}e+\frac{1}{3}\,ax^{3}e^{2}+bd^{2}x+2\,adxe-\frac{ad^{2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/7*c*x^7*e^2 + 2/5*c*d*x^5*e + 1/5*b*x^5*e^2 + 1/3*c*d^2*x^3 + 2/3*b*d*x^3
*e + 1/3*a*x^3*e^2 + b*d^2*x + 2*a*d*x*e - a*d^2/x
```

maple [A] time = 0.00, size = 75, normalized size = 1.06

$$\frac{c\,e^{2}x^{7}}{7}+\frac{b\,e^{2}x^{5}}{5}+\frac{2cde\,x^{5}}{5}+\frac{a\,e^{2}x^{3}}{3}+\frac{2bde\,x^{3}}{3}+\frac{c\,d^{2}x^{3}}{3}+2adex+b\,d^{2}x-\frac{a\,d^{2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x)
```

```
[Out] 1/7*c*e^2*x^7+1/5*x^5*b*e^2+2/5*x^5*c*d*e+1/3*x^3*a*e^2+2/3*x^3*b*d*e+1/3*x
^3*c*d^2+2*d*e*a*x+d^2*b*x-a*d^2/x
```

maxima [A] time = 1.07, size = 69, normalized size = 0.97

$$\frac{1}{7}\,ce^{2}x^{7}+\frac{1}{5}\,\left(2\,cde+be^{2}\right)x^{5}+\frac{1}{3}\,\left(cd^{2}+2\,bde+ae^{2}\right)x^{3}-\frac{ad^{2}}{x}+\left(bd^{2}+2\,ade\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*c*e^2*x^7 + 1/5*(2*c*d*e + b*e^2)*x^5 + 1/3*(c*d^2 + 2*b*d*e + a*e^2)*x
^3 - a*d^2/x + (b*d^2 + 2*a*d*e)*x
```

mupad [B] time = 0.03, size = 70, normalized size = 0.99

$$x^{3}\,\left(\frac{c\,d^{2}}{3}+\frac{2\,b\,d\,e}{3}+\frac{a\,e^{2}}{3}\right)+x\,\left(b\,d^{2}+2\,a\,e\,d\right)+x^{5}\,\left(\frac{b\,e^{2}}{5}+\frac{2\,c\,d\,e}{5}\right)-\frac{a\,d^{2}}{x}+\frac{c\,e^{2}\,x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2,x)
```

```
[Out] x^3*((a*e^2)/3 + (c*d^2)/3 + (2*b*d*e)/3) + x*(b*d^2 + 2*a*d*e) + x^5*((b*e
^2)/5 + (2*c*d*e)/5) - (a*d^2)/x + (c*e^2*x^7)/7
```

sympy [A] time = 0.16, size = 73, normalized size = 1.03

$$-\frac{ad^{2}}{x}+\frac{ce^{2}x^{7}}{7}+x^{5}\left(\frac{be^{2}}{5}+\frac{2cde}{5}\right)+x^{3}\left(\frac{ae^{2}}{3}+\frac{2bde}{3}+\frac{cd^{2}}{3}\right)+x\left(2ade+bd^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x**2,x)
```

```
[Out] -a*d**2/x + c*e**2*x**7/7 + x**5*(b*e**2/5 + 2*c*d*e/5) + x**3*(a*e**2/3 +
2*b*d*e/3 + c*d**2/3) + x*(2*a*d*e + b*d**2)
```

3.201 $\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^3}\,dx$

Optimal. Leaf size=74

$$\frac{1}{2}x^2\left(e(ae+2bd)+cd^2\right)+d\log(x)(2ae+bd)-\frac{ad^2}{2x^2}+\frac{1}{4}ex^4(be+2cd)+\frac{1}{6}ce^2x^6$$

Rubi [A] time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1251, 893}

$$\frac{1}{2}x^2\left(e(ae+2bd)+cd^2\right)+d\log(x)(2ae+bd)-\frac{ad^2}{2x^2}+\frac{1}{4}ex^4(be+2cd)+\frac{1}{6}ce^2x^6$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x]
```

```
[Out] -(a*d^2)/(2*x^2) + ((c*d^2 + e*(2*b*d + a*e))*x^2)/2 + (e*(2*c*d + b*e)*x^4
)/4 + (c*e^2*x^6)/6 + d*(b*d + 2*a*e)*Log[x]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)^2(a+bx+cx^2)}{x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(cd^2\left(1+\frac{e(2bd+ae)}{cd^2}\right)+\frac{ad^2}{x^2}+\frac{d(bd+2ae)}{x}+e(2cd+be)x+ce^2\right.\right.\\ &= -\frac{ad^2}{2x^2}+\frac{1}{2}\left(cd^2+e(2bd+ae)\right)x^2+\frac{1}{4}e(2cd+be)x^4+\frac{1}{6}ce^2x^6+d(bd+2ae)\log\end{aligned}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 0.96

$$\frac{1}{12}\left(6x^2\left(e(ae+2bd)+cd^2\right)+12d\log(x)(2ae+bd)-\frac{6ad^2}{x^2}+3ex^4(be+2cd)+2ce^2x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x]
```

```
[Out] ((-6*a*d^2)/x^2 + 6*(c*d^2 + e*(2*b*d + a*e))*x^2 + 3*e*(2*c*d + b*e)*x^4 +
 2*c*e^2*x^6 + 12*d*(b*d + 2*a*e)*Log[x])/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x]
```

```
[Out] IntegrateAlgebraic[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3, x]
```

fricas [A] time = 0.89, size = 76, normalized size = 1.03

$$\frac{2\,ce^2x^8+3\left(2\,cde+be^2\right)x^6+6\left(cd^2+2\,bde+ae^2\right)x^4+12\left(bd^2+2\,ade\right)x^2\log(x)-6\,ad^2}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(2*c*e^2*x^8 + 3*(2*c*d*e + b*e^2)*x^6 + 6*(c*d^2 + 2*b*d*e + a*e^2)*x
^4 + 12*(b*d^2 + 2*a*d*e)*x^2*log(x) - 6*a*d^2)/x^2
```

giac [A] time = 0.38, size = 97, normalized size = 1.31

$$\frac{1}{6}\,cx^6e^2+\frac{1}{2}\,cdx^4e+\frac{1}{4}\,bx^4e^2+\frac{1}{2}\,cd^2x^2+bdx^2e+\frac{1}{2}\,ax^2e^2+\frac{1}{2}\left(bd^2+2\,ade\right)\log\left(x^2\right)-\frac{bd^2x^2+2\,adx^2e+ad^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/6*c*x^6*e^2 + 1/2*c*d*x^4*e + 1/4*b*x^4*e^2 + 1/2*c*d^2*x^2 + b*d*x^2*e +
 1/2*a*x^2*e^2 + 1/2*(b*d^2 + 2*a*d*e)*log(x^2) - 1/2*(b*d^2*x^2 + 2*a*d*x^
2*e + a*d^2)/x^2
```

maple [A] time = 0.01, size = 76, normalized size = 1.03

$$\frac{c\,e^2x^6}{6}+\frac{b\,e^2x^4}{4}+\frac{cde\,x^4}{2}+\frac{a\,e^2x^2}{2}+bde\,x^2+\frac{c\,d^2x^2}{2}+2ade\ln(x)+b\,d^2\ln(x)-\frac{a\,d^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x)
```

```
[Out] 1/6*c*e^2*x^6+1/4*x^4*b*e^2+1/2*x^4*c*d*e+1/2*x^2*a*e^2+x^2*b*d*e+1/2*x^2*c
*d^2+2*ln(x)*a*d*e+ln(x)*b*d^2-1/2*a*d^2/x^2
```

maxima [A] time = 1.09, size = 73, normalized size = 0.99

$$\frac{1}{6}\,ce^2x^6+\frac{1}{4}\left(2\,cde+be^2\right)x^4+\frac{1}{2}\left(cd^2+2\,bde+ae^2\right)x^2+\frac{1}{2}\left(bd^2+2\,ade\right)\log\left(x^2\right)-\frac{ad^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*c*e^2*x^6 + 1/4*(2*c*d*e + b*e^2)*x^4 + 1/2*(c*d^2 + 2*b*d*e + a*e^2)*x
^2 + 1/2*(b*d^2 + 2*a*d*e)*log(x^2) - 1/2*a*d^2/x^2
```

mupad [B] time = 0.04, size = 70, normalized size = 0.95

$$x^2\left(\frac{c\,d^2}{2}+b\,d\,e+\frac{a\,e^2}{2}\right)+x^4\left(\frac{b\,e^2}{4}+\frac{c\,d\,e}{2}\right)+\ln(x)\,\left(b\,d^2+2\,a\,e\,d\right)-\frac{a\,d^2}{2\,x^2}+\frac{c\,e^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x)
```

```
[Out] x^2*((a*e^2)/2 + (c*d^2)/2 + b*d*e) + x^4*((b*e^2)/4 + (c*d*e)/2) + log(x)*
(b*d^2 + 2*a*d*e) - (a*d^2)/(2*x^2) + (c*e^2*x^6)/6
```

sympy [A] time = 0.26, size = 71, normalized size = 0.96

$$-\frac{ad^2}{2x^2}+\frac{ce^2x^6}{6}+d\,(2ae+bd)\log{(x)}+x^4\left(\frac{be^2}{4}+\frac{cde}{2}\right)+x^2\left(\frac{ae^2}{2}+bde+\frac{cd^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x**3,x)
```

```
[Out] -a*d**2/(2*x**2) + c*e**2*x**6/6 + d*(2*a*e + b*d)*log(x) + x**4*(b*e**2/4
+ c*d*e/2) + x**2*(a*e**2/2 + b*d*e + c*d**2/2)
```

3.202 $\int \frac{x^6\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=168

$$-\frac{dx\left(4cd^2-e(3bd-2ae)\right)}{e^5}+\frac{x^3\left(3cd^2-e(2bd-ae)\right)}{3e^4}-\frac{d^2x\left(ae^2-bde+cd^2\right)}{2e^5\left(d+ex^2\right)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(9cd^2-e(7bd-5\right.}{2e^{11/2}}$$

Rubi [A] time = 0.23, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1257, 1810, 205}

$$\frac{x^3\left(3cd^2-e(2bd-ae)\right)}{3e^4}-\frac{d^2x\left(ae^2-bde+cd^2\right)}{2e^5\left(d+ex^2\right)}-\frac{dx\left(4cd^2-e(3bd-2ae)\right)}{e^5}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(9cd^2-e(7bd-5ae)\right)}{2e^{11/2}}-\frac{x^5(2cd-be)}{5e^3}+\frac{cx^7}{7e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] -((d*(4*c*d^2 - e*(3*b*d - 2*a*e))*x)/e^5) + ((3*c*d^2 - e*(2*b*d - a*e))*x
^3)/(3*e^4) - ((2*c*d - b*e)*x^5)/(5*e^3) + (c*x^7)/(7*e^2) - (d^2*(c*d^2 -
 b*d*e + a*e^2)*x)/(2*e^5*(d + e*x^2)) + (d^(3/2)*(9*c*d^2 - e*(7*b*d - 5*a
*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^6\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^2}\,dx = -\frac{d^2\left(cd^2-bde+ae^2\right)x}{2e^5\left(d+ex^2\right)} - \frac{\int \frac{-d^2\left(cd^2-bde+ae^2\right)+2de\left(cd^2-bde+ae^2\right)x^2-2e^2\left(cd^2-bde+ae^2\right)x^4+2e^3(cd-be)x}{d+ex^2}}{2e^5}$$

$$= -\frac{d^2\left(cd^2-bde+ae^2\right)x}{2e^5\left(d+ex^2\right)} - \frac{\int \left(2d\left(4cd^2-e(3bd-2ae)\right)-2e\left(3cd^2-e(2bd-ae)\right)x^2+2}{2e^5}$$

$$= -\frac{d\left(4cd^2-e(3bd-2ae)\right)x}{e^5} + \frac{\left(3cd^2-e(2bd-ae)\right)x^3}{3e^4} - \frac{(2cd-be)x^5}{5e^3} + \frac{cx^7}{7e^2} - \frac{d^2\left(cd^2\right.}{2e^5}$$

$$= -\frac{d\left(4cd^2-e(3bd-2ae)\right)x}{e^5} + \frac{\left(3cd^2-e(2bd-ae)\right)x^3}{3e^4} - \frac{(2cd-be)x^5}{5e^3} + \frac{cx^7}{7e^2} - \frac{d^2\left(cd^2\right.}{2e^5}$$

Mathematica [A] time = 0.14, size = 165, normalized size = 0.98

$$-\frac{dx\left(2ae^2-3bde+4cd^2\right)}{e^5}+\frac{x^3\left(ae^2-2bde+3cd^2\right)}{3e^4}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(5ae^2-7bde+9cd^2\right)}{2e^{11/2}}-\frac{x\left(ad^2e^2-bd^3e+cd^4\right)}{2e^5\left(d+ex^2\right)}+\frac{x^5(be-2cd)}{5e^3}+\frac{cx^7}{7e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] -((d*(4*c*d^2 - 3*b*d*e + 2*a*e^2)*x)/e^5) + ((3*c*d^2 - 2*b*d*e + a*e^2)*x
^3)/(3*e^4) + ((-2*c*d + b*e)*x^5)/(5*e^3) + (c*x^7)/(7*e^2) - ((c*d^4 - b*
d^3*e + a*d^2*e^2)*x)/(2*e^5*(d + e*x^2)) + (d^(3/2)*(9*c*d^2 - 7*b*d*e + 5
*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(11/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2, x]
```

fricas [A] time = 0.99, size = 426, normalized size = 2.54

$$\left[\frac{60ce^4x^9-12\left(9cde^3-7be^4\right)x^7+28\left(9cd^2e^2-7bde^3+5ae^4\right)x^5-140\left(9cd^3e-7bd^2e^2+5ade^3\right)x^3+105\left(9cd^4-7bd^3e+5ad^2e^2+\left(9cd^3e-7bd^2e^2+5ade^3\right)x^2\right)\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2+2ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right)-210\left(9cd^4-7bd^3e+5ad^2e^2\right)x}{420\left(e^6x^2+de^5\right)},\ \frac{30ce^4x^9-6\left(9cde^3-7be^4\right)x^7+14\left(9cd^2e^2-7bde^3+5ae^4\right)x^5-70\left(9cd^3e-7bd^2e^2+5ade^3\right)x^3+105\left(9cd^4-7bd^3e+5ad^2e^2+\left(9cd^3e-7bd^2e^2+5ade^3\right)x^2\right)\sqrt{\frac{d}{e}}\arctan\left(\frac{ex\sqrt{\frac{d}{e}}}{d}\right)-105\left(9cd^4-7bd^3e+5ad^2e^2\right)x}{210\left(e^6x^2+de^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(60*c*e^4*x^9 - 12*(9*c*d*e^3 - 7*b*e^4)*x^7 + 28*(9*c*d^2*e^2 - 7*b
*d*e^3 + 5*a*e^4)*x^5 - 140*(9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)*x^3 + 105
*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2 + (9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)
*x^2)*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) - 210*(9*c
*d^4 - 7*b*d^3*e + 5*a*d^2*e^2)*x)/(e^6*x^2 + d*e^5), 1/210*(30*c*e^4*x^9 -
 6*(9*c*d*e^3 - 7*b*e^4)*x^7 + 14*(9*c*d^2*e^2 - 7*b*d*e^3 + 5*a*e^4)*x^5 -
 70*(9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)*x^3 + 105*(9*c*d^4 - 7*b*d^3*e +
5*a*d^2*e^2 + (9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)*x^2)*sqrt(d/e)*arctan(e
*x*sqrt(d/e)/d) - 105*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2)*x)/(e^6*x^2 + d*e
^5)]
```

giac [A] time = 0.32, size = 160, normalized size = 0.95

$$\frac{\left(9\,cd^4-7\,bd^3e+5\,ad^2e^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{11}{2}\right)}}{2\,\sqrt{d}}+\frac{1}{105}\left(15\,cx^7e^{12}-42\,cdx^5e^{11}+21\,bx^5e^{12}+105\,cd^2x^3e^{10}-70\,bdx^3e^{11}-420\,cd^3xe^9+35\,ax^3e^{12}+315\,bd^2xe^{10}-210\,adxe^{11}\right)e^{(-14)}-\frac{\left(cd^4x-bd^3xe+ad^2xe^2\right)e^{(-5)}}{2\left(x^2e+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-11/2)
/sqrt(d) + 1/105*(15*c*x^7*e^12 - 42*c*d*x^5*e^11 + 21*b*x^5*e^12 + 105*c*d
^2*x^3*e^10 - 70*b*d*x^3*e^11 - 420*c*d^3*x*e^9 + 35*a*x^3*e^12 + 315*b*d^2
*x*e^10 - 210*a*d*x*e^11)*e^(-14) - 1/2*(c*d^4*x - b*d^3*x*e + a*d^2*x*e^2)
*e^(-5)/(x^2*e + d)
```

maple [A] time = 0.01, size = 214, normalized size = 1.27

$$\frac{c\,x^7}{7e^2}+\frac{b\,x^5}{5e^2}-\frac{2cd\,x^5}{5e^3}+\frac{a\,x^3}{3e^2}-\frac{2bd\,x^3}{3e^3}+\frac{c\,d^2x^3}{e^4}-\frac{a\,d^2x}{2\left(e\,x^2+d\right)e^3}+\frac{5a\,d^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,e^3}+\frac{b\,d^3x}{2\left(e\,x^2+d\right)e^4}-\frac{7b\,d^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,e^4}-\frac{c\,d^4x}{2\left(e\,x^2+d\right)e^5}+\frac{9c\,d^4\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,e^5}-\frac{2adx}{e^3}+\frac{3b\,d^2x}{e^4}-\frac{4c\,d^3x}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x)
```

```
[Out] 1/7*c*x^7/e^2+1/5/e^2*x^5*b-2/5/e^3*x^5*c*d+1/3/e^2*x^3*a-2/3/e^3*x^3*b*d+1
/e^4*x^3*c*d^2-2/e^3*a*d*x+3/e^4*d^2*b*x-4/e^5*c*d^3*x-1/2*d^2/e^3*x/(e*x^2
+d)*a+1/2*d^3/e^4*x/(e*x^2+d)*b-1/2*d^4/e^5*x/(e*x^2+d)*c+5/2*d^2/e^3/(d*e)
^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a-7/2*d^3/e^4/(d*e)^(1/2)*arctan(1/(d*e)^(
1/2)*e*x)*b+9/2*d^4/e^5/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c
```

maxima [A] time = 2.49, size = 165, normalized size = 0.98

$$-\frac{\left(cd^4-bd^3e+ad^2e^2\right)x}{2\left(e^6x^2+de^5\right)}+\frac{\left(9\,cd^4-7\,bd^3e+5\,ad^2e^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}\,e^5}+\frac{15\,ce^3x^7-21\left(2\,cde^2-be^3\right)x^5+35\left(3\,cd^2e-2\,bde^2+ae^3\right)x^3-105\left(4\,cd^3-3\,bd^2e+2\,ade^2\right)x}{105\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(c*d^4 - b*d^3*e + a*d^2*e^2)*x/(e^6*x^2 + d*e^5) + 1/2*(9*c*d^4 - 7*b
*d^3*e + 5*a*d^2*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^5) + 1/105*(15*c*e
^3*x^7 - 21*(2*c*d*e^2 - b*e^3)*x^5 + 35*(3*c*d^2*e - 2*b*d*e^2 + a*e^3)*x^
3 - 105*(4*c*d^3 - 3*b*d^2*e + 2*a*d*e^2)*x)/e^5
```

mupad [B] time = 0.33, size = 251, normalized size = 1.49

$$x^5\left(\frac{b}{5\,e^2}-\frac{2\,c\,d}{5\,e^3}\right)-x^3\left(\frac{c\,d^2}{3\,e^4}-\frac{a}{3\,e^2}+\frac{2\,d\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right)}{3\,e}\right)+x\left(\frac{2\,d\left(\frac{c\,d^2}{e^4}-\frac{a}{e^2}+\frac{2\,d\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right)}{e}\right)}{e}-\frac{d^2\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right)}{e^2}\right)-\frac{x\left(\frac{c\,d^4}{2}-\frac{b\,d^3\,e}{2}+\frac{a\,d^2\,e^2}{2}\right)}{e^6\,x^2+d\,e^5}+\frac{c\,x^7}{7\,e^2}+\frac{d^{3/2}\,\mathrm{atan}\left(\frac{d^{3/2}\,\sqrt{e}\,x\left(9\,c\,d^2-7\,b\,d\,e+5\,a\,e^2\right)}{9\,c\,d^4-7\,b\,d^3\,e+5\,a\,d^2\,e^2}\right)\left(9\,c\,d^2-7\,b\,d\,e+5\,a\,e^2\right)}{2\,e^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x)
```

```
[Out] x^5*(b/(5*e^2) - (2*c*d)/(5*e^3)) - x^3*((c*d^2)/(3*e^4) - a/(3*e^2) + (2*d
*(b/e^2 - (2*c*d)/e^3))/(3*e)) + x*((2*d*((c*d^2)/e^4 - a/e^2 + (2*d*(b/e^2
 - (2*c*d)/e^3))/e))/e - (d^2*(b/e^2 - (2*c*d)/e^3))/e^2) - (x*((c*d^4)/2 +
 (a*d^2*e^2)/2 - (b*d^3*e)/2))/(d*e^5 + e^6*x^2) + (c*x^7)/(7*e^2) + (d^(3/
2)*atan((d^(3/2)*e^(1/2)*x*(5*a*e^2 + 9*c*d^2 - 7*b*d*e))/(9*c*d^4 + 5*a*d^
2*e^2 - 7*b*d^3*e))*(5*a*e^2 + 9*c*d^2 - 7*b*d*e))/(2*e^(11/2))
```

sympy [B] time = 1.18, size = 320, normalized size = 1.90

$$\frac{cx^7}{7e^2}+x^5\left(\frac{b}{5e^2}-\frac{2cd}{5e^3}\right)+x^3\left(\frac{a}{3e^2}-\frac{2bd}{3e^3}+\frac{cd^2}{e^4}\right)+x\left(-\frac{2ad}{e^3}+\frac{3bd^2}{e^4}-\frac{4cd^3}{e^5}\right)+\frac{x\left(-ad^2e^2+bd^3e-cd^4\right)}{2de^5+2e^6x^2}-\frac{\sqrt{-\frac{d^3}{e^{11}}}\left(5ae^2-7bde+9cd^2\right)\log\left(-\frac{e^5\sqrt{-\frac{d^3}{e^{11}}}\left(5ae^2-7bde+9cd^2\right)}{5ade^2-7bd^2e+9cd^3}+x\right)}{4}+\frac{\sqrt{-\frac{d^3}{e^{11}}}\left(5ae^2-7bde+9cd^2\right)\log\left(\frac{e^5\sqrt{-\frac{d^3}{e^{11}}}\left(5ae^2-7bde+9cd^2\right)}{5ade^2-7bd^2e+9cd^3}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x**7/(7*e**2) + x**5*(b/(5*e**2) - 2*c*d/(5*e**3)) + x**3*(a/(3*e**2) - 2
*b*d/(3*e**3) + c*d**2/e**4) + x*(-2*a*d/e**3 + 3*b*d**2/e**4 - 4*c*d**3/e*
*5) + x*(-a*d**2*e**2 + b*d**3*e - c*d**4)/(2*d*e**5 + 2*e**6*x**2) - sqrt(
-d**3/e**11)*(5*a*e**2 - 7*b*d*e + 9*c*d**2)*log(-e**5*sqrt(-d**3/e**11)*(5
*a*e**2 - 7*b*d*e + 9*c*d**2)/(5*a*d*e**2 - 7*b*d**2*e + 9*c*d**3) + x)/4 +
 sqrt(-d**3/e**11)*(5*a*e**2 - 7*b*d*e + 9*c*d**2)*log(e**5*sqrt(-d**3/e**1
1)*(5*a*e**2 - 7*b*d*e + 9*c*d**2)/(5*a*d*e**2 - 7*b*d**2*e + 9*c*d**3) + x
)/4
```

3.203 $\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2} dx$

Optimal. Leaf size=135

$$-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(7cd^2-e(5bd-3ae)\right)}{2e^{9/2}}+\frac{x\left(3cd^2-e(2bd-ae)\right)}{e^4}+\frac{dx\left(ae^2-bde+cd^2\right)}{2e^4\left(d+ex^2\right)}-\frac{x^3(2cd-be)}{3e^3}+\frac{cx^5}{5e^2}$$

Rubi [A] time = 0.16, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1257, 1810, 205}

$$\frac{dx\left(ae^2-bde+cd^2\right)}{2e^4\left(d+ex^2\right)}+\frac{x\left(3cd^2-e(2bd-ae)\right)}{e^4}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(7cd^2-e(5bd-3ae)\right)}{2e^{9/2}}-\frac{x^3(2cd-be)}{3e^3}+\frac{cx^5}{5e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] ((3*c*d^2 - e*(2*b*d - a*e))*x)/e^4 - ((2*c*d - b*e)*x^3)/(3*e^3) + (c*x^5)
/(5*e^2) + (d*(c*d^2 - b*d*e + a*e^2)*x)/(2*e^4*(d + e*x^2)) - (Sqrt[d]*(7*
c*d^2 - e*(5*b*d - 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^4\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^2}\,dx = \frac{d\left(cd^2-bde+ae^2\right)x}{2e^4\left(d+ex^2\right)} - \frac{\int \frac{d\left(cd^2-bde+ae^2\right)-2e\left(cd^2-bde+ae^2\right)x^2+2e^2(cd-be)x^4-2ce^3x^6}{d+ex^2}\,dx}{2e^4}$$

$$= \frac{d\left(cd^2-bde+ae^2\right)x}{2e^4\left(d+ex^2\right)} - \frac{\int\left(-2\left(3cd^2-2bde+ae^2\right)+2e(2cd-be)x^2-2ce^2x^4+\frac{7cd^3-5b}{d\cdot}\right.}{2e^4}$$

$$= \frac{\left(3cd^2-e(2bd-ae)\right)x}{e^4} - \frac{(2cd-be)x^3}{3e^3} + \frac{cx^5}{5e^2} + \frac{d\left(cd^2-bde+ae^2\right)x}{2e^4\left(d+ex^2\right)} - \frac{\left(d\left(7cd^2-e(5b\right.\right.}{}$$

$$= \frac{\left(3cd^2-e(2bd-ae)\right)x}{e^4} - \frac{(2cd-be)x^3}{3e^3} + \frac{cx^5}{5e^2} + \frac{d\left(cd^2-bde+ae^2\right)x}{2e^4\left(d+ex^2\right)} - \frac{\sqrt{d}\left(7cd^2-e(5\right.}{}$$

Mathematica [A] time = 0.08, size = 133, normalized size = 0.99

$$-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3ae^2-5bde+7cd^2\right)}{2e^{9/2}} + \frac{x\left(ae^2-2bde+3cd^2\right)}{e^4} + \frac{x\left(ade^2-bd^2e+cd^3\right)}{2e^4\left(d+ex^2\right)} + \frac{x^3(be-2cd)}{3e^3} + \frac{cx^5}{5e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] ((3*c*d^2 - 2*b*d*e + a*e^2)*x)/e^4 + ((-2*c*d + b*e)*x^3)/(3*e^3) + (c*x^5
)/(5*e^2) + ((c*d^3 - b*d^2*e + a*d*e^2)*x)/(2*e^4*(d + e*x^2)) - (Sqrt[d]*
(7*c*d^2 - 5*b*d*e + 3*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2, x]
```

fricas [A] time = 0.99, size = 350, normalized size = 2.59

$$\left[\frac{12ce^3x^7-4\left(7cde^2-5be^3\right)x^5+20\left(7cd^2e-5bde^2+3ae^3\right)x^3+15\left(7cd^3-5bd^2e+3ade^2+\left(7cd^2e-5bde^2+3ae^3\right)x^2\right)\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2-2ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right)+30\left(7cd^3-5bd^2e+3ade^2\right)x}{60\left(e^5x^2+de^4\right)},\ \frac{6ce^3x^7-2\left(7cde^2-5be^3\right)x^5+10\left(7cd^2e-5bde^2+3ae^3\right)x^3-15\left(7cd^3-5bd^2e+3ade^2+\left(7cd^2e-5bde^2+3ae^3\right)x^2\right)\sqrt{\frac{d}{e}}\arctan\left(\frac{ex\sqrt{\frac{d}{e}}}{d}\right)+15\left(7cd^3-5bd^2e+3ade^2\right)x}{30\left(e^5x^2+de^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*c*e^3*x^7 - 4*(7*c*d*e^2 - 5*b*e^3)*x^5 + 20*(7*c*d^2*e - 5*b*d*e
^2 + 3*a*e^3)*x^3 + 15*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2 + (7*c*d^2*e - 5*b*
d*e^2 + 3*a*e^3)*x^2)*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2
+ d)) + 30*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2)*x)/(e^5*x^2 + d*e^4), 1/30*(6*
c*e^3*x^7 - 2*(7*c*d*e^2 - 5*b*e^3)*x^5 + 10*(7*c*d^2*e - 5*b*d*e^2 + 3*a*e
^3)*x^3 - 15*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2 + (7*c*d^2*e - 5*b*d*e^2 + 3*
a*e^3)*x^2)*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 15*(7*c*d^3 - 5*b*d^2*e + 3
*a*d*e^2)*x)/(e^5*x^2 + d*e^4)]
```

giac [A] time = 0.28, size = 125, normalized size = 0.93

$$-\frac{\left(7\,cd^{3}-5\,bd^{2}e+3\,ade^{2}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{2\,\sqrt{d}}+\frac{1}{15}\,\left(3\,cx^{5}e^{8}-10\,cdx^{3}e^{7}+5\,bx^{3}e^{8}+45\,cd^{2}xe^{6}-30\,bdxe^{7}+15\,axe^{8}\right)e^{(-10)}+\frac{\left(cd^{3}x-bd^{2}xe+adxe^{2}\right)e^{(-4)}}{2\,\left(x^{2}e+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/s
qrt(d) + 1/15*(3*c*x^5*e^8 - 10*c*d*x^3*e^7 + 5*b*x^3*e^8 + 45*c*d^2*x*e^6
- 30*b*d*x*e^7 + 15*a*x*e^8)*e^(-10) + 1/2*(c*d^3*x - b*d^2*x*e + a*d*x*e^2
)*e^(-4)/(x^2*e + d)
```

maple [A] time = 0.01, size = 176, normalized size = 1.30

$$\frac{c\,x^{5}}{5e^{2}}+\frac{b\,x^{3}}{3e^{2}}-\frac{2cd\,x^{3}}{3e^{3}}+\frac{adx}{2\left(e\,x^{2}+d\right)e^{2}}-\frac{3ad\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,e^{2}}-\frac{b\,d^{2}x}{2\left(e\,x^{2}+d\right)e^{3}}+\frac{5b\,d^{2}\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,e^{3}}+\frac{c\,d^{3}x}{2\left(e\,x^{2}+d\right)e^{4}}-\frac{7c\,d^{3}\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,e^{4}}+\frac{ax}{e^{2}}-\frac{2bdx}{e^{3}}+\frac{3c\,d^{2}x}{e^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x)
```

```
[Out] 1/5*c*x^5/e^2+1/3/e^2*x^3*b-2/3/e^3*x^3*c*d+1/e^2*a*x-2/e^3*d*b*x+3/e^4*c*d
^2*x+1/2*d/e^2*x/(e*x^2+d)*a-1/2*d^2/e^3*x/(e*x^2+d)*b+1/2*d^3/e^4*x/(e*x^2
+d)*c-3/2*d/e^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a+5/2*d^2/e^3/(d*e)^(
1/2)*arctan(1/(d*e)^(1/2)*e*x)*b-7/2*d^3/e^4/(d*e)^(1/2)*arctan(1/(d*e)^(1/
2)*e*x)*c
```

maxima [A] time = 2.45, size = 130, normalized size = 0.96

$$\frac{\left(cd^{3}-bd^{2}e+ade^{2}\right)x}{2\left(e^{5}x^{2}+de^{4}\right)}-\frac{\left(7\,cd^{3}-5\,bd^{2}e+3\,ade^{2}\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}\,e^{4}}+\frac{3\,ce^{2}x^{5}-5\left(2\,cde-be^{2}\right)x^{3}+15\left(3\,cd^{2}-2\,bde+ae^{2}\right)x}{15\,e^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c*d^3 - b*d^2*e + a*d*e^2)*x/(e^5*x^2 + d*e^4) - 1/2*(7*c*d^3 - 5*b*d^
2*e + 3*a*d*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^4) + 1/15*(3*c*e^2*x^5
- 5*(2*c*d*e - b*e^2)*x^3 + 15*(3*c*d^2 - 2*b*d*e + a*e^2)*x)/e^4
```

mupad [B] time = 0.32, size = 179, normalized size = 1.33

$$x^{3}\left(\frac{b}{3\,e^{2}}-\frac{2\,c\,d}{3\,e^{3}}\right)-x\left(\frac{c\,d^{2}}{e^{4}}-\frac{a}{e^{2}}+\frac{2\,d\left(\frac{b}{e^{2}}-\frac{2\,c\,d}{e^{3}}\right)}{e}\right)+\frac{c\,x^{5}}{5\,e^{2}}+\frac{x\left(\frac{c\,d^{3}}{2}-\frac{b\,d^{2}\,e}{2}+\frac{a\,d\,e^{2}}{2}\right)}{e^{5}\,x^{2}+d\,e^{4}}-\frac{\sqrt{d}\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e}\,x\left(7\,c\,d^{2}-5\,b\,d\,e+3\,a\,e^{2}\right)}{7\,c\,d^{3}-5\,b\,d^{2}\,e+3\,a\,d\,e^{2}}\right)\left(7\,c\,d^{2}-5\,b\,d\,e+3\,a\,e^{2}\right)}{2\,e^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x)
```

```
[Out] x^3*(b/(3*e^2) - (2*c*d)/(3*e^3)) - x*((c*d^2)/e^4 - a/e^2 + (2*d*(b/e^2 -
(2*c*d)/e^3))/e) + (c*x^5)/(5*e^2) + (x*((c*d^3)/2 + (a*d*e^2)/2 - (b*d^2*e
)/2))/(d*e^4 + e^5*x^2) - (d^(1/2)*atan((d^(1/2)*e^(1/2)*x*(3*a*e^2 + 7*c*d
^2 - 5*b*d*e))/(7*c*d^3 + 3*a*d*e^2 - 5*b*d^2*e))*(3*a*e^2 + 7*c*d^2 - 5*b*
d*e))/(2*e^(9/2))
```

sympy [A] time = 1.08, size = 189, normalized size = 1.40

$$\frac{cx^{5}}{5e^{2}}+x^{3}\left(\frac{b}{3e^{2}}-\frac{2cd}{3e^{3}}\right)+x\left(\frac{a}{e^{2}}-\frac{2bd}{e^{3}}+\frac{3cd^{2}}{e^{4}}\right)+\frac{x\left(ade^{2}-bd^{2}e+cd^{3}\right)}{2de^{4}+2e^{5}x^{2}}+\frac{\sqrt{-\frac{d}{e^{9}}}\left(3ae^{2}-5bde+7cd^{2}\right)\log\left(-e^{4}\sqrt{-\frac{d}{e^{9}}}+x\right)}{4}-\frac{\sqrt{-\frac{d}{e^{9}}}\left(3ae^{2}-5bde+7cd^{2}\right)\log\left(e^{4}\sqrt{-\frac{d}{e^{9}}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x**5/(5*e**2) + x**3*(b/(3*e**2) - 2*c*d/(3*e**3)) + x*(a/e**2 - 2*b*d/e*
*3 + 3*c*d**2/e**4) + x*(a*d*e**2 - b*d**2*e + c*d**3)/(2*d*e**4 + 2*e**5*x
**2) + sqrt(-d/e**9)*(3*a*e**2 - 5*b*d*e + 7*c*d**2)*log(-e**4*sqrt(-d/e**9
) + x)/4 - sqrt(-d/e**9)*(3*a*e**2 - 5*b*d*e + 7*c*d**2)*log(e**4*sqrt(-d/e
**9) + x)/4
```

3.204 $\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=106

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(5cd^2-e(3bd-ae)\right)}{2\sqrt{d}\,e^{7/2}}-\frac{x\left(ae^2-bde+cd^2\right)}{2e^3\left(d+ex^2\right)}-\frac{x(2cd-be)}{e^3}+\frac{cx^3}{3e^2}$$

Rubi [A] time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1257, 1153, 205}

$$-\frac{x\left(ae^2-bde+cd^2\right)}{2e^3\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(5cd^2-e(3bd-ae)\right)}{2\sqrt{d}\,e^{7/2}}-\frac{x(2cd-be)}{e^3}+\frac{cx^3}{3e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] -(((2*c*d - b*e)*x)/e^3) + (c*x^3)/(3*e^2) - ((c*d^2 - b*d*e + a*e^2)*x)/(2
*e^3*(d + e*x^2)) + ((5*c*d^2 - e*(3*b*d - a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(2*Sqrt[d]*e^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^2}\,dx = -\frac{\left(cd^2-bde+ae^2\right)x}{2e^3\left(d+ex^2\right)} - \frac{\int \frac{-cd^2+bde-ae^2+2e(cd-be)x^2-2ce^2x^4}{d+ex^2}\,dx}{2e^3}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)x}{2e^3\left(d+ex^2\right)} - \frac{\int \left(2(2cd-be)-2cex^2+\frac{-5cd^2+3bde-ae^2}{d+ex^2}\right)dx}{2e^3}$$

$$= -\frac{(2cd-be)x}{e^3} + \frac{cx^3}{3e^2} - \frac{\left(cd^2-bde+ae^2\right)x}{2e^3\left(d+ex^2\right)} - \frac{\left(-5cd^2+e(3bd-ae)\right)\int \frac{1}{d+ex^2}\,dx}{2e^3}$$

$$= -\frac{(2cd-be)x}{e^3} + \frac{cx^3}{3e^2} - \frac{\left(cd^2-bde+ae^2\right)x}{2e^3\left(d+ex^2\right)} + \frac{\left(5cd^2-e(3bd-ae)\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\sqrt{d}\,e^{7/2}}$$

Mathematica [A] time = 0.06, size = 102, normalized size = 0.96

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-3bde+5cd^2\right)}{2\sqrt{d}\,e^{7/2}} - \frac{x\left(ae^2-bde+cd^2\right)}{2e^3\left(d+ex^2\right)} + \frac{x(be-2cd)}{e^3} + \frac{cx^3}{3e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] ((-2*c*d + b*e)*x)/e^3 + (c*x^3)/(3*e^2) - ((c*d^2 - b*d*e + a*e^2)*x)/(2*e
^3*(d + e*x^2)) + ((5*c*d^2 - 3*b*d*e + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/(2*Sqrt[d]*e^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2, x]
```

fricas [A] time = 1.03, size = 302, normalized size = 2.85

$$\left[\frac{4\,cde^3x^5-4\left(5\,cd^2e^2-3\,bde^3\right)x^3-3\left(5\,cd^3-3\,bd^2e+ade^2+\left(5\,cd^2e-3\,bde^2+ae^3\right)x^2\right)\sqrt{-de}\,\log\left(\frac{ex^2-2\sqrt{-de}\,x-d}{ex^2+d}\right)-6\left(5\,cd^3e-3\,bd^2e^2+ade^3\right)x}{12\left(de^5x^2+d^2e^4\right)},\ \frac{2\,cde^3x^5-2\left(5\,cd^2e^2-3\,bde^3\right)x^3+3\left(5\,cd^3-3\,bd^2e+ade^2+\left(5\,cd^2e-3\,bde^2+ae^3\right)x^2\right)\sqrt{de}\,\arctan\left(\frac{\sqrt{de}\,x}{d}\right)-3\left(5\,cd^3e-3\,bd^2e^2+ade^3\right)x}{6\left(de^5x^2+d^2e^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*c*d*e^3*x^5 - 4*(5*c*d^2*e^2 - 3*b*d*e^3)*x^3 - 3*(5*c*d^3 - 3*b*d
^2*e + a*d*e^2 + (5*c*d^2*e - 3*b*d*e^2 + a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2
 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 6*(5*c*d^3*e - 3*b*d^2*e^2 + a*d*e^3)
*x)/(d*e^5*x^2 + d^2*e^4), 1/6*(2*c*d*e^3*x^5 - 2*(5*c*d^2*e^2 - 3*b*d*e^3)
*x^3 + 3*(5*c*d^3 - 3*b*d^2*e + a*d*e^2 + (5*c*d^2*e - 3*b*d*e^2 + a*e^3)*x
^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(5*c*d^3*e - 3*b*d^2*e^2 + a*d*e^3)
*x)/(d*e^5*x^2 + d^2*e^4)]
```

giac [A] time = 0.34, size = 91, normalized size = 0.86

$$\frac{\left(5\,cd^2-3\,bde+ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)}}{2\sqrt{d}} + \frac{1}{3}\left(cx^3e^4-6\,cdxe^3+3\,bxe^4\right)e^{(-6)} - \frac{\left(cd^2x-bdxe+axe^2\right)e^{(-3)}}{2\left(x^2e+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*c*d^2 - 3*b*d*e + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-7/2)/sqrt(d)
+ 1/3*(c*x^3*e^4 - 6*c*d*x*e^3 + 3*b*x*e^4)*e^(-6) - 1/2*(c*d^2*x - b*d*x*e
 + a*x*e^2)*e^(-3)/(x^2*e + d)
```

maple [A] time = 0.01, size = 141, normalized size = 1.33

$$\frac{c\,x^3}{3e^2}-\frac{ax}{2\left(e\,x^2+d\right)e}+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;e}+\frac{bdx}{2\left(e\,x^2+d\right)e^2}-\frac{3bd\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;e^2}-\frac{c\,d^2x}{2\left(e\,x^2+d\right)e^3}+\frac{5c\,d^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;e^3}+\frac{bx}{e^2}-\frac{2cdx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x)
```

```
[Out] 1/3*c*x^3/e^2+1/e^2*b*x-2/e^3*c*d*x-1/2/e*x/(e*x^2+d)*a+1/2/e^2*x/(e*x^2+d)
*d*b-1/2/e^3*x/(e*x^2+d)*c*d^2+1/2/e/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*
a-3/2/e^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*d*b+5/2/e^3/(d*e)^(1/2)*arc
tan(1/(d*e)^(1/2)*e*x)*c*d^2
```

maxima [A] time = 2.52, size = 95, normalized size = 0.90

$$-\frac{\left(cd^2-bde+ae^2\right)x}{2\left(e^4x^2+de^3\right)}+\frac{\left(5\,cd^2-3\,bde+ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}\,e^3}+\frac{cex^3-3\,(2\,cd-be)x}{3\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(c*d^2 - b*d*e + a*e^2)*x/(e^4*x^2 + d*e^3) + 1/2*(5*c*d^2 - 3*b*d*e +
 a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^3) + 1/3*(c*e*x^3 - 3*(2*c*d - b
*e)*x)/e^3
```

mupad [B] time = 0.34, size = 95, normalized size = 0.90

$$x\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right)-\frac{x\left(\frac{c\,d^2}{2}-\frac{b\,d\,e}{2}+\frac{a\,e^2}{2}\right)}{e^4\,x^2+d\,e^3}+\frac{c\,x^3}{3\,e^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(5\,c\,d^2-3\,b\,d\,e+a\,e^2\right)}{2\,\sqrt{d}\;e^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x)
```

```
[Out] x*(b/e^2 - (2*c*d)/e^3) - (x*((a*e^2)/2 + (c*d^2)/2 - (b*d*e)/2))/(d*e^3 +
e^4*x^2) + (c*x^3)/(3*e^2) + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 + 5*c*d^2 -
3*b*d*e))/(2*d^(1/2)*e^(7/2))
```

sympy [A] time = 0.97, size = 162, normalized size = 1.53

$$\frac{cx^3}{3e^2}+x\left(\frac{b}{e^2}-\frac{2cd}{e^3}\right)+\frac{x\left(-ae^2+bde-cd^2\right)}{2de^3+2e^4x^2}-\frac{\sqrt{-\frac{1}{de^7}}\left(ae^2-3bde+5cd^2\right)\log\left(-de^3\sqrt{-\frac{1}{de^7}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{de^7}}\left(ae^2-3bde+5cd^2\right)\log\left(de^3\sqrt{-\frac{1}{de^7}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x**3/(3*e**2) + x*(b/e**2 - 2*c*d/e**3) + x*(-a*e**2 + b*d*e - c*d**2)/(2
*d*e**3 + 2*e**4*x**2) - sqrt(-1/(d*e**7))*(a*e**2 - 3*b*d*e + 5*c*d**2)*lo
g(-d*e**3*sqrt(-1/(d*e**7)) + x)/4 + sqrt(-1/(d*e**7))*(a*e**2 - 3*b*d*e +
5*c*d**2)*log(d*e**3*sqrt(-1/(d*e**7)) + x)/4
```

3.205 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=83

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1157, 388, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - e
*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2}\,dx &= \frac{\left(cd^2-bde+ae^2\right)x}{2de^2\left(d+ex^2\right)}-\frac{\int \frac{\frac{cd^2-e(bd+ae)}{e^2}-\frac{2cdx^2}{e}}{d+ex^2}\,dx}{2d}\\
&= \frac{cx}{e^2}+\frac{\left(cd^2-bde+ae^2\right)x}{2de^2\left(d+ex^2\right)}-\frac{\left(3cd^2-e(bd+ae)\right)\int \frac{1}{d+ex^2}\,dx}{2de^2}\\
&= \frac{cx}{e^2}+\frac{\left(cd^2-bde+ae^2\right)x}{2de^2\left(d+ex^2\right)}-\frac{\left(3cd^2-e(bd+ae)\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 88, normalized size = 1.06

$$\frac{x\left(ae^2-bde+cd^2\right)}{2de^2\left(d+ex^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-ae^2-bde+3cd^2\right)}{2d^{3/2}e^{5/2}}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 -
 b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(d + e*x^2)^2, x]
```

fricas [A] time = 1.08, size = 268, normalized size = 3.23

$$\left[\frac{4\,cd^2e^2x^3+\left(3\,cd^3-bd^2e-ade^2+\left(3\,cd^2e-bde^2-ae^3\right)x^2\right)\sqrt{-de}\,\log\left(\frac{ex^2-2\,\sqrt{-de}\,x-d}{ex^2+d}\right)+2\left(3\,cd^3e-bd^2e^2+ade^3\right)x}{4\left(d^2e^4x^2+d^3e^3\right)},\ \frac{2\,cd^2e^2x^3-\left(3\,cd^3-bd^2e-ade^2+\left(3\,cd^2e-bde^2-ae^3\right)x^2\right)\sqrt{de}\,\arctan\left(\frac{\sqrt{de}\,x}{d}\right)+\left(3\,cd^3e-bd^2e^2+ade^3\right)x}{2\left(d^2e^4x^2+d^3e^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2
 - a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2
*(3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*d^2
*e^2*x^3 - (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)*x^2
)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d
^2*e^4*x^2 + d^3*e^3)]
```

giac [A] time = 0.41, size = 75, normalized size = 0.90

$$cxe^{(-2)}-\frac{\left(3\,cd^2-bde-ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}}+\frac{\left(cd^2x-bdxe+axe^2\right)e^{(-2)}}{2\left(x^2e+d\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/
2)/d^(3/2) + 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-2)/((x^2*e + d)*d)
```

maple [A] time = 0.01, size = 118, normalized size = 1.42

$$\frac{ax}{2\left(e\,x^2+d\right)d}+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,d}-\frac{bx}{2\left(e\,x^2+d\right)e}+\frac{b\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,e}+\frac{cdx}{2\left(e\,x^2+d\right)e^2}-\frac{3cd\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,e^2}+\frac{cx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^2,x)
```

```
[Out] c*x/e^2+1/2/d*x/(e*x^2+d)*a-1/2/e*x/(e*x^2+d)*b+1/2/e^2*d*x/(e*x^2+d)*c+1/2
/d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a+1/2/e/(d*e)^(1/2)*arctan(1/(d*e)
^(1/2)*e*x)*b-3/2/e^2*d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c
```

maxima [A] time = 2.34, size = 84, normalized size = 1.01

$$\frac{\left(cd^2-bde+ae^2\right)x}{2\left(de^3x^2+d^2e^2\right)}+\frac{cx}{e^2}-\frac{\left(3\,cd^2-bde-ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}\,de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c*d^2 - b*d*e + a*e^2)*x/(d*e^3*x^2 + d^2*e^2) + c*x/e^2 - 1/2*(3*c*d^
2 - b*d*e - a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d*e^2)
```

mupad [B] time = 0.36, size = 77, normalized size = 0.93

$$\frac{c\,x}{e^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(-3\,c\,d^2+b\,d\,e+a\,e^2\right)}{2\,d^{3/2}\,e^{5/2}}+\frac{x\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{2\,d\,\left(e^3\,x^2+d\,e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^2,x)
```

```
[Out] (c*x)/e^2 + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 - 3*c*d^2 + b*d*e))/(2*d^(3/2
)*e^(5/2)) + (x*(a*e^2 + c*d^2 - b*d*e))/(2*d*(d*e^2 + e^3*x^2))
```

sympy [B] time = 0.77, size = 153, normalized size = 1.84

$$\frac{cx}{e^2}+\frac{x\left(ae^2-bde+cd^2\right)}{2d^2e^2+2de^3x^2}-\frac{\sqrt{-\frac{1}{d^3e^5}}\left(ae^2+bde-3cd^2\right)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{d^3e^5}}\left(ae^2+bde-3cd^2\right)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt
(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e
**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2*e*
*2*sqrt(-1/(d**3*e**5)) + x)/4
```

3.206 $\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^2}\,dx$

Optimal. Leaf size=89

$$-\frac{x\left(ae^2-bde+cd^2\right)}{2d^2e\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(e(bd-3ae)+cd^2\right)}{2d^{5/2}e^{3/2}}-\frac{a}{d^2x}$$

Rubi [A] time = 0.12, antiderivative size = 86, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1259, 453, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(e(bd-3ae)+cd^2\right)}{2d^{5/2}e^{3/2}}-\frac{x\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)}{2\left(d+ex^2\right)}-\frac{a}{d^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2),x]
```

```
[Out] -(a/(d^2*x)) - ((c/e - (b*d - a*e)/d^2)*x)/(2*(d + e*x^2)) + ((c*d^2 + e*(b
*d - 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)*e^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^2}\,dx = -\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{2\left(d+ex^2\right)} - \frac{\int \frac{-2ade^2-e\left(cd^2+e(bd-ae)\right)x^2}{x^2\left(d+ex^2\right)}\,dx}{2d^2e^2}$$

$$= -\frac{a}{d^2x} - \frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{2\left(d+ex^2\right)} + \frac{1}{2}\left(\frac{c}{e}+\frac{bd-3ae}{d^2}\right)\int \frac{1}{d+ex^2}\,dx$$

$$= -\frac{a}{d^2x} - \frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{2\left(d+ex^2\right)} + \frac{\left(cd^2+e(bd-3ae)\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{5/2}e^{3/2}}$$

Mathematica [A] time = 0.06, size = 89, normalized size = 1.00

$$-\frac{x\left(ae^2-bde+cd^2\right)}{2d^2e\left(d+ex^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(-3ae^2+bde+cd^2\right)}{2d^{5/2}e^{3/2}} - \frac{a}{d^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2),x]
```

```
[Out] -(a/(d^2*x)) - ((c*d^2 - b*d*e + a*e^2)*x)/(2*d^2*e*(d + e*x^2)) + ((c*d^2
+ b*d*e - 3*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)*e^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2), x]
```

fricas [A] time = 0.90, size = 267, normalized size = 3.00

$$\left[-\frac{4\,ad^2e^2+2\left(cd^3e-bd^2e^2+3\,ade^3\right)x^2-\left(\left(cd^2e+bde^2-3\,ae^3\right)x^3+\left(cd^3+bd^2e-3\,ade^2\right)x\right)\sqrt{-de}\log\left(\frac{ex^2+2\sqrt{-de}\,x-d}{ex^2+d}\right)}{4\left(d^3e^3x^3+d^4e^2x\right)},\ -\frac{2\,ad^2e^2+\left(cd^3e-bd^2e^2+3\,ade^3\right)x^2-\left(\left(cd^2e+bde^2-3\,ae^3\right)x^3+\left(cd^3+bd^2e-3\,ade^2\right)x\right)\sqrt{de}\arctan\left(\frac{\sqrt{de}\,x}{d}\right)}{2\left(d^3e^3x^3+d^4e^2x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*d^2*e^2 + 2*(c*d^3*e - b*d^2*e^2 + 3*a*d*e^3)*x^2 - ((c*d^2*e +
b*d*e^2 - 3*a*e^3)*x^3 + (c*d^3 + b*d^2*e - 3*a*d*e^2)*x)*sqrt(-d*e)*log((e
*x^2 + 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^3*e^3*x^3 + d^4*e^2*x), -1/2*(2
*a*d^2*e^2 + (c*d^3*e - b*d^2*e^2 + 3*a*d*e^3)*x^2 - ((c*d^2*e + b*d*e^2 -
3*a*e^3)*x^3 + (c*d^3 + b*d^2*e - 3*a*d*e^2)*x)*sqrt(d*e)*arctan(sqrt(d*e)*
x/d))/(d^3*e^3*x^3 + d^4*e^2*x)]
```

giac [A] time = 0.29, size = 83, normalized size = 0.93

$$\frac{\left(cd^2+bde-3\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{2\,d^{\frac{5}{2}}} - \frac{\left(cd^2x^2-bdx^2e+3\,ax^2e^2+2\,ade\right)e^{(-1)}}{2\left(x^3e+dx\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(c*d^2 + b*d*e - 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/d^(5/2) -
1/2*(c*d^2*x^2 - b*d*x^2*e + 3*a*x^2*e^2 + 2*a*d*e)*e^(-1)/((x^3*e + d*x)*d
^2)
```

maple [A] time = 0.01, size = 121, normalized size = 1.36

$$-\frac{aex}{2\left(e\,x^2+d\right)d^2}-\frac{3ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;d^2}+\frac{bx}{2\left(e\,x^2+d\right)d}+\frac{b\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;d}-\frac{cx}{2\left(e\,x^2+d\right)e}+\frac{c\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;e}-\frac{a}{d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x)
```

```
[Out] -a/d^2/x-1/2/d^2*e*x/(e*x^2+d)*a+1/2/d*x/(e*x^2+d)*b-1/2/e*x/(e*x^2+d)*c-3/
2/d^2*e/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a+1/2/d/(d*e)^(1/2)*arctan(1/
(d*e)^(1/2)*e*x)*b+1/2/e/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c
```

maxima [A] time = 2.46, size = 87, normalized size = 0.98

$$-\frac{2\,ade+\left(cd^2-bde+3\,ae^2\right)x^2}{2\left(d^2e^2x^3+d^3ex\right)}+\frac{\left(cd^2+bde-3\,ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}\,d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*d*e + (c*d^2 - b*d*e + 3*a*e^2)*x^2)/(d^2*e^2*x^3 + d^3*e*x) + 1/
2*(c*d^2 + b*d*e - 3*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2*e)
```

mupad [B] time = 0.37, size = 81, normalized size = 0.91

$$\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(c\,d^2+b\,d\,e-3\,a\,e^2\right)}{2\,d^{5/2}\,e^{3/2}}-\frac{\frac{a}{d}+\frac{x^2\left(c\,d^2-b\,d\,e+3\,a\,e^2\right)}{2\,d^2\,e}}{e\,x^3+d\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2),x)
```

```
[Out] (atan((e^(1/2)*x)/d^(1/2))*(c*d^2 - 3*a*e^2 + b*d*e))/(2*d^(5/2)*e^(3/2)) -
 (a/d + (x^2*(3*a*e^2 + c*d^2 - b*d*e))/(2*d^2*e))/(d*x + e*x^3)
```

sympy [A] time = 1.12, size = 155, normalized size = 1.74

$$\frac{\sqrt{-\frac{1}{d^5e^3}}\left(3ae^2-bde-cd^2\right)\log\left(-d^3e\sqrt{-\frac{1}{d^5e^3}}+x\right)}{4}-\frac{\sqrt{-\frac{1}{d^5e^3}}\left(3ae^2-bde-cd^2\right)\log\left(d^3e\sqrt{-\frac{1}{d^5e^3}}+x\right)}{4}+\frac{-2ade+x^2\left(-3ae^2+bde-cd^2\right)}{2d^3ex+2d^2e^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**2/(e*x**2+d)**2,x)
```

```
[Out] sqrt(-1/(d**5*e**3))*(3*a*e**2 - b*d*e - c*d**2)*log(-d**3*e*sqrt(-1/(d**5*
e**3)) + x)/4 - sqrt(-1/(d**5*e**3))*(3*a*e**2 - b*d*e - c*d**2)*log(d**3*e
*sqrt(-1/(d**5*e**3)) + x)/4 + (-2*a*d*e + x**2*(-3*a*e**2 + b*d*e - c*d**2
))/(2*d**3*e*x + 2*d**2*e**2*x**3)
```

3.207 $\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=106

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(cd^2-e(3bd-5ae)\right)}{2d^{7/2}\sqrt{e}}+\frac{x\left(ae^2-bde+cd^2\right)}{2d^3\left(d+ex^2\right)}-\frac{bd-2ae}{d^3x}-\frac{a}{3d^2x^3}$$

Rubi [A] time = 0.14, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1259, 1261, 205}

$$\frac{x\left(ae^2-bde+cd^2\right)}{2d^3\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(cd^2-e(3bd-5ae)\right)}{2d^{7/2}\sqrt{e}}-\frac{bd-2ae}{d^3x}-\frac{a}{3d^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2),x]
```

```
[Out] -a/(3*d^2*x^3) - (b*d - 2*a*e)/(d^3*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d^3
*(d + e*x^2)) + ((c*d^2 - e*(3*b*d - 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(
2*d^(7/2)*Sqrt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^2}\,dx=\frac{\left(cd^2-bde+ae^2\right)x}{2d^3\left(d+ex^2\right)}+\frac{\int \frac{2ad^2e^2+2de^2(bd-ae)x^2+e^2\left(cd^2-bde+ae^2\right)x^4}{x^4\left(d+ex^2\right)}\,dx}{2d^3e^2}$$

$$=\frac{\left(cd^2-bde+ae^2\right)x}{2d^3\left(d+ex^2\right)}+\frac{\int \left(\frac{2ade^2}{x^4}-\frac{2e^2(-bd+2ae)}{x^2}+\frac{e^2\left(cd^2-e(3bd-5ae)\right)}{d+ex^2}\right)dx}{2d^3e^2}$$

$$=-\frac{a}{3d^2x^3}-\frac{bd-2ae}{d^3x}+\frac{\left(cd^2-bde+ae^2\right)x}{2d^3\left(d+ex^2\right)}+\frac{\left(cd^2-e(3bd-5ae)\right)\int \frac{1}{d+ex^2}\,dx}{2d^3}$$

$$=-\frac{a}{3d^2x^3}-\frac{bd-2ae}{d^3x}+\frac{\left(cd^2-bde+ae^2\right)x}{2d^3\left(d+ex^2\right)}+\frac{\left(cd^2-e(3bd-5ae)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{7/2}\sqrt{e}}$$

Mathematica [A] time = 0.06, size = 105, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(5ae^2-3bde+cd^2\right)}{2d^{7/2}\sqrt{e}}+\frac{x\left(ae^2-bde+cd^2\right)}{2d^3\left(d+ex^2\right)}+\frac{2ae-bd}{d^3x}-\frac{a}{3d^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2),x]
```

```
[Out] -1/3*a/(d^2*x^3) + (-(b*d) + 2*a*e)/(d^3*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(
2*d^3*(d + e*x^2)) + ((c*d^2 - 3*b*d*e + 5*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d
]])/(2*d^(7/2)*Sqrt[e])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2), x]
```

fricas [A] time = 0.88, size = 316, normalized size = 2.98

$$\left[-\frac{4\,ad^3e-6\left(cd^3e-3\,bd^2e^2+5\,ade^3\right)x^4+4\left(3\,bd^3e-5\,ad^2e^2\right)x^2+3\left(\left(cd^2e-3\,bde^2+5\,ae^3\right)x^5+\left(cd^3-3\,bd^2e+5\,ade^2\right)x^3\right)\sqrt{-de}\log\left(\frac{ex^2-2\sqrt{-de}x-d}{ex^2+d}\right)}{12\left(d^4e^2x^5+d^5ex^3\right)},-\frac{2\,ad^3e-3\left(cd^3e-3\,bd^2e^2+5\,ade^3\right)x^4+2\left(3\,bd^3e-5\,ad^2e^2\right)x^2-3\left(\left(cd^2e-3\,bde^2+5\,ae^3\right)x^5+\left(cd^3-3\,bd^2e+5\,ade^2\right)x^3\right)\sqrt{de}\arctan\left(\frac{\sqrt{de}x}{d}\right)}{6\left(d^4e^2x^5+d^5ex^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a*d^3*e - 6*(c*d^3*e - 3*b*d^2*e^2 + 5*a*d*e^3)*x^4 + 4*(3*b*d^3*
e - 5*a*d^2*e^2)*x^2 + 3*((c*d^2*e - 3*b*d*e^2 + 5*a*e^3)*x^5 + (c*d^3 - 3*
b*d^2*e + 5*a*d*e^2)*x^3)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^
2 + d)))/(d^4*e^2*x^5 + d^5*e*x^3), -1/6*(2*a*d^3*e - 3*(c*d^3*e - 3*b*d^2*
e^2 + 5*a*d*e^3)*x^4 + 2*(3*b*d^3*e - 5*a*d^2*e^2)*x^2 - 3*((c*d^2*e - 3*b*
d*e^2 + 5*a*e^3)*x^5 + (c*d^3 - 3*b*d^2*e + 5*a*d*e^2)*x^3)*sqrt(d*e)*arcta
n(sqrt(d*e)*x/d))/(d^4*e^2*x^5 + d^5*e*x^3)]
```

giac [A] time = 0.26, size = 94, normalized size = 0.89

$$\frac{\left(cd^2-3\,bde+5\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,d^{\frac{7}{2}}}+\frac{cd^2x-bdxe+axe^2}{2\left(x^2e+d\right)d^3}-\frac{3\,bdx^2-6\,ax^2e+ad}{3\,d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(c*d^2 - 3*b*d*e + 5*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(7/2)
+ 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)/((x^2*e + d)*d^3) - 1/3*(3*b*d*x^2 - 6*
a*x^2*e + a*d)/(d^3*x^3)
```

maple [A] time = 0.01, size = 146, normalized size = 1.38

$$\frac{a\,e^2x}{2\left(e\,x^2+d\right)d^3}+\frac{5a\,e^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,d^3}-\frac{bex}{2\left(e\,x^2+d\right)d^2}-\frac{3be\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,d^2}+\frac{cx}{2\left(e\,x^2+d\right)d}+\frac{c\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,d}+\frac{2ae}{d^3x}-\frac{b}{d^2x}-\frac{a}{3d^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x)
```

```
[Out] -1/3*a/d^2/x^3+2/d^3/x*a*e-1/d^2/x*b+1/2/d^3*x/(e*x^2+d)*a*e^2-1/2/d^2*x/(e
*x^2+d)*e*b+1/2/d*x/(e*x^2+d)*c+5/2/d^3/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*
x)*a*e^2-3/2/d^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*e*b+1/2/d/(d*e)^(1/2
)*arctan(1/(d*e)^(1/2)*e*x)*c
```

maxima [A] time = 2.39, size = 103, normalized size = 0.97

$$\frac{3\left(cd^2-3\,bde+5\,ae^2\right)x^4-2\,ad^2-2\left(3\,bd^2-5\,ade\right)x^2}{6\left(d^3ex^5+d^4x^3\right)}+\frac{\left(cd^2-3\,bde+5\,ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(c*d^2 - 3*b*d*e + 5*a*e^2)*x^4 - 2*a*d^2 - 2*(3*b*d^2 - 5*a*d*e)*x^
2)/(d^3*e*x^5 + d^4*x^3) + 1/2*(c*d^2 - 3*b*d*e + 5*a*e^2)*arctan(e*x/sqrt(
d*e))/(sqrt(d*e)*d^3)
```

mupad [B] time = 0.36, size = 98, normalized size = 0.92

$$\frac{\frac{x^2\,\left(5\,a\,e-3\,b\,d\right)}{3\,d^2}-\frac{a}{3\,d}+\frac{x^4\,\left(c\,d^2-3\,b\,d\,e+5\,a\,e^2\right)}{2\,d^3}}{e\,x^5+d\,x^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(c\,d^2-3\,b\,d\,e+5\,a\,e^2\right)}{2\,d^{7/2}\,\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2),x)
```

```
[Out] ((x^2*(5*a*e - 3*b*d))/(3*d^2) - a/(3*d) + (x^4*(5*a*e^2 + c*d^2 - 3*b*d*e)
)/(2*d^3))/(d*x^3 + e*x^5) + (atan((e^(1/2)*x)/d^(1/2))*(5*a*e^2 + c*d^2 -
3*b*d*e))/(2*d^(7/2)*e^(1/2))
```

sympy [A] time = 1.53, size = 167, normalized size = 1.58

$$-\frac{\sqrt{-\frac{1}{d^7e}}\left(5ae^2-3bde+cd^2\right)\log\left(-d^4\sqrt{-\frac{1}{d^7e}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{d^7e}}\left(5ae^2-3bde+cd^2\right)\log\left(d^4\sqrt{-\frac{1}{d^7e}}+x\right)}{4}+\frac{-2ad^2+x^4\left(15ae^2-9bde+3cd^2\right)+x^2\left(10ade-6bd^2\right)}{6d^4x^3+6d^3ex^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**4/(e*x**2+d)**2,x)
```

```
[Out] -sqrt(-1/(d**7*e))*(5*a*e**2 - 3*b*d*e + c*d**2)*log(-d**4*sqrt(-1/(d**7*e)
) + x)/4 + sqrt(-1/(d**7*e))*(5*a*e**2 - 3*b*d*e + c*d**2)*log(d**4*sqrt(-1
/(d**7*e)) + x)/4 + (-2*a*d**2 + x**4*(15*a*e**2 - 9*b*d*e + 3*c*d**2) + x*
*2*(10*a*d*e - 6*b*d**2))/(6*d**4*x**3 + 6*d**3*e*x**5)
```

3.208 $\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=136

$$-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(5bd-7ae)\right)}{2d^{9/2}}-\frac{ex\left(ae^2-bde+cd^2\right)}{2d^4\left(d+ex^2\right)}-\frac{cd^2-e(2bd-3ae)}{d^4x}-\frac{bd-2ae}{3d^3x^3}-\frac{a}{5d^2x^5}$$

Rubi [A] time = 0.25, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1259, 1802, 205}

$$-\frac{ex\left(ae^2-bde+cd^2\right)}{2d^4\left(d+ex^2\right)}-\frac{cd^2-e(2bd-3ae)}{d^4x}-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(5bd-7ae)\right)}{2d^{9/2}}-\frac{bd-2ae}{3d^3x^3}-\frac{a}{5d^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2),x]
```

```
[Out] -a/(5*d^2*x^5) - (b*d - 2*a*e)/(3*d^3*x^3) - (c*d^2 - e*(2*b*d - 3*a*e))/(d
^4*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(2*d^4*(d + e*x^2)) - (Sqrt[e]*(3*c*d
^2 - e*(5*b*d - 7*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^2}\,dx = -\frac{e\left(cd^2-bde+ae^2\right)x}{2d^4\left(d+ex^2\right)} - \frac{\int \frac{-2ad^3e^2-2d^2e^2(bd-ae)x^2-2de^2\left(cd^2-bde+ae^2\right)x^4+e^3\left(cd^2-bde+ae^2\right)x^6}{x^6\left(d+ex^2\right)}\,dx}{2d^4e^2}$$

$$= -\frac{e\left(cd^2-bde+ae^2\right)x}{2d^4\left(d+ex^2\right)} - \frac{\int \left(-\frac{2ad^2e^2}{x^6} - \frac{2de^2(bd-2ae)}{x^4} + \frac{2e^2\left(-cd^2+e(2bd-3ae)\right)}{x^2} + \frac{e^3\left(3cd^2-e(5bd-7ae)\right)}{d+ex^2}\right.}{2d^4e^2}$$

$$= -\frac{a}{5d^2x^5} - \frac{bd-2ae}{3d^3x^3} - \frac{cd^2-e(2bd-3ae)}{d^4x} - \frac{e\left(cd^2-bde+ae^2\right)x}{2d^4\left(d+ex^2\right)} - \frac{\left(e\left(3cd^2-e(5bd-7\right.\right.}{2d^4}$$

$$= -\frac{a}{5d^2x^5} - \frac{bd-2ae}{3d^3x^3} - \frac{cd^2-e(2bd-3ae)}{d^4x} - \frac{e\left(cd^2-bde+ae^2\right)x}{2d^4\left(d+ex^2\right)} - \frac{\sqrt{e}\left(3cd^2-e(5bd-\right.}{2d^{9/}}$$

Mathematica [A] time = 0.09, size = 135, normalized size = 0.99

$$-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(7ae^2-5bde+3cd^2\right)}{2d^{9/2}} - \frac{ex\left(ae^2-bde+cd^2\right)}{2d^4\left(d+ex^2\right)} + \frac{-3ae^2+2bde-cd^2}{d^4x} + \frac{2ae-bd}{3d^3x^3} - \frac{a}{5d^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2),x]
```

```
[Out] -1/5*a/(d^2*x^5) + (-(b*d) + 2*a*e)/(3*d^3*x^3) + (-(c*d^2) + 2*b*d*e - 3*a
*e^2)/(d^4*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(2*d^4*(d + e*x^2)) - (Sqrt[e
]*(3*c*d^2 - 5*b*d*e + 7*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2), x]
```

fricas [A] time = 0.86, size = 360, normalized size = 2.65

$$\left[-\frac{30\left(3cd^2e-5bde^2+7ae^3\right)x^6+20\left(3cd^3-5bd^2e+7ade^2\right)x^4+12ad^3+4\left(5bd^3-7ad^2e\right)x^2-15\left(\left(3cd^2e-5bde^2+7ae^3\right)x^7+\left(3cd^3-5bd^2e+7ade^2\right)x^5\right)\sqrt{-\frac{e}{d}}\log\left(\frac{ex^2-2dx\sqrt{-\frac{e}{d}}-d}{ex^2+d}\right)}{60\left(d^4ex^7+d^5x^5\right)}, -\frac{15\left(3cd^2e-5bde^2+7ae^3\right)x^6+10\left(3cd^3-5bd^2e+7ade^2\right)x^4+6ad^3+2\left(5bd^3-7ad^2e\right)x^2+15\left(\left(3cd^2e-5bde^2+7ae^3\right)x^7+\left(3cd^3-5bd^2e+7ade^2\right)x^5\right)\sqrt{\frac{e}{d}}\arctan\left(x\sqrt{\frac{e}{d}}\right)}{30\left(d^4ex^7+d^5x^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(30*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*x^6 + 20*(3*c*d^3 - 5*b*d^2*e
+ 7*a*d*e^2)*x^4 + 12*a*d^3 + 4*(5*b*d^3 - 7*a*d^2*e)*x^2 - 15*((3*c*d^2*e
- 5*b*d*e^2 + 7*a*e^3)*x^7 + (3*c*d^3 - 5*b*d^2*e + 7*a*d*e^2)*x^5)*sqrt(-e
/d)*log((e*x^2 - 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)))/(d^4*e*x^7 + d^5*x^5),
 -1/30*(15*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*x^6 + 10*(3*c*d^3 - 5*b*d^2*e
+ 7*a*d*e^2)*x^4 + 6*a*d^3 + 2*(5*b*d^3 - 7*a*d^2*e)*x^2 + 15*((3*c*d^2*e -
 5*b*d*e^2 + 7*a*e^3)*x^7 + (3*c*d^3 - 5*b*d^2*e + 7*a*d*e^2)*x^5)*sqrt(e/d
)*arctan(x*sqrt(e/d)))/(d^4*e*x^7 + d^5*x^5)]
```

giac [A] time = 0.33, size = 131, normalized size = 0.96

$$-\frac{\left(3\,cd^2e-5\,bde^2+7\,ae^3\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,d^{\frac{9}{2}}}-\frac{cd^2xe-bdxe^2+axe^3}{2\left(x^2e+d\right)d^4}-\frac{15\,cd^2x^4-30\,bdx^4e+45\,ax^4e^2+5\,bd^2x^2-10\,adx^2e+3\,ad^2}{15\,d^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d
^(9/2) - 1/2*(c*d^2*x*e - b*d*x*e^2 + a*x*e^3)/((x^2*e + d)*d^4) - 1/15*(15
*c*d^2*x^4 - 30*b*d*x^4*e + 45*a*x^4*e^2 + 5*b*d^2*x^2 - 10*a*d*x^2*e + 3*a
*d^2)/(d^4*x^5)
```

maple [A] time = 0.02, size = 183, normalized size = 1.35

$$-\frac{a\,e^3x}{2\left(e\,x^2+d\right)d^4}-\frac{7a\,e^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;d^4}+\frac{b\,e^2x}{2\left(e\,x^2+d\right)d^3}+\frac{5b\,e^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;d^3}-\frac{cex}{2\left(e\,x^2+d\right)d^2}-\frac{3ce\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;d^2}-\frac{3a\,e^2}{d^4x}+\frac{2be}{d^3x}-\frac{c}{d^2x}+\frac{2ae}{3d^3x^3}-\frac{b}{3d^2x^3}-\frac{a}{5d^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x)
```

```
[Out] -1/5*a/d^2/x^5+2/3/d^3/x^3*a*e-1/3/d^2/x^3*b-3/d^4/x*a*e^2+2/d^3/x*e*b-1/d^
2/x*c-1/2*e^3/d^4*x/(e*x^2+d)*a+1/2*e^2/d^3*x/(e*x^2+d)*b-1/2*e/d^2*x/(e*x^
2+d)*c-7/2*e^3/d^4/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a+5/2*e^2/d^3/(d*e
)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b-3/2*e/d^2/(d*e)^(1/2)*arctan(1/(d*e)^(1
/2)*e*x)*c
```

maxima [A] time = 2.46, size = 139, normalized size = 1.02

$$-\frac{15\left(3\,cd^2e-5\,bde^2+7\,ae^3\right)x^6+10\left(3\,cd^3-5\,bd^2e+7\,ade^2\right)x^4+6\,ad^3+2\left(5\,bd^3-7\,ad^2e\right)x^2}{30\left(d^4ex^7+d^5x^5\right)}-\frac{\left(3\,cd^2e-5\,bde^2+7\,ae^3\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/30*(15*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*x^6 + 10*(3*c*d^3 - 5*b*d^2*e +
 7*a*d*e^2)*x^4 + 6*a*d^3 + 2*(5*b*d^3 - 7*a*d^2*e)*x^2)/(d^4*e*x^7 + d^5*x
^5) - 1/2*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*arctan(e*x/sqrt(d*e))/(sqrt(d*e
)*d^4)
```

mupad [B] time = 0.38, size = 128, normalized size = 0.94

$$-\frac{\frac{a}{5\,d}-\frac{x^2\,\left(7\,a\,e-5\,b\,d\right)}{15\,d^2}+\frac{x^4\,\left(3\,c\,d^2-5\,b\,d\,e+7\,a\,e^2\right)}{3\,d^3}+\frac{e\,x^6\,\left(3\,c\,d^2-5\,b\,d\,e+7\,a\,e^2\right)}{2\,d^4}}{e\,x^7+d\,x^5}-\frac{\sqrt{e}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(3\,c\,d^2-5\,b\,d\,e+7\,a\,e^2\right)}{2\,d^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2),x)
```

```
[Out] - (a/(5*d) - (x^2*(7*a*e - 5*b*d))/(15*d^2) + (x^4*(7*a*e^2 + 3*c*d^2 - 5*b
*d*e))/(3*d^3) + (e*x^6*(7*a*e^2 + 3*c*d^2 - 5*b*d*e))/(2*d^4))/(d*x^5 + e*
x^7) - (e^(1/2)*atan((e^(1/2)*x)/d^(1/2))*(7*a*e^2 + 3*c*d^2 - 5*b*d*e))/(2
*d^(9/2))
```

sympy [B] time = 2.13, size = 284, normalized size = 2.09

$$\frac{\sqrt{-\frac{e}{d^9}}\left(7ae^2-5bde+3cd^2\right)\log\left(-\frac{d^5\sqrt{-\frac{e}{d^9}}\left(7ae^2-5bde+3cd^2\right)}{7ae^3-5bde^2+3cd^2e}+x\right)}{4}-\frac{\sqrt{-\frac{e}{d^9}}\left(7ae^2-5bde+3cd^2\right)\log\left(\frac{d^5\sqrt{-\frac{e}{d^9}}\left(7ae^2-5bde+3cd^2\right)}{7ae^3-5bde^2+3cd^2e}+x\right)}{4}+\frac{-6ad^3+x^6\left(-105ae^3+75bde^2-45cd^2e\right)+x^4\left(-70ade^2+50bd^2e-30cd^3\right)+x^2\left(14ad^2e-10bd^3\right)}{30d^5x^5+30d^4ex^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**6/(e*x**2+d)**2,x)
```

```
[Out] sqrt(-e/d**9)*(7*a*e**2 - 5*b*d*e + 3*c*d**2)*log(-d**5*sqrt(-e/d**9)*(7*a*
e**2 - 5*b*d*e + 3*c*d**2)/(7*a*e**3 - 5*b*d*e**2 + 3*c*d**2*e) + x)/4 - sq
rt(-e/d**9)*(7*a*e**2 - 5*b*d*e + 3*c*d**2)*log(d**5*sqrt(-e/d**9)*(7*a*e**
2 - 5*b*d*e + 3*c*d**2)/(7*a*e**3 - 5*b*d*e**2 + 3*c*d**2*e) + x)/4 + (-6*a
*d**3 + x**6*(-105*a*e**3 + 75*b*d*e**2 - 45*c*d**2*e) + x**4*(-70*a*d*e**2
 + 50*b*d**2*e - 30*c*d**3) + x**2*(14*a*d**2*e - 10*b*d**3))/(30*d**5*x**5
 + 30*d**4*e*x**7)
```

3.209 $\int \frac{a+bx^2+cx^4}{x^8\left(d+ex^2\right)^2} \, dx$

Optimal. Leaf size=167

$$\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(5cd^2-e(7bd-9ae)\right)}{2d^{11/2}}+\frac{e^2x\left(ae^2-bde+cd^2\right)}{2d^5\left(d+ex^2\right)}+\frac{e\left(2cd^2-e(3bd-4ae)\right)}{d^5x}-\frac{cd^2-e(2bd-3ae)}{3d^4x^3}-\frac{bd-}{5d^3}$$

Rubi [A] time = 0.33, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1259, 1802, 205}

$$\frac{e^2x\left(ae^2-bde+cd^2\right)}{2d^5\left(d+ex^2\right)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(5cd^2-e(7bd-9ae)\right)}{2d^{11/2}}-\frac{cd^2-e(2bd-3ae)}{3d^4x^3}+\frac{e\left(2cd^2-e(3bd-4ae)\right)}{d^5x}-\frac{bd-2ae}{5d^3x^5}-\frac{a}{7d^2x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2),x]
```

```
[Out] -a/(7*d^2*x^7) - (b*d - 2*a*e)/(5*d^3*x^5) - (c*d^2 - e*(2*b*d - 3*a*e))/(3
*d^4*x^3) + (e*(2*c*d^2 - e*(3*b*d - 4*a*e)))/(d^5*x) + (e^2*(c*d^2 - b*d*e
 + a*e^2)*x)/(2*d^5*(d + e*x^2)) + (e^(3/2)*(5*c*d^2 - e*(7*b*d - 9*a*e))*A
rcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^8\left(d+ex^2\right)^2}\,dx = \frac{e^2\left(cd^2-bde+ae^2\right)x}{2d^5\left(d+ex^2\right)} + \frac{\int \frac{2ad^4e^2+2d^3e^2(bd-ae)x^2+2d^2e^2\left(cd^2-bde+ae^2\right)x^4-2de^3\left(cd^2-bde+ae^2\right)x^6+e^4\left(cd^2-\right.}{x^8\left(d+ex^2\right)}}{2d^5e^2}$$

$$= \frac{e^2\left(cd^2-bde+ae^2\right)x}{2d^5\left(d+ex^2\right)} + \frac{\int \left(\frac{2ad^3e^2}{x^8}+\frac{2d^2e^2(bd-2ae)}{x^6}+\frac{2de^2\left(cd^2-e(2bd-3ae)\right)}{x^4}+\frac{2e^3\left(-2cd^2+e(3bd-4ae\right.}{x^2}\right.}{2d^5e^2}$$

$$= -\frac{a}{7d^2x^7} - \frac{bd-2ae}{5d^3x^5} - \frac{cd^2-e(2bd-3ae)}{3d^4x^3} + \frac{e\left(2cd^2-e(3bd-4ae)\right)}{d^5x} + \frac{e^2\left(cd^2-bde+ae\right.}{2d^5\left(d+ex^2\right)}$$

$$= -\frac{a}{7d^2x^7} - \frac{bd-2ae}{5d^3x^5} - \frac{cd^2-e(2bd-3ae)}{3d^4x^3} + \frac{e\left(2cd^2-e(3bd-4ae)\right)}{d^5x} + \frac{e^2\left(cd^2-bde+ae\right.}{2d^5\left(d+ex^2\right)}$$

Mathematica [A] time = 0.10, size = 166, normalized size = 0.99

$$\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(9ae^2-7bde+5cd^2\right)}{2d^{11/2}} + \frac{e^2x\left(ae^2-bde+cd^2\right)}{2d^5\left(d+ex^2\right)} + \frac{e\left(4ae^2-3bde+2cd^2\right)}{d^5x} + \frac{-3ae^2+2bde-cd^2}{3d^4x^3} + \frac{2ae-bd}{5d^3x^5} - \frac{a}{7d^2x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2),x]
```

```
[Out] -1/7*a/(d^2*x^7) + (-(b*d) + 2*a*e)/(5*d^3*x^5) + (-(c*d^2) + 2*b*d*e - 3*a
*e^2)/(3*d^4*x^3) + (e*(2*c*d^2 - 3*b*d*e + 4*a*e^2))/(d^5*x) + (e^2*(c*d^2
 - b*d*e + a*e^2)*x)/(2*d^5*(d + e*x^2)) + (e^(3/2)*(5*c*d^2 - 7*b*d*e + 9*
a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(11/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2+cx^4}{x^8\left(d+ex^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2), x]
```

fricas [A] time = 0.92, size = 436, normalized size = 2.61

$$\left[\frac{210\left(5cd^2e^2-7bde^3+9ae^4\right)x^8+140\left(5cd^3e-7bd^2e^2+9ade^3\right)x^6-60ad^4-28\left(5cd^4-7bd^3e+9ad^2e^2\right)x^4-12\left(7bd^4-9ad^3e\right)x^2+105\left(\left(5cd^2e^2-7bde^3+9ae^4\right)x^9+\left(5cd^3e-7bd^2e^2+9ade^3\right)x^7\right)\sqrt{-\frac{e}{d}}\log\left(\frac{ex^2+2dx\sqrt{-\frac{e}{d}}-d}{ex^2+d}\right)}{420\left(d^5ex^9+d^6x^7\right)}, \frac{105\left(5cd^2e^2-7bde^3+9ae^4\right)x^8+70\left(5cd^3e-7bd^2e^2+9ade^3\right)x^6-30ad^4-14\left(5cd^4-7bd^3e+9ad^2e^2\right)x^4-6\left(7bd^4-9ad^3e\right)x^2+105\left(\left(5cd^2e^2-7bde^3+9ae^4\right)x^9+\left(5cd^3e-7bd^2e^2+9ade^3\right)x^7\right)\sqrt{\frac{e}{d}}\arctan\left(x\sqrt{\frac{e}{d}}\right)}{210\left(d^5ex^9+d^6x^7\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(210*(5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^8 + 140*(5*c*d^3*e - 7*b*
d^2*e^2 + 9*a*d*e^3)*x^6 - 60*a*d^4 - 28*(5*c*d^4 - 7*b*d^3*e + 9*a*d^2*e^2
)*x^4 - 12*(7*b*d^4 - 9*a*d^3*e)*x^2 + 105*((5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*
e^4)*x^9 + (5*c*d^3*e - 7*b*d^2*e^2 + 9*a*d*e^3)*x^7)*sqrt(-e/d)*log((e*x^2
 + 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)))/(d^5*e*x^9 + d^6*x^7), 1/210*(105*(5
*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^8 + 70*(5*c*d^3*e - 7*b*d^2*e^2 + 9*a*d
*e^3)*x^6 - 30*a*d^4 - 14*(5*c*d^4 - 7*b*d^3*e + 9*a*d^2*e^2)*x^4 - 6*(7*b*
d^4 - 9*a*d^3*e)*x^2 + 105*((5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^9 + (5*c*
d^3*e - 7*b*d^2*e^2 + 9*a*d*e^3)*x^7)*sqrt(e/d)*arctan(x*sqrt(e/d)))/(d^5*e
*x^9 + d^6*x^7)]
```

giac [A] time = 0.42, size = 164, normalized size = 0.98

$$\frac{\left(5\,cd^{2}e^{2}-7\,bde^{3}+9\,ae^{4}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,d^{\frac{11}{2}}}+\frac{cd^{2}xe^{2}-bdxe^{3}+axe^{4}}{2\,\left(x^{2}e+d\right)d^{5}}+\frac{210\,cd^{2}x^{6}e-315\,bdx^{6}e^{2}-35\,cd^{3}x^{4}+420\,ax^{6}e^{3}+70\,bd^{2}x^{4}e-105\,adx^{4}e^{2}-21\,bd^{3}x^{2}+42\,ad^{2}x^{2}e-15\,ad^{3}}{105\,d^{5}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/
d^(11/2) + 1/2*(c*d^2*x*e^2 - b*d*x*e^3 + a*x*e^4)/((x^2*e + d)*d^5) + 1/10
5*(210*c*d^2*x^6*e - 315*b*d*x^6*e^2 - 35*c*d^3*x^4 + 420*a*x^6*e^3 + 70*b*
d^2*x^4*e - 105*a*d*x^4*e^2 - 21*b*d^3*x^2 + 42*a*d^2*x^2*e - 15*a*d^3)/(d^
5*x^7)
```

maple [A] time = 0.02, size = 221, normalized size = 1.32

$$\frac{a\,e^{4}x}{2\left(e\,x^{2}+d\right)d^{5}}+\frac{9a\,e^{4}\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,d^{5}}-\frac{b\,e^{3}x}{2\left(e\,x^{2}+d\right)d^{4}}-\frac{7b\,e^{3}\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,d^{4}}+\frac{c\,e^{2}x}{2\left(e\,x^{2}+d\right)d^{3}}+\frac{5c\,e^{2}\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,d^{3}}+\frac{4a\,e^{3}}{d^{5}x}-\frac{3b\,e^{2}}{d^{4}x}+\frac{2ce}{d^{3}x}-\frac{a\,e^{2}}{d^{4}x^{3}}+\frac{2be}{3d^{3}x^{3}}-\frac{c}{3d^{2}x^{3}}+\frac{2ae}{5d^{3}x^{5}}-\frac{b}{5d^{2}x^{5}}-\frac{a}{7d^{2}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x)
```

```
[Out] -1/7*a/d^2/x^7+2/5/d^3/x^5*a*e-1/5/d^2/x^5*b-1/d^4/x^3*a*e^2+2/3/d^3/x^3*e*
b-1/3/d^2/x^3*c+4*e^3/d^5/x*a-3*e^2/d^4/x*b+2*e/d^3/x*c+1/2*e^4/d^5*x/(e*x^
2+d)*a-1/2*e^3/d^4*x/(e*x^2+d)*b+1/2*e^2/d^3*x/(e*x^2+d)*c+9/2*e^4/d^5/(d*e
)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a-7/2*e^3/d^4/(d*e)^(1/2)*arctan(1/(d*e)^
(1/2)*e*x)*b+5/2*e^2/d^3/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c
```

maxima [A] time = 2.57, size = 174, normalized size = 1.04

$$\frac{105\left(5\,cd^{2}e^{2}-7\,bde^{3}+9\,ae^{4}\right)x^{8}+70\left(5\,cd^{3}e-7\,bd^{2}e^{2}+9\,ade^{3}\right)x^{6}-30\,ad^{4}-14\left(5\,cd^{4}-7\,bd^{3}e+9\,ad^{2}e^{2}\right)x^{4}-6\left(7\,bd^{4}-9\,ad^{3}e\right)x^{2}}{210\left(d^{5}ex^{9}+d^{6}x^{7}\right)}+\frac{\left(5\,cd^{2}e^{2}-7\,bde^{3}+9\,ae^{4}\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}\,d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/210*(105*(5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^8 + 70*(5*c*d^3*e - 7*b*d^
2*e^2 + 9*a*d*e^3)*x^6 - 30*a*d^4 - 14*(5*c*d^4 - 7*b*d^3*e + 9*a*d^2*e^2)*
x^4 - 6*(7*b*d^4 - 9*a*d^3*e)*x^2)/(d^5*e*x^9 + d^6*x^7) + 1/2*(5*c*d^2*e^2
 - 7*b*d*e^3 + 9*a*e^4)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^5)
```

mupad [B] time = 0.40, size = 156, normalized size = 0.93

$$\frac{\frac{x^{2}\,(9\,a\,e-7\,b\,d)}{35\,d^{2}}-\frac{a}{7\,d}-\frac{x^{4}\,\left(5\,c\,d^{2}-7\,b\,d\,e+9\,a\,e^{2}\right)}{15\,d^{3}}+\frac{e\,x^{6}\,\left(5\,c\,d^{2}-7\,b\,d\,e+9\,a\,e^{2}\right)}{3\,d^{4}}+\frac{e^{2}\,x^{8}\,\left(5\,c\,d^{2}-7\,b\,d\,e+9\,a\,e^{2}\right)}{2\,d^{5}}}{e\,x^{9}+d\,x^{7}}+\frac{e^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(5\,c\,d^{2}-7\,b\,d\,e+9\,a\,e^{2}\right)}{2\,d^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2),x)
```

```
[Out] ((x^2*(9*a*e - 7*b*d))/(35*d^2) - a/(7*d) - (x^4*(9*a*e^2 + 5*c*d^2 - 7*b*d
*e))/(15*d^3) + (e*x^6*(9*a*e^2 + 5*c*d^2 - 7*b*d*e))/(3*d^4) + (e^2*x^8*(9
*a*e^2 + 5*c*d^2 - 7*b*d*e))/(2*d^5))/(d*x^7 + e*x^9) + (e^(3/2)*atan((e^(1
/2)*x)/d^(1/2))*(9*a*e^2 + 5*c*d^2 - 7*b*d*e))/(2*d^(11/2))
```

sympy [B] time = 2.68, size = 328, normalized size = 1.96

$$-\frac{\sqrt{-\frac{e^{3}}{d^{11}}}\left(9ae^{2}-7bde+5cd^{2}\right)\log\left(-\frac{d^{6}\sqrt{-\frac{e^{3}}{d^{11}}}\left(9ae^{2}-7bde+5cd^{2}\right)}{9ae^{4}-7bde^{3}+5cd^{2}e^{2}}+x\right)}{4}+\frac{\sqrt{-\frac{e^{3}}{d^{11}}}\left(9ae^{2}-7bde+5cd^{2}\right)\log\left(\frac{d^{6}\sqrt{-\frac{e^{3}}{d^{11}}}\left(9ae^{2}-7bde+5cd^{2}\right)}{9ae^{4}-7bde^{3}+5cd^{2}e^{2}}+x\right)}{4}+\frac{-30ad^{4}+x^{8}\left(945ae^{4}-735bde^{3}+525cd^{2}e^{2}\right)+x^{6}\left(630ade^{3}-490bd^{2}e^{2}+350cd^{3}e\right)+x^{4}\left(-126ad^{2}e^{2}+98bd^{3}e-70cd^{4}\right)+x^{2}\left(54ad^{3}e-42bd^{4}\right)}{210d^{6}x^{7}+210d^{5}ex^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**8/(e*x**2+d)**2,x)
```

```
[Out] -sqrt(-e**3/d**11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)*log(-d**6*sqrt(-e**3/d**
11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)/(9*a*e**4 - 7*b*d*e**3 + 5*c*d**2*e**2)
 + x)/4 + sqrt(-e**3/d**11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)*log(d**6*sqrt(-
e**3/d**11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)/(9*a*e**4 - 7*b*d*e**3 + 5*c*d*
*2*e**2) + x)/4 + (-30*a*d**4 + x**8*(945*a*e**4 - 735*b*d*e**3 + 525*c*d**
2*e**2) + x**6*(630*a*d*e**3 - 490*b*d**2*e**2 + 350*c*d**3*e) + x**4*(-126
*a*d**2*e**2 + 98*b*d**3*e - 70*c*d**4) + x**2*(54*a*d**3*e - 42*b*d**4))/(
210*d**6*x**7 + 210*d**5*e*x**9)
```

3.210 $\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3} dx$

Optimal. Leaf size=173

$$\frac{dx(17cd^2-e(13bd-9ae))}{8e^5(d+ex^2)}+\frac{x(6cd^2-e(3bd-ae))}{e^5}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(15ae^2-35bde+63cd^2)}{8e^{11/2}}-\frac{d^2x(ae^2-bde+}{4e^5(d+ex^2)^2}$$

Rubi [A] time = 0.32, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1257, 1814, 1810, 205}

$$\frac{dx(17cd^2-e(13bd-9ae))}{8e^5(d+ex^2)}-\frac{d^2x(ae^2-bde+cd^2)}{4e^5(d+ex^2)^2}+\frac{x(6cd^2-e(3bd-ae))}{e^5}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(15ae^2-35bde+63cd^2)}{8e^{11/2}}-\frac{x^3(3cd-be)}{3e^4}+\frac{cx^5}{5e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] ((6*c*d^2 - e*(3*b*d - a*e))*x)/e^5 - ((3*c*d - b*e)*x^3)/(3*e^4) + (c*x^5)
/(5*e^3) - (d^2*(c*d^2 - b*d*e + a*e^2)*x)/(4*e^5*(d + e*x^2)^2) + (d*(17*c
*d^2 - e*(13*b*d - 9*a*e))*x)/(8*e^5*(d + e*x^2)) - (Sqrt[d]*(63*c*d^2 - 35
*b*d*e + 15*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*e^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^6\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^3}\,dx = -\frac{d^2\left(cd^2-bde+ae^2\right)x}{4e^5\left(d+ex^2\right)^2} - \frac{\int \frac{-d^2\left(cd^2-bde+ae^2\right)+4de\left(cd^2-bde+ae^2\right)x^2-4e^2\left(cd^2-bde+ae^2\right)x^4+4e^3(cd-}{\left(d+ex^2\right)^2}}{4e^5}$$

$$= -\frac{d^2\left(cd^2-bde+ae^2\right)x}{4e^5\left(d+ex^2\right)^2} + \frac{d\left(17cd^2-e(13bd-9ae)\right)x}{8e^5\left(d+ex^2\right)} + \int \frac{-d^2\left(15cd^2-e(11bd-7ae)\right)+8de(}{}$$

$$= -\frac{d^2\left(cd^2-bde+ae^2\right)x}{4e^5\left(d+ex^2\right)^2} + \frac{d\left(17cd^2-e(13bd-9ae)\right)x}{8e^5\left(d+ex^2\right)} + \frac{\int \left(8d\left(6cd^2-e(3bd-ae)\right.\right.}{}$$

$$= \frac{\left(6cd^2-e(3bd-ae)\right)x}{e^5} - \frac{(3cd-be)x^3}{3e^4} + \frac{cx^5}{5e^3} - \frac{d^2\left(cd^2-bde+ae^2\right)x}{4e^5\left(d+ex^2\right)^2} + \frac{d\left(17cd^2-\right.}{8e^5}$$

$$= \frac{\left(6cd^2-e(3bd-ae)\right)x}{e^5} - \frac{(3cd-be)x^3}{3e^4} + \frac{cx^5}{5e^3} - \frac{d^2\left(cd^2-bde+ae^2\right)x}{4e^5\left(d+ex^2\right)^2} + \frac{d\left(17cd^2-\right.}{8e^5}$$

Mathematica [A] time = 0.11, size = 170, normalized size = 0.98

$$\frac{x\left(de(9ae-13bd)+17cd^3\right)}{8e^5\left(d+ex^2\right)} - \frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(5e(3ae-7bd)+63cd^2\right)}{8e^{11/2}} + \frac{x\left(e(ae-3bd)+6cd^2\right)}{e^5} - \frac{x\left(d^2e(ae-bd)+cd^4\right)}{4e^5\left(d+ex^2\right)^2} + \frac{x^3(be-3cd)}{3e^4} + \frac{cx^5}{5e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] ((6*c*d^2 + e*(-3*b*d + a*e))*x)/e^5 + ((-3*c*d + b*e)*x^3)/(3*e^4) + (c*x^
5)/(5*e^3) - ((c*d^4 + d^2*e*(-(b*d) + a*e))*x)/(4*e^5*(d + e*x^2)^2) + ((1
7*c*d^3 + d*e*(-13*b*d + 9*a*e))*x)/(8*e^5*(d + e*x^2)) - (Sqrt[d]*(63*c*d^
2 + 5*e*(-7*b*d + 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*e^(11/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3, x]
```

fricas [A] time = 0.73, size = 504, normalized size = 2.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(48*c*e^4*x^9 - 16*(9*c*d*e^3 - 5*b*e^4)*x^7 + 16*(63*c*d^2*e^2 - 35
*b*d*e^3 + 15*a*e^4)*x^5 + 50*(63*c*d^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^3
+ 15*(63*c*d^4 - 35*b*d^3*e + 15*a*d^2*e^2 + (63*c*d^2*e^2 - 35*b*d*e^3 + 1
5*a*e^4)*x^4 + 2*(63*c*d^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^2)*sqrt(-d/e)*l
og((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) + 30*(63*c*d^4 - 35*b*d^3*e
```

```
+ 15*a*d^2*e^2)*x)/(e^7*x^4 + 2*d*e^6*x^2 + d^2*e^5), 1/120*(24*c*e^4*x^9 -
 8*(9*c*d*e^3 - 5*b*e^4)*x^7 + 8*(63*c*d^2*e^2 - 35*b*d*e^3 + 15*a*e^4)*x^5
 + 25*(63*c*d^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^3 - 15*(63*c*d^4 - 35*b*d^
3*e + 15*a*d^2*e^2 + (63*c*d^2*e^2 - 35*b*d*e^3 + 15*a*e^4)*x^4 + 2*(63*c*d
^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^2)*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) +
15*(63*c*d^4 - 35*b*d^3*e + 15*a*d^2*e^2)*x)/(e^7*x^4 + 2*d*e^6*x^2 + d^2*e
^5)]
```

giac [A] time = 0.36, size = 160, normalized size = 0.92

$$-\frac{\left(63\,cd^3-35\,bd^2e+15\,ade^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{11}{2}\right)}}{8\,\sqrt{d}}+\frac{1}{15}\left(3\,cx^5e^{12}-15\,cdx^3e^{11}+5\,bx^3e^{12}+90\,cd^2xe^{10}-45\,bdxe^{11}+15\,axe^{12}\right)e^{(-15)}+\frac{\left(17\,cd^3x^3e-13\,bd^2x^3e^2+15\,cd^4x+9\,adx^3e^3-11\,bd^3xe+7\,ad^2xe^2\right)e^{(-5)}}{8\left(x^2e+d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/8*(63*c*d^3 - 35*b*d^2*e + 15*a*d*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-11/
2)/sqrt(d) + 1/15*(3*c*x^5*e^12 - 15*c*d*x^3*e^11 + 5*b*x^3*e^12 + 90*c*d^2
*x*e^10 - 45*b*d*x*e^11 + 15*a*x*e^12)*e^(-15) + 1/8*(17*c*d^3*x^3*e - 13*b
*d^2*x^3*e^2 + 15*c*d^4*x + 9*a*d*x^3*e^3 - 11*b*d^3*x*e + 7*a*d^2*x*e^2)*e
^(-5)/(x^2*e + d)^2
```

maple [A] time = 0.02, size = 239, normalized size = 1.38

$$\frac{9ad\,x^3}{8\left(e\,x^2+d\right)^2e^2}-\frac{13b\,d^2x^3}{8\left(e\,x^2+d\right)^2e^3}+\frac{17c\,d^3x^3}{8\left(e\,x^2+d\right)^2e^4}+\frac{c\,x^5}{5e^3}+\frac{7a\,d^2x}{8\left(e\,x^2+d\right)^2e^3}-\frac{11b\,d^3x}{8\left(e\,x^2+d\right)^2e^4}+\frac{b\,x^3}{3e^3}+\frac{15c\,d^4x}{8\left(e\,x^2+d\right)^2e^5}-\frac{cd\,x^3}{e^4}-\frac{15ad\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,e^3}+\frac{35b\,d^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,e^4}-\frac{63c\,d^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,e^5}+\frac{ax}{e^3}-\frac{3bdx}{e^4}+\frac{6c\,d^2x}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x)
```

```
[Out] 1/5*c*x^5/e^3+1/3/e^3*x^3*b-1/e^4*x^3*c*d+1/e^3*a*x-3/e^4*d*b*x+6/e^5*c*d^2
*x+9/8*d/e^2/(e*x^2+d)^2*x^3*a-13/8*d^2/e^3/(e*x^2+d)^2*x^3*b+17/8*d^3/e^4/
(e*x^2+d)^2*x^3*c+7/8*d^2/e^3/(e*x^2+d)^2*a*x-11/8*d^3/e^4/(e*x^2+d)^2*b*x+
15/8*d^4/e^5/(e*x^2+d)^2*c*x-15/8*d/e^3/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*
x)*a+35/8*d^2/e^4/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b-63/8*d^3/e^5/(d*e
)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c
```

maxima [A] time = 2.47, size = 175, normalized size = 1.01

$$\frac{\left(17\,cd^3e-13\,bd^2e^2+9\,ade^3\right)x^3+\left(15\,cd^4-11\,bd^3e+7\,ad^2e^2\right)x}{8\left(e^7x^4+2\,de^6x^2+d^2e^5\right)}-\frac{\left(63\,cd^3-35\,bd^2e+15\,ade^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}\,e^5}+\frac{3\,ce^2x^5-5\left(3\,cde-be^2\right)x^3+15\left(6\,cd^2-3\,bde+ae^2\right)x}{15\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((17*c*d^3*e - 13*b*d^2*e^2 + 9*a*d*e^3)*x^3 + (15*c*d^4 - 11*b*d^3*e +
 7*a*d^2*e^2)*x)/(e^7*x^4 + 2*d*e^6*x^2 + d^2*e^5) - 1/8*(63*c*d^3 - 35*b*d
^2*e + 15*a*d*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^5) + 1/15*(3*c*e^2*x^
5 - 5*(3*c*d*e - b*e^2)*x^3 + 15*(6*c*d^2 - 3*b*d*e + a*e^2)*x)/e^5
```

mupad [B] time = 0.35, size = 223, normalized size = 1.29

$$x^3\left(\frac{b}{3\,e^3}-\frac{c\,d}{e^4}\right)-x\left(\frac{3\,c\,d^2}{e^5}-\frac{a}{e^3}+\frac{3\,d\left(\frac{b}{e^3}-\frac{3\,c\,d}{e^4}\right)}{e}\right)+\frac{\left(\frac{17\,c\,d^3\,e}{8}-\frac{13\,b\,d^2\,e^2}{8}+\frac{9\,a\,d\,e^3}{8}\right)x^3+\left(\frac{15\,c\,d^4}{8}-\frac{11\,b\,d^3\,e}{8}+\frac{7\,a\,d^2\,e^2}{8}\right)x}{d^2\,e^5+2\,d\,e^6\,x^2+e^7\,x^4}+\frac{c\,x^5}{5\,e^3}-\frac{\sqrt{d}\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e}\,x\left(63\,c\,d^2-35\,b\,d\,e+15\,a\,e^2\right)}{63\,c\,d^3-35\,b\,d^2\,e+15\,a\,d\,e^2}\right)\left(63\,c\,d^2-35\,b\,d\,e+15\,a\,e^2\right)}{8\,e^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x)
```

```
[Out] x^3*(b/(3*e^3) - (c*d)/e^4) - x*((3*c*d^2)/e^5 - a/e^3 + (3*d*(b/e^3 - (3*c
*d)/e^4))/e) + (x^3*((9*a*d*e^3)/8 - (13*b*d^2*e^2)/8 + (17*c*d^3*e)/8) + x
```

```
*((15*c*d^4)/8 + (7*a*d^2*e^2)/8 - (11*b*d^3*e)/8))/(d^2*e^5 + e^7*x^4 + 2*
d*e^6*x^2) + (c*x^5)/(5*e^3) - (d^(1/2)*atan((d^(1/2)*e^(1/2)*x*(15*a*e^2 +
 63*c*d^2 - 35*b*d*e))/(63*c*d^3 + 15*a*d*e^2 - 35*b*d^2*e))*(15*a*e^2 + 63
*c*d^2 - 35*b*d*e))/(8*e^(11/2))
```

sympy [A] time = 3.58, size = 235, normalized size = 1.36

$$\frac{cx^5}{5e^3} + x^3\left(\frac{b}{3e^3} - \frac{cd}{e^4}\right) + x\left(\frac{a}{e^3} - \frac{3bd}{e^4} + \frac{6cd^2}{e^5}\right) + \frac{\sqrt{-\frac{d}{e^{11}}}\left(15ae^2 - 35bde + 63cd^2\right)\log\left(-e^5\sqrt{-\frac{d}{e^{11}}} + x\right)}{16} - \frac{\sqrt{-\frac{d}{e^{11}}}\left(15ae^2 - 35bde + 63cd^2\right)\log\left(e^5\sqrt{-\frac{d}{e^{11}}} + x\right)}{16} + \frac{x^3\left(9ade^3 - 13bd^2e^2 + 17cd^3e\right) + x\left(7ad^2e^2 - 11bd^3e + 15cd^4\right)}{8d^2e^5 + 16de^6x^2 + 8e^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] c*x**5/(5*e**3) + x**3*(b/(3*e**3) - c*d/e**4) + x*(a/e**3 - 3*b*d/e**4 + 6
*c*d**2/e**5) + sqrt(-d/e**11)*(15*a*e**2 - 35*b*d*e + 63*c*d**2)*log(-e**5
*sqrt(-d/e**11) + x)/16 - sqrt(-d/e**11)*(15*a*e**2 - 35*b*d*e + 63*c*d**2)
*log(e**5*sqrt(-d/e**11) + x)/16 + (x**3*(9*a*d*e**3 - 13*b*d**2*e**2 + 17*
c*d**3*e) + x*(7*a*d**2*e**2 - 11*b*d**3*e + 15*c*d**4))/(8*d**2*e**5 + 16*
d*e**6*x**2 + 8*e**7*x**4)
```

3.211 $\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=143

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(35cd^2-3e(5bd-ae))}{8\sqrt{d}\,e^{9/2}}-\frac{x(13cd^2-e(9bd-5ae))}{8e^4(d+ex^2)}+\frac{dx(ae^2-bde+cd^2)}{4e^4(d+ex^2)^2}-\frac{x(3cd-be)}{e^4}+\frac{cx^3}{3e^3}$$

Rubi [A] time = 0.21, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1257, 1814, 1153, 205}

$$-\frac{x(13cd^2-e(9bd-5ae))}{8e^4(d+ex^2)}+\frac{dx(ae^2-bde+cd^2)}{4e^4(d+ex^2)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(35cd^2-3e(5bd-ae))}{8\sqrt{d}\,e^{9/2}}-\frac{x(3cd-be)}{e^4}+\frac{cx^3}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] -(((3*c*d - b*e)*x)/e^4) + (c*x^3)/(3*e^3) + (d*(c*d^2 - b*d*e + a*e^2)*x)/
(4*e^4*(d + e*x^2)^2) - ((13*c*d^2 - e*(9*b*d - 5*a*e))*x)/(8*e^4*(d + e*x^
2)) + ((35*c*d^2 - 3*e*(5*b*d - a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[
d]*e^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^3}\,dx = \frac{d\left(cd^2-bde+ae^2\right)x}{4e^4\left(d+ex^2\right)^2} - \frac{\int \frac{d\left(cd^2-bde+ae^2\right)-4e\left(cd^2-bde+ae^2\right)x^2+4e^2(cd-be)x^4-4ce^3x^6}{\left(d+ex^2\right)^2}\,dx}{4e^4}$$

$$= \frac{d\left(cd^2-bde+ae^2\right)x}{4e^4\left(d+ex^2\right)^2} - \frac{\left(13cd^2-e(9bd-5ae)\right)x}{8e^4\left(d+ex^2\right)} + \frac{\int \frac{d\left(11cd^2-e(7bd-3ae)\right)-8de(2cd-be)x^2+}{d+ex^2}}{8de^4}$$

$$= \frac{d\left(cd^2-bde+ae^2\right)x}{4e^4\left(d+ex^2\right)^2} - \frac{\left(13cd^2-e(9bd-5ae)\right)x}{8e^4\left(d+ex^2\right)} + \frac{\int\left(-8d(3cd-be)+8cdex^2+\frac{35}{}\right.}{8de^4}$$

$$= -\frac{(3cd-be)x}{e^4} + \frac{cx^3}{3e^3} + \frac{d\left(cd^2-bde+ae^2\right)x}{4e^4\left(d+ex^2\right)^2} - \frac{\left(13cd^2-e(9bd-5ae)\right)x}{8e^4\left(d+ex^2\right)} + \frac{\left(35cd^2-\right.}{}$$

$$= -\frac{(3cd-be)x}{e^4} + \frac{cx^3}{3e^3} + \frac{d\left(cd^2-bde+ae^2\right)x}{4e^4\left(d+ex^2\right)^2} - \frac{\left(13cd^2-e(9bd-5ae)\right)x}{8e^4\left(d+ex^2\right)} + \frac{\left(35cd^2-\right.}{}$$

Mathematica [A] time = 0.09, size = 141, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3ae^2-15bde+35cd^2\right)}{8\sqrt{d}\,e^{9/2}} - \frac{x\left(5ae^2-9bde+13cd^2\right)}{8e^4\left(d+ex^2\right)} + \frac{x\left(ade^2-bd^2e+cd^3\right)}{4e^4\left(d+ex^2\right)^2} + \frac{x(be-3cd)}{e^4} + \frac{cx^3}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] ((-3*c*d + b*e)*x)/e^4 + (c*x^3)/(3*e^3) + ((c*d^3 - b*d^2*e + a*d*e^2)*x)/
(4*e^4*(d + e*x^2)^2) - ((13*c*d^2 - 9*b*d*e + 5*a*e^2)*x)/(8*e^4*(d + e*x^
2)) + ((35*c*d^2 - 15*b*d*e + 3*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt
[d]*e^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3, x]
```

fricas [A] time = 0.61, size = 462, normalized size = 3.23

$$\left[\frac{16\,cde^4x^7-16\left(7\,cd^2e^3-3\,bde^4\right)x^5-10\left(35\,cd^3e^2-15\,bd^2e^3+3\,ade^4\right)x^3-3\left(35\,cd^4-15\,bd^3e+3\,ad^2e^2+\left(35\,cd^2e^2-15\,bde^3+3\,ae^4\right)x^4+2\left(35\,cd^3e-15\,bd^2e^2+3\,ade^3\right)x^2\right)\sqrt{-de}\log\left(\frac{ex^2-2\sqrt{-de}\,x-d}{ex^2+d}\right)-6\left(35\,cd^4e-15\,bd^3e^2+3\,ad^2e^3\right)x}{48\left(de^7x^4+2\,d^2e^6x^2+d^3e^5\right)}, \frac{8\,cde^4x^7-8\left(7\,cd^2e^3-3\,bde^4\right)x^5-5\left(35\,cd^3e^2-15\,bd^2e^3+3\,ade^4\right)x^3+3\left(35\,cd^4-15\,bd^3e+3\,ad^2e^2+\left(35\,cd^2e^2-15\,bde^3+3\,ae^4\right)x^4+2\left(35\,cd^3e-15\,bd^2e^2+3\,ade^3\right)x^2\right)\sqrt{de}\arctan\left(\frac{\sqrt{de}\,x}{d}\right)-3\left(35\,cd^4e-15\,bd^3e^2+3\,ad^2e^3\right)x}{24\left(de^7x^4+2\,d^2e^6x^2+d^3e^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*c*d*e^4*x^7 - 16*(7*c*d^2*e^3 - 3*b*d*e^4)*x^5 - 10*(35*c*d^3*e^2
 - 15*b*d^2*e^3 + 3*a*d*e^4)*x^3 - 3*(35*c*d^4 - 15*b*d^3*e + 3*a*d^2*e^2 +
 (35*c*d^2*e^2 - 15*b*d*e^3 + 3*a*e^4)*x^4 + 2*(35*c*d^3*e - 15*b*d^2*e^2 +
 3*a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) -
 6*(35*c*d^4*e - 15*b*d^3*e^2 + 3*a*d^2*e^3)*x)/(d*e^7*x^4 + 2*d^2*e^6*x^2
```

```
+ d^3*e^5), 1/24*(8*c*d*e^4*x^7 - 8*(7*c*d^2*e^3 - 3*b*d*e^4)*x^5 - 5*(35*c
*d^3*e^2 - 15*b*d^2*e^3 + 3*a*d*e^4)*x^3 + 3*(35*c*d^4 - 15*b*d^3*e + 3*a*d
^2*e^2 + (35*c*d^2*e^2 - 15*b*d*e^3 + 3*a*e^4)*x^4 + 2*(35*c*d^3*e - 15*b*d
^2*e^2 + 3*a*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(35*c*d^4*e -
15*b*d^3*e^2 + 3*a*d^2*e^3)*x)/(d*e^7*x^4 + 2*d^2*e^6*x^2 + d^3*e^5)]
```

giac [A] time = 0.47, size = 125, normalized size = 0.87

$$\frac{\left(35\,cd^2-15\,bde+3\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{8\,\sqrt{d}}+\frac{1}{3}\left(cx^3e^6-9\,cdxe^5+3\,bxe^6\right)e^{(-9)}-\frac{\left(13\,cd^2x^3e-9\,bdx^3e^2+11\,cd^3x+5\,ax^3e^3-7\,bd^2xe+3\,adxe^2\right)e^{(-4)}}{8\left(x^2e+d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*c*d^2 - 15*b*d*e + 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/sqrt
(d) + 1/3*(c*x^3*e^6 - 9*c*d*x*e^5 + 3*b*x*e^6)*e^(-9) - 1/8*(13*c*d^2*x^3*
e - 9*b*d*x^3*e^2 + 11*c*d^3*x + 5*a*x^3*e^3 - 7*b*d^2*x*e + 3*a*d*x*e^2)*e
^(-4)/(x^2*e + d)^2
```

maple [A] time = 0.01, size = 202, normalized size = 1.41

$$-\frac{5a\,x^3}{8\left(e\,x^2+d\right)^2e}+\frac{9bd\,x^3}{8\left(e\,x^2+d\right)^2e^2}-\frac{13c\,d^2x^3}{8\left(e\,x^2+d\right)^2e^3}-\frac{3adx}{8\left(e\,x^2+d\right)^2e^2}+\frac{7b\,d^2x}{8\left(e\,x^2+d\right)^2e^3}-\frac{11c\,d^3x}{8\left(e\,x^2+d\right)^2e^4}+\frac{c\,x^3}{3e^3}+\frac{3a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,e^2}-\frac{15bd\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,e^3}+\frac{35c\,d^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,e^4}+\frac{bx}{e^3}-\frac{3cdx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x)
```

```
[Out] 1/3*c*x^3/e^3+1/e^3*b*x-3/e^4*c*d*x-5/8/e/(e*x^2+d)^2*x^3*a+9/8/e^2/(e*x^2+
d)^2*x^3*b*d-13/8/e^3/(e*x^2+d)^2*x^3*c*d^2-3/8/e^2/(e*x^2+d)^2*a*d*x+7/8/e
^3/(e*x^2+d)^2*d^2*b*x-11/8/e^4/(e*x^2+d)^2*c*d^3*x+3/8/e^2/(d*e)^(1/2)*arc
tan(1/(d*e)^(1/2)*e*x)*a-15/8/e^3/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*d*b
+35/8/e^4/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c*d^2
```

maxima [A] time = 2.51, size = 139, normalized size = 0.97

$$-\frac{\left(13\,cd^2e-9\,bde^2+5\,ae^3\right)x^3+\left(11\,cd^3-7\,bd^2e+3\,ade^2\right)x}{8\left(e^6x^4+2\,de^5x^2+d^2e^4\right)}+\frac{\left(35\,cd^2-15\,bde+3\,ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}\,e^4}+\frac{cex^3-3\,(3\,cd-be)x}{3\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((13*c*d^2*e - 9*b*d*e^2 + 5*a*e^3)*x^3 + (11*c*d^3 - 7*b*d^2*e + 3*a*
d*e^2)*x)/(e^6*x^4 + 2*d*e^5*x^2 + d^2*e^4) + 1/8*(35*c*d^2 - 15*b*d*e + 3*
a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^4) + 1/3*(c*e*x^3 - 3*(3*c*d - b*
e)*x)/e^4
```

mupad [B] time = 0.34, size = 137, normalized size = 0.96

$$x\left(\frac{b}{e^3}-\frac{3\,c\,d}{e^4}\right)-\frac{\left(\frac{13\,c\,d^2\,e}{8}-\frac{9\,b\,d\,e^2}{8}+\frac{5\,a\,e^3}{8}\right)\,x^3+\left(\frac{11\,c\,d^3}{8}-\frac{7\,b\,d^2\,e}{8}+\frac{3\,a\,d\,e^2}{8}\right)\,x}{d^2\,e^4+2\,d\,e^5\,x^2+e^6\,x^4}+\frac{c\,x^3}{3\,e^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(35\,c\,d^2-15\,b\,d\,e+3\,a\,e^2\right)}{8\,\sqrt{d}\,e^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x)
```

```
[Out] x*(b/e^3 - (3*c*d)/e^4) - (x*((11*c*d^3)/8 + (3*a*d*e^2)/8 - (7*b*d^2*e)/8)
 + x^3*((5*a*e^3)/8 - (9*b*d*e^2)/8 + (13*c*d^2*e)/8))/(d^2*e^4 + e^6*x^4 +
 2*d*e^5*x^2) + (c*x^3)/(3*e^3) + (atan((e^(1/2)*x)/d^(1/2))*(3*a*e^2 + 35*
c*d^2 - 15*b*d*e))/(8*d^(1/2)*e^(9/2))
```

sympy [**A**] time = 3.37, size = 212, normalized size = 1.48

$$\frac{cx^3}{3e^3}+x\left(\frac{b}{e^3}-\frac{3cd}{e^4}\right)-\frac{\sqrt{-\frac{1}{de^9}}\left(3ae^2-15bde+35cd^2\right)\log\left(-de^4\sqrt{-\frac{1}{de^9}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{de^9}}\left(3ae^2-15bde+35cd^2\right)\log\left(de^4\sqrt{-\frac{1}{de^9}}+x\right)}{16}+\frac{x^3\left(-5ae^3+9bde^2-13cd^2e\right)+x\left(-3ade^2+7bd^2e-11cd^3\right)}{8d^2e^4+16de^5x^2+8e^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] c*x**3/(3*e**3) + x*(b/e**3 - 3*c*d/e**4) - sqrt(-1/(d*e**9))*(3*a*e**2 - 1
5*b*d*e + 35*c*d**2)*log(-d*e**4*sqrt(-1/(d*e**9)) + x)/16 + sqrt(-1/(d*e**
9))*(3*a*e**2 - 15*b*d*e + 35*c*d**2)*log(d*e**4*sqrt(-1/(d*e**9)) + x)/16
+ (x**3*(-5*a*e**3 + 9*b*d*e**2 - 13*c*d**2*e) + x*(-3*a*d*e**2 + 7*b*d**2*
e - 11*c*d**3))/(8*d**2*e**4 + 16*d*e**5*x**2 + 8*e**6*x**4)
```

3.212 $\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=124

$$\frac{x\left(9cd^2-e(5bd-ae)\right)}{8de^3\left(d+ex^2\right)}-\frac{x\left(ae^2-bde+cd^2\right)}{4e^3\left(d+ex^2\right)^2}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(15cd^2-e(ae+3bd)\right)}{8d^{3/2}e^{7/2}}+\frac{cx}{e^3}$$

Rubi [A] time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1257, 1157, 388, 205}

$$\frac{x\left(9cd^2-e(5bd-ae)\right)}{8de^3\left(d+ex^2\right)}-\frac{x\left(ae^2-bde+cd^2\right)}{4e^3\left(d+ex^2\right)^2}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(15cd^2-e(ae+3bd)\right)}{8d^{3/2}e^{7/2}}+\frac{cx}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] (c*x)/e^3 - ((c*d^2 - b*d*e + a*e^2)*x)/(4*e^3*(d + e*x^2)^2) + ((9*c*d^2 -
 e*(5*b*d - a*e))*x)/(8*d*e^3*(d + e*x^2)) - ((15*c*d^2 - e*(3*b*d + a*e))*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^3}dx = -\frac{\left(cd^2-bde+ae^2\right)x}{4e^3\left(d+ex^2\right)^2} - \frac{\int \frac{-cd^2+bde-ae^2+4e(cd-be)x^2-4ce^2x^4}{\left(d+ex^2\right)^2}dx}{4e^3}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)x}{4e^3\left(d+ex^2\right)^2} + \frac{\left(9cd^2-e(5bd-ae)\right)x}{8de^3\left(d+ex^2\right)} + \frac{\int \frac{-7cd^2+e(3bd+ae)+8cdex^2}{d+ex^2}dx}{8de^3}$$

$$= \frac{cx}{e^3} - \frac{\left(cd^2-bde+ae^2\right)x}{4e^3\left(d+ex^2\right)^2} + \frac{\left(9cd^2-e(5bd-ae)\right)x}{8de^3\left(d+ex^2\right)} - \frac{\left(15cd^2-e(3bd+ae)\right)\int \frac{1}{d+ex^2}}{8de^3}$$

$$= \frac{cx}{e^3} - \frac{\left(cd^2-bde+ae^2\right)x}{4e^3\left(d+ex^2\right)^2} + \frac{\left(9cd^2-e(5bd-ae)\right)x}{8de^3\left(d+ex^2\right)} - \frac{\left(15cd^2-e(3bd+ae)\right)\tan^{-1}\left(\right.}{8d^{3/2}e^{7/2}}$$

Mathematica [**A**] time = 0.10, size = 122, normalized size = 0.98

$$\frac{x\left(ae^2-5bde+9cd^2\right)}{8de^3\left(d+ex^2\right)} - \frac{x\left(ae^2-bde+cd^2\right)}{4e^3\left(d+ex^2\right)^2} - \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-ae^2-3bde+15cd^2\right)}{8d^{3/2}e^{7/2}} + \frac{cx}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] (c*x)/e^3 - ((c*d^2 - b*d*e + a*e^2)*x)/(4*e^3*(d + e*x^2)^2) + ((9*c*d^2 -
 5*b*d*e + a*e^2)*x)/(8*d*e^3*(d + e*x^2)) - ((15*c*d^2 - 3*b*d*e - a*e^2)*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(7/2))
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^3}dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3, x]
```

fricas [**A**] time = 0.58, size = 421, normalized size = 3.40

$$\left[\frac{16\,cd^2e^3x^5 + 2\left(25\,cd^3e^2 - 5\,bd^2e^3 + ade^4\right)x^3 + \left(15\,cd^4 - 3\,bd^3e - ad^2e^2 + \left(15\,cd^2e^2 - 3\,bde^3 - ae^4\right)x^4 + 2\left(15\,cd^3e - 3\,bd^2e^2 - ade^3\right)x^2\right)\sqrt{-de}\,\log\left(\frac{ex^2-2\sqrt{-de}\,x-d}{ex^2+d}\right) + 2\left(15\,cd^4e - 3\,bd^3e^2 - ad^2e^3\right)x}{16\left(d^2e^6x^4 + 2\,d^3e^5x^2 + d^4e^4\right)}, \frac{8\,cd^2e^3x^5 + \left(25\,cd^3e^2 - 5\,bd^2e^3 + ade^4\right)x^3 - \left(15\,cd^4 - 3\,bd^3e - ad^2e^2 + \left(15\,cd^2e^2 - 3\,bde^3 - ae^4\right)x^4 + 2\left(15\,cd^3e - 3\,bd^2e^2 - ade^3\right)x^2\right)\sqrt{de}\,\arctan\left(\frac{\sqrt{de}\,x}{d}\right) + \left(15\,cd^4e - 3\,bd^3e^2 - ad^2e^3\right)x}{8\left(d^2e^6x^4 + 2\,d^3e^5x^2 + d^4e^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*c*d^2*e^3*x^5 + 2*(25*c*d^3*e^2 - 5*b*d^2*e^3 + a*d*e^4)*x^3 + (1
5*c*d^4 - 3*b*d^3*e - a*d^2*e^2 + (15*c*d^2*e^2 - 3*b*d*e^3 - a*e^4)*x^4 +
2*(15*c*d^3*e - 3*b*d^2*e^2 - a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(
-d*e)*x - d)/(e*x^2 + d)) + 2*(15*c*d^4*e - 3*b*d^3*e^2 - a*d^2*e^3)*x)/(d^
2*e^6*x^4 + 2*d^3*e^5*x^2 + d^4*e^4), 1/8*(8*c*d^2*e^3*x^5 + (25*c*d^3*e^2
- 5*b*d^2*e^3 + a*d*e^4)*x^3 - (15*c*d^4 - 3*b*d^3*e - a*d^2*e^2 + (15*c*d^
2*e^2 - 3*b*d*e^3 - a*e^4)*x^4 + 2*(15*c*d^3*e - 3*b*d^2*e^2 - a*d*e^3)*x^2
)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (15*c*d^4*e - 3*b*d^3*e^2 - a*d^2*e^3)*
x)/(d^2*e^6*x^4 + 2*d^3*e^5*x^2 + d^4*e^4)]
```

giac [A] time = 0.26, size = 107, normalized size = 0.86

$$cxe^{(-3)} - \frac{\left(15\,cd^2 - 3\,bde - ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)}}{8\,d^{\frac{3}{2}}} + \frac{\left(9\,cd^2x^3e - 5\,bdx^3e^2 + 7\,cd^3x + ax^3e^3 - 3\,bd^2xe - adxe^2\right)e^{(-3)}}{8\,\left(x^2e + d\right)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] c*x*e^(-3) - 1/8*(15*c*d^2 - 3*b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(
-7/2)/d^(3/2) + 1/8*(9*c*d^2*x^3*e - 5*b*d*x^3*e^2 + 7*c*d^3*x + a*x^3*e^3
- 3*b*d^2*x*e - a*d*x*e^2)*e^(-3)/((x^2*e + d)^2*d)
```

maple [A] time = 0.01, size = 179, normalized size = 1.44

$$\frac{a\,x^3}{8\left(e\,x^2+d\right)^2 d} - \frac{5b\,x^3}{8\left(e\,x^2+d\right)^2 e} + \frac{9cd\,x^3}{8\left(e\,x^2+d\right)^2 e^2} - \frac{ax}{8\left(e\,x^2+d\right)^2 e} - \frac{3bdx}{8\left(e\,x^2+d\right)^2 e^2} + \frac{7c\,d^2x}{8\left(e\,x^2+d\right)^2 e^3} + \frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,de} + \frac{3b\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,e^2} - \frac{15cd\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,e^3} + \frac{cx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x)
```

```
[Out] c*x/e^3+1/8/(e*x^2+d)^2/d*x^3*a-5/8/e/(e*x^2+d)^2*x^3*b+9/8/e^2/(e*x^2+d)^2
*x^3*c*d-1/8/e/(e*x^2+d)^2*a*x-3/8/e^2/(e*x^2+d)^2*d*b*x+7/8/e^3/(e*x^2+d)^
2*c*d^2*x+1/8/e/d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a+3/8/e^2/(d*e)^(1/
2)*arctan(1/(d*e)^(1/2)*e*x)*b-15/8/e^3*d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*
e*x)*c
```

maxima [A] time = 2.46, size = 126, normalized size = 1.02

$$\frac{\left(9\,cd^2e - 5\,bde^2 + ae^3\right)x^3 + \left(7\,cd^3 - 3\,bd^2e - ade^2\right)x}{8\,\left(de^5x^4 + 2\,d^2e^4x^2 + d^3e^3\right)} + \frac{cx}{e^3} - \frac{\left(15\,cd^2 - 3\,bde - ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}\,de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((9*c*d^2*e - 5*b*d*e^2 + a*e^3)*x^3 + (7*c*d^3 - 3*b*d^2*e - a*d*e^2)*
x)/(d*e^5*x^4 + 2*d^2*e^4*x^2 + d^3*e^3) + c*x/e^3 - 1/8*(15*c*d^2 - 3*b*d*
e - a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d*e^3)
```

mupad [B] time = 0.39, size = 118, normalized size = 0.95

$$\frac{c\,x}{e^3} - \frac{x\,\left(-\frac{7\,c\,d^2}{8} + \frac{3\,b\,d\,e}{8} + \frac{a\,e^2}{8}\right) - \frac{x^3\,\left(9\,c\,d^2\,e - 5\,b\,d\,e^2 + a\,e^3\right)}{8\,d}}{d^2\,e^3 + 2\,d\,e^4\,x^2 + e^5\,x^4} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(-15\,c\,d^2 + 3\,b\,d\,e + a\,e^2\right)}{8\,d^{3/2}\,e^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x)
```

```
[Out] (c*x)/e^3 - (x*((a*e^2)/8 - (7*c*d^2)/8 + (3*b*d*e)/8) - (x^3*(a*e^3 - 5*b*
d*e^2 + 9*c*d^2*e))/(8*d))/(d^2*e^3 + e^5*x^4 + 2*d*e^4*x^2) + (atan((e^(1/
2)*x)/d^(1/2))*(a*e^2 - 15*c*d^2 + 3*b*d*e))/(8*d^(3/2)*e^(7/2))
```

sympy [A] time = 2.62, size = 201, normalized size = 1.62

$$\frac{cx}{e^3} - \frac{\sqrt{-\frac{1}{d^3e^7}}\left(ae^2 + 3bde - 15cd^2\right)\log\left(-d^2e^3\sqrt{-\frac{1}{d^3e^7}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{d^3e^7}}\left(ae^2 + 3bde - 15cd^2\right)\log\left(d^2e^3\sqrt{-\frac{1}{d^3e^7}} + x\right)}{16} + \frac{x^3\left(ae^3 - 5bde^2 + 9cd^2e\right) + x\left(-ade^2 - 3bd^2e + 7cd^3\right)}{8d^3e^3 + 16d^2e^4x^2 + 8de^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] c*x/e**3 - sqrt(-1/(d**3*e**7))*(a*e**2 + 3*b*d*e - 15*c*d**2)*log(-d**2*e*
*3*sqrt(-1/(d**3*e**7)) + x)/16 + sqrt(-1/(d**3*e**7))*(a*e**2 + 3*b*d*e -
15*c*d**2)*log(d**2*e**3*sqrt(-1/(d**3*e**7)) + x)/16 + (x**3*(a*e**3 - 5*b
*d*e**2 + 9*c*d**2*e) + x*(-a*d*e**2 - 3*b*d**2*e + 7*c*d**3))/(8*d**3*e**3
 + 16*d**2*e**4*x**2 + 8*d*e**5*x**4)
```

3.213 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=115

$$-\frac{x\left(5cd^2-e(3ae+bd)\right)}{8d^2e^2\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(e(3ae+bd)+3cd^2\right)}{8d^{5/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{4d\left(d+ex^2\right)^2}$$

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1157, 385, 205}

$$-\frac{x\left(5cd^2-e(3ae+bd)\right)}{8d^2e^2\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(e(3ae+bd)+3cd^2\right)}{8d^{5/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{4d\left(d+ex^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(4*d*(d + e*x^2)^2) - ((5*c*d^2 - e*(b*d + 3*
a*e))*x)/(8*d^2*e^2*(d + e*x^2)) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^3}\,dx = \frac{\left(cd^2-bde+ae^2\right)x}{4de^2\left(d+ex^2\right)^2} - \frac{\int \frac{-3a+\frac{d(cd-be)}{e^2}-\frac{4cdx^2}{e}}{\left(d+ex^2\right)^2}\,dx}{4d}$$

$$= \frac{\left(cd^2-bde+ae^2\right)x}{4de^2\left(d+ex^2\right)^2} - \frac{\left(5cd^2-e(bd+3ae)\right)x}{8d^2e^2\left(d+ex^2\right)} - \frac{\left(-\frac{4cd^2}{e}+e\left(-3a+\frac{d(cd-be)}{e^2}\right)\right)\int \frac{1}{d+ex^2}\,dx}{8d^2e}$$

$$= \frac{\left(cd^2-bde+ae^2\right)x}{4de^2\left(d+ex^2\right)^2} - \frac{\left(5cd^2-e(bd+3ae)\right)x}{8d^2e^2\left(d+ex^2\right)} + \frac{\left(3cd^2+e(bd+3ae)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

Mathematica [A] time = 0.10, size = 110, normalized size = 0.96

$$\frac{x\left(e\left(ae\left(5d+3ex^2\right)+bd\left(ex^2-d\right)\right)-cd^2\left(3d+5ex^2\right)\right)}{8d^2e^2\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(3ae+bd)+3cd^2\right)}{8d^{5/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] (x*(-(c*d^2*(3*d + 5*e*x^2)) + e*(b*d*(-d + e*x^2) + a*e*(5*d + 3*e*x^2))))
/(8*d^2*e^2*(d + e*x^2)^2) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(Sqrt[e]*x
)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(d + e*x^2)^3, x]
```

fricas [A] time = 0.65, size = 391, normalized size = 3.40

$$\left[-\frac{2\left(5\,cd^3e^2-bd^2e^3-3\,ade^4\right)x^3+\left(3\,cd^4+bd^3e+3\,ad^2e^2+\left(3\,cd^2e^2+bde^3+3\,ae^4\right)x^4+2\left(3\,cd^3e+bd^2e^2+3\,ade^3\right)x^2\right)\sqrt{-de}\log\left(\frac{ex^2-2\sqrt{-de}x-d}{ex^2+d}\right)+2\left(3\,cd^4e+bd^3e^2-5\,ad^2e^3\right)x}{16\left(d^3e^5x^4+2\,d^4e^4x^2+d^5e^3\right)}, -\frac{\left(5\,cd^3e^2-bd^2e^3-3\,ade^4\right)x^3-\left(3\,cd^4+bd^3e+3\,ad^2e^2+\left(3\,cd^2e^2+bde^3+3\,ae^4\right)x^4+2\left(3\,cd^3e+bd^2e^2+3\,ade^3\right)x^2\right)\sqrt{de}\arctan\left(\frac{\sqrt{de}x}{d}\right)+\left(3\,cd^4e+bd^3e^2-5\,ad^2e^3\right)x}{8\left(d^3e^5x^4+2\,d^4e^4x^2+d^5e^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*c*d^3*e^2 - b*d^2*e^3 - 3*a*d*e^4)*x^3 + (3*c*d^4 + b*d^3*e +
3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*x^4 + 2*(3*c*d^3*e + b*d^2*
e^2 + 3*a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 +
d)) + 2*(3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d^4*e^4*x
^2 + d^5*e^3), -1/8*((5*c*d^3*e^2 - b*d^2*e^3 - 3*a*d*e^4)*x^3 - (3*c*d^4 +
 b*d^3*e + 3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*x^4 + 2*(3*c*d^3
*e + b*d^2*e^2 + 3*a*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^4
*e + b*d^3*e^2 - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d^4*e^4*x^2 + d^5*e^3)]
```

giac [A] time = 0.31, size = 101, normalized size = 0.88

$$\frac{\left(3\,cd^2+bde+3\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{8\,d^{\frac{5}{2}}}-\frac{\left(5\,cd^2x^3e-bdx^3e^2+3\,cd^3x-3\,ax^3e^3+bd^2xe-5\,adxe^2\right)e^{(-2)}}{8\left(x^2e+d\right)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*c*d^2 + b*d*e + 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(5/2)
- 1/8*(5*c*d^2*x^3*e - b*d*x^3*e^2 + 3*c*d^3*x - 3*a*x^3*e^3 + b*d^2*x*e -
5*a*d*x*e^2)*e^(-2)/((x^2*e + d)^2*d^2)
```

maple [A]   time = 0.01, size = 131, normalized size = 1.14

$$\frac{3a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,d^2}+\frac{b\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,de}+\frac{3c\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,e^2}+\frac{\frac{\left(3ae^2+deb-5cd^2\right)x^3}{8d^2e}+\frac{\left(5ae^2-deb-3cd^2\right)x}{8de^2}}{\left(ex^2+d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^3,x)
```

```
[Out] (1/8*(3*a*e^2+b*d*e-5*c*d^2)/d^2/e*x^3+1/8*(5*a*e^2-b*d*e-3*c*d^2)/e^2/d*x)
/(e*x^2+d)^2+3/8/d^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a+1/8/d/e/(d*e)^
(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b+3/8/e^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*
e*x)*c
```

maxima [A]   time = 2.46, size = 121, normalized size = 1.05

$$-\frac{\left(5\,cd^2e-bde^2-3\,ae^3\right)x^3+\left(3\,cd^3+bd^2e-5\,ade^2\right)x}{8\left(d^2e^4x^4+2\,d^3e^3x^2+d^4e^2\right)}+\frac{\left(3\,cd^2+bde+3\,ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,d^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*c*d^2*e - b*d*e^2 - 3*a*e^3)*x^3 + (3*c*d^3 + b*d^2*e - 5*a*d*e^2)
*x)/(d^2*e^4*x^4 + 2*d^3*e^3*x^2 + d^4*e^2) + 1/8*(3*c*d^2 + b*d*e + 3*a*e^
2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2*e^2)
```

mupad [B]   time = 0.38, size = 112, normalized size = 0.97

$$\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(3\,c\,d^2+b\,d\,e+3\,a\,e^2\right)}{8\,d^{5/2}\,e^{5/2}}-\frac{\frac{x\left(3\,c\,d^2+b\,d\,e-5\,a\,e^2\right)}{8\,d\,e^2}-\frac{x^3\left(-5\,c\,d^2+b\,d\,e+3\,a\,e^2\right)}{8\,d^2\,e}}{d^2+2\,d\,e\,x^2+e^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^3,x)
```

```
[Out] (atan((e^(1/2)*x)/d^(1/2))*(3*a*e^2 + 3*c*d^2 + b*d*e))/(8*d^(5/2)*e^(5/2))
 - ((x*(3*c*d^2 - 5*a*e^2 + b*d*e))/(8*d*e^2) - (x^3*(3*a*e^2 - 5*c*d^2 + b
*d*e))/(8*d^2*e))/(d^2 + e^2*x^4 + 2*d*e*x^2)
```

sympy [A]   time = 1.50, size = 196, normalized size = 1.70

$$-\frac{\sqrt{-\frac{1}{d^5e^5}}\left(3ae^2+bde+3cd^2\right)\log\left(-d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{d^5e^5}}\left(3ae^2+bde+3cd^2\right)\log\left(d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{16}+\frac{x^3\left(3ae^3+bde^2-5cd^2e\right)+x\left(5ade^2-bd^2e-3cd^3\right)}{8d^4e^2+16d^3e^3x^2+8d^2e^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] -sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*log(-d**3*e**2*sqrt(-1/
(d**5*e**5)) + x)/16 + sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*l
og(d**3*e**2*sqrt(-1/(d**5*e**5)) + x)/16 + (x**3*(3*a*e**3 + b*d*e**2 - 5*
c*d**2*e) + x*(5*a*d*e**2 - b*d**2*e - 3*c*d**3))/(8*d**4*e**2 + 16*d**3*e*
*3*x**2 + 8*d**2*e**4*x**4)
```

3.214 $\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^3} dx$

Optimal. Leaf size=127

$$-\frac{x\left(ae^2-bde+cd^2\right)}{4d^2e\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3e(bd-5ae)+cd^2\right)}{8d^{7/2}e^{3/2}}+\frac{x\left(e(3bd-7ae)+cd^2\right)}{8d^3e\left(d+ex^2\right)}-\frac{a}{d^3x}$$

Rubi [A] time = 0.20, antiderivative size = 124, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1259, 456, 453, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3e(bd-5ae)+cd^2\right)}{8d^{7/2}e^{3/2}}+\frac{x\left(e(3bd-7ae)+cd^2\right)}{8d^3e\left(d+ex^2\right)}-\frac{x\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)}{4\left(d+ex^2\right)^2}-\frac{a}{d^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3),x]
```

```
[Out] -(a/(d^3*x)) - ((c/e - (b*d - a*e)/d^2)*x)/(4*(d + e*x^2)^2) + ((c*d^2 + e*
(3*b*d - 7*a*e))*x)/(8*d^3*e*(d + e*x^2)) + ((c*d^2 + 3*e*(b*d - 5*a*e))*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(7/2)*e^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^3}\,dx = -\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{4\left(d+ex^2\right)^2}-\frac{\int \frac{-4ade^2-e\left(cd^2+3e(bd-ae)\right)x^2}{x^2\left(d+ex^2\right)^2}\,dx}{4d^2e^2}$$

$$= -\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{4\left(d+ex^2\right)^2}+\frac{\left(cd^2+e(3bd-7ae)\right)x}{8d^3e\left(d+ex^2\right)}+\frac{\int \frac{8ae^2+e\left(cd+e\left(3b-\frac{7ae}{d}\right)\right)x^2}{x^2\left(d+ex^2\right)}\,dx}{8d^2e^2}$$

$$= -\frac{a}{d^3x}-\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{4\left(d+ex^2\right)^2}+\frac{\left(cd^2+e(3bd-7ae)\right)x}{8d^3e\left(d+ex^2\right)}+\frac{\left(cd^2+3e(bd-5ae)\right)\int \frac{1}{d+ex^2}\,dx}{8d^3e}$$

$$= -\frac{a}{d^3x}-\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{4\left(d+ex^2\right)^2}+\frac{\left(cd^2+e(3bd-7ae)\right)x}{8d^3e\left(d+ex^2\right)}+\frac{\left(cd^2+3e(bd-5ae)\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{7/2}e^{3/2}}$$

Mathematica [A] time = 0.13, size = 124, normalized size = 0.98

$$\frac{\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(3e(bd-5ae)+cd^2\right)}{e^{3/2}}+\frac{\sqrt{d}\left(dx^2\left(be\left(5d+3ex^2\right)+cd\left(ex^2-d\right)\right)-ae\left(8d^2+25dex^2+15e^2x^4\right)\right)}{ex\left(d+ex^2\right)^2}}{8d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3),x]
```

```
[Out] ((Sqrt[d]*(-(a*e*(8*d^2 + 25*d*e*x^2 + 15*e^2*x^4)) + d*x^2*(c*d*(-d + e*x^
2) + b*e*(5*d + 3*e*x^2))))/(e*x*(d + e*x^2)^2) + ((c*d^2 + 3*e*(b*d - 5*a*
e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2))/(8*d^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3), x]
```

fricas [A] time = 0.67, size = 421, normalized size = 3.31

$$\left[-\frac{16\,ad^3e^2-2\left(cd^3e^2+3\,bd^2e^3-15\,ade^4\right)x^4+2\left(cd^4e-5\,bd^3e^2+25\,ad^2e^3\right)x^2-\left(\left(cd^2e^2+3\,bde^3-15\,ae^4\right)x^5+2\left(cd^3e+3\,bd^2e^2-15\,ade^3\right)x^3+\left(cd^4+3\,bd^3e-15\,ad^2e^2\right)x\right)\sqrt{-de}\,\log\left(\frac{ex^2+2\,\sqrt{-de}\,x-d}{ex^2+d}\right)}{16\left(d^4e^4x^5+2\,d^5e^3x^3+d^6e^2x\right)},\ -\frac{8\,ad^3e^2-\left(cd^3e^2+3\,bd^2e^3-15\,ade^4\right)x^4+\left(cd^4e-5\,bd^3e^2+25\,ad^2e^3\right)x^2-\left(\left(cd^2e^2+3\,bde^3-15\,ae^4\right)x^5+2\left(cd^3e+3\,bd^2e^2-15\,ade^3\right)x^3+\left(cd^4+3\,bd^3e-15\,ad^2e^2\right)x\right)\sqrt{de}\,\arctan\left(\frac{\sqrt{de}\,x}{d}\right)}{8\left(d^4e^4x^5+2\,d^5e^3x^3+d^6e^2x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*a*d^3*e^2 - 2*(c*d^3*e^2 + 3*b*d^2*e^3 - 15*a*d*e^4)*x^4 + 2*(c*
d^4*e - 5*b*d^3*e^2 + 25*a*d^2*e^3)*x^2 - ((c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^
4)*x^5 + 2*(c*d^3*e + 3*b*d^2*e^2 - 15*a*d*e^3)*x^3 + (c*d^4 + 3*b*d^3*e -
15*a*d^2*e^2)*x)*sqrt(-d*e)*log((e*x^2 + 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/
(d^4*e^4*x^5 + 2*d^5*e^3*x^3 + d^6*e^2*x), -1/8*(8*a*d^3*e^2 - (c*d^3*e^2 +
 3*b*d^2*e^3 - 15*a*d*e^4)*x^4 + (c*d^4*e - 5*b*d^3*e^2 + 25*a*d^2*e^3)*x^2
```

```
 - ((c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^4)*x^5 + 2*(c*d^3*e + 3*b*d^2*e^2 - 15*
a*d*e^3)*x^3 + (c*d^4 + 3*b*d^3*e - 15*a*d^2*e^2)*x)*sqrt(d*e)*arctan(sqrt(
d*e)*x/d))/(d^4*e^4*x^5 + 2*d^5*e^3*x^3 + d^6*e^2*x)]
```

giac [A] time = 0.32, size = 110, normalized size = 0.87

$$\frac{\left(cd^2 + 3\,bde - 15\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{8\,d^{\frac{7}{2}}} - \frac{a}{d^3x} + \frac{\left(cd^2x^3e + 3\,bdx^3e^2 - cd^3x - 7\,ax^3e^3 + 5\,bd^2xe - 9\,adxe^2\right)e^{(-1)}}{8\left(x^2e + d\right)^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(c*d^2 + 3*b*d*e - 15*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/d^(7/2)
 - a/(d^3*x) + 1/8*(c*d^2*x^3*e + 3*b*d*x^3*e^2 - c*d^3*x - 7*a*x^3*e^3 + 5
*b*d^2*x*e - 9*a*d*x*e^2)*e^(-1)/((x^2*e + d)^2*d^3)
```

maple [A] time = 0.01, size = 182, normalized size = 1.43

$$-\frac{7a\,e^2x^3}{8\left(e\,x^2+d\right)^2d^3} + \frac{3be\,x^3}{8\left(e\,x^2+d\right)^2d^2} + \frac{c\,x^3}{8\left(e\,x^2+d\right)^2d} - \frac{9aex}{8\left(e\,x^2+d\right)^2d^2} + \frac{5bx}{8\left(e\,x^2+d\right)^2d} - \frac{cx}{8\left(e\,x^2+d\right)^2e} - \frac{15ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,d^3} + \frac{3b\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,d^2} + \frac{c\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,de} - \frac{a}{d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x)
```

```
[Out] -a/d^3/x-7/8/d^3/(e*x^2+d)^2*x^3*a*e^2+3/8/d^2/(e*x^2+d)^2*x^3*b*e+1/8/d/(e
*x^2+d)^2*x^3*c-9/8/d^2/(e*x^2+d)^2*e*a*x+5/8/d/(e*x^2+d)^2*b*x-1/8/(e*x^2+
d)^2/e*x*c-15/8/d^3*e/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a+3/8/d^2/(d*e)
^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b+1/8/d/e/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)
*e*x)*c
```

maxima [A] time = 2.65, size = 129, normalized size = 1.02

$$\frac{\left(cd^2e + 3\,bde^2 - 15\,ae^3\right)x^4 - 8\,ad^2e - \left(cd^3 - 5\,bd^2e + 25\,ade^2\right)x^2}{8\left(d^3e^3x^5 + 2\,d^4e^2x^3 + d^5ex\right)} + \frac{\left(cd^2 + 3\,bde - 15\,ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}\,d^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((c*d^2*e + 3*b*d*e^2 - 15*a*e^3)*x^4 - 8*a*d^2*e - (c*d^3 - 5*b*d^2*e
+ 25*a*d*e^2)*x^2)/(d^3*e^3*x^5 + 2*d^4*e^2*x^3 + d^5*e*x) + 1/8*(c*d^2 + 3
*b*d*e - 15*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^3*e)
```

mupad [B] time = 0.39, size = 118, normalized size = 0.93

$$\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(c\,d^2 + 3\,b\,d\,e - 15\,a\,e^2\right)}{8\,d^{7/2}\,e^{3/2}} - \frac{\frac{a}{d} - \frac{x^4\left(c\,d^2+3\,b\,d\,e-15\,a\,e^2\right)}{8\,d^3} + \frac{x^2\left(c\,d^2-5\,b\,d\,e+25\,a\,e^2\right)}{8\,d^2\,e}}{d^2\,x + 2\,d\,e\,x^3 + e^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3),x)
```

```
[Out] (atan((e^(1/2)*x)/d^(1/2))*(c*d^2 - 15*a*e^2 + 3*b*d*e))/(8*d^(7/2)*e^(3/2)
) - (a/d - (x^4*(c*d^2 - 15*a*e^2 + 3*b*d*e))/(8*d^3) + (x^2*(25*a*e^2 + c*
d^2 - 5*b*d*e))/(8*d^2*e))/(d^2*x + e^2*x^5 + 2*d*e*x^3)
```

sympy [A] time = 2.14, size = 202, normalized size = 1.59

$$\frac{\sqrt{-\frac{1}{d^7e^3}}\left(15ae^2 - 3bde - cd^2\right)\log\left(-d^4e\sqrt{-\frac{1}{d^7e^3}} + x\right)}{16} - \frac{\sqrt{-\frac{1}{d^7e^3}}\left(15ae^2 - 3bde - cd^2\right)\log\left(d^4e\sqrt{-\frac{1}{d^7e^3}} + x\right)}{16} + \frac{-8ad^2e + x^4\left(-15ae^3 + 3bde^2 + cd^2e\right) + x^2\left(-25ade^2 + 5bd^2e - cd^3\right)}{8d^5ex + 16d^4e^2x^3 + 8d^3e^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**2/(e*x**2+d)**3,x)
```

```
[Out] sqrt(-1/(d**7*e**3))*(15*a*e**2 - 3*b*d*e - c*d**2)*log(-d**4*e*sqrt(-1/(d*
*7*e**3)) + x)/16 - sqrt(-1/(d**7*e**3))*(15*a*e**2 - 3*b*d*e - c*d**2)*log
(d**4*e*sqrt(-1/(d**7*e**3)) + x)/16 + (-8*a*d**2*e + x**4*(-15*a*e**3 + 3*
b*d*e**2 + c*d**2*e) + x**2*(-25*a*d*e**2 + 5*b*d**2*e - c*d**3))/(8*d**5*e
*x + 16*d**4*e**2*x**3 + 8*d**3*e**3*x**5)
```

3.215 $\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=142

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(35ae^2-15bde+3cd^2\right)}{8d^{9/2}\sqrt{e}}+\frac{x\left(3cd^2-e(7bd-11ae)\right)}{8d^4\left(d+ex^2\right)}+\frac{x\left(ae^2-bde+cd^2\right)}{4d^3\left(d+ex^2\right)^2}-\frac{bd-3ae}{d^4x}-\frac{a}{3d^3x^3}$$

Rubi [A] time = 0.22, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1259, 1261, 205}

$$\frac{x\left(ae^2-bde+cd^2\right)}{4d^3\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(35ae^2-15bde+3cd^2\right)}{8d^{9/2}\sqrt{e}}+\frac{x\left(3cd^2-e(7bd-11ae)\right)}{8d^4\left(d+ex^2\right)}-\frac{bd-3ae}{d^4x}-\frac{a}{3d^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3),x]
```

```
[Out] -a/(3*d^3*x^3) - (b*d - 3*a*e)/(d^4*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(4*d^3
*(d + e*x^2)^2) + ((3*c*d^2 - e*(7*b*d - 11*a*e))*x)/(8*d^4*(d + e*x^2)) +
((3*c*d^2 - 15*b*d*e + 35*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(9/2)*Sq
rt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^3}\,dx = \frac{\left(cd^2-bde+ae^2\right)x}{4d^3\left(d+ex^2\right)^2} + \frac{\int \frac{4ad^2e^2+4de^2(bd-ae)x^2+3e^2\left(cd^2-bde+ae^2\right)x^4}{x^4\left(d+ex^2\right)^2}\,dx}{4d^3e^2}$$

$$= \frac{\left(cd^2-bde+ae^2\right)x}{4d^3\left(d+ex^2\right)^2} + \frac{\left(3cd^2-e(7bd-11ae)\right)x}{8d^4\left(d+ex^2\right)} + \frac{\int \frac{8ad^4e^4+8d^3e^4(bd-2ae)x^2+d^2e^4\left(3cd^2-e(7bd-11ae)\right)}{x^4\left(d+ex^2\right)}}{8d^6e^4}$$

$$= \frac{\left(cd^2-bde+ae^2\right)x}{4d^3\left(d+ex^2\right)^2} + \frac{\left(3cd^2-e(7bd-11ae)\right)x}{8d^4\left(d+ex^2\right)} + \frac{\int \left(\frac{8ad^3e^4}{x^4}+\frac{8d^2e^4(bd-3ae)}{x^2}+\frac{d^2e^4\left(3cd^2-15bde+3\right.}{d+ex^2}\right.}{8d^6e^4}$$

$$= -\frac{a}{3d^3x^3} - \frac{bd-3ae}{d^4x} + \frac{\left(cd^2-bde+ae^2\right)x}{4d^3\left(d+ex^2\right)^2} + \frac{\left(3cd^2-e(7bd-11ae)\right)x}{8d^4\left(d+ex^2\right)} + \frac{\left(3cd^2-15bde+3\right.}{8d^4}$$

$$= -\frac{a}{3d^3x^3} - \frac{bd-3ae}{d^4x} + \frac{\left(cd^2-bde+ae^2\right)x}{4d^3\left(d+ex^2\right)^2} + \frac{\left(3cd^2-e(7bd-11ae)\right)x}{8d^4\left(d+ex^2\right)} + \frac{\left(3cd^2-15bde+3\right.}{8d^{9/2}}$$

Mathematica [A]   time = 0.09, size = 141, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(35ae^2-15bde+3cd^2\right)}{8d^{9/2}\sqrt{e}} + \frac{x\left(11ae^2-7bde+3cd^2\right)}{8d^4\left(d+ex^2\right)} + \frac{x\left(ae^2-bde+cd^2\right)}{4d^3\left(d+ex^2\right)^2} + \frac{3ae-bd}{d^4x} - \frac{a}{3d^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3),x]
```

```
[Out] -1/3*a/(d^3*x^3) + (-(b*d) + 3*a*e)/(d^4*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(
4*d^3*(d + e*x^2)^2) + ((3*c*d^2 - 7*b*d*e + 11*a*e^2)*x)/(8*d^4*(d + e*x^2
)) + ((3*c*d^2 - 15*b*d*e + 35*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(9/
2)*Sqrt[e])
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3), x]
```

fricas [A]   time = 0.78, size = 476, normalized size = 3.35

$$\left[\frac{6\left(3cd^3e^2-15bd^2e^3+35ade^4\right)x^6-16ad^4e+10\left(3cd^4e-15bd^3e^2+35ad^2e^3\right)x^4-16\left(3bd^4e-7ad^3e^2\right)x^2-3\left(\left(3cd^2e^2-15bde^3+35ae^4\right)x^7+2\left(3cd^3e-15bd^2e^2+35ade^3\right)x^5+\left(3cd^4-15bd^3e+35ad^2e^2\right)x^3\right)\sqrt{-de}\log\left(\frac{ex^2-2\sqrt{-de}x-d}{ex^2+d}\right)}{48\left(d^5e^3x^7+2d^6e^2x^5+d^7ex^3\right)}, \frac{3\left(3cd^3e^2-15bd^2e^3+35ade^4\right)x^6-8ad^4e+5\left(3cd^4e-15bd^3e^2+35ad^2e^3\right)x^4-8\left(3bd^4e-7ad^3e^2\right)x^2+3\left(\left(3cd^2e^2-15bde^3+35ae^4\right)x^7+2\left(3cd^3e-15bd^2e^2+35ade^3\right)x^5+\left(3cd^4-15bd^3e+35ad^2e^2\right)x^3\right)\sqrt{de}\arctan\left(\frac{\sqrt{de}x}{d}\right)}{24\left(d^5e^3x^7+2d^6e^2x^5+d^7ex^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(6*(3*c*d^3*e^2 - 15*b*d^2*e^3 + 35*a*d*e^4)*x^6 - 16*a*d^4*e + 10*(3
*c*d^4*e - 15*b*d^3*e^2 + 35*a*d^2*e^3)*x^4 - 16*(3*b*d^4*e - 7*a*d^3*e^2)*
x^2 - 3*((3*c*d^2*e^2 - 15*b*d*e^3 + 35*a*e^4)*x^7 + 2*(3*c*d^3*e - 15*b*d^
2*e^2 + 35*a*d*e^3)*x^5 + (3*c*d^4 - 15*b*d^3*e + 35*a*d^2*e^2)*x^3)*sqrt(-
```

```
d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^5*e^3*x^7 + 2*d^6*e^
2*x^5 + d^7*e*x^3), 1/24*(3*(3*c*d^3*e^2 - 15*b*d^2*e^3 + 35*a*d*e^4)*x^6 -
 8*a*d^4*e + 5*(3*c*d^4*e - 15*b*d^3*e^2 + 35*a*d^2*e^3)*x^4 - 8*(3*b*d^4*e
 - 7*a*d^3*e^2)*x^2 + 3*((3*c*d^2*e^2 - 15*b*d*e^3 + 35*a*e^4)*x^7 + 2*(3*c
*d^3*e - 15*b*d^2*e^2 + 35*a*d*e^3)*x^5 + (3*c*d^4 - 15*b*d^3*e + 35*a*d^2*
e^2)*x^3)*sqrt(d*e)*arctan(sqrt(d*e)*x/d))/(d^5*e^3*x^7 + 2*d^6*e^2*x^5 + d
^7*e*x^3)]
```

giac [A] time = 0.34, size = 128, normalized size = 0.90

$$\frac{\left(3\,cd^2 - 15\,bde + 35\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{8\,d^{\frac{9}{2}}} + \frac{3\,cd^2x^3e - 7\,bdx^3e^2 + 5\,cd^3x + 11\,ax^3e^3 - 9\,bd^2xe + 13\,adxe^2}{8\left(x^2e + d\right)^2d^4} - \frac{3\,bdx^2 - 9\,ax^2e + ad}{3\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*c*d^2 - 15*b*d*e + 35*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(9
/2) + 1/8*(3*c*d^2*x^3*e - 7*b*d*x^3*e^2 + 5*c*d^3*x + 11*a*x^3*e^3 - 9*b*d
^2*x*e + 13*a*d*x*e^2)/((x^2*e + d)^2*d^4) - 1/3*(3*b*d*x^2 - 9*a*x^2*e + a
*d)/(d^4*x^3)
```

maple [A] time = 0.02, size = 207, normalized size = 1.46

$$\frac{11a\,e^3x^3}{8\left(e\,x^2+d\right)^2d^4} - \frac{7b\,e^2x^3}{8\left(e\,x^2+d\right)^2d^3} + \frac{3ce\,x^3}{8\left(e\,x^2+d\right)^2d^2} + \frac{13a\,e^2x}{8\left(e\,x^2+d\right)^2d^3} - \frac{9bex}{8\left(e\,x^2+d\right)^2d^2} + \frac{5cx}{8\left(e\,x^2+d\right)^2d} + \frac{35a\,e^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,d^4} - \frac{15be\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,d^3} + \frac{3c\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,d^2} + \frac{3ae}{d^4x} - \frac{b}{d^3x} - \frac{a}{3d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x)
```

```
[Out] -1/3*a/d^3/x^3+3/d^4/x*a*e-1/d^3/x*b+11/8/d^4/(e*x^2+d)^2*x^3*a*e^3-7/8/d^3
/(e*x^2+d)^2*x^3*b*e^2+3/8/d^2/(e*x^2+d)^2*x^3*c*e+13/8/d^3/(e*x^2+d)^2*a*e
^2*x-9/8/d^2/(e*x^2+d)^2*b*e*x+5/8/d/(e*x^2+d)^2*c*x+35/8/d^4/(d*e)^(1/2)*a
rctan(1/(d*e)^(1/2)*e*x)*a*e^2-15/8/d^3/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*
x)*e*b+3/8/d^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c
```

maxima [A] time = 2.59, size = 147, normalized size = 1.04

$$\frac{3\left(3\,cd^2e - 15\,bde^2 + 35\,ae^3\right)x^6 + 5\left(3\,cd^3 - 15\,bd^2e + 35\,ade^2\right)x^4 - 8\,ad^3 - 8\left(3\,bd^3 - 7\,ad^2e\right)x^2}{24\left(d^4e^2x^7 + 2\,d^5ex^5 + d^6x^3\right)} + \frac{\left(3\,cd^2 - 15\,bde + 35\,ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(3*(3*c*d^2*e - 15*b*d*e^2 + 35*a*e^3)*x^6 + 5*(3*c*d^3 - 15*b*d^2*e +
 35*a*d*e^2)*x^4 - 8*a*d^3 - 8*(3*b*d^3 - 7*a*d^2*e)*x^2)/(d^4*e^2*x^7 + 2*
d^5*e*x^5 + d^6*x^3) + 1/8*(3*c*d^2 - 15*b*d*e + 35*a*e^2)*arctan(e*x/sqrt(
d*e))/(sqrt(d*e)*d^4)
```

mupad [B] time = 0.40, size = 138, normalized size = 0.97

$$\frac{\frac{x^2\,(7\,a\,e-3\,b\,d)}{3\,d^2} - \frac{a}{3\,d} + \frac{5\,x^4\,\left(3\,c\,d^2-15\,b\,d\,e+35\,a\,e^2\right)}{24\,d^3} + \frac{e\,x^6\,\left(3\,c\,d^2-15\,b\,d\,e+35\,a\,e^2\right)}{8\,d^4}}{d^2\,x^3 + 2\,d\,e\,x^5 + e^2\,x^7} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(3\,c\,d^2 - 15\,b\,d\,e + 35\,a\,e^2\right)}{8\,d^{9/2}\,\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3),x)
```

```
[Out] ((x^2*(7*a*e - 3*b*d))/(3*d^2) - a/(3*d) + (5*x^4*(35*a*e^2 + 3*c*d^2 - 15*
b*d*e))/(24*d^3) + (e*x^6*(35*a*e^2 + 3*c*d^2 - 15*b*d*e))/(8*d^4))/(d^2*x^
```

```
3 + e^2*x^7 + 2*d*e*x^5) + (atan((e^(1/2)*x)/d^(1/2))*(35*a*e^2 + 3*c*d^2 -
 15*b*d*e))/(8*d^(9/2)*e^(1/2))
```

sympy [A] time = 2.92, size = 214, normalized size = 1.51

$$-\frac{\sqrt{-\frac{1}{d^9e}}\left(35ae^2-15bde+3cd^2\right)\log\left(-d^5\sqrt{-\frac{1}{d^9e}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{d^9e}}\left(35ae^2-15bde+3cd^2\right)\log\left(d^5\sqrt{-\frac{1}{d^9e}}+x\right)}{16}+\frac{-8ad^3+x^6\left(105ae^3-45bde^2+9cd^2e\right)+x^4\left(175ade^2-75bd^2e+15cd^3\right)+x^2\left(56ad^2e-24bd^3\right)}{24d^6x^3+48d^5ex^5+24d^4e^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**4/(e*x**2+d)**3,x)
```

```
[Out] -sqrt(-1/(d**9*e))*(35*a*e**2 - 15*b*d*e + 3*c*d**2)*log(-d**5*sqrt(-1/(d**
9*e)) + x)/16 + sqrt(-1/(d**9*e))*(35*a*e**2 - 15*b*d*e + 3*c*d**2)*log(d**
5*sqrt(-1/(d**9*e)) + x)/16 + (-8*a*d**3 + x**6*(105*a*e**3 - 45*b*d*e**2 +
 9*c*d**2*e) + x**4*(175*a*d*e**2 - 75*b*d**2*e + 15*c*d**3) + x**2*(56*a*d
**2*e - 24*b*d**3))/(24*d**6*x**3 + 48*d**5*e*x**5 + 24*d**4*e**2*x**7)
```

3.216 $\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=171

$$-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(63ae^2-35bde+15cd^2\right)}{8d^{11/2}}-\frac{6ae^2-3bde+cd^2}{d^5x}-\frac{ex\left(7cd^2-e(11bd-15ae)\right)}{8d^5\left(d+ex^2\right)}-\frac{ex\,(ae^2-bde+c}{4d^4\left(d+ex^2\right)^2}$$

Rubi [A] time = 0.37, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1259, 1805, 1802, 205}

$$-\frac{ex\left(ae^2-bde+cd^2\right)}{4d^4\left(d+ex^2\right)^2}-\frac{6ae^2-3bde+cd^2}{d^5x}-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(63ae^2-35bde+15cd^2\right)}{8d^{11/2}}-\frac{ex\left(7cd^2-e(11bd-15ae)\right)}{8d^5\left(d+ex^2\right)}-\frac{bd-3ae}{3d^4x^3}-\frac{a}{5d^3x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3),x]
```

```
[Out] -a/(5*d^3*x^5) - (b*d - 3*a*e)/(3*d^4*x^3) - (c*d^2 - 3*b*d*e + 6*a*e^2)/(d
^5*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(4*d^4*(d + e*x^2)^2) - (e*(7*c*d^2 -
 e*(11*b*d - 15*a*e))*x)/(8*d^5*(d + e*x^2)) - (Sqrt[e]*(15*c*d^2 - 35*b*d*
e + 63*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^3}\,dx = -\frac{e\left(cd^2-bde+ae^2\right)x}{4d^4\left(d+ex^2\right)^2} - \frac{\int \frac{-4ad^3e^2-4d^2e^2(bd-ae)x^2-4de^2\left(cd^2-bde+ae^2\right)x^4+3e^3\left(cd^2-bde+ae^2\right)x^6}{x^6\left(d+ex^2\right)^2}dx}{4d^4e^2}$$

$$= -\frac{e\left(cd^2-bde+ae^2\right)x}{4d^4\left(d+ex^2\right)^2} - \frac{e\left(7cd^2-e(11bd-15ae)\right)x}{8d^5\left(d+ex^2\right)} + \frac{\int \frac{8ad^3e^2+8d^2e^2(bd-2ae)x^2+8de^2\left(cd^2-e(2bd-\right.}{x^6\left(d+ex^2\right.}}{8d^5e^2}$$

$$= -\frac{e\left(cd^2-bde+ae^2\right)x}{4d^4\left(d+ex^2\right)^2} - \frac{e\left(7cd^2-e(11bd-15ae)\right)x}{8d^5\left(d+ex^2\right)} + \frac{\int \left(\frac{8ad^2e^2}{x^6}+\frac{8de^2(bd-3ae)}{x^4}+\frac{8e^2\left(cd^2-3bde\right.}{x^2}\right.}{8d^5e}$$

$$= -\frac{a}{5d^3x^5} - \frac{bd-3ae}{3d^4x^3} - \frac{cd^2-3bde+6ae^2}{d^5x} - \frac{e\left(cd^2-bde+ae^2\right)x}{4d^4\left(d+ex^2\right)^2} - \frac{e\left(7cd^2-e(11bd-15ae)\right.}{8d^5\left(d+ex^2\right)}$$

$$= -\frac{a}{5d^3x^5} - \frac{bd-3ae}{3d^4x^3} - \frac{cd^2-3bde+6ae^2}{d^5x} - \frac{e\left(cd^2-bde+ae^2\right)x}{4d^4\left(d+ex^2\right)^2} - \frac{e\left(7cd^2-e(11bd-15ae)\right.}{8d^5\left(d+ex^2\right)}$$

Mathematica [A] time = 0.11, size = 173, normalized size = 1.01

$$-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(63ae^2-35bde+15cd^2\right)}{8d^{11/2}} + \frac{-6ae^2+3bde-cd^2}{d^5x} - \frac{x\left(15ae^3-11bde^2+7cd^2e\right)}{8d^5\left(d+ex^2\right)} - \frac{ex\left(ae^2-bde+cd^2\right)}{4d^4\left(d+ex^2\right)^2} + \frac{3ae-bd}{3d^4x^3} - \frac{a}{5d^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3),x]
```

```
[Out] -1/5*a/(d^3*x^5) + (-(b*d) + 3*a*e)/(3*d^4*x^3) + (-(c*d^2) + 3*b*d*e - 6*a
*e^2)/(d^5*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(4*d^4*(d + e*x^2)^2) - ((7*c
*d^2*e - 11*b*d*e^2 + 15*a*e^3)*x)/(8*d^5*(d + e*x^2)) - (Sqrt[e]*(15*c*d^2
 - 35*b*d*e + 63*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(11/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3), x]
```

fricas [A] time = 0.70, size = 514, normalized size = 3.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/240*(30*(15*c*d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)*x^8 + 50*(15*c*d^3*e - 3
5*b*d^2*e^2 + 63*a*d*e^3)*x^6 + 48*a*d^4 + 16*(15*c*d^4 - 35*b*d^3*e + 63*a
*d^2*e^2)*x^4 + 16*(5*b*d^4 - 9*a*d^3*e)*x^2 - 15*((15*c*d^2*e^2 - 35*b*d*e
^3 + 63*a*e^4)*x^9 + 2*(15*c*d^3*e - 35*b*d^2*e^2 + 63*a*d*e^3)*x^7 + (15*c
*d^4 - 35*b*d^3*e + 63*a*d^2*e^2)*x^5)*sqrt(-e/d)*log((e*x^2 - 2*d*x*sqrt(-
```

```
e/d) - d)/(e*x^2 + d)))/(d^5*e^2*x^9 + 2*d^6*e*x^7 + d^7*x^5), -1/120*(15*(
15*c*d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)*x^8 + 25*(15*c*d^3*e - 35*b*d^2*e^2 +
 63*a*d*e^3)*x^6 + 24*a*d^4 + 8*(15*c*d^4 - 35*b*d^3*e + 63*a*d^2*e^2)*x^4
+ 8*(5*b*d^4 - 9*a*d^3*e)*x^2 + 15*((15*c*d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)*
x^9 + 2*(15*c*d^3*e - 35*b*d^2*e^2 + 63*a*d*e^3)*x^7 + (15*c*d^4 - 35*b*d^3
*e + 63*a*d^2*e^2)*x^5)*sqrt(e/d)*arctan(x*sqrt(e/d)))/(d^5*e^2*x^9 + 2*d^6
*e*x^7 + d^7*x^5)]
```

giac [**A**] time = 0.35, size = 164, normalized size = 0.96

$$-\frac{\left(15\,cd^2e-35\,bde^2+63\,ae^3\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{8\,d^{\frac{11}{2}}}-\frac{7\,cd^2x^3e^2-11\,bdx^3e^3+9\,cd^3xe+15\,ax^3e^4-13\,bd^2xe^2+17\,adxe^3}{8\left(x^2e+d\right)^2d^5}-\frac{15\,cd^2x^4-45\,bdx^4e+90\,ax^4e^2+5\,bd^2x^2-15\,adx^2e+3\,ad^2}{15\,d^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*c*d^2*e - 35*b*d*e^2 + 63*a*e^3)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2
)/d^(11/2) - 1/8*(7*c*d^2*x^3*e^2 - 11*b*d*x^3*e^3 + 9*c*d^3*x*e + 15*a*x^3
*e^4 - 13*b*d^2*x*e^2 + 17*a*d*x*e^3)/((x^2*e + d)^2*d^5) - 1/15*(15*c*d^2*
x^4 - 45*b*d*x^4*e + 90*a*x^4*e^2 + 5*b*d^2*x^2 - 15*a*d*x^2*e + 3*a*d^2)/(
d^5*x^5)
```

maple [**A**] time = 0.02, size = 245, normalized size = 1.43

$$-\frac{15a\,e^4x^3}{8\left(e\,x^2+d\right)^2d^5}+\frac{11b\,e^3x^3}{8\left(e\,x^2+d\right)^2d^4}-\frac{7c\,e^2x^3}{8\left(e\,x^2+d\right)^2d^3}-\frac{17a\,e^3x}{8\left(e\,x^2+d\right)^2d^4}+\frac{13b\,e^2x}{8\left(e\,x^2+d\right)^2d^3}-\frac{9cex}{8\left(e\,x^2+d\right)^2d^2}-\frac{63a\,e^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,d^5}+\frac{35b\,e^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,d^4}-\frac{15ce\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,d^3}-\frac{6a\,e^2}{d^5x}+\frac{3be}{d^4x}-\frac{c}{d^3x}+\frac{ae}{d^4x^3}-\frac{b}{3d^3x^3}-\frac{a}{5d^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x)
```

```
[Out] -1/5*a/d^3/x^5+1/d^4/x^3*a*e-1/3/d^3/x^3*b-6/d^5/x*a*e^2+3/d^4/x*e*b-1/d^3/
x*c-15/8*e^4/d^5/(e*x^2+d)^2*x^3*a+11/8*e^3/d^4/(e*x^2+d)^2*x^3*b-7/8*e^2/d
^3/(e*x^2+d)^2*x^3*c-17/8*e^3/d^4/(e*x^2+d)^2*a*x+13/8*e^2/d^3/(e*x^2+d)^2*
b*x-9/8*e/d^2/(e*x^2+d)^2*c*x-63/8*e^3/d^5/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)
*e*x)*a+35/8*e^2/d^4/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b-15/8*e/d^3/(d*
e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c
```

maxima [**A**] time = 2.45, size = 183, normalized size = 1.07

$$-\frac{15\left(15\,cd^2e^2-35\,bde^3+63\,ae^4\right)x^8+25\left(15\,cd^3e-35\,bd^2e^2+63\,ade^3\right)x^6+24\,ad^4+8\left(15\,cd^4-35\,bd^3e+63\,ad^2e^2\right)x^4+8\left(5\,bd^4-9\,ad^3e\right)x^2}{120\left(d^5e^2x^9+2\,d^6ex^7+d^7x^5\right)}-\frac{\left(15\,cd^2e-35\,bde^2+63\,ae^3\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/120*(15*(15*c*d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)*x^8 + 25*(15*c*d^3*e - 35
*b*d^2*e^2 + 63*a*d*e^3)*x^6 + 24*a*d^4 + 8*(15*c*d^4 - 35*b*d^3*e + 63*a*d
^2*e^2)*x^4 + 8*(5*b*d^4 - 9*a*d^3*e)*x^2)/(d^5*e^2*x^9 + 2*d^6*e*x^7 + d^7
*x^5) - 1/8*(15*c*d^2*e - 35*b*d*e^2 + 63*a*e^3)*arctan(e*x/sqrt(d*e))/(sqr
t(d*e)*d^5)
```

mupad [**B**] time = 0.41, size = 168, normalized size = 0.98

$$-\frac{\frac{a}{5\,d}-\frac{x^2\,(9\,a\,e-5\,b\,d)}{15\,d^2}+\frac{x^4\,\left(15\,c\,d^2-35\,b\,d\,e+63\,a\,e^2\right)}{15\,d^3}+\frac{5\,e\,x^6\,\left(15\,c\,d^2-35\,b\,d\,e+63\,a\,e^2\right)}{24\,d^4}+\frac{e^2\,x^8\,\left(15\,c\,d^2-35\,b\,d\,e+63\,a\,e^2\right)}{8\,d^5}}{d^2\,x^5+2\,d\,e\,x^7+e^2\,x^9}-\frac{\sqrt{e}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(15\,c\,d^2-35\,b\,d\,e+63\,a\,e^2\right)}{8\,d^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3),x)
```

```
[Out] - (a/(5*d) - (x^2*(9*a*e - 5*b*d))/(15*d^2) + (x^4*(63*a*e^2 + 15*c*d^2 - 3
5*b*d*e))/(15*d^3) + (5*e*x^6*(63*a*e^2 + 15*c*d^2 - 35*b*d*e))/(24*d^4) +
(e^2*x^8*(63*a*e^2 + 15*c*d^2 - 35*b*d*e))/(8*d^5))/(d^2*x^5 + e^2*x^9 + 2*
d*e*x^7) - (e^(1/2)*atan((e^(1/2)*x)/d^(1/2))*(63*a*e^2 + 15*c*d^2 - 35*b*d
*e))/(8*d^(11/2))
```

sympy [**B**] time = 3.76, size = 330, normalized size = 1.93

$$\frac{\sqrt{-\frac{e}{d^{11}}}\left(63ae^2-35bde+15cd^2\right)\log\left(-\frac{d^6\sqrt{-\frac{e}{d^{11}}}\left(63ae^2-35bde+15cd^2\right)}{63ae^3-35bde^2+15cd^2e}+x\right)}{16}-\frac{\sqrt{-\frac{e}{d^{11}}}\left(63ae^2-35bde+15cd^2\right)\log\left(\frac{d^6\sqrt{-\frac{e}{d^{11}}}\left(63ae^2-35bde+15cd^2\right)}{63ae^3-35bde^2+15cd^2e}+x\right)}{16}+\frac{-24ad^4+x^8\left(-945ae^4+525bde^3-225cd^2e^2\right)+x^6\left(-1575ade^3+875bd^2e^2-375cd^3e\right)+x^4\left(-504ad^2e^2+280bd^3e-120cd^4\right)+x^2\left(72ad^3e-40bd^4\right)}{120d^7x^5+240d^6ex^7+120d^5e^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**6/(e*x**2+d)**3,x)
```

```
[Out] sqrt(-e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)*log(-d**6*sqrt(-e/d**11)*
(63*a*e**2 - 35*b*d*e + 15*c*d**2)/(63*a*e**3 - 35*b*d*e**2 + 15*c*d**2*e)
+ x)/16 - sqrt(-e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)*log(d**6*sqrt(-
e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)/(63*a*e**3 - 35*b*d*e**2 + 15*c
*d**2*e) + x)/16 + (-24*a*d**4 + x**8*(-945*a*e**4 + 525*b*d*e**3 - 225*c*d
**2*e**2) + x**6*(-1575*a*d*e**3 + 875*b*d**2*e**2 - 375*c*d**3*e) + x**4*(
-504*a*d**2*e**2 + 280*b*d**3*e - 120*c*d**4) + x**2*(72*a*d**3*e - 40*b*d*
*4))/(120*d**7*x**5 + 240*d**6*e*x**7 + 120*d**5*e**2*x**9)
```

3.217 $\int \frac{x^9}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=230

$$-\frac{(a^2ce-ab^2e-2abcd+b^3d)\log(a+bx^2+cx^4)}{4c^3(ae^2-bde+cd^2)}-\frac{(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}$$

Rubi [A] time = 0.49, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1251, 1628, 634, 618, 206, 628}

$$-\frac{(a^2ce-ab^2e-2abcd+b^3d)\log(a+bx^2+cx^4)}{4c^3(ae^2-bde+cd^2)}-\frac{(3a^2bce+2a^2c^2d-4ab^2cd-ab^3e+b^4d)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}+\frac{d^4\log(d+ex^2)}{2e^3(ae^2-bde+cd^2)}-\frac{x^2(be+cd)}{2c^2e^2}+\frac{x^4}{4ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((c*d + b*e)*x^2)/(2*c^2*e^2) + x^4/(4*c*e) - ((b^4*d - 4*a*b^2*c*d + 2*a^
2*c^2*d - a*b^3*e + 3*a^2*b*c*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(2*c^3*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + (d^4*Log[d + e*x^2])/(2
*e^3*(c*d^2 - b*d*e + a*e^2)) - ((b^3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e)*Lo
g[a + b*x^2 + c*x^4])/(4*c^3*(c*d^2 - b*d*e + a*e^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^9}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{(d+ex)\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{-cd-be}{c^2e^2} + \frac{x}{ce} + \frac{d^4}{e^2\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{-a\left(b^2d-acd-\right.}{c^2\left(cd^2\right.}\right.\right.$$

$$= -\frac{(cd+be)x^2}{2c^2e^2} + \frac{x^4}{4ce} + \frac{d^4\log\left(d+ex^2\right)}{2e^3\left(cd^2-bde+ae^2\right)} + \frac{\text{Subst}\left(\int \frac{-a\left(b^2d-acd-abe\right)-\left(b^3d-2abc\right.}{a+bx+cx^2}\right.}{2c^2\left(cd^2-bde+\right.}$$

$$= -\frac{(cd+be)x^2}{2c^2e^2} + \frac{x^4}{4ce} + \frac{d^4\log\left(d+ex^2\right)}{2e^3\left(cd^2-bde+ae^2\right)} - \frac{\left(b^3d-2abcd-ab^2e+a^2ce\right)\text{Sub}}{4c^3\left(cd^2-bde\right.}$$

$$= -\frac{(cd+be)x^2}{2c^2e^2} + \frac{x^4}{4ce} + \frac{d^4\log\left(d+ex^2\right)}{2e^3\left(cd^2-bde+ae^2\right)} - \frac{\left(b^3d-2abcd-ab^2e+a^2ce\right)\log}{4c^3\left(cd^2-bde+ae\right.}$$

$$= -\frac{(cd+be)x^2}{2c^2e^2} + \frac{x^4}{4ce} - \frac{\left(b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4a}}\right.}{2c^3\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)}$$

Mathematica [A] time = 0.24, size = 228, normalized size = 0.99

$$\frac{1}{4}\left(\frac{\left(-a^2ce+ab^2e+2abcd+b^3(-d)\right)\log\left(a+bx^2+cx^4\right)}{c^3\left(e(ae-bd)+cd^2\right)} - \frac{2\left(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{c^3\sqrt{4ac-b^2}\left(e(bd-ae)-cd^2\right)} + \frac{2d^4\log\left(d+ex^2\right)}{e^3\left(e(ae-bd)+cd^2\right)} - \frac{2x^2(be+cd)}{c^2e^2} + \frac{x^4}{ce}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*(c*d + b*e)*x^2)/(c^2*e^2) + x^4/(c*e) - (2*(b^4*d - 4*a*b^2*c*d + 2*a
^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])
/(c^3*Sqrt[-b^2 + 4*a*c]*(-(c*d^2) + e*(b*d - a*e))) + (2*d^4*Log[d + e*x^2
])/(e^3*(c*d^2 + e*(-(b*d) + a*e))) + ((-(b^3*d) + 2*a*b*c*d + a*b^2*e - a^
2*c*e)*Log[a + b*x^2 + c*x^4])/(c^3*(c*d^2 + e*(-(b*d) + a*e))))/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.73, size = 236, normalized size = 1.03

$$\frac{d^4\log\left(\left|x^2e+d\right|\right)}{2\left(cd^2e^3-bde^4+ae^5\right)}-\frac{\left(b^3d-2\,abcd-ab^2e+a^2ce\right)\log\left(cx^4+bx^2+a\right)}{4\left(c^4d^2-bc^3de+ac^3e^2\right)}+\frac{\left(b^4d-4\,ab^2cd+2\,a^2c^2d-ab^3e+3\,a^2bce\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(c^4d^2-bc^3de+ac^3e^2\right)\sqrt{-b^2+4\,ac}}+\frac{\left(cx^4e-2\,cdx^2-2\,bx^2e\right)e^{(-2)}}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d^4*log(abs(x^2*e + d))/(c*d^2*e^3 - b*d*e^4 + a*e^5) - 1/4*(b^3*d - 2*
a*b*c*d - a*b^2*e + a^2*c*e)*log(c*x^4 + b*x^2 + a)/(c^4*d^2 - b*c^3*d*e +
a*c^3*e^2) + 1/2*(b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e
)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((c^4*d^2 - b*c^3*d*e + a*c^3*e^
2)*sqrt(-b^2 + 4*a*c)) + 1/4*(c*x^4*e - 2*c*d*x^2 - 2*b*x^2*e)*e^(-2)/c^2
```

maple [B] time = 0.02, size = 538, normalized size = 2.34

$$\frac{3a^2be\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(ae^2-deb+cd^2\right)\sqrt{4ac-b^2}\,c^2}+\frac{a^2d\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\left(ae^2-deb+cd^2\right)\sqrt{4ac-b^2}\,c}-\frac{ab^3e\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(ae^2-deb+cd^2\right)\sqrt{4ac-b^2}\,c^3}-\frac{2ab^2d\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\left(ae^2-deb+cd^2\right)\sqrt{4ac-b^2}\,c^2}+\frac{b^4d\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(ae^2-deb+cd^2\right)\sqrt{4ac-b^2}\,c^3}-\frac{a^2e\ln\left(cx^4+bx^2+a\right)}{4\left(ae^2-deb+cd^2\right)c^2}+\frac{ab^2e\ln\left(cx^4+bx^2+a\right)}{4\left(ae^2-deb+cd^2\right)c^3}+\frac{abd\ln\left(cx^4+bx^2+a\right)}{2\left(ae^2-deb+cd^2\right)c^2}-\frac{b^3d\ln\left(cx^4+bx^2+a\right)}{4\left(ae^2-deb+cd^2\right)c^3}+\frac{x^4}{4ce}+\frac{d^4\ln\left(ex^2+d\right)}{2\left(ae^2-deb+cd^2\right)e^3}-\frac{bx^2}{2c^2e}-\frac{dx^2}{2ce^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4/c/e*x^4-1/2/c^2/e*x^2*b-1/2/c*d/e^2*x^2-1/4/(a*e^2-b*d*e+c*d^2)/c^2*ln(
c*x^4+b*x^2+a)*a^2*e+1/4/(a*e^2-b*d*e+c*d^2)/c^3*ln(c*x^4+b*x^2+a)*a*b^2*e+
1/2/(a*e^2-b*d*e+c*d^2)/c^2*ln(c*x^4+b*x^2+a)*a*b*d-1/4/(a*e^2-b*d*e+c*d^2)
/c^3*ln(c*x^4+b*x^2+a)*b^3*d+3/2/(a*e^2-b*d*e+c*d^2)/c^2/(4*a*c-b^2)^(1/2)*
arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*b*e+1/(a*e^2-b*d*e+c*d^2)/c/(4*a*
c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*d-2/(a*e^2-b*d*e+c*d
^2)/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^2*d-1/2
/(a*e^2-b*d*e+c*d^2)/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(
1/2))*b^3*a*e+1/2/(a*e^2-b*d*e+c*d^2)/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
+b)/(4*a*c-b^2)^(1/2))*b^4*d+1/2*d^4*ln(e*x^2+d)/e^3/(a*e^2-b*d*e+c*d^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 69.94, size = 7024, normalized size = 30.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (d^4*log(d + e*x^2))/(2*a*e^5 + 2*c*d^2*e^3 - 2*b*d*e^4) + (log((x^2*(a^7*e
^7 + b^7*d^7 - 2*a^3*b*c^3*d^7 - a^4*c^3*d^6*e - 2*a^6*c*d^2*e^5 + 7*a^2*b^
3*c^2*d^7 + 3*a^2*b^5*d^5*e^2 + 4*a^3*b^4*d^4*e^3 + 4*a^4*b^3*d^3*e^4 + 3*a
^5*b^2*d^2*e^5 + 2*a^5*c^2*d^4*e^3 - 5*a*b^5*c*d^7 + 2*a*b^6*d^6*e + 2*a^6*
b*d*e^6 - 8*a^2*b^4*c*d^6*e - 6*a^5*b*c*d^3*e^4 + 8*a^3*b^2*c^2*d^6*e - 9*a
^3*b^3*c*d^5*e^2 + 5*a^4*b*c^2*d^5*e^2 - 9*a^4*b^2*c*d^4*e^3))/(c^4*e^4) +
```

(a*d*(a^3*e^3 + b^3*d^3 - 2*a*b*c*d^3 + a*b^2*d^2*e + a^2*b*d*e^2 - a^2*c*d
^2*e)^2)/(c^4*e^4) + (((x^2*(4*a^2*c^6*d^8 + 6*a^4*b^4*e^8 + 18*a^6*c^2*e^8
 + 6*b^4*c^4*d^8 + 6*b^8*d^4*e^4 - 16*a*b^2*c^5*d^8 - 26*a^5*b^2*c*e^8 + 8*
a*b^7*d^3*e^5 + 8*a^3*b^5*d*e^7 - 2*b^5*c^3*d^7*e - 2*b^7*c*d^5*e^3 + 8*a^2
*b^6*d^2*e^6 - 20*a^3*c^5*d^6*e^2 + 40*a^4*c^4*d^4*e^4 - 36*a^5*c^3*d^2*e^6
 + 2*b^6*c^2*d^6*e^2 + 42*a^2*b^2*c^4*d^6*e^2 - 28*a^2*b^3*c^3*d^5*e^3 + 80
*a^2*b^4*c^2*d^4*e^4 - 64*a^3*b^2*c^3*d^4*e^4 + 80*a^3*b^3*c^2*d^3*e^5 + 48
*a^4*b^2*c^2*d^2*e^6 + 18*a*b^3*c^4*d^7*e - 40*a*b^6*c*d^4*e^4 - 26*a^2*b*c
^5*d^7*e - 32*a^4*b^3*c*d*e^7 + 12*a^5*b*c^2*d*e^7 - 16*a*b^4*c^3*d^6*e^2 +
 10*a*b^5*c^2*d^5*e^3 - 48*a^2*b^5*c*d^3*e^5 + 46*a^3*b*c^4*d^5*e^3 - 40*a^
3*b^4*c*d^2*e^6 - 48*a^4*b*c^3*d^3*e^5))/(c^4*e^4) + (((x^2*(8*a*b^8*e^9 +
8*b*c^8*d^9 + 8*b^9*d*e^8 + 120*a^5*c^4*e^9 - 72*a^2*b^6*c*e^9 - 8*b^2*c^7*
d^8*e - 8*b^8*c*d^2*e^7 + 212*a^3*b^4*c^2*e^9 - 240*a^4*b^2*c^3*e^9 - 112*a
^2*c^7*d^6*e^3 + 240*a^3*c^6*d^4*e^5 - 228*a^4*c^5*d^2*e^7 + 4*b^3*c^6*d^7*
e^2 - 24*b^4*c^5*d^6*e^3 + 32*b^5*c^4*d^5*e^4 - 24*b^6*c^3*d^4*e^5 + 4*b^7*
c^2*d^3*e^6 + 32*a*c^8*d^8*e - 56*a*b^7*c*d*e^8 - 428*a^2*b^2*c^5*d^4*e^5 +
 108*a^2*b^3*c^4*d^3*e^6 - 216*a^2*b^4*c^3*d^2*e^7 + 424*a^3*b^2*c^4*d^2*e^
7 - 16*a*b*c^7*d^7*e^2 + 8*a^4*b*c^4*d*e^8 + 88*a*b^2*c^6*d^6*e^3 - 116*a*b
^3*c^5*d^5*e^4 + 188*a*b^4*c^4*d^4*e^5 - 36*a*b^5*c^3*d^3*e^6 + 60*a*b^6*c^
2*d^2*e^7 + 40*a^2*b*c^6*d^5*e^4 + 100*a^2*b^5*c^2*d*e^8 - 72*a^3*b*c^5*d^3
*e^6 - 4*a^3*b^3*c^3*d*e^8))/(c^4*e^4) - (((x^2*(32*a*b^6*c^3*e^10 - 352*a^
4*c^6*e^10 + 128*a*c^9*d^6*e^4 + 32*b*c^9*d^7*e^3 + 32*b^7*c^3*d*e^9 - 256*
a^2*b^4*c^4*e^10 + 600*a^3*b^2*c^5*e^10 - 464*a^2*c^8*d^4*e^6 + 592*a^3*c^7
*d^2*e^8 - 64*b^2*c^8*d^6*e^4 + 56*b^3*c^7*d^5*e^5 - 48*b^4*c^6*d^4*e^6 + 5
6*b^5*c^5*d^3*e^7 - 64*b^6*c^4*d^2*e^8 - 688*a^2*b^2*c^6*d^2*e^8 - 192*a*b*
c^8*d^5*e^5 - 224*a*b^5*c^4*d*e^9 - 72*a^3*b*c^6*d*e^9 + 272*a*b^2*c^7*d^4*
e^6 - 200*a*b^3*c^6*d^3*e^7 + 360*a*b^4*c^5*d^2*e^8 + 136*a^2*b*c^7*d^3*e^7
 + 424*a^2*b^3*c^5*d*e^9))/(c^4*e^4) + (32*a*d*(2*b^6*e^6 + 2*c^6*d^6 - 15*
a^3*c^3*e^6 - 10*a*c^5*d^4*e^2 + 29*a^2*b^2*c^2*e^6 + 17*a^2*c^4*d^2*e^4 +
3*b^2*c^4*d^4*e^2 - b^3*c^3*d^3*e^3 + 3*b^4*c^2*d^2*e^4 - 14*a*b^4*c*e^6 -
2*b*c^5*d^5*e - 2*b^5*c*d*e^5 + 2*a*b*c^4*d^3*e^3 + 6*a*b^3*c^2*d*e^5 + a^2
*b*c^3*d*e^5 - 13*a*b^2*c^3*d^2*e^4))/(c*e) - (8*e^2*(b^2*e^2 + c^2*d^2 - 3
*a*c*e^2 - b*c*d*e)*(b^5*d + b^4*d*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*e - a*b^
4*e - 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d + 5*a^2*b^2
*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) +
3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2))*(2*a*c^2*d^3 + a*b^2*e^3*x^2 + b*c^2*d^3*x
^2 - 4*a^2*c*e^3*x^2 + b^3*d*e^2*x^2 + 2*a*b^2*d*e^2 - 6*a^2*c*d*e^2 + 4*a*
c^2*d^2*e*x^2 - 2*b^2*c*d^2*e*x^2 - 2*a*b*c*d^2*e - 3*a*b*c*d*e^2*x^2))/(c*
(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^5*d + b^4*d*(b^2 - 4*a*c)^(1/2)
 - 4*a^3*c^2*e - a*b^4*e - 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) + 8*a^2
*b*c^2*d + 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b
^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*c^3*(4*a*c - b^2)*
(a*e^2 + c*d^2 - b*d*e)) + (4*a*d*(4*b^8*e^8 + 4*c^8*d^8 + 37*a^4*c^4*e^8 -
 16*a*c^7*d^6*e^2 + 84*a^2*b^4*c^2*e^8 - 84*a^3*b^2*c^3*e^8 + 40*a^2*c^6*d^
4*e^4 - 56*a^3*c^5*d^2*e^6 + 4*b^2*c^6*d^6*e^2 - 4*b^3*c^5*d^5*e^3 + 13*b^4
*c^4*d^4*e^4 - 4*b^5*c^3*d^3*e^5 + 4*b^6*c^2*d^2*e^6 - 32*a*b^6*c*e^8 + 98*
a^2*b^2*c^4*d^2*e^6 - 8*a*b^5*c^2*d*e^7 - 4*a^3*b*c^4*d*e^7 - 52*a*b^2*c^5*
d^4*e^4 + 20*a*b^3*c^4*d^3*e^5 - 36*a*b^4*c^3*d^2*e^6 - 16*a^2*b*c^5*d^3*e^
5 + 28*a^2*b^3*c^3*d*e^7))/(c^4*e^4))*(b^5*d + b^4*d*(b^2 - 4*a*c)^(1/2) -
4*a^3*c^2*e - a*b^4*e - 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b
*c^2*d + 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2
 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*c^3*(4*a*c - b^2)*(a
*e^2 + c*d^2 - b*d*e)) + (4*a*d*(2*a^3*b^4*e^7 + 5*a^5*c^2*e^7 + 2*b^3*c^4*
d^7 + 2*b^7*d^3*e^4 - 8*a^4*b^2*c*e^7 + 2*a*b^6*d^2*e^5 + 2*a^2*b^5*d*e^6 -
 2*a^2*c^5*d^6*e + 6*a^3*c^4*d^4*e^3 - 9*a^4*c^3*d^2*e^5 + b^5*c^2*d^5*e^2
- 4*a*b*c^5*d^7 - a^2*b^2*c^3*d^4*e^3 + 20*a^2*b^3*c^2*d^3*e^4 + 12*a^3*b^2
*c^2*d^2*e^5 + 2*a*b^2*c^4*d^6*e - 12*a*b^5*c*d^3*e^4 - 8*a^3*b^3*c*d*e^6 +
 3*a^4*b*c^2*d*e^6 - 6*a*b^3*c^3*d^5*e^2 - a*b^4*c^2*d^4*e^3 + 10*a^2*b*c^4
*d^5*e^2 - 10*a^2*b^4*c*d^2*e^5 - 12*a^3*b*c^3*d^3*e^4))/(c^4*e^4))*(b^5*d

```
+ b^4*d*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*e - a*b^4*e - 6*a*b^3*c*d - a*b^3*e
*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d + 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4
*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(
1/2)))/(4*c^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^5*d + b^4*d*(b^2 -
 4*a*c)^(1/2) - 4*a^3*c^2*e - a*b^4*e - 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)
^(1/2) + 8*a^2*b*c^2*d + 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) -
4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*(4*a
*c^5*d^2 + 4*a^2*c^4*e^2 - b^2*c^4*d^2 - a*b^2*c^3*e^2 + b^3*c^3*d*e - 4*a*
b*c^4*d*e)) - (log((x^2*(a^7*e^7 + b^7*d^7 - 2*a^3*b*c^3*d^7 - a^4*c^3*d^6*
e - 2*a^6*c*d^2*e^5 + 7*a^2*b^3*c^2*d^7 + 3*a^2*b^5*d^5*e^2 + 4*a^3*b^4*d^4
*e^3 + 4*a^4*b^3*d^3*e^4 + 3*a^5*b^2*d^2*e^5 + 2*a^5*c^2*d^4*e^3 - 5*a*b^5*
c*d^7 + 2*a*b^6*d^6*e + 2*a^6*b*d*e^6 - 8*a^2*b^4*c*d^6*e - 6*a^5*b*c*d^3*e
^4 + 8*a^3*b^2*c^2*d^6*e - 9*a^3*b^3*c*d^5*e^2 + 5*a^4*b*c^2*d^5*e^2 - 9*a^
4*b^2*c*d^4*e^3))/(c^4*e^4) + (a*d*(a^3*e^3 + b^3*d^3 - 2*a*b*c*d^3 + a*b^2
*d^2*e + a^2*b*d*e^2 - a^2*c*d^2*e)^2)/(c^4*e^4) - (((x^2*(4*a^2*c^6*d^8 +
6*a^4*b^4*e^8 + 18*a^6*c^2*e^8 + 6*b^4*c^4*d^8 + 6*b^8*d^4*e^4 - 16*a*b^2*c
^5*d^8 - 26*a^5*b^2*c*e^8 + 8*a*b^7*d^3*e^5 + 8*a^3*b^5*d*e^7 - 2*b^5*c^3*d
^7*e - 2*b^7*c*d^5*e^3 + 8*a^2*b^6*d^2*e^6 - 20*a^3*c^5*d^6*e^2 + 40*a^4*c^
4*d^4*e^4 - 36*a^5*c^3*d^2*e^6 + 2*b^6*c^2*d^6*e^2 + 42*a^2*b^2*c^4*d^6*e^2
 - 28*a^2*b^3*c^3*d^5*e^3 + 80*a^2*b^4*c^2*d^4*e^4 - 64*a^3*b^2*c^3*d^4*e^4
 + 80*a^3*b^3*c^2*d^3*e^5 + 48*a^4*b^2*c^2*d^2*e^6 + 18*a*b^3*c^4*d^7*e - 4
0*a*b^6*c*d^4*e^4 - 26*a^2*b*c^5*d^7*e - 32*a^4*b^3*c*d*e^7 + 12*a^5*b*c^2*
d*e^7 - 16*a*b^4*c^3*d^6*e^2 + 10*a*b^5*c^2*d^5*e^3 - 48*a^2*b^5*c*d^3*e^5
+ 46*a^3*b*c^4*d^5*e^3 - 40*a^3*b^4*c*d^2*e^6 - 48*a^4*b*c^3*d^3*e^5))/(c^4
*e^4) - (((x^2*(8*a*b^8*e^9 + 8*b*c^8*d^9 + 8*b^9*d*e^8 + 120*a^5*c^4*e^9 -
 72*a^2*b^6*c*e^9 - 8*b^2*c^7*d^8*e - 8*b^8*c*d^2*e^7 + 212*a^3*b^4*c^2*e^9
 - 240*a^4*b^2*c^3*e^9 - 112*a^2*c^7*d^6*e^3 + 240*a^3*c^6*d^4*e^5 - 228*a^
4*c^5*d^2*e^7 + 4*b^3*c^6*d^7*e^2 - 24*b^4*c^5*d^6*e^3 + 32*b^5*c^4*d^5*e^4
 - 24*b^6*c^3*d^4*e^5 + 4*b^7*c^2*d^3*e^6 + 32*a*c^8*d^8*e - 56*a*b^7*c*d*e
^8 - 428*a^2*b^2*c^5*d^4*e^5 + 108*a^2*b^3*c^4*d^3*e^6 - 216*a^2*b^4*c^3*d^
2*e^7 + 424*a^3*b^2*c^4*d^2*e^7 - 16*a*b*c^7*d^7*e^2 + 8*a^4*b*c^4*d*e^8 +
88*a*b^2*c^6*d^6*e^3 - 116*a*b^3*c^5*d^5*e^4 + 188*a*b^4*c^4*d^4*e^5 - 36*a
*b^5*c^3*d^3*e^6 + 60*a*b^6*c^2*d^2*e^7 + 40*a^2*b*c^6*d^5*e^4 + 100*a^2*b^
5*c^2*d*e^8 - 72*a^3*b*c^5*d^3*e^6 - 4*a^3*b^3*c^3*d*e^8))/(c^4*e^4) + (((x
^2*(32*a*b^6*c^3*e^10 - 352*a^4*c^6*e^10 + 128*a*c^9*d^6*e^4 + 32*b*c^9*d^7
*e^3 + 32*b^7*c^3*d*e^9 - 256*a^2*b^4*c^4*e^10 + 600*a^3*b^2*c^5*e^10 - 464
*a^2*c^8*d^4*e^6 + 592*a^3*c^7*d^2*e^8 - 64*b^2*c^8*d^6*e^4 + 56*b^3*c^7*d^
5*e^5 - 48*b^4*c^6*d^4*e^6 + 56*b^5*c^5*d^3*e^7 - 64*b^6*c^4*d^2*e^8 - 688*
a^2*b^2*c^6*d^2*e^8 - 192*a*b*c^8*d^5*e^5 - 224*a*b^5*c^4*d*e^9 - 72*a^3*b*
c^6*d*e^9 + 272*a*b^2*c^7*d^4*e^6 - 200*a*b^3*c^6*d^3*e^7 + 360*a*b^4*c^5*d
^2*e^8 + 136*a^2*b*c^7*d^3*e^7 + 424*a^2*b^3*c^5*d*e^9))/(c^4*e^4) + (32*a*
d*(2*b^6*e^6 + 2*c^6*d^6 - 15*a^3*c^3*e^6 - 10*a*c^5*d^4*e^2 + 29*a^2*b^2*c
^2*e^6 + 17*a^2*c^4*d^2*e^4 + 3*b^2*c^4*d^4*e^2 - b^3*c^3*d^3*e^3 + 3*b^4*c
^2*d^2*e^4 - 14*a*b^4*c*e^6 - 2*b*c^5*d^5*e - 2*b^5*c*d*e^5 + 2*a*b*c^4*d^3
*e^3 + 6*a*b^3*c^2*d*e^5 + a^2*b*c^3*d*e^5 - 13*a*b^2*c^3*d^2*e^4))/(c*e) +
 (8*e^2*(b^2*e^2 + c^2*d^2 - 3*a*c*e^2 - b*c*d*e)*(b^4*d*(b^2 - 4*a*c)^(1/2
) - b^5*d + 4*a^3*c^2*e + a*b^4*e + 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/
2) - 8*a^2*b*c^2*d - 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*
b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2))*(2*a*c^2*d^3
 + a*b^2*e^3*x^2 + b*c^2*d^3*x^2 - 4*a^2*c*e^3*x^2 + b^3*d*e^2*x^2 + 2*a*b^
2*d*e^2 - 6*a^2*c*d*e^2 + 4*a*c^2*d^2*e*x^2 - 2*b^2*c*d^2*e*x^2 - 2*a*b*c*d
^2*e - 3*a*b*c*d*e^2*x^2))/(c*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^4*
d*(b^2 - 4*a*c)^(1/2) - b^5*d + 4*a^3*c^2*e + a*b^4*e + 6*a*b^3*c*d - a*b^3
*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c^2*d - 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 -
 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)
^(1/2)))/(4*c^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*a*d*(4*b^8*e^8
+ 4*c^8*d^8 + 37*a^4*c^4*e^8 - 16*a*c^7*d^6*e^2 + 84*a^2*b^4*c^2*e^8 - 84*a
^3*b^2*c^3*e^8 + 40*a^2*c^6*d^4*e^4 - 56*a^3*c^5*d^2*e^6 + 4*b^2*c^6*d^6*e^
2 - 4*b^3*c^5*d^5*e^3 + 13*b^4*c^4*d^4*e^4 - 4*b^5*c^3*d^3*e^5 + 4*b^6*c^2*
```

```
d^2*e^6 - 32*a*b^6*c*e^8 + 98*a^2*b^2*c^4*d^2*e^6 - 8*a*b^5*c^2*d*e^7 - 4*a
^3*b*c^4*d*e^7 - 52*a*b^2*c^5*d^4*e^4 + 20*a*b^3*c^4*d^3*e^5 - 36*a*b^4*c^3
*d^2*e^6 - 16*a^2*b*c^5*d^3*e^5 + 28*a^2*b^3*c^3*d*e^7))/(c^4*e^4))*(b^4*d*
(b^2 - 4*a*c)^(1/2) - b^5*d + 4*a^3*c^2*e + a*b^4*e + 6*a*b^3*c*d - a*b^3*e
*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c^2*d - 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4
*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(
1/2)))/(4*c^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*a*d*(2*a^3*b^4*e^
7 + 5*a^5*c^2*e^7 + 2*b^3*c^4*d^7 + 2*b^7*d^3*e^4 - 8*a^4*b^2*c*e^7 + 2*a*b
^6*d^2*e^5 + 2*a^2*b^5*d*e^6 - 2*a^2*c^5*d^6*e + 6*a^3*c^4*d^4*e^3 - 9*a^4*
c^3*d^2*e^5 + b^5*c^2*d^5*e^2 - 4*a*b*c^5*d^7 - a^2*b^2*c^3*d^4*e^3 + 20*a^
2*b^3*c^2*d^3*e^4 + 12*a^3*b^2*c^2*d^2*e^5 + 2*a*b^2*c^4*d^6*e - 12*a*b^5*c
*d^3*e^4 - 8*a^3*b^3*c*d*e^6 + 3*a^4*b*c^2*d*e^6 - 6*a*b^3*c^3*d^5*e^2 - a*
b^4*c^2*d^4*e^3 + 10*a^2*b*c^4*d^5*e^2 - 10*a^2*b^4*c*d^2*e^5 - 12*a^3*b*c^
3*d^3*e^4))/(c^4*e^4))*(b^4*d*(b^2 - 4*a*c)^(1/2) - b^5*d + 4*a^3*c^2*e + a
*b^4*e + 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c^2*d - 5*a^2*
b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2)
 + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*c^3*(4*a*c - b^2)*(a*e^2 + c*d^2 -
b*d*e)))*(b^4*d*(b^2 - 4*a*c)^(1/2) - b^5*d + 4*a^3*c^2*e + a*b^4*e + 6*a*b
^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c^2*d - 5*a^2*b^2*c*e + 2*a^
2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e
*(b^2 - 4*a*c)^(1/2)))/(4*(4*a*c^5*d^2 + 4*a^2*c^4*e^2 - b^2*c^4*d^2 - a*b^
2*c^3*e^2 + b^3*c^3*d*e - 4*a*b*c^4*d*e)) + x^4/(4*c*e) - (x^2*(b*e + c*d))
/(2*c^2*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.218 $\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=189

$$\frac{\left(2a^2ce-ab^2e-3abcd+b^3d\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}+\frac{\left(-abe-acd+b^2d\right)\log\left(a+bx^2+cx^4\right)}{4c^2\left(ae^2-bde+cd^2\right)}-\frac{d^3\log\left(d+ex^2\right)}{2e^2\left(ae^2-bde+cd^2\right)}$$

Rubi [A] time = 0.33, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1251, 1628, 634, 618, 206, 628}

$$\frac{\left(2a^2ce-ab^2e-3abcd+b^3d\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}+\frac{\left(-abe-acd+b^2d\right)\log\left(a+bx^2+cx^4\right)}{4c^2\left(ae^2-bde+cd^2\right)}-\frac{d^3\log\left(d+ex^2\right)}{2e^2\left(ae^2-bde+cd^2\right)}+\frac{x^2}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] x^2/(2*c*e) + ((b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) -
 (d^3*Log[d + e*x^2])/(2*e^2*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d - a*c*d - a
*b*e)*Log[a + b*x^2 + c*x^4])/(4*c^2*(c*d^2 - b*d*e + a*e^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^7}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(d+ex)\left(a+bx+cx^2\right)}\,dx,x,x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ce} - \frac{d^3}{e\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{a(bd-ae)+\left(b^2d-acd-ab\right.}{c\left(cd^2-bde+ae^2\right)\left(a+bx+\right.}\right.\right.$$

$$= \frac{x^2}{2ce} - \frac{d^3\log\left(d+ex^2\right)}{2e^2\left(cd^2-bde+ae^2\right)} + \frac{\text{Subst}\left(\int \frac{a(bd-ae)+\left(b^2d-acd-abe\right)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2c\left(cd^2-bde+ae^2\right)}$$

$$= \frac{x^2}{2ce} - \frac{d^3\log\left(d+ex^2\right)}{2e^2\left(cd^2-bde+ae^2\right)} + \frac{\left(b^2d-acd-abe\right)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^2\left(cd^2-bde+ae^2\right)} -$$

$$= \frac{x^2}{2ce} - \frac{d^3\log\left(d+ex^2\right)}{2e^2\left(cd^2-bde+ae^2\right)} + \frac{\left(b^2d-acd-abe\right)\log\left(a+bx^2+cx^4\right)}{4c^2\left(cd^2-bde+ae^2\right)} + \frac{\left(b^3d-\right.}{}$$

$$= \frac{x^2}{2ce} + \frac{\left(b^3d-3abcd-ab^2e+2a^2ce\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)} - \frac{d^3\log\left(d+ex^2\right)}{2e^2\left(cd^2-bde+ae^2\right)}$$

Mathematica [A] time = 0.19, size = 186, normalized size = 0.98

$$\frac{2e^2\left(2a^2ce-ab^2e-3abcd+b^3d\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)+\sqrt{4ac-b^2}\left(e\left(e\left(abe+acd+b^2(-d)\right)\log\left(a+bx^2+cx^4\right)-2cx^2\left(ae^2-bde+cd^2\right)\right)+2c^2d^3\log\left(d+ex^2\right)\right)}{4c^2e^2\sqrt{4ac-b^2}\left(e(bd-ae)-cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*e^2*(b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*ArcTan[(b + 2*c*x^2)/Sqrt[
-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*(2*c^2*d^3*Log[d + e*x^2] + e*(-2*c*(c*
d^2 - b*d*e + a*e^2)*x^2 + e*(-(b^2*d) + a*c*d + a*b*e)*Log[a + b*x^2 + c*x
^4])))/(4*c^2*Sqrt[-b^2 + 4*a*c]*e^2*(-(c*d^2) + e*(b*d - a*e)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 2.19, size = 194, normalized size = 1.03

$$-\frac{d^{3} \log\left(\left|x^{2} e + d\right|\right)}{2\left(c d^{2} e^{2} - b d e^{3} + a e^{4}\right)} + \frac{x^{2} e^{(-1)}}{2c} + \frac{\left(b^{2} d - a c d - a b e\right) \log\left(c x^{4} + b x^{2} + a\right)}{4\left(c^{3} d^{2} - b c^{2} d e + a c^{2} e^{2}\right)} - \frac{\left(b^{3} d - 3 a b c d - a b^{2} e + 2 a^{2} c e\right) \arctan\left(\frac{2 c x^{2} + b}{\sqrt{-b^{2} + 4 a c}}\right)}{2\left(c^{3} d^{2} - b c^{2} d e + a c^{2} e^{2}\right)\sqrt{-b^{2} + 4 a c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*d^3*log(abs(x^2*e + d))/(c*d^2*e^2 - b*d*e^3 + a*e^4) + 1/2*x^2*e^(-1)
/c + 1/4*(b^2*d - a*c*d - a*b*e)*log(c*x^4 + b*x^2 + a)/(c^3*d^2 - b*c^2*d*
e + a*c^2*e^2) - 1/2*(b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*arctan((2*c*
x^2 + b)/sqrt(-b^2 + 4*a*c))/((c^3*d^2 - b*c^2*d*e + a*c^2*e^2)*sqrt(-b^2 +
 4*a*c))
```

maple [B] time = 0.01, size = 408, normalized size = 2.16

$$-\frac{a^{2} e \arctan\left(\frac{2 c x^{2}+b}{\sqrt{4 a c-b^{2}}}\right)}{\left(a e^{2}-d e b+c d^{2}\right) \sqrt{4 a c-b^{2}}\, c}+\frac{a b^{2} e \arctan\left(\frac{2 c x^{2}+b}{\sqrt{4 a c-b^{2}}}\right)}{2\left(a e^{2}-d e b+c d^{2}\right) \sqrt{4 a c-b^{2}}\, c^{2}}+\frac{3 a b d \arctan\left(\frac{2 c x^{2}+b}{\sqrt{4 a c-b^{2}}}\right)}{2\left(a e^{2}-d e b+c d^{2}\right) \sqrt{4 a c-b^{2}}\, c}-\frac{b^{3} d \arctan\left(\frac{2 c x^{2}+b}{\sqrt{4 a c-b^{2}}}\right)}{2\left(a e^{2}-d e b+c d^{2}\right) \sqrt{4 a c-b^{2}}\, c^{2}}-\frac{a b e \ln\left(c x^{4}+b x^{2}+a\right)}{4\left(a e^{2}-d e b+c d^{2}\right) c^{2}}-\frac{a d \ln\left(c x^{4}+b x^{2}+a\right)}{4\left(a e^{2}-d e b+c d^{2}\right) c}+\frac{b^{2} d \ln\left(c x^{4}+b x^{2}+a\right)}{4\left(a e^{2}-d e b+c d^{2}\right) c^{2}}-\frac{d^{3} \ln\left(e x^{2}+d\right)}{2\left(a e^{2}-d e b+c d^{2}\right) e^{2}}+\frac{x^{2}}{2 c e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/c/e*x^2-1/4/(a*e^2-b*d*e+c*d^2)/c^2*ln(c*x^4+b*x^2+a)*a*b*e-1/4/(a*e^2-
b*d*e+c*d^2)/c*ln(c*x^4+b*x^2+a)*a*d+1/4/(a*e^2-b*d*e+c*d^2)/c^2*ln(c*x^4+b
*x^2+a)*b^2*d-1/(a*e^2-b*d*e+c*d^2)/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/
(4*a*c-b^2)^(1/2))*a^2*e+3/2/(a*e^2-b*d*e+c*d^2)/c/(4*a*c-b^2)^(1/2)*arctan
((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*d+1/2/(a*e^2-b*d*e+c*d^2)/c^2/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*a*e-1/2/(a*e^2-b*d*e+c*d
^2)/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*d-1/2*d
^3*ln(e*x^2+d)/e^2/(a*e^2-b*d*e+c*d^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 15.21, size = 2304, normalized size = 12.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] x^2/(2*c*e) - (d^3*log(d + e*x^2))/(2*(a*e^4 + c*d^2*e^2 - b*d*e^3)) - (log
(a*e^5*x^2*(b^2 - 4*a*c)^(7/2) - 128*a^5*c^3*e^5 - 8*c^3*d^5*(b^2 - 4*a*c)^
(5/2) - 512*a^3*c^5*d^4*e + 8*b^2*c^3*d^5*(b^2 - 4*a*c)^(3/2) + 6*b^3*d^2*e
^3*(b^2 - 4*a*c)^(5/2) - 3*b^5*d^2*e^3*(b^2 - 4*a*c)^(3/2) + 32*a^4*b^2*c^2
*e^5 + 384*a^4*c^4*d^2*e^3 + 256*a^2*c^6*d^5*x^2 + 16*b^4*c^4*d^5*x^2 + 3*a
*d*e^4*(b^2 - 4*a*c)^(7/2) - 3*b*d^2*e^3*(b^2 - 4*a*c)^(7/2) - 3*c*d^3*e^2*
(b^2 - 4*a*c)^(7/2) - 16*a^2*b^3*c^3*d^3*e^2 + 48*a^2*b^4*c^2*d^2*e^3 - 288
*a^3*b^2*c^3*d^2*e^3 + 16*a^3*b^3*c^2*e^5*x^2 - 384*a^3*c^5*d^3*e^2*x^2 + 1
```

```
6*b^6*c^2*d^3*e^2*x^2 - 6*a*b^2*d*e^4*(b^2 - 4*a*c)^(5/2) + 3*a*b^4*d*e^4*(
b^2 - 4*a*c)^(3/2) + 8*b*c^2*d^4*e*(b^2 - 4*a*c)^(5/2) - 32*a*b^4*c^3*d^4*e
 + 192*a^4*b*c^3*d*e^4 - 2*b^2*c*d^3*e^2*(b^2 - 4*a*c)^(5/2) - 8*b^3*c^2*d^
4*e*(b^2 - 4*a*c)^(3/2) + 5*b^4*c*d^3*e^2*(b^2 - 4*a*c)^(3/2) - 2*a*b^2*e^5
*x^2*(b^2 - 4*a*c)^(5/2) + a*b^4*e^5*x^2*(b^2 - 4*a*c)^(3/2) + 16*b*c^4*d^5
*x^2*(b^2 - 4*a*c)^(3/2) - 3*c*d^2*e^3*x^2*(b^2 - 4*a*c)^(7/2) - 16*c^3*d^4
*e*x^2*(b^2 - 4*a*c)^(5/2) + 256*a^2*b^2*c^4*d^4*e + 64*a^3*b*c^4*d^3*e^2 -
 48*a^3*b^3*c^2*d*e^4 - 128*a*b^2*c^5*d^5*x^2 - 64*a^4*b*c^3*e^5*x^2 + 384*
a^4*c^4*d*e^4*x^2 - 32*b^5*c^3*d^4*e*x^2 + 480*a^2*b^2*c^4*d^3*e^2*x^2 + 48
*a^2*b^3*c^3*d^2*e^3*x^2 + 256*a*b^3*c^4*d^4*e*x^2 - 512*a^2*b*c^5*d^4*e*x^
2 + 8*b*c^2*d^3*e^2*x^2*(b^2 - 4*a*c)^(5/2) + 6*b^2*c*d^2*e^3*x^2*(b^2 - 4*
a*c)^(5/2) - 16*b^2*c^3*d^4*e*x^2*(b^2 - 4*a*c)^(3/2) - 3*b^4*c*d^2*e^3*x^2
*(b^2 - 4*a*c)^(3/2) - 160*a*b^4*c^3*d^3*e^2*x^2 - 192*a^3*b*c^4*d^2*e^3*x^
2 - 96*a^3*b^2*c^3*d*e^4*x^2 + 8*b^3*c^2*d^3*e^2*x^2*(b^2 - 4*a*c)^(3/2))*(
b^4*d - b^3*d*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*d - a*b^3*e - 5*a*b^2*c*d + 4
*a^2*b*c*e + a*b^2*e*(b^2 - 4*a*c)^(1/2) - 2*a^2*c*e*(b^2 - 4*a*c)^(1/2) +
3*a*b*c*d*(b^2 - 4*a*c)^(1/2)))/(4*(4*a*c^4*d^2 + 4*a^2*c^3*e^2 - b^2*c^3*d
^2 - a*b^2*c^2*e^2 + b^3*c^2*d*e - 4*a*b*c^3*d*e)) - (log(8*c^3*d^5*(b^2 -
4*a*c)^(5/2) - 128*a^5*c^3*e^5 - a*e^5*x^2*(b^2 - 4*a*c)^(7/2) - 512*a^3*c^
5*d^4*e - 8*b^2*c^3*d^5*(b^2 - 4*a*c)^(3/2) - 6*b^3*d^2*e^3*(b^2 - 4*a*c)^(
5/2) + 3*b^5*d^2*e^3*(b^2 - 4*a*c)^(3/2) + 32*a^4*b^2*c^2*e^5 + 384*a^4*c^4
*d^2*e^3 + 256*a^2*c^6*d^5*x^2 + 16*b^4*c^4*d^5*x^2 - 3*a*d*e^4*(b^2 - 4*a*
c)^(7/2) + 3*b*d^2*e^3*(b^2 - 4*a*c)^(7/2) + 3*c*d^3*e^2*(b^2 - 4*a*c)^(7/2
) - 16*a^2*b^3*c^3*d^3*e^2 + 48*a^2*b^4*c^2*d^2*e^3 - 288*a^3*b^2*c^3*d^2*e
^3 + 16*a^3*b^3*c^2*e^5*x^2 - 384*a^3*c^5*d^3*e^2*x^2 + 16*b^6*c^2*d^3*e^2*
x^2 + 6*a*b^2*d*e^4*(b^2 - 4*a*c)^(5/2) - 3*a*b^4*d*e^4*(b^2 - 4*a*c)^(3/2)
 - 8*b*c^2*d^4*e*(b^2 - 4*a*c)^(5/2) - 32*a*b^4*c^3*d^4*e + 192*a^4*b*c^3*d
*e^4 + 2*b^2*c*d^3*e^2*(b^2 - 4*a*c)^(5/2) + 8*b^3*c^2*d^4*e*(b^2 - 4*a*c)^
(3/2) - 5*b^4*c*d^3*e^2*(b^2 - 4*a*c)^(3/2) + 2*a*b^2*e^5*x^2*(b^2 - 4*a*c)
^(5/2) - a*b^4*e^5*x^2*(b^2 - 4*a*c)^(3/2) - 16*b*c^4*d^5*x^2*(b^2 - 4*a*c)
^(3/2) + 3*c*d^2*e^3*x^2*(b^2 - 4*a*c)^(7/2) + 16*c^3*d^4*e*x^2*(b^2 - 4*a*
c)^(5/2) + 256*a^2*b^2*c^4*d^4*e + 64*a^3*b*c^4*d^3*e^2 - 48*a^3*b^3*c^2*d*
e^4 - 128*a*b^2*c^5*d^5*x^2 - 64*a^4*b*c^3*e^5*x^2 + 384*a^4*c^4*d*e^4*x^2
- 32*b^5*c^3*d^4*e*x^2 + 480*a^2*b^2*c^4*d^3*e^2*x^2 + 48*a^2*b^3*c^3*d^2*e
^3*x^2 + 256*a*b^3*c^4*d^4*e*x^2 - 512*a^2*b*c^5*d^4*e*x^2 - 8*b*c^2*d^3*e^
2*x^2*(b^2 - 4*a*c)^(5/2) - 6*b^2*c*d^2*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 16*b^
2*c^3*d^4*e*x^2*(b^2 - 4*a*c)^(3/2) + 3*b^4*c*d^2*e^3*x^2*(b^2 - 4*a*c)^(3/
2) - 160*a*b^4*c^3*d^3*e^2*x^2 - 192*a^3*b*c^4*d^2*e^3*x^2 - 96*a^3*b^2*c^3
*d*e^4*x^2 - 8*b^3*c^2*d^3*e^2*x^2*(b^2 - 4*a*c)^(3/2))*(b^4*d + b^3*d*(b^2
 - 4*a*c)^(1/2) + 4*a^2*c^2*d - a*b^3*e - 5*a*b^2*c*d + 4*a^2*b*c*e - a*b^2
*e*(b^2 - 4*a*c)^(1/2) + 2*a^2*c*e*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*d*(b^2 - 4
*a*c)^(1/2)))/(4*(4*a*c^4*d^2 + 4*a^2*c^3*e^2 - b^2*c^3*d^2 - a*b^2*c^2*e^2
 + b^3*c^2*d*e - 4*a*b*c^3*d*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.219 $\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=158

$$-\frac{\left(-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}+\frac{d^2\log\left(d+ex^2\right)}{2e\left(ae^2-bde+cd^2\right)}-\frac{(bd-ae)\log\left(a+bx^2+cx^4\right)}{4c\left(ae^2-bde+cd^2\right)}$$

Rubi [A] time = 0.26, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1251, 1628, 634, 618, 206, 628}

$$-\frac{\left(-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}+\frac{d^2\log\left(d+ex^2\right)}{2e\left(ae^2-bde+cd^2\right)}-\frac{(bd-ae)\log\left(a+bx^2+cx^4\right)}{4c\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((b^2*d - 2*a*c*d - a*b*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*
Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + (d^2*Log[d + e*x^2])/(2*e*(c*d
^2 - b*d*e + a*e^2)) - ((b*d - a*e)*Log[a + b*x^2 + c*x^4])/(4*c*(c*d^2 - b
*d*e + a*e^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(d+ex)(a+bx+cx^2)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{d^2}{(cd^2-bde+ae^2)(d+ex)} + \frac{-ad-(bd-ae)x}{(cd^2-bde+ae^2)(a+bx+cx^2)}\right) d$$

$$= \frac{d^2\log(d+ex^2)}{2e(cd^2-bde+ae^2)} + \frac{\operatorname{Subst}\left(\int \frac{-ad-(bd-ae)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2(cd^2-bde+ae^2)}$$

$$= \frac{d^2\log(d+ex^2)}{2e(cd^2-bde+ae^2)} - \frac{(bd-ae)\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c(cd^2-bde+ae^2)} + \frac{(b^2d-2acd-a}{4c}$$

$$= \frac{d^2\log(d+ex^2)}{2e(cd^2-bde+ae^2)} - \frac{(bd-ae)\log(a+bx^2+cx^4)}{4c(cd^2-bde+ae^2)} - \frac{(b^2d-2acd-abe)\operatorname{Subs}}{2c(cd^2}$$

$$= -\frac{(b^2d-2acd-abe)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} + \frac{d^2\log(d+ex^2)}{2e(cd^2-bde+ae^2)} - \frac{(bd-ae)\log(}{4c(cd^2-}$$

Mathematica [A] time = 0.11, size = 139, normalized size = 0.88

$$-\frac{\sqrt{4ac-b^2}\left(e(bd-ae)\log(a+bx^2+cx^4)-2cd^2\log(d+ex^2)\right)+2e\left(abe+2acd+b^2(-d)\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{4ce\sqrt{4ac-b^2}\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*(2*e*(-(b^2*d) + 2*a*c*d + a*b*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a
*c]] + Sqrt[-b^2 + 4*a*c]*(-2*c*d^2*Log[d + e*x^2] + e*(b*d - a*e)*Log[a +
b*x^2 + c*x^4]))/(c*Sqrt[-b^2 + 4*a*c]*e*(c*d^2 + e*(-(b*d) + a*e)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]
```

fricas [A] time = 128.56, size = 421, normalized size = 2.66

$$\left[\frac{2(b^2c-4ac^2)d^2\log(ex^2+d)+(abe^2-(b^2-2ac)de)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac+(2cx^2+b)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)-((b^3-4abc)de-(ab^2-4a^2c)e^2)\log(cx^4+bx^2+a)}{4((b^2c^2-4ac^3)d^2e-(b^3c-4abc^2)de^2+(ab^2c-4a^2c^2)e^3)}, \frac{2(b^2c-4ac^2)d^2\log(ex^2+d)+2(abe^2-(b^2-2ac)de)\sqrt{-b^2+4ac}\arctan\left(-\frac{(2cx^2+b)\sqrt{-b^2+4ac}}{b^2-4ac}\right)-((b^3-4abc)de-(ab^2-4a^2c)e^2)\log(cx^4+bx^2+a)}{4((b^2c^2-4ac^3)d^2e-(b^3c-4abc^2)de^2+(ab^2c-4a^2c^2)e^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c - 4*a*c^2)*d^2*log(e*x^2 + d) + (a*b*e^2 - (b^2 - 2*a*c)*d*e
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b
)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((b^3 - 4*a*b*c)*d*e - (a*b^2 -
 4*a^2*c)*e^2)*log(c*x^4 + b*x^2 + a))/((b^2*c^2 - 4*a*c^3)*d^2*e - (b^3*c
- 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3), 1/4*(2*(b^2*c - 4*a*c^2)*d
^2*log(e*x^2 + d) + 2*(a*b*e^2 - (b^2 - 2*a*c)*d*e)*sqrt(-b^2 + 4*a*c)*arct
an(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^3 - 4*a*b*c)*d*e
- (a*b^2 - 4*a^2*c)*e^2)*log(c*x^4 + b*x^2 + a))/((b^2*c^2 - 4*a*c^3)*d^2*e
 - (b^3*c - 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)]
```

giac [A] time = 1.84, size = 157, normalized size = 0.99

$$\frac{d^2\log\left(\left|x^2e+d\right|\right)}{2\left(cd^2e-bde^2+ae^3\right)}-\frac{(bd-ae)\log\left(cx^4+bx^2+a\right)}{4\left(c^2d^2-bcde+ace^2\right)}+\frac{\left(b^2d-2\,acd-abe\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(c^2d^2-bcde+ace^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d^2*log(abs(x^2*e + d))/(c*d^2*e - b*d*e^2 + a*e^3) - 1/4*(b*d - a*e)*l
og(c*x^4 + b*x^2 + a)/(c^2*d^2 - b*c*d*e + a*c*e^2) + 1/2*(b^2*d - 2*a*c*d
- a*b*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((c^2*d^2 - b*c*d*e + a*c
*e^2)*sqrt(-b^2 + 4*a*c))
```

maple [A] time = 0.01, size = 289, normalized size = 1.83

$$-\frac{abe\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(ae^2-deb+cd^2\right)\sqrt{4ac-b^2}\,c}-\frac{ad\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\left(ae^2-deb+cd^2\right)\sqrt{4ac-b^2}}+\frac{b^2d\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(ae^2-deb+cd^2\right)\sqrt{4ac-b^2}\,c}+\frac{ae\ln\left(cx^4+bx^2+a\right)}{4\left(ae^2-deb+cd^2\right)c}-\frac{bd\ln\left(cx^4+bx^2+a\right)}{4\left(ae^2-deb+cd^2\right)c}+\frac{d^2\ln\left(ex^2+d\right)}{2\left(ae^2-deb+cd^2\right)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4/(a*e^2-b*d*e+c*d^2)/c*ln(c*x^4+b*x^2+a)*a*e-1/4/(a*e^2-b*d*e+c*d^2)/c*l
n(c*x^4+b*x^2+a)*b*d-1/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^
2+b)/(4*a*c-b^2)^(1/2))*a*d-1/2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b/c*a*e+1/2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2/c*d+1/2*d^2*ln(e*x^2+d)/e
/(a*e^2-b*d*e+c*d^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 11.05, size = 1853, normalized size = 11.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (d^2*log(d + e*x^2))/(2*a*e^3 - 2*b*d*e^2 + 2*c*d^2*e) + (log(4*a^2*e^4*(b^
2 - 4*a*c)^(5/2) + 8*c^2*d^4*(b^2 - 4*a*c)^(5/2) + 5*d^2*e^2*(b^2 - 4*a*c)^
```

```
(7/2) + 3*d*e^3*x^2*(b^2 - 4*a*c)^(7/2) - 16*a^3*b^3*c*e^4 + 64*a^4*b*c^2*e
^4 + 640*a^3*c^4*d^3*e - 384*a^4*c^3*d*e^3 - 4*a^2*b^2*e^4*(b^2 - 4*a*c)^(3
/2) - 8*b^2*c^2*d^4*(b^2 - 4*a*c)^(3/2) - 6*b^2*d^2*e^2*(b^2 - 4*a*c)^(5/2)
 + b^4*d^2*e^2*(b^2 - 4*a*c)^(3/2) - 256*a^2*c^5*d^4*x^2 - 128*a^4*c^3*e^4*
x^2 - 16*b^4*c^3*d^4*x^2 + 80*a^2*b^3*c^2*d^2*e^2 + 96*a^3*b^2*c^2*e^4*x^2
+ 640*a^3*c^4*d^2*e^2*x^2 + 4*b^3*c*d^3*e*(b^2 - 4*a*c)^(3/2) + 4*a*b*e^4*x
^2*(b^2 - 4*a*c)^(5/2) + 48*a*b^4*c^2*d^3*e - 16*a*b^5*c*d^2*e^2 - 4*a*b^3*
e^4*x^2*(b^2 - 4*a*c)^(3/2) - 16*b*c^3*d^4*x^2*(b^2 - 4*a*c)^(3/2) - 6*b^2*
d*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 3*b^4*d*e^3*x^2*(b^2 - 4*a*c)^(3/2) + 20*c^
2*d^3*e*x^2*(b^2 - 4*a*c)^(5/2) - 352*a^2*b^2*c^3*d^3*e - 64*a^3*b*c^3*d^2*
e^2 + 96*a^3*b^2*c^2*d*e^3 + 128*a*b^2*c^4*d^4*x^2 - 16*a^2*b^4*c*e^4*x^2 +
 32*b^5*c^2*d^3*e*x^2 - 16*b^6*c*d^2*e^2*x^2 - 4*b*c*d^3*e*(b^2 - 4*a*c)^(5
/2) - 480*a^2*b^2*c^3*d^2*e^2*x^2 - 12*b*c*d^2*e^2*x^2*(b^2 - 4*a*c)^(5/2)
- 240*a*b^3*c^3*d^3*e*x^2 + 448*a^2*b*c^4*d^3*e*x^2 - 192*a^3*b*c^3*d*e^3*x
^2 + 12*b^2*c^2*d^3*e*x^2*(b^2 - 4*a*c)^(3/2) - 4*b^3*c*d^2*e^2*x^2*(b^2 -
4*a*c)^(3/2) + 144*a*b^4*c^2*d^2*e^2*x^2 + 48*a^2*b^3*c^2*d*e^3*x^2)*((b^3*
d)/4 + e*(a^2*c - (a*b^2)/4 + (a*b*(b^2 - 4*a*c)^(1/2))/4) - (b^2*d*(b^2 -
4*a*c)^(1/2))/4 + (a*c*d*(b^2 - 4*a*c)^(1/2))/2 - a*b*c*d))/(4*a*c^3*d^2 +
4*a^2*c^2*e^2 - b^2*c^2*d^2 + b^3*c*d*e - a*b^2*c*e^2 - 4*a*b*c^2*d*e) - (l
og(4*a^2*e^4*(b^2 - 4*a*c)^(5/2) + 8*c^2*d^4*(b^2 - 4*a*c)^(5/2) + 5*d^2*e^
2*(b^2 - 4*a*c)^(7/2) + 3*d*e^3*x^2*(b^2 - 4*a*c)^(7/2) + 16*a^3*b^3*c*e^4
- 64*a^4*b*c^2*e^4 - 640*a^3*c^4*d^3*e + 384*a^4*c^3*d*e^3 - 4*a^2*b^2*e^4*
(b^2 - 4*a*c)^(3/2) - 8*b^2*c^2*d^4*(b^2 - 4*a*c)^(3/2) - 6*b^2*d^2*e^2*(b^
2 - 4*a*c)^(5/2) + b^4*d^2*e^2*(b^2 - 4*a*c)^(3/2) + 256*a^2*c^5*d^4*x^2 +
128*a^4*c^3*e^4*x^2 + 16*b^4*c^3*d^4*x^2 - 80*a^2*b^3*c^2*d^2*e^2 - 96*a^3*
b^2*c^2*e^4*x^2 - 640*a^3*c^4*d^2*e^2*x^2 + 4*b^3*c*d^3*e*(b^2 - 4*a*c)^(3/
2) + 4*a*b*e^4*x^2*(b^2 - 4*a*c)^(5/2) - 48*a*b^4*c^2*d^3*e + 16*a*b^5*c*d^
2*e^2 - 4*a*b^3*e^4*x^2*(b^2 - 4*a*c)^(3/2) - 16*b*c^3*d^4*x^2*(b^2 - 4*a*c
)^(3/2) - 6*b^2*d*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 3*b^4*d*e^3*x^2*(b^2 - 4*a*
c)^(3/2) + 20*c^2*d^3*e*x^2*(b^2 - 4*a*c)^(5/2) + 352*a^2*b^2*c^3*d^3*e + 6
4*a^3*b*c^3*d^2*e^2 - 96*a^3*b^2*c^2*d*e^3 - 128*a*b^2*c^4*d^4*x^2 + 16*a^2
*b^4*c*e^4*x^2 - 32*b^5*c^2*d^3*e*x^2 + 16*b^6*c*d^2*e^2*x^2 - 4*b*c*d^3*e*
(b^2 - 4*a*c)^(5/2) + 480*a^2*b^2*c^3*d^2*e^2*x^2 - 12*b*c*d^2*e^2*x^2*(b^2
 - 4*a*c)^(5/2) + 240*a*b^3*c^3*d^3*e*x^2 - 448*a^2*b*c^4*d^3*e*x^2 + 192*a
^3*b*c^3*d*e^3*x^2 + 12*b^2*c^2*d^3*e*x^2*(b^2 - 4*a*c)^(3/2) - 4*b^3*c*d^2
*e^2*x^2*(b^2 - 4*a*c)^(3/2) - 144*a*b^4*c^2*d^2*e^2*x^2 - 48*a^2*b^3*c^2*d
*e^3*x^2)*(e*((a*b^2)/4 - a^2*c + (a*b*(b^2 - 4*a*c)^(1/2))/4) - (b^3*d)/4
- (b^2*d*(b^2 - 4*a*c)^(1/2))/4 + (a*c*d*(b^2 - 4*a*c)^(1/2))/2 + a*b*c*d))
/(4*a*c^3*d^2 + 4*a^2*c^2*e^2 - b^2*c^2*d^2 + b^3*c*d*e - a*b^2*c*e^2 - 4*a
*b*c^2*d*e)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.220 $\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=132

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}-\frac{d\log\left(d+ex^2\right)}{2\left(ae^2-bde+cd^2\right)}+\frac{d\log\left(a+bx^2+cx^4\right)}{4\left(ae^2-bde+cd^2\right)}$$

Rubi [A] time = 0.16, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1251, 800, 634, 618, 206, 628}

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}-\frac{d\log\left(d+ex^2\right)}{2\left(ae^2-bde+cd^2\right)}+\frac{d\log\left(a+bx^2+cx^4\right)}{4\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*Sqrt[b^2 - 4*a*
c]*(c*d^2 - b*d*e + a*e^2)) - (d*Log[d + e*x^2])/(2*(c*d^2 - b*d*e + a*e^2)
) + (d*Log[a + b*x^2 + c*x^4])/(4*(c*d^2 - b*d*e + a*e^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
```

```
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(d+ex)\left(a+bx+cx^2\right)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{de}{\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{ae+cdx}{\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}\right)\right.\\
&= -\frac{d\log\left(d+ex^2\right)}{2\left(cd^2-bde+ae^2\right)}+\frac{\operatorname{Subst}\left(\int \frac{ae+cdx}{a+bx+cx^2}\,dx, x, x^2\right)}{2\left(cd^2-bde+ae^2\right)}\\
&= -\frac{d\log\left(d+ex^2\right)}{2\left(cd^2-bde+ae^2\right)}+\frac{d\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4\left(cd^2-bde+ae^2\right)}-\frac{(bd-2ae)\operatorname{Subst}\left(\int \frac{}{a+}\right.}{4\left(cd^2-bde\right.}\\
&= -\frac{d\log\left(d+ex^2\right)}{2\left(cd^2-bde+ae^2\right)}+\frac{d\log\left(a+bx^2+cx^4\right)}{4\left(cd^2-bde+ae^2\right)}+\frac{(bd-2ae)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,d\right.}{2\left(cd^2-bde+ae^2\right.}\\
&= \frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)}-\frac{d\log\left(d+ex^2\right)}{2\left(cd^2-bde+ae^2\right)}+\frac{d\log\left(a+bx^2+cx^4\right)}{4\left(cd^2-bde+ae^2\right)}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 114, normalized size = 0.86

$$
\frac{2(bd-2ae)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)+d\sqrt{4ac-b^2}\left(2\log\left(d+ex^2\right)-\log\left(a+bx^2+cx^4\right)\right)}{4\sqrt{4ac-b^2}\left(e(bd-ae)-cd^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*(b*d - 2*a*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a
*c]*d*(2*Log[d + e*x^2] - Log[a + b*x^2 + c*x^4]))/(4*Sqrt[-b^2 + 4*a*c]*(-
(c*d^2) + e*(b*d - a*e)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^3}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]
```

fricas [A] time = 37.37, size = 321, normalized size = 2.43

$$
\left[\frac{\left(b^2-4\,ac\right)d\log\left(cx^4+bx^2+a\right)-2\left(b^2-4\,ac\right)d\log\left(ex^2+d\right)-\sqrt{b^2-4\,ac}\,(bd-2\,ae)\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{4\left(\left(b^2c-4\,ac^2\right)d^2-\left(b^3-4\,abc\right)de+\left(ab^2-4\,a^2c\right)e^2\right)}, \frac{\left(b^2-4\,ac\right)d\log\left(cx^4+bx^2+a\right)-2\left(b^2-4\,ac\right)d\log\left(ex^2+d\right)+2\sqrt{-b^2+4\,ac}\,(bd-2\,ae)\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{4\left(\left(b^2c-4\,ac^2\right)d^2-\left(b^3-4\,abc\right)de+\left(ab^2-4\,a^2c\right)e^2\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2 - 4*a*c)*d*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*d*log(e*x^2
+ d) - sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2
*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^2*c - 4*a
*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2), 1/4*((b^2 - 4*a*c
)*d*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*d*log(e*x^2 + d) + 2*sqrt(-b^2
 + 4*a*c)*(b*d - 2*a*e)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a
*c)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)
]
```

giac [A] time = 1.72, size = 133, normalized size = 1.01

$$-\frac{de\log\left(\left|x^2e+d\right|\right)}{2\left(cd^2e-bde^2+ae^3\right)}+\frac{d\log\left(cx^4+bx^2+a\right)}{4\left(cd^2-bde+ae^2\right)}-\frac{(bd-2\,ae)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(cd^2-bde+ae^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*d*e*log(abs(x^2*e + d))/(c*d^2*e - b*d*e^2 + a*e^3) + 1/4*d*log(c*x^4
+ b*x^2 + a)/(c*d^2 - b*d*e + a*e^2) - 1/2*(b*d - 2*a*e)*arctan((2*c*x^2 +
b)/sqrt(-b^2 + 4*a*c))/((c*d^2 - b*d*e + a*e^2)*sqrt(-b^2 + 4*a*c))
```

maple [A] time = 0.01, size = 176, normalized size = 1.33

$$\frac{ae\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,e^2-deb+c\,d^2\right)\sqrt{4ac-b^2}}-\frac{bd\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(a\,e^2-deb+c\,d^2\right)\sqrt{4ac-b^2}}-\frac{d\ln\left(e\,x^2+d\right)}{2\left(a\,e^2-deb+c\,d^2\right)}+\frac{d\ln\left(c\,x^4+b\,x^2+a\right)}{4a\,e^2-4deb+4c\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*d*ln(c*x^4+b*x^2+a)/(a*e^2-b*d*e+c*d^2)+1/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*e-1/2/(a*e^2-b*d*e+c*d^2)/
(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*d-1/2*d*ln(e*x^2+
d)/(a*e^2-b*d*e+c*d^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 9.75, size = 3704, normalized size = 28.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(76*d^3*e^3*(b^2 - 4*a*c)^(9/2) - 64*a^3*b^6*e^6 - 4608*a^3*c^6*d^6 + 5
12*a^6*c^3*e^6 - 320*a*b^4*c^4*d^6 + 512*a^4*b^4*c*e^6 - 64*a*b^8*d^2*e^4 -
 128*a^2*b^7*d*e^5 + 32*a^3*b^3*e^6*(b^2 - 4*a*c)^(3/2) - 48*b^3*c^3*d^6*(b
^2 - 4*a*c)^(3/2) - 68*b^2*d^3*e^3*(b^2 - 4*a*c)^(7/2) - 28*b^4*d^3*e^3*(b^
2 - 4*a*c)^(5/2) + 20*b^6*d^3*e^3*(b^2 - 4*a*c)^(3/2) + 4*a^2*e^6*x^2*(b^2
```

- 4*a*c)^(7/2) + 144*c^4*d^6*x^2*(b^2 - 4*a*c)^(5/2) + 39*d^2*e^4*x^2*(b^2
- 4*a*c)^(9/2) + 2432*a^2*b^2*c^5*d^6 - 1152*a^5*b^2*c^2*e^6 + 40448*a^4*c^
5*d^4*e^2 - 19968*a^5*c^4*d^2*e^4 - 64*a^2*b^7*e^6*x^2 - 64*b^5*c^4*d^6*x^2
- 64*b^9*d^2*e^4*x^2 + 32*a^3*b*e^6*(b^2 - 4*a*c)^(5/2) + 48*b*c^3*d^6*(b^
2 - 4*a*c)^(5/2) + 40*a^2*d*e^5*(b^2 - 4*a*c)^(7/2) + 168*c^2*d^5*e*(b^2 -
4*a*c)^(7/2) + 40*a^2*b^2*e^6*x^2*(b^2 - 4*a*c)^(5/2) + 20*a^2*b^4*e^6*x^2*
(b^2 - 4*a*c)^(3/2) - 80*b^2*c^4*d^6*x^2*(b^2 - 4*a*c)^(3/2) + 155*b^2*d^2*
e^4*x^2*(b^2 - 4*a*c)^(7/2) - 155*b^4*d^2*e^4*x^2*(b^2 - 4*a*c)^(5/2) + 25*
b^6*d^2*e^4*x^2*(b^2 - 4*a*c)^(3/2) + 316*c^2*d^4*e^2*x^2*(b^2 - 4*a*c)^(7/
2) + 5120*a^2*b^4*c^3*d^4*e^2 - 4096*a^2*b^5*c^2*d^3*e^3 - 24448*a^3*b^2*c^
4*d^4*e^2 + 21760*a^3*b^3*c^3*d^3*e^3 - 9920*a^3*b^4*c^2*d^2*e^4 + 26240*a^
4*b^2*c^3*d^2*e^4 - 1600*a^4*b^3*c^2*e^6*x^2 + 38912*a^4*c^5*d^3*e^3*x^2 -
384*b^7*c^2*d^4*e^2*x^2 + 212*a*b*d^2*e^4*(b^2 - 4*a*c)^(7/2) - 176*b*c*d^4
e^2(b^2 - 4*a*c)^(7/2) + 256*a*b^5*c^3*d^5*e + 256*a*b^7*c*d^3*e^3 + 2560
*a^3*b*c^5*d^5*e + 1664*a^3*b^5*c*d*e^5 + 8704*a^5*b*c^3*d*e^5 - 128*a*b^8*
d*e^5*x^2 - 168*a*b^3*d^2*e^4*(b^2 - 4*a*c)^(5/2) + 20*a*b^5*d^2*e^4*(b^2 -
4*a*c)^(3/2) + 144*a^2*b^2*d*e^5*(b^2 - 4*a*c)^(5/2) - 56*a^2*b^4*d*e^5*(b
^2 - 4*a*c)^(3/2) - 272*b^2*c^2*d^5*e*(b^2 - 4*a*c)^(5/2) + 256*b^3*c*d^4*e
^2*(b^2 - 4*a*c)^(5/2) + 104*b^4*c^2*d^5*e*(b^2 - 4*a*c)^(3/2) - 80*b^5*c*d
^4*e^2*(b^2 - 4*a*c)^(3/2) - 384*a*b^6*c^2*d^4*e^2 - 1664*a^2*b^3*c^4*d^5*e
+ 1408*a^2*b^6*c*d^2*e^4 - 37888*a^4*b*c^4*d^3*e^3 - 6784*a^4*b^3*c^2*d*e^
5 + 448*a*b^3*c^5*d^6*x^2 - 768*a^2*b*c^6*d^6*x^2 + 576*a^3*b^5*c*e^6*x^2 +
1280*a^5*b*c^3*e^6*x^2 - 21504*a^3*c^6*d^5*e*x^2 - 5120*a^5*c^4*d*e^5*x^2
+ 256*b^6*c^3*d^5*e*x^2 + 256*b^8*c*d^3*e^3*x^2 - 26560*a^2*b^3*c^4*d^4*e^2
*x^2 + 25600*a^2*b^4*c^3*d^3*e^3*x^2 - 11264*a^2*b^5*c^2*d^2*e^4*x^2 - 5888
0*a^3*b^2*c^4*d^3*e^3*x^2 + 34880*a^3*b^3*c^3*d^2*e^4*x^2 + 80*a*b^3*d*e^5*
x^2*(b^2 - 4*a*c)^(5/2) - 40*a*b^5*d*e^5*x^2*(b^2 - 4*a*c)^(3/2) - 448*b*c*
d^3*e^3*x^2*(b^2 - 4*a*c)^(7/2) - 416*b*c^3*d^5*e*x^2*(b^2 - 4*a*c)^(5/2) -
3200*a*b^4*c^4*d^5*e*x^2 + 1472*a*b^7*c*d^2*e^4*x^2 + 1792*a^2*b^6*c*d*e^5
*x^2 + 192*b^3*c*d^3*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 160*b^3*c^3*d^5*e*x^2*(b
^2 - 4*a*c)^(3/2) + 5504*a*b^5*c^3*d^4*e^2*x^2 - 4352*a*b^6*c^2*d^3*e^3*x^2
+ 14080*a^2*b^2*c^5*d^5*e*x^2 + 42752*a^3*b*c^5*d^4*e^2*x^2 - 8320*a^3*b^4
*c^2*d*e^5*x^2 - 37120*a^4*b*c^4*d^2*e^4*x^2 + 14080*a^4*b^2*c^3*d*e^5*x^2
+ 88*a*b*d*e^5*x^2*(b^2 - 4*a*c)^(7/2) + 168*b^2*c^2*d^4*e^2*x^2*(b^2 - 4*a
*c)^(5/2) - 100*b^4*c^2*d^4*e^2*x^2*(b^2 - 4*a*c)^(3/2))*(d*((b*(b^2 - 4*a*
c)^(1/2))/4 - a*c + b^2/4) - (a*e*(b^2 - 4*a*c)^(1/2))/2))/(a*b^2*e^2 - 4*a
*c^2*d^2 - 4*a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d*e) - (log(76*d^3*e
^3*(b^2 - 4*a*c)^(9/2) + 64*a^3*b^6*e^6 + 4608*a^3*c^6*d^6 - 512*a^6*c^3*e^
6 + 320*a*b^4*c^4*d^6 - 512*a^4*b^4*c*e^6 + 64*a*b^8*d^2*e^4 + 128*a^2*b^7*
d*e^5 + 32*a^3*b^3*e^6*(b^2 - 4*a*c)^(3/2) - 48*b^3*c^3*d^6*(b^2 - 4*a*c)^(
3/2) - 68*b^2*d^3*e^3*(b^2 - 4*a*c)^(7/2) - 28*b^4*d^3*e^3*(b^2 - 4*a*c)^(5
/2) + 20*b^6*d^3*e^3*(b^2 - 4*a*c)^(3/2) + 4*a^2*e^6*x^2*(b^2 - 4*a*c)^(7/2
) + 144*c^4*d^6*x^2*(b^2 - 4*a*c)^(5/2) + 39*d^2*e^4*x^2*(b^2 - 4*a*c)^(9/2
) - 2432*a^2*b^2*c^5*d^6 + 1152*a^5*b^2*c^2*e^6 - 40448*a^4*c^5*d^4*e^2 + 1
9968*a^5*c^4*d^2*e^4 + 64*a^2*b^7*e^6*x^2 + 64*b^5*c^4*d^6*x^2 + 64*b^9*d^2
*e^4*x^2 + 32*a^3*b*e^6*(b^2 - 4*a*c)^(5/2) + 48*b*c^3*d^6*(b^2 - 4*a*c)^(5
/2) + 40*a^2*d*e^5*(b^2 - 4*a*c)^(7/2) + 168*c^2*d^5*e*(b^2 - 4*a*c)^(7/2)
+ 40*a^2*b^2*e^6*x^2*(b^2 - 4*a*c)^(5/2) + 20*a^2*b^4*e^6*x^2*(b^2 - 4*a*c)
^(3/2) - 80*b^2*c^4*d^6*x^2*(b^2 - 4*a*c)^(3/2) + 155*b^2*d^2*e^4*x^2*(b^2
- 4*a*c)^(7/2) - 155*b^4*d^2*e^4*x^2*(b^2 - 4*a*c)^(5/2) + 25*b^6*d^2*e^4*x
^2*(b^2 - 4*a*c)^(3/2) + 316*c^2*d^4*e^2*x^2*(b^2 - 4*a*c)^(7/2) - 5120*a^2
*b^4*c^3*d^4*e^2 + 4096*a^2*b^5*c^2*d^3*e^3 + 24448*a^3*b^2*c^4*d^4*e^2 - 2
1760*a^3*b^3*c^3*d^3*e^3 + 9920*a^3*b^4*c^2*d^2*e^4 - 26240*a^4*b^2*c^3*d^2
*e^4 + 1600*a^4*b^3*c^2*e^6*x^2 - 38912*a^4*c^5*d^3*e^3*x^2 + 384*b^7*c^2*d
^4*e^2*x^2 + 212*a*b*d^2*e^4*(b^2 - 4*a*c)^(7/2) - 176*b*c*d^4*e^2*(b^2 - 4
*a*c)^(7/2) - 256*a*b^5*c^3*d^5*e - 256*a*b^7*c*d^3*e^3 - 2560*a^3*b*c^5*d^
5*e - 1664*a^3*b^5*c*d*e^5 - 8704*a^5*b*c^3*d*e^5 + 128*a*b^8*d*e^5*x^2 - 1
68*a*b^3*d^2*e^4*(b^2 - 4*a*c)^(5/2) + 20*a*b^5*d^2*e^4*(b^2 - 4*a*c)^(3/2)
+ 144*a^2*b^2*d*e^5*(b^2 - 4*a*c)^(5/2) - 56*a^2*b^4*d*e^5*(b^2 - 4*a*c)^(

```
3/2) - 272*b^2*c^2*d^5*e*(b^2 - 4*a*c)^(5/2) + 256*b^3*c*d^4*e^2*(b^2 - 4*a
*c)^(5/2) + 104*b^4*c^2*d^5*e*(b^2 - 4*a*c)^(3/2) - 80*b^5*c*d^4*e^2*(b^2 -
 4*a*c)^(3/2) + 384*a*b^6*c^2*d^4*e^2 + 1664*a^2*b^3*c^4*d^5*e - 1408*a^2*b
^6*c*d^2*e^4 + 37888*a^4*b*c^4*d^3*e^3 + 6784*a^4*b^3*c^2*d*e^5 - 448*a*b^3
*c^5*d^6*x^2 + 768*a^2*b*c^6*d^6*x^2 - 576*a^3*b^5*c*e^6*x^2 - 1280*a^5*b*c
^3*e^6*x^2 + 21504*a^3*c^6*d^5*e*x^2 + 5120*a^5*c^4*d*e^5*x^2 - 256*b^6*c^3
*d^5*e*x^2 - 256*b^8*c*d^3*e^3*x^2 + 26560*a^2*b^3*c^4*d^4*e^2*x^2 - 25600*
a^2*b^4*c^3*d^3*e^3*x^2 + 11264*a^2*b^5*c^2*d^2*e^4*x^2 + 58880*a^3*b^2*c^4
*d^3*e^3*x^2 - 34880*a^3*b^3*c^3*d^2*e^4*x^2 + 80*a*b^3*d*e^5*x^2*(b^2 - 4*
a*c)^(5/2) - 40*a*b^5*d*e^5*x^2*(b^2 - 4*a*c)^(3/2) - 448*b*c*d^3*e^3*x^2*(
b^2 - 4*a*c)^(7/2) - 416*b*c^3*d^5*e*x^2*(b^2 - 4*a*c)^(5/2) + 3200*a*b^4*c
^4*d^5*e*x^2 - 1472*a*b^7*c*d^2*e^4*x^2 - 1792*a^2*b^6*c*d*e^5*x^2 + 192*b^
3*c*d^3*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 160*b^3*c^3*d^5*e*x^2*(b^2 - 4*a*c)^(
3/2) - 5504*a*b^5*c^3*d^4*e^2*x^2 + 4352*a*b^6*c^2*d^3*e^3*x^2 - 14080*a^2*
b^2*c^5*d^5*e*x^2 - 42752*a^3*b*c^5*d^4*e^2*x^2 + 8320*a^3*b^4*c^2*d*e^5*x^
2 + 37120*a^4*b*c^4*d^2*e^4*x^2 - 14080*a^4*b^2*c^3*d*e^5*x^2 + 88*a*b*d*e^
5*x^2*(b^2 - 4*a*c)^(7/2) + 168*b^2*c^2*d^4*e^2*x^2*(b^2 - 4*a*c)^(5/2) - 1
00*b^4*c^2*d^4*e^2*x^2*(b^2 - 4*a*c)^(3/2))*(d*(a*c + (b*(b^2 - 4*a*c)^(1/2
))/4 - b^2/4) - (a*e*(b^2 - 4*a*c)^(1/2))/2))/(a*b^2*e^2 - 4*a*c^2*d^2 - 4*
a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d*e) - (d*log(d + e*x^2))/(2*(a*e
^2 + c*d^2 - b*d*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.221 $\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=133

$$-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}+\frac{e\log\left(d+ex^2\right)}{2\left(ae^2-bde+cd^2\right)}-\frac{e\log\left(a+bx^2+cx^4\right)}{4\left(ae^2-bde+cd^2\right)}$$

Rubi [A] time = 0.12, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = $\{1247, 705, 31, 634, 618, 206, 628\}$

$$-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}+\frac{e\log\left(d+ex^2\right)}{2\left(ae^2-bde+cd^2\right)}-\frac{e\log\left(a+bx^2+cx^4\right)}{4\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((2*c*d - b*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*Sqrt[b^2 - 4*a
*c]*(c*d^2 - b*d*e + a*e^2)) + (e*Log[d + e*x^2])/(2*(c*d^2 - b*d*e + a*e^2
)) - (e*Log[a + b*x^2 + c*x^4])/(4*(c*d^2 - b*d*e + a*e^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
```

```
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)}\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{cd-be-cex}{a+bx+cx^2}\,dx,x,x^2\right)}{2\left(cd^2-bde+ae^2\right)}+\frac{e^2\,\text{Subst}\left(\int \frac{1}{d+ex}\,dx,x,x^2\right)}{2\left(cd^2-bde+ae^2\right)}\\
&= \frac{e\log\left(d+ex^2\right)}{2\left(cd^2-bde+ae^2\right)}-\frac{e\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4\left(cd^2-bde+ae^2\right)}+\frac{(2cd-be)\,\text{Subst}\left(\int \frac{\ }{a}\right.}{4\left(cd^2-bde\right.}\\
&= \frac{e\log\left(d+ex^2\right)}{2\left(cd^2-bde+ae^2\right)}-\frac{e\log\left(a+bx^2+cx^4\right)}{4\left(cd^2-bde+ae^2\right)}-\frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\right.}{2\left(cd^2-bde+ae\right.}\\
&= -\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)}+\frac{e\log\left(d+ex^2\right)}{2\left(cd^2-bde+ae^2\right)}-\frac{e\log\left(a+bx^2+cx\right.}{4\left(cd^2-bde+ae^2\right.}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 112, normalized size = 0.84

$$\frac{(2be-4cd)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)+e\sqrt{4ac-b^2}\left(\log\left(a+bx^2+cx^4\right)-2\log\left(d+ex^2\right)\right)}{4\sqrt{4ac-b^2}\left(e(bd-ae)-cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-4*c*d + 2*b*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*
a*c]*e*(-2*Log[d + e*x^2] + Log[a + b*x^2 + c*x^4]))/(4*Sqrt[-b^2 + 4*a*c]*
(-(c*d^2) + e*(b*d - a*e)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]
```

fricas [A] time = 26.75, size = 321, normalized size = 2.41

$$\left[-\frac{\left(b^2-4ac\right)e\log\left(cx^4+bx^2+a\right)-2\left(b^2-4ac\right)e\log\left(ex^2+d\right)+\sqrt{b^2-4ac}\,(2cd-be)\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac+\left(2cx^2+b\right)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)}{4\left(\left(b^2c-4ac^2\right)d^2-\left(b^3-4abc\right)de+\left(ab^2-4a^2c\right)e^2\right)},-\frac{\left(b^2-4ac\right)e\log\left(cx^4+bx^2+a\right)-2\left(b^2-4ac\right)e\log\left(ex^2+d\right)+2\sqrt{-b^2+4ac}\,(2cd-be)\arctan\left(-\frac{\left(2cx^2+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)}{4\left(\left(b^2c-4ac^2\right)d^2-\left(b^3-4abc\right)de+\left(ab^2-4a^2c\right)e^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2 - 4*a*c)*e*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*e*log(e*x^2
 + d) + sqrt(b^2 - 4*a*c)*(2*c*d - b*e)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 -
2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^2*c - 4*
a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2), -1/4*((b^2 - 4*a
*c)*e*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*e*log(e*x^2 + d) + 2*sqrt(-b
^2 + 4*a*c)*(2*c*d - b*e)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4
*a*c)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^
2)]
```

giac [A] time = 1.91, size = 134, normalized size = 1.01

$$-\frac{e \log \left(cx^{4}+bx^{2}+a\right)}{4\left(cd^{2}-bde+ae^{2}\right)}+\frac{e^{2} \log \left(\left|x^{2}e+d\right|\right)}{2\left(cd^{2}e-bde^{2}+ae^{3}\right)}+\frac{(2\,cd-be)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\left(cd^{2}-bde+ae^{2}\right)\sqrt{-b^{2}+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*e*log(c*x^4 + b*x^2 + a)/(c*d^2 - b*d*e + a*e^2) + 1/2*e^2*log(abs(x^2
*e + d))/(c*d^2*e - b*d*e^2 + a*e^3) + 1/2*(2*c*d - b*e)*arctan((2*c*x^2 +
b)/sqrt(-b^2 + 4*a*c))/((c*d^2 - b*d*e + a*e^2)*sqrt(-b^2 + 4*a*c))
```

maple [A] time = 0.01, size = 176, normalized size = 1.32

$$-\frac{be\arctan\left(\frac{2cx^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{2\left(ae^{2}-deb+cd^{2}\right)\sqrt{4ac-b^{2}}}+\frac{cd\arctan\left(\frac{2cx^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(ae^{2}-deb+cd^{2}\right)\sqrt{4ac-b^{2}}}+\frac{e\ln\left(ex^{2}+d\right)}{2ae^{2}-2deb+2cd^{2}}-\frac{e\ln\left(cx^{4}+bx^{2}+a\right)}{4\left(ae^{2}-deb+cd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4*e*ln(c*x^4+b*x^2+a)/(a*e^2-b*d*e+c*d^2)-1/2/(a*e^2-b*d*e+c*d^2)/(4*a*c
-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*e+1/(a*e^2-b*d*e+c*d^2)
/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c*d+1/2*e*ln(e*x^2
+d)/(a*e^2-b*d*e+c*d^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 8.71, size = 2434, normalized size = 18.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (e*log(d + e*x^2))/(2*a*e^2 + 2*c*d^2 - 2*b*d*e) - (log(36*a^4*c^3*e^5 - 4*
a*b^6*e^5 - 4*b^7*e^5*x^2 + 32*a^2*b^4*c*e^5 + 36*a^2*c^5*d^4*e - 4*a*c^6*d
^5*x^2 - 4*b^6*e^5*x^2*(b^2 - 4*a*c)^(1/2) - 73*a^3*b^2*c^2*e^5 - 184*a^3*c
^4*d^2*e^3 + b^2*c^5*d^5*x^2 - 4*a*b^5*e^5*(b^2 - 4*a*c)^(1/2) + 2*a*c^5*d^
```

```
5*(b^2 - 4*a*c)^(1/2) + 16*a*b^5*c*d*e^4 - 60*a^2*c^4*d^3*e^2*(b^2 - 4*a*c)
^(1/2) + 18*a^3*c^3*e^5*x^2*(b^2 - 4*a*c)^(1/2) + 146*a^2*b^2*c^3*d^2*e^3 -
 101*a^2*b^3*c^2*e^5*x^2 + 120*a^2*c^5*d^3*e^2*x^2 + 19*b^4*c^3*d^3*e^2*x^2
 - 25*b^5*c^2*d^2*e^3*x^2 - 9*a*b^2*c^4*d^4*e + 184*a^3*b*c^3*d*e^4 + 36*a*
b^5*c*e^5*x^2 + 16*b^6*c*d*e^4*x^2 + 24*a^2*b^3*c*e^5*(b^2 - 4*a*c)^(1/2) -
 33*a^3*b*c^2*e^5*(b^2 - 4*a*c)^(1/2) + 66*a^3*c^3*d*e^4*(b^2 - 4*a*c)^(1/2
) + b*c^5*d^5*x^2*(b^2 - 4*a*c)^(1/2) + 18*a*b^3*c^3*d^3*e^2 - 25*a*b^4*c^2
*d^2*e^3 - 72*a^2*b*c^4*d^3*e^2 - 110*a^2*b^3*c^2*d*e^4 + 84*a^3*b*c^3*e^5*
x^2 - 132*a^3*c^4*d*e^4*x^2 - 7*b^3*c^4*d^4*e*x^2 + 28*a*b^4*c*e^5*x^2*(b^2
 - 4*a*c)^(1/2) + 18*a*c^5*d^4*e*x^2*(b^2 - 4*a*c)^(1/2) + 16*b^5*c*d*e^4*x
^2*(b^2 - 4*a*c)^(1/2) - 126*a*b^4*c^2*d*e^4*x^2 + 20*a*b^2*c^3*d^3*e^2*(b^
2 - 4*a*c)^(1/2) - 25*a*b^3*c^2*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 90*a^2*b*c^3*
d^2*e^3*(b^2 - 4*a*c)^(1/2) - 78*a^2*b^2*c^2*d*e^4*(b^2 - 4*a*c)^(1/2) - 7*
b^2*c^4*d^4*e*x^2*(b^2 - 4*a*c)^(1/2) - 106*a*b^2*c^4*d^3*e^2*x^2 + 168*a*b
^3*c^3*d^2*e^3*x^2 - 272*a^2*b*c^4*d^2*e^3*x^2 + 281*a^2*b^2*c^3*d*e^4*x^2
- 5*a*b*c^4*d^4*e*(b^2 - 4*a*c)^(1/2) + 16*a*b^4*c*d*e^4*(b^2 - 4*a*c)^(1/2
) - 53*a^2*b^2*c^2*e^5*x^2*(b^2 - 4*a*c)^(1/2) + 28*a*b*c^5*d^4*e*x^2 - 92*
a^2*c^4*d^2*e^3*x^2*(b^2 - 4*a*c)^(1/2) + 19*b^3*c^3*d^3*e^2*x^2*(b^2 - 4*a
*c)^(1/2) - 25*b^4*c^2*d^2*e^3*x^2*(b^2 - 4*a*c)^(1/2) + 118*a*b^2*c^3*d^2*
e^3*x^2*(b^2 - 4*a*c)^(1/2) - 66*a*b*c^4*d^3*e^2*x^2*(b^2 - 4*a*c)^(1/2) -
94*a*b^3*c^2*d*e^4*x^2*(b^2 - 4*a*c)^(1/2) + 125*a^2*b*c^3*d*e^4*x^2*(b^2 -
 4*a*c)^(1/2))*(e*((b*(b^2 - 4*a*c)^(1/2))/4 - a*c + b^2/4) - (c*d*(b^2 - 4
*a*c)^(1/2))/2))/(a*b^2*e^2 - 4*a*c^2*d^2 - 4*a^2*c*e^2 + b^2*c*d^2 - b^3*d
*e + 4*a*b*c*d*e) + (log(4*a*b^6*e^5 - 36*a^4*c^3*e^5 + 4*b^7*e^5*x^2 - 32*
a^2*b^4*c*e^5 - 36*a^2*c^5*d^4*e + 4*a*c^6*d^5*x^2 - 4*b^6*e^5*x^2*(b^2 - 4
*a*c)^(1/2) + 73*a^3*b^2*c^2*e^5 + 184*a^3*c^4*d^2*e^3 - b^2*c^5*d^5*x^2 -
4*a*b^5*e^5*(b^2 - 4*a*c)^(1/2) + 2*a*c^5*d^5*(b^2 - 4*a*c)^(1/2) - 16*a*b^
5*c*d*e^4 - 60*a^2*c^4*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 18*a^3*c^3*e^5*x^2*(b^
2 - 4*a*c)^(1/2) - 146*a^2*b^2*c^3*d^2*e^3 + 101*a^2*b^3*c^2*e^5*x^2 - 120*
a^2*c^5*d^3*e^2*x^2 - 19*b^4*c^3*d^3*e^2*x^2 + 25*b^5*c^2*d^2*e^3*x^2 + 9*a
*b^2*c^4*d^4*e - 184*a^3*b*c^3*d*e^4 - 36*a*b^5*c*e^5*x^2 - 16*b^6*c*d*e^4*
x^2 + 24*a^2*b^3*c*e^5*(b^2 - 4*a*c)^(1/2) - 33*a^3*b*c^2*e^5*(b^2 - 4*a*c)
^(1/2) + 66*a^3*c^3*d*e^4*(b^2 - 4*a*c)^(1/2) + b*c^5*d^5*x^2*(b^2 - 4*a*c)
^(1/2) - 18*a*b^3*c^3*d^3*e^2 + 25*a*b^4*c^2*d^2*e^3 + 72*a^2*b*c^4*d^3*e^2
 + 110*a^2*b^3*c^2*d*e^4 - 84*a^3*b*c^3*e^5*x^2 + 132*a^3*c^4*d*e^4*x^2 + 7
*b^3*c^4*d^4*e*x^2 + 28*a*b^4*c*e^5*x^2*(b^2 - 4*a*c)^(1/2) + 18*a*c^5*d^4*
e*x^2*(b^2 - 4*a*c)^(1/2) + 16*b^5*c*d*e^4*x^2*(b^2 - 4*a*c)^(1/2) + 126*a*
b^4*c^2*d*e^4*x^2 + 20*a*b^2*c^3*d^3*e^2*(b^2 - 4*a*c)^(1/2) - 25*a*b^3*c^2
*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 90*a^2*b*c^3*d^2*e^3*(b^2 - 4*a*c)^(1/2) - 7
8*a^2*b^2*c^2*d*e^4*(b^2 - 4*a*c)^(1/2) - 7*b^2*c^4*d^4*e*x^2*(b^2 - 4*a*c)
^(1/2) + 106*a*b^2*c^4*d^3*e^2*x^2 - 168*a*b^3*c^3*d^2*e^3*x^2 + 272*a^2*b*
c^4*d^2*e^3*x^2 - 281*a^2*b^2*c^3*d*e^4*x^2 - 5*a*b*c^4*d^4*e*(b^2 - 4*a*c)
^(1/2) + 16*a*b^4*c*d*e^4*(b^2 - 4*a*c)^(1/2) - 53*a^2*b^2*c^2*e^5*x^2*(b^2
 - 4*a*c)^(1/2) - 28*a*b*c^5*d^4*e*x^2 - 92*a^2*c^4*d^2*e^3*x^2*(b^2 - 4*a*
c)^(1/2) + 19*b^3*c^3*d^3*e^2*x^2*(b^2 - 4*a*c)^(1/2) - 25*b^4*c^2*d^2*e^3*
x^2*(b^2 - 4*a*c)^(1/2) + 118*a*b^2*c^3*d^2*e^3*x^2*(b^2 - 4*a*c)^(1/2) - 6
6*a*b*c^4*d^3*e^2*x^2*(b^2 - 4*a*c)^(1/2) - 94*a*b^3*c^2*d*e^4*x^2*(b^2 - 4
*a*c)^(1/2) + 125*a^2*b*c^3*d*e^4*x^2*(b^2 - 4*a*c)^(1/2))*(e*(a*c + (b*(b^
2 - 4*a*c)^(1/2))/4 - b^2/4) - (c*d*(b^2 - 4*a*c)^(1/2))/2))/(a*b^2*e^2 - 4
*a*c^2*d^2 - 4*a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d*e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

3.222 $\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=167

$$\frac{(2ace+b^2(-e)+bcd)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(ae^2-bde+cd^2\right)}-\frac{(cd-be)\log\left(a+bx^2+cx^4\right)}{4a\left(ae^2-bde+cd^2\right)}+\frac{\log(x)}{ad}$$

Rubi [A] time = 0.31, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1251, 893, 634, 618, 206, 628}

$$\frac{(2ace+b^2(-e)+bcd)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(ae^2-bde+cd^2\right)}-\frac{(cd-be)\log\left(a+bx^2+cx^4\right)}{4a\left(ae^2-bde+cd^2\right)}+\frac{\log(x)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*S
qrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + Log[x]/(a*d) - (e^2*Log[d + e*x
^2])/(2*d*(c*d^2 - b*d*e + a*e^2)) - ((c*d - b*e)*Log[a + b*x^2 + c*x^4])/(
4*a*(c*d^2 - b*d*e + a*e^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{adx}-\frac{e^3}{d\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{-bcd+b^2e-ace-c(\ldots}{a\left(cd^2-bde+ae^2\right)\left(a\ldots\right.}\right.\right.\\
&= \frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2-bde+ae^2\right)}+\frac{\text{Subst}\left(\int \frac{-bcd+b^2e-ace-c(cd-be)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2a\left(cd^2-bde+ae^2\right)}\\
&= \frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2-bde+ae^2\right)}-\frac{(cd-be)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a\left(cd^2-bde+ae^2\right)}-\ldots\\
&= \frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2-bde+ae^2\right)}-\frac{(cd-be)\log\left(a+bx^2+cx^4\right)}{4a\left(cd^2-bde+ae^2\right)}+\frac{\left(bcd-b^2\ldots\right.}{\ldots}\\
&= \frac{\left(bcd-b^2e+2ace\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\,\left(cd^2-bde+ae^2\right)}+\frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2-bde+ae^2\right)}-\ldots
\end{aligned}
$$

Mathematica [A] time = 0.32, size = 242, normalized size = 1.45

$$
\frac{4\log(x)\sqrt{b^2-4ac}\left(e(ae-bd)+cd^2\right)-2ae^2\sqrt{b^2-4ac}\log\left(d+ex^2\right)-d\left(cd\sqrt{b^2-4ac}-be\sqrt{b^2-4ac}+2ace+b^2(-e)+bcd\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)+d\left(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}+2ace+b^2(-e)+bcd\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{4ad\sqrt{b^2-4ac}\left(e(ae-bd)+cd^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*(c*d^2 + e*(-(b*d) + a*e))*Log[x] - d*(b*c*d + c*Sqrt[
b^2 - 4*a*c]*d - b^2*e + 2*a*c*e - b*Sqrt[b^2 - 4*a*c]*e)*Log[b - Sqrt[b^2
- 4*a*c] + 2*c*x^2] + d*(b*c*d - c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e +
b*Sqrt[b^2 - 4*a*c]*e)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2] - 2*a*Sqrt[b^2
- 4*a*c]*e^2*Log[d + e*x^2])/(4*a*Sqrt[b^2 - 4*a*c]*d*(c*d^2 + e*(-(b*d) +
a*e)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.92, size = 172, normalized size = 1.03

$$-\frac{(cd-be)\log\left(cx^4+bx^2+a\right)}{4\left(acd^2-abde+a^2e^2\right)}-\frac{e^3\log\left(\left|x^2e+d\right|\right)}{2\left(cd^3e-bd^2e^2+ade^3\right)}-\frac{\left(bcd-b^2e+2\,ace\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(acd^2-abde+a^2e^2\right)\sqrt{-b^2+4\,ac}}+\frac{\log\left(x^2\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(c*d - b*e)*log(c*x^4 + b*x^2 + a)/(a*c*d^2 - a*b*d*e + a^2*e^2) - 1/2
*e^3*log(abs(x^2*e + d))/(c*d^3*e - b*d^2*e^2 + a*d*e^3) - 1/2*(b*c*d - b^2
*e + 2*a*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a*c*d^2 - a*b*d*e
+ a^2*e^2)*sqrt(-b^2 + 4*a*c)) + 1/2*log(x^2)/(a*d)
```

maple [A] time = 0.01, size = 298, normalized size = 1.78

$$\frac{b^2e\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(ae^2-deb+cd^2\right)\sqrt{4ac-b^2}\,a}-\frac{bcd\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(ae^2-deb+cd^2\right)\sqrt{4ac-b^2}\,a}-\frac{ce\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\left(ae^2-deb+cd^2\right)\sqrt{4ac-b^2}}+\frac{be\ln\left(cx^4+bx^2+a\right)}{4\left(ae^2-deb+cd^2\right)a}-\frac{cd\ln\left(cx^4+bx^2+a\right)}{4\left(ae^2-deb+cd^2\right)a}-\frac{e^2\ln\left(ex^2+d\right)}{2\left(ae^2-deb+cd^2\right)d}+\frac{\ln(x)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/a/d*ln(x)+1/4/(a*e^2-b*d*e+c*d^2)/a*ln(c*x^4+b*x^2+a)*b*e-1/4/(a*e^2-b*d*
e+c*d^2)/a*c*ln(c*x^4+b*x^2+a)*d-1/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*ar
ctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c*e+1/2/(a*e^2-b*d*e+c*d^2)/a/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*e-1/2/(a*e^2-b*d*e+c*d^2
)/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*c*d-1/2*e^2*l
n(e*x^2+d)/d/(a*e^2-b*d*e+c*d^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 17.20, size = 6285, normalized size = 37.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(256*a^4*e^8*(4*a*c - b^2)^4 - 80*c^4*d^8*(4*a*c - b^2)^4 - 61*d^4*e^4*
(4*a*c - b^2)^6 + 160*b^3*c^4*d^8*(b^2 - 4*a*c)^(5/2) + 16*b^5*c^4*d^8*(b^2
 - 4*a*c)^(3/2) - 184*b^3*d^4*e^4*(b^2 - 4*a*c)^(9/2) + 370*b^5*d^4*e^4*(b^
2 - 4*a*c)^(7/2) + 128*b^7*d^4*e^4*(b^2 - 4*a*c)^(5/2) + 5*b^9*d^4*e^4*(b^2
 - 4*a*c)^(3/2) + 128*a^3*e^8*x^2*(b^2 - 4*a*c)^(9/2) + 160*c^5*d^8*x^2*(b^
2 - 4*a*c)^(7/2) - 256*a^4*b^2*e^8*(4*a*c - b^2)^3 + 32*b^2*c^4*d^8*(4*a*c
- b^2)^3 + 112*b^4*c^4*d^8*(4*a*c - b^2)^2 - 144*a^2*d^2*e^6*(4*a*c - b^2)^
```

5 + 544*b^2*d^4*e^4*(4*a*c - b^2)^5 + 382*b^4*d^4*e^4*(4*a*c - b^2)^4 - 152
*b^6*d^4*e^4*(4*a*c - b^2)^3 + 71*b^8*d^4*e^4*(4*a*c - b^2)^2 + 200*c^2*d^6
e^2(4*a*c - b^2)^5 - 13*d^3*e^5*x^2*(4*a*c - b^2)^6 + 512*a^4*b*e^8*(b^2
- 4*a*c)^(7/2) - 176*b*c^4*d^8*(b^2 - 4*a*c)^(7/2) - 26*a*d^3*e^5*(b^2 - 4*
a*c)^(11/2) + 352*a^3*d*e^7*(b^2 - 4*a*c)^(9/2) - 319*b*d^4*e^4*(b^2 - 4*a*
c)^(11/2) + 148*c*d^5*e^3*(b^2 - 4*a*c)^(11/2) + 168*c^3*d^7*e*(b^2 - 4*a*c
)^(9/2) - 768*a*b^3*d^3*e^5*(4*a*c - b^2)^4 - 368*a*b^5*d^3*e^5*(4*a*c - b^
2)^3 + 128*a^3*b^3*d*e^7*(4*a*c - b^2)^3 - 32*a*b^7*d^3*e^5*(4*a*c - b^2)^2
- 672*a^2*b^3*d^2*e^6*(b^2 - 4*a*c)^(7/2) - 272*a^2*b^5*d^2*e^6*(b^2 - 4*a
*c)^(5/2) + 408*b^3*c*d^5*e^3*(4*a*c - b^2)^4 + 256*b^3*c^3*d^7*e*(4*a*c -
b^2)^3 + 792*b^5*c*d^5*e^3*(4*a*c - b^2)^3 - 352*b^5*c^3*d^7*e*(4*a*c - b^2
)^2 - 248*b^7*c*d^5*e^3*(4*a*c - b^2)^2 - 328*b^3*c^2*d^6*e^2*(b^2 - 4*a*c)
^(7/2) + 1064*b^5*c^2*d^6*e^2*(b^2 - 4*a*c)^(5/2) + 40*b^7*c^2*d^6*e^2*(b^2
- 4*a*c)^(3/2) + 384*a^3*b*e^8*x^2*(4*a*c - b^2)^4 + 384*a^3*b^2*e^8*x^2*(
b^2 - 4*a*c)^(7/2) - 512*b*c^5*d^8*x^2*(4*a*c - b^2)^3 + 576*b^2*c^5*d^8*x^
2*(b^2 - 4*a*c)^(5/2) + 32*b^4*c^5*d^8*x^2*(b^2 - 4*a*c)^(3/2) - 176*a^2*d*
e^7*x^2*(4*a*c - b^2)^5 - 800*b^3*d^3*e^5*x^2*(b^2 - 4*a*c)^(9/2) + 158*b^5
*d^3*e^5*x^2*(b^2 - 4*a*c)^(7/2) + 56*b^7*d^3*e^5*x^2*(b^2 - 4*a*c)^(5/2) -
b^9*d^3*e^5*x^2*(b^2 - 4*a*c)^(3/2) - 336*c^4*d^7*e*x^2*(4*a*c - b^2)^4 +
400*c^3*d^6*e^2*x^2*(b^2 - 4*a*c)^(9/2) - 608*a^2*b^2*d^2*e^6*(4*a*c - b^2)
^4 + 560*a^2*b^4*d^2*e^6*(4*a*c - b^2)^3 - 1096*b^2*c^2*d^6*e^2*(4*a*c - b^
2)^4 - 872*b^4*c^2*d^6*e^2*(4*a*c - b^2)^3 + 424*b^6*c^2*d^6*e^2*(4*a*c - b
^2)^2 - 128*a^3*b^3*e^8*x^2*(4*a*c - b^2)^3 + 256*b^3*c^5*d^8*x^2*(4*a*c -
b^2)^2 + 584*b^2*d^3*e^5*x^2*(4*a*c - b^2)^5 - 410*b^4*d^3*e^5*x^2*(4*a*c -
b^2)^4 - 256*b^6*d^3*e^5*x^2*(4*a*c - b^2)^3 - 17*b^8*d^3*e^5*x^2*(4*a*c -
b^2)^2 + 296*c^2*d^5*e^3*x^2*(4*a*c - b^2)^5 + 336*a*b*d^3*e^5*(4*a*c - b^
2)^5 + 384*a^3*b*d*e^7*(4*a*c - b^2)^4 - 832*a*b^2*d^3*e^5*(b^2 - 4*a*c)^(9
/2) - 52*a*b^4*d^3*e^5*(b^2 - 4*a*c)^(7/2) + 144*a*b^6*d^3*e^5*(b^2 - 4*a*c
)^(5/2) - 2*a*b^8*d^3*e^5*(b^2 - 4*a*c)^(3/2) - 80*a^2*b*d^2*e^6*(b^2 - 4*a
*c)^(9/2) - 192*a^3*b^2*d*e^7*(b^2 - 4*a*c)^(7/2) + 96*a^3*b^4*d*e^7*(b^2 -
4*a*c)^(5/2) - 632*b*c*d^5*e^3*(4*a*c - b^2)^5 + 608*b*c^3*d^7*e*(4*a*c -
b^2)^4 - 776*b*c^2*d^6*e^2*(b^2 - 4*a*c)^(9/2) + 920*b^2*c*d^5*e^3*(b^2 - 4
*a*c)^(9/2) + 584*b^2*c^3*d^7*e*(b^2 - 4*a*c)^(7/2) - 384*b^4*c*d^5*e^3*(b^
2 - 4*a*c)^(7/2) - 712*b^4*c^3*d^7*e*(b^2 - 4*a*c)^(5/2) - 664*b^6*c*d^5*e^
3*(b^2 - 4*a*c)^(5/2) - 40*b^6*c^3*d^7*e*(b^2 - 4*a*c)^(3/2) - 20*b^8*c*d^5
e^3(b^2 - 4*a*c)^(3/2) + 72*a*d^2*e^6*x^2*(b^2 - 4*a*c)^(11/2) - 181*b*d^
3*e^5*x^2*(b^2 - 4*a*c)^(11/2) + 122*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(11/2) + 3
68*a^2*b*d*e^7*x^2*(b^2 - 4*a*c)^(9/2) - 1552*b*c^4*d^7*e*x^2*(b^2 - 4*a*c)
^(7/2) - 3400*b^2*c^2*d^5*e^3*x^2*(4*a*c - b^2)^4 - 4800*b^3*c^3*d^6*e^2*x^
2*(4*a*c - b^2)^3 + 3448*b^4*c^2*d^5*e^3*x^2*(4*a*c - b^2)^3 + 928*b^5*c^3*
d^6*e^2*x^2*(4*a*c - b^2)^2 - 536*b^6*c^2*d^5*e^3*x^2*(4*a*c - b^2)^2 - 32*
a*b*d^2*e^6*x^2*(4*a*c - b^2)^5 - 344*a*b^2*d^2*e^6*x^2*(b^2 - 4*a*c)^(9/2)
- 616*a*b^4*d^2*e^6*x^2*(b^2 - 4*a*c)^(7/2) - 136*a*b^6*d^2*e^6*x^2*(b^2 -
4*a*c)^(5/2) - 160*a^2*b^3*d*e^7*x^2*(b^2 - 4*a*c)^(7/2) + 48*a^2*b^5*d*e^
7*x^2*(b^2 - 4*a*c)^(5/2) - 760*b*c*d^4*e^4*x^2*(4*a*c - b^2)^5 - 1560*b*c^
2*d^5*e^3*x^2*(b^2 - 4*a*c)^(9/2) + 1848*b^2*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(9
/2) - 2208*b^3*c^4*d^7*e*x^2*(b^2 - 4*a*c)^(5/2) + 1452*b^4*c*d^4*e^4*x^2*(
b^2 - 4*a*c)^(7/2) - 80*b^5*c^4*d^7*e*x^2*(b^2 - 4*a*c)^(3/2) + 408*b^6*c*d
^4*e^4*x^2*(b^2 - 4*a*c)^(5/2) + 10*b^8*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(3/2) -
640*a*b^3*d^2*e^6*x^2*(4*a*c - b^2)^4 + 96*a^2*b^2*d*e^7*x^2*(4*a*c - b^2)
^4 + 416*a*b^5*d^2*e^6*x^2*(4*a*c - b^2)^3 + 16*a^2*b^4*d*e^7*x^2*(4*a*c -
b^2)^3 + 1952*b*c^3*d^6*e^2*x^2*(4*a*c - b^2)^4 + 2216*b^3*c*d^4*e^4*x^2*(4
*a*c - b^2)^4 + 2720*b^2*c^4*d^7*e*x^2*(4*a*c - b^2)^3 - 712*b^5*c*d^4*e^4*
x^2*(4*a*c - b^2)^3 - 784*b^4*c^4*d^7*e*x^2*(4*a*c - b^2)^2 + 152*b^7*c*d^4
*e^4*x^2*(4*a*c - b^2)^2 + 4144*b^2*c^3*d^6*e^2*x^2*(b^2 - 4*a*c)^(7/2) - 4
216*b^3*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(7/2) + 3056*b^4*c^3*d^6*e^2*x^2*(b^2
- 4*a*c)^(5/2) - 1864*b^5*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 80*b^6*c^3
*d^6*e^2*x^2*(b^2 - 4*a*c)^(3/2) - 40*b^7*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(3/
2))*(d*((b^2*c)/4 - a*c^2 + (b*c*(b^2 - 4*a*c)^(1/2))/4) - (b^3*e)/4 - (b^2

```
*e*(b^2 - 4*a*c)^(1/2))/4 + (a*c*e*(b^2 - 4*a*c)^(1/2))/2 + a*b*c*e))/(4*a^
3*c*e^2 - a^2*b^2*e^2 + 4*a^2*c^2*d^2 + a*b^3*d*e - a*b^2*c*d^2 - 4*a^2*b*c
*d*e) - (log(80*c^4*d^8*(4*a*c - b^2)^4 - 256*a^4*e^8*(4*a*c - b^2)^4 + 61*
d^4*e^4*(4*a*c - b^2)^6 + 160*b^3*c^4*d^8*(b^2 - 4*a*c)^(5/2) + 16*b^5*c^4*
d^8*(b^2 - 4*a*c)^(3/2) - 184*b^3*d^4*e^4*(b^2 - 4*a*c)^(9/2) + 370*b^5*d^4
*e^4*(b^2 - 4*a*c)^(7/2) + 128*b^7*d^4*e^4*(b^2 - 4*a*c)^(5/2) + 5*b^9*d^4*
e^4*(b^2 - 4*a*c)^(3/2) + 128*a^3*e^8*x^2*(b^2 - 4*a*c)^(9/2) + 160*c^5*d^8
*x^2*(b^2 - 4*a*c)^(7/2) + 256*a^4*b^2*e^8*(4*a*c - b^2)^3 - 32*b^2*c^4*d^8
*(4*a*c - b^2)^3 - 112*b^4*c^4*d^8*(4*a*c - b^2)^2 + 144*a^2*d^2*e^6*(4*a*c
 - b^2)^5 - 544*b^2*d^4*e^4*(4*a*c - b^2)^5 - 382*b^4*d^4*e^4*(4*a*c - b^2)
^4 + 152*b^6*d^4*e^4*(4*a*c - b^2)^3 - 71*b^8*d^4*e^4*(4*a*c - b^2)^2 - 200
*c^2*d^6*e^2*(4*a*c - b^2)^5 + 13*d^3*e^5*x^2*(4*a*c - b^2)^6 + 512*a^4*b*e
^8*(b^2 - 4*a*c)^(7/2) - 176*b*c^4*d^8*(b^2 - 4*a*c)^(7/2) - 26*a*d^3*e^5*(
b^2 - 4*a*c)^(11/2) + 352*a^3*d*e^7*(b^2 - 4*a*c)^(9/2) - 319*b*d^4*e^4*(b^
2 - 4*a*c)^(11/2) + 148*c*d^5*e^3*(b^2 - 4*a*c)^(11/2) + 168*c^3*d^7*e*(b^2
 - 4*a*c)^(9/2) + 768*a*b^3*d^3*e^5*(4*a*c - b^2)^4 + 368*a*b^5*d^3*e^5*(4*
a*c - b^2)^3 - 128*a^3*b^3*d*e^7*(4*a*c - b^2)^3 + 32*a*b^7*d^3*e^5*(4*a*c
 - b^2)^2 - 672*a^2*b^3*d^2*e^6*(b^2 - 4*a*c)^(7/2) - 272*a^2*b^5*d^2*e^6*(b
^2 - 4*a*c)^(5/2) - 408*b^3*c*d^5*e^3*(4*a*c - b^2)^4 - 256*b^3*c^3*d^7*e*(
4*a*c - b^2)^3 - 792*b^5*c*d^5*e^3*(4*a*c - b^2)^3 + 352*b^5*c^3*d^7*e*(4*a
*c - b^2)^2 + 248*b^7*c*d^5*e^3*(4*a*c - b^2)^2 - 328*b^3*c^2*d^6*e^2*(b^2
 - 4*a*c)^(7/2) + 1064*b^5*c^2*d^6*e^2*(b^2 - 4*a*c)^(5/2) + 40*b^7*c^2*d^6*
e^2*(b^2 - 4*a*c)^(3/2) - 384*a^3*b*e^8*x^2*(4*a*c - b^2)^4 + 384*a^3*b^2*e
^8*x^2*(b^2 - 4*a*c)^(7/2) + 512*b*c^5*d^8*x^2*(4*a*c - b^2)^3 + 576*b^2*c^
5*d^8*x^2*(b^2 - 4*a*c)^(5/2) + 32*b^4*c^5*d^8*x^2*(b^2 - 4*a*c)^(3/2) + 17
6*a^2*d*e^7*x^2*(4*a*c - b^2)^5 - 800*b^3*d^3*e^5*x^2*(b^2 - 4*a*c)^(9/2) +
 158*b^5*d^3*e^5*x^2*(b^2 - 4*a*c)^(7/2) + 56*b^7*d^3*e^5*x^2*(b^2 - 4*a*c)
^(5/2) - b^9*d^3*e^5*x^2*(b^2 - 4*a*c)^(3/2) + 336*c^4*d^7*e*x^2*(4*a*c - b
^2)^4 + 400*c^3*d^6*e^2*x^2*(b^2 - 4*a*c)^(9/2) + 608*a^2*b^2*d^2*e^6*(4*a*
c - b^2)^4 - 560*a^2*b^4*d^2*e^6*(4*a*c - b^2)^3 + 1096*b^2*c^2*d^6*e^2*(4*
a*c - b^2)^4 + 872*b^4*c^2*d^6*e^2*(4*a*c - b^2)^3 - 424*b^6*c^2*d^6*e^2*(4
*a*c - b^2)^2 + 128*a^3*b^3*e^8*x^2*(4*a*c - b^2)^3 - 256*b^3*c^5*d^8*x^2*(
4*a*c - b^2)^2 - 584*b^2*d^3*e^5*x^2*(4*a*c - b^2)^5 + 410*b^4*d^3*e^5*x^2*
(4*a*c - b^2)^4 + 256*b^6*d^3*e^5*x^2*(4*a*c - b^2)^3 + 17*b^8*d^3*e^5*x^2*
(4*a*c - b^2)^2 - 296*c^2*d^5*e^3*x^2*(4*a*c - b^2)^5 - 336*a*b*d^3*e^5*(4*
a*c - b^2)^5 - 384*a^3*b*d*e^7*(4*a*c - b^2)^4 - 832*a*b^2*d^3*e^5*(b^2 - 4
*a*c)^(9/2) - 52*a*b^4*d^3*e^5*(b^2 - 4*a*c)^(7/2) + 144*a*b^6*d^3*e^5*(b^2
 - 4*a*c)^(5/2) - 2*a*b^8*d^3*e^5*(b^2 - 4*a*c)^(3/2) - 80*a^2*b*d^2*e^6*(b
^2 - 4*a*c)^(9/2) - 192*a^3*b^2*d*e^7*(b^2 - 4*a*c)^(7/2) + 96*a^3*b^4*d*e^
7*(b^2 - 4*a*c)^(5/2) + 632*b*c*d^5*e^3*(4*a*c - b^2)^5 - 608*b*c^3*d^7*e*(
4*a*c - b^2)^4 - 776*b*c^2*d^6*e^2*(b^2 - 4*a*c)^(9/2) + 920*b^2*c*d^5*e^3*
(b^2 - 4*a*c)^(9/2) + 584*b^2*c^3*d^7*e*(b^2 - 4*a*c)^(7/2) - 384*b^4*c*d^5
*e^3*(b^2 - 4*a*c)^(7/2) - 712*b^4*c^3*d^7*e*(b^2 - 4*a*c)^(5/2) - 664*b^6*
c*d^5*e^3*(b^2 - 4*a*c)^(5/2) - 40*b^6*c^3*d^7*e*(b^2 - 4*a*c)^(3/2) - 20*b
^8*c*d^5*e^3*(b^2 - 4*a*c)^(3/2) + 72*a*d^2*e^6*x^2*(b^2 - 4*a*c)^(11/2) -
181*b*d^3*e^5*x^2*(b^2 - 4*a*c)^(11/2) + 122*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(1
1/2) + 368*a^2*b*d*e^7*x^2*(b^2 - 4*a*c)^(9/2) - 1552*b*c^4*d^7*e*x^2*(b^2
 - 4*a*c)^(7/2) + 3400*b^2*c^2*d^5*e^3*x^2*(4*a*c - b^2)^4 + 4800*b^3*c^3*d^
6*e^2*x^2*(4*a*c - b^2)^3 - 3448*b^4*c^2*d^5*e^3*x^2*(4*a*c - b^2)^3 - 928*
b^5*c^3*d^6*e^2*x^2*(4*a*c - b^2)^2 + 536*b^6*c^2*d^5*e^3*x^2*(4*a*c - b^2)
^2 + 32*a*b*d^2*e^6*x^2*(4*a*c - b^2)^5 - 344*a*b^2*d^2*e^6*x^2*(b^2 - 4*a*
c)^(9/2) - 616*a*b^4*d^2*e^6*x^2*(b^2 - 4*a*c)^(7/2) - 136*a*b^6*d^2*e^6*x^
2*(b^2 - 4*a*c)^(5/2) - 160*a^2*b^3*d*e^7*x^2*(b^2 - 4*a*c)^(7/2) + 48*a^2*
b^5*d*e^7*x^2*(b^2 - 4*a*c)^(5/2) + 760*b*c*d^4*e^4*x^2*(4*a*c - b^2)^5 - 1
560*b*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(9/2) + 1848*b^2*c*d^4*e^4*x^2*(b^2 - 4
*a*c)^(9/2) - 2208*b^3*c^4*d^7*e*x^2*(b^2 - 4*a*c)^(5/2) + 1452*b^4*c*d^4*e
^4*x^2*(b^2 - 4*a*c)^(7/2) - 80*b^5*c^4*d^7*e*x^2*(b^2 - 4*a*c)^(3/2) + 408
*b^6*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(5/2) + 10*b^8*c*d^4*e^4*x^2*(b^2 - 4*a*c)
^(3/2) + 640*a*b^3*d^2*e^6*x^2*(4*a*c - b^2)^4 - 96*a^2*b^2*d*e^7*x^2*(4*a*
```

```
c - b^2)^4 - 416*a*b^5*d^2*e^6*x^2*(4*a*c - b^2)^3 - 16*a^2*b^4*d*e^7*x^2*(
4*a*c - b^2)^3 - 1952*b*c^3*d^6*e^2*x^2*(4*a*c - b^2)^4 - 2216*b^3*c*d^4*e^
4*x^2*(4*a*c - b^2)^4 - 2720*b^2*c^4*d^7*e*x^2*(4*a*c - b^2)^3 + 712*b^5*c*
d^4*e^4*x^2*(4*a*c - b^2)^3 + 784*b^4*c^4*d^7*e*x^2*(4*a*c - b^2)^2 - 152*b
^7*c*d^4*e^4*x^2*(4*a*c - b^2)^2 + 4144*b^2*c^3*d^6*e^2*x^2*(b^2 - 4*a*c)^(
7/2) - 4216*b^3*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(7/2) + 3056*b^4*c^3*d^6*e^2*
x^2*(b^2 - 4*a*c)^(5/2) - 1864*b^5*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 80
*b^6*c^3*d^6*e^2*x^2*(b^2 - 4*a*c)^(3/2) - 40*b^7*c^2*d^5*e^3*x^2*(b^2 - 4*
a*c)^(3/2))*((b^3*e)/4 + d*(a*c^2 - (b^2*c)/4 + (b*c*(b^2 - 4*a*c)^(1/2))/4
) - (b^2*e*(b^2 - 4*a*c)^(1/2))/4 + (a*c*e*(b^2 - 4*a*c)^(1/2))/2 - a*b*c*e
))/(4*a^3*c*e^2 - a^2*b^2*e^2 + 4*a^2*c^2*d^2 + a*b^3*d*e - a*b^2*c*d^2 - 4
*a^2*b*c*d*e) - (e^2*log(d + e*x^2))/(2*c*d^3 + 2*a*d*e^2 - 2*b*d^2*e) + lo
g(x)/(a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.223 $\int \frac{1}{x^3(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=205

$$\frac{(ace+b^2(-e)+bcd)\log(a+bx^2+cx^4)}{4a^2(ae^2-bde+cd^2)}-\frac{(3abce-2ac^2d+b^3(-e)+b^2cd)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}-\frac{\log(x)(ae+bd)}{a^2d^2}+\ldots 2$$

Rubi [A] time = 0.47, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1251, 893, 634, 618, 206, 628}

$$-\frac{(3abce-2ac^2d+b^2cd+b^3(-e))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}+\frac{(ace+b^2(-e)+bcd)\log(a+bx^2+cx^4)}{4a^2(ae^2-bde+cd^2)}-\frac{\log(x)(ae+bd)}{a^2d^2}+\frac{e^3\log(d+ex^2)}{2d^2(ae^2-bde+cd^2)}-\frac{1}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(2*a*d*x^2) - ((b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)
) - ((b*d + a*e)*Log[x])/(a^2*d^2) + (e^3*Log[d + e*x^2])/(2*d^2*(c*d^2 - b
*d*e + a*e^2)) + ((b*c*d - b^2*e + a*c*e)*Log[a + b*x^2 + c*x^4])/(4*a^2*(c
*d^2 - b*d*e + a*e^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{adx^2} + \frac{-bd-ae}{a^2d^2x} + \frac{e^4}{d^2\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{b^2cd - \cdots}{\cdots}\right.\right.$$

$$= -\frac{1}{2adx^2} - \frac{(bd+ae)\log(x)}{a^2d^2} + \frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2-bde+ae^2\right)} + \frac{\text{Subst}\left(\int \frac{b^2cd-ac^2d-\cdots}{\cdots}\right.}{2a^2\cdots}$$

$$= -\frac{1}{2adx^2} - \frac{(bd+ae)\log(x)}{a^2d^2} + \frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2-bde+ae^2\right)} + \frac{\left(bcd-b^2e+ace\right)\text{S}\cdots}{4a^2\left(c\cdots\right.}$$

$$= -\frac{1}{2adx^2} - \frac{(bd+ae)\log(x)}{a^2d^2} + \frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2-bde+ae^2\right)} + \frac{\left(bcd-b^2e+ace\right)\text{l}\cdots}{4a^2\left(cd^2-\cdots\right.}$$

$$= -\frac{1}{2adx^2} - \frac{\left(b^2cd-2ac^2d-b^3e+3abce\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)} - \frac{(bd+ae)\log\cdots}{a^2d^2}$$

Mathematica [A] time = 0.34, size = 331, normalized size = 1.61

$$\frac{1}{4}\left(\frac{\left(b^2\left(e\sqrt{b^2-4ac}-cd\right)-bc\left(d\sqrt{b^2-4ac}+3ae\right)+ac\left(2cd-e\sqrt{b^2-4ac}\right)+b^3e\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{a^2\sqrt{b^2-4ac}\left(e(bd-ae)-cd^2\right)}+\frac{\left(b^2\left(e\sqrt{b^2-4ac}+cd\right)+bc\left(3ae-d\sqrt{b^2-4ac}\right)-ac\left(e\sqrt{b^2-4ac}+2cd\right)+b^3(-e)\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{a^2\sqrt{b^2-4ac}\left(e(bd-ae)-cd^2\right)}-\frac{4\log(x)(ae+bd)}{a^2d^2}+\frac{2e^3\log\left(d+ex^2\right)}{d^2e(ae-bd)+cd^4}-\frac{2}{adx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-2/(a*d*x^2) - (4*(b*d + a*e)*Log[x])/(a^2*d^2) + ((b^3*e - b*c*(Sqrt[b^2
- 4*a*c]*d + 3*a*e) + a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b^2*(-(c*d) + Sqr
t[b^2 - 4*a*c]*e))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(a^2*Sqrt[b^2 - 4*
a*c]*(-(c*d^2) + e*(b*d - a*e))) + ((-(b^3*e) + b*c*(-(Sqrt[b^2 - 4*a*c]*d)
 + 3*a*e) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e) - a*c*(2*c*d + Sqrt[b^2 - 4*a*c
]*e))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(a^2*Sqrt[b^2 - 4*a*c]*(-(c*d^2
) + e*(b*d - a*e))) + (2*e^3*Log[d + e*x^2])/(c*d^4 + d^2*e*(-(b*d) + a*e))
)/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.96, size = 237, normalized size = 1.16

$$\frac{\left(bcd-b^{2}e+ace\right)\log\left(cx^{4}+bx^{2}+a\right)}{4\left(a^{2}cd^{2}-a^{2}bde+a^{3}e^{2}\right)}+\frac{e^{4}\log\left(\left|x^{2}e+d\right|\right)}{2\left(cd^{4}e-bd^{3}e^{2}+ad^{2}e^{3}\right)}+\frac{\left(b^{2}cd-2\,ac^{2}d-b^{3}e+3\,abce\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\left(a^{2}cd^{2}-a^{2}bde+a^{3}e^{2}\right)\sqrt{-b^{2}+4\,ac}}-\frac{(bd+ae)\log\left(x^{2}\right)}{2\,a^{2}d^{2}}+\frac{bdx^{2}+ax^{2}e-ad}{2\,a^{2}d^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(b*c*d - b^2*e + a*c*e)*log(c*x^4 + b*x^2 + a)/(a^2*c*d^2 - a^2*b*d*e +
 a^3*e^2) + 1/2*e^4*log(abs(x^2*e + d))/(c*d^4*e - b*d^3*e^2 + a*d^2*e^3) +
 1/2*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*arctan((2*c*x^2 + b)/sqrt(-b
^2 + 4*a*c))/((a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*sqrt(-b^2 + 4*a*c)) - 1/2*(
b*d + a*e)*log(x^2)/(a^2*d^2) + 1/2*(b*d*x^2 + a*x^2*e - a*d)/(a^2*d^2*x^2)
```

maple [B] time = 0.02, size = 430, normalized size = 2.10

$$\frac{3bce\arctan\left(\frac{2cx^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{2\left(ae^{2}-deb+cd^{2}\right)\sqrt{4ac-b^{2}}\,a}-\frac{c^{2}d\arctan\left(\frac{2cx^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(ae^{2}-deb+cd^{2}\right)\sqrt{4ac-b^{2}}\,a}-\frac{b^{3}e\arctan\left(\frac{2cx^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{2\left(ae^{2}-deb+cd^{2}\right)\sqrt{4ac-b^{2}}\,a^{2}}+\frac{b^{2}cd\arctan\left(\frac{2cx^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{2\left(ae^{2}-deb+cd^{2}\right)\sqrt{4ac-b^{2}}\,a^{2}}+\frac{ce\ln\left(cx^{4}+bx^{2}+a\right)}{4\left(ae^{2}-deb+cd^{2}\right)a}-\frac{b^{2}e\ln\left(cx^{4}+bx^{2}+a\right)}{4\left(ae^{2}-deb+cd^{2}\right)a^{2}}+\frac{bcd\ln\left(cx^{4}+bx^{2}+a\right)}{4\left(ae^{2}-deb+cd^{2}\right)a^{2}}+\frac{e^{3}\ln\left(ex^{2}+d\right)}{2\left(ae^{2}-deb+cd^{2}\right)d^{2}}-\frac{e\ln(x)}{ad^{2}}-\frac{b\ln(x)}{a^{2}d}-\frac{1}{2adx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/a/d/x^2-1/a/d^2*e*ln(x)-1/d/a^2*ln(x)*b+1/4/(a*e^2-b*d*e+c*d^2)/a*c*ln
(c*x^4+b*x^2+a)*e-1/4/(a*e^2-b*d*e+c*d^2)/a^2*ln(c*x^4+b*x^2+a)*b^2*e+1/4/(
a*e^2-b*d*e+c*d^2)/a^2*c*ln(c*x^4+b*x^2+a)*b*d+3/2/(a*e^2-b*d*e+c*d^2)/a/(4
*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*c*e-1/(a*e^2-b*d*e+
c*d^2)/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c^2*d-1/2/
(a*e^2-b*d*e+c*d^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1
/2))*b^3*e+1/2/(a*e^2-b*d*e+c*d^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)
/(4*a*c-b^2)^(1/2))*b^2*c*d+1/2*e^3*ln(e*x^2+d)/d^2/(a*e^2-b*d*e+c*d^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 62.95, size = 5368, normalized size = 26.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log((((((4*c^2*e^2*(a*c^6*d^7 - 4*a^2*b^5*e^7 - 4*b^2*c^5*d^7 - 4*b^7*d^2*
e^5 + 28*a^3*b^3*c*e^7 - 48*a^4*b*c^2*e^7 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e
```

^4 - 16*a^2*c^5*d^5*e^2 + 16*a^3*c^4*d^3*e^4 - 4*b^4*c^3*d^5*e^2 - 4*b^5*c^2*d^4*e^3 - 7*a*b^6*d*e^6 - 20*a*b*c^5*d^6*e + 56*a^2*b^2*c^3*d^3*e^4 - 76*a^2*b^3*c^2*d^2*e^5 + 32*a*b^5*c*d^2*e^5 + 46*a^2*b^4*c*d*e^6 + 20*a*b^2*c^4*d^5*e^2 + 6*a*b^3*c^3*d^4*e^3 - 44*a*b^4*c^2*d^3*e^4 + 22*a^2*b*c^4*d^4*e^3 + 48*a^3*b*c^3*d^2*e^5 - 75*a^3*b^2*c^2*d*e^6))/(a^2*d^2) + (((16*c^2*e^2*(a^3*b^4*e^7 + 16*a^5*c^2*e^7 + b^3*c^4*d^7 + b^7*d^3*e^4 - 8*a^4*b^2*c*e^7 + 2*a*b^6*d^2*e^5 + 2*a^2*b^5*d*e^6 - 4*a^2*c^5*d^6*e - 4*b^4*c^3*d^6*e - 4*b^6*c*d^4*e^3 + 20*a^3*c^4*d^4*e^3 - 32*a^4*c^3*d^2*e^5 + 6*b^5*c^2*d^5*e^2 - a*b*c^5*d^7 - 52*a^2*b^2*c^3*d^4*e^3 + 45*a^2*b^3*c^2*d^3*e^4 + 48*a^3*b^2*c^2*d^2*e^5 + 11*a*b^2*c^4*d^6*e - 12*a*b^5*c*d^3*e^4 - 15*a^3*b^3*c*d*e^6 + 28*a^4*b*c^2*d*e^6 - 27*a*b^3*c^3*d^5*e^2 + 27*a*b^4*c^2*d^4*e^3 + 27*a^2*b*c^4*d^5*e^2 - 18*a^2*b^4*c*d^2*e^5 - 52*a^3*b*c^3*d^3*e^4))/(a*d) + (8*c^2*e^2*x^2*(10*a*c^6*d^7 + a^2*b^5*e^7 + b^2*c^5*d^7 + b^7*d^2*e^5 - 11*a^3*b^3*c*e^7 + 28*a^4*b*c^2*e^7 - 88*a^4*c^3*d*e^6 - 6*b^3*c^4*d^6*e - 6*b^6*c*d^3*e^4 + 26*a^2*c^5*d^5*e^2 + 88*a^3*c^4*d^3*e^4 + 5*b^4*c^3*d^5*e^2 + 5*b^5*c^2*d^4*e^3 + 12*a*b^6*d*e^6 - 3*a*b*c^5*d^6*e - 110*a^2*b^2*c^3*d^3*e^4 + 155*a^2*b^3*c^2*d^2*e^5 - 28*a*b^5*c*d^2*e^5 - 93*a^2*b^4*c*d*e^6 - 10*a*b^2*c^4*d^5*e^2 - 27*a*b^3*c^3*d^4*e^3 + 46*a*b^4*c^2*d^3*e^4 + 15*a^2*b*c^4*d^4*e^3 - 236*a^3*b*c^3*d^2*e^5 + 202*a^3*b^2*c^2*d*e^6))/(a*d) + (4*c^2*e^2*(a*b^2*e^3 + b*c^2*d^3 - 4*a^2*c*e^3 + b^3*d*e^2 + 4*a*c^2*d^2*e - 2*b^2*c*d^2*e - 3*a*b*c*d*e^2)*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) - b^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2))*(a*b^3*d^2*e^2 + a^2*b^2*d*e^3 + 4*a^2*c^2*d^3*e - 10*a*c^3*d^4*x^2 - 12*a^3*c*e^4*x^2 + 3*a^2*b^2*e^4*x^2 + 3*b^2*c^2*d^4*x^2 + 3*b^4*d^2*e^2*x^2 + a*b*c^2*d^4 - 4*a^3*c*d*e^3 - 2*a*b^2*c*d^3*e - 14*a^2*c^2*d^2*e^2*x^2 - 3*a^2*b*c*d^2*e^2 - 4*a*b^3*d*e^3*x^2 - 6*b^3*c*d^3*e*x^2 - 8*a*b^2*c*d^2*e^2*x^2 + 22*a*b*c^2*d^3*e*x^2 + 16*a^2*b*c*d*e^3*x^2))/(a^2*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) - b^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) - (4*c^2*e^2*x^2*(6*a*b^6*e^7 + 6*b*c^6*d^7 + 6*b^7*d*e^6 - 16*a^4*c^3*e^7 - 44*a^2*b^4*c*e^7 - 8*b^2*c^5*d^6*e - 8*b^6*c*d^2*e^5 + 84*a^3*b^2*c^2*e^7 + 30*a^2*c^5*d^4*e^3 - 2*b^3*c^4*d^5*e^2 + 8*b^4*c^3*d^4*e^3 - 2*b^5*c^2*d^3*e^4 + 11*a*c^6*d^6*e - 47*a*b^5*c*d*e^6 - 96*a^2*b^2*c^3*d^2*e^5 + 14*a*b*c^5*d^5*e^2 - 94*a^3*b*c^3*d*e^6 - 35*a*b^2*c^4*d^4*e^3 + 7*a*b^3*c^3*d^3*e^4 + 56*a*b^4*c^2*d^2*e^5 - 17*a^2*b*c^4*d^3*e^4 + 117*a^2*b^3*c^2*d*e^6))/(a^2*d^2))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) - b^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*c^2*e^2*x^2*(b^7*e^7 + c^7*d^7 - 6*a^3*b*c^3*e^7 + 2*a*c^6*d^5*e^2 - 4*a^3*c^4*d*e^6 + 14*a^2*b^3*c^2*e^7 + 6*a^2*c^5*d^3*e^4 + b^3*c^4*d^4*e^3 + b^4*c^3*d^3*e^4 - 7*a*b^5*c*e^7 + 2*a*b^4*c^2*d*e^6 - 6*a*b^2*c^4*d^3*e^4 + 3*a*b^3*c^3*d^2*e^5 - 9*a^2*b*c^4*d^2*e^5 - 5*a^2*b^2*c^3*d*e^6))/(a^3*d^3) + (4*c^2*e^2*(a*e + b*d)*(b^3*e^3 + c^3*d^3 - 3*a*b*c*e^3)^2)/(a^3*d^3))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) - b^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) - (2*c^5*e^5*x^2*(b^3*e^3 + c^3*d^3 - 3*a*b*c*e^3))/(a^3*d^3))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) - b^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*(4*a^4*c*e^2 - a^3*b^2*e^2 + 4*a^3*c^2*d^2 - a^2*b^2*c*d^2 + a^2*b^3*d*e - 4*a^3*b*c*d*e)) + (log((((((4*c^2*e^2*(a*c^6*d^7 - 4*a^2*b^5*e^7 - 4*b^2*c^5*d^7 - 4*b^7*d^2*e^5 + 28*a^3*b^3*c*e^7 - 48*a^4*b*c^2*e^7 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 16*a^2*c^5*d^5*e^2 + 16*a^3*c^4*d^3*e^4 - 4*b^4*c^3*d^5*e^2 - 4*b^5*c^2*d^4*e^3 - 7*a*b^6*d*e^6 - 20*a*b*c^5*d^6*e + 56*a^2*b^2*c^3*d^3*e^4 - 76*a^2*b^3*c^2*d^2*e^5 + 32*a*b^5*c*d^2*e^5 + 46*a^2*b^4*c*d*e^6 + 20*a*b^2*c^4*d^5*e^2 + 6*a*b^3*c^3*d^4*e^3 - 44*a*b^4*c^2*d^3*e^4 + 22*a^2*b*

```
c^4*d^4*e^3 + 48*a^3*b*c^3*d^2*e^5 - 75*a^3*b^2*c^2*d*e^6))/(a^2*d^2) + (((
16*c^2*e^2*(a^3*b^4*e^7 + 16*a^5*c^2*e^7 + b^3*c^4*d^7 + b^7*d^3*e^4 - 8*a^
4*b^2*c*e^7 + 2*a*b^6*d^2*e^5 + 2*a^2*b^5*d*e^6 - 4*a^2*c^5*d^6*e - 4*b^4*c
^3*d^6*e - 4*b^6*c*d^4*e^3 + 20*a^3*c^4*d^4*e^3 - 32*a^4*c^3*d^2*e^5 + 6*b^
5*c^2*d^5*e^2 - a*b*c^5*d^7 - 52*a^2*b^2*c^3*d^4*e^3 + 45*a^2*b^3*c^2*d^3*e
^4 + 48*a^3*b^2*c^2*d^2*e^5 + 11*a*b^2*c^4*d^6*e - 12*a*b^5*c*d^3*e^4 - 15*
a^3*b^3*c*d*e^6 + 28*a^4*b*c^2*d*e^6 - 27*a*b^3*c^3*d^5*e^2 + 27*a*b^4*c^2*
d^4*e^3 + 27*a^2*b*c^4*d^5*e^2 - 18*a^2*b^4*c*d^2*e^5 - 52*a^3*b*c^3*d^3*e^
4))/(a*d) + (8*c^2*e^2*x^2*(10*a*c^6*d^7 + a^2*b^5*e^7 + b^2*c^5*d^7 + b^7*
d^2*e^5 - 11*a^3*b^3*c*e^7 + 28*a^4*b*c^2*e^7 - 88*a^4*c^3*d*e^6 - 6*b^3*c^
4*d^6*e - 6*b^6*c*d^3*e^4 + 26*a^2*c^5*d^5*e^2 + 88*a^3*c^4*d^3*e^4 + 5*b^4
*c^3*d^5*e^2 + 5*b^5*c^2*d^4*e^3 + 12*a*b^6*d*e^6 - 3*a*b*c^5*d^6*e - 110*a
^2*b^2*c^3*d^3*e^4 + 155*a^2*b^3*c^2*d^2*e^5 - 28*a*b^5*c*d^2*e^5 - 93*a^2*
b^4*c*d*e^6 - 10*a*b^2*c^4*d^5*e^2 - 27*a*b^3*c^3*d^4*e^3 + 46*a*b^4*c^2*d^
3*e^4 + 15*a^2*b*c^4*d^4*e^3 - 236*a^3*b*c^3*d^2*e^5 + 202*a^3*b^2*c^2*d*e^
6))/(a*d) + (4*c^2*e^2*(a*b^2*e^3 + b*c^2*d^3 - 4*a^2*c*e^3 + b^3*d*e^2 + 4
*a*c^2*d^2*e - 2*b^2*c*d^2*e - 3*a*b*c*d*e^2)*(b^4*e - b^3*e*(b^2 - 4*a*c)^
(1/2) + 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e - 2*a*c^2*d*(b^2
- 4*a*c)^(1/2) + b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/2
))*(a*b^3*d^2*e^2 + a^2*b^2*d*e^3 + 4*a^2*c^2*d^3*e - 10*a*c^3*d^4*x^2 - 12
*a^3*c*e^4*x^2 + 3*a^2*b^2*e^4*x^2 + 3*b^2*c^2*d^4*x^2 + 3*b^4*d^2*e^2*x^2
+ a*b*c^2*d^4 - 4*a^3*c*d*e^3 - 2*a*b^2*c*d^3*e - 14*a^2*c^2*d^2*e^2*x^2 -
3*a^2*b*c*d^2*e^2 - 4*a*b^3*d*e^3*x^2 - 6*b^3*c*d^3*e*x^2 - 8*a*b^2*c*d^2*e
^2*x^2 + 22*a*b*c^2*d^3*e*x^2 + 16*a^2*b*c*d*e^3*x^2))/(a^2*(4*a*c - b^2)*(
a*e^2 + c*d^2 - b*d*e)))*(b^4*e - b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e -
 b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e - 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) + b^2*
c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4*a*c - b
^2)*(a*e^2 + c*d^2 - b*d*e)) - (4*c^2*e^2*x^2*(6*a*b^6*e^7 + 6*b*c^6*d^7 +
6*b^7*d*e^6 - 16*a^4*c^3*e^7 - 44*a^2*b^4*c*e^7 - 8*b^2*c^5*d^6*e - 8*b^6*c
*d^2*e^5 + 84*a^3*b^2*c^2*e^7 + 30*a^2*c^5*d^4*e^3 - 2*b^3*c^4*d^5*e^2 + 8*
b^4*c^3*d^4*e^3 - 2*b^5*c^2*d^3*e^4 + 11*a*c^6*d^6*e - 47*a*b^5*c*d*e^6 - 9
6*a^2*b^2*c^3*d^2*e^5 + 14*a*b*c^5*d^5*e^2 - 94*a^3*b*c^3*d*e^6 - 35*a*b^2*
c^4*d^4*e^3 + 7*a*b^3*c^3*d^3*e^4 + 56*a*b^4*c^2*d^2*e^5 - 17*a^2*b*c^4*d^3
*e^4 + 117*a^2*b^3*c^2*d*e^6))/(a^2*d^2))*(b^4*e - b^3*e*(b^2 - 4*a*c)^(1/2
) + 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e - 2*a*c^2*d*(b^2 - 4*
a*c)^(1/2) + b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/
(4*a^2*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*c^2*e^2*x^2*(b^7*e^7 + c
^7*d^7 - 6*a^3*b*c^3*e^7 + 2*a*c^6*d^5*e^2 - 4*a^3*c^4*d*e^6 + 14*a^2*b^3*c
^2*e^7 + 6*a^2*c^5*d^3*e^4 + b^3*c^4*d^4*e^3 + b^4*c^3*d^3*e^4 - 7*a*b^5*c*
e^7 + 2*a*b^4*c^2*d*e^6 - 6*a*b^2*c^4*d^3*e^4 + 3*a*b^3*c^3*d^2*e^5 - 9*a^2
*b*c^4*d^2*e^5 - 5*a^2*b^2*c^3*d*e^6))/(a^3*d^3) + (4*c^2*e^2*(a*e + b*d)*(
b^3*e^3 + c^3*d^3 - 3*a*b*c*e^3)^2)/(a^3*d^3))*(b^4*e - b^3*e*(b^2 - 4*a*c)
^(1/2) + 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e - 2*a*c^2*d*(b^2
 - 4*a*c)^(1/2) + b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/
2)))/(4*a^2*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) - (2*c^5*e^5*x^2*(b^3*e^
3 + c^3*d^3 - 3*a*b*c*e^3))/(a^3*d^3))*(b^4*e - b^3*e*(b^2 - 4*a*c)^(1/2) +
 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e - 2*a*c^2*d*(b^2 - 4*a*c
)^(1/2) + b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*
(4*a^4*c*e^2 - a^3*b^2*e^2 + 4*a^3*c^2*d^2 - a^2*b^2*c*d^2 + a^2*b^3*d*e -
4*a^3*b*c*d*e)) + (e^3*log(d + e*x^2))/(2*c*d^4 + 2*a*d^2*e^2 - 2*b*d^3*e)
- 1/(2*a*d*x^2) - (log(x)*(a*e + b*d))/(a^2*d^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.224 $\int \frac{1}{x^5(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=268

$$\frac{\log(x)\left(abde-a\left(cd^2-ae^2\right)+b^2d^2\right)}{a^3d^3}-\frac{\left(2abce-ac^2d+b^3(-e)+b^2cd\right)\log\left(a+bx^2+cx^4\right)}{4a^3\left(ae^2-bde+cd^2\right)}+\frac{ae+bd}{2a^2d^2x^2}+\frac{\left(-2a^2c^2e\right.}{}$$

Rubi [A] time = 0.60, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1251, 893, 634, 618, 206, 628}

$$-\frac{\left(2abce-ac^2d+b^2cd+b^3(-e)\right)\log\left(a+bx^2+cx^4\right)}{4a^3\left(ae^2-bde+cd^2\right)}+\frac{\left(-2a^2c^2e+4ab^2ce-3abc^2d+b^3cd+b^4(-e)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}+\frac{\log(x)\left(abde-a\left(cd^2-ae^2\right)+b^2d^2\right)}{a^3d^3}+\frac{ae+bd}{2a^2d^2x^2}-\frac{e^4\log\left(d+ex^2\right)}{2d^3\left(ae^2-bde+cd^2\right)}-\frac{1}{4adx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(4*a*d*x^4) + (b*d + a*e)/(2*a^2*d^2*x^2) + ((b^3*c*d - 3*a*b*c^2*d - b^
4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(2*a^3*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d^2 + a*b*d*e - a
*(c*d^2 - a*e^2))*Log[x])/(a^3*d^3) - (e^4*Log[d + e*x^2])/(2*d^3*(c*d^2 -
b*d*e + a*e^2)) - ((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*Log[a + b*x^2 +
c*x^4])/(4*a^3*(c*d^2 - b*d*e + a*e^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^5\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^3(d+ex)\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{adx^3} + \frac{-bd-ae}{a^2d^2x^2} + \frac{b^2d^2+abde-a\left(cd^2-ae^2\right)}{a^3d^3x} - \frac{\cdots}{d^3\left(cd^2-bd\cdots\right.}\right.\right.$$

$$= -\frac{1}{4adx^4} + \frac{bd+ae}{2a^2d^2x^2} + \frac{\left(b^2d^2+abde-a\left(cd^2-ae^2\right)\right)\log(x)}{a^3d^3} - \frac{e^4\log\left(d+\cdots\right.}{2d^3\left(cd^2-bde\cdots\right.}$$

$$= -\frac{1}{4adx^4} + \frac{bd+ae}{2a^2d^2x^2} + \frac{\left(b^2d^2+abde-a\left(cd^2-ae^2\right)\right)\log(x)}{a^3d^3} - \frac{e^4\log\left(d+\cdots\right.}{2d^3\left(cd^2-bde\cdots\right.}$$

$$= -\frac{1}{4adx^4} + \frac{bd+ae}{2a^2d^2x^2} + \frac{\left(b^2d^2+abde-a\left(cd^2-ae^2\right)\right)\log(x)}{a^3d^3} - \frac{e^4\log\left(d+\cdots\right.}{2d^3\left(cd^2-bde\cdots\right.}$$

$$= -\frac{1}{4adx^4} + \frac{bd+ae}{2a^2d^2x^2} + \frac{\left(b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e\right)\tanh^{-1}\left(\frac{b+2\cdots}{\sqrt{b^2-\cdots}}\right.}{2a^3\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)}$$

Mathematica [A] time = 0.43, size = 426, normalized size = 1.59

$$\frac{1}{4}\left(\frac{4\log(x)\left(abde+a\left(ae^2-cd^2\right)+b^2d^2\right)}{a^3d^3} - \frac{\left(ac^2\left(d\sqrt{b^2-4ac}+2ae\right)-b^2c\left(d\sqrt{b^2-4ac}+4ae\right)+abc\left(3cd-2e\sqrt{b^2-4ac}\right)+b^3\left(e\sqrt{b^2-4ac}-cd\right)+b^4e\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{a^3\sqrt{b^2-4ac}\left(e(bd-ae)-cd^2\right)} - \frac{\left(ac^2\left(d\sqrt{b^2-4ac}-2ae\right)+b^2c\left(4ae-d\sqrt{b^2-4ac}\right)-abc\left(2e\sqrt{b^2-4ac}+3cd\right)+b^3\left(e\sqrt{b^2-4ac}+cd\right)+b^4(-e)\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{a^3\sqrt{b^2-4ac}\left(e(bd-ae)-cd^2\right)} + \frac{2(ae+bd)}{a^2d^2x^2} - \frac{2e^4\log\left(d+ex^2\right)}{d^3e(ae-bd)+cd^5} - \frac{1}{adx^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-(1/(a*d*x^4)) + (2*(b*d + a*e))/(a^2*d^2*x^2) + (4*(b^2*d^2 + a*b*d*e + a
*(-(c*d^2) + a*e^2))*Log[x])/(a^3*d^3) - ((b^4*e + a*c^2*(Sqrt[b^2 - 4*a*c]
*d + 2*a*e) - b^2*c*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + a*b*c*(3*c*d - 2*Sqrt[b
^2 - 4*a*c]*e) + b^3*(-(c*d) + Sqrt[b^2 - 4*a*c]*e))*Log[b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2])/(a^3*Sqrt[b^2 - 4*a*c]*(-(c*d^2) + e*(b*d - a*e))) - ((-(b^
4*e) + a*c^2*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*c*(-(Sqrt[b^2 - 4*a*c]*d)
+ 4*a*e) + b^3*(c*d + Sqrt[b^2 - 4*a*c]*e) - a*b*c*(3*c*d + 2*Sqrt[b^2 - 4*
a*c]*e))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(a^3*Sqrt[b^2 - 4*a*c]*(-(c*
d^2) + e*(b*d - a*e))) - (2*e^4*Log[d + e*x^2])/(c*d^5 + d^3*e*(-(b*d) + a*
e)))/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)), x]
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**A**] time = 1.46, size = 332, normalized size = 1.24

$$-\frac{\left(b^2cd - ac^2d - b^3e + 2\,abce\right)\log\left(cx^4 + bx^2 + a\right)}{4\left(a^3cd^2 - a^3bde + a^4e^2\right)} - \frac{e^5\log\left(\left|x^2e + d\right|\right)}{2\left(cd^5e - bd^4e^2 + ad^3e^3\right)} - \frac{\left(b^3cd - 3\,abc^2d - b^4e + 4\,ab^2ce - 2\,a^2c^2e\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^3cd^2 - a^3bde + a^4e^2\right)\sqrt{-b^2 + 4\,ac}} + \frac{\left(b^2d^2 - acd^2 + abde + a^2e^2\right)\log\left(x^2\right)}{2\,a^3d^3} - \frac{3\,b^2d^2x^4 - 3\,acd^2x^4 + 3\,abdx^4e + 3\,a^2x^4e^2 - 2\,abd^2x^2 - 2\,a^2dx^2e + a^2d^2}{4\,a^3d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*log(c*x^4 + b*x^2 + a)/(a^3*c*
d^2 - a^3*b*d*e + a^4*e^2) - 1/2*e^5*log(abs(x^2*e + d))/(c*d^5*e - b*d^4*e
^2 + a*d^3*e^3) - 1/2*(b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*
c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^3*c*d^2 - a^3*b*d*e + a
^4*e^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(b^2*d^2 - a*c*d^2 + a*b*d*e + a^2*e^2)*l
og(x^2)/(a^3*d^3) - 1/4*(3*b^2*d^2*x^4 - 3*a*c*d^2*x^4 + 3*a*b*d*x^4*e + 3*
a^2*x^4*e^2 - 2*a*b*d^2*x^2 - 2*a^2*d*x^2*e + a^2*d^2)/(a^3*d^3*x^4)
```

maple [**B**] time = 0.02, size = 584, normalized size = 2.18

$$\frac{c^2e\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,e^2 - deb + c\,d^2\right)\sqrt{4ac-b^2}\;a} - \frac{2b^2ce\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,e^2 - deb + c\,d^2\right)\sqrt{4ac-b^2}\;a^2} + \frac{3bc^2d\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(a\,e^2 - deb + c\,d^2\right)\sqrt{4ac-b^2}\;a^2} + \frac{b^4e\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(a\,e^2 - deb + c\,d^2\right)\sqrt{4ac-b^2}\;a^3} - \frac{b^3cd\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(a\,e^2 - deb + c\,d^2\right)\sqrt{4ac-b^2}\;a^3} - \frac{bce\ln\left(c\,x^4 + b\,x^2 + a\right)}{2\left(a\,e^2 - deb + c\,d^2\right)a^2} + \frac{c^2d\ln\left(c\,x^4 + b\,x^2 + a\right)}{4\left(a\,e^2 - deb + c\,d^2\right)a^2} + \frac{b^3e\ln\left(c\,x^4 + b\,x^2 + a\right)}{4\left(a\,e^2 - deb + c\,d^2\right)a^3} - \frac{b^2cd\ln\left(c\,x^4 + b\,x^2 + a\right)}{4\left(a\,e^2 - deb + c\,d^2\right)a^3} - \frac{e^4\ln\left(e\,x^2 + d\right)}{2\left(a\,e^2 - deb + c\,d^2\right)d^3} + \frac{e^2\ln(x)}{a\,d^3} + \frac{be\ln(x)}{a^2d^2} - \frac{c\ln(x)}{a^2d} + \frac{b^2\ln(x)}{a^3d} + \frac{e}{2a\,d^2x^2} + \frac{b}{2a^2d\,x^2} - \frac{1}{4ad\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4/a/d/x^4+1/2/a/d^2*e/x^2+1/2/d/a^2/x^2*b+1/a/d^3*e^2*ln(x)+1/d^2/a^2*ln
(x)*b*e-1/a^2*c/d*ln(x)+1/d/a^3*ln(x)*b^2-1/2/(a*e^2-b*d*e+c*d^2)/a^2*c*ln(
c*x^4+b*x^2+a)*b*e+1/4/(a*e^2-b*d*e+c*d^2)/a^2*c^2*ln(c*x^4+b*x^2+a)*d+1/4/
(a*e^2-b*d*e+c*d^2)/a^3*ln(c*x^4+b*x^2+a)*b^3*e-1/4/(a*e^2-b*d*e+c*d^2)/a^3
*c*ln(c*x^4+b*x^2+a)*b^2*d+1/(a*e^2-b*d*e+c*d^2)/a/(4*a*c-b^2)^(1/2)*arctan
((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*e*c^2-2/(a*e^2-b*d*e+c*d^2)/a^2/(4*a*c-b^2)
^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*c*e+3/2/(a*e^2-b*d*e+c*d^2
)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*d*b*c^2+1/2/(
a*e^2-b*d*e+c*d^2)/a^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/
2))*b^4*e-1/2/(a*e^2-b*d*e+c*d^2)/a^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/
(4*a*c-b^2)^(1/2))*b^3*c*d-1/2*e^4*ln(e*x^2+d)/d^3/(a*e^2-b*d*e+c*d^2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [**B**] time = 144.76, size = 10300, normalized size = 38.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)

[Out] (log((c^8*e^8*(a^2*e^2 + b^2*d^2 - a*c*d^2 + a*b*d*e))/(a^6*d^6) - (c^9*e^9
*x^2)/(a^5*d^5) - (((c^5*e^5*(4*a^3*b^3*e^6 + 4*b^3*c^3*d^6 + 4*b^6*d^3*e^3
+ 8*a*b^5*d^2*e^4 + 8*a^2*b^4*d*e^5 + 4*a^2*c^4*d^5*e + 16*a^4*c^2*d*e^5 -
19*a^3*c^3*d^3*e^3 - 4*a*b*c^4*d^6 - 12*a^4*b*c*e^6 + 36*a^2*b^2*c^2*d^3*e
^3 - 24*a*b^4*c*d^3*e^3 - 32*a^3*b^2*c*d*e^5 - 36*a^2*b^3*c*d^2*e^4 + 28*a^
3*b*c^2*d^2*e^4))/(a^6*d^6) - (((4*a^4*b^6*c^2*e^12 - 24*a^5*b^4*c^3*e^12 +
36*a^6*b^2*c^4*e^12 - 4*a^3*c^9*d^8*e^4 + 64*a^4*c^8*d^6*e^6 - 144*a^5*c^7
*d^4*e^8 + 96*a^6*c^6*d^2*e^10 + 4*b^4*c^8*d^10*e^2 + 8*b^7*c^5*d^7*e^5 + 4
*b^10*c^2*d^4*e^8 + 64*a^2*b^3*c^7*d^7*e^5 - 8*a^2*b^4*c^6*d^6*e^6 - 8*a^2*
b^5*c^5*d^5*e^7 + 172*a^2*b^6*c^4*d^4*e^8 - 112*a^2*b^7*c^3*d^3*e^9 + 16*a^
2*b^8*c^2*d^2*e^10 - 72*a^3*b^2*c^7*d^6*e^6 + 56*a^3*b^3*c^6*d^5*e^7 - 312*
a^3*b^4*c^5*d^4*e^8 + 348*a^3*b^5*c^4*d^3*e^9 - 132*a^3*b^6*c^3*d^2*e^10 +
324*a^4*b^2*c^6*d^4*e^8 - 428*a^4*b^3*c^5*d^3*e^9 + 344*a^4*b^4*c^4*d^2*e^1
0 - 300*a^5*b^2*c^5*d^2*e^10 - 96*a^6*b*c^5*d*e^11 - 4*a*b^2*c^9*d^10*e^2 -
4*a*b^3*c^8*d^9*e^3 - 48*a*b^5*c^6*d^7*e^5 + 8*a*b^6*c^5*d^6*e^6 - 44*a*b^
8*c^3*d^4*e^8 + 12*a*b^9*c^2*d^3*e^9 + 8*a^2*b*c^9*d^9*e^3 - 24*a^3*b*c^8*d
^7*e^5 + 12*a^3*b^7*c^2*d*e^11 - 88*a^4*b*c^7*d^5*e^7 - 88*a^4*b^5*c^3*d*e^
11 + 228*a^5*b*c^6*d^3*e^9 + 188*a^5*b^3*c^4*d*e^11)/(a^6*d^6) + (x^2*(32*a
^6*c^6*d*e^11 - 24*a^6*b*c^5*e^12 + 4*a^3*b^7*c^2*e^12 - 28*a^4*b^5*c^3*e^1
2 + 56*a^5*b^3*c^4*e^12 + 2*a^3*c^9*d^7*e^5 + 104*a^4*c^8*d^5*e^7 - 156*a^5
*c^7*d^3*e^9 + 4*b^3*c^9*d^10*e^2 + 4*b^6*c^6*d^7*e^5 + 4*b^7*c^5*d^6*e^6 +
4*b^10*c^2*d^3*e^9 + 8*a^2*b^2*c^8*d^7*e^5 + 40*a^2*b^3*c^7*d^6*e^6 - 12*a
^2*b^5*c^5*d^4*e^8 + 180*a^2*b^6*c^4*d^3*e^9 - 116*a^2*b^7*c^3*d^2*e^10 - 9
2*a^3*b^2*c^7*d^5*e^7 + 84*a^3*b^3*c^6*d^4*e^8 - 350*a^3*b^4*c^5*d^3*e^9 +
388*a^3*b^5*c^4*d^2*e^10 + 348*a^4*b^2*c^6*d^3*e^9 - 524*a^4*b^3*c^5*d^2*e^
10 - 4*a*b^2*c^9*d^9*e^3 - 20*a*b^4*c^7*d^7*e^5 - 20*a*b^5*c^6*d^6*e^6 + 4*
a*b^6*c^5*d^5*e^7 - 44*a*b^8*c^3*d^3*e^9 + 12*a*b^9*c^2*d^2*e^10 + 8*a^2*b*
c^9*d^8*e^4 + 12*a^2*b^8*c^2*d*e^11 - 36*a^3*b*c^8*d^6*e^6 - 100*a^3*b^6*c^
3*d*e^11 - 132*a^4*b*c^7*d^4*e^8 + 264*a^4*b^4*c^4*d*e^11 + 264*a^5*b*c^6*d
^2*e^10 - 224*a^5*b^2*c^5*d*e^11))/(a^6*d^6) + (((192*a^6*b*c^4*e^11 - 256*
a^6*c^5*d*e^10 + 16*a^4*b^5*c^2*e^11 - 112*a^5*b^3*c^3*e^11 + 60*a^3*c^8*d^
7*e^4 - 320*a^4*c^7*d^5*e^6 + 480*a^5*c^6*d^3*e^8 + 16*b^4*c^7*d^9*e^2 - 32
*b^5*c^6*d^8*e^3 + 16*b^6*c^5*d^7*e^4 + 16*b^7*c^4*d^6*e^5 - 32*b^8*c^3*d^5
*e^6 + 16*b^9*c^2*d^4*e^7 + 16*a^2*b^2*c^7*d^7*e^4 + 120*a^2*b^3*c^6*d^6*e^
5 - 816*a^2*b^4*c^5*d^5*e^6 + 880*a^2*b^5*c^4*d^4*e^7 - 424*a^2*b^6*c^3*d^3
*e^8 + 56*a^2*b^7*c^2*d^2*e^9 + 832*a^3*b^2*c^6*d^5*e^6 - 1424*a^3*b^3*c^5*
d^4*e^7 + 1340*a^3*b^4*c^4*d^3*e^8 - 464*a^3*b^5*c^3*d^2*e^9 - 1512*a^4*b^2
*c^5*d^3*e^8 + 1144*a^4*b^3*c^4*d^2*e^9 - 20*a*b^2*c^8*d^9*e^2 + 96*a*b^3*c
^7*d^8*e^3 - 64*a*b^4*c^6*d^7*e^4 - 88*a*b^5*c^5*d^6*e^5 + 288*a*b^6*c^4*d^
5*e^6 - 208*a*b^7*c^3*d^4*e^7 + 44*a*b^8*c^2*d^3*e^8 - 40*a^2*b*c^8*d^8*e^3
- 88*a^3*b*c^7*d^6*e^5 + 44*a^3*b^6*c^2*d*e^10 + 704*a^4*b*c^6*d^4*e^7 - 3
28*a^4*b^4*c^3*d*e^10 - 736*a^5*b*c^5*d^2*e^9 + 684*a^5*b^2*c^4*d*e^10)/(a^
4*d^4) + (((256*a^6*c^4*e^10 + 16*a^4*b^4*c^2*e^10 - 128*a^5*b^2*c^3*e^10 -
192*a^3*c^7*d^6*e^4 + 448*a^4*c^6*d^4*e^6 - 512*a^5*c^5*d^2*e^8 + 16*b^4*c
^6*d^8*e^2 - 64*b^5*c^5*d^7*e^3 + 96*b^6*c^4*d^6*e^4 - 64*b^7*c^3*d^5*e^5 +
16*b^8*c^2*d^4*e^6 + 768*a^2*b^2*c^6*d^6*e^4 - 1200*a^2*b^3*c^5*d^5*e^5 +
896*a^2*b^4*c^4*d^4*e^6 - 320*a^2*b^5*c^3*d^3*e^7 + 32*a^2*b^6*c^2*d^2*e^8
- 1392*a^3*b^2*c^5*d^4*e^6 + 1024*a^3*b^3*c^4*d^3*e^7 - 288*a^3*b^4*c^3*d^2
*e^8 + 768*a^4*b^2*c^4*d^2*e^8 + 448*a^5*b*c^4*d*e^9 - 32*a*b^2*c^7*d^8*e^2
+ 240*a*b^3*c^6*d^7*e^3 - 528*a*b^4*c^5*d^6*e^4 + 496*a*b^5*c^4*d^5*e^5 -
208*a*b^6*c^3*d^4*e^6 + 32*a*b^7*c^2*d^3*e^7 - 176*a^2*b*c^7*d^7*e^3 + 848*
a^3*b*c^6*d^5*e^5 + 32*a^3*b^5*c^2*d*e^9 - 1024*a^4*b*c^5*d^3*e^7 - 240*a^4
*b^3*c^3*d*e^9)/(a^2*d^2) + (8*c^2*e^2*x^2*(a^3*b^5*e^8 + b^3*c^5*d^8 + b^8
*d^3*e^5 - 11*a^4*b^3*c*e^8 + 28*a^5*b*c^2*e^8 + 8*a*b^7*d^2*e^6 + 8*a^2*b^
6*d*e^7 - 30*a^2*c^6*d^7*e - 24*a^5*c^3*d*e^7 - 6*b^4*c^4*d^7*e - 6*b^7*c*d
^4*e^4 - 18*a^3*c^5*d^5*e^3 + 180*a^4*c^4*d^3*e^5 + 5*b^5*c^3*d^6*e^2 + 5*b
^6*c^2*d^5*e^3 + 5*a*b*c^6*d^8 + 13*a^2*b^2*c^4*d^5*e^3 - 82*a^2*b^3*c^3*d^
4*e^4 + 110*a^2*b^4*c^2*d^3*e^5 - 277*a^3*b^2*c^3*d^3*e^5 + 328*a^3*b^3*c^2

*d^2*e^6 + 15*a*b^2*c^5*d^7*e - 17*a*b^6*c*d^3*e^5 - 57*a^3*b^4*c*d*e^7 - 27*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 40*a*b^5*c^2*d^4*e^4 + 67*a^2*b*c^5*d^6*e^2 - 92*a^2*b^5*c*d^2*e^6 + 72*a^3*b*c^4*d^4*e^4 - 352*a^4*b*c^3*d^2*e^6 + 106*a^4*b^2*c^2*d*e^7))/(a^2*d^2) - (4*c^2*e^2*(a*b^2*e^3 + b*c^2*d^3 - 4*a^2*c*e^3 + b^3*d*e^2 + 4*a*c^2*d^2*e - 2*b^2*c*d^2*e - 3*a*b*c*d*e^2)*(b^4*e*(b^2 - 4*a*c)^(1/2) - b^5*e + 4*a^2*c^3*d + b^4*c*d + 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) - 5*a*b^2*c^2*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2))*(a*b^3*d^2*e^2 + a^2*b^2*d*e^3 + 4*a^2*c^2*d^3*e - 10*a*c^3*d^4*x^2 - 12*a^3*c*e^4*x^2 + 3*a^2*b^2*e^4*x^2 + 3*b^2*c^2*d^4*x^2 + 3*b^4*d^2*e^2*x^2 + a*b*c^2*d^4 - 4*a^3*c*d*e^3 - 2*a*b^2*c*d^3*e - 14*a^2*c^2*d^2*e^2*x^2 - 3*a^2*b*c*d^2*e^2 - 4*a*b^3*d*e^3*x^2 - 6*b^3*c*d^3*e*x^2 - 8*a*b^2*c*d^2*e^2*x^2 + 22*a*b*c^2*d^3*e*x^2 + 16*a^2*b*c*d*e^3*x^2))/(a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^4*e*(b^2 - 4*a*c)^(1/2) - b^5*e + 4*a^2*c^3*d + b^4*c*d + 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) - 5*a*b^2*c^2*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*c^2*e^2*x^2*(6*a^3*b^6*e^9 - 16*a^6*c^3*e^9 + 6*b^3*c^6*d^9 + 6*b^9*d^3*e^6 - 44*a^4*b^4*c*e^9 + 13*a*b^8*d^2*e^7 + 13*a^2*b^7*d*e^8 + 5*a^2*c^7*d^8*e - 8*b^4*c^5*d^8*e - 8*b^8*c*d^4*e^5 + 84*a^5*b^2*c^2*e^9 + 2*a^3*c^6*d^6*e^3 - 160*a^4*c^5*d^4*e^5 + 124*a^5*c^4*d^2*e^7 - 2*b^5*c^4*d^7*e^2 + 8*b^6*c^3*d^6*e^3 - 2*b^7*c^2*d^5*e^4 - 5*a*b*c^7*d^9 + 40*a^2*b^2*c^5*d^6*e^3 - 45*a^2*b^3*c^4*d^5*e^4 - 220*a^2*b^4*c^3*d^4*e^5 + 316*a^2*b^5*c^2*d^3*e^6 + 264*a^3*b^2*c^4*d^4*e^5 - 546*a^3*b^3*c^3*d^3*e^6 + 388*a^3*b^4*c^2*d^2*e^7 - 447*a^4*b^2*c^3*d^2*e^7 + 12*a*b^2*c^6*d^8*e - 74*a*b^7*c*d^3*e^6 - 111*a^3*b^5*c*d*e^8 - 210*a^5*b*c^3*d*e^8 + 18*a*b^3*c^5*d^7*e^2 - 43*a*b^4*c^4*d^6*e^3 + 19*a*b^5*c^3*d^5*e^4 + 72*a*b^6*c^2*d^4*e^5 - 20*a^2*b*c^6*d^7*e^2 - 123*a^2*b^6*c*d^2*e^7 + 31*a^3*b*c^5*d^5*e^4 + 328*a^4*b*c^4*d^3*e^6 + 290*a^4*b^3*c^2*d*e^8))/(a^4*d^4))*(b^4*e*(b^2 - 4*a*c)^(1/2) - b^5*e + 4*a^2*c^3*d + b^4*c*d + 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) - 5*a*b^2*c^2*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^4*e*(b^2 - 4*a*c)^(1/2) - b^5*e + 4*a^2*c^3*d + b^4*c*d + 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) - 5*a*b^2*c^2*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (2*c^5*e^5*x^2*(a^2*b^4*e^6 + 2*a^4*c^2*e^6 + b^2*c^4*d^6 + b^6*d^2*e^4 - 4*a^3*b^2*c*e^6 + 3*a^2*c^4*d^4*e^2 - 10*a^3*c^3*d^2*e^4 + 2*a*b^5*d*e^5 - a*b*c^4*d^5*e + 16*a^2*b^2*c^2*d^2*e^4 - 7*a*b^4*c*d^2*e^4 - 11*a^2*b^3*c*d*e^5 + 13*a^3*b*c^2*d*e^5))/(a^6*d^6))*(b^4*e*(b^2 - 4*a*c)^(1/2) - b^5*e + 4*a^2*c^3*d + b^4*c*d + 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) - 5*a*b^2*c^2*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^4*e*(b^2 - 4*a*c)^(1/2) - b^5*e + 4*a^2*c^3*d + b^4*c*d + 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) - 5*a*b^2*c^2*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*(4*a^5*c*e^2 - a^4*b^2*e^2 + 4*a^4*c^2*d^2 - a^3*b^2*c*d^2 + a^3*b^3*d*e - 4*a^4*b*c*d*e)) - (log((c^8*e^8*(a^2*e^2 + b^2*d^2 - a*c*d^2 + a*b*d*e))/(a^6*d^6) - (c^9*e^9*x^2)/(a^5*d^5) + (((((4*a^4*b^6*c^2*e^12 - 24*a^5*b^4*c^3*e^12 + 36*a^6*b^2*c^4*e^12 - 4*a^3*c^9*d^8*e^4 + 64*a^4*c^8*d^6*e^6 - 144*a^5*c^7*d^4*e^8 + 96*a^6*c^6*d^2*e^10 + 4*b^4*c^8*d^10*e^2 + 8*b^7*c^5*d^7*e^5 + 4*b^10*c^2*d^4*e^8 + 64*a^2*b^3*c^7*d^7*e^5 - 8*a^2*b^4*c^6*d^6*e^6 - 8*a^2*b^5*c^5*d^5*e^7 + 172*a^2*b^6*c^4*d^4*e^8 - 112*a^2*b^7*c^3*d^3*e^9 + 16*a^2*b^8*c^2*d^2*e^10 - 72*a^3*b^2*c^7*d^6*e^6 + 56*a^3*b^3*c^6*d^5*e^7 - 312*a^3*b^4*c^5*d^4*e^8 + 348*a^3*b^5*c^4*d^3*e^9 - 132*a^3*b^6*c^3*d^2*e^10 + 324*a^4*b^2*c^6*d^4*e^8 - 428*a^4*b^3*c^5*d^3*e^9 + 344*a^4*b^4*c^4*d^2*e^10 - 300*a^5*b^2*c^5*d^2*e^10 - 96*a^6*b*c^5*d*e^11 - 4*a*b^2*c^9*d^10*e^2 - 4*a*b^3*c^8*d^9*e^3 - 48*a*b^5*c^6*d^7*e^5 + 8*a*b^6*c^5*d

^6*e^6 - 44*a*b^8*c^3*d^4*e^8 + 12*a*b^9*c^2*d^3*e^9 + 8*a^2*b*c^9*d^9*e^3 - 24*a^3*b*c^8*d^7*e^5 + 12*a^3*b^7*c^2*d*e^11 - 88*a^4*b*c^7*d^5*e^7 - 88*a^4*b^5*c^3*d*e^11 + 228*a^5*b*c^6*d^3*e^9 + 188*a^5*b^3*c^4*d*e^11)/(a^6*d^6) + (x^2*(32*a^6*c^6*d*e^11 - 24*a^6*b*c^5*e^12 + 4*a^3*b^7*c^2*e^12 - 28*a^4*b^5*c^3*e^12 + 56*a^5*b^3*c^4*e^12 + 2*a^3*c^9*d^7*e^5 + 104*a^4*c^8*d^5*e^7 - 156*a^5*c^7*d^3*e^9 + 4*b^3*c^9*d^10*e^2 + 4*b^6*c^6*d^7*e^5 + 4*b^7*c^5*d^6*e^6 + 4*b^10*c^2*d^3*e^9 + 8*a^2*b^2*c^8*d^7*e^5 + 40*a^2*b^3*c^7*d^6*e^6 - 12*a^2*b^5*c^5*d^4*e^8 + 180*a^2*b^6*c^4*d^3*e^9 - 116*a^2*b^7*c^3*d^2*e^10 - 92*a^3*b^2*c^7*d^5*e^7 + 84*a^3*b^3*c^6*d^4*e^8 - 350*a^3*b^4*c^5*d^3*e^9 + 388*a^3*b^5*c^4*d^2*e^10 + 348*a^4*b^2*c^6*d^3*e^9 - 524*a^4*b^3*c^5*d^2*e^10 - 4*a*b^2*c^9*d^9*e^3 - 20*a*b^4*c^7*d^7*e^5 - 20*a*b^5*c^6*d^6*e^6 + 4*a*b^6*c^5*d^5*e^7 - 44*a*b^8*c^3*d^3*e^9 + 12*a*b^9*c^2*d^2*e^10 + 8*a^2*b*c^9*d^8*e^4 + 12*a^2*b^8*c^2*d*e^11 - 36*a^3*b*c^8*d^6*e^6 - 100*a^3*b^6*c^3*d*e^11 - 132*a^4*b*c^7*d^4*e^8 + 264*a^4*b^4*c^4*d*e^11 + 264*a^5*b*c^6*d^2*e^10 - 224*a^5*b^2*c^5*d*e^11))/(a^6*d^6) - (((192*a^6*b*c^4*e^11 - 256*a^6*c^5*d*e^10 + 16*a^4*b^5*c^2*e^11 - 112*a^5*b^3*c^3*e^11 + 60*a^3*c^8*d^7*e^4 - 320*a^4*c^7*d^5*e^6 + 480*a^5*c^6*d^3*e^8 + 16*b^4*c^7*d^9*e^2 - 32*b^5*c^6*d^8*e^3 + 16*b^6*c^5*d^7*e^4 + 16*b^7*c^4*d^6*e^5 - 32*b^8*c^3*d^5*e^6 + 16*b^9*c^2*d^4*e^7 + 16*a^2*b^2*c^7*d^7*e^4 + 120*a^2*b^3*c^6*d^6*e^5 - 816*a^2*b^4*c^5*d^5*e^6 + 880*a^2*b^5*c^4*d^4*e^7 - 424*a^2*b^6*c^3*d^3*e^8 + 56*a^2*b^7*c^2*d^2*e^9 + 832*a^3*b^2*c^6*d^5*e^6 - 1424*a^3*b^3*c^5*d^4*e^7 + 1340*a^3*b^4*c^4*d^3*e^8 - 464*a^3*b^5*c^3*d^2*e^9 - 1512*a^4*b^2*c^5*d^3*e^8 + 1144*a^4*b^3*c^4*d^2*e^9 - 20*a*b^2*c^8*d^9*e^2 + 96*a*b^3*c^7*d^8*e^3 - 64*a*b^4*c^6*d^7*e^4 - 88*a*b^5*c^5*d^6*e^5 + 288*a*b^6*c^4*d^5*e^6 - 208*a*b^7*c^3*d^4*e^7 + 44*a*b^8*c^2*d^3*e^8 - 40*a^2*b*c^8*d^8*e^3 - 88*a^3*b*c^7*d^6*e^5 + 44*a^3*b^6*c^2*d*e^10 + 704*a^4*b*c^6*d^4*e^7 - 328*a^4*b^4*c^3*d*e^10 - 736*a^5*b*c^5*d^2*e^9 + 684*a^5*b^2*c^4*d*e^10)/(a^4*d^4) - (((256*a^6*c^4*e^10 + 16*a^4*b^4*c^2*e^10 - 128*a^5*b^2*c^3*e^10 - 192*a^3*c^7*d^6*e^4 + 448*a^4*c^6*d^4*e^6 - 512*a^5*c^5*d^2*e^8 + 16*b^4*c^6*d^8*e^2 - 64*b^5*c^5*d^7*e^3 + 96*b^6*c^4*d^6*e^4 - 64*b^7*c^3*d^5*e^5 + 16*b^8*c^2*d^4*e^6 + 768*a^2*b^2*c^6*d^6*e^4 - 1200*a^2*b^3*c^5*d^5*e^5 + 896*a^2*b^4*c^4*d^4*e^6 - 320*a^2*b^5*c^3*d^3*e^7 + 32*a^2*b^6*c^2*d^2*e^8 - 1392*a^3*b^2*c^5*d^4*e^6 + 1024*a^3*b^3*c^4*d^3*e^7 - 288*a^3*b^4*c^3*d^2*e^8 + 768*a^4*b^2*c^4*d^2*e^8 + 448*a^5*b*c^4*d*e^9 - 32*a*b^2*c^7*d^8*e^2 + 240*a*b^3*c^6*d^7*e^3 - 528*a*b^4*c^5*d^6*e^4 + 496*a*b^5*c^4*d^5*e^5 - 208*a*b^6*c^3*d^4*e^6 + 32*a*b^7*c^2*d^3*e^7 - 176*a^2*b*c^7*d^7*e^3 + 848*a^3*b*c^6*d^5*e^5 + 32*a^3*b^5*c^2*d*e^9 - 1024*a^4*b*c^5*d^3*e^7 - 240*a^4*b^3*c^3*d*e^9)/(a^2*d^2) + (8*c^2*e^2*x^2*(a^3*b^5*e^8 + b^3*c^5*d^8 + b^8*d^3*e^5 - 11*a^4*b^3*c*e^8 + 28*a^5*b*c^2*e^8 + 8*a*b^7*d^2*e^6 + 8*a^2*b^6*d*e^7 - 30*a^2*c^6*d^7*e - 24*a^5*c^3*d*e^7 - 6*b^4*c^4*d^7*e - 6*b^7*c*d^4*e^4 - 18*a^3*c^5*d^5*e^3 + 180*a^4*c^4*d^3*e^5 + 5*b^5*c^3*d^6*e^2 + 5*b^6*c^2*d^5*e^3 + 5*a*b*c^6*d^8 + 13*a^2*b^2*c^4*d^5*e^3 - 82*a^2*b^3*c^3*d^4*e^4 + 110*a^2*b^4*c^2*d^3*e^5 - 277*a^3*b^2*c^3*d^3*e^5 + 328*a^3*b^3*c^2*d^2*e^6 + 15*a*b^2*c^5*d^7*e - 17*a*b^6*c*d^3*e^5 - 57*a^3*b^4*c*d*e^7 - 27*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 40*a*b^5*c^2*d^4*e^4 + 67*a^2*b*c^5*d^6*e^2 - 92*a^2*b^5*c*d^2*e^6 + 72*a^3*b*c^4*d^4*e^4 - 352*a^4*b*c^3*d^2*e^6 + 106*a^4*b^2*c^2*d*e^7))/(a^2*d^2) + (4*c^2*e^2*(a*b^2*e^3 + b*c^2*d^3 - 4*a^2*c*e^3 + b^3*d*e^2 + 4*a*c^2*d^2*e - 2*b^2*c*d^2*e - 3*a*b*c*d*e^2)*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c^3*d - b^4*c*d - 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) + 5*a*b^2*c^2*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2))*(a*b^3*d^2*e^2 + a^2*b^2*d*e^3 + 4*a^2*c^2*d^3*e - 10*a*c^3*d^4*x^2 - 12*a^3*c*e^4*x^2 + 3*a^2*b^2*e^4*x^2 + 3*b^2*c^2*d^4*x^2 + 3*b^4*d^2*e^2*x^2 + a*b*c^2*d^4 - 4*a^3*c*d*e^3 - 2*a*b^2*c*d^3*e - 14*a^2*c^2*d^2*e^2*x^2 - 3*a^2*b*c*d^2*e^2 - 4*a*b^3*d*e^3*x^2 - 6*b^3*c*d^3*e*x^2 - 8*a*b^2*c*d^2*e^2*x^2 + 22*a*b*c^2*d^3*e*x^2 + 16*a^2*b*c*d*e^3*x^2))/(a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c^3*d - b^4*c*d - 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) + 5*a*b^2*c^2*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2)

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 + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a
^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*c^2*e^2*x^2*(6*a^3*b^6*e^9 -
 16*a^6*c^3*e^9 + 6*b^3*c^6*d^9 + 6*b^9*d^3*e^6 - 44*a^4*b^4*c*e^9 + 13*a*b
^8*d^2*e^7 + 13*a^2*b^7*d*e^8 + 5*a^2*c^7*d^8*e - 8*b^4*c^5*d^8*e - 8*b^8*c
*d^4*e^5 + 84*a^5*b^2*c^2*e^9 + 2*a^3*c^6*d^6*e^3 - 160*a^4*c^5*d^4*e^5 + 1
24*a^5*c^4*d^2*e^7 - 2*b^5*c^4*d^7*e^2 + 8*b^6*c^3*d^6*e^3 - 2*b^7*c^2*d^5*
e^4 - 5*a*b*c^7*d^9 + 40*a^2*b^2*c^5*d^6*e^3 - 45*a^2*b^3*c^4*d^5*e^4 - 220
*a^2*b^4*c^3*d^4*e^5 + 316*a^2*b^5*c^2*d^3*e^6 + 264*a^3*b^2*c^4*d^4*e^5 -
546*a^3*b^3*c^3*d^3*e^6 + 388*a^3*b^4*c^2*d^2*e^7 - 447*a^4*b^2*c^3*d^2*e^7
 + 12*a*b^2*c^6*d^8*e - 74*a*b^7*c*d^3*e^6 - 111*a^3*b^5*c*d*e^8 - 210*a^5*
b*c^3*d*e^8 + 18*a*b^3*c^5*d^7*e^2 - 43*a*b^4*c^4*d^6*e^3 + 19*a*b^5*c^3*d^
5*e^4 + 72*a*b^6*c^2*d^4*e^5 - 20*a^2*b*c^6*d^7*e^2 - 123*a^2*b^6*c*d^2*e^7
 + 31*a^3*b*c^5*d^5*e^4 + 328*a^4*b*c^4*d^3*e^6 + 290*a^4*b^3*c^2*d*e^8))/(
a^4*d^4))*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c^3*d - b^4*c*d - 6*a*
b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) + 5*a*b^2*c^2*d + 8*a^2*b*c^2*e + 2*a
^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*
e*(b^2 - 4*a*c)^(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^5
*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c^3*d - b^4*c*d - 6*a*b^3*c*e - b^3*
c*d*(b^2 - 4*a*c)^(1/2) + 5*a*b^2*c^2*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2
- 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c
)^(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (c^5*e^5*(4*a^3*b
^3*e^6 + 4*b^3*c^3*d^6 + 4*b^6*d^3*e^3 + 8*a*b^5*d^2*e^4 + 8*a^2*b^4*d*e^5
+ 4*a^2*c^4*d^5*e + 16*a^4*c^2*d*e^5 - 19*a^3*c^3*d^3*e^3 - 4*a*b*c^4*d^6 -
 12*a^4*b*c*e^6 + 36*a^2*b^2*c^2*d^3*e^3 - 24*a*b^4*c*d^3*e^3 - 32*a^3*b^2*
c*d*e^5 - 36*a^2*b^3*c*d^2*e^4 + 28*a^3*b*c^2*d^2*e^4))/(a^6*d^6) + (2*c^5*
e^5*x^2*(a^2*b^4*e^6 + 2*a^4*c^2*e^6 + b^2*c^4*d^6 + b^6*d^2*e^4 - 4*a^3*b^
2*c*e^6 + 3*a^2*c^4*d^4*e^2 - 10*a^3*c^3*d^2*e^4 + 2*a*b^5*d*e^5 - a*b*c^4*
d^5*e + 16*a^2*b^2*c^2*d^2*e^4 - 7*a*b^4*c*d^2*e^4 - 11*a^2*b^3*c*d*e^5 + 1
3*a^3*b*c^2*d*e^5))/(a^6*d^6))*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c
^3*d - b^4*c*d - 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) + 5*a*b^2*c^2*d
+ 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*
c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 +
c*d^2 - b*d*e)))*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c^3*d - b^4*c*d
 - 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) + 5*a*b^2*c^2*d + 8*a^2*b*c^2*
e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a
*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*(4*a^5*c*e^2 - a^4*b^2*e^2 + 4*a^4*c^2*d^
2 - a^3*b^2*c*d^2 + a^3*b^3*d*e - 4*a^4*b*c*d*e)) - (1/(4*a*d) - (x^2*(a*e
+ b*d))/(2*a^2*d^2))/x^4 - (e^4*log(d + e*x^2))/(2*(c*d^5 + a*d^3*e^2 - b*d
^4*e)) + (log(x)*(a^2*e^2 + b^2*d^2 - a*c*d^2 + a*b*d*e))/(a^3*d^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.225 $\int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=387

$$-\frac{\left(-\frac{3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d}{\sqrt{b^2-4ac}}+a^2ce-ab^2e-2abcd+b^3d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\,\left(ae^2-bde+cd^2\right)}-\frac{\left(\frac{3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d}{\sqrt{b^2-4ac}}+a\cdots\right.}{\sqrt{2}\,c^{5/2}\sqrt{\sqrt{b^2\cdots}}}$$

Rubi [A] time = 4.03, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1287, 205, 1166}

$$-\frac{\left(-\frac{3a^2bce+2a^2c^2d-4ab^2cd-ab^3e+b^4d}{\sqrt{b^2-4ac}}+a^2ce-ab^2e-2abcd+b^3d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\,\left(ae^2-bde+cd^2\right)}-\frac{\left(\frac{3a^2bce+2a^2c^2d-4ab^2cd-ab^3e+b^4d}{\sqrt{b^2-4ac}}+a^2ce-ab^2e-2abcd+b^3d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}\,\left(ae^2-bde+cd^2\right)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{5/2}\left(ae^2-bde+cd^2\right)}-\frac{x(be+cd)}{c^2e^2}+\frac{x^3}{3ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(((c*d + b*e)*x)/(c^2*e^2)) + x^3/(3*c*e) - ((b^3*d - 2*a*b*c*d - a*b^2*e
+ a^2*c*e - (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(S
qrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - ((b^3
*d - 2*a*b*c*d - a*b^2*e + a^2*c*e + (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a
*b^3*e + 3*a^2*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2)) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(5/2)*(c*d^2 -
 b*d*e + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^8}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{-cd-be}{c^2e^2}+\frac{x^2}{ce}+\frac{d^4}{e^2\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)}+\frac{-a\left(b^2d-acd-abe\right)}{c^2\left(cd^2-bde\right.}\right.$$

$$= -\frac{(cd+be)x}{c^2e^2}+\frac{x^3}{3ce}+\frac{\int \frac{-a\left(b^2d-acd-abe\right)+\left(-b^3d+2abcd+ab^2e-a^2ce\right)x^2}{a+bx^2+cx^4}dx}{c^2\left(cd^2-bde+ae^2\right)}+\frac{d^4\int \frac{}{d}}{e^2\left(cd^2-\right.}$$

$$= -\frac{(cd+be)x}{c^2e^2}+\frac{x^3}{3ce}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}\left(cd^2-bde+ae^2\right)}-\frac{\left(b^3d-2abcd-ab^2e+a^2ce-\right.}{2}$$

$$= -\frac{(cd+be)x}{c^2e^2}+\frac{x^3}{3ce}-\frac{\left(b^3d-2abcd-ab^2e+a^2ce-\frac{b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2b}{\sqrt{b^2-4ac}}\right.}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2-bde+\right.}$$

Mathematica [A] time = 0.61, size = 463, normalized size = 1.20

$$\frac{\left(a^2c\left(e\sqrt{b^2-4ac}-2cd\right)+ab^2\left(4cd-e\sqrt{b^2-4ac}\right)-abc\left(2d\sqrt{b^2-4ac}+3ae\right)+b^3\left(d\sqrt{b^2-4ac}+ae\right)+b^4(-d)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(e(bd-ae)-cd^2\right)}+\frac{\left(a^2c\left(e\sqrt{b^2-4ac}+2cd\right)-ab^2\left(e\sqrt{b^2-4ac}+4cd\right)+abc\left(3ae-2d\sqrt{b^2-4ac}\right)+b^3\left(d\sqrt{b^2-4ac}-ae\right)+b^4d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\left(e(bd-ae)-cd^2\right)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}\left(ae^2-bde+cd^2\right)}-\frac{x(be+cd)}{c^2e^2}+\frac{x^3}{3ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(((c*d + b*e)*x)/(c^2*e^2)) + x^3/(3*c*e) + ((-(b^4*d) + b^3*(Sqrt[b^2 - 4
*a*c]*d + a*e) - a*b*c*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a*b^2*(4*c*d - Sqr
t[b^2 - 4*a*c]*e) + a^2*c*(-2*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*
Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))) + ((b^4*d + b^3*(Sq
rt[b^2 - 4*a*c]*d - a*e) + a*b*c*(-2*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a^2*c*(
2*c*d + Sqrt[b^2 - 4*a*c]*e) - a*b^2*(4*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2
 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))) + (d^(7/2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(5/2)*(c*d^2 - b*d*e + a*e^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 12.65, size = 12506, normalized size = 32.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] d^(7/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2*e^2 - b*d*e^3 + a*e^4) +
1/8*((2*b^7*c^8 - 16*a*b^5*c^9 + 36*a^2*b^3*c^10 - 16*a^3*b*c^11 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^7 - 18*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 - 8*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^8 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^3*c^9 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b*c^10 - 2*(b^2 - 4*a*c)*b^5*c^8 + 8*(b^2 - 4*a*c)*
a*b^3*c^9 - 4*(b^2 - 4*a*c)*a^2*b*c^10)*d^5 - (4*b^8*c^7 - 30*a*b^6*c^8 + 5
8*a^2*b^4*c^9 - 8*a^3*b^2*c^10 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^8*c^5 + 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^6*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*b^7*c^6 - 29*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b^4*c^7 - 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^5*c^7 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^6*c^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^3*b^2*c^8 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^3*c^8 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^4*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*
c^9 - 4*(b^2 - 4*a*c)*b^6*c^7 + 14*(b^2 - 4*a*c)*a*b^4*c^8 - 2*(b^2 - 4*a*c
)*a^2*b^2*c^9)*d^4*e - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4
 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 - 2*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^5 + 2*a*b^6*c^5 + 24*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 + 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^3*c^6 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^6 - 18*a^2*
b^4*c^6 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^7 - 8*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^7 - 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^2*c^7 + 48*a^3*b^2*c^7 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*c^8 - 32*a^4*c^8 - 2*(b^2 - 4*a*c)*a*b^4*c^5 + 10*(b^2 - 4*a*c)*
a^2*b^2*c^6 - 8*(b^2 - 4*a*c)*a^3*c^7)*d^3*abs(-c^3*d^2 + b*c^2*d*e - a*c^2
*e^2) + (2*b^9*c^6 - 8*a*b^7*c^7 - 24*a^2*b^5*c^8 + 104*a^3*b^3*c^9 - 32*a^
4*b*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^9*c^
4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^5 + 12*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 - 52*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 - 24*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 16*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 + 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 12*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 - 4*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 2*(b^2 - 4*a*c)*b
^7*c^6 + 24*(b^2 - 4*a*c)*a^2*b^3*c^8 - 8*(b^2 - 4*a*c)*a^3*b*c^9)*d^3*e^2
+ 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 - 8*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^6*c^4 + 2*a*b^7*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^3*b^3*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 + sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^5 - 16*a^2*b^5*c^5 - 4*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^6 + 32*a^3*b^3*c^6 - 2*(b^2 - 4*a*c
)*a*b^5*c^4 + 8*(b^2 - 4*a*c)*a^2*b^3*c^5)*d^2*abs(-c^3*d^2 + b*c^2*d*e - a
*c^2*e^2)*e - (2*b^7*c^2 - 20*a*b^5*c^3 + 64*a^2*b^3*c^4 - 64*a^3*b*c^5 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 10*sqrt(2)*s
```

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qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c - 32*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 12*(b^2 - 4*a*
c)*a*b^3*c^3 - 16*(b^2 - 4*a*c)*a^2*b*c^4)*(c^3*d^2 - b*c^2*d*e + a*c^2*e^2
)^2*d - (6*a*b^8*c^6 - 42*a^2*b^6*c^7 + 68*a^3*b^4*c^8 + 16*a^4*b^2*c^9 - 3
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^8*c^4 + 21*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^5 + 6*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 - 34*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 - 18*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - 3*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 - 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 - 4*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 9*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 6*(b^2 - 4*a*c
)*a*b^6*c^6 + 18*(b^2 - 4*a*c)*a^2*b^4*c^7 + 4*(b^2 - 4*a*c)*a^3*b^2*c^8)*d
^2*e^3 - 2*(2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^3 - 17*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b^5*c^4 + 4*a^2*b^6*c^4 + 40*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^4*b^2*c^5 + 18*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3
*b^3*c^5 + 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 - 34*a^3*b
^4*c^5 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^6 - 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^6 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^3*b^2*c^6 + 80*a^4*b^2*c^6 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^4*c^7 - 32*a^5*c^7 - 4*(b^2 - 4*a*c)*a^2*b^4*c^4 + 18*(b^2 - 4*a*c)
*a^3*b^2*c^5 - 8*(b^2 - 4*a*c)*a^4*c^6)*d*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*
e^2)*e^2 + (2*a*b^6*c^2 - 18*a^2*b^4*c^3 + 48*a^3*b^2*c^4 - 32*a^4*c^5 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6 + 9*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 24*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 10*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c^2 + 10*(b^2 -
 4*a*c)*a^2*b^2*c^3 - 8*(b^2 - 4*a*c)*a^3*c^4)*(c^3*d^2 - b*c^2*d*e + a*c^2
*e^2)^2*e + (6*a^2*b^7*c^6 - 44*a^3*b^5*c^7 + 84*a^4*b^3*c^8 - 16*a^5*b*c^9
 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^4
+ 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5
+ 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^5 -
 42*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^6 -
 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 -
 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 +
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^7 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 + 10*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 - 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 6*(b^2
- 4*a*c)*a^2*b^5*c^6 + 20*(b^2 - 4*a*c)*a^3*b^3*c^7 - 4*(b^2 - 4*a*c)*a^4*b
*c^8)*d*e^4 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^3 - 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^4 - 2*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b^4*c^4 + 2*a^3*b^5*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt
```

(b^2 - 4*a*c)*c)*a^5*b*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 - 16*a^4*b^3*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^6 + 32*a^5*b*c^6 - 2*(b^2 - 4*a*c)*a^3*b^3*c^4 + 8*(b^2 - 4*a*c)*a^4*b*c^5)*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*e^3 - (2*a^3*b^6*c^6 - 14*a^4*b^4*c^7 + 24*a^5*b^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 - 2*(b^2 - 4*a*c)*a^3*b^4*c^6 + 6*(b^2 - 4*a*c)*a^4*b^2*c^7)*e^5)*arctan(2*sqrt(1/2)*x/sqrt((b*c^3*d^2 - b^2*c^2*d*e + a*b*c^2*e^2 + sqrt((b*c^3*d^2 - b^2*c^2*d*e + a*b*c^2*e^2)^2 - 4*(a*c^3*d^2 - a*b*c^2*d*e + a^2*c^2*e^2)*(c^4*d^2 - b*c^3*d*e + a*c^3*e^2)))/(c^4*d^2 - b*c^3*d*e + a*c^3*e^2)))/((a*b^4*c^7 - 8*a^2*b^2*c^8 - 2*a*b^3*c^8 + 16*a^3*c^9 + 8*a^2*b*c^9 + a*b^2*c^9 - 4*a^2*c^10)*d^4*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*abs(c) - 2*(a*b^5*c^6 - 8*a^2*b^3*c^7 - 2*a*b^4*c^7 + 16*a^3*b*c^8 + 8*a^2*b^2*c^8 + a*b^3*c^8 - 4*a^2*b*c^9)*d^3*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*abs(c)*e + (a*b^6*c^5 - 6*a^2*b^4*c^6 - 2*a*b^5*c^6 + 4*a^2*b^3*c^7 + a*b^4*c^7 + 32*a^4*c^8 + 16*a^3*b*c^8 - 2*a^2*b^2*c^8 - 8*a^3*c^9)*d^2*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*abs(c)*e^2 - 2*(a^2*b^5*c^5 - 8*a^3*b^3*c^6 - 2*a^2*b^4*c^6 + 16*a^4*b*c^7 + 8*a^3*b^2*c^7 + a^2*b^3*c^7 - 4*a^3*b*c^8)*d*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*abs(c)*e^3 + (a^3*b^4*c^5 - 8*a^4*b^2*c^6 - 2*a^3*b^3*c^6 + 16*a^5*c^7 + 8*a^4*b*c^7 + a^3*b^2*c^7 - 4*a^4*c^8)*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*abs(c)*e^4) - 1/8*((2*b^7*c^8 - 16*a*b^5*c^9 + 36*a^2*b^3*c^10 - 16*a^3*b*c^11 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^7 - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^8 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^9 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^10 - 2*(b^2 - 4*a*c)*b^5*c^8 + 8*(b^2 - 4*a*c)*a*b^3*c^9 - 4*(b^2 - 4*a*c)*a^2*b*c^10)*d^5 - (4*b^8*c^7 - 30*a*b^6*c^8 + 58*a^2*b^4*c^9 - 8*a^3*b^2*c^10 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^5 + 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^6 - 29*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 - 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 - 4*(b^2 - 4*a*c)*b^6*c^7 + 14*(b^2 - 4*a*c)*a*b^4*c^8 - 2*(b^2 - 4*a*c)*a^2*b^2*c^9)*d^4*e + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 - 9*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^5 - 2*a*b^6*c^5 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 + 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^6 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^6 + 18*a^2*b^4*c^6 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^7 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^7 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^7 - 48*a^3*b^2*c^7 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^8 + 32*a^4*c^8 + 2*(b^2 - 4*a*c)*a*b^4*c^5 - 10*(b^2 - 4*a*c)*a^2*b^2*c^6 + 8*(b^2 - 4*a*c)*a^3*c^7)*d^3*abs(

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-c^3*d^2 + b*c^2*d*e - a*c^2*e^2) + (2*b^9*c^6 - 8*a*b^7*c^7 - 24*a^2*b^5*c
^8 + 104*a^3*b^3*c^9 - 32*a^4*b*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^9*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^7*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^8*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^5*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^7*c^6 - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^3*b^3*c^7 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^4*c^7 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^4*b*c^8 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*b^2*c^8 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b^3*c^8 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b*c^9 - 2*(b^2 - 4*a*c)*b^7*c^6 + 24*(b^2 - 4*a*c)*a^2*b^3*c^8 - 8*(b^2
- 4*a*c)*a^3*b*c^9)*d^3*e^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^7*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^4 - 2*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 - 2*a*b^7*c^4 + 16*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^4*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^5 + 1
6*a^2*b^5*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^6 - 32*
a^3*b^3*c^6 + 2*(b^2 - 4*a*c)*a*b^5*c^4 - 8*(b^2 - 4*a*c)*a^2*b^3*c^5)*d^2*
abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*e - (2*b^7*c^2 - 20*a*b^5*c^3 + 64*a^
2*b^3*c^4 - 64*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^7 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6
*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c
^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 32*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 16*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 6*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a
*c)*b^5*c^2 + 12*(b^2 - 4*a*c)*a*b^3*c^3 - 16*(b^2 - 4*a*c)*a^2*b*c^4)*(c^3
*d^2 - b*c^2*d*e + a*c^2*e^2)^2*d - (6*a*b^8*c^6 - 42*a^2*b^6*c^7 + 68*a^3*
b^4*c^8 + 16*a^4*b^2*c^9 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^8*c^4 + 21*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^6*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^7*c^5 - 34*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b^4*c^6 - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^5*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^6*c^6 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^4*b^2*c^7 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^3*b^3*c^7 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b^4*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*b^2*c^8 - 6*(b^2 - 4*a*c)*a*b^6*c^6 + 18*(b^2 - 4*a*c)*a^2*b^4*c^7 + 4*
(b^2 - 4*a*c)*a^3*b^2*c^8)*d^2*e^3 + 2*(2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^6*c^3 - 17*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^4
 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^4 - 4*a^2*b^6*c^4 +
40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^5 + 18*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^4*c^5 + 34*a^3*b^4*c^5 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^5*c^6 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^6 - 9*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 - 80*a^4*b^2*c^6 + 4*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^7 + 32*a^5*c^7 + 4*(b^2 - 4*a*c)*a^
2*b^4*c^4 - 18*(b^2 - 4*a*c)*a^3*b^2*c^5 + 8*(b^2 - 4*a*c)*a^4*c^6)*d*abs(-
c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*e^2 + (2*a*b^6*c^2 - 18*a^2*b^4*c^3 + 48*a
^3*b^2*c^4 - 32*a^4*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b
```

```
^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b
^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 +
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 5*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 4*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 2*(b^2 -
4*a*c)*a*b^4*c^2 + 10*(b^2 - 4*a*c)*a^2*b^2*c^3 - 8*(b^2 - 4*a*c)*a^3*c^4)*
(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)^2*e + (6*a^2*b^7*c^6 - 44*a^3*b^5*c^7 + 8
4*a^4*b^3*c^8 - 16*a^5*b*c^9 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^7*c^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^5*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^6*c^5 - 42*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^4*b^3*c^6 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^3*b^4*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^5*c^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^5*b*c^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^4*b^2*c^7 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*b^3*c^7 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^4*b*c^8 - 6*(b^2 - 4*a*c)*a^2*b^5*c^6 + 20*(b^2 - 4*a*c)*a^3*b^3
*c^7 - 4*(b^2 - 4*a*c)*a^4*b*c^8)*d*e^4 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*b^5*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c
^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^4 - 2*a^3*b^5*c^4
+ 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^5 + 8*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^3*b^3*c^5 + 16*a^4*b^3*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^4*b*c^6 - 32*a^5*b*c^6 + 2*(b^2 - 4*a*c)*a^3*b^3*c^4 - 8*(b^2 - 4*a*c)*a
^4*b*c^5)*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*e^3 - (2*a^3*b^6*c^6 - 14*a
^4*b^4*c^7 + 24*a^5*b^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b^6*c^4 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^4*b^4*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*b^5*c^5 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^5*b^2*c^6 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^4*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b^4*c^6 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^4*b^2*c^7 - 2*(b^2 - 4*a*c)*a^3*b^4*c^6 + 6*(b^2 - 4*a*c)*a^4*b^2*c
^7)*e^5)*arctan(2*sqrt(1/2)*x/sqrt((b*c^3*d^2 - b^2*c^2*d*e + a*b*c^2*e^2 -
 sqrt((b*c^3*d^2 - b^2*c^2*d*e + a*b*c^2*e^2)^2 - 4*(a*c^3*d^2 - a*b*c^2*d*
e + a^2*c^2*e^2)*(c^4*d^2 - b*c^3*d*e + a*c^3*e^2)))/(c^4*d^2 - b*c^3*d*e +
 a*c^3*e^2)))/((a*b^4*c^7 - 8*a^2*b^2*c^8 - 2*a*b^3*c^8 + 16*a^3*c^9 + 8*a^
2*b*c^9 + a*b^2*c^9 - 4*a^2*c^10)*d^4*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)
*abs(c) - 2*(a*b^5*c^6 - 8*a^2*b^3*c^7 - 2*a*b^4*c^7 + 16*a^3*b*c^8 + 8*a^2
*b^2*c^8 + a*b^3*c^8 - 4*a^2*b*c^9)*d^3*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^
2)*abs(c)*e + (a*b^6*c^5 - 6*a^2*b^4*c^6 - 2*a*b^5*c^6 + 4*a^2*b^3*c^7 + a*
b^4*c^7 + 32*a^4*c^8 + 16*a^3*b*c^8 - 2*a^2*b^2*c^8 - 8*a^3*c^9)*d^2*abs(-c
^3*d^2 + b*c^2*d*e - a*c^2*e^2)*abs(c)*e^2 - 2*(a^2*b^5*c^5 - 8*a^3*b^3*c^6
 - 2*a^2*b^4*c^6 + 16*a^4*b*c^7 + 8*a^3*b^2*c^7 + a^2*b^3*c^7 - 4*a^3*b*c^8
)*d*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*abs(c)*e^3 + (a^3*b^4*c^5 - 8*a^4
*b^2*c^6 - 2*a^3*b^3*c^6 + 16*a^5*c^7 + 8*a^4*b*c^7 + a^3*b^2*c^7 - 4*a^4*c
^8)*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*abs(c)*e^4) + 1/3*(c^2*x^3*e^2 -
3*c^2*d*x*e - 3*b*c*x*e^2)*e^(-3)/c^3
```

maple [B] time = 0.04, size = 1449, normalized size = 3.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3/c/e*x^3-1/c^2/e*b*x-1/c*d/e^2*x+1/2/(a*e^2-b*d*e+c*d^2)/c*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
```

```
/2)*c*x)*a^2*e-1/2/(a*e^2-b*d*e+c*d^2)/c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2*e-1/
(a*e^2-b*d*e+c*d^2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*d+1/2/(a*e^2-b*d*e+c*d^2)/c
^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*c*x)*b^3*d-3/2/(a*e^2-b*d*e+c*d^2)/c/(-4*a*c+b^2)^(1/2)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*a^2*b*e-1/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x)*a^2*d+1/2/(a*e^2-b*d*e+c*d^2)/c^2/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x)*a*b^3*e+2/(a*e^2-b*d*e+c*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x)*a*b^2*d-1/2/(a*e^2-b*d*e+c*d^2)/c^2/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x)*b^4*d-1/2/(a*e^2-b*d*e+c*d^2)/c*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*e
+1/2/(a*e^2-b*d*e+c*d^2)/c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2*e+1/(a*e^2-b*d*e+c*d^
2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*c*x)*a*b*d-1/2/(a*e^2-b*d*e+c*d^2)/c^2*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*b^3*d-3/2/(a*e^2-b*d*e+c*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2
*b*e-1/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*d+1/2/
(a*e^2-b*d*e+c*d^2)/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^3*e+2/(a*
e^2-b*d*e+c*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2*d-1/2/(a*e^2
-b*d*e+c*d^2)/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*d+1/e^2*d^4/(a*
e^2-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{d^4 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(cd^2e^2 - bde^3 + ae^4\right)\sqrt{de}} - \frac{-\int \frac{a^2be - \left(\left(b^3 - 2\,abc\right)d - \left(ab^2 - a^2c\right)e\right)x^2 - \left(ab^2 - a^2c\right)d}{cx^4 + bx^2 + a}\,dx}{c^3d^2 - bc^2de + ac^2e^2} + \frac{cex^3 - 3\,(cd + be)x}{3\,c^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] d^4*arctan(e*x/sqrt(d*e))/((c*d^2*e^2 - b*d*e^3 + a*e^4)*sqrt(d*e)) - integ
rate(-(a^2*b*e - ((b^3 - 2*a*b*c)*d - (a*b^2 - a^2*c)*e)*x^2 - (a*b^2 - a^2
*c)*d)/(c*x^4 + b*x^2 + a), x)/(c^3*d^2 - b*c^2*d*e + a*c^2*e^2) + 1/3*(c*e
*x^3 - 3*(c*d + b*e)*x)/(c^2*e^2)
```

mupad [B] time = 7.13, size = 41755, normalized size = 107.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan(((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9
- 48*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7
+ 96*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9
- 384*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3*e^3) - (2*x*(-(b^9*d^
2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a
```

^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3
*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c
- b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) +
20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*
d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*
a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)
^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*
a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5
*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*
c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6
*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*(128*a^4*b^2*c^6*e^12 -
16*a^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9
*d^4*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 -
96*b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2
*c^8*d^4*e^8 + 144*a^2*b^3*c^7*d^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*
b^2*c^7*d^2*e^10 + 64*a*b*c^10*d^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c
^9*d^6*e^6 + 528*a*b^3*c^8*d^5*e^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d
^3*e^9 + 16*a*b^6*c^5*d^2*e^10 - 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e
^11 - 320*a^3*b*c^8*d^3*e^9 - 144*a^3*b^3*c^6*d*e^11))/(c^3*e^3))*(-(b^9*d^
2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a
^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3
*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c
- b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) +
20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*
d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*
a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)
^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*
a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5
*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*
c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6
*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*x*(4*a^3*b^7*e^10 +
4*b^3*c^7*d^10 + 4*b^10*d^3*e^7 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 -
4*a*b^9*d^2*e^8 - 4*a^2*b^8*d*e^9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8
*b^4*c^6*d^9*e - 8*b^9*c*d^4*e^6 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5
+ 16*a^5*c^5*d^3*e^7 + 4*b^5*c^5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*
d^10 + 80*a^2*b^4*c^4*d^5*e^5 - 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^
3*e^7 - 64*a^3*b^2*c^5*d^5*e^5 + 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d
^3*e^7 + 8*a^3*b^5*c^2*d^2*e^8 - 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*
d^2*e^8 + 48*a*b^2*c^7*d^9*e - 24*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28
*a*b^3*c^6*d^8*e^2 - 32*a*b^6*c^3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b
*c^7*d^8*e^2 + 20*a^2*b^7*c*d^2*e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^
2*d*e^9 + 96*a^5*b*c^4*d^2*e^8 + 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*
d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9
*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a
^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*
c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2)
+ 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*
c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e -
3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^
3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 1
6*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c
^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^
3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 -
6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (16*a^3*c^6*d^9 + 4*
a*b^4*c^4*d^9 + 4*a*b^8*d^5*e^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^
2*b^2*c^5*d^9 - 4*a^2*b^7*d^4*e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2

+ 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5*c^2*d^6*e^3 + 96*a^3*b^2*c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^3*b^4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5*b^2*c^2*d^3*e^6 + 4*a*b^5*c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8*e - 12*a^6*b^2*c*d*e^8 + 4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^2*b^6*c*d^5*e^4 + 36*a^3*b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4*c*d^3*e^6 + 88*a^5*b*c^3*d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^7)/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*x*(a^8*e^8 + b^8*d^8 + 2*a^4*c^4*d^8 + 20*a^2*b^4*c^2*d^8 - 16*a^3*b^2*c^3*d^8 - 8*a*b^6*c*d^8))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*1i - (((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9 - 48*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7 + 96*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9 - 384*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3*e^3) + (2*x*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*(128*a^4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^8*d^4*e^8 + 144*a^2*b^3*c^7*d^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2*c^7*d^2*e^10 + 64*a*b*c^10*d^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9*d^6*e^6 + 528*a*b^3*c^8*d^5*e^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3*e^9 + 16*a*b^6*c^5*d^2*e^10 - 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11 - 320*a^3*b*c^8*d^3*e^9 - 144*a^3*b^3*c^6*d*e^

11))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (2*x*(4*a^3*b^7*e^10 + 4*b^3*c^7*d^10 + 4*b^10*d^3*e^7 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a*b^9*d^2*e^8 - 4*a^2*b^8*d*e^9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^4*c^6*d^9*e - 8*b^9*c*d^4*e^6 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 + 16*a^5*c^5*d^3*e^7 + 4*b^5*c^5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^10 + 80*a^2*b^4*c^4*d^5*e^5 - 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e^7 - 64*a^3*b^2*c^5*d^5*e^5 + 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3*e^7 + 8*a^3*b^5*c^2*d^2*e^8 - 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2*e^8 + 48*a*b^2*c^7*d^9*e - 24*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a*b^3*c^6*d^8*e^2 - 32*a*b^6*c^3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^7*d^8*e^2 + 20*a^2*b^7*c*d^2*e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d*e^9 + 96*a^5*b*c^4*d^2*e^8 + 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (16*a^3*c^6*d^9 + 4*a*b^4*c^4*d^9 + 4*a*b^8*d^5*e^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^2*b^2*c^5*d^9 - 4*a^2*b^7*d^4*e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5*c^2*d^6*e^3 + 96*a^3*b^2*c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^3*b^4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5*b^2*c^2*d^3*e^6 + 4*a*b^5*c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8*e - 12*a^6*b^2*c*d*e^8 + 4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^2*b^6*c*d^5*e^4 + 36*a^3*b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4*c*d^3*e^6 + 88*a^5*b*c^3*d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^7)/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16

*a^2*b^3*c^6*d*e^3)))^(1/2) - (2*x*(a^8*e^8 + b^8*d^8 + 2*a^4*c^4*d^8 + 20*
a^2*b^4*c^2*d^8 - 16*a^3*b^2*c^3*d^8 - 8*a*b^6*c*d^8))/(c^3*e^3))*(-(b^9*d^
2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a
^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3
*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c
- b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) +
 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*
d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*
a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)
^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*
a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5
*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*
c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6
*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*1i)/((((((192*a^3*c^8*d^
6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9 - 48*a^2*b^2*c^7*d^6*e^5
+ 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7 + 96*a^3*b^2*c^6*d^4*e^7
+ 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9 - 384*a^3*b*c^7*d^5*e^6 -
 384*a^4*b*c^6*d^3*e^8)/(c^3*e^3) - (2*x*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2
*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c
^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^
2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*
b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^
5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2
*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c -
b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4
 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4
+ 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d
*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a
^2*b^3*c^6*d*e^3)))^(1/2)*(128*a^4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256
*a^5*c^7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^
2*e^10 - 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*
b^6*c^6*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^8*d^4*e^8 + 144*a^2*b^
3*c^7*d^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2*c^7*d^2*e^10 + 64*a*b
*c^10*d^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9*d^6*e^6 + 528*a*b^3*c^
8*d^5*e^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3*e^9 + 16*a*b^6*c^5*d^2
*e^10 - 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11 - 320*a^3*b*c^8*d^3*e
^9 - 144*a^3*b^3*c^6*d*e^11))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2
*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c
^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^
2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*
b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^
5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2
*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c -
b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4
 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4
+ 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d
*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a
^2*b^3*c^6*d*e^3)))^(1/2) + (2*x*(4*a^3*b^7*e^10 + 4*b^3*c^7*d^10 + 4*b^10*
d^3*e^7 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a*b^9*d^2*e^8 - 4*a^2*b
^8*d*e^9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^4*c^6*d^9*e - 8*b^9*c*
d^4*e^6 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 + 16*a^5*c^5*d^3*e^7 + 4
*b^5*c^5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^10 + 80*a^2*b^4*c^4*d^5
*e^5 - 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e^7 - 64*a^3*b^2*c^5*d^
5*e^5 + 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3*e^7 + 8*a^3*b^5*c^2*d^

2*e^8 - 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2*e^8 + 48*a*b^2*c^7*d^9*e - 24*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a*b^3*c^6*d^8*e^2 - 32*a*b^6*c^3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^7*d^8*e^2 + 20*a^2*b^7*c*d^2*e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d*e^9 + 96*a^5*b*c^4*d^2*e^8 + 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (16*a^3*c^6*d^9 + 4*a*b^4*c^4*d^9 + 4*a*b^8*d^5*e^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^2*b^2*c^5*d^9 - 4*a^2*b^7*d^4*e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5*c^2*d^6*e^3 + 96*a^3*b^2*c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^3*b^4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5*b^2*c^2*d^3*e^6 + 4*a*b^5*c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8*e - 12*a^6*b^2*c*d*e^8 + 4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^2*b^6*c*d^5*e^4 + 36*a^3*b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4*c*d^3*e^6 + 88*a^5*b*c^3*d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^7)/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*x*(a^8*e^8 + b^8*d^8 + 2*a^4*c^4*d^8 + 20*a^2*b^4*c^2*d^8 - 16*a^3*b^2*c^3*d^8 - 8*a*b^6*c*d^8))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9 - 48*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7 + 96*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9 - 384*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3*e^3) + (2*x*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3* b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*(128*a^4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^8*d^4*e^8 + 144*a^2*b^3*c^7*d^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2*c^7*d^2*e^10 + 64*a*b*c^10*d^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9*d^6*e^6 + 528*a*b^3*c^8*d^5*e^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3*e^9 + 16*a*b^6*c^5*d^2*e^10 - 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11 - 320*a^3*b*c^8*d^3*e^9 - 144*a^3*b^3*c^6*d*e^11))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (2*x*(4*a^3*b^7*e^10 + 4*b^3*c^7*d^10 + 4*b^10*d^3*e^7 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a*b^9*d^2*e^8 - 4*a^2*b^8*d*e^9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^4*c^6*d^9*e - 8*b^9*c*d^4*e^6 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 + 16*a^5*c^5*d^3*e^7 + 4*b^5*c^5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^10 + 80*a^2*b^4*c^4*d^5*e^5 - 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e^7 - 64*a^3*b^2*c^5*d^5*e^5 + 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3*e^7 + 8*a^3*b^5*c^2*d^2*e^8 - 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2*e^8 + 48*a*b^2*c^7*d^9*e - 24*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a*b^3*c^6*d^8*e^2 - 32*a*b^6*c^3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^7*d^8*e^2 + 20*a^2*b^7*c*d^2*e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d*e^9 + 96*a^5*b*c^4*d^2*e^8 + 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (16*a^3*c^6*d^9 + 4*a*b^4*c^4*d^9 + 4*a*b^8*d^5*e^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^2*b^2*c^5*d^9 - 4*a^2*b^7*d^4*e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2 + 6

4*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5*c^2*d^6*e^3 + 96*a^3*b^2*c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^3*b^4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5*b^2*c^2*d^3*e^6 + 4*a*b^5*c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8*e - 12*a^6*b^2*c*d*e^8 + 4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^2*b^6*c*d^5*e^4 + 36*a^3*b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4*c*d^3*e^6 + 88*a^5*b*c^3*d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^7)/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (2*x*(a^8*e^8 + b^8*d^8 + 2*a^4*c^4*d^8 + 20*a^2*b^4*c^2*d^8 - 16*a^3*b^2*c^3*d^8 - 8*a*b^6*c*d^8))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*(a^4*b^3*d^7 + a^7*d^4*e^3 + a^5*b^2*d^6*e + a^6*b*d^5*e^2 - 2*a^5*b*c*d^7 - a^6*c*d^6*e))/(c^3*e^3)))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*2i + atan(((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9 - 48*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7 + 96*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9 - 384*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3*e^3) - (2*x*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2)

- 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*(128*a^4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^8*d^4*e^8 + 144*a^2*b^3*c^7*d^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2*c^7*d^2*e^10 + 64*a*b*c^10*d^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9*d^6*e^6 + 528*a*b^3*c^8*d^5*e^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3*e^9 + 16*a*b^6*c^5*d^2*e^10 - 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11 - 320*a^3*b*c^8*d^3*e^9 - 144*a^3*b^3*c^6*d*e^11))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*x*(4*a^3*b^7*e^10 + 4*b^3*c^7*d^10 + 4*b^10*d^3*e^7 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a*b^9*d^2*e^8 - 4*a^2*b^8*d*e^9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^4*c^6*d^9*e - 8*b^9*c*d^4*e^6 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 + 16*a^5*c^5*d^3*e^7 + 4*b^5*c^5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^10 + 80*a^2*b^4*c^4*d^5*e^5 - 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e^7 - 64*a^3*b^2*c^5*d^5*e^5 + 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3*e^7 + 8*a^3*b^5*c^2*d^2*e^8 - 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2*e^8 + 48*a*b^2*c^7*d^9*e - 24*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a*b^3*c^6*d^8*e^2 - 32*a*b^6*c^3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^7*d^8*e^2 + 20*a^2*b^7*c*d^2*e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d*e^9 + 96*a^5*b*c^4*d^2*e^8 + 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (16*a^3*c^6*d^9 + 4*a*b^4*c^4*d^9 + 4*a*b^8*d^5*e^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^2*b^2*c^5*d^9 - 4*a^2*b^7*d^4*e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5*c^2*d^6*e^3 + 96*a^3*b^2*c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^3*b^4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5*b^2*c^2*d^3*e^6 + 4*a*b^5*c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8*e - 12*a^6*b^2*c*d*e^8 + 4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^2*b^6*c*d^5*e^4 + 36*a^3*

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b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4*c*d^3*e^6 + 88*a^5*b*c^3*
d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^7)/(c^3*e^3))*(-(b^9*d^2
+ a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3
*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b
^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c -
b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2
0*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^
2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^
3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(
1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^
4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e
^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^
7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a
*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*x*(a^8*e^8 + b^8*d^8
+ 2*a^4*c^4*d^8 + 20*a^2*b^4*c^2*d^8 - 16*a^3*b^2*c^3*d^8 - 8*a*b^6*c*d^8))
/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 2
8*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2
*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) +
a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*
a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^
3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76
*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d
*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*
(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^
6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^
5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 3
2*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*1i
- (((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9 - 48
*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7 + 96
*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9 - 38
4*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3*e^3) + (2*x*(-(b^9*d^2 +
a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b
^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3
*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^
2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 1
1*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*
a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*
b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/
2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*
c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4
 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*
d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b
^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*(128*a^4*b^2*c^6*e^12 - 16*a
^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^4
*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96*
b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^8
*d^4*e^8 + 144*a^2*b^3*c^7*d^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2*
c^7*d^2*e^10 + 64*a*b*c^10*d^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9*d
^6*e^6 + 528*a*b^3*c^8*d^5*e^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3*e
^9 + 16*a*b^6*c^5*d^2*e^10 - 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11
- 320*a^3*b*c^8*d^3*e^9 - 144*a^3*b^3*c^6*d*e^11))/(c^3*e^3))*(-(b^9*d^2 +
a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b
^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3
*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^
2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 1
1*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*
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a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*
b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/
2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*
c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4
 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*
d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b
^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (2*x*(4*a^3*b^7*e^10 + 4*b
^3*c^7*d^10 + 4*b^10*d^3*e^7 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a*
b^9*d^2*e^8 - 4*a^2*b^8*d*e^9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^4
*c^6*d^9*e - 8*b^9*c*d^4*e^6 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 + 1
6*a^5*c^5*d^3*e^7 + 4*b^5*c^5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^10
 + 80*a^2*b^4*c^4*d^5*e^5 - 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e^
7 - 64*a^3*b^2*c^5*d^5*e^5 + 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3*e
^7 + 8*a^3*b^5*c^2*d^2*e^8 - 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2*
e^8 + 48*a*b^2*c^7*d^9*e - 24*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a*b
^3*c^6*d^8*e^2 - 32*a*b^6*c^3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^7
*d^8*e^2 + 20*a^2*b^7*c*d^2*e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d*
e^9 + 96*a^5*b*c^4*d^2*e^8 + 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2
+ a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3
*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b
^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c -
b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2
0*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^
2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^
3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(
1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^
4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e
^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^
7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a
*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (16*a^3*c^6*d^9 + 4*a*b^
4*c^4*d^9 + 4*a*b^8*d^5*e^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^2*b^
2*c^5*d^9 - 4*a^2*b^7*d^4*e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2 + 64
*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5*
c^2*d^6*e^3 + 96*a^3*b^2*c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^3*b^
4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5*
b^2*c^2*d^3*e^6 + 4*a*b^5*c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8*
e - 12*a^6*b^2*c*d*e^8 + 4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^
2*b^6*c*d^5*e^4 + 36*a^3*b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4*
c*d^3*e^6 + 88*a^5*b*c^3*d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^
7)/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) +
 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a
^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(
4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2
)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e +
76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c
*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*
c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*
c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e -
 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) -
 (2*x*(a^8*e^8 + b^8*d^8 + 2*a^4*c^4*d^8 + 20*a^2*b^4*c^2*d^8 - 16*a^3*b^2*
c^3*d^8 - 8*a*b^6*c*d^8))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(
4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e
^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-
(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*
```

c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*1i)/((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9 - 48*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7 + 96*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9 - 384*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3*e^3) - (2*x*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*(128*a^4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^8*d^4*e^8 + 144*a^2*b^3*c^7*d^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2*c^7*d^2*e^10 + 64*a*b*c^10*d^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9*d^6*e^6 + 528*a*b^3*c^8*d^5*e^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3*e^9 + 16*a*b^6*c^5*d^2*e^10 - 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11 - 320*a^3*b*c^8*d^3*e^9 - 144*a^3*b^3*c^6*d*e^11))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*x*(4*a^3*b^7*e^10 + 4*b^3*c^7*d^10 + 4*b^10*d^3*e^7 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a*b^9*d^2*e^8 - 4*a^2*b^8*d*e^9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^4*c^6*d^9*e - 8*b^9*c*d^4*e^6 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 + 16*a^5*c^5*d^3*e^7 + 4*b^5*c^5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^10 + 80*a^2*b^4*c^4*d^5*e^5 - 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e^7 - 64*a^3*b^2*c^5*d^5*e^5 + 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3*e^7 + 8*a^3*b^5*c^2*d^2*e^8 - 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2*e^8 + 48*a*b^2*c^7*d^9*e - 24*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a*b^3*c^6*d^8*e^2 - 32*a*b^6*c^3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^7*d^8*e^2 + 20*a^2*b^7*c*d^2*e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d*e^9 + 96*a^5*b*c^4*d^2*e^8 + 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) +

28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (16*a^3*c^6*d^9 + 4*a*b^4*c^4*d^9 + 4*a*b^8*d^5*e^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^2*b^2*c^5*d^9 - 4*a^2*b^7*d^4*e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5*c^2*d^6*e^3 + 96*a^3*b^2*c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^3*b^4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5*b^2*c^2*d^3*e^6 + 4*a*b^5*c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8*e - 12*a^6*b^2*c*d*e^8 + 4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^2*b^6*c*d^5*e^4 + 36*a^3*b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4*c*d^3*e^6 + 88*a^5*b*c^3*d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^7)/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*x*(a^8*e^8 + b^8*d^8 + 2*a^4*c^4*d^8 + 20*a^2*b^4*c^2*d^8 - 16*a^3*b^2*c^3*d^8 - 8*a*b^6*c*d^8))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9 - 48*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7 + 96*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9 - 384*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3*e^3) + (2*x*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 6

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6*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*
b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^
3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 -
32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*
c^6*d*e^3)))^(1/2)*(128*a^4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256*a^5*c^
7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^2*e^10
- 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*b^6*c^6
*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^8*d^4*e^8 + 144*a^2*b^3*c^7*d
^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2*c^7*d^2*e^10 + 64*a*b*c^10*d
^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9*d^6*e^6 + 528*a*b^3*c^8*d^5*e
^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3*e^9 + 16*a*b^6*c^5*d^2*e^10 -
 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11 - 320*a^3*b*c^8*d^3*e^9 - 14
4*a^3*b^3*c^6*d*e^11))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a
*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2
- 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*
a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2
*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d
*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^
2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 6
6*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*
b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^
3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 -
32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*
c^6*d*e^3)))^(1/2) - (2*x*(4*a^3*b^7*e^10 + 4*b^3*c^7*d^10 + 4*b^10*d^3*e^7
 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a*b^9*d^2*e^8 - 4*a^2*b^8*d*e^
9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^4*c^6*d^9*e - 8*b^9*c*d^4*e^6
 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 + 16*a^5*c^5*d^3*e^7 + 4*b^5*c^
5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^10 + 80*a^2*b^4*c^4*d^5*e^5 -
160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e^7 - 64*a^3*b^2*c^5*d^5*e^5 +
 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3*e^7 + 8*a^3*b^5*c^2*d^2*e^8 -
 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2*e^8 + 48*a*b^2*c^7*d^9*e - 2
4*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a*b^3*c^6*d^8*e^2 - 32*a*b^6*c^
3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^7*d^8*e^2 + 20*a^2*b^7*c*d^2*
e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d*e^9 + 96*a^5*b*c^4*d^2*e^8 +
 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4
*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^
2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(
4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c
^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4
*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*
c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^
3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*
a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*
a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3
- 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^
3*c^6*d*e^3)))^(1/2) - (16*a^3*c^6*d^9 + 4*a*b^4*c^4*d^9 + 4*a*b^8*d^5*e^4
+ 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^2*b^2*c^5*d^9 - 4*a^2*b^7*d^4*e^
5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3
*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5*c^2*d^6*e^3 + 96*a^3*b^2*c^4
*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^3*b^4*c^2*d^5*e^4 - 224*a^4*b^2*
c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5*b^2*c^2*d^3*e^6 + 4*a*b^5*c^
3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8*e - 12*a^6*b^2*c*d*e^8 + 4*a
*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^2*b^6*c*d^5*e^4 + 36*a^3*b^5
```

*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4*c*d^3*e^6 + 88*a^5*b*c^3*d^4
*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^7)/(c^3*e^3))*(-(b^9*d^2 + a
^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^
5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*
c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2
)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11
*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a
^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(
-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b
^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2
) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c
^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4
- 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d
^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^
4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (2*x*(a^8*e^8 + b^8*d^8 + 2
*a^4*c^4*d^8 + 20*a^2*b^4*c^2*d^8 - 16*a^3*b^2*c^3*d^8 - 8*a*b^6*c*d^8))/(c
^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a
^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^
5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3
*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c
 - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^
(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^
4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*
(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16
*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d
^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d
^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a
^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*(
a^4*b^3*d^7 + a^7*d^4*e^3 + a^5*b^2*d^6*e + a^6*b*d^5*e^2 - 2*a^5*b*c*d^7 -
 a^6*c*d^6*e))/(c^3*e^3)))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^
2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b
^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b
^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 -
a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*
a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(
-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b
^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2
)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8
*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d
^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*
b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e
^3)))^(1/2)*2i - (log(a^9*d^4*e^26 - b^9*d^13*e^17 + 2*a*b^8*d^12*e^18 - 2*
a^8*b*d^5*e^25 + 2*a^8*c*d^6*e^24 - a^2*b^7*d^11*e^19 + a^7*b^2*d^6*e^24 +
16*a^2*c^7*d^18*e^12 + 16*a^5*c^4*d^12*e^18 + a^7*c^2*d^8*e^22 + b^4*c^5*d^
18*e^12 + 16*a^2*c^7*x*(-d^7*e^5)^(5/2) + b^4*c^5*x*(-d^7*e^5)^(5/2) + a^9*
e^24*x*(-d^7*e^5)^(1/2) - 8*a*b^2*c^6*x*(-d^7*e^5)^(5/2) - 42*a^2*b^5*c^2*d
^13*e^17 + 63*a^3*b^3*c^3*d^13*e^17 + 66*a^3*b^4*c^2*d^12*e^18 - 76*a^4*b^2
*c^3*d^12*e^18 - 25*a^4*b^3*c^2*d^11*e^19 + a^2*b^7*e^12*x*(-d^7*e^5)^(3/2)
 + b^9*d^2*e^10*x*(-d^7*e^5)^(3/2) + 11*a*b^7*c*d^13*e^17 - 2*a^7*b*c*d^7*e
^23 - 8*a*b^2*c^6*d^18*e^12 - 20*a^2*b^6*c*d^12*e^18 + 9*a^3*b^5*c*d^11*e^1
9 - 28*a^4*b*c^4*d^13*e^17 + 20*a^5*b*c^3*d^11*e^19 + 25*a^4*b^3*c^2*e^12*x
*(-d^7*e^5)^(3/2) + a^7*b^2*d^2*e^22*x*(-d^7*e^5)^(1/2) + a^7*c^2*d^4*e^20*
x*(-d^7*e^5)^(1/2) - 2*a*b^8*d*e^11*x*(-d^7*e^5)^(3/2) - 2*a^8*b*d*e^23*x*(
-d^7*e^5)^(1/2) - 9*a^3*b^5*c*e^12*x*(-d^7*e^5)^(3/2) - 20*a^5*b*c^3*e^12*x
*(-d^7*e^5)^(3/2) - 16*a^5*c^4*d*e^11*x*(-d^7*e^5)^(3/2) + 2*a^8*c*d^2*e^22
x(-d^7*e^5)^(1/2) - 11*a*b^7*c*d^2*e^10*x*(-d^7*e^5)^(3/2) + 20*a^2*b^6*c
*d*e^11*x*(-d^7*e^5)^(3/2) - 2*a^7*b*c*d^3*e^21*x*(-d^7*e^5)^(1/2) - 66*a^3

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*b^4*c^2*d*e^11*x*(-d^7*e^5)^(3/2) + 28*a^4*b*c^4*d^2*e^10*x*(-d^7*e^5)^(3/
2) + 76*a^4*b^2*c^3*d*e^11*x*(-d^7*e^5)^(3/2) + 42*a^2*b^5*c^2*d^2*e^10*x*(
-d^7*e^5)^(3/2) - 63*a^3*b^3*c^3*d^2*e^10*x*(-d^7*e^5)^(3/2))*(-d^7*e^5)^(1
/2))/(2*(a*e^7 + c*d^2*e^5 - b*d*e^6)) + (log(a^9*d^4*e^26 - b^9*d^13*e^17
+ 2*a*b^8*d^12*e^18 - 2*a^8*b*d^5*e^25 + 2*a^8*c*d^6*e^24 - a^2*b^7*d^11*e^
19 + a^7*b^2*d^6*e^24 + 16*a^2*c^7*d^18*e^12 + 16*a^5*c^4*d^12*e^18 + a^7*c
^2*d^8*e^22 + b^4*c^5*d^18*e^12 - 16*a^2*c^7*x*(-d^7*e^5)^(5/2) - b^4*c^5*x
*(-d^7*e^5)^(5/2) - a^9*e^24*x*(-d^7*e^5)^(1/2) + 8*a*b^2*c^6*x*(-d^7*e^5)^
(5/2) - 42*a^2*b^5*c^2*d^13*e^17 + 63*a^3*b^3*c^3*d^13*e^17 + 66*a^3*b^4*c^
2*d^12*e^18 - 76*a^4*b^2*c^3*d^12*e^18 - 25*a^4*b^3*c^2*d^11*e^19 - a^2*b^7
*e^12*x*(-d^7*e^5)^(3/2) - b^9*d^2*e^10*x*(-d^7*e^5)^(3/2) + 11*a*b^7*c*d^1
3*e^17 - 2*a^7*b*c*d^7*e^23 - 8*a*b^2*c^6*d^18*e^12 - 20*a^2*b^6*c*d^12*e^1
8 + 9*a^3*b^5*c*d^11*e^19 - 28*a^4*b*c^4*d^13*e^17 + 20*a^5*b*c^3*d^11*e^19
 - 25*a^4*b^3*c^2*e^12*x*(-d^7*e^5)^(3/2) - a^7*b^2*d^2*e^22*x*(-d^7*e^5)^(
1/2) - a^7*c^2*d^4*e^20*x*(-d^7*e^5)^(1/2) + 2*a*b^8*d*e^11*x*(-d^7*e^5)^(3
/2) + 2*a^8*b*d*e^23*x*(-d^7*e^5)^(1/2) + 9*a^3*b^5*c*e^12*x*(-d^7*e^5)^(3/
2) + 20*a^5*b*c^3*e^12*x*(-d^7*e^5)^(3/2) + 16*a^5*c^4*d*e^11*x*(-d^7*e^5)^
(3/2) - 2*a^8*c*d^2*e^22*x*(-d^7*e^5)^(1/2) + 11*a*b^7*c*d^2*e^10*x*(-d^7*e
^5)^(3/2) - 20*a^2*b^6*c*d*e^11*x*(-d^7*e^5)^(3/2) + 2*a^7*b*c*d^3*e^21*x*(
-d^7*e^5)^(1/2) + 66*a^3*b^4*c^2*d*e^11*x*(-d^7*e^5)^(3/2) - 28*a^4*b*c^4*d
^2*e^10*x*(-d^7*e^5)^(3/2) - 76*a^4*b^2*c^3*d*e^11*x*(-d^7*e^5)^(3/2) - 42*
a^2*b^5*c^2*d^2*e^10*x*(-d^7*e^5)^(3/2) + 63*a^3*b^3*c^3*d^2*e^10*x*(-d^7*e
^5)^(3/2))*(-d^7*e^5)^(1/2))/(2*a*e^7 + 2*c*d^2*e^5 - 2*b*d*e^6) + x^3/(3*c
*e) - (x*(b*e + c*d))/(c^2*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.226 $\int \frac{x^6}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=323

$$\frac{\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,\left(ae^2-bde+cd^2\right)}+\frac{\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(-\right.}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}\,\left(ae^2-bde+c\right.}$$

Rubi [A] time = 1.37, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1287, 205, 1166}

$$\frac{\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,\left(ae^2-bde+cd^2\right)}+\frac{\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}\,\left(ae^2-bde+cd^2\right)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}\left(ae^2-bde+cd^2\right)}+\frac{x}{ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] x/(c*e) + ((b^2*d - a*c*d - a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*
e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) +
 ((b^2*d - a*c*d - a*b*e + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (d^(5/2)
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 - b*d*e + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \int \left(\frac{1}{ce} - \frac{d^3}{e(cd^2-bde+ae^2)(d+ex^2)} + \frac{a(bd-ae)+(b^2d-acd-abe)x^2}{c(cd^2-bde+ae^2)(a+bx^2+cx^4)}\right) d$$

$$= \frac{x}{ce} + \frac{\int \frac{a(bd-ae)+(b^2d-acd-abe)x^2}{a+bx^2+cx^4}\,dx}{c(cd^2-bde+ae^2)} - \frac{d^3\int \frac{1}{d+ex^2}\,dx}{e(cd^2-bde+ae^2)}$$

$$= \frac{x}{ce} - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}(cd^2-bde+ae^2)} + \frac{\left(b^2d-acd-abe-\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-}}}{2c(cd^2-bde+ae^2)}$$

$$= \frac{x}{ce} + \frac{\left(b^2d-acd-abe-\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} + \frac{(b^2d-acd}{\sqrt{2}}$$

Mathematica [A] time = 0.52, size = 385, normalized size = 1.19

$$\frac{\left(-b^2\left(d\sqrt{b^2-4ac}+ae\right)+ab\left(e\sqrt{b^2-4ac}-3cd\right)+ac\left(d\sqrt{b^2-4ac}+2ae\right)+b^3d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(e(bd-ae)-cd^2)} + \frac{\left(b^2\left(d\sqrt{b^2-4ac}-ae\right)-ab\left(e\sqrt{b^2-4ac}+3cd\right)+ac\left(2ae-d\sqrt{b^2-4ac}\right)+b^3d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\,(e(ae-bd)+cd^2)} - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}(ae^2-bde+cd^2)} + \frac{x}{ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] x/(c*e) + ((b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c
]*d + 2*a*e) + a*b*(-3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b
- Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))) + ((b^3*d + b^2*(Sqrt[b^2
- 4*a*c]*d - a*e) + a*c*(-(Sqrt[b^2 - 4*a*c]*d) + 2*a*e) - a*b*(3*c*d + Sqr
t[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*
(-(b*d) + a*e))) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 -
b*d*e + a*e^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 13.87, size = 11030, normalized size = 34.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] -d^(5/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2*e - b*d*e^2 + a*e^3) - 1/8*((2*b^6*c^6 - 14*a*b^4*c^7 + 24*a^2*b^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^6 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^7 - 2*(b^2 - 4*a*c)*b^4*c^6 + 6*(b^2 - 4*a*c)*a*b^2*c^7)*d^5 - (4*b^7*c^5 - 26*a*b^5*c^6 + 36*a^2*b^3*c^7 + 16*a^3*b*c^8 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 - 4*(b^2 - 4*a*c)*b^5*c^5 + 10*(b^2 - 4*a*c)*a*b^3*c^6 + 4*(b^2 - 4*a*c)*a^2*b*c^7)*d^4*e - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 + 2*a*b^5*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 - 16*a^2*b^3*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 + 32*a^3*b*c^6 - 2*(b^2 - 4*a*c)*a*b^3*c^4 + 8*(b^2 - 4*a*c)*a^2*b*c^5)*d^3*abs(-c^2*d^2 + b*c*d*e - a*c*e^2) + (2*b^8*c^4 - 6*a*b^6*c^5 - 28*a^2*b^4*c^6 + 80*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c^5 + 20*(b^2 - 4*a*c)*a^2*b^2*c^6)*d^3*e^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^2 - 7*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 + 2*a*b^6*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 14*a^2*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 - 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + 16*a^3*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^6 + 32*a^4*c^6 - 2*(b^2 - 4*a*c)*a*b^4*c^3 + 6*(b^2 - 4*a*c)*a^2*b^2*c^4 + 8*(b^2 - 4*a*c)*a^3*c^5)*d^2*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*e - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(

```
b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^
2 - 4*a*c)*a^2*c^4)*(c^2*d^2 - b*c*d*e + a*c*e^2)^2*d - (6*a*b^7*c^4 - 36*a
^2*b^5*c^5 + 40*a^3*b^3*c^6 + 32*a^4*b*c^7 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^2 + 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b*c^6 - 6*(b^2 - 4*a*c)*a*b^5*c^4 + 12*(b^2 - 4*a*c)*a
^2*b^3*c^5 + 8*(b^2 - 4*a*c)*a^3*b*c^6)*d^2*e^3 - 4*(sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^3*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 + 2*a^2*
b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 + 8*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b^3*c^4 - 16*a^3*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*b*c^5 + 32*a^4*b*c^5 - 2*(b^2 - 4*a*c)*a^2*b^3*c^3 + 8*(b^2 -
4*a*c)*a^3*b*c^4)*d*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*e^2 + (2*a*b^5*c^2 -
16*a^2*b^3*c^3 + 32*a^3*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^3*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3
*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^
3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^3)*(c^2*d^2 - b*c*d
*e + a*c*e^2)^2*e + (6*a^2*b^6*c^4 - 38*a^3*b^4*c^5 + 56*a^4*b^2*c^6 - 3*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 + 19*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 6*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 - 28*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 14*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 3*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 + 7*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 - 6*(b^2 -
 4*a*c)*a^2*b^4*c^4 + 14*(b^2 - 4*a*c)*a^3*b^2*c^5)*d*e^4 + 2*(sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^4*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^
3 + 2*a^3*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^4 + 8*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 + sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b^2*c^4 - 16*a^4*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^4*c^5 + 32*a^5*c^5 - 2*(b^2 - 4*a*c)*a^3*b^2*c^3 + 8*(b^2
- 4*a*c)*a^4*c^4)*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*e^3 - (2*a^3*b^5*c^4 -
12*a^4*b^3*c^5 + 16*a^5*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^3*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^4*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^3*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^5*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^4*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^4*b*c^5 - 2*(b^2 - 4*a*c)*a^3*b^3*c^4 + 4*(b^2 - 4*a*c)*a^4*b*c^5)*e
^5)*arctan(2*sqrt(1/2)*x/sqrt((b*c^2*d^2 - b^2*c*d*e + a*b*c*e^2 + sqrt((b*
c^2*d^2 - b^2*c*d*e + a*b*c*e^2)^2 - 4*(a*c^2*d^2 - a*b*c*d*e + a^2*c*e^2)*
(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)))/(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)))/((a
```

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*b^4*c^5 - 8*a^2*b^2*c^6 - 2*a*b^3*c^6 + 16*a^3*c^7 + 8*a^2*b*c^7 + a*b^2*c
^7 - 4*a^2*c^8)*d^4*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*abs(c) - 2*(a*b^5*c^4
 - 8*a^2*b^3*c^5 - 2*a*b^4*c^5 + 16*a^3*b*c^6 + 8*a^2*b^2*c^6 + a*b^3*c^6 -
 4*a^2*b*c^7)*d^3*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*abs(c)*e + (a*b^6*c^3 -
 6*a^2*b^4*c^4 - 2*a*b^5*c^4 + 4*a^2*b^3*c^5 + a*b^4*c^5 + 32*a^4*c^6 + 16*
a^3*b*c^6 - 2*a^2*b^2*c^6 - 8*a^3*c^7)*d^2*abs(-c^2*d^2 + b*c*d*e - a*c*e^2
)*abs(c)*e^2 - 2*(a^2*b^5*c^3 - 8*a^3*b^3*c^4 - 2*a^2*b^4*c^4 + 16*a^4*b*c^
5 + 8*a^3*b^2*c^5 + a^2*b^3*c^5 - 4*a^3*b*c^6)*d*abs(-c^2*d^2 + b*c*d*e - a
*c*e^2)*abs(c)*e^3 + (a^3*b^4*c^3 - 8*a^4*b^2*c^4 - 2*a^3*b^3*c^4 + 16*a^5*
c^5 + 8*a^4*b*c^5 + a^3*b^2*c^5 - 4*a^4*c^6)*abs(-c^2*d^2 + b*c*d*e - a*c*e
^2)*abs(c)*e^4) + 1/8*((2*b^6*c^6 - 14*a*b^4*c^7 + 24*a^2*b^2*c^8 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 7*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 12*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 6*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^6 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^7 - 2*(b^2 - 4*a*c)*b^4*c^6 + 6*(b^2 -
4*a*c)*a*b^2*c^7)*d^5 - (4*b^7*c^5 - 26*a*b^5*c^6 + 36*a^2*b^3*c^7 + 16*a^3
*b*c^8 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^
3 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4
 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 - 18*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - 10*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 5*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 - 4*(b^2 - 4*a*c)*b^5*
c^5 + 10*(b^2 - 4*a*c)*a*b^3*c^6 + 4*(b^2 - 4*a*c)*a^2*b*c^7)*d^4*e + 2*(sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^4*c^4 - 2*a*b^5*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^
5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 + 16*a^2*b^3*c^5 - 4*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 - 32*a^3*b*c^6 + 2*(b^2 - 4*a*c)*a*b^3*c^4 -
 8*(b^2 - 4*a*c)*a^2*b*c^5)*d^3*abs(-c^2*d^2 + b*c*d*e - a*c*e^2) + (2*b^8*
c^4 - 6*a*b^6*c^5 - 28*a^2*b^4*c^6 + 80*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^6*c^4 - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^3*b^2*c^5 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^4*c^5 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c^5
 + 20*(b^2 - 4*a*c)*a^2*b^2*c^6)*d^3*e^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^6*c^2 - 7*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^
3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 - 2*a*b^6*c^3 + 8*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 6*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^4*c^4 + 14*a^2*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*
c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 - 3*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 - 16*a^3*b^2*c^5 - 4*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*c^6 - 32*a^4*c^6 + 2*(b^2 - 4*a*c)*a*b^4*c^3 -
 6*(b^2 - 4*a*c)*a^2*b^2*c^4 - 8*(b^2 - 4*a*c)*a^3*c^5)*d^2*abs(-c^2*d^2 +
b*c*d*e - a*c*e^2)*e - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*
```

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c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c
)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*(c^2*d^2 - b*c*d*e + a*c*e^2)^2*d -
(6*a*b^7*c^4 - 36*a^2*b^5*c^5 + 40*a^3*b^3*c^6 + 32*a^4*b*c^7 - 3*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^2 + 18*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 + 6*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 20*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 12*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 3*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 16*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 8*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 6*(b^2 - 4*a*c)*a*b^5*c^4 +
 12*(b^2 - 4*a*c)*a^2*b^3*c^5 + 8*(b^2 - 4*a*c)*a^3*b*c^6)*d^2*e^3 + 4*(sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^4*c^3 - 2*a^2*b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4
*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 + 16*a^3*b^3*c^4 - 4*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 - 32*a^4*b*c^5 + 2*(b^2 - 4*a*c)*a^2*
b^3*c^3 - 8*(b^2 - 4*a*c)*a^3*b*c^4)*d*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*e^
2 + (2*a*b^5*c^2 - 16*a^2*b^3*c^3 + 32*a^3*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^
3)*(c^2*d^2 - b*c*d*e + a*c*e^2)^2*e + (6*a^2*b^6*c^4 - 38*a^3*b^4*c^5 + 56
*a^4*b^2*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b^6*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*b^4*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^5*c^3 - 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^4*b^2*c^4 - 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^4*c^4 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3
*b^2*c^5 - 6*(b^2 - 4*a*c)*a^2*b^4*c^4 + 14*(b^2 - 4*a*c)*a^3*b^2*c^5)*d*e^
4 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - 8*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*b^3*c^3 - 2*a^3*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^5*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 + sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 16*a^4*b^2*c^4 - 4*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^5 - 32*a^5*c^5 + 2*(b^2 - 4*a*c)*a^
3*b^2*c^3 - 8*(b^2 - 4*a*c)*a^4*c^4)*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*e^3
 - (2*a^3*b^5*c^4 - 12*a^4*b^3*c^5 + 16*a^5*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
```

```
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 2*(b^2 - 4*a*c)*a^3*b^3*c^4 + 4*(b^2 -
4*a*c)*a^4*b*c^5)*e^5)*arctan(2*sqrt(1/2)*x/sqrt((b*c^2*d^2 - b^2*c*d*e + a
*b*c*e^2 - sqrt((b*c^2*d^2 - b^2*c*d*e + a*b*c*e^2)^2 - 4*(a*c^2*d^2 - a*b*
c*d*e + a^2*c*e^2)*(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)))/(c^3*d^2 - b*c^2*d*e
 + a*c^2*e^2)))/((a*b^4*c^5 - 8*a^2*b^2*c^6 - 2*a*b^3*c^6 + 16*a^3*c^7 + 8*
a^2*b*c^7 + a*b^2*c^7 - 4*a^2*c^8)*d^4*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*ab
s(c) - 2*(a*b^5*c^4 - 8*a^2*b^3*c^5 - 2*a*b^4*c^5 + 16*a^3*b*c^6 + 8*a^2*b^
2*c^6 + a*b^3*c^6 - 4*a^2*b*c^7)*d^3*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*abs(
c)*e + (a*b^6*c^3 - 6*a^2*b^4*c^4 - 2*a*b^5*c^4 + 4*a^2*b^3*c^5 + a*b^4*c^5
 + 32*a^4*c^6 + 16*a^3*b*c^6 - 2*a^2*b^2*c^6 - 8*a^3*c^7)*d^2*abs(-c^2*d^2
+ b*c*d*e - a*c*e^2)*abs(c)*e^2 - 2*(a^2*b^5*c^3 - 8*a^3*b^3*c^4 - 2*a^2*b^
4*c^4 + 16*a^4*b*c^5 + 8*a^3*b^2*c^5 + a^2*b^3*c^5 - 4*a^3*b*c^6)*d*abs(-c^
2*d^2 + b*c*d*e - a*c*e^2)*abs(c)*e^3 + (a^3*b^4*c^3 - 8*a^4*b^2*c^4 - 2*a^
3*b^3*c^4 + 16*a^5*c^5 + 8*a^4*b*c^5 + a^3*b^2*c^5 - 4*a^4*c^6)*abs(-c^2*d^
2 + b*c*d*e - a*c*e^2)*abs(c)*e^4) + x*e^(-1)/c
```

maple [B] time = 0.04, size = 1098, normalized size = 3.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/c/e*x+1/2/(a*e^2-b*d*e+c*d^2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*e+1/2/(a*e^2-b*
d*e+c*d^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*d-1/2/(a*e^2-b*d*e+c*d^2)/c*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*c*x)*b^2*d+1/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*a^2*e-1/2/(a*e^2-b*d*e+c*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*a*b^2*e-3/2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*
a*b*d+1/2/(a*e^2-b*d*e+c*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^
3*d-1/2/(a*e^2-b*d*e+c*d^2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*e-1/2/(a*e^2-b*d*e+c*d
^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*a*d+1/2/(a*e^2-b*d*e+c*d^2)/c*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2
*d+1/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*e-1/2/(a
*e^2-b*d*e+c*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2*e-3/2/(a*e^
2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*d+1/2/(a*e^2-b*d*e
+c*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*d-1/e*d^3/(a*e^2-b*d*e+
c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{d^3 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(cd^2e - bde^2 + ae^3\right)\sqrt{de}} - \frac{-\int \frac{abd - a^2e - \left(abe - \left(b^2 - ac\right)d\right)x^2}{cx^4 + bx^2 + a}\, dx}{c^2d^2 - bcde + ace^2} + \frac{x}{ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -d^3*arctan(e*x/sqrt(d*e))/((c*d^2*e - b*d*e^2 + a*e^3)*sqrt(d*e)) - integr
ate(-(a*b*d - a^2*e - (a*b*e - (b^2 - a*c)*d)*x^2)/(c*x^4 + b*x^2 + a), x)/
(c^2*d^2 - b*c*d*e + a*c*e^2) + x/(c*e)
```

mupad [B] time = 6.45, size = 33892, normalized size = 104.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] x/(c*e) - atan((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^
3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d
^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6
*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 -
 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*e) - (2*x*(-(b^7*d^2 + a^2
*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*
c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e
 + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(
4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*
d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*
d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*
a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*(128*
a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 + 256*a^2*c^8*d^6
*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*
b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*
e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^
2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9
- 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 4
8*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2
*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*b^3*c^4*d*e^9))/(c*e))*(-(
b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2
 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
 a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*
a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4
 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e
^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^
6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3))
)^(1/2) + (2*x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*
c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d
^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5
*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*
c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*
d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^
7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a
*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5
*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*
d*e^7))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*
c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 1
6*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3
*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*
```

```
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4
 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4
+ 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d
*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a
^2*b^3*c^4*d*e^3)))^(1/2) - (4*a*b^3*c^3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d
^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4
*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^
2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*
d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^
2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e
))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c
^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(
1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*
e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c
^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6
*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^
2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d
*e^3)))^(1/2) + (2*x*(a^6*e^6 + b^6*d^6 - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6
 - 6*a*b^4*c*d^6))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2
)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*
e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e
^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^
5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^
4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a
^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 +
b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b
^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*
a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2
*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*1i - (((((64*a^5*c^4*d*e^8 + 64*a^3*c^
6*d^5*e^4 + 128*a^4*c^5*d^3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*
d^4*e^5 + 16*a^2*b^4*c^3*d^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*
d^2*e^7 - 16*a*b^3*c^5*d^6*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^
5 + 64*a^2*b*c^6*d^6*e^3 - 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*
e) + (2*x*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*
a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^
2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4
*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^
2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*
a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*
a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3
- 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^
3*c^4*d*e^3)))^(1/2)*(128*a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*
c^5*e^10 + 256*a^2*c^8*d^6*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8
- 16*b^3*c^7*d^7*e^3 + 64*b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4
*d^4*e^6 - 16*b^7*c^3*d^3*e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d
^3*e^7 - 96*a^2*b^4*c^4*d^2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*
e^3 + 320*a^4*b*c^5*d*e^9 - 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 -
 336*a*b^4*c^5*d^4*e^6 + 48*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*
a^2*b*c^7*d^5*e^5 + 16*a^2*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*
b^3*c^4*d*e^9))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3
)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2
*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*
(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c
*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c
```

*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (2*x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*d*e^7))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (4*a*b^3*c^3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (2*x*(a^6*e^6 + b^6*d^6 - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*d^6))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*1i)/((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 - 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*e) - (2*x*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 -

a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*(128*a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 + 256*a^2*c^8*d^6*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9 - 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 48*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*b^3*c^4*d*e^9))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) + (2*x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*d*e^7))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (4*a*b^3*c^3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2* e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) + (2*x*(a^6*e^6 + b^6*d^6 - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*d^6))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) + (((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 - 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*e) + (2*x*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*(128*a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 + 256*a^2*c^8*d^6*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9 - 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 48*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*b^3*c^4*d*e^9))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (2*x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*d*e^7))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 1

6*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (4*a*b^3*c^3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (2*x*(a^6*e^6 + b^6*d^6 - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*d^6))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) + (2*(a^3*b^2*d^5 - a^4*c*d^5 + a^5*d^3*e^2 + a^4*b*d^4*e))/(c*e)))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*2i - atan((((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 - 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*e) - (2*x*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 +

16*a^2*b^3*c^4*d*e^3)))^(1/2)*(128*a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 + 256*a^2*c^8*d^6*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9 - 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 48*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*b^3*c^4*d*e^9))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) + (2*x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*d*e^7))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (4*a*b^3*c^3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) + (2*x*(a^6*e^6 + b^6*d^6 - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*d^6))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*

```
(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)
^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8
*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c
^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c
^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e -
32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*1i
 - (((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6 - 144*a
^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d^3*e^6 - 96*a
^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6*e^3 + 32*a*b
^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 - 64*a^4*b*c^4
*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*e) + (2*x*(-(b^7*d^2 + a^2*b^5*e^2 + b^
4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^
4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3
*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^
2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^
3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a
^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4
 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b
^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*
a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e
^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*(128*a^4*b^2*c^4*e
^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 + 256*a^2*c^8*d^6*e^4 + 256*a^
3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*b^4*c^6*d^6*e
^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*e^7 + 256*a^2
*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^2*e^8 + 192*a
^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9 - 320*a*b^2*c
^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 48*a*b^5*c^4*d
^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2*b^5*c^3*d*e^
9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*b^3*c^4*d*e^9))/(c*e))*(-(b^7*d^2 + a^2
*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*
c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e
 + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(
4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*
d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*
d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*
a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (2*
x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*c*e^8 + 48*a^
5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d^7*e + 56*a^5
*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5*e^3 - 16*a^4
*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*c^6*d^8 + 36*
a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*d^3*e^5 + 64*
a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^7*e - 16*a*b^
6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*
e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5*c*d^2*e^6 +
16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*d*e^7))/(c*e)
)*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^
3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(
1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1
/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e
 - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^
6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*
d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2
*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*
e^3)))^(1/2) - (4*a*b^3*c^3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d^4*e^3 + 4*a^
```

4*b^3*d*e^6 + 48*a^3*c^4*d^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (2*x*(a^6*e^6 + b^6*d^6 - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*d^6))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*1i)/((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 - 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*e) - (2*x*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*(128*a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 + 256*a^2*c^8*d^6*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9 - 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 48*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*b^3*c^4*d*e^9))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^

3*c^4*d*e^3)))^(1/2) + (2*x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*d*e^7))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (4*a*b^3*c^3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) + (2*x*(a^6*e^6 + b^6*d^6 - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*d^6))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) + (((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 - 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*e) + (2*x*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4

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*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*
c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 +
 16*a^2*b^3*c^4*d*e^3)))^(1/2)*(128*a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10
- 256*a^5*c^5*e^10 + 256*a^2*c^8*d^6*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^
6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 +
64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2
*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*
b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9 - 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6
*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 48*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*
e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7
- 144*a^3*b^3*c^4*d*e^9))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*
c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 -
 a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^
2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16
*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*
e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5
*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 -
8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3
*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*
c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (2*x*(4*a^3*b^5*e^8 + 4*b^3*c
^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*
e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7
*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d
^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a
^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a
^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c
*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4
 + 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*
a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*d*e^7))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2
 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 +
12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^
2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c
 - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c -
b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^
7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e +
a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2
+ 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^
5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (4*a*b^3*c^
3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d
^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c
^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*
e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2
*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2
*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(
-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2
*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^
2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(
1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2
) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*
c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a
^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*
e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c
^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*
a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (2*x*(a^6*e^6 + b^6*d^6
 - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*d^6))/(c*e))*(-(b^7*d^2 +
a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b
^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*
```

```
d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d
^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*
(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)
^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8
*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c
^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c
^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e -
32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) +
(2*(a^3*b^2*d^5 - a^4*c*d^5 + a^5*d^3*e^2 + a^4*b*d^4*e))/(c*e)))*(-(b^7*d^
2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a
^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*
b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c
^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*
d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c -
b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b
^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b
^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*
e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2
)*2i - (log(b^7*d^10*e^10 - a^7*d^3*e^17 - 2*a*b^6*d^9*e^11 + 2*a^6*b*d^4*e
^16 - 2*a^6*c*d^5*e^15 + a^2*b^5*d^8*e^12 - a^5*b^2*d^5*e^15 - 16*a^2*c^5*d
^13*e^7 + 16*a^4*c^3*d^9*e^11 - a^5*c^2*d^7*e^13 - b^4*c^3*d^13*e^7 + 16*a^
2*c^5*x*(-d^5*e^3)^(5/2) + b^4*c^3*x*(-d^5*e^3)^(5/2) + a^7*e^16*x*(-d^5*e^
3)^(1/2) - 8*a*b^2*c^4*x*(-d^5*e^3)^(5/2) + 25*a^2*b^3*c^2*d^10*e^10 - 36*a
^3*b^2*c^2*d^9*e^11 + a^2*b^5*e^8*x*(-d^5*e^3)^(3/2) + b^7*d^2*e^6*x*(-d^5*
e^3)^(3/2) - 9*a*b^5*c*d^10*e^10 + 2*a^5*b*c*d^6*e^14 + 8*a*b^2*c^4*d^13*e^
7 + 16*a^2*b^4*c*d^9*e^11 - 20*a^3*b*c^3*d^10*e^10 - 7*a^3*b^3*c*d^8*e^12 +
 12*a^4*b*c^2*d^8*e^12 + a^5*b^2*d^2*e^14*x*(-d^5*e^3)^(1/2) + a^5*c^2*d^4*
e^12*x*(-d^5*e^3)^(1/2) - 2*a*b^6*d*e^7*x*(-d^5*e^3)^(3/2) - 2*a^6*b*d*e^15
*x*(-d^5*e^3)^(1/2) - 7*a^3*b^3*c*e^8*x*(-d^5*e^3)^(3/2) + 12*a^4*b*c^2*e^8
*x*(-d^5*e^3)^(3/2) + 16*a^4*c^3*d*e^7*x*(-d^5*e^3)^(3/2) + 2*a^6*c*d^2*e^1
4*x*(-d^5*e^3)^(1/2) - 9*a*b^5*c*d^2*e^6*x*(-d^5*e^3)^(3/2) + 16*a^2*b^4*c*
d*e^7*x*(-d^5*e^3)^(3/2) - 2*a^5*b*c*d^3*e^13*x*(-d^5*e^3)^(1/2) - 20*a^3*b
*c^3*d^2*e^6*x*(-d^5*e^3)^(3/2) - 36*a^3*b^2*c^2*d*e^7*x*(-d^5*e^3)^(3/2) +
 25*a^2*b^3*c^2*d^2*e^6*x*(-d^5*e^3)^(3/2))*(-d^5*e^3)^(1/2))/(2*(a*e^5 + c
*d^2*e^3 - b*d*e^4)) + (log(a^7*d^3*e^17 - b^7*d^10*e^10 + 2*a*b^6*d^9*e^11
 - 2*a^6*b*d^4*e^16 + 2*a^6*c*d^5*e^15 - a^2*b^5*d^8*e^12 + a^5*b^2*d^5*e^1
5 + 16*a^2*c^5*d^13*e^7 - 16*a^4*c^3*d^9*e^11 + a^5*c^2*d^7*e^13 + b^4*c^3*
d^13*e^7 + 16*a^2*c^5*x*(-d^5*e^3)^(5/2) + b^4*c^3*x*(-d^5*e^3)^(5/2) + a^7
*e^16*x*(-d^5*e^3)^(1/2) - 8*a*b^2*c^4*x*(-d^5*e^3)^(5/2) - 25*a^2*b^3*c^2*
d^10*e^10 + 36*a^3*b^2*c^2*d^9*e^11 + a^2*b^5*e^8*x*(-d^5*e^3)^(3/2) + b^7*
d^2*e^6*x*(-d^5*e^3)^(3/2) + 9*a*b^5*c*d^10*e^10 - 2*a^5*b*c*d^6*e^14 - 8*a
*b^2*c^4*d^13*e^7 - 16*a^2*b^4*c*d^9*e^11 + 20*a^3*b*c^3*d^10*e^10 + 7*a^3*
b^3*c*d^8*e^12 - 12*a^4*b*c^2*d^8*e^12 + a^5*b^2*d^2*e^14*x*(-d^5*e^3)^(1/2
) + a^5*c^2*d^4*e^12*x*(-d^5*e^3)^(1/2) - 2*a*b^6*d*e^7*x*(-d^5*e^3)^(3/2)
- 2*a^6*b*d*e^15*x*(-d^5*e^3)^(1/2) - 7*a^3*b^3*c*e^8*x*(-d^5*e^3)^(3/2) +
12*a^4*b*c^2*e^8*x*(-d^5*e^3)^(3/2) + 16*a^4*c^3*d*e^7*x*(-d^5*e^3)^(3/2) +
 2*a^6*c*d^2*e^14*x*(-d^5*e^3)^(1/2) - 9*a*b^5*c*d^2*e^6*x*(-d^5*e^3)^(3/2)
 + 16*a^2*b^4*c*d*e^7*x*(-d^5*e^3)^(3/2) - 2*a^5*b*c*d^3*e^13*x*(-d^5*e^3)^
(1/2) - 20*a^3*b*c^3*d^2*e^6*x*(-d^5*e^3)^(3/2) - 36*a^3*b^2*c^2*d*e^7*x*(-
d^5*e^3)^(3/2) + 25*a^2*b^3*c^2*d^2*e^6*x*(-d^5*e^3)^(3/2))*(-d^5*e^3)^(1/2
))/(2*a*e^5 + 2*c*d^2*e^3 - 2*b*d*e^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.227 $\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=280

$$-\frac{\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}-\frac{\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}+\frac{d^{3/2}\tan^{-1}\left(\right.}{\sqrt{e}\,(ae^2-bd}$$

Rubi [A] time = 0.89, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1287, 205, 1166}

$$-\frac{\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}-\frac{\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(((b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b
^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2))) - ((b*d - a*e + (b^2*d - 2*a*c*d - a
*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)
) + (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \int \left(\frac{d^2}{(cd^2-bde+ae^2)(d+ex^2)} + \frac{-ad-(bd-ae)x^2}{(cd^2-bde+ae^2)(a+bx^2+cx^4)} \right) dx$$

$$= \frac{\int \frac{-ad+(-bd+ae)x^2}{a+bx^2+cx^4}\,dx}{cd^2-bde+ae^2} + \frac{d^2\int \frac{1}{d+ex^2}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,(cd^2-bde+ae^2)} - \frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2(cd^2-bde+ae^2)} - \frac{(bd-a\ldots}{\ldots}$$

$$= -\frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}(\ldots}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2\ldots}$$

Mathematica [A] time = 0.33, size = 323, normalized size = 1.15

$$\frac{\left(bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+abe+2acd+b^2(-d)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-ae^2+bde-cd^2)} + \frac{\left(bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}-abe-2acd+b^2d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\,(-ae^2+bde-cd^2)} + \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-(b^2*d) + 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d + a*b*e - a*Sqrt[b^2 - 4*a*c]*
e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c
]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2))
 + ((b^2*d - 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d - a*b*e - a*Sqrt[b^2 - 4*a*c]*
e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c
]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2))
 + (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 - b*d*e + a*e^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]
```

fricas [B] time = 9.25, size = 15553, normalized size = 55.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*
d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a
*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*
a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*
e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^
2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*
d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b
^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 -
```

```
24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 +
2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4
*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d
^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*
e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))*log(-
2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d^2)*x + sqrt(1/2)*((b^4 - 5*a*b
^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4*a^3*c)*d
*e^2 - ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d
^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^
2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2
*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^
2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b
^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a
*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*
e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*
b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b
^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^
2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2
 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e
+ (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e
^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2
*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)
*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b
^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^
2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^
4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c
^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*
e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^
2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3
*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) - sqrt(1/2)*(c*d^2
 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2
*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c
 - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^
2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c
 + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2
)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 -
 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)
*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 -
4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*
a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a
^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*
b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a
^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))*log(-2*(2*a^2*b*d*e - a^3*e
^2 - (a*b^2 - a^2*c)*d^2)*x - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3
- 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4*a^3*c)*d*e^2 - ((b^3*c^3 - 4*a
*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e + (b^5*c + 2*a*b^
3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2
*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*s
qrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3
- a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8
- 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*
d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b
^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3
- 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^
2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8
)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^
2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2
 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^
3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^
```

```
4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 -
4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5
- 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^
6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 -
 a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 -
8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4
*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e +
 (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^
3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) + sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt
(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 -
4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2
*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e
^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a
*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)
*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*
c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 +
8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3
*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^
4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3
)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c
- 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2
*b^2*c - 4*a^3*c^2)*e^4))*log(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d
^2)*x + sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c
)*d^2*e + (a^2*b^2 - 4*a^3*c)*d*e^2 + ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c
^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*
d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c -
4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 -
a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(
3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c
^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 -
 a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^
4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^
5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2
 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 +
 (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 -
 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2
- 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a
^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c
)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*
c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 -
 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 -
42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*
b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4
*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2
*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2
- 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3
*c^2)*e^4))) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 -
3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*
c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b
^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e
^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e
- 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*
a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*
c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b
^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d
^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^
3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4
*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*
c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^
```

4))*log(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d^2)*x - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4*a^3*c)*d*e^2 + ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) + d*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)))/(c*d^2 - b*d*e + a*e^2), 1/2*(sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))*log(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d^2)*x + sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4*a^3*c)*d*e^2 - ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5

)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))*log(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d^2)*x - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4*a^3*c)*d*e^2 - ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) + sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))*log(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d^2)*x + sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4*a^3*c)*d*e^2 + ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c

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^3)*d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*
c - 4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^
3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e -
 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a
*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c
^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^
2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^
3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3
*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e
^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d
^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*
e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-
(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2
*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(
b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e
^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^
3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*
a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6
 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/(
(b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*
c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4
*a^3*c^2)*e^4))) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^
3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(
b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*
(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b
*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^
3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5
 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(
b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a
^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^
4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^
3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3
 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*
a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2
)*e^4))*log(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d^2)*x - sqrt(1/2)*
((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2
 - 4*a^3*c)*d*e^2 + ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 -
 4*a^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4
*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4
 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2
*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)
*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b
^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^
2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^
4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c
^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*
e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d
^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*
b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a
^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e
^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2
*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d
^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^
3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 2
4*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2
*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*
a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^
4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e
^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) + 2*d
```

```
*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d))/(c*d^2 - b*d*e + a*e^2)]
```

giac [B] time = 11.24, size = 8658, normalized size = 30.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] d^(3/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2 - b*d*e + a*e^2) + 1/8*((
2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*d^5 - (4*b^6*c^
3 - 22*a*b^4*c^4 + 24*a^2*b^2*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^6*c + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^5*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^2*c^3 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^3*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b^4*c^3 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c^4 - 4*(b^2 - 4*a*c)*b^4*c^3 + 6*(b^2 - 4*a*c)*a*b^2*c^4)*d^4*e - 2*(
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^
4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^
2 - 4*a*c)*a^2*c^4)*d^3*abs(c*d^2 - b*d*e + a*e^2) + (2*b^7*c^2 - 4*a*b^5*c
^3 - 24*a^2*b^3*c^4 + 32*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^6*c + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^
2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3
- 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 -
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 4*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^
2 - 4*a*c)*b^5*c^2 - 4*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)
*d^3*e^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 8*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^4*c^2 - 2*a*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 16*a^2*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 32*a^3*b*c^4 + 2*(b^2 - 4*a*
c)*a*b^3*c^2 - 8*(b^2 - 4*a*c)*a^2*b*c^3)*d^2*abs(c*d^2 - b*d*e + a*e^2)*e
- (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*(c*d^2 - b*d*e + a*
e^2)^2*d - (6*a*b^6*c^2 - 28*a^2*b^4*c^3 + 16*a^3*b^2*c^4 - 3*sqrt(2)*sqrt(
```

b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 6*(b^2 - 4*a*c)*a*b^4*c^2 + 4*(b^2 - 4*a*c)*a^2*b^2*c^3)*d^2*e^3 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2*a^2*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*a^3*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 32*a^4*c^4 + 2*(b^2 - 4*a*c)*a^2*b^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3)*d*abs(c*d^2 - b*d*e + a*e^2)*e^2 + (2*a*b^4*c^2 - 16*a^2*b^2*c^3 + 32*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*(c*d^2 - b*d*e + a*e^2)^2*e + (6*a^2*b^5*c^2 - 28*a^3*b^3*c^3 + 16*a^4*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 6*(b^2 - 4*a*c)*a^2*b^3*c^2 + 4*(b^2 - 4*a*c)*a^3*b*c^3)*d*e^4 - (2*a^3*b^4*c^2 - 8*a^4*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 2*(b^2 - 4*a*c)*a^3*b^2*c^2)*e^5)*arctan(2*sqrt(1/2)*x/sqrt((b*c*d^2 - b^2*d*e + a*b*e^2 + sqrt((b*c*d^2 - b^2*d*e + a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*e + a^2*e^2)*(c^2*d^2 - b*c*d*e + a*c*e^2)))/(c^2*d^2 - b*c*d*e + a*c*e^2)))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*d^4*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 - 2*a*b^4*c^3 + 16*a^3*b*c^4 + 8*a^2*b^2*c^4 + a*b^3*c^4 - 4*a^2*b*c^5)*d^3*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e + (a*b^6*c - 6*a^2*b^4*c^2 - 2*a*b^5*c^2 + 4*a^2*b^3*c^3 + a*b^4*c^3 + 32*a^4*c^4 + 16*a^3*b*c^4 - 2*a^2*b^2*c^4 - 8*a^3*c^5)*d^2*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^2 - 2*(a^2*b^5*c - 8*a^3*b^3*c^2 - 2*a^2*b^4*c^2 + 16*a^4*b*c^3 + 8*a^3*b^2*c^3 + a^2*b^3*c^3 - 4*a^3*b*c^4)*d*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^3 + (a^3*b^4*c - 8*a^4*b^2*c^2 - 2*a^3*b^3*c^2 + 16*a^5*c^3 + 8*a^4*b*c^3 + a^3*b^2*c^3 - 4*a^4*c^4)*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^4) - 1/8*((2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*d^5 - (4*b^6*c^3 - 22*a*b^4*c^4 + 24*a^2*b^2*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)

*c)*b^5*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^2*c^3 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*
c^3 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4
 - 4*(b^2 - 4*a*c)*b^4*c^3 + 6*(b^2 - 4*a*c)*a*b^2*c^4)*d^4*e + 2*(sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^
3 + 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*
c)*a^2*c^4)*d^3*abs(c*d^2 - b*d*e + a*e^2) + (2*b^7*c^2 - 4*a*b^5*c^3 - 24*
a^2*b^3*c^4 + 32*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
6*c + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*
c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 8*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 4*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*
c)*b^5*c^2 - 4*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*d^3*e^2
 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 8*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^4*c^2 + 2*a*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 16*a^2*b^3*c^3 - 4*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 32*a^3*b*c^4 - 2*(b^2 - 4*a*c)*a*b^3
c^2 + 8(b^2 - 4*a*c)*a^2*b*c^3)*d^2*abs(c*d^2 - b*d*e + a*e^2)*e - (2*b^5
*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(
b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*(c*d^2 - b*d*e + a*e^2)^2*d
 - (6*a*b^6*c^2 - 28*a^2*b^4*c^3 + 16*a^3*b^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 6*(b^2 - 4*a*c)*a*b^4*c^2 + 4*(b^2 - 4*a*c)
*a^2*b^2*c^3)*d^2*e^3 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*
c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 2*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 2*a^2*b^4*c^2 + 16*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 16*a^3*
b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 32*a^4*c^4 -
2*(b^2 - 4*a*c)*a^2*b^2*c^2 + 8*(b^2 - 4*a*c)*a^3*c^3)*d*abs(c*d^2 - b*d*e
+ a*e^2)*e^2 + (2*a*b^4*c^2 - 16*a^2*b^2*c^3 + 32*a^3*c^4 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq

```
rt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c
^3)*(c*d^2 - b*d*e + a*e^2)^2*e + (6*a^2*b^5*c^2 - 28*a^3*b^3*c^3 + 16*a^4*
b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5
 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c +
 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 8*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^2 - 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 3*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 6*(b^2 - 4
*a*c)*a^2*b^3*c^2 + 4*(b^2 - 4*a*c)*a^3*b*c^3)*d*e^4 - (2*a^3*b^4*c^2 - 8*a
^4*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*
b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 2*(
b^2 - 4*a*c)*a^3*b^2*c^2)*e^5)*arctan(2*sqrt(1/2)*x/sqrt((b*c*d^2 - b^2*d*e
 + a*b*e^2 - sqrt((b*c*d^2 - b^2*d*e + a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*e +
a^2*e^2)*(c^2*d^2 - b*c*d*e + a*c*e^2)))/(c^2*d^2 - b*c*d*e + a*c*e^2)))/((
a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*
c^5 - 4*a^2*c^6)*d^4*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a*b^5*c^2 - 8*a
^2*b^3*c^3 - 2*a*b^4*c^3 + 16*a^3*b*c^4 + 8*a^2*b^2*c^4 + a*b^3*c^4 - 4*a^2
*b*c^5)*d^3*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e + (a*b^6*c - 6*a^2*b^4*c^2
- 2*a*b^5*c^2 + 4*a^2*b^3*c^3 + a*b^4*c^3 + 32*a^4*c^4 + 16*a^3*b*c^4 - 2*a
^2*b^2*c^4 - 8*a^3*c^5)*d^2*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^2 - 2*(a^2*
b^5*c - 8*a^3*b^3*c^2 - 2*a^2*b^4*c^2 + 16*a^4*b*c^3 + 8*a^3*b^2*c^3 + a^2*
b^3*c^3 - 4*a^3*b*c^4)*d*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^3 + (a^3*b^4*c
 - 8*a^4*b^2*c^2 - 2*a^3*b^3*c^2 + 16*a^5*c^3 + 8*a^4*b*c^3 + a^3*b^2*c^3 -
 4*a^4*c^4)*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^4)
```

maple [B] time = 0.03, size = 764, normalized size = 2.73

$$\frac{\sqrt{2}\,abe\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(ae^2-deb+cd^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,abe\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(ae^2-deb+cd^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,acd\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(ae^2-deb+cd^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,acd\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(ae^2-deb+cd^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,b^2d\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(ae^2-deb+cd^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,b^2d\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(ae^2-deb+cd^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,ae\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(ae^2-deb+cd^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,ae\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(ae^2-deb+cd^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,bd\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(ae^2-deb+cd^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,bd\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(ae^2-deb+cd^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{d^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(ae^2-deb+cd^2\right)\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/(a*e^2-b*d*e+c*d^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*e+1/2/(a*e^2-b*d*e+c*d^2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*b*d+1/2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*c*x)*a*b*e+1/(a*e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*c*x)*a*d-1/2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x)*b^2*d+1/2/(a*e^2-b*d*e+c*d^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*e-1/2/(a*e^2-b*d*e
+c*d^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)*b*d+1/2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*c*x)*a*b*e+1/(a*e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*a*d-1/2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*b^2*d+d^2/(a*e^2-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{d^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(cd^2-bde+ae^2\right)\sqrt{de}}+\frac{-\int\frac{(bd-ae)x^2+ad}{cx^4+bx^2+a}\,dx}{cd^2-bde+ae^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] d^2*arctan(e*x/sqrt(d*e))/((c*d^2 - b*d*e + a*e^2)*sqrt(d*e)) + integrate(-
((b*d - a*e)*x^2 + a*d)/(c*x^4 + b*x^2 + a), x)/(c*d^2 - b*d*e + a*e^2)
```

mupad [B] time = 5.80, size = 25202, normalized size = 90.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2
 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a
^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*
a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3
*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e
^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4
*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*((x*(8*a^3*b^3*c*e^7 - 32*a^4*
b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2
*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^4*e^3 -
 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^
5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^
3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64
*a^3*b^2*c^2*d*e^6) + (-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)
^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e
- 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3
*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^
2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4
- 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2
- 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*
d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(64*a^2*c^6*d^6
*e^2 - x*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*
d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d
^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12
*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 1
6*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d
^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d
*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b
^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^
3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5
 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5
*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d
^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^
4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e
^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6
+ 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 64
0*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 128*a^3*c^5*d^4*e^4 + 64*a^4
*c^4*d^2*e^6 - 96*a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4
*c^2*d^2*e^6 - 144*a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*
d^6*e^2 + 16*a*b^3*c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^
5 - 64*a^2*b*c^5*d^5*e^3 - 16*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2
+ a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*
b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c
^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2
```

```
*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*
e^3)))^(1/2) + 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3 + 24
*a^2*b^2*c^2*d^3*e^3 - 4*a*b^2*c^3*d^5*e - 4*a*b^4*c*d^3*e^3 - 4*a^3*b^2*c*
d*e^5 - 4*a*b^3*c^2*d^4*e^2 + 20*a^2*b*c^3*d^4*e^2 + 8*a^2*b^3*c*d^2*e^4 -
16*a^3*b*c^2*d^2*e^4) + x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d^4*e -
8*a*b^2*c^2*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(
1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e -
7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c
^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*
c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 -
2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 -
2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*
e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*1i + ((-(b^5*d^2
+ a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*
c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a
*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*
c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2
 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*
d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*
a^2*b^3*c^2*d*e^3)))^(1/2)*((x*(8*a^3*b^3*c*e^7 - 32*a^4*b*c^2*e^7 - 112*a^
4*c^3*d*e^6 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2*c^5*d^5*e^2 + 32*
a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^4*e^3 - 32*a*b*c^5*d^6*e
- 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^
2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d
^3*e^4 + 64*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6
) - (-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(
-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 -
a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*
b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4
*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e
+ b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3
+ 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^
2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(x*(-(b^5*d^2 + a^2*b^3*e^2 + a^2
*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b
*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c
^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^
4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^
4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3))
)^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a
^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7
*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32
*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*
a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a
^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4
*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6
*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d
*e^8) + 64*a^2*c^6*d^6*e^2 + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6 - 96*
a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^6 - 144
*a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 16*a*b^3
*c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^5 - 64*a^2*b*c^5*d
^5*e^3 - 16*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c
 - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*
a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^
2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*
b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^
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4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2) - 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3 - 24*a^2*b^2*c^2*d^3*e^3 + 4*a*b^2*c^3*d^5*e + 4*a*b^4*c*d^3*e^3 + 4*a^3*b^2*c*d*e^5 + 4*a*b^3*c^2*d^4*e^2 - 20*a^2*b*c^3*d^4*e^2 - 8*a^2*b^3*c*d^2*e^4 + 16*a^3*b*c^2*d^2*e^4) + x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d^4*e - 8*a*b^2*c^2*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*1i)/(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*((x*(8*a^3*b^3*c*e^7 - 32*a^4*b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^4*e^3 - 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6) + (-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(64*a^2*c^6*d^6*e^2 - x*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6 - 96*a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^6 - 144*a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 16*a*b^3*c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^5 - 64*a^2*b*c^5*d^5*e^3 - 16*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5* c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2) + 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3 + 24*a^2*b^2*c^2*d^3*e^3 - 4*a*b^2*c^3*d^5*e - 4*a*b^4*c*d^3*e^3 - 4*a^3*b^2*c*d*e^5 - 4*a*b^3*c^2*d^4*e^2 + 20*a^2*b*c^3*d^4*e^2 + 8*a^2*b^3*c*d^2*e^4 - 16*a^3*b*c^2*d^2*e^4) + x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d^4*e - 8*a*b^2*c^2*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2) - ((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*((x*(8*a^3*b^3*c*e^7 - 32*a^4*b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^4*e^3 - 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6) - (-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(x*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 64*a^2*c^6*d^6*e^2 + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6 - 96*a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^6 - 144*a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 16*a*b^3*c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^5 - 64*a^2*b*c^5*d^5*e^3 - 16*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 +

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b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2
*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*
c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 +
 16*a^2*b^3*c^2*d*e^3)))^(1/2) - 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^
3*c^3*d^3*e^3 - 24*a^2*b^2*c^2*d^3*e^3 + 4*a*b^2*c^3*d^5*e + 4*a*b^4*c*d^3*
e^3 + 4*a^3*b^2*c*d*e^5 + 4*a*b^3*c^2*d^4*e^2 - 20*a^2*b*c^3*d^4*e^2 - 8*a^
2*b^3*c*d^2*e^4 + 16*a^3*b*c^2*d^2*e^4) + x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e
 + 2*b^4*c*d^4*e - 8*a*b^2*c^2*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-
(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^
2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*
b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^
(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4
 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*
a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e
 - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)
 + 2*a^3*c*d^2*e^2 + 2*a^2*b*c*d^3*e))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(
-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d
^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3
*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4
 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32
*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*
e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2
)*2i + atan((((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
 b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^
3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e
 + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^
4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*
c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^
5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 -
6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*((x*(8*a^3*b^3*c*e^7 -
32*a^4*b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 -
112*a^2*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^
4*e^3 - 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 -
 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3
*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e
^5 + 64*a^3*b^2*c^2*d*e^6) + (-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c -
b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b
^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 -
 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8
*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4
*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d
^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3
*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(64*a^2*
c^6*d^6*e^2 - x*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*
b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d
*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*
d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^
5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*
b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3
- 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9
- 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*
d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 -
192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^
2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3
*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^
6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d
^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e
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^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6 - 96*a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^6 - 144*a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 16*a*b^3*c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^5 - 64*a^2*b*c^5*d^5*e^3 - 16*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2) + 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3 + 24*a^2*b^2*c^2*d^3*e^3 - 4*a*b^2*c^3*d^5*e - 4*a*b^4*c*d^3*e^3 - 4*a^3*b^2*c*d*e^5 - 4*a*b^3*c^2*d^4*e^2 + 20*a^2*b*c^3*d^4*e^2 + 8*a^2*b^3*c*d^2*e^4 - 16*a^3*b*c^2*d^2*e^4) + x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d^4*e - 8*a*b^2*c^2*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*1i + ((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*((x*(8*a^3*b^3*c*e^7 - 32*a^4*b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^4*e^3 - 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6) - (-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(x*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^

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2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^
3*c^3*d*e^8) + 64*a^2*c^6*d^6*e^2 + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^
6 - 96*a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^
6 - 144*a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 1
6*a*b^3*c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^5 - 64*a^2*
b*c^5*d^5*e^3 - 16*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(
-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d
^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3
*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4
 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32
*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*
e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2
) - 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3 - 24*a^2*b^2*c^
2*d^3*e^3 + 4*a*b^2*c^3*d^5*e + 4*a*b^4*c*d^3*e^3 + 4*a^3*b^2*c*d*e^5 + 4*a
*b^3*c^2*d^4*e^2 - 20*a^2*b*c^3*d^4*e^2 - 8*a^2*b^3*c*d^2*e^4 + 16*a^3*b*c^
2*d^2*e^4) + x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d^4*e - 8*a*b^2*c^2
*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*
d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d
^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12
*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 1
6*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d
^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d
*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b
^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*1i)/(((-(b^5*d^2 + a^2*b^3*e
^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8
*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^
2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*
a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2
*d*e^3)))^(1/2)*((x*(8*a^3*b^3*c*e^7 - 32*a^4*b*c^2*e^7 - 112*a^4*c^3*d*e^6
 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2*c^5*d^5*e^2 + 32*a^3*c^4*d^3
*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^4*e^3 - 32*a*b*c^5*d^6*e - 48*a^2*b^
2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e
^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64
*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6) + (-(b^5*
d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b
^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(
4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e +
 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 +
b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2
*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*
c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 +
 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(64*a^2*c^6*d^6*e^2 - x*(-(b^5*d^2 + a^2*b^3
*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^
3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 -
 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*
b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 3
2*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c
^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e
^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b
^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^
4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*
e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e
^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 -
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672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152
*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3
*b^3*c^3*d*e^8) + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6 - 96*a^2*b^2*c^4
*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^6 - 144*a^3*b^2*c^
3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 16*a*b^3*c^4*d^5*e^
3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^5 - 64*a^2*b*c^5*d^5*e^3 - 16
*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^
(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e -
 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*
c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2
*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 -
 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 -
 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d
*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2) + 16*a^2*c^4*d^5
*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3 + 24*a^2*b^2*c^2*d^3*e^3 - 4*a*b^
2*c^3*d^5*e - 4*a*b^4*c*d^3*e^3 - 4*a^3*b^2*c*d*e^5 - 4*a*b^3*c^2*d^4*e^2 +
 20*a^2*b*c^3*d^4*e^2 + 8*a^2*b^3*c*d^2*e^4 - 16*a^3*b*c^2*d^2*e^4) + x*(2*
a^4*c*e^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d^4*e - 8*a*b^2*c^2*d^4*e))*(-(b^5*d^
2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2
*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^
4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e
^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^
3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 1
6*a^2*b^3*c^2*d*e^3)))^(1/2) - ((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*
a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^
2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*
b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^
4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*
a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*((x*(
8*a^3*b^3*c*e^7 - 32*a^4*b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^3*c^4*d^6*e +
8*b^6*c*d^3*e^4 - 112*a^2*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*
e^2 - 8*b^5*c^2*d^4*e^3 - 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2
*b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5
*e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*c^4*d^4*e^3 +
64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6) - (-(b^5*d^2 + a^2*b^3*e^2 - a
^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2
*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2)
 - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c -
 b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2
*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*
e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*
c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3
)))^(1/2)*(x*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3
*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e
+ 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4
 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c
^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5
*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6
*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 3
2*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4
*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192
*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c
^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^
2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d

^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 64*a^2*c^6*d^6*e^2 + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6 - 96*a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^6 - 144*a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 16*a*b^3*c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^5 - 64*a^2*b*c^5*d^5*e^3 - 16*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2) - 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3 - 24*a^2*b^2*c^2*d^3*e^3 + 4*a*b^2*c^3*d^5*e + 4*a*b^4*c*d^3*e^3 + 4*a^3*b^2*c*d*e^5 + 4*a*b^3*c^2*d^4*e^2 - 20*a^2*b*c^3*d^4*e^2 - 8*a^2*b^3*c*d^2*e^4 + 16*a^3*b*c^2*d^2*e^4) + x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d^4*e - 8*a*b^2*c^2*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2) + 2*a^3*c*d^2*e^2 + 2*a^2*b*c*d^3*e))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*2i - (log(a^5*d^2*e^8 - b^5*d^7*e^3 + 2*a*b^4*d^6*e^4 - 2*a^4*b*d^3*e^7 + 2*a^4*c*d^4*e^6 + b^4*c*d^8*e^2 + 16*a^2*c^3*x*(-d^3*e)^(5/2) + a^5*e^8*x*(-d^3*e)^(1/2) - a^2*b^3*d^5*e^5 + a^3*b^2*d^4*e^6 + 16*a^2*c^3*d^8*e^2 + 17*a^3*c^2*d^6*e^4 + b^4*c*x*(-d^3*e)^(5/2) + a^2*b^3*e^4*x*(-d^3*e)^(3/2) + b^5*d^2*e^2*x*(-d^3*e)^(3/2) + 7*a*b^3*c*d^7*e^3 + 2*a^3*b*c*d^5*e^5 - 8*a*b^2*c^2*x*(-d^3*e)^(5/2) - 8*a*b^2*c^2*d^8*e^2 - 12*a^2*b*c^2*d^7*e^3 - 12*a^2*b^2*c*d^6*e^4 - 2*a^3*b*c*e^4*x*(-d^3*e)^(3/2) - 2*a*b^4*d*e^3*x*(-d^3*e)^(3/2) - 2*a^4*b*d*e^7*x*(-d^3*e)^(1/2) - 17*a^3*c^2*d*e^3*x*(-d^3*e)^(3/2) + 2*a^4*c*d^2*e^6*x*(-d^3*e)^(1/2) + a^3*b^2*d^2*e^6*x*(-d^3*e)^(1/2) + 12*a^2*b*c^2*d^2*e^2*x*(-d^3*e)^(3/2) - 7*a*b^3*c*d^2*e^2*x*(-d^3*e)^(3/2) + 12*a^2*b^2*c*d*e^3*x*(-d^3*e)^(3/2))*(-d^3*e)^(1/2))/(2*(a*e^3 - b*d*e^2 + c*d^2*e)) + (log(a^5*d^2*e^8 - b^5*d^7*e^3 + 2*a*b^4*d^6*e^4 - 2*a^4*b*d^3*e^7 + 2*a^4*c*d^4*e^6 + b^4*c*d^8*e^2 - 16*a^2*c^3*x*(-d^3*e)^(5/2) - a^5*e^8*x*(-d^3*e)^(1/2) - a^2*b^3*d^5*e^5 + a^3*b^2*d^4*e^6 + 16*a^2*c^3*d^8*e^2 + 17*a^3*c^2*d^6*e^4 - b^4*c*x*(-d^3*e)^(5/2) - a^2*b^3*e^4*x*(-d^3*e)^(3/2) - b^5*d^2*e^2*x*(-d^3*e)^(3/2) + 7*a*b^3*c*d^7*e^3 + 2*a^3*b*c*d^5*e^5 + 8*a*b^2*c^2*x*(-d^3*e)^(5/2) - 8*a*b^2*c^2*d^8*e^2 - 12*a^2*b*c^2*d^7*e^3 - 12*a^2*b^2*c*d^6*e^4 + 2*a^3*b*c*e^4*x*(-d^3*e)^(3/2) + 2*a*b^4*d*e^3*x*(-d^3*e)^(3/2) + 2*a^4*b*d*e^7*x*(-d^3*e)^(1/2) + 17*a^3*c^2*d*e^3*x*(-d^3*e)^(3/2) - 2*a^4*c*d^2*e^6*x*(-d^3*e)^(1/2) - a^3*b^2*d^2*e^6*x*(-d^3*e)^(1/2) - 12*a^2*b*c^2*d^2*e^2*x*(-d^3*e)^(3/2) + 7*a*b^3*c*d^2*e^2*x*(-d^3*e)^(3/2) - 12*a^2*b^2*c*d*e^3*x*(-d^3*e)^(3/2))*(-d^3*e)^(1/2))/(2*a*e^3 - 2*b*d*e^2 + 2*c*d^2*e)

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x**2+d)/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

3.228 $\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)} dx$

Optimal. Leaf size=251

$$\frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}+\frac{\sqrt{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}-\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{ae^2-bde+cd^2}$$

Rubi [A] time = 0.45, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{1287, 205, 1166\}$

$$\frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}+\frac{\sqrt{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}-\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{ae^2-bde+cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[c]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 -
b*d*e + a*e^2)) + (Sqrt[c]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqr
t[d]])/(c*d^2 - b*d*e + a*e^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \int \left(-\frac{de}{(cd^2-bde+ae^2)(d+ex^2)} + \frac{ae+cdx^2}{(cd^2-bde+ae^2)(a+bx^2+cx^4)}\right) dx$$

$$= \frac{\int \frac{ae+cdx^2}{a+bx^2+cx^4}\,dx}{cd^2-bde+ae^2} - \frac{(de)\int \frac{1}{d+ex^2}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2-bde+ae^2} + \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2(cd^2-bde+ae^2)} + \cdots$$

$$= \frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} + \frac{\sqrt{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\cdots\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+\cdots}$$

Mathematica [A] time = 0.50, size = 277, normalized size = 1.10

$$-\frac{\sqrt{c}\left(d\sqrt{b^2-4ac}+2ae-bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-ae^2+bde-cd^2)} - \frac{\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\,(-ae^2+bde-cd^2)} - \frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{ae^2-bde+cd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[c]*(-(b*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^
2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2))) - (Sqrt[c]*(b*d + Sqrt[b^2 - 4*a*c
]*d - 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2
)) - (Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 - b*d*e + a*e^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]
```

fricas [B] time = 3.76, size = 12269, normalized size = 48.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^
2 + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2
*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)
*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4
*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6
*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42
*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d
^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*
```

```
a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b
^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3
- 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*e^2)*x
 + sqrt(1/2)*((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 - (2*(b^2*c^3
 - 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^2 -
4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*b^4 -
3*a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*sqrt((c^2*d^4 -
 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4
)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b
^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3
*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4
 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4
*b^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4
*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d
^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d
^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b
*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c -
 a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24
*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2
*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 +
(a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)
*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^
3 + (a^2*b^2 - 4*a^3*c)*e^4))) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b
*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*
c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*
d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(
(b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10
*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*
e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 -
a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2
)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^
2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a
^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*l
og(-2*(c^2*d^2 - a*c*e^2)*x - sqrt(1/2)*((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 -
 4*a^2*c)*e^3 - (2*(b^2*c^3 - 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e
+ 4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b
*c^2)*d^2*e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^
3*b*c)*e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d
^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^
4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*
c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*
c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^
3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*
e + a*b*e^2 + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4
 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2
- 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^
5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^
2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*
b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^
3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^
3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*
d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 -
 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))) + sqrt(1/2)*(c*d^
2 - b*d*e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4*a*c
^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e
^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 -
 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4
)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b
^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3
```

```
*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4
 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4
*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3
*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 +
(a^2*b^2 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*e^2)*x + sqrt(1/2)*((b^2*c
- 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 + (2*(b^2*c^3 - 4*a*c^4)*d^5 - 5*(
b^3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 -
(b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c
^2)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*
e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^
2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3
)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*
b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a
^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8))
)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*
c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4
*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 +
a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^
4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b
*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4
*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c -
 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e
^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b
^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*
c)*e^4))) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e +
a*b*e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2
*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*
a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d
^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^
4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*
c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*
c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^
3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4
- 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(
a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*
e^2)*x - sqrt(1/2)*((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 + (2*(b
^2*c^3 - 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*
c^2 - 4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*
b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*sqrt((c^2
*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a
*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c
 - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 -
24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a
^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7
+ (a^4*b^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c
^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*
c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt(
(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 -
 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b
^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^
2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*
(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*
e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*
b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c
)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))) + sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)
*x - d)/(e*x^2 + d)))/(c*d^2 - b*d*e + a*e^2), 1/2*(sqrt(1/2)*(c*d^2 - b*d*
e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4*a*c^3)*d^4
- 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(
a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d
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^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*e^2)*x + sqrt(1/2)*((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 - (2*(b^2*c^3 - 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*e^2)*x - sqrt(1/2)*((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 - (2*(b^2*c^3 - 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))) + sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^

2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*
e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b
^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a
*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^
3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^
2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*
d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*
c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2
*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*log
(-2*(c^2*d^2 - a*c*e^2)*x + sqrt(1/2)*((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4
*a^2*c)*e^3 + (2*(b^2*c^3 - 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e +
4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c
^2)*d^2*e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*
b*c)*e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8
 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)
*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c
- 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^
2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3
- 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*e
+ a*b*e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 -
 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 -
4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)
d^8 - 4(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*
c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^
4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*
b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*
b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^
4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2
*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))) - sqrt(1/2)*(c*d^2
- b*d*e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4*a*c^3
)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2
 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2
*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*
d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3
*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c
^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 -
 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b
^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e
 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a
^2*b^2 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*e^2)*x - sqrt(1/2)*((b^2*c -
4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 + (2*(b^2*c^3 - 4*a*c^4)*d^5 - 5*(b^
3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 - (b
^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2
)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^
4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2
- 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*
d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^
5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4
*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))*
sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c
- 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a
^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^
2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*
c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c
^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(
a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8
*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8
)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2
*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)

```
*e^4))) - 2*sqrt(d*e)*arctan(sqrt(d*e)*x/d))/(c*d^2 - b*d*e + a*e^2)]
```

giac [B] time = 8.45, size = 6921, normalized size = 27.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/(c*d^2 - b*d*e + a*e^2) - 1/8*((
2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^
4 - 2*(b^2 - 4*a*c)*b^2*c^4)*d^5 - 2*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*c^5
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 3*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^2 -
 4*a*c)*a*b*c^4)*d^4*e + (2*b^6*c^2 + 4*a*b^4*c^3 - 48*a^2*b^2*c^4 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 - 12*(b^2 - 4*a*c
)*a*b^2*c^3)*d^3*e^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c -
 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b
*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 -
 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 -
4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*d^2*abs(c*d^2 - b*d*e + a*e^2)*
e - (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*
c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2
*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*(c*d^2 - b*d*e + a*e^2)^2*d
 - 4*(2*a*b^5*c^2 - 6*a^2*b^3*c^3 - 8*a^3*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^3)*d
^2*e^3 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*
b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^2 - 4*a*c)*a*b^3*c
 - 8*(b^2 - 4*a*c)*a^2*b*c^2)*d*abs(c*d^2 - b*d*e + a*e^2)*e^2 + 5*(2*a^2*b
```

^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2)*d*e^4 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*a^2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 16*a^3*b^2*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 32*a^4*c^3 + 2*(b^2 - 4*a*c)*a^2*b^2*c - 8*(b^2 - 4*a*c)*a^3*c^2)*abs(c*d^2 - b*d*e + a*e^2)*e^3 - 2*(2*a^3*b^3*c^2 - 8*a^4*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^3*b*c^2)*e^5)*arctan(2*sqrt(1/2)*x/sqrt((b*c*d^2 - b^2*d*e + a*b*e^2 + sqrt((b*c*d^2 - b^2*d*e + a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*e + a^2*e^2)*(c^2*d^2 - b*c*d*e + a*c*e^2)))/(c^2*d^2 - b*c*d*e + a*c*e^2)))/((a*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*c^5)*d^4*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*d^3*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e + (a*b^6 - 6*a^2*b^4*c - 2*a*b^5*c + 4*a^2*b^3*c^2 + a*b^4*c^2 + 32*a^4*c^3 + 16*a^3*b*c^3 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^2 - 2*(a^2*b^5 - 8*a^3*b^3*c - 2*a^2*b^4*c + 16*a^4*b*c^2 + 8*a^3*b^2*c^2 + a^2*b^3*c^2 - 4*a^3*b*c^3)*d*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^3 + (a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^4) + 1/8*((2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*d^5 - 2*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^2 - 4*a*c)*a*b*c^4)*d^4*e + (2*b^6*c^2 + 4*a*b^4*c^3 - 48*a^2*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 - 12*(b^2 - 4*a*c)*a*b^2*c^3)*d^3*e^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*d^2*abs(c*d^2 - b*d*e + a*e^2)*e - (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -

```
 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3
- 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*(c*d^2 - b*d*e + a*e^2)^
2*d - 4*(2*a*b^5*c^2 - 6*a^2*b^3*c^3 - 8*a^3*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^3
)*d^2*e^3 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^3*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^3*b*c^3 - 2*(b^2 - 4*a*c)*a*b^
3*c + 8*(b^2 - 4*a*c)*a^2*b*c^2)*d*abs(c*d^2 - b*d*e + a*e^2)*e^2 + 5*(2*a^
2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2)*d*e^4 + 2*(sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^
2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*a^2*b^4*c + 1
6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b^2*c^2 - 16*a^3*b^2*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^
3 + 32*a^4*c^3 - 2*(b^2 - 4*a*c)*a^2*b^2*c + 8*(b^2 - 4*a*c)*a^3*c^2)*abs(c
*d^2 - b*d*e + a*e^2)*e^3 - 2*(2*a^3*b^3*c^2 - 8*a^4*b*c^3 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^3*b*c^2)*e^5)*ar
ctan(2*sqrt(1/2)*x/sqrt((b*c*d^2 - b^2*d*e + a*b*e^2 - sqrt((b*c*d^2 - b^2*
d*e + a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*e + a^2*e^2)*(c^2*d^2 - b*c*d*e + a*c
*e^2)))/(c^2*d^2 - b*c*d*e + a*c*e^2)))/((a*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a*b
^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*c^5)*d^4*abs(c*d^2 -
b*d*e + a*e^2)*abs(c) - 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16*a^3*b
*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*d^3*abs(c*d^2 - b*d*e + a*e
^2)*abs(c)*e + (a*b^6 - 6*a^2*b^4*c - 2*a*b^5*c + 4*a^2*b^3*c^2 + a*b^4*c^2
 + 32*a^4*c^3 + 16*a^3*b*c^3 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*abs(c*d^2 - b
*d*e + a*e^2)*abs(c)*e^2 - 2*(a^2*b^5 - 8*a^3*b^3*c - 2*a^2*b^4*c + 16*a^4*
b*c^2 + 8*a^3*b^2*c^2 + a^2*b^3*c^2 - 4*a^3*b*c^3)*d*abs(c*d^2 - b*d*e + a*
e^2)*abs(c)*e^3 + (a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4
*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^4)
```

maple [B] time = 0.02, size = 478, normalized size = 1.90

$$-\frac{\sqrt{2}\, a c e \arctanh\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(a e^2-d e b+c d^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, a c e \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(a e^2-d e b+c d^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, b c d \arctanh\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(a e^2-d e b+c d^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, b c d \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(a e^2-d e b+c d^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, c d \arctanh\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(a e^2-d e b+c d^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, c d \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(a e^2-d e b+c d^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{d e \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{\left(a e^2-d e b+c d^2\right)\sqrt{d e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/(a*e^2-b*d*e+c*d^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
h(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d-1/(a*e^2-b*d*e+c*d^2)*c/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*e+1/2/(a*e^2-b*d*e+c*d^2)*c/(-4*
```

```
a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d+1/2/(a*e^2-b*d*e+c*d^2)*c*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*d-1/(a*e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*
a*e+1/2/(a*e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d-d*e
/(a*e^2-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -d*e*arctan(e*x/sqrt(d*e))/((c*d^2 - b*d*e + a*e^2)*sqrt(d*e)) + integrate(
(c*d*x^2 + a*e)/(c*x^4 + b*x^2 + a), x)/(c*d^2 - b*d*e + a*e^2)
```

mupad [B] time = 4.96, size = 19401, normalized size = 77.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(b^4*d^3*e^5 - a*b^3*d^2*e^6 + a*c^3*d^5*e^3 - b^3*c*d^4*e^4 + 2*a^2*c^
2*d^3*e^5 + a^3*c*d*e^7 + b^4*e^3*x*(-d*e)^(5/2) + a*b^3*e^5*x*(-d*e)^(3/2)
 + a^3*c*e^7*x*(-d*e)^(1/2) + 2*a*b*c^2*d^4*e^4 - 3*a*b^2*c*d^3*e^5 + 2*a^2
*b*c*d^2*e^6 + 2*a^2*c^2*e^3*x*(-d*e)^(5/2) - a*c^3*d*x*(-d*e)^(7/2) + b^3*
c*e*x*(-d*e)^(7/2) - 2*a*b*c^2*e*x*(-d*e)^(7/2) - 3*a*b^2*c*e^3*x*(-d*e)^(5
/2) - 2*a^2*b*c*e^5*x*(-d*e)^(3/2))*(-d*e)^(1/2))/(2*a*e^2 + 2*c*d^2 - 2*b*
d*e) - atan(((x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (-(
a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b
^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d
*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d
^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d
*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^
3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*((x*(32*a^3*b*c^
3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*
c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8
*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c
^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6) + (-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3
)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^
2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^
4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^
2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2
*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a
^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) +
b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2
+ 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4
*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 +
32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6
*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d
*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 +
 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c
^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^
5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6
- 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 +
 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672
*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2
```

*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 192*a^4*c^4*d*e^7 - 192*a^2*c^6*d^5*e^3 - 384*a^3*c^5*d^3*e^5 - 96*a^2*b^2*c^4*d^3*e^5 - 96*a^2*b^3*c^3*d^2*e^6 + 48*a*b^2*c^5*d^5*e^3 - 96*a*b^3*c^4*d^4*e^4 + 48*a*b^4*c^3*d^3*e^5 + 384*a^2*b*c^5*d^4*e^4 + 384*a^3*b*c^4*d^2*e^6 + 48*a^3*b^2*c^3*d*e^7))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) - 4*a*c^5*d^4*e^2 - 52*a^2*c^4*d^2*e^4 + 8*a*b*c^4*d^3*e^3 - 4*a*b^3*c^2*d*e^5 + 20*a^2*b*c^3*d*e^5 + 8*a*b^2*c^3*d^2*e^4))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*1i + (x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*((x*(32*a^3*b*c^3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6) + (-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 192*a^4*c^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5 + 96*a^2*b^2*c^4*d^3*e^5 + 96*a^2*b^3*c^3*d^2*e^6 - 48*a*b^2*c^5*d^5*e^3 + 96*a*b^3*c^4*d^4*e^4 - 48*a*b^4*c^3*d^3*e^5 - 384*a^2*b*c^5*d^4*e^4 - 384*a^3*b*c^4*d^2*e^6 - 48*a^3*b^2*c^3*d*e^7))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4 - 8*a*b*c^4*d^3*e^3 + 4*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 8*a*b^2*c^3*d^2*e^4))*(-(a*b^3*e^2

- a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*1i)/((x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*((x*(32*a^3*b*c^3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6) + (-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 192*a^4*c^4*d*e^7 - 192*a^2*c^6*d^5*e^3 - 384*a^3*c^5*d^3*e^5 - 96*a^2*b^2*c^4*d^3*e^5 - 96*a^2*b^3*c^3*d^2*e^6 + 48*a*b^2*c^5*d^5*e^3 - 96*a*b^3*c^4*d^4*e^4 + 48*a*b^4*c^3*d^3*e^5 + 384*a^2*b*c^5*d^4*e^4 + 384*a^3*b*c^4*d^2*e^6 + 48*a^3*b^2*c^3*d*e^7))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) - 4*a*c^5*d^4*e^2 - 52*a^2*c^4*d^2*e^4 + 8*a*b*c^4*d^3*e^3 - 4*a*b^3*c^2*d*e^5 + 20*a^2*b*c^3*d*e^5 + 8*a*b^2*c^3*d^2*e^4))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) - (x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(

1/2)*((x*(32*a^3*b*c^3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6) + (-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 192*a^4*c^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5 + 96*a^2*b^2*c^4*d^3*e^5 + 96*a^2*b^3*c^3*d^2*e^6 - 48*a*b^2*c^5*d^5*e^3 + 96*a*b^3*c^4*d^4*e^4 - 48*a*b^4*c^3*d^3*e^5 - 384*a^2*b*c^5*d^4*e^4 - 384*a^3*b*c^4*d^2*e^6 - 48*a^3*b^2*c^3*d*e^7))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4 - 8*a*b*c^4*d^3*e^3 + 4*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 8*a*b^2*c^3*d^2*e^4))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) + 2*a*c^3*d*e^3))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*2i - (log(b^4*d^3*e^5 - a*b^3*d^2*e^6 + a*c^3*d^5*e^3 - b^3*c*d^4*e^4 + 2*a^2*c^2*d^3*e^5 + a^3*c*d*e^7 - b^4*e^3*x*(-d*e)^(5/2) - a*b^3*e^5*x*(-d*e)^(3/2) - a^3*c*e^7*x*(-d*e)^(1/2) + 2*a*b*c^2*d^4*e^4 - 3*a*b^2*c*d^3*e^5 + 2*a^2*b*c*d^2*e^6 - 2*a^2*c^2*e^3*x*(-d*e)^(5/2) + a*c^3*d*x*(-d*e)^(7/2) - b^3*c*e*x*(-d*e)^(7/2) + 2*a*b*c^2*e*x*(-d*e)^(7/2) + 3*a*b^2*c*e^3*x*(-d*e)^(5/2) + 2*a^2*b*c*e^5*x*(-d*e)^(3/2))*(-d*e)^(1/2))/(2*(a*e^2 + c*d^2 - b*d*e)) - atan(((x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1

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/2)*((x*(32*a^3*b*c^3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^
3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8
*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*
d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6) + (-(a*b^3*e^2 + a*e
^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4
*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^
2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*
d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^
2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^3*e^2 + a*e^2*(-(4*a*
c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*
d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*
a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4
- 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16
*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*
e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9
 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^
5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4
+ 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*
a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 1
28*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b
^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^
2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d
^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 192*a^4*c^4*d*e^7 - 192*a^2*c^6*d^5*e^3 -
 384*a^3*c^5*d^3*e^5 - 96*a^2*b^2*c^4*d^3*e^5 - 96*a^2*b^3*c^3*d^2*e^6 + 48
*a*b^2*c^5*d^5*e^3 - 96*a*b^3*c^4*d^4*e^4 + 48*a*b^4*c^3*d^3*e^5 + 384*a^2*
b*c^5*d^4*e^4 + 384*a^3*b*c^4*d^2*e^6 + 48*a^3*b^2*c^3*d*e^7))*(-(a*b^3*e^2
 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a
^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 -
8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*
b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e +
 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) - 4*a*c^5*d^4*e^2 - 52*a^
2*c^4*d^2*e^4 + 8*a*b*c^4*d^3*e^3 - 4*a*b^3*c^2*d*e^5 + 20*a^2*b*c^3*d*e^5
+ 8*a*b^2*c^3*d^2*e^4))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3
*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 1
6*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^
2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*
a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*
b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^
3)))^(1/2)*1i + (x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) -
(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c
- b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*
c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^
6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^
5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b
*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*((x*(32*a^3*b
*c^3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a
^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4
- 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*
b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6) + (-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2
)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4
*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4
*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3
*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*
c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 3
2*a^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2)
 + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e
^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*
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a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 192*a^4*c^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5 + 96*a^2*b^2*c^4*d^3*e^5 + 96*a^2*b^3*c^3*d^2*e^6 - 48*a*b^2*c^5*d^5*e^3 + 96*a*b^3*c^4*d^4*e^4 - 48*a*b^4*c^3*d^3*e^5 - 384*a^2*b*c^5*d^4*e^4 - 384*a^3*b*c^4*d^2*e^6 - 48*a^3*b^2*c^3*d*e^7))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4 - 8*a*b*c^4*d^3*e^3 + 4*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 8*a*b^2*c^3*d^2*e^4))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*1i)/((x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*((x*(32*a^3*b*c^3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6) + (-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 192*a^4*c^4*d*e^7 - 192*a^2*c^6*d^5*e^3 - 384*a^3*c^5*d^3*e^5 - 96*a^2*b^2*

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c^4*d^3*e^5 - 96*a^2*b^3*c^3*d^2*e^6 + 48*a*b^2*c^5*d^5*e^3 - 96*a*b^3*c^4*
d^4*e^4 + 48*a*b^4*c^3*d^3*e^5 + 384*a^2*b*c^5*d^4*e^4 + 384*a^3*b*c^4*d^2*
e^6 + 48*a^3*b^2*c^3*d*e^7))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^
2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a
^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4
+ 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e -
 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2
*d*e^3)))^(1/2) - 4*a*c^5*d^4*e^2 - 52*a^2*c^4*d^2*e^4 + 8*a*b*c^4*d^3*e^3
- 4*a*b^3*c^2*d*e^5 + 20*a^2*b*c^3*d*e^5 + 8*a*b^2*c^3*d^2*e^4))*(-(a*b^3*e
^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^
(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*
(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2
- 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 -
2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e
 + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) - (x*(2*a^2*c^3*e^5 - 4
*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^
3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a
^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d
^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b
^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^
2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*
a^3*b*c^2*d*e^3)))^(1/2)*((x*(32*a^3*b*c^3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3
*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 +
8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^
3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6)
 + (-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a
*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b
^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 +
 b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a
*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^
2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^
3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^
3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/
(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e
^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3
 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^
3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9
 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6
*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 -
 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b
^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^
3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c
^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*
d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*
e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 192*a^4*c^4*d*e^7 +
192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5 + 96*a^2*b^2*c^4*d^3*e^5 + 96*a^2
*b^3*c^3*d^2*e^6 - 48*a*b^2*c^5*d^5*e^3 + 96*a*b^3*c^4*d^4*e^4 - 48*a*b^4*c
^3*d^3*e^5 - 384*a^2*b*c^5*d^4*e^4 - 384*a^3*b*c^4*d^2*e^6 - 48*a^3*b^2*c^3
*d*e^7))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e -
 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2
*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2
 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 -
 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) + 4*
a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4 - 8*a*b*c^4*d^3*e^3 + 4*a*b^3*c^2*d*e^5
- 20*a^2*b*c^3*d*e^5 - 8*a*b^2*c^3*d^2*e^4))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c
- b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^
2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^
```

```
2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 -
8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a
*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^
3 - 32*a^3*b*c^2*d*e^3)))^(1/2) + 2*a*c^3*d*e^3))*(-(a*b^3*e^2 + a*e^2*(-(4
*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c
^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 +
16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d
^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e +
 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c
*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.229 $\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=254

$$-\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,\left(ae^2-bde+cd^2\right)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2-bde+cd^2\right)}$$

Rubi [A] time = 0.52, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1170, 205, 1166}

$$-\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,\left(ae^2-bde+cd^2\right)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[c]*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2))) - (Sqrt[c]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^
2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \int \left(\frac{e^2}{(cd^2-bde+ae^2)(d+ex^2)} + \frac{cd-be-cex^2}{(cd^2-bde+ae^2)(a+bx^2+cx^4)}\right) dx$$

$$= \frac{\int \frac{cd-be-cex^2}{a+bx^2+cx^4}\,dx}{cd^2-bde+ae^2} + \frac{e^2\int \frac{1}{d+ex^2}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)} - \frac{\left(c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2(cd^2-bde+ae^2)} - \frac{\left(c\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right.\right.\cdots}{2(c\cdots}$$

$$= -\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\sqrt{c}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4a\cdots}}}\right.}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+a\cdots}$$

Mathematica [A] time = 0.25, size = 274, normalized size = 1.08

$$\frac{\sqrt{c}\left(e\sqrt{b^2-4ac}+be-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(-ae^2+bde-cd^2\right)} + \frac{\sqrt{c}\left(e\sqrt{b^2-4ac}-be+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\left(-ae^2+bde-cd^2\right)} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[c]*(-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 -
4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) + (Sqrt[c]*(2*c*d - b*e + Sqrt[b^2 - 4*
a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) +
(e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 9.13, size = 7650, normalized size = 30.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*(2*(2*b^3*c^5 - 8*a*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*
c^5 - 2*(b^2 - 4*a*c)*b*c^5)*d^5 - 5*(2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*d^4*e +
2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^3*c^3 - 2*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 8
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^2*c^4 + 16*a*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*c^5 - 32*a^2*c^5 + 2*(b^2 - 4*a*c)*b^2*c^3 - 8*(b^2 - 4*a*c)*a*c^
4)*d^3*abs(c*d^2 - b*d*e + a*e^2) + 4*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*c^
5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 3*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^2
- 4*a*c)*a*b*c^4)*d^3*e^2 - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*
c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a
*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*d^2*abs(c*d^2 - b*d*e + a*e^2)*e - (
2*b^6*c^2 + 4*a*b^4*c^3 - 48*a^2*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^5*c + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^2*c^2 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^4*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 - 12*(b^2 - 4*a*c)*a*b^2*c^3)*d^2*e^3 + 2*(
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 7*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 2*
b^6*c + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 6*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^4*c^2 + 14*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 3*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*b^4*c -
6*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*d*abs(c*d^2 - b*d*e +
a*e^2)*e^2 - (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*(c*d^2 - b*d*e + a
*e^2)^2*e + 2*(2*a*b^5*c^2 - 6*a^2*b^3*c^3 - 8*a^3*b*c^4 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 + 3*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
```

```
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 - 2*(b^2 - 4*a*c)*a^2
*b*c^3)*d*e^4 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 - 4*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^2 - 4*a*c)*
a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2)*abs(c*d^2 - b*d*e + a*e^2)*e^3 - (2*a^
2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2)*e^5)*arctan(2*sqrt(1/2)*x/sqrt((b*
c*d^2 - b^2*d*e + a*b*e^2 + sqrt((b*c*d^2 - b^2*d*e + a*b*e^2)^2 - 4*(a*c*d
^2 - a*b*d*e + a^2*e^2)*(c^2*d^2 - b*c*d*e + a*c*e^2)))/(c^2*d^2 - b*c*d*e
+ a*c*e^2)))/((a*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2
*b*c^4 + a*b^2*c^4 - 4*a^2*c^5)*d^4*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(
a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^
3*c^3 - 4*a^2*b*c^4)*d^3*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e + (a*b^6 - 6*a
^2*b^4*c - 2*a*b^5*c + 4*a^2*b^3*c^2 + a*b^4*c^2 + 32*a^4*c^3 + 16*a^3*b*c^
3 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^2 -
2*(a^2*b^5 - 8*a^3*b^3*c - 2*a^2*b^4*c + 16*a^4*b*c^2 + 8*a^3*b^2*c^2 + a^2
*b^3*c^2 - 4*a^3*b*c^3)*d*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^3 + (a^3*b^4
- 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^
4*c^3)*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^4) - 1/8*(2*(2*b^3*c^5 - 8*a*b*c
^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^5 - 2*(b^2 - 4*a*c)*b*c^5
)*d^5 - 5*(2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*d^4*e - 2*(sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 2*b^4*c^3 + 16*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 1
6*a*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^5 + 32*a^2*c^5
- 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*d^3*abs(c*d^2 - b*d*e +
a*e^2) + 4*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^2 - 4*a*c)*a*b*c^4)*d^3*e^2 +
 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 +
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*
c)*a*b*c^3)*d^2*abs(c*d^2 - b*d*e + a*e^2)*e - (2*b^6*c^2 + 4*a*b^4*c^3 - 4
8*a^2*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
```

```
^6 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 24*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 12*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 6*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^4
*c^2 - 12*(b^2 - 4*a*c)*a*b^2*c^3)*d^2*e^3 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b^6 - 7*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 2*b^6*c + 8*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 14*a*b^4*
c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*b^4*c + 6*(b^2 - 4*a*c)*a*b^2*c^2 +
 8*(b^2 - 4*a*c)*a^2*c^3)*d*abs(c*d^2 - b*d*e + a*e^2)*e^2 - (2*b^4*c^2 - 1
6*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*
c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 -
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2
*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*(c*d^2 - b*d*e + a*e^2)^2*e + 2*(2*a*b^5*c^2
- 6*a^2*b^3*c^3 - 8*a^3*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^5 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^3*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*
c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 -
 2*(b^2 - 4*a*c)*a*b^3*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^3)*d*e^4 + 2*(sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*a*
b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^3*c^2 - 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^3 + 32*a^3*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a
^2*b*c^2)*abs(c*d^2 - b*d*e + a*e^2)*e^3 - (2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)
*a^2*b^2*c^2)*e^5)*arctan(2*sqrt(1/2)*x/sqrt((b*c*d^2 - b^2*d*e + a*b*e^2 -
 sqrt((b*c*d^2 - b^2*d*e + a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*e + a^2*e^2)*(c^
2*d^2 - b*c*d*e + a*c*e^2)))/(c^2*d^2 - b*c*d*e + a*c*e^2)))/((a*b^4*c^2 -
8*a^2*b^2*c^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*
c^5)*d^4*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2
*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*d^3*ab
s(c*d^2 - b*d*e + a*e^2)*abs(c)*e + (a*b^6 - 6*a^2*b^4*c - 2*a*b^5*c + 4*a^
2*b^3*c^2 + a*b^4*c^2 + 32*a^4*c^3 + 16*a^3*b*c^3 - 2*a^2*b^2*c^3 - 8*a^3*c
^4)*d^2*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^2 - 2*(a^2*b^5 - 8*a^3*b^3*c -
2*a^2*b^4*c + 16*a^4*b*c^2 + 8*a^3*b^2*c^2 + a^2*b^3*c^2 - 4*a^3*b*c^3)*d*a
bs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^3 + (a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c
 + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c*d^2 - b*d*e +
a*e^2)*abs(c)*e^4) + arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/((c*d^2 - b*d*e + a*
e^2)*sqrt(d))
```

maple [B] time = 0.03, size = 480, normalized size = 1.89

$$\frac{\sqrt{2}\, bce \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(a e^2 - deb + c d^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}} + \frac{\sqrt{2}\, bce \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(a e^2 - deb + c d^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}} - \frac{\sqrt{2}\, c^2 d \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(a e^2 - deb + c d^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}} - \frac{\sqrt{2}\, c^2 d \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(a e^2 - deb + c d^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}} + \frac{\sqrt{2}\, ce \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(a e^2 - deb + c d^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}} - \frac{\sqrt{2}\, ce \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(a e^2 - deb + c d^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}} + \frac{e^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(a e^2 - deb + c d^2\right)\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/(a*e^2-b*d*e+c*d^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e+1/2/(a*e^2-b*d*e+c*d^2)*c
/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e-1/(a*e^2-b*d*e+c*d^2)*c^2/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d-1/2/(a*e^2-b*d*e+c*d^2)*c*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*c*x)*e+1/2/(a*e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*b*e-1/(a*e^2-b*d*e+c*d^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d+e^2/
(a*e^2-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] e^2*arctan(e*x/sqrt(d*e))/((c*d^2 - b*d*e + a*e^2)*sqrt(d*e)) - integrate((
c*e*x^2 - c*d + b*e)/(c*x^4 + b*x^2 + a), x)/(c*d^2 - b*d*e + a*e^2)
```

mupad [B] time = 5.61, size = 23640, normalized size = 93.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2
 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a
*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3
*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2
 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3
*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*
b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d
^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2
*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*
b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 +
b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a
*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c
*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^
2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 -
 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^
3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^
2*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b
^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*
c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) -
16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*
(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*
```

c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c^7*d^6*e^2 - 16*a^2*b^4*c^2*e^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4 - 448*a^3*c^5*d^2*e^6 - 16*b^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c^4*d^4*e^4 + 64*b^5*c^3*d^3*e^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*e^6 - 256*a*b*c^6*d^5*e^3 + 32*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*a*b^2*c^5*d^4*e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*b*c^5*d^3*e^5 - 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - 4*b^3*c^3*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16*a*b*c^4*e^6 - 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*1i + ((-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e

```
 - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 -
 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*
d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 +
128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^
2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128
*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3
*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*
d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3
*e^6 - 288*a^3*b^3*c^3*d*e^8) - 64*a*c^7*d^6*e^2 + 16*a^2*b^4*c^2*e^8 - 128
*a^3*b^2*c^3*e^8 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6 + 16*b^2*c^6*d
^6*e^2 - 64*b^3*c^5*d^5*e^3 + 96*b^4*c^4*d^4*e^4 - 64*b^5*c^3*d^3*e^5 + 16*
b^6*c^2*d^2*e^6 - 240*a^2*b^2*c^4*d^2*e^6 + 256*a*b*c^6*d^5*e^3 - 32*a*b^5*
c^2*d*e^7 - 384*a^3*b*c^4*d*e^7 - 416*a*b^2*c^5*d^4*e^4 + 288*a*b^3*c^4*d^3
*e^5 - 32*a*b^4*c^3*d^2*e^6 - 128*a^2*b*c^5*d^3*e^5 + 224*a^2*b^3*c^3*d*e^7
))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-
(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7
*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2
*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4
*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2
*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e -
 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*
c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) + 4*b^3*c^3*e^6 + 4*c^6*d^3*e^3 -
4*b*c^5*d^2*e^4 - 4*b^2*c^4*d*e^5 - 16*a*b*c^4*e^6 + 20*a*c^5*d*e^5) + 6*c^
5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*
d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d
^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12
*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a
^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e
^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d
^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^
2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*1i)/(((-(b^5*e^2 + b^3*c^2*d
^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a
*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^
2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*
a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d
^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 +
192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3
 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*
d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*
c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2
*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4
*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c
^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32
*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(x*(
-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b
^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2
*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4
 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2
*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*
a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*
d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*
e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4
*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e
```

^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 2
88*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7
+ 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*
a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6
*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^
5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c^7*d^6*e^2 - 1
6*a^2*b^4*c^2*e^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4 - 448*a^3*c^5
*d^2*e^6 - 16*b^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c^4*d^4*e^4 + 6
4*b^5*c^3*d^3*e^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*e^6 - 256*a*b*
c^6*d^5*e^3 + 32*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*a*b^2*c^5*d^4*
e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*b*c^5*d^3*e^5
- 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2
)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*
d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16
*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*
e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*
e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*
c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - 4*b^3*c^3
*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16*a*b*c^4*e^6 -
20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c
- b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*
b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2
) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))
/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*
b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^
3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*
a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - ((-
(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*
c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^
3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*
d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4
+ 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*
b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a
^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d
^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 -
112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^
6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d
^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d
^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a
*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^
2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*
a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d
^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2
- 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(
1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8
*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a
^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3
- 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*
(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d
^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 1
28*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2
*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*
c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4

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*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4
*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4
 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) -
64*a*c^7*d^6*e^2 + 16*a^2*b^4*c^2*e^8 - 128*a^3*b^2*c^3*e^8 + 128*a^2*c^6*d
^4*e^4 + 448*a^3*c^5*d^2*e^6 + 16*b^2*c^6*d^6*e^2 - 64*b^3*c^5*d^5*e^3 + 96
*b^4*c^4*d^4*e^4 - 64*b^5*c^3*d^3*e^5 + 16*b^6*c^2*d^2*e^6 - 240*a^2*b^2*c^
4*d^2*e^6 + 256*a*b*c^6*d^5*e^3 - 32*a*b^5*c^2*d*e^7 - 384*a^3*b*c^4*d*e^7
 - 416*a*b^2*c^5*d^4*e^4 + 288*a*b^3*c^4*d^3*e^5 - 32*a*b^4*c^3*d^2*e^6 - 12
8*a^2*b*c^5*d^3*e^5 + 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 + b^
2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^
2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d
^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3
*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)
))^(1/2) + 4*b^3*c^3*e^6 + 4*c^6*d^3*e^3 - 4*b*c^5*d^2*e^4 - 4*b^2*c^4*d*e^
5 - 16*a*b*c^4*e^6 + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^
2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*
b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2
*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a
^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^
2*e^2)))^(1/2)))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2
) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a
*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*
d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^
4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b
^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a
*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3
 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*2i + atan((((-(b^5*e
^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^
2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^
2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e +
2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a
^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*
e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c
^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e -
 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*
b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32
*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5
 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 - b
^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2
*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c -
 b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c
^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*
d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^
3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2
)))^(1/2)*(x*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c
^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e
+ 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 +
16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d
^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5
*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 1
6*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 3
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2*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c^7*d^6*e^2 - 16*a^2*b^4*c^2*e^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4 - 448*a^3*c^5*d^2*e^6 - 16*b^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c^4*d^4*e^4 + 64*b^5*c^3*d^3*e^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*e^6 - 256*a*b*c^6*d^5*e^3 + 32*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*a*b^2*c^5*d^4*e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*b*c^5*d^3*e^5 - 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - 4*b^3*c^3*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16*a*b*c^4*e^6 - 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*1i + ((-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4

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*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e
^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^
2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 -
672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*
a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*
b^3*c^3*d*e^8) - 64*a*c^7*d^6*e^2 + 16*a^2*b^4*c^2*e^8 - 128*a^3*b^2*c^3*e^
8 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6 + 16*b^2*c^6*d^6*e^2 - 64*b^3
*c^5*d^5*e^3 + 96*b^4*c^4*d^4*e^4 - 64*b^5*c^3*d^3*e^5 + 16*b^6*c^2*d^2*e^6
 - 240*a^2*b^2*c^4*d^2*e^6 + 256*a*b*c^6*d^5*e^3 - 32*a*b^5*c^2*d*e^7 - 384
*a^3*b*c^4*d*e^7 - 416*a*b^2*c^5*d^4*e^4 + 288*a*b^3*c^4*d^3*e^5 - 32*a*b^4
*c^3*d^2*e^6 - 128*a^2*b*c^5*d^3*e^5 + 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 +
 b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 +
a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*
c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c
^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3
- 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d
^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a
^2*b^4*c*d^2*e^2)))^(1/2) + 4*b^3*c^3*e^6 + 4*c^6*d^3*e^3 - 4*b*c^5*d^2*e^4
 - 4*b^2*c^4*d*e^5 - 16*a*b*c^4*e^6 + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5
*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*
e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e
+ 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16
*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*
d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b
*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e
 - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*1i)/(((-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-
(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^
2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^
(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 -
8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*
a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3
 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)
*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e
^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d
*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a
*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/
2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*
a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3
*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e
^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*
b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*
a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^
3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^5*e^2 + b^3
*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*
e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*
e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e
^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*
a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e
 + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b
^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c
^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 -
32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^
3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*
d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d
^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e
```

^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 -
1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288
*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c^7*d^6*e^2 - 16*a^2*b^4*c^2*e
^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4 - 448*a^3*c^5*d^2*e^6 - 16*b
^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c^4*d^4*e^4 + 64*b^5*c^3*d^3*e
^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*e^6 - 256*a*b*c^6*d^5*e^3 + 3
2*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*a*b^2*c^5*d^4*e^4 - 288*a*b^3
*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*b*c^5*d^3*e^5 - 224*a^2*b^3*c
^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^
2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3
*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e +
12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16
*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2
*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c
*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*
a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - 4*b^3*c^3*e^6 - 4*c^6*d^
3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16*a*b*c^4*e^6 - 20*a*c^5*d*e^5
) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2
) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a
*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*
d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^
4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b
^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a
*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3
 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - ((-(b^5*e^2 + b^3*
c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/
2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e
^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^
4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a
^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e
+ 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^
4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e
^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^
4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b
*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-
(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^
2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^
(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 -
8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*
a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3
 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)
*(256*a^4*c^4*e^8 + x*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)
^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e
- 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2
*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b
^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4
+ a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2
- 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*
d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^
3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^
3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*
e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*
a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 3
84*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*
b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5
*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c
^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 64*a*c^7*d^6*e^

```
2 + 16*a^2*b^4*c^2*e^8 - 128*a^3*b^2*c^3*e^8 + 128*a^2*c^6*d^4*e^4 + 448*a^
3*c^5*d^2*e^6 + 16*b^2*c^6*d^6*e^2 - 64*b^3*c^5*d^5*e^3 + 96*b^4*c^4*d^4*e^
4 - 64*b^5*c^3*d^3*e^5 + 16*b^6*c^2*d^2*e^6 - 240*a^2*b^2*c^4*d^2*e^6 + 256
*a*b*c^6*d^5*e^3 - 32*a*b^5*c^2*d*e^7 - 384*a^3*b*c^4*d*e^7 - 416*a*b^2*c^5
*d^4*e^4 + 288*a*b^3*c^4*d^3*e^5 - 32*a*b^4*c^3*d^2*e^6 - 128*a^2*b*c^5*d^3
*e^5 + 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b
^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b
^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3
*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a
^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) + 4*b^
3*c^3*e^6 + 4*c^6*d^3*e^3 - 4*b*c^5*d^2*e^4 - 4*b^2*c^4*d*e^5 - 16*a*b*c^4*
e^6 + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2
 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(
1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8
*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a
^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3
 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2))
)*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*
a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*
c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*
d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*
a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e -
32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c
^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*2i - (log(b^5*d*(-d*e^3)^(5/2) - b^
5*d^3*e^8*x + c^5*d^8*e^3*x + 2*a*c^4*d^5*(-d*e^3)^(3/2) - 16*a^3*c^2*e*(-d
*e^3)^(5/2) - c^5*d^8*e*(-d*e^3)^(1/2) + b^2*c^3*d^5*(-d*e^3)^(3/2) - a*b^4
*e*(-d*e^3)^(5/2) - 7*a*b^3*c*d*(-d*e^3)^(5/2) + 17*a^2*c^3*d^3*e^2*(-d*e^3
)^(3/2) + a*b^4*d^2*e^9*x + 2*a*c^4*d^6*e^5*x - 2*b*c^4*d^7*e^4*x + 2*b^4*c
*d^4*e^7*x + 12*a^2*b*c^2*d*(-d*e^3)^(5/2) + 8*a^2*b^2*c*e*(-d*e^3)^(5/2) +
 17*a^2*c^3*d^4*e^7*x + 16*a^3*c^2*d^2*e^9*x + b^2*c^3*d^6*e^5*x - b^3*c^2*
d^5*e^6*x - b^3*c^2*d^4*e*(-d*e^3)^(3/2) + 2*b^4*c*d^3*e^2*(-d*e^3)^(3/2) +
 2*b*c^4*d^7*e^2*(-d*e^3)^(1/2) - 12*a*b^2*c^2*d^4*e^7*x - 12*a^2*b*c^2*d^3
*e^8*x - 8*a^2*b^2*c*d^2*e^9*x - 12*a*b^2*c^2*d^3*e^2*(-d*e^3)^(3/2) + 2*a*
b*c^3*d^5*e^6*x + 7*a*b^3*c*d^3*e^8*x + 2*a*b*c^3*d^4*e*(-d*e^3)^(3/2))*(-d
*e^3)^(1/2))/(2*(c*d^3 + a*d*e^2 - b*d^2*e)) + (log(b^5*d*(-d*e^3)^(5/2) +
b^5*d^3*e^8*x - c^5*d^8*e^3*x + 2*a*c^4*d^5*(-d*e^3)^(3/2) - 16*a^3*c^2*e*(
-d*e^3)^(5/2) - c^5*d^8*e*(-d*e^3)^(1/2) + b^2*c^3*d^5*(-d*e^3)^(3/2) - a*b
^4*e*(-d*e^3)^(5/2) - 7*a*b^3*c*d*(-d*e^3)^(5/2) + 17*a^2*c^3*d^3*e^2*(-d*e
^3)^(3/2) - a*b^4*d^2*e^9*x - 2*a*c^4*d^6*e^5*x + 2*b*c^4*d^7*e^4*x - 2*b^4
*c*d^4*e^7*x + 12*a^2*b*c^2*d*(-d*e^3)^(5/2) + 8*a^2*b^2*c*e*(-d*e^3)^(5/2)
 - 17*a^2*c^3*d^4*e^7*x - 16*a^3*c^2*d^2*e^9*x - b^2*c^3*d^6*e^5*x + b^3*c^
2*d^5*e^6*x - b^3*c^2*d^4*e*(-d*e^3)^(3/2) + 2*b^4*c*d^3*e^2*(-d*e^3)^(3/2)
 + 2*b*c^4*d^7*e^2*(-d*e^3)^(1/2) + 12*a*b^2*c^2*d^4*e^7*x + 12*a^2*b*c^2*d
^3*e^8*x + 8*a^2*b^2*c*d^2*e^9*x - 12*a*b^2*c^2*d^3*e^2*(-d*e^3)^(3/2) - 2*
a*b*c^3*d^5*e^6*x - 7*a*b^3*c*d^3*e^8*x + 2*a*b*c^3*d^4*e*(-d*e^3)^(3/2))*(
-d*e^3)^(1/2))/(2*c*d^3 + 2*a*d*e^2 - 2*b*d^2*e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.230 $\int \frac{1}{x^2(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=298

$$-\frac{\sqrt{c}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}-\frac{e^{5/2}\,\text{t}}{d^{3/2}\left(ae\right.}$$

Rubi [A] time = 0.96, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1287, 205, 1166}

$$-\frac{\sqrt{c}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(ae^2-bde+cd^2\right)}-\frac{1}{adx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(1/(a*d*x)) - (Sqrt[c]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*
Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*(c*d - b*e
- (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2)) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 -
 b*d*e + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^2\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \int\left(\frac{1}{adx^2} - \frac{e^3}{d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} + \frac{-bcd+b^2e-ace-c(cd-b}{a\left(cd^2-bde+ae^2\right)\left(a+bx^2\right.}\right.$$

$$= -\frac{1}{adx} + \frac{\int \frac{-bcd+b^2e-ace-c(cd-be)x^2}{a+bx^2+cx^4}\,dx}{a\left(cd^2-bde+ae^2\right)} - \frac{e^3\int \frac{1}{d+ex^2}\,dx}{d\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{1}{adx} - \frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2-bde+ae^2\right)} - \frac{\left(c\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}}}{2a\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{1}{adx} - \frac{\sqrt{c}\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)} - \frac{\sqrt{c}\left(cd-be-\right.}{\sqrt{2}\,a\sqrt{b+}}$$

Mathematica [A] time = 0.40, size = 340, normalized size = 1.14

$$-\frac{\sqrt{c}\left(cd\sqrt{b^2-4ac}-be\sqrt{b^2-4ac}+2ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(e(ae-bd)+cd^2\right)}+\frac{\sqrt{c}\left(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}+2ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\left(e(ae-bd)+cd^2\right)}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(ae^2-bde+cd^2\right)}-\frac{1}{adx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(1/(a*d*x)) - (Sqrt[c]*(b*c*d + c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e -
b*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(-
(b*d) + a*e))) + (Sqrt[c]*(b*c*d - c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e
+ b*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*
(-(b*d) + a*e))) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 -
b*d*e + a*e^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 12.82, size = 10058, normalized size = 33.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] -1/8*((2*a^2*b^4*c^5 - 8*a^3*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c^5 - 2*(b^2 - 4*a*c)*a^2*b^2*c^5)*d^5 - (6*a^2*b^
5*c^4 - 28*a^3*b^3*c^5 + 16*a^4*b*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b*c^5 - 6*(b^2 - 4*a*c)*a^2*b^3*c^4 + 4*(b^2 - 4*a*c)
*a^3*b*c^5)*d^4*e + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 -
8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^
4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*
(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*d^3*abs(a*c*d^2 - a*b*
d*e + a^2*e^2) + (6*a^2*b^6*c^3 - 28*a^3*b^4*c^4 + 16*a^4*b^2*c^5 - 3*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c + 14*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 6*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 8*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 4*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 3*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 - 6*(b^2 - 4*a*c)
*a^2*b^4*c^3 + 4*(b^2 - 4*a*c)*a^3*b^2*c^4)*d^3*e^2 - 2*(2*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 17*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^4*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 4*a*b
^6*c^2 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 18*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + 2*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^4*c^3 + 34*a^2*b^4*c^3 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^4*c^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 -
 9*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 80*a^3*b^2*c^4 + 4
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 32*a^4*c^5 + 4*(b^2 - 4*
a*c)*a*b^4*c^2 - 18*(b^2 - 4*a*c)*a^2*b^2*c^3 + 8*(b^2 - 4*a*c)*a^3*c^4)*d^
2*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*e + (2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 -
4*a*c)*a*c^4)*(a*c*d^2 - a*b*d*e + a^2*e^2)^2*d - (2*a^2*b^7*c^2 - 4*a^3*b^
5*c^3 - 24*a^4*b^3*c^4 + 32*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^6*c + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^4*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^5*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^4*b^2*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^3*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 - 4*(b^2 - 4*a*c)*a^3*b^3*c^3 +
```

```
8*(b^2 - 4*a*c)*a^4*b*c^4)*d^2*e^3 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^7 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 2*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*a*b^7*c + 16*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^4*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + 16*
a^2*b^5*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 32*a^
3*b^3*c^3 + 2*(b^2 - 4*a*c)*a*b^5*c - 8*(b^2 - 4*a*c)*a^2*b^3*c^2)*d*abs(a*
c*d^2 - a*b*d*e + a^2*e^2)*e^2 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*
c)*a*b*c^3)*(a*c*d^2 - a*b*d*e + a^2*e^2)^2*e + (4*a^3*b^6*c^2 - 22*a^4*b^4
*c^3 + 24*a^5*b^2*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*b^6 + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^4*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^3*b^5*c - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^5*b^2*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^4*b^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*b^4*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4
*b^2*c^3 - 4*(b^2 - 4*a*c)*a^3*b^4*c^2 + 6*(b^2 - 4*a*c)*a^4*b^2*c^3)*d*e^4
 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6 - 9*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^5*c - 2*a^2*b^6*c + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*
b^2*c^2 + 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 18*a^3*b^4*c^2 - 16*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^4*b*c^3 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 -
 48*a^4*b^2*c^3 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 32*a^
5*c^4 + 2*(b^2 - 4*a*c)*a^2*b^4*c - 10*(b^2 - 4*a*c)*a^3*b^2*c^2 + 8*(b^2 -
 4*a*c)*a^4*c^3)*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*e^3 - (2*a^4*b^5*c^2 - 12
*a^5*b^3*c^3 + 16*a^6*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^4*b^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^5*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^4*b^4*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^6*b*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^5*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*
b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b
*c^3 - 2*(b^2 - 4*a*c)*a^4*b^3*c^2 + 4*(b^2 - 4*a*c)*a^5*b*c^3)*e^5)*arctan
(2*sqrt(1/2)*x/sqrt((a*b*c*d^2 - a*b^2*d*e + a^2*b*e^2 + sqrt((a*b*c*d^2 -
a*b^2*d*e + a^2*b*e^2)^2 - 4*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*(a*c^2*d^2 -
 a*b*c*d*e + a^2*c*e^2)))/(a*c^2*d^2 - a*b*c*d*e + a^2*c*e^2)))/((a^3*b^4*c
^2 - 8*a^4*b^2*c^3 - 2*a^3*b^3*c^3 + 16*a^5*c^4 + 8*a^4*b*c^4 + a^3*b^2*c^4
 - 4*a^4*c^5)*d^4*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c) - 2*(a^3*b^5*c -
8*a^4*b^3*c^2 - 2*a^3*b^4*c^2 + 16*a^5*b*c^3 + 8*a^4*b^2*c^3 + a^3*b^3*c^3
- 4*a^4*b*c^4)*d^3*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c)*e + (a^3*b^6 - 6
*a^4*b^4*c - 2*a^3*b^5*c + 4*a^4*b^3*c^2 + a^3*b^4*c^2 + 32*a^6*c^3 + 16*a^
5*b*c^3 - 2*a^4*b^2*c^3 - 8*a^5*c^4)*d^2*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*a
bs(c)*e^2 - 2*(a^4*b^5 - 8*a^5*b^3*c - 2*a^4*b^4*c + 16*a^6*b*c^2 + 8*a^5*b
^2*c^2 + a^4*b^3*c^2 - 4*a^5*b*c^3)*d*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(
c)*e^3 + (a^5*b^4 - 8*a^6*b^2*c - 2*a^5*b^3*c + 16*a^7*c^2 + 8*a^6*b*c^2 +
a^5*b^2*c^2 - 4*a^6*c^3)*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c)*e^4) + 1/8
*((2*a^2*b^4*c^5 - 8*a^3*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
```

- 4*a*c)*c)*a^2*b^2*c^5 - 2*(b^2 - 4*a*c)*a^2*b^2*c^5)*d^5 - (6*a^2*b^5*c^4 - 28*a^3*b^3*c^5 + 16*a^4*b*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 - 6*(b^2 - 4*a*c)*a^2*b^3*c^4 + 4*(b^2 - 4*a*c)*a^3*b*c^5)*d^4*e - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*d^3*abs(a*c*d^2 - a*b*d*e + a^2*e^2) + (6*a^2*b^6*c^3 - 28*a^3*b^4*c^4 + 16*a^4*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 - 6*(b^2 - 4*a*c)*a^2*b^4*c^3 + 4*(b^2 - 4*a*c)*a^3*b^2*c^4)*d^3*e^2 + 2*(2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 17*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + 4*a*b^6*c^2 + 40*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 18*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 34*a^2*b^4*c^3 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 80*a^3*b^2*c^4 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^5 - 32*a^4*c^5 - 4*(b^2 - 4*a*c)*a*b^4*c^2 + 18*(b^2 - 4*a*c)*a^2*b^2*c^3 - 8*(b^2 - 4*a*c)*a^3*c^4)*d^2*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*e + (2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*(a*c*d^2 - a*b*d*e + a^2*e^2)^2*d - (2*a^2*b^7*c^2 - 4*a^3*b^5*c^3 - 24*a^4*b^3*c^4 + 32*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 - 4*(b^2 - 4*a*c)*a^3*b^3*c^3 + 8*(b^2 - 4*a*c)*a^4*b*c^4)*d^2*e^3 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c + 2*a*b^7*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 16*a^2*

```
b^5*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + 32*a^3*b^
3*c^3 - 2*(b^2 - 4*a*c)*a*b^5*c + 8*(b^2 - 4*a*c)*a^2*b^3*c^2)*d*abs(a*c*d^
2 - a*b*d*e + a^2*e^2)*e^2 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a
*b*c^3)*(a*c*d^2 - a*b*d*e + a^2*e^2)^2*e + (4*a^3*b^6*c^2 - 22*a^4*b^4*c^3
 + 24*a^5*b^2*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^3*b^6 + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^4*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*b^5*c - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*
b^2*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b
^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^
4*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2
*c^3 - 4*(b^2 - 4*a*c)*a^3*b^4*c^2 + 6*(b^2 - 4*a*c)*a^4*b^2*c^3)*d*e^4 + 2
*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6 - 9*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^5*c + 2*a^2*b^6*c + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*
c^2 + 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 18*a^3*b^4*c^2 - 16*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^4*b*c^3 - 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 48*
a^4*b^2*c^3 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^4 - 32*a^5*c^
4 - 2*(b^2 - 4*a*c)*a^2*b^4*c + 10*(b^2 - 4*a*c)*a^3*b^2*c^2 - 8*(b^2 - 4*a
*c)*a^4*c^3)*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*e^3 - (2*a^4*b^5*c^2 - 12*a^5
*b^3*c^3 + 16*a^6*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^4*b^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^5*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^4*b^4*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
6*b*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b
^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*
c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3
 - 2*(b^2 - 4*a*c)*a^4*b^3*c^2 + 4*(b^2 - 4*a*c)*a^5*b*c^3)*e^5)*arctan(2*s
qrt(1/2)*x/sqrt((a*b*c*d^2 - a*b^2*d*e + a^2*b*e^2 - sqrt((a*b*c*d^2 - a*b^
2*d*e + a^2*b*e^2)^2 - 4*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*(a*c^2*d^2 - a*b
*c*d*e + a^2*c*e^2)))/(a*c^2*d^2 - a*b*c*d*e + a^2*c*e^2)))/((a^3*b^4*c^2 -
 8*a^4*b^2*c^3 - 2*a^3*b^3*c^3 + 16*a^5*c^4 + 8*a^4*b*c^4 + a^3*b^2*c^4 - 4
*a^4*c^5)*d^4*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c) - 2*(a^3*b^5*c - 8*a^
4*b^3*c^2 - 2*a^3*b^4*c^2 + 16*a^5*b*c^3 + 8*a^4*b^2*c^3 + a^3*b^3*c^3 - 4*
a^4*b*c^4)*d^3*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c)*e + (a^3*b^6 - 6*a^4
*b^4*c - 2*a^3*b^5*c + 4*a^4*b^3*c^2 + a^3*b^4*c^2 + 32*a^6*c^3 + 16*a^5*b*
c^3 - 2*a^4*b^2*c^3 - 8*a^5*c^4)*d^2*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c
)*e^2 - 2*(a^4*b^5 - 8*a^5*b^3*c - 2*a^4*b^4*c + 16*a^6*b*c^2 + 8*a^5*b^2*c
^2 + a^4*b^3*c^2 - 4*a^5*b*c^3)*d*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c)*e
^3 + (a^5*b^4 - 8*a^6*b^2*c - 2*a^5*b^3*c + 16*a^7*c^2 + 8*a^6*b*c^2 + a^5*
b^2*c^2 - 4*a^6*c^3)*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c)*e^4) - arctan(
x*e^(1/2)/sqrt(d))*e^(5/2)/((c*d^3 - b*d^2*e + a*d*e^2)*sqrt(d)) - 1/(a*d*x
)
```

maple [B] time = 0.03, size = 817, normalized size = 2.74

$$-\frac{\sqrt{2}\, b^2 c e \arctan\!\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(a e^2-d e b+c d^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\, a}-\frac{\sqrt{2}\, b^2 c e \arctan\!\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(a e^2-d e b+c d^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\, a}+\frac{\sqrt{2}\, b c^2 d \,\mathrm{arctanh}\!\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(a e^2-d e b+c d^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\, a}+\frac{\sqrt{2}\, b c^2 d \arctan\!\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(a e^2-d e b+c d^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\, a}+\frac{\sqrt{2}\, c^2 e \,\mathrm{arctanh}\!\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(a e^2-d e b+c d^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, c^2 e \arctan\!\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(a e^2-d e b+c d^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, b c e \,\mathrm{arctanh}\!\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(a e^2-d e b+c d^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\, a}+\frac{\sqrt{2}\, b c e \arctan\!\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(a e^2-d e b+c d^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\, a}+\frac{\sqrt{2}\, c^2 d \,\mathrm{arctanh}\!\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(a e^2-d e b+c d^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\, a}-\frac{\sqrt{2}\, c^2 d \arctan\!\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(a e^2-d e b+c d^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\, a}-\frac{e^3 \arctan\!\left(\frac{e x}{\sqrt{d e}}\right)}{\left(a e^2-d e b+c d^2\right)\sqrt{d e}\, d}-\frac{1}{a d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/a/d/x-1/2/(a*e^2-b*d*e+c*d^2)/a*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e+1/2/(a*e^2-b
*d*e+c*d^2)/a*c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d+1/(a*e^2-b*d*e+c*d^2)*c^2/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e-1/2/(a*e^2-b*d*e+c*d^2)/a*c/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*e+1/2/(a*e^2-b*d*e+c*d^2)/a*c^2/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d+1/2/(a*e^2-b*d*e+c*d^2)/a*c*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*b*e-1/2/(a*e^2-b*d*e+c*d^2)/a*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d+1/(a*e^2-b*d
*e+c*d^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e-1/2/(a*e^2-b*d*e+c*d^
2)/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*e+1/2/(a*e^2-b*d*e+c*d^2)/
a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d-1/d*e^3/(a*e^2-b*d*e+c*d^2)
/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{e^{3}\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(cd^{3}-bd^{2}e+ade^{2}\right)\sqrt{de}}+\frac{-\int\frac{bcd+\left(c^{2}d-bce\right)x^{2}-\left(b^{2}-ac\right)e}{cx^{4}+bx^{2}+a}\,dx}{acd^{2}-abde+a^{2}e^{2}}-\frac{1}{adx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -e^3*arctan(e*x/sqrt(d*e))/((c*d^3 - b*d^2*e + a*d*e^2)*sqrt(d*e)) + integr
ate(-(b*c*d + (c^2*d - b*c*e)*x^2 - (b^2 - a*c)*e)/(c*x^4 + b*x^2 + a), x)/
(a*c*d^2 - a*b*d*e + a^2*e^2) - 1/(a*d*x)
```

mupad [B] time = 5.89, size = 33644, normalized size = 112.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^
3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*
b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^
3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^
4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c
*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6
*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^
6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 1
6*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c
*d^2*e^2)))^(1/2)*(((-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(
1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^
2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2
) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2
*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^
2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3
*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*
```

c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(192*a^10*c^7*d^14*e^3 - x*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9 - 32*a^9*b^3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d^15*e^3 - 192*a^9*b^5*c^4*d^14*e^4 + 128*a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*c^2*d^12*e^6 - 640*a^10*b^2*c^6*d^15*e^3 + 1056*a^10*b^3*c^5*d^14*e^4 - 672*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*c^3*d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^7 + 512*a^11*b^2*c^5*d^13*e^5 + 288*a^11*b^3*c^4*d^12*e^6 - 192*a^11*b^4*c^3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8 + 384*a^12*b^2*c^4*d^11*e^7 - 288*a^12*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*d^9*e^9 + 256*a^13*b^2*c^3*d^9*e^9 + 128*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*c^6*d^14*e^4 - 640*a^12*b*c^5*d^12*e^6 + 640*a^13*b*c^4*d^10*e^8) + 128*a^11*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*c^6*d^15*e^2 + 64*a^8*b^4*c^5*d^14*e^3 - 96*a^8*b^5*c^4*d^13*e^4 + 64*a^8*b^6*c^3*d^12*e^5 - 16*a^8*b^7*c^2*d^11*e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*a^9*b^3*c^5*d^13*e^4 - 352*a^9*b^4*c^4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 + 16*a^9*b^6*c^2*d^10*e^7 + 352*a^10*b^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11*e^6 - 128*a^10*b^4*c^3*d^10*e^7 + 16*a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c^4*d^10*e^7 - 128*a^11*b^3*c^3*d^9*e^8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12*b^2*c^3*d^8*e^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^11*b*c^5*d^11*e^6 + 256*a^12*b*c^4*d^9*e^8) + x*(112*a^10*c^6*d^10*e^6 - 32*a^9*c^7*d^12*e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2 - 16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5 - 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^5 + 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^10*e^6 + 208*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^7*e^9 + 96*a^10*b^2*c^4*d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^13*e^3 + 128*a^9*b*c^6*d^11*e^5 - 192*a^10*b*c^5*d^9*e^7 + 96*a^11*b*c^4*d^7*e^9))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8 - 4*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^6*c^2*d^7*e^8 + 24*a^8*b^3*c^4*d^8*e^7 - 28*a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^2*c^4*d^7*e^8 - 4*a^7*b*c^7*d^12*e^3 - 32*a^9*b*c^5*d^8*e^7) + x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7 + 2*a^7*b^2*c^5*d^7*e^7))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^

3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*1i - ((-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(((-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9 - 32*a^9*b^3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d^15*e^3 - 192*a^9*b^5*c^4*d^14*e^4 + 128*a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*c^2*d^12*e^6 - 640*a^10*b^2*c^6*d^15*e^3 + 1056*a^10*b^3*c^5*d^14*e^4 - 672*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*c^3*d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^7 + 512*a^11*b^2*c^5*d^13*e^5 + 288*a^11*b^3*c^4*d^12*e^6 - 192*a^11*b^4*c^3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8 + 384*a^12*b^2*c^4*d^11*e^7 - 288*a^12*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*d^9*e^9 + 256*a^13*b^2*c^3*d^9*e^9 + 128*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*c^6*d^14*e^4 - 640*a^12*b*c^5*d^12*e^6 + 640*a^13*b*c^4*d^10*e^8) + 192*a^10*c^7*d^14*e^3 + 128*a^11*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*c^6*d^15*e^2 + 64*a^8*b^4*c^5*d^14*e^3 - 96*a^8*b^5*c^4*d^13*e^4 + 64*a^8*b^6*c^3*d^12*e^5 - 16*a^8*b^7*c^2*d^11*e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*a^9*b^3*c^5*d^13*e^4 - 352*a^9*b^4*c^4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 + 16*a^9*b^6*c^2*d^10*e^7 + 352*a^10*b^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11*e^6 - 128*a^10*b^4*c^3*d^10*e^7 + 16*a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c^4*d^10*e^7 - 128*a^11*b^3*c^3*d^9*e^8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12*b^2*c^3*d^8*e^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^11*b*c^5*d^11*e^6 + 256*a^12*b*c^4*d^9*e^8) - x*(112*a^10*c^6*d^10*e^6 - 32*a^9*c^7*d^12*e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2 - 16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5 - 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^5 + 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^10*e^6 + 208*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^7*e^9 + 96*a^10*b^2*c^4*d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^13*e^3 + 128*a^9*b*c^6*d^11*e^5 - 192*a^10*b*c^5*d^9

```
*e^7 + 96*a^11*b*c^4*d^7*e^9))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c -
 b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*
c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9
*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b
^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e
+ 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4
+ 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*
a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e
- 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3
*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^8*c^7*
d^11*e^4 - 16*a^10*c^5*d^7*e^8 - 4*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^6*c^2*d^7*
e^8 + 24*a^8*b^3*c^4*d^8*e^7 - 28*a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^2*c^4*d^7*
e^8 - 4*a^7*b*c^7*d^12*e^3 - 32*a^9*b*c^5*d^8*e^7) - x*(2*a^7*c^7*d^9*e^5 -
 4*a^8*c^6*d^7*e^7 + 2*a^7*b^2*c^5*d^7*e^7))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4
*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*
d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*
c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2
)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*
e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^
2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 1
6*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^
4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^
3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^
3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*1i)/(((-(b^7*e^2 +
b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b
*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*
d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d
^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2
*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8
*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b
^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c
^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 -
32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*((
(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d
^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e
^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2
) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e +
 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)
^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*
e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e
^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b
^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^
2)))^(1/2)*(192*a^10*c^7*d^14*e^3 - x*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(
4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2
 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*
a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c
^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c
^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d
^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c
*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*
a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 5
12*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9 - 32*a^
9*b^3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d^15*e^3 - 192*a^9*b^5*c^4*d^14*e^4 +
128*a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*c^2*d^12*e^6 - 640*a^10*b^2*c^6*d^15*
```

e^3 + 1056*a^10*b^3*c^5*d^14*e^4 - 672*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*
c^3*d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^7 + 512*a^11*b^2*c^5*d^13*e^5 + 288*a
^11*b^3*c^4*d^12*e^6 - 192*a^11*b^4*c^3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8
 + 384*a^12*b^2*c^4*d^11*e^7 - 288*a^12*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*
d^9*e^9 + 256*a^13*b^2*c^3*d^9*e^9 + 128*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*
c^6*d^14*e^4 - 640*a^12*b*c^5*d^12*e^6 + 640*a^13*b*c^4*d^10*e^8) + 128*a^1
1*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*
c^6*d^15*e^2 + 64*a^8*b^4*c^5*d^14*e^3 - 96*a^8*b^5*c^4*d^13*e^4 + 64*a^8*b
^6*c^3*d^12*e^5 - 16*a^8*b^7*c^2*d^11*e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*
a^9*b^3*c^5*d^13*e^4 - 352*a^9*b^4*c^4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 +
 16*a^9*b^6*c^2*d^10*e^7 + 352*a^10*b^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11
*e^6 - 128*a^10*b^4*c^3*d^10*e^7 + 16*a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c
^4*d^10*e^7 - 128*a^11*b^3*c^3*d^9*e^8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12
*b^2*c^3*d^8*e^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^
11*b*c^5*d^11*e^6 + 256*a^12*b*c^4*d^9*e^8) + x*(112*a^10*c^6*d^10*e^6 - 32
*a^9*c^7*d^12*e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*
c^7*d^14*e^2 - 16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5
*c^4*d^11*e^5 - 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^
3*c^5*d^11*e^5 + 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^
9*b^2*c^5*d^10*e^6 + 208*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a
^9*b^5*c^2*d^7*e^9 + 96*a^10*b^2*c^4*d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32
*a^8*b*c^7*d^13*e^3 + 128*a^9*b*c^6*d^11*e^5 - 192*a^10*b*c^5*d^9*e^7 + 96*
a^11*b*c^4*d^7*e^9))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(
1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-
(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e
^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/
2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^
2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c
^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^
3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b
*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e
 - 6*a^4*b^4*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^8*c^7*d^11*e^4 -
 16*a^10*c^5*d^7*e^8 - 4*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^6*c^2*d^7*e^8 + 24*a
^8*b^3*c^4*d^8*e^7 - 28*a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^2*c^4*d^7*e^8 - 4*a^
7*b*c^7*d^12*e^3 - 32*a^9*b*c^5*d^8*e^7) + x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6
*d^7*e^7 + 2*a^7*b^2*c^5*d^7*e^7))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a
*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 +
a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
 - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c
 - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*
d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*
d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4
 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^
3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4
*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2) + ((-(b^7*e^2 + b^5*c^2*d^2 +
 b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*
c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*
b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c -
 b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*
c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3
6*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4
 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^
3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 -
2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*
d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(((-(b^7*e^2 +
b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b
*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*

```
d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d
^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2
*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8
*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b
^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c
^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 -
32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(x
*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*
d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*
e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/
2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e
+ 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c
*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*
e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*
b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e
^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d
^11*e^7 - 512*a^14*c^4*d^9*e^9 - 32*a^9*b^3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*
d^15*e^3 - 192*a^9*b^5*c^4*d^14*e^4 + 128*a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7
*c^2*d^12*e^6 - 640*a^10*b^2*c^6*d^15*e^3 + 1056*a^10*b^3*c^5*d^14*e^4 - 67
2*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*c^3*d^12*e^6 + 32*a^10*b^6*c^2*d^11*e
^7 + 512*a^11*b^2*c^5*d^13*e^5 + 288*a^11*b^3*c^4*d^12*e^6 - 192*a^11*b^4*c
^3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8 + 384*a^12*b^2*c^4*d^11*e^7 - 288*a^
12*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*d^9*e^9 + 256*a^13*b^2*c^3*d^9*e^9 +
128*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*c^6*d^14*e^4 - 640*a^12*b*c^5*d^12*e^
6 + 640*a^13*b*c^4*d^10*e^8) + 192*a^10*c^7*d^14*e^3 + 128*a^11*c^6*d^12*e^
5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*c^6*d^15*e^2
+ 64*a^8*b^4*c^5*d^14*e^3 - 96*a^8*b^5*c^4*d^13*e^4 + 64*a^8*b^6*c^3*d^12*e
^5 - 16*a^8*b^7*c^2*d^11*e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*a^9*b^3*c^5*d
^13*e^4 - 352*a^9*b^4*c^4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 + 16*a^9*b^6*c
^2*d^10*e^7 + 352*a^10*b^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11*e^6 - 128*a^
10*b^4*c^3*d^10*e^7 + 16*a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c^4*d^10*e^7 -
 128*a^11*b^3*c^3*d^9*e^8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12*b^2*c^3*d^8*
e^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^11*b*c^5*d^11
*e^6 + 256*a^12*b*c^4*d^9*e^8) - x*(112*a^10*c^6*d^10*e^6 - 32*a^9*c^7*d^12
*e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2
- 16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5
 - 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^
5 + 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^1
0*e^6 + 208*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^
7*e^9 + 96*a^10*b^2*c^4*d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^
13*e^3 + 128*a^9*b*c^6*d^11*e^5 - 192*a^10*b*c^5*d^9*e^7 + 96*a^11*b*c^4*d^
7*e^9))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b
^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3
*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)
^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c
^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*
c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*
c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^
6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b
^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e +
16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*
c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*
d^7*e^8 - 4*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^6*c^2*d^7*e^8 + 24*a^8*b^3*c^4*d^
8*e^7 - 28*a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^2*c^4*d^7*e^8 - 4*a^7*b*c^7*d^12*
e^3 - 32*a^9*b*c^5*d^8*e^7) - x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7 + 2*
a^7*b^2*c^5*d^7*e^7))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)
```

^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^
(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(
-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*
e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1
/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c
^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*
c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c
^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*
b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*
e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a
*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 +
a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
 - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c
 - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*
d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*
d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4
- 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^
3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4
*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*2i + atan((((-(b^7*e^2 + b^5*
c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4
*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e
+ 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(
-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e
 + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^
5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d
*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d
^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a
^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(((-(b
^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 +
 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 -
 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*
a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^
(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4
- 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 +
 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c
*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))
^(1/2)*(192*a^10*c^7*d^14*e^3 - x*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*
c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a
^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c
- b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d
*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d
^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 -
 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3
*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*
b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a
^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9 - 32*a^9*b^
3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d^15*e^3 - 192*a^9*b^5*c^4*d^14*e^4 + 128*
a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*c^2*d^12*e^6 - 640*a^10*b^2*c^6*d^15*e^3
+ 1056*a^10*b^3*c^5*d^14*e^4 - 672*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*c^3*
d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^7 + 512*a^11*b^2*c^5*d^13*e^5 + 288*a^11*
b^3*c^4*d^12*e^6 - 192*a^11*b^4*c^3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8 + 3
84*a^12*b^2*c^4*d^11*e^7 - 288*a^12*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*d^9*
e^9 + 256*a^13*b^2*c^3*d^9*e^9 + 128*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*c^6*

d^14*e^4 - 640*a^12*b*c^5*d^12*e^6 + 640*a^13*b*c^4*d^10*e^8) + 128*a^11*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*c^6*d^15*e^2 + 64*a^8*b^4*c^5*d^14*e^3 - 96*a^8*b^5*c^4*d^13*e^4 + 64*a^8*b^6*c^3*d^12*e^5 - 16*a^8*b^7*c^2*d^11*e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*a^9*b^3*c^5*d^13*e^4 - 352*a^9*b^4*c^4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 + 16*a^9*b^6*c^2*d^10*e^7 + 352*a^10*b^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11*e^6 - 128*a^10*b^4*c^3*d^10*e^7 + 16*a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c^4*d^10*e^7 - 128*a^11*b^3*c^3*d^9*e^8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12*b^2*c^3*d^8*e^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^11*b*c^5*d^11*e^6 + 256*a^12*b*c^4*d^9*e^8) + x*(112*a^10*c^6*d^10*e^6 - 32*a^9*c^7*d^12*e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2 - 16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5 - 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^5 + 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^10*e^6 + 208*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^7*e^9 + 96*a^10*b^2*c^4*d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^13*e^3 + 128*a^9*b*c^6*d^11*e^5 - 192*a^10*b*c^5*d^9*e^7 + 96*a^11*b*c^4*d^7*e^9))*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8 - 4*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^6*c^2*d^7*e^8 + 24*a^8*b^3*c^4*d^8*e^7 - 28*a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^2*c^4*d^7*e^8 - 4*a^7*b*c^7*d^12*e^3 - 32*a^9*b*c^5*d^8*e^7) + x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7 + 2*a^7*b^2*c^5*d^7*e^7))*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*1i - ((-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(((-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^

5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9 - 32*a^9*b^3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d^15*e^3 - 192*a^9*b^5*c^4*d^14*e^4 + 128*a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*c^2*d^12*e^6 - 640*a^10*b^2*c^6*d^15*e^3 + 1056*a^10*b^3*c^5*d^14*e^4 - 672*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*c^3*d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^7 + 512*a^11*b^2*c^5*d^13*e^5 + 288*a^11*b^3*c^4*d^12*e^6 - 192*a^11*b^4*c^3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8 + 384*a^12*b^2*c^4*d^11*e^7 - 288*a^12*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*d^9*e^9 + 256*a^13*b^2*c^3*d^9*e^9 + 128*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*c^6*d^14*e^4 - 640*a^12*b*c^5*d^12*e^6 + 640*a^13*b*c^4*d^10*e^8) + 192*a^10*c^7*d^14*e^3 + 128*a^11*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*c^6*d^15*e^2 + 64*a^8*b^4*c^5*d^14*e^3 - 96*a^8*b^5*c^4*d^13*e^4 + 64*a^8*b^6*c^3*d^12*e^5 - 16*a^8*b^7*c^2*d^11*e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*a^9*b^3*c^5*d^13*e^4 - 352*a^9*b^4*c^4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 + 16*a^9*b^6*c^2*d^10*e^7 + 352*a^10*b^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11*e^6 - 128*a^10*b^4*c^3*d^10*e^7 + 16*a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c^4*d^10*e^7 - 128*a^11*b^3*c^3*d^9*e^8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12*b^2*c^3*d^8*e^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^11*b*c^5*d^11*e^6 + 256*a^12*b*c^4*d^9*e^8) - x*(112*a^10*c^6*d^10*e^6 - 32*a^9*c^7*d^12*e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2 - 16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5 - 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^5 + 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^10*e^6 + 208*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^7*e^9 + 96*a^10*b^2*c^4*d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^13*e^3 + 128*a^9*b*c^6*d^11*e^5 - 192*a^10*b*c^5*d^9*e^7 + 96*a^11*b*c^4*d^7*e^9))*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8 - 4*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^6*c^2*d^7*e^8 + 24*a^8*b^3*c^4*d^8*e^7 - 28*a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^2*c^4*d^7*e^8 - 4*a^7*b*c^7*d^12*e^3 - 32*a^9*b*c^5*d^8*e^7) - x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7 + 2*a^7*b^2*c^5*d^7*e^7))*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^

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2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c
^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^
3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b
*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e
 - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*1i)/(((-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(
4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2
 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*
a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c
^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c
^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d
^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c
*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*
a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(((-(b^7*e^2 + b^5*c^2*d^2
 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 +
a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^
2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c
 - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^
3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) -
 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e
^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 +
a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2
- 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^
2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(192*a^10*c^7
*d^14*e^3 - x*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) -
 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) -
20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c
- b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16
*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*
a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(
-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4
- 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 +
 a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^
3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^
4*b^4*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 5
12*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9 - 32*a^9*b^3*c^6*d^16*e^2 + 128
*a^9*b^4*c^5*d^15*e^3 - 192*a^9*b^5*c^4*d^14*e^4 + 128*a^9*b^6*c^3*d^13*e^5
 - 32*a^9*b^7*c^2*d^12*e^6 - 640*a^10*b^2*c^6*d^15*e^3 + 1056*a^10*b^3*c^5*
d^14*e^4 - 672*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*c^3*d^12*e^6 + 32*a^10*b
^6*c^2*d^11*e^7 + 512*a^11*b^2*c^5*d^13*e^5 + 288*a^11*b^3*c^4*d^12*e^6 - 1
92*a^11*b^4*c^3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8 + 384*a^12*b^2*c^4*d^11
*e^7 - 288*a^12*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*d^9*e^9 + 256*a^13*b^2*c
^3*d^9*e^9 + 128*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*c^6*d^14*e^4 - 640*a^12*
b*c^5*d^12*e^6 + 640*a^13*b*c^4*d^10*e^8) + 128*a^11*c^6*d^12*e^5 - 320*a^1
2*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*c^6*d^15*e^2 + 64*a^8*b^
4*c^5*d^14*e^3 - 96*a^8*b^5*c^4*d^13*e^4 + 64*a^8*b^6*c^3*d^12*e^5 - 16*a^8
*b^7*c^2*d^11*e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*a^9*b^3*c^5*d^13*e^4 - 3
52*a^9*b^4*c^4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 + 16*a^9*b^6*c^2*d^10*e^7
 + 352*a^10*b^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11*e^6 - 128*a^10*b^4*c^3*
d^10*e^7 + 16*a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c^4*d^10*e^7 - 128*a^11*b
^3*c^3*d^9*e^8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12*b^2*c^3*d^8*e^9 + 64*a^
9*b*c^7*d^15*e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^11*b*c^5*d^11*e^6 + 256*
a^12*b*c^4*d^9*e^8) + x*(112*a^10*c^6*d^10*e^6 - 32*a^9*c^7*d^12*e^4 - 16*a
^8*c^8*d^14*e^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2 - 16*a^7*b^
3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5 - 16*a^7*b
^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^5 + 128*a^8
*b^4*c^4*d^10*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^10*e^6 + 208
*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^7*e^9 + 96*
```

a^10*b^2*c^4*d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^13*e^3 + 12
8*a^9*b*c^6*d^11*e^5 - 192*a^10*b*c^5*d^9*e^7 + 96*a^11*b*c^4*d^7*e^9))*(-(
b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2
+ 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2
- 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16
*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4
- 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2
+ 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*
c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2))
)^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8 - 4
*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^6*c^2*d^7*e^8 + 24*a^8*b^3*c^4*d^8*e^7 - 28*
a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^2*c^4*d^7*e^8 - 4*a^7*b*c^7*d^12*e^3 - 32*a^
9*b*c^5*d^8*e^7) + x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7 + 2*a^7*b^2*c^5
*d^7*e^7))*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*
a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*
a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b
^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^
3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b
^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8
*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^
3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e
+ 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b
^4*c*d^2*e^2)))^(1/2) + ((-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^
3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^
2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5
*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)
^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*
b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a
^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^
2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a
^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d
^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(((-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(
4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2
- a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*
a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c
^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c
^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d
^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c
*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*
a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^7*e^2 + b^5*c^2*d^
2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 +
a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a
^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*
c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b
^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*
e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 +
a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2
- 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c
^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(512*a^11*c^
7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d
^9*e^9 - 32*a^9*b^3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d^15*e^3 - 192*a^9*b^5*c

^4*d^14*e^4 + 128*a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*c^2*d^12*e^6 - 640*a^10 *b^2*c^6*d^15*e^3 + 1056*a^10*b^3*c^5*d^14*e^4 - 672*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*c^3*d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^7 + 512*a^11*b^2*c^5*d^ 13*e^5 + 288*a^11*b^3*c^4*d^12*e^6 - 192*a^11*b^4*c^3*d^11*e^7 + 32*a^11*b^ 5*c^2*d^10*e^8 + 384*a^12*b^2*c^4*d^11*e^7 - 288*a^12*b^3*c^3*d^10*e^8 - 32 *a^12*b^4*c^2*d^9*e^9 + 256*a^13*b^2*c^3*d^9*e^9 + 128*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*c^6*d^14*e^4 - 640*a^12*b*c^5*d^12*e^6 + 640*a^13*b*c^4*d^10* e^8) + 192*a^10*c^7*d^14*e^3 + 128*a^11*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^ 7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*c^6*d^15*e^2 + 64*a^8*b^4*c^5*d^14*e^ 3 - 96*a^8*b^5*c^4*d^13*e^4 + 64*a^8*b^6*c^3*d^12*e^5 - 16*a^8*b^7*c^2*d^11 *e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*a^9*b^3*c^5*d^13*e^4 - 352*a^9*b^4*c^ 4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 + 16*a^9*b^6*c^2*d^10*e^7 + 352*a^10*b ^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11*e^6 - 128*a^10*b^4*c^3*d^10*e^7 + 16 *a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c^4*d^10*e^7 - 128*a^11*b^3*c^3*d^9*e^ 8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12*b^2*c^3*d^8*e^9 + 64*a^9*b*c^7*d^15* e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^11*b*c^5*d^11*e^6 + 256*a^12*b*c^4*d^ 9*e^8) - x*(112*a^10*c^6*d^10*e^6 - 32*a^9*c^7*d^12*e^4 - 16*a^8*c^8*d^14*e ^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2 - 16*a^7*b^3*c^6*d^13*e^ 3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5 - 16*a^7*b^6*c^3*d^10*e ^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^5 + 128*a^8*b^4*c^4*d^10 *e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^10*e^6 + 208*a^9*b^3*c^4* d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^7*e^9 + 96*a^10*b^2*c^4* d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^13*e^3 + 128*a^9*b*c^6*d ^11*e^5 - 192*a^10*b*c^5*d^9*e^7 + 96*a^11*b*c^4*d^7*e^9))*(-(b^7*e^2 + b^5 *c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^ 4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2* (-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d* e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3) ^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a ^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5* d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3* d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32* a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2) + 4*a ^7*c^8*d^13*e^2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8 - 4*a^7*b^5*c^3* d^8*e^7 + 4*a^7*b^6*c^2*d^7*e^8 + 24*a^8*b^3*c^4*d^8*e^7 - 28*a^8*b^4*c^3*d ^7*e^8 + 52*a^9*b^2*c^4*d^7*e^8 - 4*a^7*b*c^7*d^12*e^3 - 32*a^9*b*c^5*d^8*e ^7) - x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7 + 2*a^7*b^2*c^5*d^7*e^7))*(- (b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 1 6*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4 *a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^ 4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3 *c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)))*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7* a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20* a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b ^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^ 3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b ^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4 *a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8 *a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^ 3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b ^4*c*d^2*e^2)))^(1/2)*2i - (log(c^6*d^11*(-d^3*e^5)^(1/2) + b^6*d^6*e^9*x +

```
 c^6*d^12*e^3*x + b^5*c*d^3*(-d^3*e^5)^(3/2) - b^6*d^2*e*(-d^3*e^5)^(3/2) -
 a^2*b^4*e^3*(-d^3*e^5)^(3/2) - 16*a^4*c^2*e^3*(-d^3*e^5)^(3/2) - 7*a*b^3*c
^2*d^3*(-d^3*e^5)^(3/2) + 12*a^2*b*c^3*d^3*(-d^3*e^5)^(3/2) + 8*a^3*b^2*c*e
^3*(-d^3*e^5)^(3/2) + 16*a^3*c^3*d^2*e*(-d^3*e^5)^(3/2) + a*c^5*d^9*e^2*(-d
^3*e^5)^(1/2) + a*b^5*d^5*e^10*x + a*c^5*d^10*e^5*x - b*c^5*d^11*e^4*x - b^
5*c*d^7*e^8*x + a^2*b^4*d^4*e^11*x - 16*a^3*c^3*d^6*e^9*x + 16*a^4*c^2*d^4*
e^11*x - a*b^5*d*e^2*(-d^3*e^5)^(3/2) - b*c^5*d^10*e*(-d^3*e^5)^(1/2) - 24*
a^2*b^2*c^2*d^2*e*(-d^3*e^5)^(3/2) + 7*a*b^3*c^2*d^7*e^8*x - 12*a^2*b*c^3*d
^7*e^8*x - 8*a^2*b^3*c*d^5*e^10*x + 16*a^3*b*c^2*d^5*e^10*x - 8*a^3*b^2*c*d
^4*e^11*x + 9*a*b^4*c*d^2*e*(-d^3*e^5)^(3/2) + 24*a^2*b^2*c^2*d^6*e^9*x + 8
*a^2*b^3*c*d*e^2*(-d^3*e^5)^(3/2) - 16*a^3*b*c^2*d*e^2*(-d^3*e^5)^(3/2) - 9
*a*b^4*c*d^6*e^9*x)*(-d^3*e^5)^(1/2))/(2*(c*d^5 + a*d^3*e^2 - b*d^4*e)) + (
log(b^6*d^6*e^9*x - c^6*d^11*(-d^3*e^5)^(1/2) + c^6*d^12*e^3*x - b^5*c*d^3*
(-d^3*e^5)^(3/2) + b^6*d^2*e*(-d^3*e^5)^(3/2) + a^2*b^4*e^3*(-d^3*e^5)^(3/2
) + 16*a^4*c^2*e^3*(-d^3*e^5)^(3/2) + 7*a*b^3*c^2*d^3*(-d^3*e^5)^(3/2) - 12
*a^2*b*c^3*d^3*(-d^3*e^5)^(3/2) - 8*a^3*b^2*c*e^3*(-d^3*e^5)^(3/2) - 16*a^3
*c^3*d^2*e*(-d^3*e^5)^(3/2) - a*c^5*d^9*e^2*(-d^3*e^5)^(1/2) + a*b^5*d^5*e^
10*x + a*c^5*d^10*e^5*x - b*c^5*d^11*e^4*x - b^5*c*d^7*e^8*x + a^2*b^4*d^4*
e^11*x - 16*a^3*c^3*d^6*e^9*x + 16*a^4*c^2*d^4*e^11*x + a*b^5*d*e^2*(-d^3*e
^5)^(3/2) + b*c^5*d^10*e*(-d^3*e^5)^(1/2) + 24*a^2*b^2*c^2*d^2*e*(-d^3*e^5)
^(3/2) + 7*a*b^3*c^2*d^7*e^8*x - 12*a^2*b*c^3*d^7*e^8*x - 8*a^2*b^3*c*d^5*e
^10*x + 16*a^3*b*c^2*d^5*e^10*x - 8*a^3*b^2*c*d^4*e^11*x - 9*a*b^4*c*d^2*e*
(-d^3*e^5)^(3/2) + 24*a^2*b^2*c^2*d^6*e^9*x - 8*a^2*b^3*c*d*e^2*(-d^3*e^5)^
(3/2) + 16*a^3*b*c^2*d*e^2*(-d^3*e^5)^(3/2) - 9*a*b^4*c*d^6*e^9*x)*(-d^3*e^
5)^(1/2))/(2*c*d^5 + 2*a*d^3*e^2 - 2*b*d^4*e) - 1/(a*d*x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.231 $\int \frac{1}{x^4(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=348

$$\frac{\sqrt{c}\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{c}\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-\right.}{\sqrt{2}a^2\sqrt{\sqrt{b^2-4ac}+b}\,\left(a\right.}$$

Rubi [A] time = 1.55, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1287, 205, 1166}

$$\frac{\sqrt{c}\left(\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{c}\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}+\frac{ae+bd}{a^2d^2x}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(ae^2-bde+cd^2\right)}-\frac{1}{3adx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(3*a*d*x^3) + (b*d + a*e)/(a^2*d^2*x) + (Sqrt[c]*(b*c*d - b^2*e + a*c*e
+ (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2
 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (Sqrt[c]*(b*c*d - b^2*e + a*c*e - (b^
2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*
a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(
5/2)*(c*d^2 - b*d*e + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^4\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{1}{adx^4} + \frac{-bd-ae}{a^2d^2x^2} + \frac{e^4}{d^2\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} + \frac{b^2cd-ac^2d-b^3e+\cdots}{a^2\left(cd^2-bd\cdots\right.}\right.$$

$$= -\frac{1}{3adx^3} + \frac{bd+ae}{a^2d^2x} + \frac{\int \frac{b^2cd-ac^2d-b^3e+2abce+c\left(bcd-b^2e+ace\right)x^2}{a+bx^2+cx^4}\,dx}{a^2\left(cd^2-bde+ae^2\right)} + \frac{e^4\int \frac{1}{d+ex^2}\cdots}{d^2\left(cd^2-bde\cdots\right.}$$

$$= -\frac{1}{3adx^3} + \frac{bd+ae}{a^2d^2x} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2-bde+ae^2\right)} + \frac{\left(c\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2\cdots}{\sqrt{b^2\cdots}}\right.\right.}{2a^2\left(cd^2\cdots\right.}$$

$$= -\frac{1}{3adx^3} + \frac{bd+ae}{a^2d^2x} + \frac{\sqrt{c}\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\cdots}{\sqrt{b-\sqrt{\cdots}}}\right.}{\sqrt{2}\,a^2\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)}$$

Mathematica [A] time = 0.52, size = 410, normalized size = 1.18

$$\frac{\sqrt{c}\left(b^2\left(cd-e\sqrt{b^2-4ac}\right)+bc\left(d\sqrt{b^2-4ac}+3ae\right)+ac\left(e\sqrt{b^2-4ac}-2cd\right)+b^3(-e)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(e(ae-bd)+cd^2\right)}+\frac{\sqrt{c}\left(-b^2\left(e\sqrt{b^2-4ac}+cd\right)+bc\left(d\sqrt{b^2-4ac}-3ae\right)+ac\left(e\sqrt{b^2-4ac}+2cd\right)+b^3e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\left(e(ae-bd)+cd^2\right)}+\frac{ae+bd}{a^2d^2x}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(ae^2-bde+cd^2\right)}-\frac{1}{3adx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/3*1/(a*d*x^3) + (b*d + a*e)/(a^2*d^2*x) + (Sqrt[c]*(-(b^3*e) + b*c*(Sqrt
[b^2 - 4*a*c]*d + 3*a*e) + b^2*(c*d - Sqrt[b^2 - 4*a*c]*e) + a*c*(-2*c*d +
Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(
-(b*d) + a*e))) + (Sqrt[c]*(b^3*e + b*c*(Sqrt[b^2 - 4*a*c]*d - 3*a*e) - b^2
*(c*d + Sqrt[b^2 - 4*a*c]*e) + a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a
*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(-(b*d) + a*e))) + (e^(7/2)*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/2)*(c*d^2 - b*d*e + a*e^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 12.00, size = 12268, normalized size = 35.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*((2*a^4*b^5*c^5 - 12*a^5*b^3*c^6 + 16*a^6*b*c^7 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^5 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^6 - 2*(b^2 - 4*a*c)*a^4*b^3*c^5 + 4*(b^2
 - 4*a*c)*a^5*b*c^6)*d^5 - (6*a^4*b^6*c^4 - 38*a^5*b^4*c^5 + 56*a^6*b^2*c^6
 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2
+ 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3
+ 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^3 -
 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 -
 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^4 -
 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^4 +
7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^5 - 6
*(b^2 - 4*a*c)*a^4*b^4*c^4 + 14*(b^2 - 4*a*c)*a^5*b^2*c^5)*d^4*e + 2*(sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 - 9*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^5*c^3 - 2*a^2*b^6*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b
^2*c^4 + 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 + 18*a^3*b^4*c^4 - 16*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^4*b*c^5 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 -
48*a^4*b^2*c^5 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^6 + 32*a^5
*c^6 + 2*(b^2 - 4*a*c)*a^2*b^4*c^3 - 10*(b^2 - 4*a*c)*a^3*b^2*c^4 + 8*(b^2
- 4*a*c)*a^4*c^5)*d^3*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2) + (6*a^4*b^7*c^3
 - 36*a^5*b^5*c^4 + 40*a^6*b^3*c^5 + 32*a^7*b*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c + 18*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^5 - 6*(b^2 - 4*a*c)*a^4*b^5*c^3 + 12*(b
^2 - 4*a*c)*a^5*b^3*c^4 + 8*(b^2 - 4*a*c)*a^6*b*c^5)*d^3*e^2 - 2*(2*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 19*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b^5*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
6*c^2 - 4*a^2*b^7*c^2 + 56*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*
c^3 + 22*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 2*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 + 38*a^3*b^5*c^3 - 48*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^4*b^2*c^4 - 11*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c
^4 - 112*a^4*b^3*c^4 + 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5
 + 96*a^5*b*c^5 + 4*(b^2 - 4*a*c)*a^2*b^5*c^2 - 22*(b^2 - 4*a*c)*a^3*b^3*c^
3 + 24*(b^2 - 4*a*c)*a^4*b*c^4)*d^2*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*e
+ (2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*(a^2*c*d^2 -
a^2*b*d*e + a^3*e^2)^2*d - (2*a^4*b^8*c^2 - 6*a^5*b^6*c^3 - 28*a^6*b^4*c^4
```

```
+ 80*a^7*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^4*b^8 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5
*b^6*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^
7*c + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*
c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c
^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2
- 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3
- 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 + 1
0*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 2
*(b^2 - 4*a*c)*a^4*b^6*c^2 - 2*(b^2 - 4*a*c)*a^5*b^4*c^3 + 20*(b^2 - 4*a*c)
*a^6*b^2*c^4)*d^2*e^3 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8
- 9*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c - 2*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 2*a^2*b^8*c + 23*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^4*b^4*c^2 + 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b^5*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 + 18*a^3*b
^6*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 - 6*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b^4*c^3 - 46*a^4*b^4*c^3 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^6*c^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 +
 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + 16*a^5*b^2*c^4 + 4
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^5 + 32*a^6*c^5 + 2*(b^2 - 4*
a*c)*a^2*b^6*c - 10*(b^2 - 4*a*c)*a^3*b^4*c^2 + 6*(b^2 - 4*a*c)*a^4*b^2*c^3
 + 8*(b^2 - 4*a*c)*a^5*c^4)*d*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*e^2 - (2
*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*
a*c)*a^2*c^4)*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)^2*e + (4*a^5*b^7*c^2 - 26*a
^6*b^5*c^3 + 36*a^7*b^3*c^4 + 16*a^8*b*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^7 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^8*b*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^6*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^7*b*c^4 - 4*(b^2 - 4*a*c)*a^5*b^5*c^2 + 10*(b^2 - 4*a*c)*a^
6*b^3*c^3 + 4*(b^2 - 4*a*c)*a^7*b*c^4)*d*e^4 - 2*(sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^3*b^7 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5
*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c - 2*a^3*b^7*c + 32
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^2 + 12*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^3*b^5*c^2 + 20*a^4*b^5*c^2 - 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^6*b*c^3 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 - 6*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 64*a^5*b^3*c^3 + 8*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 + 64*a^6*b*c^4 + 2*(b^2 - 4*a*
c)*a^3*b^5*c - 12*(b^2 - 4*a*c)*a^4*b^3*c^2 + 16*(b^2 - 4*a*c)*a^5*b*c^3)*a
bs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*e^3 - (2*a^6*b^6*c^2 - 14*a^7*b^4*c^3 +
 24*a^8*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
```

```
*a^6*b^6 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*
b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5
*c - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^2*c
^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^
2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 +
 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 -
2*(b^2 - 4*a*c)*a^6*b^4*c^2 + 6*(b^2 - 4*a*c)*a^7*b^2*c^3)*e^5)*arctan(2*sq
rt(1/2)*x/sqrt((a^2*b*c*d^2 - a^2*b^2*d*e + a^3*b*e^2 + sqrt((a^2*b*c*d^2 -
 a^2*b^2*d*e + a^3*b*e^2)^2 - 4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*(a^2*c^2*
d^2 - a^2*b*c*d*e + a^3*c*e^2)))/(a^2*c^2*d^2 - a^2*b*c*d*e + a^3*c*e^2)))/
((a^5*b^4*c^2 - 8*a^6*b^2*c^3 - 2*a^5*b^3*c^3 + 16*a^7*c^4 + 8*a^6*b*c^4 +
a^5*b^2*c^4 - 4*a^6*c^5)*d^4*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*abs(c) -
2*(a^5*b^5*c - 8*a^6*b^3*c^2 - 2*a^5*b^4*c^2 + 16*a^7*b*c^3 + 8*a^6*b^2*c^3
 + a^5*b^3*c^3 - 4*a^6*b*c^4)*d^3*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*abs(
c)*e + (a^5*b^6 - 6*a^6*b^4*c - 2*a^5*b^5*c + 4*a^6*b^3*c^2 + a^5*b^4*c^2 +
 32*a^8*c^3 + 16*a^7*b*c^3 - 2*a^6*b^2*c^3 - 8*a^7*c^4)*d^2*abs(a^2*c*d^2 -
 a^2*b*d*e + a^3*e^2)*abs(c)*e^2 - 2*(a^6*b^5 - 8*a^7*b^3*c - 2*a^6*b^4*c +
 16*a^8*b*c^2 + 8*a^7*b^2*c^2 + a^6*b^3*c^2 - 4*a^7*b*c^3)*d*abs(a^2*c*d^2
- a^2*b*d*e + a^3*e^2)*abs(c)*e^3 + (a^7*b^4 - 8*a^8*b^2*c - 2*a^7*b^3*c +
16*a^9*c^2 + 8*a^8*b*c^2 + a^7*b^2*c^2 - 4*a^8*c^3)*abs(a^2*c*d^2 - a^2*b*d
*e + a^3*e^2)*abs(c)*e^4) - 1/8*((2*a^4*b^5*c^5 - 12*a^5*b^3*c^6 + 16*a^6*b
*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^
3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^4
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^4
- 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^5 - 4
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^5 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^5 + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^6 - 2*(b^2 -
 4*a*c)*a^4*b^3*c^5 + 4*(b^2 - 4*a*c)*a^5*b*c^6)*d^5 - (6*a^4*b^6*c^4 - 38*
a^5*b^4*c^5 + 56*a^6*b^2*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^5*b^4*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^4*b^5*c^3 - 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^6*b^2*c^4 - 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^5*b^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^4*b^4*c^4 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^5*b^2*c^5 - 6*(b^2 - 4*a*c)*a^4*b^4*c^4 + 14*(b^2 - 4*a*c)*a
^5*b^2*c^5)*d^4*e - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2
- 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 2*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 + 2*a^2*b^6*c^3 + 24*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^3*b^3*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 18
*a^3*b^4*c^4 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^5 - 8*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 5*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^3*b^2*c^5 + 48*a^4*b^2*c^5 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^4*c^6 - 32*a^5*c^6 - 2*(b^2 - 4*a*c)*a^2*b^4*c^3 + 10*(b^2 -
4*a*c)*a^3*b^2*c^4 - 8*(b^2 - 4*a*c)*a^4*c^5)*d^3*abs(a^2*c*d^2 - a^2*b*d*e
 + a^3*e^2) + (6*a^4*b^7*c^3 - 36*a^5*b^5*c^4 + 40*a^6*b^3*c^5 + 32*a^7*b*c
^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c
+ 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2
+ 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 -
 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 -
 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 -
 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^3 -
16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b*c^4 - 8*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 + 6*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^4 + 4*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^5 - 6*(b^2
- 4*a*c)*a^4*b^5*c^3 + 12*(b^2 - 4*a*c)*a^5*b^3*c^4 + 8*(b^2 - 4*a*c)*a^6*b
```

```
*c^5)*d^3*e^2 + 2*(2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 19
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 4*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 + 4*a^2*b^7*c^2 + 56*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 + 22*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b^4*c^3 + 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 - 38*
a^3*b^5*c^3 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 24*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 11*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + 112*a^4*b^3*c^4 + 12*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 96*a^5*b*c^5 - 4*(b^2 - 4*a*c)*a^2*b^5*c^2 +
 22*(b^2 - 4*a*c)*a^3*b^3*c^3 - 24*(b^2 - 4*a*c)*a^4*b*c^4)*d^2*abs(a^2*c*d
^2 - a^2*b*d*e + a^3*e^2)*e + (2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4
*a*c)*a*b*c^4)*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)^2*d - (2*a^4*b^8*c^2 - 6*a
^5*b^6*c^3 - 28*a^6*b^4*c^4 + 80*a^7*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^4*b^6*c^2 - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^7*b^2*c^3 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^6*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^5*b^4*c^3 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^6*b^2*c^4 - 2*(b^2 - 4*a*c)*a^4*b^6*c^2 - 2*(b^2 - 4*a*c)*a^
5*b^4*c^3 + 20*(b^2 - 4*a*c)*a^6*b^2*c^4)*d^2*e^3 - 2*(sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b^8 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3
*b^6*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c + 2*a^2*b^8*c
 + 23*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2 + 10*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b^6*c^2 - 18*a^3*b^6*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^5*b^2*c^3 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 5
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 46*a^4*b^4*c^3 - 16*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^5*b*c^4 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*
b^2*c^4 - 16*a^5*b^2*c^4 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^
5 - 32*a^6*c^5 - 2*(b^2 - 4*a*c)*a^2*b^6*c + 10*(b^2 - 4*a*c)*a^3*b^4*c^2 -
 6*(b^2 - 4*a*c)*a^4*b^2*c^3 - 8*(b^2 - 4*a*c)*a^5*c^4)*d*abs(a^2*c*d^2 - a
^2*b*d*e + a^3*e^2)*e^2 - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a
^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 9*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*
a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)
^2*e + (4*a^5*b^7*c^2 - 26*a^6*b^5*c^3 + 36*a^7*b^3*c^4 + 16*a^8*b*c^5 - 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^7 + 13*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c + 4*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c - 18*sqrt(2)*s
```

```
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^2 - 10*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b*c^3 - 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 5*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b*c^4 - 4*(b^2 - 4*a*c)*a^5*b
^5*c^2 + 10*(b^2 - 4*a*c)*a^6*b^3*c^3 + 4*(b^2 - 4*a*c)*a^7*b*c^4)*d*e^4 +
2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7 - 10*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^3*b^6*c + 2*a^3*b^7*c + 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^
3*c^2 + 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2 + sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 20*a^4*b^5*c^2 - 32*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^3 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^5*b^2*c^3 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^
3 + 64*a^5*b^3*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 -
64*a^6*b*c^4 - 2*(b^2 - 4*a*c)*a^3*b^5*c + 12*(b^2 - 4*a*c)*a^4*b^3*c^2 - 1
6*(b^2 - 4*a*c)*a^5*b*c^3)*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*e^3 - (2*a^
6*b^6*c^2 - 14*a^7*b^4*c^3 + 24*a^8*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^6 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^7*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^8*b^2*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^7*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^6*b^4*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^7*b^2*c^3 - 2*(b^2 - 4*a*c)*a^6*b^4*c^2 + 6*(b^2 - 4*a*c)*a
^7*b^2*c^3)*e^5)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b*c*d^2 - a^2*b^2*d*e + a^3
*b*e^2 - sqrt((a^2*b*c*d^2 - a^2*b^2*d*e + a^3*b*e^2)^2 - 4*(a^3*c*d^2 - a^
3*b*d*e + a^4*e^2)*(a^2*c^2*d^2 - a^2*b*c*d*e + a^3*c*e^2)))/(a^2*c^2*d^2 -
 a^2*b*c*d*e + a^3*c*e^2)))/((a^5*b^4*c^2 - 8*a^6*b^2*c^3 - 2*a^5*b^3*c^3 +
 16*a^7*c^4 + 8*a^6*b*c^4 + a^5*b^2*c^4 - 4*a^6*c^5)*d^4*abs(a^2*c*d^2 - a^
2*b*d*e + a^3*e^2)*abs(c) - 2*(a^5*b^5*c - 8*a^6*b^3*c^2 - 2*a^5*b^4*c^2 +
16*a^7*b*c^3 + 8*a^6*b^2*c^3 + a^5*b^3*c^3 - 4*a^6*b*c^4)*d^3*abs(a^2*c*d^2
 - a^2*b*d*e + a^3*e^2)*abs(c)*e + (a^5*b^6 - 6*a^6*b^4*c - 2*a^5*b^5*c + 4
*a^6*b^3*c^2 + a^5*b^4*c^2 + 32*a^8*c^3 + 16*a^7*b*c^3 - 2*a^6*b^2*c^3 - 8*
a^7*c^4)*d^2*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*abs(c)*e^2 - 2*(a^6*b^5 -
 8*a^7*b^3*c - 2*a^6*b^4*c + 16*a^8*b*c^2 + 8*a^7*b^2*c^2 + a^6*b^3*c^2 - 4
*a^7*b*c^3)*d*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*abs(c)*e^3 + (a^7*b^4 -
8*a^8*b^2*c - 2*a^7*b^3*c + 16*a^9*c^2 + 8*a^8*b*c^2 + a^7*b^2*c^2 - 4*a^8*
c^3)*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*abs(c)*e^4) + arctan(x*e^(1/2)/sq
rt(d))*e^(7/2)/((c*d^4 - b*d^3*e + a*d^2*e^2)*sqrt(d)) + 1/3*(3*b*d*x^2 + 3
*a*x^2*e - a*d)/(a^2*d^2*x^3)
```

maple [B] time = 0.04, size = 1160, normalized size = 3.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/3/a/d/x^3+1/a/d^2*e/x+1/d/a^2/x*b-1/2/(a*e^2-b*d*e+c*d^2)/a*c^2*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*c*x)*e+1/2/(a*e^2-b*d*e+c*d^2)/a^2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*e-1
/2/(a*e^2-b*d*e+c*d^2)/a^2*c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d-3/2/(a*e^2-b*d*e+c
*d^2)/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e+1/(a*e^2-b*d*e+c*d
^2)/a*c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d+1/2/(a*e^2-b*d*e+c*d^2
```

```
)/a^2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
h(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*e-1/2/(a*e^2-b*d*e+c*d
^2)/a^2*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d+1/2/(a*e^2-b*d*e
+c*d^2)/a*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e-1/2/(a*e^2-b*d*e+c*d^2)/a^2*c*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*b^2*e+1/2/(a*e^2-b*d*e+c*d^2)/a^2*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d-3/2/(
a*e^2-b*d*e+c*d^2)/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e+1/(a*e^2
-b*d*e+c*d^2)/a*c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d+1/2/(a*e^2-b*d*
e+c*d^2)/a^2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*e-1/2/(a*e^2-b*d*e
+c*d^2)/a^2*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d+1/d^2*e^4/(a*e^
2-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] e^4*arctan(e*x/sqrt(d*e))/((c*d^4 - b*d^3*e + a*d^2*e^2)*sqrt(d*e)) + integ
rate(((b*c^2*d - (b^2*c - a*c^2)*e)*x^2 + (b^2*c - a*c^2)*d - (b^3 - 2*a*b*
c)*e)/(c*x^4 + b*x^2 + a), x)/(a^2*c*d^2 - a^2*b*d*e + a^3*e^2) + 1/3*(3*(b
*d + a*e)*x^2 - a*d)/(a^2*d^2*x^3)
```

mupad [B] time = 6.73, size = 42882, normalized size = 123.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(c^9*d^27*e^6 - b^9*d^18*e^15 + 2*a*c^8*d^25*e^8 - 2*b*c^8*d^26*e^7 + 2
*b^8*c*d^19*e^14 + a^5*b^4*d^13*e^20 + a^2*c^7*d^23*e^10 + 16*a^4*c^5*d^19*
e^14 + 16*a^7*c^2*d^13*e^20 + b^2*c^7*d^25*e^8 - b^7*c^2*d^20*e^13 - 25*a^2
*b^3*c^4*d^20*e^13 + 66*a^2*b^4*c^3*d^19*e^14 - 42*a^2*b^5*c^2*d^18*e^15 -
76*a^3*b^2*c^4*d^19*e^14 + 63*a^3*b^3*c^3*d^18*e^15 - a^5*b^4*e^3*x*(-d^5*e
^7)^(5/2) + a^2*c^7*d^15*x*(-d^5*e^7)^(3/2) - 16*a^7*c^2*e^3*x*(-d^5*e^7)^(
5/2) - b^9*d^10*e^5*x*(-d^5*e^7)^(3/2) - c^9*d^24*e^3*x*(-d^5*e^7)^(1/2) -
2*a*b*c^7*d^24*e^9 + 11*a*b^7*c*d^18*e^15 + 9*a*b^5*c^3*d^20*e^13 - 20*a*b^
6*c^2*d^19*e^14 + 20*a^3*b*c^5*d^20*e^13 - 28*a^4*b*c^4*d^18*e^15 - 8*a^6*b
^2*c*d^13*e^20 + 16*a^4*c^5*d^11*e^4*x*(-d^5*e^7)^(3/2) - b^7*c^2*d^12*e^3*
x*(-d^5*e^7)^(3/2) - b^2*c^7*d^22*e^5*x*(-d^5*e^7)^(1/2) + 8*a^6*b^2*c*e^3*
x*(-d^5*e^7)^(5/2) - 2*a*c^8*d^22*e^5*x*(-d^5*e^7)^(1/2) + 2*b^8*c*d^11*e^4
*x*(-d^5*e^7)^(3/2) + 2*b*c^8*d^23*e^4*x*(-d^5*e^7)^(1/2) + 11*a*b^7*c*d^10
*e^5*x*(-d^5*e^7)^(3/2) + 2*a*b*c^7*d^21*e^6*x*(-d^5*e^7)^(1/2) + 9*a*b^5*c
^3*d^12*e^3*x*(-d^5*e^7)^(3/2) - 20*a*b^6*c^2*d^11*e^4*x*(-d^5*e^7)^(3/2) +
 20*a^3*b*c^5*d^12*e^3*x*(-d^5*e^7)^(3/2) - 28*a^4*b*c^4*d^10*e^5*x*(-d^5*e
^7)^(3/2) - 25*a^2*b^3*c^4*d^12*e^3*x*(-d^5*e^7)^(3/2) + 66*a^2*b^4*c^3*d^1
1*e^4*x*(-d^5*e^7)^(3/2) - 42*a^2*b^5*c^2*d^10*e^5*x*(-d^5*e^7)^(3/2) - 76*
a^3*b^2*c^4*d^11*e^4*x*(-d^5*e^7)^(3/2) + 63*a^3*b^3*c^3*d^10*e^5*x*(-d^5*e
^7)^(3/2))*(-d^5*e^7)^(1/2))/(2*c*d^7 + 2*a*d^5*e^2 - 2*b*d^6*e) - atan(((
-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^
2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2
- a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^
```

3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(((-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(512*a^20*c^7*d^24*e^3 + 512*a^21*c^6*d^22*e^5 - 512*a^22*c^5*d^20*e^7 - 512*a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*d^25*e^2 + 128*a^18*b^4*c^5*d^24*e^3 - 192*a^18*b^5*c^4*d^23*e^4 + 128*a^18*b^6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^21*e^6 - 640*a^19*b^2*c^6*d^24*e^3 + 1056*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^21*e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^3*c^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20*e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384*a^21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e^9 + 256*a^22*b^2*c^3*d^18*e^9 + 128*a^19*b*c^7*d^25*e^2 - 1152*a^20*b*c^6*d^23*e^4 - 640*a^21*b*c^5*d^21*e^6 + 640*a^22*b*c^4*d^19*e^8) - 64*a^18*c^8*d^24*e^2 + 128*a^19*c^7*d^22*e^4 + 192*a^20*c^6*d^20*e^6 - 256*a^21*c^5*d^18*e^8 - 256*a^22*c^4*d^16*e^10 - 16*a^16*b^4*c^6*d^24*e^2 + 64*a^16*b^5*c^5*d^23*e^3 - 96*a^16*b^6*c^4*d^22*e^4 + 64*a^16*b^7*c^3*d^21*e^5 - 16*a^16*b^8*c^2*d^20*e^6 + 80*a^17*b^2*c^7*d^24*e^2 - 368*a^17*b^3*c^6*d^23*e^3 + 608*a^17*b^4*c^5*d^22*e^4 - 416*a^17*b^5*c^4*d^21*e^5 + 80*a^17*b^6*c^3*d^20*e^6 + 16*a^17*b^7*c^2*d^19*e^7 - 928*a^18*b^2*c^6*d^22*e^4 + 640*a^18*b^3*c^5*d^21*e^5 + 32*a^18*b^4*c^4*d^20*e^6 - 128*a^18*b^5*c^3*d^19*e^7 - 432*a^19*b^2*c^5*d^20*e^6 + 304*a^19*b^3*c^4*d^19*e^7 - 16*a^19*b^4*c^3*d^18*e^8 + 16*a^19*b^5*c^2*d^17*e^9 + 128*a^20*b^2*c^4*d^18*e^8 - 128*a^20*b^3*c^3*d^17*e^9 - 16*a^20*b^4*c^2*d^16*e^10 + 128*a^21*b^2*c^3*d^16*e^10 + 448*a^18*b*c^7*d^23*e^3 - 192*a^20*b*c^5*d^19*e^7 + 256*a^21*b*c^4*d^17*e^9) - x*(16*a^16*c^9*d^23*e^2 + 32*a^17*c^8*d^21*e^4 - 112*a^18*c^7*d^19*e^6 - 128*a^20*c^5*d^15*e^10 + 8*a^14*b^4*c^7*d^23*e^2 - 16*a^14*b^5*c^6*d^22*e^3 + 8*a^14*b^6*c^5*d^21*e^4 + 8*a^14*b^7*c^4*d^20*e^5 - 16*a^14*b^8*c^3*d^19*e^6 + 8

*a^14*b^9*c^2*d^18*e^7 - 32*a^15*b^2*c^8*d^23*e^2 + 64*a^15*b^3*c^7*d^22*e^
3 - 16*a^15*b^4*c^6*d^21*e^4 - 88*a^15*b^5*c^5*d^20*e^5 + 160*a^15*b^6*c^4*
d^19*e^6 - 88*a^15*b^7*c^3*d^18*e^7 - 48*a^16*b^2*c^7*d^21*e^4 + 264*a^16*b
^3*c^6*d^20*e^5 - 520*a^16*b^4*c^5*d^19*e^6 + 336*a^16*b^5*c^4*d^18*e^7 + 5
76*a^17*b^2*c^6*d^19*e^6 - 504*a^17*b^3*c^5*d^18*e^7 + 8*a^18*b^3*c^4*d^16*
e^9 - 16*a^18*b^4*c^3*d^15*e^10 + 8*a^18*b^5*c^2*d^14*e^11 + 96*a^19*b^2*c^
4*d^15*e^10 - 56*a^19*b^3*c^3*d^14*e^11 - 32*a^16*b*c^8*d^22*e^3 - 192*a^17
*b*c^7*d^20*e^5 + 224*a^18*b*c^6*d^18*e^7 - 32*a^19*b*c^5*d^16*e^9 + 96*a^2
0*b*c^4*d^14*e^11))*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(
1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e
+ 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^
2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2
d^2(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*
c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c
 - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*
d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b
^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1
/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 -
2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 3
2*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d
*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(
1/2) - 4*a^15*c^9*d^21*e^3 - 4*a^16*c^8*d^19*e^5 + 48*a^18*c^6*d^15*e^9 - 4
*a^14*b^2*c^8*d^21*e^3 - 4*a^14*b^7*c^3*d^16*e^8 + 4*a^14*b^8*c^2*d^15*e^9
+ 36*a^15*b^5*c^4*d^16*e^8 - 44*a^15*b^6*c^3*d^15*e^9 + 4*a^15*b^7*c^2*d^14
*e^10 - 100*a^16*b^3*c^5*d^16*e^8 + 160*a^16*b^4*c^4*d^15*e^9 - 32*a^16*b^5
*c^3*d^14*e^10 - 204*a^17*b^2*c^5*d^15*e^9 + 76*a^17*b^3*c^4*d^14*e^10 + 4*
a^14*b*c^9*d^22*e^2 + 8*a^15*b*c^8*d^20*e^4 + 80*a^17*b*c^6*d^16*e^8 - 48*a
^18*b*c^5*d^14*e^10) - x*(2*a^14*c^9*d^18*e^5 + 4*a^16*c^7*d^14*e^9 + 2*a^1
4*b^4*c^5*d^14*e^9 - 8*a^15*b^2*c^6*d^14*e^9))*(-(b^9*e^2 + b^7*c^2*d^2 - b
^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a
^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^
2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*
c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 -
 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) -
 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^
3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c
^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^
3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b
*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e
 - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*1i + ((-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*
e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/
2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^
3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c
^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^
2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2
)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(
4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 -
8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a
^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*
e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*
b^4*c*d^2*e^2)))^(1/2)*(((-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c
*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b
^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^
4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a

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*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4
*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) -
8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^
3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e
^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^
2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^
3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2
)))^(1/2)*(x*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) -
9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^
2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 -
 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-
(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e
 + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 7
6*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*
d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8
*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b
^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c
^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 -
32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(5
12*a^20*c^7*d^24*e^3 + 512*a^21*c^6*d^22*e^5 - 512*a^22*c^5*d^20*e^7 - 512*
a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*d^25*e^2 + 128*a^18*b^4*c^5*d^24*e^3 -
192*a^18*b^5*c^4*d^23*e^4 + 128*a^18*b^6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^2
1*e^6 - 640*a^19*b^2*c^6*d^24*e^3 + 1056*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b
^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^21*e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512
*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^3*c^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20*
e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384*a^21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c
^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e^9 + 256*a^22*b^2*c^3*d^18*e^9 + 128*a^
19*b*c^7*d^25*e^2 - 1152*a^20*b*c^6*d^23*e^4 - 640*a^21*b*c^5*d^21*e^6 + 64
0*a^22*b*c^4*d^19*e^8) + 64*a^18*c^8*d^24*e^2 - 128*a^19*c^7*d^22*e^4 - 192
*a^20*c^6*d^20*e^6 + 256*a^21*c^5*d^18*e^8 + 256*a^22*c^4*d^16*e^10 + 16*a^
16*b^4*c^6*d^24*e^2 - 64*a^16*b^5*c^5*d^23*e^3 + 96*a^16*b^6*c^4*d^22*e^4 -
 64*a^16*b^7*c^3*d^21*e^5 + 16*a^16*b^8*c^2*d^20*e^6 - 80*a^17*b^2*c^7*d^24
*e^2 + 368*a^17*b^3*c^6*d^23*e^3 - 608*a^17*b^4*c^5*d^22*e^4 + 416*a^17*b^5
*c^4*d^21*e^5 - 80*a^17*b^6*c^3*d^20*e^6 - 16*a^17*b^7*c^2*d^19*e^7 + 928*a
^18*b^2*c^6*d^22*e^4 - 640*a^18*b^3*c^5*d^21*e^5 - 32*a^18*b^4*c^4*d^20*e^6
 + 128*a^18*b^5*c^3*d^19*e^7 + 432*a^19*b^2*c^5*d^20*e^6 - 304*a^19*b^3*c^4
*d^19*e^7 + 16*a^19*b^4*c^3*d^18*e^8 - 16*a^19*b^5*c^2*d^17*e^9 - 128*a^20*
b^2*c^4*d^18*e^8 + 128*a^20*b^3*c^3*d^17*e^9 + 16*a^20*b^4*c^2*d^16*e^10 -
128*a^21*b^2*c^3*d^16*e^10 - 448*a^18*b*c^7*d^23*e^3 + 192*a^20*b*c^5*d^19*
e^7 - 256*a^21*b*c^4*d^17*e^9) - x*(16*a^16*c^9*d^23*e^2 + 32*a^17*c^8*d^21
*e^4 - 112*a^18*c^7*d^19*e^6 - 128*a^20*c^5*d^15*e^10 + 8*a^14*b^4*c^7*d^23
*e^2 - 16*a^14*b^5*c^6*d^22*e^3 + 8*a^14*b^6*c^5*d^21*e^4 + 8*a^14*b^7*c^4*
d^20*e^5 - 16*a^14*b^8*c^3*d^19*e^6 + 8*a^14*b^9*c^2*d^18*e^7 - 32*a^15*b^2
*c^8*d^23*e^2 + 64*a^15*b^3*c^7*d^22*e^3 - 16*a^15*b^4*c^6*d^21*e^4 - 88*a^
15*b^5*c^5*d^20*e^5 + 160*a^15*b^6*c^4*d^19*e^6 - 88*a^15*b^7*c^3*d^18*e^7
- 48*a^16*b^2*c^7*d^21*e^4 + 264*a^16*b^3*c^6*d^20*e^5 - 520*a^16*b^4*c^5*d
^19*e^6 + 336*a^16*b^5*c^4*d^18*e^7 + 576*a^17*b^2*c^6*d^19*e^6 - 504*a^17*
b^3*c^5*d^18*e^7 + 8*a^18*b^3*c^4*d^16*e^9 - 16*a^18*b^4*c^3*d^15*e^10 + 8*
a^18*b^5*c^2*d^14*e^11 + 96*a^19*b^2*c^4*d^15*e^10 - 56*a^19*b^3*c^3*d^14*e
^11 - 32*a^16*b*c^8*d^22*e^3 - 192*a^17*b*c^7*d^20*e^5 + 224*a^18*b*c^6*d^1
8*e^7 - 32*a^19*b*c^5*d^16*e^9 + 96*a^20*b*c^4*d^14*e^11))*(-(b^9*e^2 + b^7
*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^
5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(
-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3
*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a
*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2
)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(
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4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c
^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2)
+ 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^
4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 -
8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*
e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b
^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) + 4*a^15*c^9*d^21*e^3 + 4*a^16*
c^8*d^19*e^5 - 48*a^18*c^6*d^15*e^9 + 4*a^14*b^2*c^8*d^21*e^3 + 4*a^14*b^7*
c^3*d^16*e^8 - 4*a^14*b^8*c^2*d^15*e^9 - 36*a^15*b^5*c^4*d^16*e^8 + 44*a^15
*b^6*c^3*d^15*e^9 - 4*a^15*b^7*c^2*d^14*e^10 + 100*a^16*b^3*c^5*d^16*e^8 -
160*a^16*b^4*c^4*d^15*e^9 + 32*a^16*b^5*c^3*d^14*e^10 + 204*a^17*b^2*c^5*d^
15*e^9 - 76*a^17*b^3*c^4*d^14*e^10 - 4*a^14*b*c^9*d^22*e^2 - 8*a^15*b*c^8*d
^20*e^4 - 80*a^17*b*c^6*d^16*e^8 + 48*a^18*b*c^5*d^14*e^10) - x*(2*a^14*c^9
*d^18*e^5 + 4*a^16*c^7*d^14*e^9 + 2*a^14*b^4*c^5*d^14*e^9 - 8*a^15*b^2*c^6*
d^14*e^9))*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*
a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*
b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 6
3*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4
*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e +
 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3
)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*
a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*
e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(
a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5
*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3
*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32
*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*1i)/
(((-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3
*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d
^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3
*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^
2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*
d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) +
 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c
^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*
c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e
^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 +
a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2
- 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^
2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(((-(b^9*e^2
+ b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3
*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*
d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^
3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c
- b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*
b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(
1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c
^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d
^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c
*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*
a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^9*e^2 + b^7*c^2*d^
2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 +
 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c
 - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-
(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*
e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1
/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c -
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b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(512*a^20*c^7*d^24*e^3 + 512*a^21*c^6*d^22*e^5 - 512*a^22*c^5*d^20*e^7 - 512*a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*d^25*e^2 + 128*a^18*b^4*c^5*d^24*e^3 - 192*a^18*b^5*c^4*d^23*e^4 + 128*a^18*b^6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^21*e^6 - 640*a^19*b^2*c^6*d^24*e^3 + 1056*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^21*e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^3*c^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20*e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384*a^21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e^9 + 256*a^22*b^2*c^3*d^18*e^9 + 128*a^19*b*c^7*d^25*e^2 - 1152*a^20*b*c^6*d^23*e^4 - 640*a^21*b*c^5*d^21*e^6 + 640*a^22*b*c^4*d^19*e^8) + 64*a^18*c^8*d^24*e^2 - 128*a^19*c^7*d^22*e^4 - 192*a^20*c^6*d^20*e^6 + 256*a^21*c^5*d^18*e^8 + 256*a^22*c^4*d^16*e^10 + 16*a^16*b^4*c^6*d^24*e^2 - 64*a^16*b^5*c^5*d^23*e^3 + 96*a^16*b^6*c^4*d^22*e^4 - 64*a^16*b^7*c^3*d^21*e^5 + 16*a^16*b^8*c^2*d^20*e^6 - 80*a^17*b^2*c^7*d^24*e^2 + 368*a^17*b^3*c^6*d^23*e^3 - 608*a^17*b^4*c^5*d^22*e^4 + 416*a^17*b^5*c^4*d^21*e^5 - 80*a^17*b^6*c^3*d^20*e^6 - 16*a^17*b^7*c^2*d^19*e^7 + 928*a^18*b^2*c^6*d^22*e^4 - 640*a^18*b^3*c^5*d^21*e^5 - 32*a^18*b^4*c^4*d^20*e^6 + 128*a^18*b^5*c^3*d^19*e^7 + 432*a^19*b^2*c^5*d^20*e^6 - 304*a^19*b^3*c^4*d^19*e^7 + 16*a^19*b^4*c^3*d^18*e^8 - 16*a^19*b^5*c^2*d^17*e^9 - 128*a^20*b^2*c^4*d^18*e^8 + 128*a^20*b^3*c^3*d^17*e^9 + 16*a^20*b^4*c^2*d^16*e^10 - 128*a^21*b^2*c^3*d^16*e^10 - 448*a^18*b*c^7*d^23*e^3 + 192*a^20*b*c^5*d^19*e^7 - 256*a^21*b*c^4*d^17*e^9) - x*(16*a^16*c^9*d^23*e^2 + 32*a^17*c^8*d^21*e^4 - 112*a^18*c^7*d^19*e^6 - 128*a^20*c^5*d^15*e^10 + 8*a^14*b^4*c^7*d^23*e^2 - 16*a^14*b^5*c^6*d^22*e^3 + 8*a^14*b^6*c^5*d^21*e^4 + 8*a^14*b^7*c^4*d^20*e^5 - 16*a^14*b^8*c^3*d^19*e^6 + 8*a^14*b^9*c^2*d^18*e^7 - 32*a^15*b^2*c^8*d^23*e^2 + 64*a^15*b^3*c^7*d^22*e^3 - 16*a^15*b^4*c^6*d^21*e^4 - 88*a^15*b^5*c^5*d^20*e^5 + 160*a^15*b^6*c^4*d^19*e^6 - 88*a^15*b^7*c^3*d^18*e^7 - 48*a^16*b^2*c^7*d^21*e^4 + 264*a^16*b^3*c^6*d^20*e^5 - 520*a^16*b^4*c^5*d^19*e^6 + 336*a^16*b^5*c^4*d^18*e^7 + 576*a^17*b^2*c^6*d^19*e^6 - 504*a^17*b^3*c^5*d^18*e^7 + 8*a^18*b^3*c^4*d^16*e^9 - 16*a^18*b^4*c^3*d^15*e^10 + 8*a^18*b^5*c^2*d^14*e^11 + 96*a^19*b^2*c^4*d^15*e^10 - 56*a^19*b^3*c^3*d^14*e^11 - 32*a^16*b*c^8*d^22*e^3 - 192*a^17*b*c^7*d^20*e^5 + 224*a^18*b*c^6*d^18*e^7 - 32*a^19*b*c^5*d^16*e^9 + 96*a^20*b*c^4*d^14*e^11))*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) + 4*a^15*c^9*d^21*e^3 + 4*a^16*c^8*d^19*e^5 - 48*a^18*c^6*d^15*e^9 + 4*a^14*b^2*c^8*d^21*e^3 + 4*a^14*b^7*c^3*d^16*e^8 - 4*a^14*b^8*c^2*d^15*e^9 - 36*a^15*b^5*c^4*d^16*e^8 + 44*a^15*b^6*c^3*d^15*e^9 - 4*a^15*b^7*c^2*d^14*e^10 + 100*a^16*b^3*c^5*d^16*e^8 - 160*a^16*b^4*c^4*d^15*e^9 + 32*a^16*b^5*c^3*d^14*e^10 + 204*a^17*b^2*c^5*d^15*e^9 - 76*a^17*b^3*c^4*d^14*e^10 - 4*a^14*b*c^9*d^22*e^2 - 8*a^15*b*c^8*d^20*e^4 - 80*a^17*b*c^6*d^16*e^8 + 48*a^18*b*c^5*d^14*e^10) - x*(2*a^14*c^9*d^18*e^5 + 4*a^16*c^7*d^14*e^9 + 2*a^14*b^4*c^5*d^14*e^9 - 8*a^15*b^2*c^6*d^14*e^9))*(-(b^9*e^2 + b^7*c^2*d^2

- b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 2
8*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c -
 b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4
*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^
2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2
) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-
(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b
*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^
9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2
*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^
7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^
3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) - ((-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*
e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/
2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^
3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c
^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^
2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2
)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(
4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 -
8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a
^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*
e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*
b^4*c*d^2*e^2)))^(1/2)*(((-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c
*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b
^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^
4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a
*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4
*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) -
8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^
3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e
^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^
2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^
3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2
)))^(1/2)*(x*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) -
9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^
2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 -
 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-
(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e
 + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 7
6*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*
d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8
*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b
^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c
^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 -
32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(5
12*a^20*c^7*d^24*e^3 + 512*a^21*c^6*d^22*e^5 - 512*a^22*c^5*d^20*e^7 - 512*
a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*d^25*e^2 + 128*a^18*b^4*c^5*d^24*e^3 -
192*a^18*b^5*c^4*d^23*e^4 + 128*a^18*b^6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^2
1*e^6 - 640*a^19*b^2*c^6*d^24*e^3 + 1056*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b
^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^21*e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512
*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^3*c^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20*
e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384*a^21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c
^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e^9 + 256*a^22*b^2*c^3*d^18*e^9 + 128*a^

19*b*c^7*d^25*e^2 - 1152*a^20*b*c^6*d^23*e^4 - 640*a^21*b*c^5*d^21*e^6 + 64
0*a^22*b*c^4*d^19*e^8) - 64*a^18*c^8*d^24*e^2 + 128*a^19*c^7*d^22*e^4 + 192
*a^20*c^6*d^20*e^6 - 256*a^21*c^5*d^18*e^8 - 256*a^22*c^4*d^16*e^10 - 16*a^
16*b^4*c^6*d^24*e^2 + 64*a^16*b^5*c^5*d^23*e^3 - 96*a^16*b^6*c^4*d^22*e^4 +
 64*a^16*b^7*c^3*d^21*e^5 - 16*a^16*b^8*c^2*d^20*e^6 + 80*a^17*b^2*c^7*d^24
*e^2 - 368*a^17*b^3*c^6*d^23*e^3 + 608*a^17*b^4*c^5*d^22*e^4 - 416*a^17*b^5
*c^4*d^21*e^5 + 80*a^17*b^6*c^3*d^20*e^6 + 16*a^17*b^7*c^2*d^19*e^7 - 928*a
^18*b^2*c^6*d^22*e^4 + 640*a^18*b^3*c^5*d^21*e^5 + 32*a^18*b^4*c^4*d^20*e^6
 - 128*a^18*b^5*c^3*d^19*e^7 - 432*a^19*b^2*c^5*d^20*e^6 + 304*a^19*b^3*c^4
*d^19*e^7 - 16*a^19*b^4*c^3*d^18*e^8 + 16*a^19*b^5*c^2*d^17*e^9 + 128*a^20*
b^2*c^4*d^18*e^8 - 128*a^20*b^3*c^3*d^17*e^9 - 16*a^20*b^4*c^2*d^16*e^10 +
128*a^21*b^2*c^3*d^16*e^10 + 448*a^18*b*c^7*d^23*e^3 - 192*a^20*b*c^5*d^19*
e^7 + 256*a^21*b*c^4*d^17*e^9) - x*(16*a^16*c^9*d^23*e^2 + 32*a^17*c^8*d^21
*e^4 - 112*a^18*c^7*d^19*e^6 - 128*a^20*c^5*d^15*e^10 + 8*a^14*b^4*c^7*d^23
*e^2 - 16*a^14*b^5*c^6*d^22*e^3 + 8*a^14*b^6*c^5*d^21*e^4 + 8*a^14*b^7*c^4*
d^20*e^5 - 16*a^14*b^8*c^3*d^19*e^6 + 8*a^14*b^9*c^2*d^18*e^7 - 32*a^15*b^2
*c^8*d^23*e^2 + 64*a^15*b^3*c^7*d^22*e^3 - 16*a^15*b^4*c^6*d^21*e^4 - 88*a^
15*b^5*c^5*d^20*e^5 + 160*a^15*b^6*c^4*d^19*e^6 - 88*a^15*b^7*c^3*d^18*e^7
- 48*a^16*b^2*c^7*d^21*e^4 + 264*a^16*b^3*c^6*d^20*e^5 - 520*a^16*b^4*c^5*d
^19*e^6 + 336*a^16*b^5*c^4*d^18*e^7 + 576*a^17*b^2*c^6*d^19*e^6 - 504*a^17*
b^3*c^5*d^18*e^7 + 8*a^18*b^3*c^4*d^16*e^9 - 16*a^18*b^4*c^3*d^15*e^10 + 8*
a^18*b^5*c^2*d^14*e^11 + 96*a^19*b^2*c^4*d^15*e^10 - 56*a^19*b^3*c^3*d^14*e
^11 - 32*a^16*b*c^8*d^22*e^3 - 192*a^17*b*c^7*d^20*e^5 + 224*a^18*b*c^6*d^1
8*e^7 - 32*a^19*b*c^5*d^16*e^9 + 96*a^20*b*c^4*d^14*e^11))*(-(b^9*e^2 + b^7
*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^
5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(
-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3
e^2(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a
*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2
)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c
^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2)
+ 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^
4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 -
8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*
e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b
^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) - 4*a^15*c^9*d^21*e^3 - 4*a^16*
c^8*d^19*e^5 + 48*a^18*c^6*d^15*e^9 - 4*a^14*b^2*c^8*d^21*e^3 - 4*a^14*b^7*
c^3*d^16*e^8 + 4*a^14*b^8*c^2*d^15*e^9 + 36*a^15*b^5*c^4*d^16*e^8 - 44*a^15
*b^6*c^3*d^15*e^9 + 4*a^15*b^7*c^2*d^14*e^10 - 100*a^16*b^3*c^5*d^16*e^8 +
160*a^16*b^4*c^4*d^15*e^9 - 32*a^16*b^5*c^3*d^14*e^10 - 204*a^17*b^2*c^5*d^
15*e^9 + 76*a^17*b^3*c^4*d^14*e^10 + 4*a^14*b*c^9*d^22*e^2 + 8*a^15*b*c^8*d
^20*e^4 + 80*a^17*b*c^6*d^16*e^8 - 48*a^18*b*c^5*d^14*e^10) - x*(2*a^14*c^9
*d^18*e^5 + 4*a^16*c^7*d^14*e^9 + 2*a^14*b^4*c^5*d^14*e^9 - 8*a^15*b^2*c^6*
d^14*e^9))*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*
a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*
b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 6
3*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4
*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e +
 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3
)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*
a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*
e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(
a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5
*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3
*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32
*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) + 2*
a^14*c^8*d^14*e^8))*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(
1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e

+ 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*2i - (1/(3*a*d) - (x^2*(a*e + b*d))/(a^2*d^2))/x^3 - (log(c^9*d^27*e^6 - b^9*d^18*e^15 + 2*a*c^8*d^25*e^8 - 2*b*c^8*d^26*e^7 + 2*b^8*c*d^19*e^14 + a^5*b^4*d^13*e^20 + a^2*c^7*d^23*e^10 + 16*a^4*c^5*d^19*e^14 + 16*a^7*c^2*d^13*e^20 + b^2*c^7*d^25*e^8 - b^7*c^2*d^20*e^13 - 25*a^2*b^3*c^4*d^20*e^13 + 66*a^2*b^4*c^3*d^19*e^14 - 42*a^2*b^5*c^2*d^18*e^15 - 76*a^3*b^2*c^4*d^19*e^14 + 63*a^3*b^3*c^3*d^18*e^15 + a^5*b^4*e^3*x*(-d^5*e^7)^(5/2) - a^2*c^7*d^15*x*(-d^5*e^7)^(3/2) + 16*a^7*c^2*e^3*x*(-d^5*e^7)^(5/2) + b^9*d^10*e^5*x*(-d^5*e^7)^(3/2) + c^9*d^24*e^3*x*(-d^5*e^7)^(1/2) - 2*a*b*c^7*d^24*e^9 + 11*a*b^7*c*d^18*e^15 + 9*a*b^5*c^3*d^20*e^13 - 20*a*b^6*c^2*d^19*e^14 + 20*a^3*b*c^5*d^20*e^13 - 28*a^4*b*c^4*d^18*e^15 - 8*a^6*b^2*c*d^13*e^20 - 16*a^4*c^5*d^11*e^4*x*(-d^5*e^7)^(3/2) + b^7*c^2*d^12*e^3*x*(-d^5*e^7)^(3/2) + b^2*c^7*d^22*e^5*x*(-d^5*e^7)^(1/2) - 8*a^6*b^2*c*e^3*x*(-d^5*e^7)^(5/2) + 2*a*c^8*d^22*e^5*x*(-d^5*e^7)^(1/2) - 2*b^8*c*d^11*e^4*x*(-d^5*e^7)^(3/2) - 2*b*c^8*d^23*e^4*x*(-d^5*e^7)^(1/2) - 11*a*b^7*c*d^10*e^5*x*(-d^5*e^7)^(3/2) - 2*a*b*c^7*d^21*e^6*x*(-d^5*e^7)^(1/2) - 9*a*b^5*c^3*d^12*e^3*x*(-d^5*e^7)^(3/2) + 20*a*b^6*c^2*d^11*e^4*x*(-d^5*e^7)^(3/2) - 20*a^3*b*c^5*d^12*e^3*x*(-d^5*e^7)^(3/2) + 28*a^4*b*c^4*d^10*e^5*x*(-d^5*e^7)^(3/2) + 25*a^2*b^3*c^4*d^12*e^3*x*(-d^5*e^7)^(3/2) - 66*a^2*b^4*c^3*d^11*e^4*x*(-d^5*e^7)^(3/2) + 42*a^2*b^5*c^2*d^10*e^5*x*(-d^5*e^7)^(3/2) + 76*a^3*b^2*c^4*d^11*e^4*x*(-d^5*e^7)^(3/2) - 63*a^3*b^3*c^3*d^10*e^5*x*(-d^5*e^7)^(3/2))*(-d^5*e^7)^(1/2))/(2*(c*d^7 + a*d^5*e^2 - b*d^6*e)) - atan((((-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(((-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*

```
b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*
a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2)
 + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e +
 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^
2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/
2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^
2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d
^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a
^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^
2*e^2)))^(1/2)*(512*a^20*c^7*d^24*e^3 + 512*a^21*c^6*d^22*e^5 - 512*a^22*c^
5*d^20*e^7 - 512*a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*d^25*e^2 + 128*a^18*b^
4*c^5*d^24*e^3 - 192*a^18*b^5*c^4*d^23*e^4 + 128*a^18*b^6*c^3*d^22*e^5 - 32
*a^18*b^7*c^2*d^21*e^6 - 640*a^19*b^2*c^6*d^24*e^3 + 1056*a^19*b^3*c^5*d^23
*e^4 - 672*a^19*b^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^21*e^6 + 32*a^19*b^6*c
^2*d^20*e^7 + 512*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^3*c^4*d^21*e^6 - 192*a
^20*b^4*c^3*d^20*e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384*a^21*b^2*c^4*d^20*e^7
 - 288*a^21*b^3*c^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e^9 + 256*a^22*b^2*c^3*
d^18*e^9 + 128*a^19*b*c^7*d^25*e^2 - 1152*a^20*b*c^6*d^23*e^4 - 640*a^21*b*
c^5*d^21*e^6 + 640*a^22*b*c^4*d^19*e^8) - 64*a^18*c^8*d^24*e^2 + 128*a^19*c
^7*d^22*e^4 + 192*a^20*c^6*d^20*e^6 - 256*a^21*c^5*d^18*e^8 - 256*a^22*c^4*
d^16*e^10 - 16*a^16*b^4*c^6*d^24*e^2 + 64*a^16*b^5*c^5*d^23*e^3 - 96*a^16*b
^6*c^4*d^22*e^4 + 64*a^16*b^7*c^3*d^21*e^5 - 16*a^16*b^8*c^2*d^20*e^6 + 80*
a^17*b^2*c^7*d^24*e^2 - 368*a^17*b^3*c^6*d^23*e^3 + 608*a^17*b^4*c^5*d^22*e
^4 - 416*a^17*b^5*c^4*d^21*e^5 + 80*a^17*b^6*c^3*d^20*e^6 + 16*a^17*b^7*c^2
*d^19*e^7 - 928*a^18*b^2*c^6*d^22*e^4 + 640*a^18*b^3*c^5*d^21*e^5 + 32*a^18
*b^4*c^4*d^20*e^6 - 128*a^18*b^5*c^3*d^19*e^7 - 432*a^19*b^2*c^5*d^20*e^6 +
 304*a^19*b^3*c^4*d^19*e^7 - 16*a^19*b^4*c^3*d^18*e^8 + 16*a^19*b^5*c^2*d^1
7*e^9 + 128*a^20*b^2*c^4*d^18*e^8 - 128*a^20*b^3*c^3*d^17*e^9 - 16*a^20*b^4
*c^2*d^16*e^10 + 128*a^21*b^2*c^3*d^16*e^10 + 448*a^18*b*c^7*d^23*e^3 - 192
*a^20*b*c^5*d^19*e^7 + 256*a^21*b*c^4*d^17*e^9) - x*(16*a^16*c^9*d^23*e^2 +
 32*a^17*c^8*d^21*e^4 - 112*a^18*c^7*d^19*e^6 - 128*a^20*c^5*d^15*e^10 + 8*
a^14*b^4*c^7*d^23*e^2 - 16*a^14*b^5*c^6*d^22*e^3 + 8*a^14*b^6*c^5*d^21*e^4
 + 8*a^14*b^7*c^4*d^20*e^5 - 16*a^14*b^8*c^3*d^19*e^6 + 8*a^14*b^9*c^2*d^18*
e^7 - 32*a^15*b^2*c^8*d^23*e^2 + 64*a^15*b^3*c^7*d^22*e^3 - 16*a^15*b^4*c^6
*d^21*e^4 - 88*a^15*b^5*c^5*d^20*e^5 + 160*a^15*b^6*c^4*d^19*e^6 - 88*a^15*
b^7*c^3*d^18*e^7 - 48*a^16*b^2*c^7*d^21*e^4 + 264*a^16*b^3*c^6*d^20*e^5 - 5
20*a^16*b^4*c^5*d^19*e^6 + 336*a^16*b^5*c^4*d^18*e^7 + 576*a^17*b^2*c^6*d^1
9*e^6 - 504*a^17*b^3*c^5*d^18*e^7 + 8*a^18*b^3*c^4*d^16*e^9 - 16*a^18*b^4*c
^3*d^15*e^10 + 8*a^18*b^5*c^2*d^14*e^11 + 96*a^19*b^2*c^4*d^15*e^10 - 56*a^
19*b^3*c^3*d^14*e^11 - 32*a^16*b*c^8*d^22*e^3 - 192*a^17*b*c^7*d^20*e^5 + 2
24*a^18*b*c^6*d^18*e^7 - 32*a^19*b*c^5*d^16*e^9 + 96*a^20*b*c^4*d^14*e^11))
*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*
d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^
2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*
c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2
)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d
*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^
4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c
 - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^
4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a
^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 -
 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2
*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) - 4*a^15*c^9*d
^21*e^3 - 4*a^16*c^8*d^19*e^5 + 48*a^18*c^6*d^15*e^9 - 4*a^14*b^2*c^8*d^21*
e^3 - 4*a^14*b^7*c^3*d^16*e^8 + 4*a^14*b^8*c^2*d^15*e^9 + 36*a^15*b^5*c^4*d
^16*e^8 - 44*a^15*b^6*c^3*d^15*e^9 + 4*a^15*b^7*c^2*d^14*e^10 - 100*a^16*b^
```

3*c^5*d^16*e^8 + 160*a^16*b^4*c^4*d^15*e^9 - 32*a^16*b^5*c^3*d^14*e^10 - 20
4*a^17*b^2*c^5*d^15*e^9 + 76*a^17*b^3*c^4*d^14*e^10 + 4*a^14*b*c^9*d^22*e^2
 + 8*a^15*b*c^8*d^20*e^4 + 80*a^17*b*c^6*d^16*e^8 - 48*a^18*b*c^5*d^14*e^10
) - x*(2*a^14*c^9*d^18*e^5 + 4*a^16*c^7*d^14*e^9 + 2*a^14*b^4*c^5*d^14*e^9
- 8*a^15*b^2*c^6*d^14*e^9))*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^
8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^
2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) +
 b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 2
0*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*
(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*
b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2)
 + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^
2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*
c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2
*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7
*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*
e^2)))^(1/2)*1i + ((-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1
/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e +
 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2
*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*
d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c
^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d
*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^
3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/
2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2
*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32
*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*
e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1
/2)*(((-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5
*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c
^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3
*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c
- b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^
5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b
^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(
4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b
^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^
3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*
e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*
b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^9
*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 2
0*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2
*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2
 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4
*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4
*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e -
 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)
^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*
a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*
c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*
b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3
+ 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(512*a^20*c^7*d^24*e^
3 + 512*a^21*c^6*d^22*e^5 - 512*a^22*c^5*d^20*e^7 - 512*a^23*c^4*d^18*e^9 -
 32*a^18*b^3*c^6*d^25*e^2 + 128*a^18*b^4*c^5*d^24*e^3 - 192*a^18*b^5*c^4*d^
23*e^4 + 128*a^18*b^6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^21*e^6 - 640*a^19*b^

2*c^6*d^24*e^3 + 1056*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^21*e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^3*c^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20*e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384*a^21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e^9 + 256*a^22*b^2*c^3*d^18*e^9 + 128*a^19*b*c^7*d^25*e^2 - 1152*a^20*b*c^6*d^23*e^4 - 640*a^21*b*c^5*d^21*e^6 + 640*a^22*b*c^4*d^19*e^8) + 64*a^18*c^8*d^24*e^2 - 128*a^19*c^7*d^22*e^4 - 192*a^20*c^6*d^20*e^6 + 256*a^21*c^5*d^18*e^8 + 256*a^22*c^4*d^16*e^10 + 16*a^16*b^4*c^6*d^24*e^2 - 64*a^16*b^5*c^5*d^23*e^3 + 96*a^16*b^6*c^4*d^22*e^4 - 64*a^16*b^7*c^3*d^21*e^5 + 16*a^16*b^8*c^2*d^20*e^6 - 80*a^17*b^2*c^7*d^24*e^2 + 368*a^17*b^3*c^6*d^23*e^3 - 608*a^17*b^4*c^5*d^22*e^4 + 416*a^17*b^5*c^4*d^21*e^5 - 80*a^17*b^6*c^3*d^20*e^6 - 16*a^17*b^7*c^2*d^19*e^7 + 928*a^18*b^2*c^6*d^22*e^4 - 640*a^18*b^3*c^5*d^21*e^5 - 32*a^18*b^4*c^4*d^20*e^6 + 128*a^18*b^5*c^3*d^19*e^7 + 432*a^19*b^2*c^5*d^20*e^6 - 304*a^19*b^3*c^4*d^19*e^7 + 16*a^19*b^4*c^3*d^18*e^8 - 16*a^19*b^5*c^2*d^17*e^9 - 128*a^20*b^2*c^4*d^18*e^8 + 128*a^20*b^3*c^3*d^17*e^9 + 16*a^20*b^4*c^2*d^16*e^10 - 128*a^21*b^2*c^3*d^16*e^10 - 448*a^18*b*c^7*d^23*e^3 + 192*a^20*b*c^5*d^19*e^7 - 256*a^21*b*c^4*d^17*e^9) - x*(16*a^16*c^9*d^23*e^2 + 32*a^17*c^8*d^21*e^4 - 112*a^18*c^7*d^19*e^6 - 128*a^20*c^5*d^15*e^10 + 8*a^14*b^4*c^7*d^23*e^2 - 16*a^14*b^5*c^6*d^22*e^3 + 8*a^14*b^6*c^5*d^21*e^4 + 8*a^14*b^7*c^4*d^20*e^5 - 16*a^14*b^8*c^3*d^19*e^6 + 8*a^14*b^9*c^2*d^18*e^7 - 32*a^15*b^2*c^8*d^23*e^2 + 64*a^15*b^3*c^7*d^22*e^3 - 16*a^15*b^4*c^6*d^21*e^4 - 88*a^15*b^5*c^5*d^20*e^5 + 160*a^15*b^6*c^4*d^19*e^6 - 88*a^15*b^7*c^3*d^18*e^7 - 48*a^16*b^2*c^7*d^21*e^4 + 264*a^16*b^3*c^6*d^20*e^5 - 520*a^16*b^4*c^5*d^19*e^6 + 336*a^16*b^5*c^4*d^18*e^7 + 576*a^17*b^2*c^6*d^19*e^6 - 504*a^17*b^3*c^5*d^18*e^7 + 8*a^18*b^3*c^4*d^16*e^9 - 16*a^18*b^4*c^3*d^15*e^10 + 8*a^18*b^5*c^2*d^14*e^11 + 96*a^19*b^2*c^4*d^15*e^10 - 56*a^19*b^3*c^3*d^14*e^11 - 32*a^16*b*c^8*d^22*e^3 - 192*a^17*b*c^7*d^20*e^5 + 224*a^18*b*c^6*d^18*e^7 - 32*a^19*b*c^5*d^16*e^9 + 96*a^20*b*c^4*d^14*e^11))*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) + 4*a^15*c^9*d^21*e^3 + 4*a^16*c^8*d^19*e^5 - 48*a^18*c^6*d^15*e^9 + 4*a^14*b^2*c^8*d^21*e^3 + 4*a^14*b^7*c^3*d^16*e^8 - 4*a^14*b^8*c^2*d^15*e^9 - 36*a^15*b^5*c^4*d^16*e^8 + 44*a^15*b^6*c^3*d^15*e^9 - 4*a^15*b^7*c^2*d^14*e^10 + 100*a^16*b^3*c^5*d^16*e^8 - 160*a^16*b^4*c^4*d^15*e^9 + 32*a^16*b^5*c^3*d^14*e^10 + 204*a^17*b^2*c^5*d^15*e^9 - 76*a^17*b^3*c^4*d^14*e^10 - 4*a^14*b*c^9*d^22*e^2 - 8*a^15*b*c^8*d^20*e^4 - 80*a^17*b*c^6*d^16*e^8 + 48*a^18*b*c^5*d^14*e^10) - x*(2*a^14*c^9*d^18*e^5 + 4*a^16*c^7*d^14*e^9 + 2*a^14*b^4*c^5*d^14*e^9 - 8*a^15*b^2*c^6*d^14*e^9))*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^

2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*1i)/(((-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(((-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(512*a^20*c^7*d^24*e^3 + 512*a^21*c^6*d^22*e^5 - 512*a^22*c^5*d^20*e^7 - 512*a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*d^25*e^2 + 128*a^18*b^4*c^5*d^24*e^3 - 192*a^18*b^5*c^4*d^23*e^4 + 128*a^18*b^6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^21*e^6 - 640*a^19*b^2*c^6*d^24*e^3 + 1056*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^21*e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^3*c^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20*e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384*a^21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e^9 + 256*a^22*b^2*c^3*d^18*e^9 + 128*a^19*b*c^7*d^25*e^2 - 1152*a^20*b*c^6*d^23*e^4 - 640*a^21*b*c^5*d^21*e^6 + 640*a^22*b*c^4*d^19*e^8) + 64*a^18*c^8*d^24*e^2 - 128*a^19*c^7*d^22*e^4 - 192*a^20*c^6*d^20*e^6 + 256*a^21*c^5*d^18*e^8 + 256*a^22*c^4*d^16*e^10 + 16*a^16*b^4*c^6*d^24*e^2 - 64*a^16*b^5*c^5*d^23*e^3 + 96*a^16*b^6*c^4*d^22*e^4 - 64*a^16*b^7*c^3*d^21*e^5 + 16*a^16*b^8*c^2*d^20*e^6 - 80*a^17*b^2*c^7*d^24*e^2 + 368*a^17*b^3*c^6*d^23*e^3 - 608*a^17*b^4*c^5*d^22*e^4 + 416*a^17*b^5*c^4*d^21*e^5 - 80*a^17*b^6*c^3*d^20*e^6 - 16*a^17*b^7*c^2*d^19*e^7 + 928*a^18*b^2*c^6*d^22*e^4 - 640*a^18*b^3*c^5*d^21*e^5 - 32*a^18*b^4*c^4*d^20*e^6 + 128*a^18*b^5*c^3*d^19*e^7 + 432*a^19*b^2*c^5*d^20*e^6 - 304*a^19*b^3*c^4*d^19*e^7 + 16*a^19*b^4*c^3*d^18*e^8 - 16*a^19*b^5*c^2* d^17*e^9 - 128*a^20*b^2*c^4*d^18*e^8 + 128*a^20*b^3*c^3*d^17*e^9 + 16*a^20*b^4*c^2*d^16*e^10 - 128*a^21*b^2*c^3*d^16*e^10 - 448*a^18*b*c^7*d^23*e^3 + 192*a^20*b*c^5*d^19*e^7 - 256*a^21*b*c^4*d^17*e^9) - x*(16*a^16*c^9*d^23*e^2 + 32*a^17*c^8*d^21*e^4 - 112*a^18*c^7*d^19*e^6 - 128*a^20*c^5*d^15*e^10 + 8*a^14*b^4*c^7*d^23*e^2 - 16*a^14*b^5*c^6*d^22*e^3 + 8*a^14*b^6*c^5*d^21*e^4 + 8*a^14*b^7*c^4*d^20*e^5 - 16*a^14*b^8*c^3*d^19*e^6 + 8*a^14*b^9*c^2*d^18*e^7 - 32*a^15*b^2*c^8*d^23*e^2 + 64*a^15*b^3*c^7*d^22*e^3 - 16*a^15*b^4*c^6*d^21*e^4 - 88*a^15*b^5*c^5*d^20*e^5 + 160*a^15*b^6*c^4*d^19*e^6 - 88*a^15*b^7*c^3*d^18*e^7 - 48*a^16*b^2*c^7*d^21*e^4 + 264*a^16*b^3*c^6*d^20*e^5 - 520*a^16*b^4*c^5*d^19*e^6 + 336*a^16*b^5*c^4*d^18*e^7 + 576*a^17*b^2*c^6*d^19*e^6 - 504*a^17*b^3*c^5*d^18*e^7 + 8*a^18*b^3*c^4*d^16*e^9 - 16*a^18*b^4*c^3*d^15*e^10 + 8*a^18*b^5*c^2*d^14*e^11 + 96*a^19*b^2*c^4*d^15*e^10 - 56*a^19*b^3*c^3*d^14*e^11 - 32*a^16*b*c^8*d^22*e^3 - 192*a^17*b*c^7*d^20*e^5 + 224*a^18*b*c^6*d^18*e^7 - 32*a^19*b*c^5*d^16*e^9 + 96*a^20*b*c^4*d^14*e^11))*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) + 4*a^15*c^9*d^21*e^3 + 4*a^16*c^8*d^19*e^5 - 48*a^18*c^6*d^15*e^9 + 4*a^14*b^2*c^8*d^21*e^3 + 4*a^14*b^7*c^3*d^16*e^8 - 4*a^14*b^8*c^2*d^15*e^9 - 36*a^15*b^5*c^4*d^16*e^8 + 44*a^15*b^6*c^3*d^15*e^9 - 4*a^15*b^7*c^2*d^14*e^10 + 100*a^16*b^3*c^5*d^16*e^8 - 160*a^16*b^4*c^4*d^15*e^9 + 32*a^16*b^5*c^3*d^14*e^10 + 204*a^17*b^2*c^5*d^15*e^9 - 76*a^17*b^3*c^4*d^14*e^10 - 4*a^14*b*c^9*d^22*e^2 - 8*a^15*b*c^8*d^20*e^4 - 80*a^17*b*c^6*d^16*e^8 + 48*a^18*b*c^5*d^14*e^10) - x*(2*a^14*c^9*d^18*e^5 + 4*a^16*c^7*d^14*e^9 + 2*a^14*b^4*c^5*d^14*e^9 - 8*a^15*b^2*c^6*d^14*e^9))*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) - ((-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d* e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(((-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(512*a^20*c^7*d^24*e^3 + 512*a^21*c^6*d^22*e^5 - 512*a^22*c^5*d^20*e^7 - 512*a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*d^25*e^2 + 128*a^18*b^4*c^5*d^24*e^3 - 192*a^18*b^5*c^4*d^23*e^4 + 128*a^18*b^6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^21*e^6 - 640*a^19*b^2*c^6*d^24*e^3 + 1056*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^21*e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^3*c^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20*e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384*a^21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e^9 + 256*a^22*b^2*c^3*d^18*e^9 + 128*a^19*b*c^7*d^25*e^2 - 1152*a^20*b*c^6*d^23*e^4 - 640*a^21*b*c^5*d^21*e^6 + 640*a^22*b*c^4*d^19*e^8) - 64*a^18*c^8*d^24*e^2 + 128*a^19*c^7*d^22*e^4 + 192*a^20*c^6*d^20*e^6 - 256*a^21*c^5*d^18*e^8 - 256*a^22*c^4*d^16*e^10 - 16*a^16*b^4*c^6*d^24*e^2 + 64*a^16*b^5*c^5*d^23*e^3 - 96*a^16*b^6*c^4*d^22*e^4 + 64*a^16*b^7*c^3*d^21*e^5 - 16*a^16*b^8*c^2*d^20*e^6 + 80*a^17*b^2*c^7*d^24*e^2 - 368*a^17*b^3*c^6*d^23*e^3 + 608*a^17*b^4*c^5*d^22*e^4 - 416*a^17*b^5*c^4*d^21*e^5 + 80*a^17*b^6*c^3*d^20*e^6 + 16*a^17*b^7*c^2*d^19*e^7 - 928*a^18*b^2*c^6*d^22*e^4 + 640*a^18*b^3*c^5*d^21*e^5 + 32*a^18*b^4*c^4*d^20*e^6 - 128*a^18*b^5*c^3*d^19*e^7 - 432*a^19*b^2*c^5*d^20*e^6 + 304*a^19*b^3*c^4*d^19*e^7 - 16*a^19*b^4*c^3*d^18*e^8 + 16*a^19*b^5*c^2*d^17*e^9 + 128*a^20*b^2*c^4*d^18*e^8 - 128*a^20*b^3*c^3*d^17*e^9 - 16*a^20*b^4*c^2*d^16*e^10 + 128*a^21*b^2*c^3*d^16*e^10 + 448*a^18*b*c^7*d^23*e^3 - 192*a^20*b*c^5*d^19*e^7 + 256*a^21*b*c^4*d^17*e^9) - x*(16*a^16*c^9*d^23*e^2 + 32*a^17*c^8*d^21*e^4 - 112*a^18*c^7*d^19*e^6 - 128*a^20*c^5*d^15*e^10 + 8*a^14*b^4*c^7*d^23*e^2 - 16*a^14*b^5*c^6*d^22*e^3 + 8*a^14*b^6*c^5*d^21*e^4 + 8*a^14*b^7*c^4*d^20*e^5 - 16*a^14*b^8*c^3*d^19*e^6 + 8*a^14*b^9*c^2*d^18*e^7 - 32*a^15*b^2*c^8*d^23*e^2 + 64*a^15*b^3*c^7*d^22*e^3 - 16*a^15*b^4*c^6*d^21*e^4 - 88*a^15*b^5*c^5*d^20*e^5 + 160*a^15*b^6*c^4*d^19*e^6 - 88*a^15*b^7*c^3*d^18*e^7 - 48*a^16*b^2*c^7*d^21*e^4 + 264*a^16*b^3*c^6*d^20*e^5 - 520*a^16*b^4*c^5*d^19*e^6 + 336*a^16*b^5*c^4*d^18*e^7 + 576*a^17*b^2*c^6*d^19*e^6 - 504*a^17*b^3*c^5*d^18*e^7 + 8*a^18*b^3*c^4*d^16*e^9 - 16*a^18*b^4*c^3*d^15*e^10 + 8*a^18*b^5*c^2*d^14*e^11 + 96*a^19*b^2*c^4*d^15*e^10 - 56*a^19*b^3*c^3*d^14*e^11 - 32*a^16*b*c^8*d^22*e^3 - 192*a^17*b*c^7*d^20*e^5 + 224*a^18*b*c^6*d^18*e^7 - 32*a^19*b*c^5*d^16*e^9 + 96*a^20*b*c^4*d^14*e^11))*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*

```
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^
4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(
1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2
)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4
*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*
b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b
^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(
-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4
- 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 +
 a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^
3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^
6*b^4*c*d^2*e^2)))^(1/2) - 4*a^15*c^9*d^21*e^3 - 4*a^16*c^8*d^19*e^5 + 48*a
^18*c^6*d^15*e^9 - 4*a^14*b^2*c^8*d^21*e^3 - 4*a^14*b^7*c^3*d^16*e^8 + 4*a^
14*b^8*c^2*d^15*e^9 + 36*a^15*b^5*c^4*d^16*e^8 - 44*a^15*b^6*c^3*d^15*e^9 +
 4*a^15*b^7*c^2*d^14*e^10 - 100*a^16*b^3*c^5*d^16*e^8 + 160*a^16*b^4*c^4*d^
15*e^9 - 32*a^16*b^5*c^3*d^14*e^10 - 204*a^17*b^2*c^5*d^15*e^9 + 76*a^17*b^
3*c^4*d^14*e^10 + 4*a^14*b*c^9*d^22*e^2 + 8*a^15*b*c^8*d^20*e^4 + 80*a^17*b
*c^6*d^16*e^8 - 48*a^18*b*c^5*d^14*e^10) - x*(2*a^14*c^9*d^18*e^5 + 4*a^16*
c^7*d^14*e^9 + 2*a^14*b^4*c^5*d^14*e^9 - 8*a^15*b^2*c^6*d^14*e^9))*(-(b^9*e
^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*
a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c
^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 -
 a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2
) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a
*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c
*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3
*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3
)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^
7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^
2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^
5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 +
16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) + 2*a^14*c^8*d^14*e^8))
*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*
d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^
2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*
c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2
)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d
*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^
4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c
 - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^
4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a
^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 -
 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2
*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.232 $\int \frac{1}{\sqrt{fx}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=866

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}\right)e^{7/4}}{\sqrt{2}\,d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}+1\right)e^{7/4}}{\sqrt{2}\,d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}-\frac{\log\left(\sqrt{e}\sqrt{f}\,x+\sqrt{d}\sqrt{f}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{fx}\right)e^{7/4}}{2\sqrt{2}\,d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}+$$

Rubi [A] time = 2.51, antiderivative size = 866, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {1269, 1424, 211, 1165, 628, 1162, 617, 204, 1422, 212, 208, 205}

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}\right)e^{7/4}}{\sqrt{2}\,d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}+1\right)e^{7/4}}{\sqrt{2}\,d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}-\frac{\log\left(\sqrt{e}\sqrt{f}\,x+\sqrt{d}\sqrt{f}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{fx}\right)e^{7/4}}{2\sqrt{2}\,d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}+\frac{\log\left(\sqrt{e}\sqrt{f}\,x+\sqrt{d}\sqrt{f}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{fx}\right)e^{7/4}}{2\sqrt{2}\,d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}+\frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}-\frac{c^{3/4}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}+\frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}-\frac{c^{3/4}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (c^(3/4)*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[f
*x])/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*
(-b - Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (c^(3/4)*
(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[f*x])/((-b
 + Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqr
t[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (e^(7/4)*ArcTan[1
 - (Sqrt[2]*e^(1/4)*Sqrt[f*x])/(d^(1/4)*Sqrt[f])])/(Sqrt[2]*d^(3/4)*(c*d^2 -
 b*d*e + a*e^2)*Sqrt[f]) + (e^(7/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[f*x])/
(d^(1/4)*Sqrt[f])])/(Sqrt[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) + (c^
(3/4)*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[f*x
])/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-
b - Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (c^(3/4)*(2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[f*x])/((-b
+ Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt
[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (e^(7/4)*Log[Sqrt[d
]*Sqrt[f] + Sqrt[e]*Sqrt[f]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[f*x]])/(2*Sqrt
[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) + (e^(7/4)*Log[Sqrt[d]*Sqrt[f]
 + Sqrt[e]*Sqrt[f]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[f*x]])/(2*Sqrt[2]*d^(3/
4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1269

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/f, Subst[I
nt[x^(k*(m + 1) - 1)*(d + (e*x^(2*k))/f^2)^q*(a + (b*x^(2*k))/f^k + (c*x^(4
*k))/f^4)^p, x], x, (f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, p, q}, x]
 && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1424

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),
```

```
x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{\sqrt{fx}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}dx = \frac{2\,\text{Subst}\left(\int \frac{1}{\left(d+\frac{ex^4}{f^2}\right)\left(a+\frac{bx^4}{f^2}+\frac{cx^8}{f^4}\right)}dx, x, \sqrt{fx}\right)}{f}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{e^2f^2}{\left(cd^2-bde+ae^2\right)\left(df^2+ex^4\right)} + \frac{cdf^4-bef^4-cef^2x^4}{\left(cd^2-bde+ae^2\right)\left(af^4+bf^2x^4+cx^8\right)}\right)dx, x, \sqrt{fx}\right.}{f}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{cdf^4-bef^4-cef^2x^4}{af^4+bf^2x^4+cx^8}dx, x, \sqrt{fx}\right)}{\left(cd^2-bde+ae^2\right)f} + \frac{\left(2e^2f\right)\text{Subst}\left(\int \frac{1}{df^2+ex^4}dx, x, \right.}{cd^2-bde+ae^2}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{\sqrt{d}f-\sqrt{e}x^2}{df^2+ex^4}dx, x, \sqrt{fx}\right)}{\sqrt{d}\left(cd^2-bde+ae^2\right)} + \frac{e^2\,\text{Subst}\left(\int \frac{\sqrt{d}f+\sqrt{e}x^2}{df^2+ex^4}dx, x, \sqrt{fx}\right)}{\sqrt{d}\left(cd^2-bde+ae^2\right)}$$

$$= \frac{e^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}f}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{f}x}{\sqrt[4]{e}}+x^2}dx, x, \sqrt{fx}\right)}{2\sqrt{d}\left(cd^2-bde+ae^2\right)} + \frac{e^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}f}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{f}x}{\sqrt[4]{e}}}\right.}{2\sqrt{d}\left(cd^2-bde\right.}$$

$$= \frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\left(cd^2-bde+ae^2\right)\sqrt{f}} - \frac{c^{3/4}\left(2cd-\right.}{\sqrt[4]{2}\sqrt{b^2-4a}}$$

$$= \frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\left(cd^2-bde+ae^2\right)\sqrt{f}} - \frac{c^{3/4}\left(2cd-\right.}{\sqrt[4]{2}\sqrt{b^2-4a}}$$

Mathematica [C] time = 0.38, size = 267, normalized size = 0.31

$$\frac{\sqrt{x}\left(\sqrt{2}e^{7/4}\left(-\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{d}+\sqrt{e}x\right)+\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{d}+\sqrt{e}x\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}+1\right)\right)-2d^{3/4}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4ce\log\left(\sqrt{x}-\#1\right)+be\log\left(\sqrt{x}-\#1\right)-cd\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]\right)}{4d^{3/4}\sqrt{fx}\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[x]*(Sqrt[2]*e^(7/4)*(-2*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)]
 + 2*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - Log[Sqrt[d] - Sqrt[2]*
d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] + Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[x] + Sqrt[e]*x]) - 2*d^(3/4)*RootSum[a + b*#1^4 + c*#1^8 & , (-(c*d*
Log[Sqrt[x] - #1]) + b*e*Log[Sqrt[x] - #1] + c*e*Log[Sqrt[x] - #1]*#1^4)/(b
*#1^3 + 2*c*#1^7) & ]))/(4*d^(3/4)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[f*x])
```

IntegrateAlgebraic [C] time = 0.40, size = 308, normalized size = 0.36

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4bf^2+af^4\&,\frac{\#1^4cef\log\left(\sqrt{fx}-\#1\right)+bef^3\log\left(\sqrt{fx}-\#1\right)-cdf^3\log\left(\sqrt{fx}-\#1\right)}{2\#1^7c+\#1^3bf^2}\&\right]}{2\left(ae^2-bde+cd^2\right)}-\frac{e^{7/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{d}\sqrt{f}}{\sqrt{2}\sqrt[4]{e}}-\frac{\sqrt[4]{e}\sqrt{f}x}{\sqrt{2}\sqrt[4]{d}}}{\sqrt{fx}}\right)}{\sqrt{2}d^{3/4}\sqrt{f}\left(ae^2-bde+cd^2\right)}+\frac{e^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{f}\sqrt{fx}}{\sqrt{d}f+\sqrt{e}fx}\right)}{\sqrt{2}d^{3/4}\sqrt{f}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((e^(7/4)*ArcTan[((d^(1/4)*Sqrt[f])/(Sqrt[2]*e^(1/4)) - (e^(1/4)*Sqrt[f]*x
)/(Sqrt[2]*d^(1/4)))/Sqrt[f*x]])/(Sqrt[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)*S
qrt[f])) + (e^(7/4)*ArcTanh[(Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[f]*Sqrt[f*x])/(Sq
rt[d]*f + Sqrt[e]*f*x)])/(Sqrt[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f])
- RootSum[a*f^4 + b*f^2*#1^4 + c*#1^8 & , (-(c*d*f^3*Log[Sqrt[f*x] - #1]) +
 b*e*f^3*Log[Sqrt[f*x] - #1] + c*e*f*Log[Sqrt[f*x] - #1]*#1^4)/(b*f^2*#1^3
+ 2*c*#1^7) & ]/(2*(c*d^2 - b*d*e + a*e^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage2
```

maple [C] time = 0.10, size = 336, normalized size = 0.39

$$\frac{f\left(-\operatorname{RootOf}\left(c\_Z^8+b f^2\_Z^4+a f^4\right)^4 ce-be f^2+cd f^2\right)\ln\left(-\operatorname{RootOf}\left(c\_Z^8+b f^2\_Z^4+a f^4\right)+\sqrt{fx}\right)}{2\left(a e^2-deb+c d^2\right)\left(2\operatorname{RootOf}\left(c\_Z^8+b f^2\_Z^4+a f^4\right)^7 c+\operatorname{RootOf}\left(c\_Z^8+b f^2\_Z^4+a f^4\right)^3 b f^2\right)}+\frac{\left(\frac{d f^2}{e}\right)^{\frac{1}{4}}\sqrt{2}\, e^2\arctan\left(\frac{\sqrt{2}\,\sqrt{fx}}{\left(\frac{d f^2}{e}\right)^{\frac{1}{4}}}-1\right)}{2\left(a e^2-deb+c d^2\right)df}+\frac{\left(\frac{d f^2}{e}\right)^{\frac{1}{4}}\sqrt{2}\, e^2\arctan\left(\frac{\sqrt{2}\,\sqrt{fx}}{\left(\frac{d f^2}{e}\right)^{\frac{1}{4}}}+1\right)}{2\left(a e^2-deb+c d^2\right)df}+\frac{\left(\frac{d f^2}{e}\right)^{\frac{1}{4}}\sqrt{2}\, e^2\ln\left(\frac{fx+\left(\frac{d f^2}{e}\right)^{\frac{1}{4}}\sqrt{fx}\,\sqrt{2}+\sqrt{\frac{d f^2}{e}}}{fx-\left(\frac{d f^2}{e}\right)^{\frac{1}{4}}\sqrt{fx}\,\sqrt{2}+\sqrt{\frac{d f^2}{e}}}\right)}{4\left(a e^2-deb+c d^2\right)df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x)
```

```
[Out] 1/2*f/(a*e^2-b*d*e+c*d^2)*sum((-_R^4*c*e-b*e*f^2+c*d*f^2)/(2*_R^7*c+_R^3*b*
f^2)*ln((f*x)^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b*f^2+a*f^4))+1/4/f*e^2/(a*e^
2-b*d*e+c*d^2)*(d*f^2/e)^(1/4)/d*2^(1/2)*ln((f*x+(d*f^2/e)^(1/4)*(f*x)^(1/2
)*2^(1/2)+(d*f^2/e)^(1/2))/(f*x-(d*f^2/e)^(1/4)*(f*x)^(1/2)*2^(1/2)+(d*f^2/
e)^(1/2)))+1/2/f*e^2/(a*e^2-b*d*e+c*d^2)*(d*f^2/e)^(1/4)/d*2^(1/2)*arctan(2
^(1/2)/(d*f^2/e)^(1/4)*(f*x)^(1/2)+1)+1/2/f*e^2/(a*e^2-b*d*e+c*d^2)*(d*f^2/
e)^(1/4)/d*2^(1/2)*arctan(2^(1/2)/(d*f^2/e)^(1/4)*(f*x)^(1/2)-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2 e^2\sqrt{x}}{cd^3\sqrt{f}-bd^2e\sqrt{f}+ade^2\sqrt{f}}+\frac{\frac{2\sqrt{2}e^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}d^{\frac{1}{4}}e^{\frac{1}{4}}+2\sqrt{e}\sqrt{x}\right)}{2\sqrt{\sqrt{d}\sqrt{e}}}\right)}{\sqrt{d}\sqrt{\sqrt{d}\sqrt{e}}}+\frac{2\sqrt{2}e^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}d^{\frac{1}{4}}e^{\frac{1}{4}}-2\sqrt{e}\sqrt{x}\right)}{2\sqrt{\sqrt{d}\sqrt{e}}}\right)}{\sqrt{d}\sqrt{\sqrt{d}\sqrt{e}}}+\frac{\sqrt{2}e^{\frac{7}{4}}\log\left(\sqrt{2}d^{\frac{1}{4}}e^{\frac{1}{4}}\sqrt{x}+\sqrt{e}x+\sqrt{d}\right)}{d^{\frac{3}{4}}}-\frac{\sqrt{2}e^{\frac{7}{4}}\log\left(-\sqrt{2}d^{\frac{1}{4}}e^{\frac{1}{4}}\sqrt{x}+\sqrt{e}x+\sqrt{d}\right)}{d^{\frac{3}{4}}}}{4\left(cd^2\sqrt{f}-bde\sqrt{f}+ae^2\sqrt{f}\right)}+\frac{2\sqrt{x}}{ad\sqrt{f}}+\int -\frac{\left(c^2d-bce\right)x^{\frac{7}{2}}+\left(bcd-b^2e+ace\right)x^{\frac{3}{2}}}{a^3e^2\sqrt{f}+\left(a^2ce^2\sqrt{f}+\left(c^2d^2\sqrt{f}-bcde\sqrt{f}\right)a\right)x^4+\left(cd^2\sqrt{f}-bde\sqrt{f}\right)a^2+\left(a^2be^2\sqrt{f}+\left(bcd^2\sqrt{f}-b^2de\sqrt{f}\right)a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*e^2*sqrt(x)/(c*d^3*sqrt(f) - b*d^2*e*sqrt(f) + a*d*e^2*sqrt(f)) + 1/4*(2
*sqrt(2)*e^2*arctan(1/2*sqrt(2)*(sqrt(2)*d^(1/4)*e^(1/4) + 2*sqrt(e)*sqrt(x
))/sqrt(sqrt(d)*sqrt(e)))/(sqrt(d)*sqrt(sqrt(d)*sqrt(e))) + 2*sqrt(2)*e^2*a
rctan(-1/2*sqrt(2)*(sqrt(2)*d^(1/4)*e^(1/4) - 2*sqrt(e)*sqrt(x))/sqrt(sqrt(
d)*sqrt(e)))/(sqrt(d)*sqrt(sqrt(d)*sqrt(e))) + sqrt(2)*e^(7/4)*log(sqrt(2)*
```

```
d^(1/4)*e^(1/4)*sqrt(x) + sqrt(e)*x + sqrt(d))/d^(3/4) - sqrt(2)*e^(7/4)*lo
g(-sqrt(2)*d^(1/4)*e^(1/4)*sqrt(x) + sqrt(e)*x + sqrt(d))/d^(3/4))/(c*d^2*s
qrt(f) - b*d*e*sqrt(f) + a*e^2*sqrt(f)) + 2*sqrt(x)/(a*d*sqrt(f)) + integra
te(-((c^2*d - b*c*e)*x^(7/2) + (b*c*d - b^2*e + a*c*e)*x^(3/2))/(a^3*e^2*sq
rt(f) + (a^2*c*e^2*sqrt(f) + (c^2*d^2*sqrt(f) - b*c*d*e*sqrt(f))*a)*x^4 + (
c*d^2*sqrt(f) - b*d*e*sqrt(f))*a^2 + (a^2*b*e^2*sqrt(f) + (b*c*d^2*sqrt(f)
- b^2*d*e*sqrt(f))*a)*x^2), x)
```

mupad [B] time = 6.84, size = 43112, normalized size = 49.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f*x)^(1/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] symsum(log(-root(8388608*a^7*b*c^11*d^18*e*f^6*h^12 - 513802240*a^10*b^2*c^
7*d^11*e^8*f^6*h^12 - 381681664*a^11*b^2*c^6*d^9*e^10*f^6*h^12 - 381681664*
a^9*b^2*c^8*d^13*e^6*f^6*h^12 - 300941312*a^9*b^5*c^5*d^10*e^9*f^6*h^12 - 3
00941312*a^8*b^5*c^6*d^12*e^7*f^6*h^12 + 293601280*a^10*b^3*c^6*d^10*e^9*f^
6*h^12 + 293601280*a^9*b^3*c^7*d^12*e^7*f^6*h^12 - 168820736*a^10*b^5*c^4*d
^8*e^11*f^6*h^12 - 168820736*a^7*b^5*c^7*d^14*e^5*f^6*h^12 + 166068224*a^8*
b^6*c^5*d^11*e^8*f^6*h^12 - 146800640*a^12*b^2*c^5*d^7*e^12*f^6*h^12 - 1468
00640*a^8*b^2*c^9*d^15*e^4*f^6*h^12 + 124780544*a^10*b^4*c^5*d^9*e^10*f^6*h
^12 + 124780544*a^8*b^4*c^7*d^13*e^6*f^6*h^12 + 119275520*a^9*b^4*c^6*d^11*
e^8*f^6*h^12 + 117440512*a^11*b^3*c^5*d^8*e^11*f^6*h^12 + 117440512*a^8*b^3
*c^8*d^14*e^5*f^6*h^12 + 102760448*a^9*b^6*c^4*d^9*e^10*f^6*h^12 + 10276044
8*a^7*b^6*c^6*d^13*e^6*f^6*h^12 + 91750400*a^11*b^4*c^4*d^7*e^12*f^6*h^12 +
 91750400*a^7*b^4*c^8*d^15*e^4*f^6*h^12 - 71065600*a^7*b^8*c^4*d^11*e^8*f^6
*h^12 - 53444608*a^8*b^8*c^3*d^9*e^10*f^6*h^12 - 53444608*a^6*b^8*c^5*d^13*
e^6*f^6*h^12 + 40370176*a^9*b^7*c^3*d^8*e^11*f^6*h^12 + 40370176*a^6*b^7*c^
6*d^14*e^5*f^6*h^12 - 36700160*a^11*b^5*c^3*d^6*e^13*f^6*h^12 - 36700160*a^
6*b^5*c^8*d^16*e^3*f^6*h^12 + 34078720*a^8*b^7*c^4*d^10*e^9*f^6*h^12 + 3407
8720*a^7*b^7*c^5*d^12*e^7*f^6*h^12 + 26214400*a^12*b^4*c^3*d^5*e^14*f^6*h^1
2 + 26214400*a^6*b^4*c^9*d^17*e^2*f^6*h^12 + 22118400*a^7*b^9*c^3*d^10*e^9*
f^6*h^12 + 22118400*a^6*b^9*c^4*d^12*e^7*f^6*h^12 - 20971520*a^13*b^2*c^4*d
^5*e^14*f^6*h^12 - 20971520*a^7*b^2*c^10*d^17*e^2*f^6*h^12 + 18350080*a^10*
b^7*c^2*d^6*e^13*f^6*h^12 + 18350080*a^5*b^7*c^7*d^16*e^3*f^6*h^12 - 166297
60*a^9*b^8*c^2*d^7*e^12*f^6*h^12 - 16629760*a^5*b^8*c^6*d^15*e^4*f^6*h^12 -
 10485760*a^11*b^6*c^2*d^5*e^14*f^6*h^12 - 10485760*a^5*b^6*c^8*d^17*e^2*f^
6*h^12 + 9175040*a^10*b^6*c^3*d^7*e^12*f^6*h^12 + 9175040*a^6*b^6*c^7*d^15*
e^4*f^6*h^12 - 8388608*a^13*b^3*c^3*d^4*e^15*f^6*h^12 + 5619712*a^7*b^10*c^
2*d^9*e^10*f^6*h^12 + 5619712*a^5*b^10*c^4*d^13*e^6*f^6*h^12 - 5570560*a^6*
b^11*c^2*d^10*e^9*f^6*h^12 - 5570560*a^5*b^11*c^3*d^12*e^7*f^6*h^12 + 43581
44*a^8*b^9*c^2*d^8*e^11*f^6*h^12 + 4358144*a^5*b^9*c^5*d^14*e^5*f^6*h^12 +
4259840*a^6*b^10*c^3*d^11*e^8*f^6*h^12 + 3899392*a^4*b^10*c^5*d^15*e^4*f^6*
h^12 - 3440640*a^4*b^9*c^6*d^16*e^3*f^6*h^12 + 3145728*a^12*b^5*c^2*d^4*e^1
5*f^6*h^12 - 2523136*a^4*b^11*c^4*d^14*e^5*f^6*h^12 + 1802240*a^4*b^8*c^7*d
^17*e^2*f^6*h^12 + 1556480*a^5*b^12*c^2*d^11*e^8*f^6*h^12 + 1048576*a^14*b^
2*c^3*d^3*e^16*f^6*h^12 + 688128*a^4*b^12*c^3*d^13*e^6*f^6*h^12 - 393216*a^
13*b^4*c^2*d^3*e^16*f^6*h^12 - 286720*a^3*b^12*c^4*d^15*e^4*f^6*h^12 + 2293
76*a^3*b^13*c^3*d^14*e^5*f^6*h^12 + 229376*a^3*b^11*c^5*d^16*e^3*f^6*h^12 +
 163840*a^4*b^13*c^2*d^12*e^7*f^6*h^12 - 114688*a^3*b^14*c^2*d^13*e^6*f^6*h
^12 - 114688*a^3*b^10*c^6*d^17*e^2*f^6*h^12 + 293601280*a^11*b*c^7*d^10*e^9
*f^6*h^12 + 293601280*a^10*b*c^8*d^12*e^7*f^6*h^12 + 176160768*a^12*b*c^6*d
^8*e^11*f^6*h^12 + 176160768*a^9*b*c^9*d^14*e^5*f^6*h^12 + 58720256*a^13*b*
c^5*d^6*e^13*f^6*h^12 + 58720256*a^8*b*c^10*d^16*e^3*f^6*h^12 + 8388608*a^1
4*b*c^4*d^4*e^15*f^6*h^12 - 8388608*a^6*b^3*c^10*d^18*e*f^6*h^12 + 3899392*
a^8*b^10*c*d^7*e^12*f^6*h^12 - 3440640*a^9*b^9*c*d^6*e^13*f^6*h^12 + 314572
8*a^5*b^5*c^9*d^18*e*f^6*h^12 - 2523136*a^7*b^11*c*d^8*e^11*f^6*h^12 + 1802
240*a^10*b^8*c*d^5*e^14*f^6*h^12 + 688128*a^6*b^12*c*d^9*e^10*f^6*h^12 - 52
```

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4288*a^11*b^7*c*d^4*e^15*f^6*h^12 - 524288*a^4*b^7*c^8*d^18*e*f^6*h^12 + 16
3840*a^5*b^13*c*d^10*e^9*f^6*h^12 - 163840*a^4*b^14*c*d^11*e^8*f^6*h^12 + 6
5536*a^12*b^6*c*d^3*e^16*f^6*h^12 + 32768*a^3*b^15*c*d^12*e^7*f^6*h^12 + 32
768*a^3*b^9*c^7*d^18*e*f^6*h^12 - 73400320*a^11*c^8*d^11*e^8*f^6*h^12 - 587
20256*a^12*c^7*d^9*e^10*f^6*h^12 - 58720256*a^10*c^9*d^13*e^6*f^6*h^12 - 29
360128*a^13*c^6*d^7*e^12*f^6*h^12 - 29360128*a^9*c^10*d^15*e^4*f^6*h^12 - 8
388608*a^14*c^5*d^5*e^14*f^6*h^12 - 8388608*a^8*c^11*d^17*e^2*f^6*h^12 - 10
48576*a^15*c^4*d^3*e^16*f^6*h^12 - 286720*a^7*b^12*d^7*e^12*f^6*h^12 + 2293
76*a^8*b^11*d^6*e^13*f^6*h^12 + 229376*a^6*b^13*d^8*e^11*f^6*h^12 - 114688*
a^9*b^10*d^5*e^14*f^6*h^12 - 114688*a^5*b^14*d^9*e^10*f^6*h^12 + 32768*a^10
*b^9*d^4*e^15*f^6*h^12 + 32768*a^4*b^15*d^10*e^9*f^6*h^12 - 4096*a^11*b^8*d
^3*e^16*f^6*h^12 - 4096*a^3*b^16*d^11*e^8*f^6*h^12 + 1048576*a^6*b^2*c^11*d
^19*f^6*h^12 - 393216*a^5*b^4*c^10*d^19*f^6*h^12 + 65536*a^4*b^6*c^9*d^19*f
^6*h^12 - 4096*a^3*b^8*c^8*d^19*f^6*h^12 - 1048576*a^7*c^12*d^19*f^6*h^12 +
 262144*a^10*b*c^4*d*e^14*f^4*h^8 - 23552*a*b^6*c^8*d^14*e*f^4*h^8 - 16384*
a^7*b^7*c*d*e^14*f^4*h^8 - 3328*a*b^13*c*d^7*e^8*f^4*h^8 + 2429952*a^4*b^5*
c^6*d^9*e^6*f^4*h^8 - 1865728*a^6*b^3*c^6*d^7*e^8*f^4*h^8 - 1716224*a^4*b^4
*c^7*d^10*e^5*f^4*h^8 + 1605632*a^6*b^2*c^7*d^8*e^7*f^4*h^8 + 1584384*a^5*b
^5*c^5*d^7*e^8*f^4*h^8 + 1572864*a^5*b^2*c^8*d^10*e^5*f^4*h^8 - 1433600*a^5
*b^3*c^7*d^9*e^6*f^4*h^8 - 1261568*a^4*b^6*c^5*d^8*e^7*f^4*h^8 - 1124352*a^
3*b^4*c^8*d^12*e^3*f^4*h^8 - 1110016*a^7*b^3*c^5*d^5*e^10*f^4*h^8 + 1106176
*a^3*b^5*c^7*d^11*e^4*f^4*h^8 - 936960*a^5*b^6*c^4*d^6*e^9*f^4*h^8 - 838656
*a^2*b^7*c^6*d^11*e^4*f^4*h^8 - 795648*a^3*b^7*c^5*d^9*e^6*f^4*h^8 + 730880
*a^3*b^8*c^4*d^8*e^7*f^4*h^8 + 714752*a^2*b^6*c^7*d^12*e^3*f^4*h^8 + 686080
*a^7*b^4*c^4*d^4*e^11*f^4*h^8 + 641024*a^6*b^4*c^5*d^6*e^9*f^4*h^8 - 595968
*a^8*b^3*c^4*d^3*e^12*f^4*h^8 + 544768*a^3*b^3*c^9*d^13*e^2*f^4*h^8 + 51609
6*a^2*b^8*c^5*d^10*e^5*f^4*h^8 + 441856*a^6*b^5*c^4*d^5*e^10*f^4*h^8 + 3932
16*a^7*b^2*c^6*d^6*e^9*f^4*h^8 + 376832*a^4*b^2*c^9*d^12*e^3*f^4*h^8 - 3665
92*a^6*b^6*c^3*d^4*e^11*f^4*h^8 + 363520*a^4*b^8*c^3*d^6*e^9*f^4*h^8 - 3563
52*a^5*b^4*c^6*d^8*e^7*f^4*h^8 - 348672*a^2*b^5*c^8*d^13*e^2*f^4*h^8 - 3440
64*a^8*b^2*c^5*d^4*e^11*f^4*h^8 + 294912*a^8*b^4*c^3*d^2*e^13*f^4*h^8 + 210
944*a^4*b^3*c^8*d^11*e^4*f^4*h^8 - 198400*a^3*b^9*c^3*d^7*e^8*f^4*h^8 - 144
640*a^4*b^7*c^4*d^7*e^8*f^4*h^8 - 131072*a^9*b^2*c^4*d^2*e^13*f^4*h^8 - 131
072*a^7*b^6*c^2*d^2*e^13*f^4*h^8 - 129024*a^3*b^6*c^6*d^10*e^5*f^4*h^8 - 10
4448*a^2*b^10*c^3*d^8*e^7*f^4*h^8 + 96768*a^5*b^8*c^2*d^4*e^11*f^4*h^8 + 91
904*a^7*b^5*c^3*d^3*e^12*f^4*h^8 - 74240*a^4*b^9*c^2*d^5*e^10*f^4*h^8 - 716
80*a^2*b^9*c^4*d^9*e^6*f^4*h^8 + 58368*a^2*b^11*c^2*d^7*e^8*f^4*h^8 + 36864
*a^5*b^7*c^3*d^5*e^10*f^4*h^8 - 35328*a^3*b^10*c^2*d^6*e^9*f^4*h^8 + 27136*
a^6*b^7*c^2*d^3*e^12*f^4*h^8 + 909312*a^8*b*c^6*d^5*e^10*f^4*h^8 + 815104*a
^9*b*c^5*d^3*e^12*f^4*h^8 - 651264*a^5*b*c^9*d^11*e^4*f^4*h^8 - 573440*a^6*
b*c^8*d^9*e^6*f^4*h^8 - 262144*a^9*b^3*c^3*d*e^14*f^4*h^8 + 217088*a^7*b*c^
7*d^7*e^8*f^4*h^8 + 211456*a*b^9*c^5*d^11*e^4*f^4*h^8 - 204800*a^4*b*c^10*d
^13*e^2*f^4*h^8 - 172032*a*b^8*c^6*d^12*e^3*f^4*h^8 - 157696*a*b^10*c^4*d^1
0*e^5*f^4*h^8 - 131072*a^3*b^2*c^10*d^14*e*f^4*h^8 + 98304*a^8*b^5*c^2*d*e^
14*f^4*h^8 + 92160*a^2*b^4*c^9*d^14*e*f^4*h^8 + 84992*a*b^7*c^7*d^13*e^2*f^
4*h^8 + 64512*a*b^11*c^3*d^9*e^6*f^4*h^8 + 23552*a^6*b^8*c*d^2*e^13*f^4*h^8
 + 18944*a^3*b^11*c*d^5*e^10*f^4*h^8 - 13312*a^4*b^10*c*d^4*e^11*f^4*h^8 -
9472*a^5*b^9*c*d^3*e^12*f^4*h^8 - 8192*a*b^12*c^2*d^8*e^7*f^4*h^8 - 6144*a^
2*b^12*c*d^6*e^9*f^4*h^8 - 17920*b^11*c^4*d^11*e^4*f^4*h^8 + 14336*b^12*c^3
*d^10*e^5*f^4*h^8 + 14336*b^10*c^5*d^12*e^3*f^4*h^8 - 7168*b^13*c^2*d^9*e^6
*f^4*h^8 - 7168*b^9*c^6*d^13*e^2*f^4*h^8 - 425984*a^9*c^6*d^4*e^11*f^4*h^8
- 360448*a^8*c^7*d^6*e^9*f^4*h^8 - 262144*a^10*c^5*d^2*e^13*f^4*h^8 - 13107
2*a^7*c^8*d^8*e^7*f^4*h^8 + 98304*a^5*c^10*d^12*e^3*f^4*h^8 + 65536*a^6*c^9
*d^10*e^5*f^4*h^8 - 1536*a^5*b^10*d^2*e^13*f^4*h^8 - 1536*a^2*b^13*d^5*e^10
*f^4*h^8 + 768*a^4*b^11*d^3*e^12*f^4*h^8 + 768*a^3*b^12*d^4*e^11*f^4*h^8 +
65536*a^10*b^2*c^3*e^15*f^4*h^8 - 24576*a^9*b^4*c^2*e^15*f^4*h^8 - 10240*a^
2*b^3*c^10*d^15*f^4*h^8 + 2048*b^14*c*d^8*e^7*f^4*h^8 + 2048*b^8*c^7*d^14*e
*f^4*h^8 + 32768*a^4*c^11*d^14*e*f^4*h^8 + 1024*a^6*b^9*d*e^14*f^4*h^8 + 10
24*a*b^14*d^6*e^9*f^4*h^8 + 4096*a^8*b^6*c*e^15*f^4*h^8 + 12288*a^3*b*c^11*
```

d^15*f^4*h^8 + 2816*a*b^5*c^9*d^15*f^4*h^8 - 256*b^15*d^7*e^8*f^4*h^8 - 655
36*a^11*c^4*e^15*f^4*h^8 - 256*b^7*c^8*d^15*f^4*h^8 - 256*a^7*b^8*e^15*f^4*
h^8 - 896*a*b^8*c^2*d*e^10*f^2*h^4 + 192*a*b*c^9*d^8*e^3*f^2*h^4 + 11520*a^
3*b^3*c^5*d^2*e^9*f^2*h^4 - 5856*a^2*b^5*c^4*d^2*e^9*f^2*h^4 - 5120*a^3*b^2
*c^6*d^3*e^8*f^2*h^4 + 3200*a^2*b^4*c^5*d^3*e^8*f^2*h^4 - 640*a^2*b^3*c^6*d
^4*e^7*f^2*h^4 - 96*a^2*b^2*c^7*d^5*e^6*f^2*h^4 - 10880*a^3*b^4*c^4*d*e^10*
f^2*h^4 + 10240*a^4*b^2*c^5*d*e^10*f^2*h^4 - 7680*a^4*b*c^6*d^2*e^9*f^2*h^4
 + 4672*a^2*b^6*c^3*d*e^10*f^2*h^4 + 1248*a*b^7*c^3*d^2*e^9*f^2*h^4 + 832*a
^3*b*c^7*d^4*e^7*f^2*h^4 - 768*a*b^6*c^4*d^3*e^8*f^2*h^4 + 192*a^2*b*c^8*d^
6*e^5*f^2*h^4 - 192*a*b^2*c^8*d^7*e^4*f^2*h^4 + 176*a*b^5*c^5*d^4*e^7*f^2*h
^4 + 64*a*b^3*c^7*d^6*e^5*f^2*h^4 - 96*b^9*c^2*d^2*e^9*f^2*h^4 - 96*b^2*c^9
*d^9*e^2*f^2*h^4 + 64*b^8*c^3*d^3*e^8*f^2*h^4 + 64*b^3*c^8*d^8*e^3*f^2*h^4
 - 16*b^7*c^4*d^4*e^7*f^2*h^4 - 16*b^4*c^7*d^7*e^4*f^2*h^4 + 2032*a^4*c^7*d^
3*e^8*f^2*h^4 - 96*a^2*c^9*d^7*e^4*f^2*h^4 - 64*a^3*c^8*d^5*e^6*f^2*h^4 - 4
480*a^4*b^3*c^4*e^11*f^2*h^4 + 3696*a^3*b^5*c^3*e^11*f^2*h^4 - 1376*a^2*b^7
*c^2*e^11*f^2*h^4 - 2048*a^5*c^6*d*e^10*f^2*h^4 - 64*a*c^10*d^9*e^2*f^2*h^4
 + 1792*a^5*b*c^5*e^11*f^2*h^4 + 64*b^10*c*d*e^10*f^2*h^4 + 64*b*c^10*d^10*
e*f^2*h^4 + 240*a*b^9*c*e^11*f^2*h^4 - 16*c^11*d^11*f^2*h^4 - 16*b^11*e^11*
f^2*h^4 - c^7*e^7, h, k)*(root(8388608*a^7*b*c^11*d^18*e*f^6*h^12 - 5138022
40*a^10*b^2*c^7*d^11*e^8*f^6*h^12 - 381681664*a^11*b^2*c^6*d^9*e^10*f^6*h^1
2 - 381681664*a^9*b^2*c^8*d^13*e^6*f^6*h^12 - 300941312*a^9*b^5*c^5*d^10*e^
9*f^6*h^12 - 300941312*a^8*b^5*c^6*d^12*e^7*f^6*h^12 + 293601280*a^10*b^3*c
^6*d^10*e^9*f^6*h^12 + 293601280*a^9*b^3*c^7*d^12*e^7*f^6*h^12 - 168820736*
a^10*b^5*c^4*d^8*e^11*f^6*h^12 - 168820736*a^7*b^5*c^7*d^14*e^5*f^6*h^12 +
166068224*a^8*b^6*c^5*d^11*e^8*f^6*h^12 - 146800640*a^12*b^2*c^5*d^7*e^12*f
^6*h^12 - 146800640*a^8*b^2*c^9*d^15*e^4*f^6*h^12 + 124780544*a^10*b^4*c^5*
d^9*e^10*f^6*h^12 + 124780544*a^8*b^4*c^7*d^13*e^6*f^6*h^12 + 119275520*a^9
*b^4*c^6*d^11*e^8*f^6*h^12 + 117440512*a^11*b^3*c^5*d^8*e^11*f^6*h^12 + 117
440512*a^8*b^3*c^8*d^14*e^5*f^6*h^12 + 102760448*a^9*b^6*c^4*d^9*e^10*f^6*h
^12 + 102760448*a^7*b^6*c^6*d^13*e^6*f^6*h^12 + 91750400*a^11*b^4*c^4*d^7*e
^12*f^6*h^12 + 91750400*a^7*b^4*c^8*d^15*e^4*f^6*h^12 - 71065600*a^7*b^8*c^
4*d^11*e^8*f^6*h^12 - 53444608*a^8*b^8*c^3*d^9*e^10*f^6*h^12 - 53444608*a^6
*b^8*c^5*d^13*e^6*f^6*h^12 + 40370176*a^9*b^7*c^3*d^8*e^11*f^6*h^12 + 40370
176*a^6*b^7*c^6*d^14*e^5*f^6*h^12 - 36700160*a^11*b^5*c^3*d^6*e^13*f^6*h^12
 - 36700160*a^6*b^5*c^8*d^16*e^3*f^6*h^12 + 34078720*a^8*b^7*c^4*d^10*e^9*f
^6*h^12 + 34078720*a^7*b^7*c^5*d^12*e^7*f^6*h^12 + 26214400*a^12*b^4*c^3*d^
5*e^14*f^6*h^12 + 26214400*a^6*b^4*c^9*d^17*e^2*f^6*h^12 + 22118400*a^7*b^9
*c^3*d^10*e^9*f^6*h^12 + 22118400*a^6*b^9*c^4*d^12*e^7*f^6*h^12 - 20971520*
a^13*b^2*c^4*d^5*e^14*f^6*h^12 - 20971520*a^7*b^2*c^10*d^17*e^2*f^6*h^12 +
18350080*a^10*b^7*c^2*d^6*e^13*f^6*h^12 + 18350080*a^5*b^7*c^7*d^16*e^3*f^6
*h^12 - 16629760*a^9*b^8*c^2*d^7*e^12*f^6*h^12 - 16629760*a^5*b^8*c^6*d^15*
e^4*f^6*h^12 - 10485760*a^11*b^6*c^2*d^5*e^14*f^6*h^12 - 10485760*a^5*b^6*c
^8*d^17*e^2*f^6*h^12 + 9175040*a^10*b^6*c^3*d^7*e^12*f^6*h^12 + 9175040*a^6
*b^6*c^7*d^15*e^4*f^6*h^12 - 8388608*a^13*b^3*c^3*d^4*e^15*f^6*h^12 + 56197
12*a^7*b^10*c^2*d^9*e^10*f^6*h^12 + 5619712*a^5*b^10*c^4*d^13*e^6*f^6*h^12
 - 5570560*a^6*b^11*c^2*d^10*e^9*f^6*h^12 - 5570560*a^5*b^11*c^3*d^12*e^7*f^
6*h^12 + 4358144*a^8*b^9*c^2*d^8*e^11*f^6*h^12 + 4358144*a^5*b^9*c^5*d^14*e
^5*f^6*h^12 + 4259840*a^6*b^10*c^3*d^11*e^8*f^6*h^12 + 3899392*a^4*b^10*c^5
*d^15*e^4*f^6*h^12 - 3440640*a^4*b^9*c^6*d^16*e^3*f^6*h^12 + 3145728*a^12*b
^5*c^2*d^4*e^15*f^6*h^12 - 2523136*a^4*b^11*c^4*d^14*e^5*f^6*h^12 + 1802240
*a^4*b^8*c^7*d^17*e^2*f^6*h^12 + 1556480*a^5*b^12*c^2*d^11*e^8*f^6*h^12 + 1
048576*a^14*b^2*c^3*d^3*e^16*f^6*h^12 + 688128*a^4*b^12*c^3*d^13*e^6*f^6*h^
12 - 393216*a^13*b^4*c^2*d^3*e^16*f^6*h^12 - 286720*a^3*b^12*c^4*d^15*e^4*f
^6*h^12 + 229376*a^3*b^13*c^3*d^14*e^5*f^6*h^12 + 229376*a^3*b^11*c^5*d^16*
e^3*f^6*h^12 + 163840*a^4*b^13*c^2*d^12*e^7*f^6*h^12 - 114688*a^3*b^14*c^2*
d^13*e^6*f^6*h^12 - 114688*a^3*b^10*c^6*d^17*e^2*f^6*h^12 + 293601280*a^11*
b*c^7*d^10*e^9*f^6*h^12 + 293601280*a^10*b*c^8*d^12*e^7*f^6*h^12 + 17616076
8*a^12*b*c^6*d^8*e^11*f^6*h^12 + 176160768*a^9*b*c^9*d^14*e^5*f^6*h^12 + 58
720256*a^13*b*c^5*d^6*e^13*f^6*h^12 + 58720256*a^8*b*c^10*d^16*e^3*f^6*h^12

+ 8388608*a^14*b*c^4*d^4*e^15*f^6*h^12 - 8388608*a^6*b^3*c^10*d^18*e*f^6*h
^12 + 3899392*a^8*b^10*c*d^7*e^12*f^6*h^12 - 3440640*a^9*b^9*c*d^6*e^13*f^6
*h^12 + 3145728*a^5*b^5*c^9*d^18*e*f^6*h^12 - 2523136*a^7*b^11*c*d^8*e^11*f
^6*h^12 + 1802240*a^10*b^8*c*d^5*e^14*f^6*h^12 + 688128*a^6*b^12*c*d^9*e^10
*f^6*h^12 - 524288*a^11*b^7*c*d^4*e^15*f^6*h^12 - 524288*a^4*b^7*c^8*d^18*e
*f^6*h^12 + 163840*a^5*b^13*c*d^10*e^9*f^6*h^12 - 163840*a^4*b^14*c*d^11*e^
8*f^6*h^12 + 65536*a^12*b^6*c*d^3*e^16*f^6*h^12 + 32768*a^3*b^15*c*d^12*e^7
*f^6*h^12 + 32768*a^3*b^9*c^7*d^18*e*f^6*h^12 - 73400320*a^11*c^8*d^11*e^8*
f^6*h^12 - 58720256*a^12*c^7*d^9*e^10*f^6*h^12 - 58720256*a^10*c^9*d^13*e^6
*f^6*h^12 - 29360128*a^13*c^6*d^7*e^12*f^6*h^12 - 29360128*a^9*c^10*d^15*e^
4*f^6*h^12 - 8388608*a^14*c^5*d^5*e^14*f^6*h^12 - 8388608*a^8*c^11*d^17*e^2
*f^6*h^12 - 1048576*a^15*c^4*d^3*e^16*f^6*h^12 - 286720*a^7*b^12*d^7*e^12*f
^6*h^12 + 229376*a^8*b^11*d^6*e^13*f^6*h^12 + 229376*a^6*b^13*d^8*e^11*f^6*
h^12 - 114688*a^9*b^10*d^5*e^14*f^6*h^12 - 114688*a^5*b^14*d^9*e^10*f^6*h^1
2 + 32768*a^10*b^9*d^4*e^15*f^6*h^12 + 32768*a^4*b^15*d^10*e^9*f^6*h^12 - 4
096*a^11*b^8*d^3*e^16*f^6*h^12 - 4096*a^3*b^16*d^11*e^8*f^6*h^12 + 1048576*
a^6*b^2*c^11*d^19*f^6*h^12 - 393216*a^5*b^4*c^10*d^19*f^6*h^12 + 65536*a^4*
b^6*c^9*d^19*f^6*h^12 - 4096*a^3*b^8*c^8*d^19*f^6*h^12 - 1048576*a^7*c^12*d
^19*f^6*h^12 + 262144*a^10*b*c^4*d*e^14*f^4*h^8 - 23552*a*b^6*c^8*d^14*e*f^
4*h^8 - 16384*a^7*b^7*c*d*e^14*f^4*h^8 - 3328*a*b^13*c*d^7*e^8*f^4*h^8 + 24
29952*a^4*b^5*c^6*d^9*e^6*f^4*h^8 - 1865728*a^6*b^3*c^6*d^7*e^8*f^4*h^8 - 1
716224*a^4*b^4*c^7*d^10*e^5*f^4*h^8 + 1605632*a^6*b^2*c^7*d^8*e^7*f^4*h^8 +
1584384*a^5*b^5*c^5*d^7*e^8*f^4*h^8 + 1572864*a^5*b^2*c^8*d^10*e^5*f^4*h^8
- 1433600*a^5*b^3*c^7*d^9*e^6*f^4*h^8 - 1261568*a^4*b^6*c^5*d^8*e^7*f^4*h^
8 - 1124352*a^3*b^4*c^8*d^12*e^3*f^4*h^8 - 1110016*a^7*b^3*c^5*d^5*e^10*f^4
*h^8 + 1106176*a^3*b^5*c^7*d^11*e^4*f^4*h^8 - 936960*a^5*b^6*c^4*d^6*e^9*f^
4*h^8 - 838656*a^2*b^7*c^6*d^11*e^4*f^4*h^8 - 795648*a^3*b^7*c^5*d^9*e^6*f^
4*h^8 + 730880*a^3*b^8*c^4*d^8*e^7*f^4*h^8 + 714752*a^2*b^6*c^7*d^12*e^3*f^
4*h^8 + 686080*a^7*b^4*c^4*d^4*e^11*f^4*h^8 + 641024*a^6*b^4*c^5*d^6*e^9*f^
4*h^8 - 595968*a^8*b^3*c^4*d^3*e^12*f^4*h^8 + 544768*a^3*b^3*c^9*d^13*e^2*f
^4*h^8 + 516096*a^2*b^8*c^5*d^10*e^5*f^4*h^8 + 441856*a^6*b^5*c^4*d^5*e^10*
f^4*h^8 + 393216*a^7*b^2*c^6*d^6*e^9*f^4*h^8 + 376832*a^4*b^2*c^9*d^12*e^3*
f^4*h^8 - 366592*a^6*b^6*c^3*d^4*e^11*f^4*h^8 + 363520*a^4*b^8*c^3*d^6*e^9*
f^4*h^8 - 356352*a^5*b^4*c^6*d^8*e^7*f^4*h^8 - 348672*a^2*b^5*c^8*d^13*e^2*
f^4*h^8 - 344064*a^8*b^2*c^5*d^4*e^11*f^4*h^8 + 294912*a^8*b^4*c^3*d^2*e^13
*f^4*h^8 + 210944*a^4*b^3*c^8*d^11*e^4*f^4*h^8 - 198400*a^3*b^9*c^3*d^7*e^8
*f^4*h^8 - 144640*a^4*b^7*c^4*d^7*e^8*f^4*h^8 - 131072*a^9*b^2*c^4*d^2*e^13
*f^4*h^8 - 131072*a^7*b^6*c^2*d^2*e^13*f^4*h^8 - 129024*a^3*b^6*c^6*d^10*e^
5*f^4*h^8 - 104448*a^2*b^10*c^3*d^8*e^7*f^4*h^8 + 96768*a^5*b^8*c^2*d^4*e^1
1*f^4*h^8 + 91904*a^7*b^5*c^3*d^3*e^12*f^4*h^8 - 74240*a^4*b^9*c^2*d^5*e^10
*f^4*h^8 - 71680*a^2*b^9*c^4*d^9*e^6*f^4*h^8 + 58368*a^2*b^11*c^2*d^7*e^8*f
^4*h^8 + 36864*a^5*b^7*c^3*d^5*e^10*f^4*h^8 - 35328*a^3*b^10*c^2*d^6*e^9*f^
4*h^8 + 27136*a^6*b^7*c^2*d^3*e^12*f^4*h^8 + 909312*a^8*b*c^6*d^5*e^10*f^4*
h^8 + 815104*a^9*b*c^5*d^3*e^12*f^4*h^8 - 651264*a^5*b*c^9*d^11*e^4*f^4*h^8
- 573440*a^6*b*c^8*d^9*e^6*f^4*h^8 - 262144*a^9*b^3*c^3*d*e^14*f^4*h^8 + 2
17088*a^7*b*c^7*d^7*e^8*f^4*h^8 + 211456*a*b^9*c^5*d^11*e^4*f^4*h^8 - 20480
0*a^4*b*c^10*d^13*e^2*f^4*h^8 - 172032*a*b^8*c^6*d^12*e^3*f^4*h^8 - 157696*
a*b^10*c^4*d^10*e^5*f^4*h^8 - 131072*a^3*b^2*c^10*d^14*e*f^4*h^8 + 98304*a^
8*b^5*c^2*d*e^14*f^4*h^8 + 92160*a^2*b^4*c^9*d^14*e*f^4*h^8 + 84992*a*b^7*c
^7*d^13*e^2*f^4*h^8 + 64512*a*b^11*c^3*d^9*e^6*f^4*h^8 + 23552*a^6*b^8*c*d^
2*e^13*f^4*h^8 + 18944*a^3*b^11*c*d^5*e^10*f^4*h^8 - 13312*a^4*b^10*c*d^4*e
^11*f^4*h^8 - 9472*a^5*b^9*c*d^3*e^12*f^4*h^8 - 8192*a*b^12*c^2*d^8*e^7*f^4
*h^8 - 6144*a^2*b^12*c*d^6*e^9*f^4*h^8 - 17920*b^11*c^4*d^11*e^4*f^4*h^8 +
14336*b^12*c^3*d^10*e^5*f^4*h^8 + 14336*b^10*c^5*d^12*e^3*f^4*h^8 - 7168*b^
13*c^2*d^9*e^6*f^4*h^8 - 7168*b^9*c^6*d^13*e^2*f^4*h^8 - 425984*a^9*c^6*d^4
*e^11*f^4*h^8 - 360448*a^8*c^7*d^6*e^9*f^4*h^8 - 262144*a^10*c^5*d^2*e^13*f
^4*h^8 - 131072*a^7*c^8*d^8*e^7*f^4*h^8 + 98304*a^5*c^10*d^12*e^3*f^4*h^8 +
65536*a^6*c^9*d^10*e^5*f^4*h^8 - 1536*a^5*b^10*d^2*e^13*f^4*h^8 - 1536*a^2
*b^13*d^5*e^10*f^4*h^8 + 768*a^4*b^11*d^3*e^12*f^4*h^8 + 768*a^3*b^12*d^4*e

^11*f^4*h^8 + 65536*a^10*b^2*c^3*e^15*f^4*h^8 - 24576*a^9*b^4*c^2*e^15*f^4*h^8 - 10240*a^2*b^3*c^10*d^15*f^4*h^8 + 2048*b^14*c*d^8*e^7*f^4*h^8 + 2048*b^8*c^7*d^14*e*f^4*h^8 + 32768*a^4*c^11*d^14*e*f^4*h^8 + 1024*a^6*b^9*d*e^14*f^4*h^8 + 1024*a*b^14*d^6*e^9*f^4*h^8 + 4096*a^8*b^6*c*e^15*f^4*h^8 + 12288*a^3*b*c^11*d^15*f^4*h^8 + 2816*a*b^5*c^9*d^15*f^4*h^8 - 256*b^15*d^7*e^8*f^4*h^8 - 65536*a^11*c^4*e^15*f^4*h^8 - 256*b^7*c^8*d^15*f^4*h^8 - 256*a^7*b^8*e^15*f^4*h^8 - 896*a*b^8*c^2*d*e^10*f^2*h^4 + 192*a*b*c^9*d^8*e^3*f^2*h^4 + 11520*a^3*b^3*c^5*d^2*e^9*f^2*h^4 - 5856*a^2*b^5*c^4*d^2*e^9*f^2*h^4 - 5120*a^3*b^2*c^6*d^3*e^8*f^2*h^4 + 3200*a^2*b^4*c^5*d^3*e^8*f^2*h^4 - 640*a^2*b^3*c^6*d^4*e^7*f^2*h^4 - 96*a^2*b^2*c^7*d^5*e^6*f^2*h^4 - 10880*a^3*b^4*c^4*d*e^10*f^2*h^4 + 10240*a^4*b^2*c^5*d*e^10*f^2*h^4 - 7680*a^4*b*c^6*d^2*e^9*f^2*h^4 + 4672*a^2*b^6*c^3*d*e^10*f^2*h^4 + 1248*a*b^7*c^3*d^2*e^9*f^2*h^4 + 832*a^3*b*c^7*d^4*e^7*f^2*h^4 - 768*a*b^6*c^4*d^3*e^8*f^2*h^4 + 192*a^2*b*c^8*d^6*e^5*f^2*h^4 - 192*a*b^2*c^8*d^7*e^4*f^2*h^4 + 176*a*b^5*c^5*d^4*e^7*f^2*h^4 + 64*a*b^3*c^7*d^6*e^5*f^2*h^4 - 96*b^9*c^2*d^2*e^9*f^2*h^4 - 96*b^2*c^9*d^9*e^2*f^2*h^4 + 64*b^8*c^3*d^3*e^8*f^2*h^4 + 64*b^3*c^8*d^8*e^3*f^2*h^4 - 16*b^7*c^4*d^4*e^7*f^2*h^4 - 16*b^4*c^7*d^7*e^4*f^2*h^4 + 2032*a^4*c^7*d^3*e^8*f^2*h^4 - 96*a^2*c^9*d^7*e^4*f^2*h^4 - 64*a^3*c^8*d^5*e^6*f^2*h^4 - 4480*a^4*b^3*c^4*e^11*f^2*h^4 + 3696*a^3*b^5*c^3*e^11*f^2*h^4 - 1376*a^2*b^7*c^2*e^11*f^2*h^4 - 2048*a^5*c^6*d*e^10*f^2*h^4 - 64*a*c^10*d^9*e^2*f^2*h^4 + 1792*a^5*b*c^5*e^11*f^2*h^4 + 64*b^10*c*d*e^10*f^2*h^4 + 64*b*c^10*d^10*e*f^2*h^4 + 240*a*b^9*c*e^11*f^2*h^4 - 16*c^11*d^11*f^2*h^4 - 16*b^11*e^11*f^2*h^4 - c^7*e^7, h, k)*(root(8388608*a^7*b*c^11*d^18*e*f^6*h^12 - 513802240*a^10*b^2*c^7*d^11*e^8*f^6*h^12 - 381681664*a^11*b^2*c^6*d^9*e^10*f^6*h^12 - 381681664*a^9*b^2*c^8*d^13*e^6*f^6*h^12 - 300941312*a^9*b^5*c^5*d^10*e^9*f^6*h^12 - 300941312*a^8*b^5*c^6*d^12*e^7*f^6*h^12 + 293601280*a^10*b^3*c^6*d^10*e^9*f^6*h^12 + 293601280*a^9*b^3*c^7*d^12*e^7*f^6*h^12 - 168820736*a^10*b^5*c^4*d^8*e^11*f^6*h^12 - 168820736*a^7*b^5*c^7*d^14*e^5*f^6*h^12 + 166068224*a^8*b^6*c^5*d^11*e^8*f^6*h^12 - 146800640*a^12*b^2*c^5*d^7*e^12*f^6*h^12 - 146800640*a^8*b^2*c^9*d^15*e^4*f^6*h^12 + 124780544*a^10*b^4*c^5*d^9*e^10*f^6*h^12 + 124780544*a^8*b^4*c^7*d^13*e^6*f^6*h^12 + 119275520*a^9*b^4*c^6*d^11*e^8*f^6*h^12 + 117440512*a^11*b^3*c^5*d^8*e^11*f^6*h^12 + 117440512*a^8*b^3*c^8*d^14*e^5*f^6*h^12 + 102760448*a^9*b^6*c^4*d^9*e^10*f^6*h^12 + 102760448*a^7*b^6*c^6*d^13*e^6*f^6*h^12 + 91750400*a^11*b^4*c^4*d^7*e^12*f^6*h^12 + 91750400*a^7*b^4*c^8*d^15*e^4*f^6*h^12 - 71065600*a^7*b^8*c^4*d^11*e^8*f^6*h^12 - 53444608*a^8*b^8*c^3*d^9*e^10*f^6*h^12 - 53444608*a^6*b^8*c^5*d^13*e^6*f^6*h^12 + 40370176*a^9*b^7*c^3*d^8*e^11*f^6*h^12 + 40370176*a^6*b^7*c^6*d^14*e^5*f^6*h^12 - 36700160*a^11*b^5*c^3*d^6*e^13*f^6*h^12 - 36700160*a^6*b^5*c^8*d^16*e^3*f^6*h^12 + 34078720*a^8*b^7*c^4*d^10*e^9*f^6*h^12 + 34078720*a^7*b^7*c^5*d^12*e^7*f^6*h^12 + 26214400*a^12*b^4*c^3*d^5*e^14*f^6*h^12 + 26214400*a^6*b^4*c^9*d^17*e^2*f^6*h^12 + 22118400*a^7*b^9*c^3*d^10*e^9*f^6*h^12 + 22118400*a^6*b^9*c^4*d^12*e^7*f^6*h^12 - 20971520*a^13*b^2*c^4*d^5*e^14*f^6*h^12 - 20971520*a^7*b^2*c^10*d^17*e^2*f^6*h^12 + 18350080*a^10*b^7*c^2*d^6*e^13*f^6*h^12 + 18350080*a^5*b^7*c^7*d^16*e^3*f^6*h^12 - 16629760*a^9*b^8*c^2*d^7*e^12*f^6*h^12 - 16629760*a^5*b^8*c^6*d^15*e^4*f^6*h^12 - 10485760*a^11*b^6*c^2*d^5*e^14*f^6*h^12 - 10485760*a^5*b^6*c^8*d^17*e^2*f^6*h^12 + 9175040*a^10*b^6*c^3*d^7*e^12*f^6*h^12 + 9175040*a^6*b^6*c^7*d^15*e^4*f^6*h^12 - 8388608*a^13*b^3*c^3*d^4*e^15*f^6*h^12 + 5619712*a^7*b^10*c^2*d^9*e^10*f^6*h^12 + 5619712*a^5*b^10*c^4*d^13*e^6*f^6*h^12 - 5570560*a^6*b^11*c^2*d^10*e^9*f^6*h^12 - 5570560*a^5*b^11*c^3*d^12*e^7*f^6*h^12 + 4358144*a^8*b^9*c^2*d^8*e^11*f^6*h^12 + 4358144*a^5*b^9*c^5*d^14*e^5*f^6*h^12 + 4259840*a^6*b^10*c^3*d^11*e^8*f^6*h^12 + 3899392*a^4*b^10*c^5*d^15*e^4*f^6*h^12 - 3440640*a^4*b^9*c^6*d^16*e^3*f^6*h^12 + 3145728*a^12*b^5*c^2*d^4*e^15*f^6*h^12 - 2523136*a^4*b^11*c^4*d^14*e^5*f^6*h^12 + 1802240*a^4*b^8*c^7*d^17*e^2*f^6*h^12 + 1556480*a^5*b^12*c^2*d^11*e^8*f^6*h^12 + 1048576*a^14*b^2*c^3*d^3*e^16*f^6*h^12 + 688128*a^4*b^12*c^3*d^13*e^6*f^6*h^12 - 393216*a^13*b^4*c^2*d^3*e^16*f^6*h^12 - 286720*a^3*b^12*c^4*d^15*e^4*f^6*h^12 + 229376*a^3*b^13*c^3*d^14*e^5*f^6*h^12 + 229376*a^3*b^11*c^5*d^16*e^3*f^6*h^12 + 163840*a^4*b^13*c^2*d^12*e^7*f^6*h^12 - 114688

*a^3*b^14*c^2*d^13*e^6*f^6*h^12 - 114688*a^3*b^10*c^6*d^17*e^2*f^6*h^12 + 2
93601280*a^11*b*c^7*d^10*e^9*f^6*h^12 + 293601280*a^10*b*c^8*d^12*e^7*f^6*h
^12 + 176160768*a^12*b*c^6*d^8*e^11*f^6*h^12 + 176160768*a^9*b*c^9*d^14*e^5
*f^6*h^12 + 58720256*a^13*b*c^5*d^6*e^13*f^6*h^12 + 58720256*a^8*b*c^10*d^1
6*e^3*f^6*h^12 + 8388608*a^14*b*c^4*d^4*e^15*f^6*h^12 - 8388608*a^6*b^3*c^1
0*d^18*e*f^6*h^12 + 3899392*a^8*b^10*c*d^7*e^12*f^6*h^12 - 3440640*a^9*b^9*
c*d^6*e^13*f^6*h^12 + 3145728*a^5*b^5*c^9*d^18*e*f^6*h^12 - 2523136*a^7*b^1
1*c*d^8*e^11*f^6*h^12 + 1802240*a^10*b^8*c*d^5*e^14*f^6*h^12 + 688128*a^6*b
^12*c*d^9*e^10*f^6*h^12 - 524288*a^11*b^7*c*d^4*e^15*f^6*h^12 - 524288*a^4*
b^7*c^8*d^18*e*f^6*h^12 + 163840*a^5*b^13*c*d^10*e^9*f^6*h^12 - 163840*a^4*
b^14*c*d^11*e^8*f^6*h^12 + 65536*a^12*b^6*c*d^3*e^16*f^6*h^12 + 32768*a^3*b
^15*c*d^12*e^7*f^6*h^12 + 32768*a^3*b^9*c^7*d^18*e*f^6*h^12 - 73400320*a^11
*c^8*d^11*e^8*f^6*h^12 - 58720256*a^12*c^7*d^9*e^10*f^6*h^12 - 58720256*a^1
0*c^9*d^13*e^6*f^6*h^12 - 29360128*a^13*c^6*d^7*e^12*f^6*h^12 - 29360128*a^
9*c^10*d^15*e^4*f^6*h^12 - 8388608*a^14*c^5*d^5*e^14*f^6*h^12 - 8388608*a^8
*c^11*d^17*e^2*f^6*h^12 - 1048576*a^15*c^4*d^3*e^16*f^6*h^12 - 286720*a^7*b
^12*d^7*e^12*f^6*h^12 + 229376*a^8*b^11*d^6*e^13*f^6*h^12 + 229376*a^6*b^13
*d^8*e^11*f^6*h^12 - 114688*a^9*b^10*d^5*e^14*f^6*h^12 - 114688*a^5*b^14*d^
9*e^10*f^6*h^12 + 32768*a^10*b^9*d^4*e^15*f^6*h^12 + 32768*a^4*b^15*d^10*e^
9*f^6*h^12 - 4096*a^11*b^8*d^3*e^16*f^6*h^12 - 4096*a^3*b^16*d^11*e^8*f^6*h
^12 + 1048576*a^6*b^2*c^11*d^19*f^6*h^12 - 393216*a^5*b^4*c^10*d^19*f^6*h^1
2 + 65536*a^4*b^6*c^9*d^19*f^6*h^12 - 4096*a^3*b^8*c^8*d^19*f^6*h^12 - 1048
576*a^7*c^12*d^19*f^6*h^12 + 262144*a^10*b*c^4*d*e^14*f^4*h^8 - 23552*a*b^6
*c^8*d^14*e*f^4*h^8 - 16384*a^7*b^7*c*d*e^14*f^4*h^8 - 3328*a*b^13*c*d^7*e^
8*f^4*h^8 + 2429952*a^4*b^5*c^6*d^9*e^6*f^4*h^8 - 1865728*a^6*b^3*c^6*d^7*e
^8*f^4*h^8 - 1716224*a^4*b^4*c^7*d^10*e^5*f^4*h^8 + 1605632*a^6*b^2*c^7*d^8
*e^7*f^4*h^8 + 1584384*a^5*b^5*c^5*d^7*e^8*f^4*h^8 + 1572864*a^5*b^2*c^8*d^
10*e^5*f^4*h^8 - 1433600*a^5*b^3*c^7*d^9*e^6*f^4*h^8 - 1261568*a^4*b^6*c^5*
d^8*e^7*f^4*h^8 - 1124352*a^3*b^4*c^8*d^12*e^3*f^4*h^8 - 1110016*a^7*b^3*c^
5*d^5*e^10*f^4*h^8 + 1106176*a^3*b^5*c^7*d^11*e^4*f^4*h^8 - 936960*a^5*b^6*
c^4*d^6*e^9*f^4*h^8 - 838656*a^2*b^7*c^6*d^11*e^4*f^4*h^8 - 795648*a^3*b^7*
c^5*d^9*e^6*f^4*h^8 + 730880*a^3*b^8*c^4*d^8*e^7*f^4*h^8 + 714752*a^2*b^6*c
^7*d^12*e^3*f^4*h^8 + 686080*a^7*b^4*c^4*d^4*e^11*f^4*h^8 + 641024*a^6*b^4*
c^5*d^6*e^9*f^4*h^8 - 595968*a^8*b^3*c^4*d^3*e^12*f^4*h^8 + 544768*a^3*b^3*
c^9*d^13*e^2*f^4*h^8 + 516096*a^2*b^8*c^5*d^10*e^5*f^4*h^8 + 441856*a^6*b^5
*c^4*d^5*e^10*f^4*h^8 + 393216*a^7*b^2*c^6*d^6*e^9*f^4*h^8 + 376832*a^4*b^2
*c^9*d^12*e^3*f^4*h^8 - 366592*a^6*b^6*c^3*d^4*e^11*f^4*h^8 + 363520*a^4*b^
8*c^3*d^6*e^9*f^4*h^8 - 356352*a^5*b^4*c^6*d^8*e^7*f^4*h^8 - 348672*a^2*b^5
*c^8*d^13*e^2*f^4*h^8 - 344064*a^8*b^2*c^5*d^4*e^11*f^4*h^8 + 294912*a^8*b^
4*c^3*d^2*e^13*f^4*h^8 + 210944*a^4*b^3*c^8*d^11*e^4*f^4*h^8 - 198400*a^3*b
^9*c^3*d^7*e^8*f^4*h^8 - 144640*a^4*b^7*c^4*d^7*e^8*f^4*h^8 - 131072*a^9*b^
2*c^4*d^2*e^13*f^4*h^8 - 131072*a^7*b^6*c^2*d^2*e^13*f^4*h^8 - 129024*a^3*b
^6*c^6*d^10*e^5*f^4*h^8 - 104448*a^2*b^10*c^3*d^8*e^7*f^4*h^8 + 96768*a^5*b
^8*c^2*d^4*e^11*f^4*h^8 + 91904*a^7*b^5*c^3*d^3*e^12*f^4*h^8 - 74240*a^4*b^
9*c^2*d^5*e^10*f^4*h^8 - 71680*a^2*b^9*c^4*d^9*e^6*f^4*h^8 + 58368*a^2*b^11
*c^2*d^7*e^8*f^4*h^8 + 36864*a^5*b^7*c^3*d^5*e^10*f^4*h^8 - 35328*a^3*b^10*
c^2*d^6*e^9*f^4*h^8 + 27136*a^6*b^7*c^2*d^3*e^12*f^4*h^8 + 909312*a^8*b*c^6
*d^5*e^10*f^4*h^8 + 815104*a^9*b*c^5*d^3*e^12*f^4*h^8 - 651264*a^5*b*c^9*d^
11*e^4*f^4*h^8 - 573440*a^6*b*c^8*d^9*e^6*f^4*h^8 - 262144*a^9*b^3*c^3*d*e^
14*f^4*h^8 + 217088*a^7*b*c^7*d^7*e^8*f^4*h^8 + 211456*a*b^9*c^5*d^11*e^4*f
^4*h^8 - 204800*a^4*b*c^10*d^13*e^2*f^4*h^8 - 172032*a*b^8*c^6*d^12*e^3*f^4
*h^8 - 157696*a*b^10*c^4*d^10*e^5*f^4*h^8 - 131072*a^3*b^2*c^10*d^14*e*f^4*
h^8 + 98304*a^8*b^5*c^2*d*e^14*f^4*h^8 + 92160*a^2*b^4*c^9*d^14*e*f^4*h^8 +
 84992*a*b^7*c^7*d^13*e^2*f^4*h^8 + 64512*a*b^11*c^3*d^9*e^6*f^4*h^8 + 2355
2*a^6*b^8*c*d^2*e^13*f^4*h^8 + 18944*a^3*b^11*c*d^5*e^10*f^4*h^8 - 13312*a^
4*b^10*c*d^4*e^11*f^4*h^8 - 9472*a^5*b^9*c*d^3*e^12*f^4*h^8 - 8192*a*b^12*c
^2*d^8*e^7*f^4*h^8 - 6144*a^2*b^12*c*d^6*e^9*f^4*h^8 - 17920*b^11*c^4*d^11*
e^4*f^4*h^8 + 14336*b^12*c^3*d^10*e^5*f^4*h^8 + 14336*b^10*c^5*d^12*e^3*f^4
*h^8 - 7168*b^13*c^2*d^9*e^6*f^4*h^8 - 7168*b^9*c^6*d^13*e^2*f^4*h^8 - 4259

84*a^9*c^6*d^4*e^11*f^4*h^8 - 360448*a^8*c^7*d^6*e^9*f^4*h^8 - 262144*a^10*
c^5*d^2*e^13*f^4*h^8 - 131072*a^7*c^8*d^8*e^7*f^4*h^8 + 98304*a^5*c^10*d^12
*e^3*f^4*h^8 + 65536*a^6*c^9*d^10*e^5*f^4*h^8 - 1536*a^5*b^10*d^2*e^13*f^4*
h^8 - 1536*a^2*b^13*d^5*e^10*f^4*h^8 + 768*a^4*b^11*d^3*e^12*f^4*h^8 + 768*
a^3*b^12*d^4*e^11*f^4*h^8 + 65536*a^10*b^2*c^3*e^15*f^4*h^8 - 24576*a^9*b^4
*c^2*e^15*f^4*h^8 - 10240*a^2*b^3*c^10*d^15*f^4*h^8 + 2048*b^14*c*d^8*e^7*f
^4*h^8 + 2048*b^8*c^7*d^14*e*f^4*h^8 + 32768*a^4*c^11*d^14*e*f^4*h^8 + 1024
*a^6*b^9*d*e^14*f^4*h^8 + 1024*a*b^14*d^6*e^9*f^4*h^8 + 4096*a^8*b^6*c*e^15
*f^4*h^8 + 12288*a^3*b*c^11*d^15*f^4*h^8 + 2816*a*b^5*c^9*d^15*f^4*h^8 - 25
6*b^15*d^7*e^8*f^4*h^8 - 65536*a^11*c^4*e^15*f^4*h^8 - 256*b^7*c^8*d^15*f^4
*h^8 - 256*a^7*b^8*e^15*f^4*h^8 - 896*a*b^8*c^2*d*e^10*f^2*h^4 + 192*a*b*c^
9*d^8*e^3*f^2*h^4 + 11520*a^3*b^3*c^5*d^2*e^9*f^2*h^4 - 5856*a^2*b^5*c^4*d^
2*e^9*f^2*h^4 - 5120*a^3*b^2*c^6*d^3*e^8*f^2*h^4 + 3200*a^2*b^4*c^5*d^3*e^8
*f^2*h^4 - 640*a^2*b^3*c^6*d^4*e^7*f^2*h^4 - 96*a^2*b^2*c^7*d^5*e^6*f^2*h^4
 - 10880*a^3*b^4*c^4*d*e^10*f^2*h^4 + 10240*a^4*b^2*c^5*d*e^10*f^2*h^4 - 76
80*a^4*b*c^6*d^2*e^9*f^2*h^4 + 4672*a^2*b^6*c^3*d*e^10*f^2*h^4 + 1248*a*b^7
*c^3*d^2*e^9*f^2*h^4 + 832*a^3*b*c^7*d^4*e^7*f^2*h^4 - 768*a*b^6*c^4*d^3*e^
8*f^2*h^4 + 192*a^2*b*c^8*d^6*e^5*f^2*h^4 - 192*a*b^2*c^8*d^7*e^4*f^2*h^4 +
 176*a*b^5*c^5*d^4*e^7*f^2*h^4 + 64*a*b^3*c^7*d^6*e^5*f^2*h^4 - 96*b^9*c^2*
d^2*e^9*f^2*h^4 - 96*b^2*c^9*d^9*e^2*f^2*h^4 + 64*b^8*c^3*d^3*e^8*f^2*h^4 +
 64*b^3*c^8*d^8*e^3*f^2*h^4 - 16*b^7*c^4*d^4*e^7*f^2*h^4 - 16*b^4*c^7*d^7*e
^4*f^2*h^4 + 2032*a^4*c^7*d^3*e^8*f^2*h^4 - 96*a^2*c^9*d^7*e^4*f^2*h^4 - 64
*a^3*c^8*d^5*e^6*f^2*h^4 - 4480*a^4*b^3*c^4*e^11*f^2*h^4 + 3696*a^3*b^5*c^3
*e^11*f^2*h^4 - 1376*a^2*b^7*c^2*e^11*f^2*h^4 - 2048*a^5*c^6*d*e^10*f^2*h^4
 - 64*a*c^10*d^9*e^2*f^2*h^4 + 1792*a^5*b*c^5*e^11*f^2*h^4 + 64*b^10*c*d*e^
10*f^2*h^4 + 64*b*c^10*d^10*e*f^2*h^4 + 240*a*b^9*c*e^11*f^2*h^4 - 16*c^11*
d^11*f^2*h^4 - 16*b^11*e^11*f^2*h^4 - c^7*e^7, h, k)^3*(root(8388608*a^7*b*
c^11*d^18*e*f^6*h^12 - 513802240*a^10*b^2*c^7*d^11*e^8*f^6*h^12 - 381681664
*a^11*b^2*c^6*d^9*e^10*f^6*h^12 - 381681664*a^9*b^2*c^8*d^13*e^6*f^6*h^12 -
 300941312*a^9*b^5*c^5*d^10*e^9*f^6*h^12 - 300941312*a^8*b^5*c^6*d^12*e^7*f
^6*h^12 + 293601280*a^10*b^3*c^6*d^10*e^9*f^6*h^12 + 293601280*a^9*b^3*c^7*
d^12*e^7*f^6*h^12 - 168820736*a^10*b^5*c^4*d^8*e^11*f^6*h^12 - 168820736*a^
7*b^5*c^7*d^14*e^5*f^6*h^12 + 166068224*a^8*b^6*c^5*d^11*e^8*f^6*h^12 - 146
800640*a^12*b^2*c^5*d^7*e^12*f^6*h^12 - 146800640*a^8*b^2*c^9*d^15*e^4*f^6*
h^12 + 124780544*a^10*b^4*c^5*d^9*e^10*f^6*h^12 + 124780544*a^8*b^4*c^7*d^1
3*e^6*f^6*h^12 + 119275520*a^9*b^4*c^6*d^11*e^8*f^6*h^12 + 117440512*a^11*b
^3*c^5*d^8*e^11*f^6*h^12 + 117440512*a^8*b^3*c^8*d^14*e^5*f^6*h^12 + 102760
448*a^9*b^6*c^4*d^9*e^10*f^6*h^12 + 102760448*a^7*b^6*c^6*d^13*e^6*f^6*h^12
 + 91750400*a^11*b^4*c^4*d^7*e^12*f^6*h^12 + 91750400*a^7*b^4*c^8*d^15*e^4*
f^6*h^12 - 71065600*a^7*b^8*c^4*d^11*e^8*f^6*h^12 - 53444608*a^8*b^8*c^3*d^
9*e^10*f^6*h^12 - 53444608*a^6*b^8*c^5*d^13*e^6*f^6*h^12 + 40370176*a^9*b^7
*c^3*d^8*e^11*f^6*h^12 + 40370176*a^6*b^7*c^6*d^14*e^5*f^6*h^12 - 36700160*
a^11*b^5*c^3*d^6*e^13*f^6*h^12 - 36700160*a^6*b^5*c^8*d^16*e^3*f^6*h^12 + 3
4078720*a^8*b^7*c^4*d^10*e^9*f^6*h^12 + 34078720*a^7*b^7*c^5*d^12*e^7*f^6*h
^12 + 26214400*a^12*b^4*c^3*d^5*e^14*f^6*h^12 + 26214400*a^6*b^4*c^9*d^17*e
^2*f^6*h^12 + 22118400*a^7*b^9*c^3*d^10*e^9*f^6*h^12 + 22118400*a^6*b^9*c^4
*d^12*e^7*f^6*h^12 - 20971520*a^13*b^2*c^4*d^5*e^14*f^6*h^12 - 20971520*a^7
*b^2*c^10*d^17*e^2*f^6*h^12 + 18350080*a^10*b^7*c^2*d^6*e^13*f^6*h^12 + 183
50080*a^5*b^7*c^7*d^16*e^3*f^6*h^12 - 16629760*a^9*b^8*c^2*d^7*e^12*f^6*h^1
2 - 16629760*a^5*b^8*c^6*d^15*e^4*f^6*h^12 - 10485760*a^11*b^6*c^2*d^5*e^14
*f^6*h^12 - 10485760*a^5*b^6*c^8*d^17*e^2*f^6*h^12 + 9175040*a^10*b^6*c^3*d
^7*e^12*f^6*h^12 + 9175040*a^6*b^6*c^7*d^15*e^4*f^6*h^12 - 8388608*a^13*b^3
*c^3*d^4*e^15*f^6*h^12 + 5619712*a^7*b^10*c^2*d^9*e^10*f^6*h^12 + 5619712*a
^5*b^10*c^4*d^13*e^6*f^6*h^12 - 5570560*a^6*b^11*c^2*d^10*e^9*f^6*h^12 - 55
70560*a^5*b^11*c^3*d^12*e^7*f^6*h^12 + 4358144*a^8*b^9*c^2*d^8*e^11*f^6*h^1
2 + 4358144*a^5*b^9*c^5*d^14*e^5*f^6*h^12 + 4259840*a^6*b^10*c^3*d^11*e^8*f
^6*h^12 + 3899392*a^4*b^10*c^5*d^15*e^4*f^6*h^12 - 3440640*a^4*b^9*c^6*d^16
*e^3*f^6*h^12 + 3145728*a^12*b^5*c^2*d^4*e^15*f^6*h^12 - 2523136*a^4*b^11*c
^4*d^14*e^5*f^6*h^12 + 1802240*a^4*b^8*c^7*d^17*e^2*f^6*h^12 + 1556480*a^5*

b^12*c^2*d^11*e^8*f^6*h^12 + 1048576*a^14*b^2*c^3*d^3*e^16*f^6*h^12 + 68812
8*a^4*b^12*c^3*d^13*e^6*f^6*h^12 - 393216*a^13*b^4*c^2*d^3*e^16*f^6*h^12 -
286720*a^3*b^12*c^4*d^15*e^4*f^6*h^12 + 229376*a^3*b^13*c^3*d^14*e^5*f^6*h^
12 + 229376*a^3*b^11*c^5*d^16*e^3*f^6*h^12 + 163840*a^4*b^13*c^2*d^12*e^7*f
^6*h^12 - 114688*a^3*b^14*c^2*d^13*e^6*f^6*h^12 - 114688*a^3*b^10*c^6*d^17*
e^2*f^6*h^12 + 293601280*a^11*b*c^7*d^10*e^9*f^6*h^12 + 293601280*a^10*b*c^
8*d^12*e^7*f^6*h^12 + 176160768*a^12*b*c^6*d^8*e^11*f^6*h^12 + 176160768*a^
9*b*c^9*d^14*e^5*f^6*h^12 + 58720256*a^13*b*c^5*d^6*e^13*f^6*h^12 + 5872025
6*a^8*b*c^10*d^16*e^3*f^6*h^12 + 8388608*a^14*b*c^4*d^4*e^15*f^6*h^12 - 838
8608*a^6*b^3*c^10*d^18*e*f^6*h^12 + 3899392*a^8*b^10*c*d^7*e^12*f^6*h^12 -
3440640*a^9*b^9*c*d^6*e^13*f^6*h^12 + 3145728*a^5*b^5*c^9*d^18*e*f^6*h^12 -
 2523136*a^7*b^11*c*d^8*e^11*f^6*h^12 + 1802240*a^10*b^8*c*d^5*e^14*f^6*h^1
2 + 688128*a^6*b^12*c*d^9*e^10*f^6*h^12 - 524288*a^11*b^7*c*d^4*e^15*f^6*h^
12 - 524288*a^4*b^7*c^8*d^18*e*f^6*h^12 + 163840*a^5*b^13*c*d^10*e^9*f^6*h^
12 - 163840*a^4*b^14*c*d^11*e^8*f^6*h^12 + 65536*a^12*b^6*c*d^3*e^16*f^6*h^
12 + 32768*a^3*b^15*c*d^12*e^7*f^6*h^12 + 32768*a^3*b^9*c^7*d^18*e*f^6*h^12
 - 73400320*a^11*c^8*d^11*e^8*f^6*h^12 - 58720256*a^12*c^7*d^9*e^10*f^6*h^1
2 - 58720256*a^10*c^9*d^13*e^6*f^6*h^12 - 29360128*a^13*c^6*d^7*e^12*f^6*h^
12 - 29360128*a^9*c^10*d^15*e^4*f^6*h^12 - 8388608*a^14*c^5*d^5*e^14*f^6*h^
12 - 8388608*a^8*c^11*d^17*e^2*f^6*h^12 - 1048576*a^15*c^4*d^3*e^16*f^6*h^1
2 - 286720*a^7*b^12*d^7*e^12*f^6*h^12 + 229376*a^8*b^11*d^6*e^13*f^6*h^12 +
 229376*a^6*b^13*d^8*e^11*f^6*h^12 - 114688*a^9*b^10*d^5*e^14*f^6*h^12 - 11
4688*a^5*b^14*d^9*e^10*f^6*h^12 + 32768*a^10*b^9*d^4*e^15*f^6*h^12 + 32768*
a^4*b^15*d^10*e^9*f^6*h^12 - 4096*a^11*b^8*d^3*e^16*f^6*h^12 - 4096*a^3*b^1
6*d^11*e^8*f^6*h^12 + 1048576*a^6*b^2*c^11*d^19*f^6*h^12 - 393216*a^5*b^4*c
^10*d^19*f^6*h^12 + 65536*a^4*b^6*c^9*d^19*f^6*h^12 - 4096*a^3*b^8*c^8*d^19
*f^6*h^12 - 1048576*a^7*c^12*d^19*f^6*h^12 + 262144*a^10*b*c^4*d*e^14*f^4*h
^8 - 23552*a*b^6*c^8*d^14*e*f^4*h^8 - 16384*a^7*b^7*c*d*e^14*f^4*h^8 - 3328
*a*b^13*c*d^7*e^8*f^4*h^8 + 2429952*a^4*b^5*c^6*d^9*e^6*f^4*h^8 - 1865728*a
^6*b^3*c^6*d^7*e^8*f^4*h^8 - 1716224*a^4*b^4*c^7*d^10*e^5*f^4*h^8 + 1605632
*a^6*b^2*c^7*d^8*e^7*f^4*h^8 + 1584384*a^5*b^5*c^5*d^7*e^8*f^4*h^8 + 157286
4*a^5*b^2*c^8*d^10*e^5*f^4*h^8 - 1433600*a^5*b^3*c^7*d^9*e^6*f^4*h^8 - 1261
568*a^4*b^6*c^5*d^8*e^7*f^4*h^8 - 1124352*a^3*b^4*c^8*d^12*e^3*f^4*h^8 - 11
10016*a^7*b^3*c^5*d^5*e^10*f^4*h^8 + 1106176*a^3*b^5*c^7*d^11*e^4*f^4*h^8 -
 936960*a^5*b^6*c^4*d^6*e^9*f^4*h^8 - 838656*a^2*b^7*c^6*d^11*e^4*f^4*h^8 -
 795648*a^3*b^7*c^5*d^9*e^6*f^4*h^8 + 730880*a^3*b^8*c^4*d^8*e^7*f^4*h^8 +
714752*a^2*b^6*c^7*d^12*e^3*f^4*h^8 + 686080*a^7*b^4*c^4*d^4*e^11*f^4*h^8 +
 641024*a^6*b^4*c^5*d^6*e^9*f^4*h^8 - 595968*a^8*b^3*c^4*d^3*e^12*f^4*h^8 +
 544768*a^3*b^3*c^9*d^13*e^2*f^4*h^8 + 516096*a^2*b^8*c^5*d^10*e^5*f^4*h^8
+ 441856*a^6*b^5*c^4*d^5*e^10*f^4*h^8 + 393216*a^7*b^2*c^6*d^6*e^9*f^4*h^8
+ 376832*a^4*b^2*c^9*d^12*e^3*f^4*h^8 - 366592*a^6*b^6*c^3*d^4*e^11*f^4*h^8
 + 363520*a^4*b^8*c^3*d^6*e^9*f^4*h^8 - 356352*a^5*b^4*c^6*d^8*e^7*f^4*h^8
- 348672*a^2*b^5*c^8*d^13*e^2*f^4*h^8 - 344064*a^8*b^2*c^5*d^4*e^11*f^4*h^8
 + 294912*a^8*b^4*c^3*d^2*e^13*f^4*h^8 + 210944*a^4*b^3*c^8*d^11*e^4*f^4*h^
8 - 198400*a^3*b^9*c^3*d^7*e^8*f^4*h^8 - 144640*a^4*b^7*c^4*d^7*e^8*f^4*h^8
 - 131072*a^9*b^2*c^4*d^2*e^13*f^4*h^8 - 131072*a^7*b^6*c^2*d^2*e^13*f^4*h^
8 - 129024*a^3*b^6*c^6*d^10*e^5*f^4*h^8 - 104448*a^2*b^10*c^3*d^8*e^7*f^4*h
^8 + 96768*a^5*b^8*c^2*d^4*e^11*f^4*h^8 + 91904*a^7*b^5*c^3*d^3*e^12*f^4*h^
8 - 74240*a^4*b^9*c^2*d^5*e^10*f^4*h^8 - 71680*a^2*b^9*c^4*d^9*e^6*f^4*h^8
+ 58368*a^2*b^11*c^2*d^7*e^8*f^4*h^8 + 36864*a^5*b^7*c^3*d^5*e^10*f^4*h^8 -
 35328*a^3*b^10*c^2*d^6*e^9*f^4*h^8 + 27136*a^6*b^7*c^2*d^3*e^12*f^4*h^8 +
909312*a^8*b*c^6*d^5*e^10*f^4*h^8 + 815104*a^9*b*c^5*d^3*e^12*f^4*h^8 - 651
264*a^5*b*c^9*d^11*e^4*f^4*h^8 - 573440*a^6*b*c^8*d^9*e^6*f^4*h^8 - 262144*
a^9*b^3*c^3*d*e^14*f^4*h^8 + 217088*a^7*b*c^7*d^7*e^8*f^4*h^8 + 211456*a*b^
9*c^5*d^11*e^4*f^4*h^8 - 204800*a^4*b*c^10*d^13*e^2*f^4*h^8 - 172032*a*b^8*
c^6*d^12*e^3*f^4*h^8 - 157696*a*b^10*c^4*d^10*e^5*f^4*h^8 - 131072*a^3*b^2*
c^10*d^14*e*f^4*h^8 + 98304*a^8*b^5*c^2*d*e^14*f^4*h^8 + 92160*a^2*b^4*c^9*
d^14*e*f^4*h^8 + 84992*a*b^7*c^7*d^13*e^2*f^4*h^8 + 64512*a*b^11*c^3*d^9*e^
6*f^4*h^8 + 23552*a^6*b^8*c*d^2*e^13*f^4*h^8 + 18944*a^3*b^11*c*d^5*e^10*f^

4*h^8 - 13312*a^4*b^10*c*d^4*e^11*f^4*h^8 - 9472*a^5*b^9*c*d^3*e^12*f^4*h^8 - 8192*a*b^12*c^2*d^8*e^7*f^4*h^8 - 6144*a^2*b^12*c*d^6*e^9*f^4*h^8 - 17920*b^11*c^4*d^11*e^4*f^4*h^8 + 14336*b^12*c^3*d^10*e^5*f^4*h^8 + 14336*b^10*c^5*d^12*e^3*f^4*h^8 - 7168*b^13*c^2*d^9*e^6*f^4*h^8 - 7168*b^9*c^6*d^13*e^2*f^4*h^8 - 425984*a^9*c^6*d^4*e^11*f^4*h^8 - 360448*a^8*c^7*d^6*e^9*f^4*h^8 - 262144*a^10*c^5*d^2*e^13*f^4*h^8 - 131072*a^7*c^8*d^8*e^7*f^4*h^8 + 98304*a^5*c^10*d^12*e^3*f^4*h^8 + 65536*a^6*c^9*d^10*e^5*f^4*h^8 - 1536*a^5*b^10*d^2*e^13*f^4*h^8 - 1536*a^2*b^13*d^5*e^10*f^4*h^8 + 768*a^4*b^11*d^3*e^12*f^4*h^8 + 768*a^3*b^12*d^4*e^11*f^4*h^8 + 65536*a^10*b^2*c^3*e^15*f^4*h^8 - 24576*a^9*b^4*c^2*e^15*f^4*h^8 - 10240*a^2*b^3*c^10*d^15*f^4*h^8 + 2048*b^14*c*d^8*e^7*f^4*h^8 + 2048*b^8*c^7*d^14*e*f^4*h^8 + 32768*a^4*c^11*d^14*e*f^4*h^8 + 1024*a^6*b^9*d*e^14*f^4*h^8 + 1024*a*b^14*d^6*e^9*f^4*h^8 + 4096*a^8*b^6*c*e^15*f^4*h^8 + 12288*a^3*b*c^11*d^15*f^4*h^8 + 2816*a*b^5*c^9*d^15*f^4*h^8 - 256*b^15*d^7*e^8*f^4*h^8 - 65536*a^11*c^4*e^15*f^4*h^8 - 256*b^7*c^8*d^15*f^4*h^8 - 256*a^7*b^8*e^15*f^4*h^8 - 896*a*b^8*c^2*d*e^10*f^2*h^4 + 192*a*b*c^9*d^8*e^3*f^2*h^4 + 11520*a^3*b^3*c^5*d^2*e^9*f^2*h^4 - 5856*a^2*b^5*c^4*d^2*e^9*f^2*h^4 - 5120*a^3*b^2*c^6*d^3*e^8*f^2*h^4 + 3200*a^2*b^4*c^5*d^3*e^8*f^2*h^4 - 640*a^2*b^3*c^6*d^4*e^7*f^2*h^4 - 96*a^2*b^2*c^7*d^5*e^6*f^2*h^4 - 10880*a^3*b^4*c^4*d*e^10*f^2*h^4 + 10240*a^4*b^2*c^5*d*e^10*f^2*h^4 - 7680*a^4*b*c^6*d^2*e^9*f^2*h^4 + 4672*a^2*b^6*c^3*d*e^10*f^2*h^4 + 1248*a*b^7*c^3*d^2*e^9*f^2*h^4 + 832*a^3*b*c^7*d^4*e^7*f^2*h^4 - 768*a*b^6*c^4*d^3*e^8*f^2*h^4 + 192*a^2*b*c^8*d^6*e^5*f^2*h^4 - 192*a*b^2*c^8*d^7*e^4*f^2*h^4 + 176*a*b^5*c^5*d^4*e^7*f^2*h^4 + 64*a*b^3*c^7*d^6*e^5*f^2*h^4 - 96*b^9*c^2*d^2*e^9*f^2*h^4 - 96*b^2*c^9*d^9*e^2*f^2*h^4 + 64*b^8*c^3*d^3*e^8*f^2*h^4 + 64*b^3*c^8*d^8*e^3*f^2*h^4 - 16*b^7*c^4*d^4*e^7*f^2*h^4 - 16*b^4*c^7*d^7*e^4*f^2*h^4 + 2032*a^4*c^7*d^3*e^8*f^2*h^4 - 96*a^2*c^9*d^7*e^4*f^2*h^4 - 64*a^3*c^8*d^5*e^6*f^2*h^4 - 4480*a^4*b^3*c^4*e^11*f^2*h^4 + 3696*a^3*b^5*c^3*e^11*f^2*h^4 - 1376*a^2*b^7*c^2*e^11*f^2*h^4 - 2048*a^5*c^6*d*e^10*f^2*h^4 - 64*a*c^10*d^9*e^2*f^2*h^4 + 1792*a^5*b*c^5*e^11*f^2*h^4 + 64*b^10*c*d*e^10*f^2*h^4 + 64*b*c^10*d^10*e*f^2*h^4 + 240*a*b^9*c*e^11*f^2*h^4 - 16*c^11*d^11*f^2*h^4 - 16*b^11*e^11*f^2*h^4 - c^7*e^7, h, k)*(root(8388608*a^7*b*c^11*d^18*e*f^6*h^12 - 513802240*a^10*b^2*c^7*d^11*e^8*f^6*h^12 - 381681664*a^11*b^2*c^6*d^9*e^10*f^6*h^12 - 381681664*a^9*b^2*c^8*d^13*e^6*f^6*h^12 - 300941312*a^9*b^5*c^5*d^10*e^9*f^6*h^12 - 300941312*a^8*b^5*c^6*d^12*e^7*f^6*h^12 + 293601280*a^10*b^3*c^6*d^10*e^9*f^6*h^12 + 293601280*a^9*b^3*c^7*d^12*e^7*f^6*h^12 - 168820736*a^10*b^5*c^4*d^8*e^11*f^6*h^12 - 168820736*a^7*b^5*c^7*d^14*e^5*f^6*h^12 + 166068224*a^8*b^6*c^5*d^11*e^8*f^6*h^12 - 146800640*a^12*b^2*c^5*d^7*e^12*f^6*h^12 - 146800640*a^8*b^2*c^9*d^15*e^4*f^6*h^12 + 124780544*a^10*b^4*c^5*d^9*e^10*f^6*h^12 + 124780544*a^8*b^4*c^7*d^13*e^6*f^6*h^12 + 119275520*a^9*b^4*c^6*d^11*e^8*f^6*h^12 + 117440512*a^11*b^3*c^5*d^8*e^11*f^6*h^12 + 117440512*a^8*b^3*c^8*d^14*e^5*f^6*h^12 + 102760448*a^9*b^6*c^4*d^9*e^10*f^6*h^12 + 102760448*a^7*b^6*c^6*d^13*e^6*f^6*h^12 + 91750400*a^11*b^4*c^4*d^7*e^12*f^6*h^12 + 91750400*a^7*b^4*c^8*d^15*e^4*f^6*h^12 - 71065600*a^7*b^8*c^4*d^11*e^8*f^6*h^12 - 53444608*a^8*b^8*c^3*d^9*e^10*f^6*h^12 - 53444608*a^6*b^8*c^5*d^13*e^6*f^6*h^12 + 40370176*a^9*b^7*c^3*d^8*e^11*f^6*h^12 + 40370176*a^6*b^7*c^6*d^14*e^5*f^6*h^12 - 36700160*a^11*b^5*c^3*d^6*e^13*f^6*h^12 - 36700160*a^6*b^5*c^8*d^16*e^3*f^6*h^12 + 34078720*a^8*b^7*c^4*d^10*e^9*f^6*h^12 + 34078720*a^7*b^7*c^5*d^12*e^7*f^6*h^12 + 26214400*a^12*b^4*c^3*d^5*e^14*f^6*h^12 + 26214400*a^6*b^4*c^9*d^17*e^2*f^6*h^12 + 22118400*a^7*b^9*c^3*d^10*e^9*f^6*h^12 + 22118400*a^6*b^9*c^4*d^12*e^7*f^6*h^12 - 20971520*a^13*b^2*c^4*d^5*e^14*f^6*h^12 - 20971520*a^7*b^2*c^10*d^17*e^2*f^6*h^12 + 18350080*a^10*b^7*c^2*d^6*e^13*f^6*h^12 + 18350080*a^5*b^7*c^7*d^16*e^3*f^6*h^12 - 16629760*a^9*b^8*c^2*d^7*e^12*f^6*h^12 - 16629760*a^5*b^8*c^6*d^15*e^4*f^6*h^12 - 10485760*a^11*b^6*c^2*d^5*e^14*f^6*h^12 - 10485760*a^5*b^6*c^8*d^17*e^2*f^6*h^12 + 9175040*a^10*b^6*c^3*d^7*e^12*f^6*h^12 + 9175040*a^6*b^6*c^7*d^15*e^4*f^6*h^12 - 8388608*a^13*b^3*c^3*d^4*e^15*f^6*h^12 + 5619712*a^7*b^10*c^2*d^9*e^10*f^6*h^12 + 5619712*a^5*b^10*c^4*d^13*e^6*f^6*h^12 - 5570560*a^6*b^11*c^2*d^10*e^9*f^6*h^12 - 5570560*a^5*b^11*c^3*d^12*e^7*f^6*h^12 + 4358144*a^8*b^9*c^2*d^

8*e^11*f^6*h^12 + 4358144*a^5*b^9*c^5*d^14*e^5*f^6*h^12 + 4259840*a^6*b^10*c^3*d^11*e^8*f^6*h^12 + 3899392*a^4*b^10*c^5*d^15*e^4*f^6*h^12 - 3440640*a^4*b^9*c^6*d^16*e^3*f^6*h^12 + 3145728*a^12*b^5*c^2*d^4*e^15*f^6*h^12 - 2523136*a^4*b^11*c^4*d^14*e^5*f^6*h^12 + 1802240*a^4*b^8*c^7*d^17*e^2*f^6*h^12 + 1556480*a^5*b^12*c^2*d^11*e^8*f^6*h^12 + 1048576*a^14*b^2*c^3*d^3*e^16*f^6*h^12 + 688128*a^4*b^12*c^3*d^13*e^6*f^6*h^12 - 393216*a^13*b^4*c^2*d^3*e^16*f^6*h^12 - 286720*a^3*b^12*c^4*d^15*e^4*f^6*h^12 + 229376*a^3*b^13*c^3*d^14*e^5*f^6*h^12 + 229376*a^3*b^11*c^5*d^16*e^3*f^6*h^12 + 163840*a^4*b^13*c^2*d^12*e^7*f^6*h^12 - 114688*a^3*b^14*c^2*d^13*e^6*f^6*h^12 - 114688*a^3*b^10*c^6*d^17*e^2*f^6*h^12 + 293601280*a^11*b*c^7*d^10*e^9*f^6*h^12 + 293601280*a^10*b*c^8*d^12*e^7*f^6*h^12 + 176160768*a^12*b*c^6*d^8*e^11*f^6*h^12 + 176160768*a^9*b*c^9*d^14*e^5*f^6*h^12 + 58720256*a^13*b*c^5*d^6*e^13*f^6*h^12 + 58720256*a^8*b*c^10*d^16*e^3*f^6*h^12 + 8388608*a^14*b*c^4*d^4*e^15*f^6*h^12 - 8388608*a^6*b^3*c^10*d^18*e*f^6*h^12 + 3899392*a^8*b^10*c*d^7*e^12*f^6*h^12 - 3440640*a^9*b^9*c*d^6*e^13*f^6*h^12 + 3145728*a^5*b^5*c^9*d^18*e*f^6*h^12 - 2523136*a^7*b^11*c*d^8*e^11*f^6*h^12 + 1802240*a^10*b^8*c*d^5*e^14*f^6*h^12 + 688128*a^6*b^12*c*d^9*e^10*f^6*h^12 - 524288*a^11*b^7*c*d^4*e^15*f^6*h^12 - 524288*a^4*b^7*c^8*d^18*e*f^6*h^12 + 163840*a^5*b^13*c*d^10*e^9*f^6*h^12 - 163840*a^4*b^14*c*d^11*e^8*f^6*h^12 + 65536*a^12*b^6*c*d^3*e^16*f^6*h^12 + 32768*a^3*b^15*c*d^12*e^7*f^6*h^12 + 32768*a^3*b^9*c^7*d^18*e*f^6*h^12 - 73400320*a^11*c^8*d^11*e^8*f^6*h^12 - 58720256*a^12*c^7*d^9*e^10*f^6*h^12 - 58720256*a^10*c^9*d^13*e^6*f^6*h^12 - 29360128*a^13*c^6*d^7*e^12*f^6*h^12 - 29360128*a^9*c^10*d^15*e^4*f^6*h^12 - 8388608*a^14*c^5*d^5*e^14*f^6*h^12 - 8388608*a^8*c^11*d^17*e^2*f^6*h^12 - 1048576*a^15*c^4*d^3*e^16*f^6*h^12 - 286720*a^7*b^12*d^7*e^12*f^6*h^12 + 229376*a^8*b^11*d^6*e^13*f^6*h^12 + 229376*a^6*b^13*d^8*e^11*f^6*h^12 - 114688*a^9*b^10*d^5*e^14*f^6*h^12 - 114688*a^5*b^14*d^9*e^10*f^6*h^12 + 32768*a^10*b^9*d^4*e^15*f^6*h^12 + 32768*a^4*b^15*d^10*e^9*f^6*h^12 - 4096*a^11*b^8*d^3*e^16*f^6*h^12 - 4096*a^3*b^16*d^11*e^8*f^6*h^12 + 1048576*a^6*b^2*c^11*d^19*f^6*h^12 - 393216*a^5*b^4*c^10*d^19*f^6*h^12 + 65536*a^4*b^6*c^9*d^19*f^6*h^12 - 4096*a^3*b^8*c^8*d^19*f^6*h^12 - 1048576*a^7*c^12*d^19*f^6*h^12 + 262144*a^10*b*c^4*d*e^14*f^4*h^8 - 23552*a*b^6*c^8*d^14*e*f^4*h^8 - 16384*a^7*b^7*c*d*e^14*f^4*h^8 - 3328*a*b^13*c*d^7*e^8*f^4*h^8 + 2429952*a^4*b^5*c^6*d^9*e^6*f^4*h^8 - 1865728*a^6*b^3*c^6*d^7*e^8*f^4*h^8 - 1716224*a^4*b^4*c^7*d^10*e^5*f^4*h^8 + 1605632*a^6*b^2*c^7*d^8*e^7*f^4*h^8 + 1584384*a^5*b^5*c^5*d^7*e^8*f^4*h^8 + 1572864*a^5*b^2*c^8*d^10*e^5*f^4*h^8 - 1433600*a^5*b^3*c^7*d^9*e^6*f^4*h^8 - 1261568*a^4*b^6*c^5*d^8*e^7*f^4*h^8 - 1124352*a^3*b^4*c^8*d^12*e^3*f^4*h^8 - 1110016*a^7*b^3*c^5*d^5*e^10*f^4*h^8 + 1106176*a^3*b^5*c^7*d^11*e^4*f^4*h^8 - 936960*a^5*b^6*c^4*d^6*e^9*f^4*h^8 - 838656*a^2*b^7*c^6*d^11*e^4*f^4*h^8 - 795648*a^3*b^7*c^5*d^9*e^6*f^4*h^8 + 730880*a^3*b^8*c^4*d^8*e^7*f^4*h^8 + 714752*a^2*b^6*c^7*d^12*e^3*f^4*h^8 + 686080*a^7*b^4*c^4*d^4*e^11*f^4*h^8 + 641024*a^6*b^4*c^5*d^6*e^9*f^4*h^8 - 595968*a^8*b^3*c^4*d^3*e^12*f^4*h^8 + 544768*a^3*b^3*c^9*d^13*e^2*f^4*h^8 + 516096*a^2*b^8*c^5*d^10*e^5*f^4*h^8 + 441856*a^6*b^5*c^4*d^5*e^10*f^4*h^8 + 393216*a^7*b^2*c^6*d^6*e^9*f^4*h^8 + 376832*a^4*b^2*c^9*d^12*e^3*f^4*h^8 - 366592*a^6*b^6*c^3*d^4*e^11*f^4*h^8 + 363520*a^4*b^8*c^3*d^6*e^9*f^4*h^8 - 356352*a^5*b^4*c^6*d^8*e^7*f^4*h^8 - 348672*a^2*b^5*c^8*d^13*e^2*f^4*h^8 - 344064*a^8*b^2*c^5*d^4*e^11*f^4*h^8 + 294912*a^8*b^4*c^3*d^2*e^13*f^4*h^8 + 210944*a^4*b^3*c^8*d^11*e^4*f^4*h^8 - 198400*a^3*b^9*c^3*d^7*e^8*f^4*h^8 - 144640*a^4*b^7*c^4*d^7*e^8*f^4*h^8 - 131072*a^9*b^2*c^4*d^2*e^13*f^4*h^8 - 131072*a^7*b^6*c^2*d^2*e^13*f^4*h^8 - 129024*a^3*b^6*c^6*d^10*e^5*f^4*h^8 - 104448*a^2*b^10*c^3*d^8*e^7*f^4*h^8 + 96768*a^5*b^8*c^2*d^4*e^11*f^4*h^8 + 91904*a^7*b^5*c^3*d^3*e^12*f^4*h^8 - 74240*a^4*b^9*c^2*d^5*e^10*f^4*h^8 - 71680*a^2*b^9*c^4*d^9*e^6*f^4*h^8 + 58368*a^2*b^11*c^2*d^7*e^8*f^4*h^8 + 36864*a^5*b^7*c^3*d^5*e^10*f^4*h^8 - 35328*a^3*b^10*c^2*d^6*e^9*f^4*h^8 + 27136*a^6*b^7*c^2*d^3*e^12*f^4*h^8 + 909312*a^8*b*c^6*d^5*e^10*f^4*h^8 + 815104*a^9*b*c^5*d^3*e^12*f^4*h^8 - 651264*a^5*b*c^9*d^11*e^4*f^4*h^8 - 573440*a^6*b*c^8*d^9*e^6*f^4*h^8 - 262144*a^9*b^3*c^3*d*e^14*f^4*h^8 + 217088*a^7*b*c^7*d^7*e^8*f^4*h^8 + 211456*a*b^9*c^5*d^11*e^4*f^4*h^8 - 204800*a^4*b*c^10*d^13*e^2*f^4*h^8 -

172032*a*b^8*c^6*d^12*e^3*f^4*h^8 - 157696*a*b^10*c^4*d^10*e^5*f^4*h^8 - 1
31072*a^3*b^2*c^10*d^14*e*f^4*h^8 + 98304*a^8*b^5*c^2*d*e^14*f^4*h^8 + 9216
0*a^2*b^4*c^9*d^14*e*f^4*h^8 + 84992*a*b^7*c^7*d^13*e^2*f^4*h^8 + 64512*a*b
^11*c^3*d^9*e^6*f^4*h^8 + 23552*a^6*b^8*c*d^2*e^13*f^4*h^8 + 18944*a^3*b^11
*c*d^5*e^10*f^4*h^8 - 13312*a^4*b^10*c*d^4*e^11*f^4*h^8 - 9472*a^5*b^9*c*d^
3*e^12*f^4*h^8 - 8192*a*b^12*c^2*d^8*e^7*f^4*h^8 - 6144*a^2*b^12*c*d^6*e^9*
f^4*h^8 - 17920*b^11*c^4*d^11*e^4*f^4*h^8 + 14336*b^12*c^3*d^10*e^5*f^4*h^8
 + 14336*b^10*c^5*d^12*e^3*f^4*h^8 - 7168*b^13*c^2*d^9*e^6*f^4*h^8 - 7168*b
^9*c^6*d^13*e^2*f^4*h^8 - 425984*a^9*c^6*d^4*e^11*f^4*h^8 - 360448*a^8*c^7*
d^6*e^9*f^4*h^8 - 262144*a^10*c^5*d^2*e^13*f^4*h^8 - 131072*a^7*c^8*d^8*e^7
*f^4*h^8 + 98304*a^5*c^10*d^12*e^3*f^4*h^8 + 65536*a^6*c^9*d^10*e^5*f^4*h^8
 - 1536*a^5*b^10*d^2*e^13*f^4*h^8 - 1536*a^2*b^13*d^5*e^10*f^4*h^8 + 768*a^
4*b^11*d^3*e^12*f^4*h^8 + 768*a^3*b^12*d^4*e^11*f^4*h^8 + 65536*a^10*b^2*c^
3*e^15*f^4*h^8 - 24576*a^9*b^4*c^2*e^15*f^4*h^8 - 10240*a^2*b^3*c^10*d^15*f
^4*h^8 + 2048*b^14*c*d^8*e^7*f^4*h^8 + 2048*b^8*c^7*d^14*e*f^4*h^8 + 32768*
a^4*c^11*d^14*e*f^4*h^8 + 1024*a^6*b^9*d*e^14*f^4*h^8 + 1024*a*b^14*d^6*e^9
*f^4*h^8 + 4096*a^8*b^6*c*e^15*f^4*h^8 + 12288*a^3*b*c^11*d^15*f^4*h^8 + 28
16*a*b^5*c^9*d^15*f^4*h^8 - 256*b^15*d^7*e^8*f^4*h^8 - 65536*a^11*c^4*e^15*
f^4*h^8 - 256*b^7*c^8*d^15*f^4*h^8 - 256*a^7*b^8*e^15*f^4*h^8 - 896*a*b^8*c
^2*d*e^10*f^2*h^4 + 192*a*b*c^9*d^8*e^3*f^2*h^4 + 11520*a^3*b^3*c^5*d^2*e^9
*f^2*h^4 - 5856*a^2*b^5*c^4*d^2*e^9*f^2*h^4 - 5120*a^3*b^2*c^6*d^3*e^8*f^2*
h^4 + 3200*a^2*b^4*c^5*d^3*e^8*f^2*h^4 - 640*a^2*b^3*c^6*d^4*e^7*f^2*h^4 -
96*a^2*b^2*c^7*d^5*e^6*f^2*h^4 - 10880*a^3*b^4*c^4*d*e^10*f^2*h^4 + 10240*a
^4*b^2*c^5*d*e^10*f^2*h^4 - 7680*a^4*b*c^6*d^2*e^9*f^2*h^4 + 4672*a^2*b^6*c
^3*d*e^10*f^2*h^4 + 1248*a*b^7*c^3*d^2*e^9*f^2*h^4 + 832*a^3*b*c^7*d^4*e^7*
f^2*h^4 - 768*a*b^6*c^4*d^3*e^8*f^2*h^4 + 192*a^2*b*c^8*d^6*e^5*f^2*h^4 - 1
92*a*b^2*c^8*d^7*e^4*f^2*h^4 + 176*a*b^5*c^5*d^4*e^7*f^2*h^4 + 64*a*b^3*c^7
*d^6*e^5*f^2*h^4 - 96*b^9*c^2*d^2*e^9*f^2*h^4 - 96*b^2*c^9*d^9*e^2*f^2*h^4
+ 64*b^8*c^3*d^3*e^8*f^2*h^4 + 64*b^3*c^8*d^8*e^3*f^2*h^4 - 16*b^7*c^4*d^4*
e^7*f^2*h^4 - 16*b^4*c^7*d^7*e^4*f^2*h^4 + 2032*a^4*c^7*d^3*e^8*f^2*h^4 - 9
6*a^2*c^9*d^7*e^4*f^2*h^4 - 64*a^3*c^8*d^5*e^6*f^2*h^4 - 4480*a^4*b^3*c^4*e
^11*f^2*h^4 + 3696*a^3*b^5*c^3*e^11*f^2*h^4 - 1376*a^2*b^7*c^2*e^11*f^2*h^4
 - 2048*a^5*c^6*d*e^10*f^2*h^4 - 64*a*c^10*d^9*e^2*f^2*h^4 + 1792*a^5*b*c^5
*e^11*f^2*h^4 + 64*b^10*c*d*e^10*f^2*h^4 + 64*b*c^10*d^10*e*f^2*h^4 + 240*a
*b^9*c*e^11*f^2*h^4 - 16*c^11*d^11*f^2*h^4 - 16*b^11*e^11*f^2*h^4 - c^7*e^7
, h, k)^3*(root(8388608*a^7*b*c^11*d^18*e*f^6*h^12 - 513802240*a^10*b^2*c^7
*d^11*e^8*f^6*h^12 - 381681664*a^11*b^2*c^6*d^9*e^10*f^6*h^12 - 381681664*a
^9*b^2*c^8*d^13*e^6*f^6*h^12 - 300941312*a^9*b^5*c^5*d^10*e^9*f^6*h^12 - 30
0941312*a^8*b^5*c^6*d^12*e^7*f^6*h^12 + 293601280*a^10*b^3*c^6*d^10*e^9*f^6
*h^12 + 293601280*a^9*b^3*c^7*d^12*e^7*f^6*h^12 - 168820736*a^10*b^5*c^4*d^
8*e^11*f^6*h^12 - 168820736*a^7*b^5*c^7*d^14*e^5*f^6*h^12 + 166068224*a^8*b
^6*c^5*d^11*e^8*f^6*h^12 - 146800640*a^12*b^2*c^5*d^7*e^12*f^6*h^12 - 14680
0640*a^8*b^2*c^9*d^15*e^4*f^6*h^12 + 124780544*a^10*b^4*c^5*d^9*e^10*f^6*h^
12 + 124780544*a^8*b^4*c^7*d^13*e^6*f^6*h^12 + 119275520*a^9*b^4*c^6*d^11*e
^8*f^6*h^12 + 117440512*a^11*b^3*c^5*d^8*e^11*f^6*h^12 + 117440512*a^8*b^3*
c^8*d^14*e^5*f^6*h^12 + 102760448*a^9*b^6*c^4*d^9*e^10*f^6*h^12 + 102760448
*a^7*b^6*c^6*d^13*e^6*f^6*h^12 + 91750400*a^11*b^4*c^4*d^7*e^12*f^6*h^12 +
91750400*a^7*b^4*c^8*d^15*e^4*f^6*h^12 - 71065600*a^7*b^8*c^4*d^11*e^8*f^6*
h^12 - 53444608*a^8*b^8*c^3*d^9*e^10*f^6*h^12 - 53444608*a^6*b^8*c^5*d^13*e
^6*f^6*h^12 + 40370176*a^9*b^7*c^3*d^8*e^11*f^6*h^12 + 40370176*a^6*b^7*c^6
*d^14*e^5*f^6*h^12 - 36700160*a^11*b^5*c^3*d^6*e^13*f^6*h^12 - 36700160*a^6
*b^5*c^8*d^16*e^3*f^6*h^12 + 34078720*a^8*b^7*c^4*d^10*e^9*f^6*h^12 + 34078
720*a^7*b^7*c^5*d^12*e^7*f^6*h^12 + 26214400*a^12*b^4*c^3*d^5*e^14*f^6*h^12
 + 26214400*a^6*b^4*c^9*d^17*e^2*f^6*h^12 + 22118400*a^7*b^9*c^3*d^10*e^9*f
^6*h^12 + 22118400*a^6*b^9*c^4*d^12*e^7*f^6*h^12 - 20971520*a^13*b^2*c^4*d^
5*e^14*f^6*h^12 - 20971520*a^7*b^2*c^10*d^17*e^2*f^6*h^12 + 18350080*a^10*b
^7*c^2*d^6*e^13*f^6*h^12 + 18350080*a^5*b^7*c^7*d^16*e^3*f^6*h^12 - 1662976
0*a^9*b^8*c^2*d^7*e^12*f^6*h^12 - 16629760*a^5*b^8*c^6*d^15*e^4*f^6*h^12 -
10485760*a^11*b^6*c^2*d^5*e^14*f^6*h^12 - 10485760*a^5*b^6*c^8*d^17*e^2*f^6

```
*h^12 + 9175040*a^10*b^6*c^3*d^7*e^12*f^6*h^12 + 9175040*a^6*b^6*c^7*d^15*e
^4*f^6*h^12 - 8388608*a^13*b^3*c^3*d^4*e^15*f^6*h^12 + 5619712*a^7*b^10*c^2
*d^9*e^10*f^6*h^12 + 5619712*a^5*b^10*c^4*d^13*e^6*f^6*h^12 - 5570560*a^6*b
^11*c^2*d^10*e^9*f^6*h^12 - 5570560*a^5*b^11*c^3*d^12*e^7*f^6*h^12 + 435814
4*a^8*b^9*c^2*d^8*e^11*f^6*h^12 + 4358144*a^5*b^9*c^5*d^14*e^5*f^6*h^12 + 4
259840*a^6*b^10*c^3*d^11*e^8*f^6*h^12 + 3899392*a^4*b^10*c^5*d^15*e^4*f^6*h
^12 - 3440640*a^4*b^9*c^6*d^16*e^3*f^6*h^12 + 3145728*a^12*b^5*c^2*d^4*e^15
*f^6*h^12 - 2523136*a^4*b^11*c^4*d^14*e^5*f^6*h^12 + 1802240*a^4*b^8*c^7*d^
17*e^2*f^6*h^12 + 1556480*a^5*b^12*c^2*d^11*e^8*f^6*h^12 + 1048576*a^14*b^2
*c^3*d^3*e^16*f^6*h^12 + 688128*a^4*b^12*c^3*d^13*e^6*f^6*h^12 - 393216*a^1
3*b^4*c^2*d^3*e^16*f^6*h^12 - 286720*a^3*b^12*c^4*d^15*e^4*f^6*h^12 + 22937
6*a^3*b^13*c^3*d^14*e^5*f^6*h^12 + 229376*a^3*b^11*c^5*d^16*e^3*f^6*h^12 +
163840*a^4*b^13*c^2*d^12*e^7*f^6*h^12 - 114688*a^3*b^14*c^2*d^13*e^6*f^6*h^
12 - 114688*a^3*b^10*c^6*d^17*e^2*f^6*h^12 + 293601280*a^11*b*c^7*d^10*e^9*
f^6*h^12 + 293601280*a^10*b*c^8*d^12*e^7*f^6*h^12 + 176160768*a^12*b*c^6*d^
8*e^11*f^6*h^12 + 176160768*a^9*b*c^9*d^14*e^5*f^6*h^12 + 58720256*a^13*b*c
^5*d^6*e^13*f^6*h^12 + 58720256*a^8*b*c^10*d^16*e^3*f^6*h^12 + 8388608*a^14
*b*c^4*d^4*e^15*f^6*h^12 - 8388608*a^6*b^3*c^10*d^18*e*f^6*h^12 + 3899392*a
^8*b^10*c*d^7*e^12*f^6*h^12 - 3440640*a^9*b^9*c*d^6*e^13*f^6*h^12 + 3145728
*a^5*b^5*c^9*d^18*e*f^6*h^12 - 2523136*a^7*b^11*c*d^8*e^11*f^6*h^12 + 18022
40*a^10*b^8*c*d^5*e^14*f^6*h^12 + 688128*a^6*b^12*c*d^9*e^10*f^6*h^12 - 524
288*a^11*b^7*c*d^4*e^15*f^6*h^12 - 524288*a^4*b^7*c^8*d^18*e*f^6*h^12 + 163
840*a^5*b^13*c*d^10*e^9*f^6*h^12 - 163840*a^4*b^14*c*d^11*e^8*f^6*h^12 + 65
536*a^12*b^6*c*d^3*e^16*f^6*h^12 + 32768*a^3*b^15*c*d^12*e^7*f^6*h^12 + 327
68*a^3*b^9*c^7*d^18*e*f^6*h^12 - 73400320*a^11*c^8*d^11*e^8*f^6*h^12 - 5872
0256*a^12*c^7*d^9*e^10*f^6*h^12 - 58720256*a^10*c^9*d^13*e^6*f^6*h^12 - 293
60128*a^13*c^6*d^7*e^12*f^6*h^12 - 29360128*a^9*c^10*d^15*e^4*f^6*h^12 - 83
88608*a^14*c^5*d^5*e^14*f^6*h^12 - 8388608*a^8*c^11*d^17*e^2*f^6*h^12 - 104
8576*a^15*c^4*d^3*e^16*f^6*h^12 - 286720*a^7*b^12*d^7*e^12*f^6*h^12 + 22937
6*a^8*b^11*d^6*e^13*f^6*h^12 + 229376*a^6*b^13*d^8*e^11*f^6*h^12 - 114688*a
^9*b^10*d^5*e^14*f^6*h^12 - 114688*a^5*b^14*d^9*e^10*f^6*h^12 + 32768*a^10*
b^9*d^4*e^15*f^6*h^12 + 32768*a^4*b^15*d^10*e^9*f^6*h^12 - 4096*a^11*b^8*d^
3*e^16*f^6*h^12 - 4096*a^3*b^16*d^11*e^8*f^6*h^12 + 1048576*a^6*b^2*c^11*d^
19*f^6*h^12 - 393216*a^5*b^4*c^10*d^19*f^6*h^12 + 65536*a^4*b^6*c^9*d^19*f^
6*h^12 - 4096*a^3*b^8*c^8*d^19*f^6*h^12 - 1048576*a^7*c^12*d^19*f^6*h^12 +
262144*a^10*b*c^4*d*e^14*f^4*h^8 - 23552*a*b^6*c^8*d^14*e*f^4*h^8 - 16384*a
^7*b^7*c*d*e^14*f^4*h^8 - 3328*a*b^13*c*d^7*e^8*f^4*h^8 + 2429952*a^4*b^5*c
^6*d^9*e^6*f^4*h^8 - 1865728*a^6*b^3*c^6*d^7*e^8*f^4*h^8 - 1716224*a^4*b^4*
c^7*d^10*e^5*f^4*h^8 + 1605632*a^6*b^2*c^7*d^8*e^7*f^4*h^8 + 1584384*a^5*b^
5*c^5*d^7*e^8*f^4*h^8 + 1572864*a^5*b^2*c^8*d^10*e^5*f^4*h^8 - 1433600*a^5*
b^3*c^7*d^9*e^6*f^4*h^8 - 1261568*a^4*b^6*c^5*d^8*e^7*f^4*h^8 - 1124352*a^3
*b^4*c^8*d^12*e^3*f^4*h^8 - 1110016*a^7*b^3*c^5*d^5*e^10*f^4*h^8 + 1106176*
a^3*b^5*c^7*d^11*e^4*f^4*h^8 - 936960*a^5*b^6*c^4*d^6*e^9*f^4*h^8 - 838656*
a^2*b^7*c^6*d^11*e^4*f^4*h^8 - 795648*a^3*b^7*c^5*d^9*e^6*f^4*h^8 + 730880*
a^3*b^8*c^4*d^8*e^7*f^4*h^8 + 714752*a^2*b^6*c^7*d^12*e^3*f^4*h^8 + 686080*
a^7*b^4*c^4*d^4*e^11*f^4*h^8 + 641024*a^6*b^4*c^5*d^6*e^9*f^4*h^8 - 595968*
a^8*b^3*c^4*d^3*e^12*f^4*h^8 + 544768*a^3*b^3*c^9*d^13*e^2*f^4*h^8 + 516096
*a^2*b^8*c^5*d^10*e^5*f^4*h^8 + 441856*a^6*b^5*c^4*d^5*e^10*f^4*h^8 + 39321
6*a^7*b^2*c^6*d^6*e^9*f^4*h^8 + 376832*a^4*b^2*c^9*d^12*e^3*f^4*h^8 - 36659
2*a^6*b^6*c^3*d^4*e^11*f^4*h^8 + 363520*a^4*b^8*c^3*d^6*e^9*f^4*h^8 - 35635
2*a^5*b^4*c^6*d^8*e^7*f^4*h^8 - 348672*a^2*b^5*c^8*d^13*e^2*f^4*h^8 - 34406
4*a^8*b^2*c^5*d^4*e^11*f^4*h^8 + 294912*a^8*b^4*c^3*d^2*e^13*f^4*h^8 + 2109
44*a^4*b^3*c^8*d^11*e^4*f^4*h^8 - 198400*a^3*b^9*c^3*d^7*e^8*f^4*h^8 - 1446
40*a^4*b^7*c^4*d^7*e^8*f^4*h^8 - 131072*a^9*b^2*c^4*d^2*e^13*f^4*h^8 - 1310
72*a^7*b^6*c^2*d^2*e^13*f^4*h^8 - 129024*a^3*b^6*c^6*d^10*e^5*f^4*h^8 - 104
448*a^2*b^10*c^3*d^8*e^7*f^4*h^8 + 96768*a^5*b^8*c^2*d^4*e^11*f^4*h^8 + 919
04*a^7*b^5*c^3*d^3*e^12*f^4*h^8 - 74240*a^4*b^9*c^2*d^5*e^10*f^4*h^8 - 7168
0*a^2*b^9*c^4*d^9*e^6*f^4*h^8 + 58368*a^2*b^11*c^2*d^7*e^8*f^4*h^8 + 36864*
a^5*b^7*c^3*d^5*e^10*f^4*h^8 - 35328*a^3*b^10*c^2*d^6*e^9*f^4*h^8 + 27136*a
```

^6*b^7*c^2*d^3*e^12*f^4*h^8 + 909312*a^8*b*c^6*d^5*e^10*f^4*h^8 + 815104*a^
9*b*c^5*d^3*e^12*f^4*h^8 - 651264*a^5*b*c^9*d^11*e^4*f^4*h^8 - 573440*a^6*b
*c^8*d^9*e^6*f^4*h^8 - 262144*a^9*b^3*c^3*d*e^14*f^4*h^8 + 217088*a^7*b*c^7
*d^7*e^8*f^4*h^8 + 211456*a*b^9*c^5*d^11*e^4*f^4*h^8 - 204800*a^4*b*c^10*d^
13*e^2*f^4*h^8 - 172032*a*b^8*c^6*d^12*e^3*f^4*h^8 - 157696*a*b^10*c^4*d^10
*e^5*f^4*h^8 - 131072*a^3*b^2*c^10*d^14*e*f^4*h^8 + 98304*a^8*b^5*c^2*d*e^1
4*f^4*h^8 + 92160*a^2*b^4*c^9*d^14*e*f^4*h^8 + 84992*a*b^7*c^7*d^13*e^2*f^4
*h^8 + 64512*a*b^11*c^3*d^9*e^6*f^4*h^8 + 23552*a^6*b^8*c*d^2*e^13*f^4*h^8
+ 18944*a^3*b^11*c*d^5*e^10*f^4*h^8 - 13312*a^4*b^10*c*d^4*e^11*f^4*h^8 - 9
472*a^5*b^9*c*d^3*e^12*f^4*h^8 - 8192*a*b^12*c^2*d^8*e^7*f^4*h^8 - 6144*a^2
*b^12*c*d^6*e^9*f^4*h^8 - 17920*b^11*c^4*d^11*e^4*f^4*h^8 + 14336*b^12*c^3*
d^10*e^5*f^4*h^8 + 14336*b^10*c^5*d^12*e^3*f^4*h^8 - 7168*b^13*c^2*d^9*e^6*
f^4*h^8 - 7168*b^9*c^6*d^13*e^2*f^4*h^8 - 425984*a^9*c^6*d^4*e^11*f^4*h^8 -
360448*a^8*c^7*d^6*e^9*f^4*h^8 - 262144*a^10*c^5*d^2*e^13*f^4*h^8 - 131072
*a^7*c^8*d^8*e^7*f^4*h^8 + 98304*a^5*c^10*d^12*e^3*f^4*h^8 + 65536*a^6*c^9*
d^10*e^5*f^4*h^8 - 1536*a^5*b^10*d^2*e^13*f^4*h^8 - 1536*a^2*b^13*d^5*e^10*
f^4*h^8 + 768*a^4*b^11*d^3*e^12*f^4*h^8 + 768*a^3*b^12*d^4*e^11*f^4*h^8 + 6
5536*a^10*b^2*c^3*e^15*f^4*h^8 - 24576*a^9*b^4*c^2*e^15*f^4*h^8 - 10240*a^2
*b^3*c^10*d^15*f^4*h^8 + 2048*b^14*c*d^8*e^7*f^4*h^8 + 2048*b^8*c^7*d^14*e*
f^4*h^8 + 32768*a^4*c^11*d^14*e*f^4*h^8 + 1024*a^6*b^9*d*e^14*f^4*h^8 + 102
4*a*b^14*d^6*e^9*f^4*h^8 + 4096*a^8*b^6*c*e^15*f^4*h^8 + 12288*a^3*b*c^11*d
^15*f^4*h^8 + 2816*a*b^5*c^9*d^15*f^4*h^8 - 256*b^15*d^7*e^8*f^4*h^8 - 6553
6*a^11*c^4*e^15*f^4*h^8 - 256*b^7*c^8*d^15*f^4*h^8 - 256*a^7*b^8*e^15*f^4*h
^8 - 896*a*b^8*c^2*d*e^10*f^2*h^4 + 192*a*b*c^9*d^8*e^3*f^2*h^4 + 11520*a^3
*b^3*c^5*d^2*e^9*f^2*h^4 - 5856*a^2*b^5*c^4*d^2*e^9*f^2*h^4 - 5120*a^3*b^2*
c^6*d^3*e^8*f^2*h^4 + 3200*a^2*b^4*c^5*d^3*e^8*f^2*h^4 - 640*a^2*b^3*c^6*d^
4*e^7*f^2*h^4 - 96*a^2*b^2*c^7*d^5*e^6*f^2*h^4 - 10880*a^3*b^4*c^4*d*e^10*f
^2*h^4 + 10240*a^4*b^2*c^5*d*e^10*f^2*h^4 - 7680*a^4*b*c^6*d^2*e^9*f^2*h^4
+ 4672*a^2*b^6*c^3*d*e^10*f^2*h^4 + 1248*a*b^7*c^3*d^2*e^9*f^2*h^4 + 832*a^
3*b*c^7*d^4*e^7*f^2*h^4 - 768*a*b^6*c^4*d^3*e^8*f^2*h^4 + 192*a^2*b*c^8*d^6
*e^5*f^2*h^4 - 192*a*b^2*c^8*d^7*e^4*f^2*h^4 + 176*a*b^5*c^5*d^4*e^7*f^2*h^
4 + 64*a*b^3*c^7*d^6*e^5*f^2*h^4 - 96*b^9*c^2*d^2*e^9*f^2*h^4 - 96*b^2*c^9*
d^9*e^2*f^2*h^4 + 64*b^8*c^3*d^3*e^8*f^2*h^4 + 64*b^3*c^8*d^8*e^3*f^2*h^4 -
16*b^7*c^4*d^4*e^7*f^2*h^4 - 16*b^4*c^7*d^7*e^4*f^2*h^4 + 2032*a^4*c^7*d^3
*e^8*f^2*h^4 - 96*a^2*c^9*d^7*e^4*f^2*h^4 - 64*a^3*c^8*d^5*e^6*f^2*h^4 - 44
80*a^4*b^3*c^4*e^11*f^2*h^4 + 3696*a^3*b^5*c^3*e^11*f^2*h^4 - 1376*a^2*b^7*
c^2*e^11*f^2*h^4 - 2048*a^5*c^6*d*e^10*f^2*h^4 - 64*a*c^10*d^9*e^2*f^2*h^4
+ 1792*a^5*b*c^5*e^11*f^2*h^4 + 64*b^10*c*d*e^10*f^2*h^4 + 64*b*c^10*d^10*e
*f^2*h^4 + 240*a*b^9*c*e^11*f^2*h^4 - 16*c^11*d^11*f^2*h^4 - 16*b^11*e^11*f
^2*h^4 - c^7*e^7, h, k)*(4697620480*a^9*c^11*d^7*e^13*f^55 - 1879048192*a^6
*c^14*d^13*e^7*f^55 - 2818572288*a^7*c^13*d^11*e^9*f^55 - 402653184*a^5*c^1
5*d^15*e^5*f^55 + 5637144576*a^10*c^10*d^5*e^15*f^55 + 2818572288*a^11*c^9*
d^3*e^17*f^55 + 536870912*a^12*c^8*d*e^19*f^55 + 2097152*a*b^7*c^12*d^16*e^
4*f^55 - 16777216*a*b^8*c^11*d^15*e^5*f^55 + 58720256*a*b^9*c^10*d^14*e^6*f
^55 - 117440512*a*b^10*c^9*d^13*e^7*f^55 + 146800640*a*b^11*c^8*d^12*e^8*f^
55 - 117440512*a*b^12*c^7*d^11*e^9*f^55 + 58720256*a*b^13*c^6*d^10*e^10*f^5
5 - 16777216*a*b^14*c^5*d^9*e^11*f^55 + 2097152*a*b^15*c^4*d^8*e^12*f^55 -
134217728*a^4*b*c^15*d^16*e^4*f^55 + 2147483648*a^5*b*c^14*d^14*e^6*f^55 +
10066329600*a^6*b*c^13*d^12*e^8*f^55 + 13421772800*a^7*b*c^12*d^10*e^10*f^5
5 + 671088640*a^8*b*c^11*d^8*e^12*f^55 + 2097152*a^8*b^8*c^4*d*e^19*f^55 -
12884901888*a^9*b*c^10*d^6*e^14*f^55 - 33554432*a^9*b^6*c^5*d*e^19*f^55 - 1
0603200512*a^10*b*c^9*d^4*e^16*f^55 + 201326592*a^10*b^4*c^6*d*e^19*f^55 -
2684354560*a^11*b*c^8*d^2*e^18*f^55 - 536870912*a^11*b^2*c^7*d*e^19*f^55 -
25165824*a^2*b^5*c^13*d^16*e^4*f^55 + 207618048*a^2*b^6*c^12*d^15*e^5*f^55
- 738197504*a^2*b^7*c^11*d^14*e^6*f^55 + 1468006400*a^2*b^8*c^10*d^13*e^7*f
^55 - 1761607680*a^2*b^9*c^9*d^12*e^8*f^55 + 1262485504*a^2*b^10*c^8*d^11*e
^9*f^55 - 469762048*a^2*b^11*c^7*d^10*e^10*f^55 + 25165824*a^2*b^12*c^6*d^9
*e^11*f^55 + 41943040*a^2*b^13*c^5*d^8*e^12*f^55 - 10485760*a^2*b^14*c^4*d^
7*e^13*f^55 + 100663296*a^3*b^3*c^14*d^16*e^4*f^55 - 880803840*a^3*b^4*c^13

*d^15*e^5*f^55 + 3221225472*a^3*b^5*c^12*d^14*e^6*f^55 - 6312427520*a^3*b^6*c^11*d^13*e^7*f^55 + 6889144320*a^3*b^7*c^10*d^12*e^8*f^55 - 3548381184*a^3*b^8*c^9*d^11*e^9*f^55 - 304087040*a^3*b^9*c^8*d^10*e^10*f^55 + 1371537408*a^3*b^10*c^7*d^9*e^11*f^55 - 597688320*a^3*b^11*c^6*d^8*e^12*f^55 + 41943040*a^3*b^12*c^5*d^7*e^13*f^55 + 18874368*a^3*b^13*c^4*d^6*e^14*f^55 + 1375731712*a^4*b^2*c^14*d^15*e^5*f^55 - 5368709120*a^4*b^3*c^13*d^14*e^6*f^55 + 9982443520*a^4*b^4*c^12*d^13*e^7*f^55 - 7507804160*a^4*b^5*c^11*d^12*e^8*f^55 - 3412066304*a^4*b^6*c^10*d^11*e^9*f^55 + 10955522048*a^4*b^7*c^9*d^10*e^10*f^55 - 7748976640*a^4*b^8*c^8*d^9*e^11*f^55 + 1468006400*a^4*b^9*c^7*d^8*e^12*f^55 + 618659840*a^4*b^10*c^6*d^7*e^13*f^55 - 218103808*a^4*b^11*c^5*d^6*e^14*f^55 - 10485760*a^4*b^12*c^4*d^5*e^15*f^55 - 2348810240*a^5*b^2*c^13*d^13*e^7*f^55 - 7549747200*a^5*b^3*c^12*d^12*e^8*f^55 + 24570232832*a^5*b^4*c^11*d^11*e^9*f^55 - 27111981056*a^5*b^5*c^10*d^10*e^10*f^55 + 9638510592*a^5*b^6*c^9*d^9*e^11*f^55 + 4854906880*a^5*b^7*c^8*d^8*e^12*f^55 - 4697620480*a^5*b^8*c^7*d^7*e^13*f^55 + 742391808*a^5*b^9*c^6*d^6*e^14*f^55 + 167772160*a^5*b^10*c^5*d^5*e^15*f^55 - 10485760*a^5*b^11*c^4*d^4*e^16*f^55 - 18824036352*a^6*b^2*c^12*d^11*e^9*f^55 + 9395240960*a^6*b^3*c^11*d^10*e^10*f^55 + 14596177920*a^6*b^4*c^10*d^9*e^11*f^55 - 22825402368*a^6*b^5*c^9*d^8*e^12*f^55 + 10328473600*a^6*b^6*c^8*d^7*e^13*f^55 + 150994944*a^6*b^7*c^7*d^6*e^14*f^55 - 1170210816*a^6*b^8*c^6*d^5*e^15*f^55 + 142606336*a^6*b^9*c^5*d^4*e^16*f^55 + 18874368*a^6*b^10*c^4*d^3*e^17*f^55 - 24830279680*a^7*b^2*c^11*d^9*e^11*f^55 + 20971520000*a^7*b^3*c^10*d^8*e^12*f^55 - 4487905280*a^7*b^4*c^9*d^7*e^13*f^55 - 5972688896*a^7*b^5*c^8*d^6*e^14*f^55 + 4559208448*a^7*b^6*c^7*d^5*e^15*f^55 - 538968064*a^7*b^7*c^6*d^4*e^16*f^55 - 293601280*a^7*b^8*c^5*d^3*e^17*f^55 - 10485760*a^7*b^9*c^4*d^2*e^18*f^55 - 6207569920*a^8*b^2*c^10*d^7*e^13*f^55 + 13690208256*a^8*b^3*c^9*d^6*e^14*f^55 - 9479127040*a^8*b^4*c^8*d^5*e^15*f^55 - 511705088*a^8*b^5*c^7*d^4*e^16*f^55 + 1667235840*a^8*b^6*c^6*d^3*e^17*f^55 + 167772160*a^8*b^7*c^5*d^2*e^18*f^55 + 6241124352*a^9*b^2*c^9*d^5*e^15*f^55 + 6878658560*a^9*b^3*c^8*d^4*e^16*f^55 - 3900702720*a^9*b^4*c^7*d^3*e^17*f^55 - 1006632960*a^9*b^5*c^6*d^2*e^18*f^55 + 2181038080*a^10*b^2*c^8*d^3*e^17*f^55 + 2684354560*a^10*b^3*c^7*d^2*e^18*f^55) + (f*x)^(1/2)*(268435456*a^11*c^8*e^19*f^54 + 1048576*a^7*b^8*c^4*e^19*f^54 - 16777216*a^8*b^6*c^5*e^19*f^54 + 100663296*a^9*b^4*c^6*e^19*f^54 - 268435456*a^10*b^2*c^7*e^19*f^54 - 134217728*a^4*c^15*d^14*e^5*f^54 - 402653184*a^5*c^14*d^12*e^7*f^54 - 268435456*a^6*c^13*d^10*e^9*f^54 + 536870912*a^7*c^12*d^8*e^11*f^54 + 1476395008*a^8*c^11*d^6*e^13*f^54 + 1744830464*a^9*c^10*d^4*e^15*f^54 + 1073741824*a^10*c^9*d^2*e^17*f^54 + 1048576*b^7*c^12*d^15*e^4*f^54 - 8388608*b^8*c^11*d^14*e^5*f^54 + 29360128*b^9*c^10*d^13*e^6*f^54 - 58720256*b^10*c^9*d^12*e^7*f^54 + 73400320*b^11*c^8*d^11*e^8*f^54 - 58720256*b^12*c^7*d^10*e^9*f^54 + 29360128*b^13*c^6*d^9*e^10*f^54 - 8388608*b^14*c^5*d^8*e^11*f^54 + 1048576*b^15*c^4*d^7*e^12*f^54 - 1073741824*a^10*b*c^8*d*e^18*f^54 - 11534336*a*b^5*c^13*d^15*e^4*f^54 + 96468992*a*b^6*c^12*d^14*e^5*f^54 - 348127232*a*b^7*c^11*d^13*e^6*f^54 + 704643072*a*b^8*c^10*d^12*e^7*f^54 - 866123776*a*b^9*c^9*d^11*e^8*f^54 + 645922816*a*b^10*c^8*d^10*e^9*f^54 - 264241152*a*b^11*c^7*d^9*e^10*f^54 + 33554432*a*b^12*c^6*d^8*e^11*f^54 + 13631488*a*b^13*c^5*d^7*e^12*f^54 - 4194304*a*b^14*c^4*d^6*e^13*f^54 - 50331648*a^3*b*c^15*d^15*e^4*f^54 + 838860800*a^4*b*c^14*d^13*e^6*f^54 + 2667577344*a^5*b*c^13*d^11*e^8*f^54 + 2348810240*a^6*b*c^12*d^9*e^10*f^54 - 4194304*a^6*b^9*c^4*d*e^18*f^54 - 889192448*a^7*b*c^11*d^7*e^12*f^54 + 67108864*a^7*b^7*c^5*d*e^18*f^54 - 3724541952*a^8*b*c^10*d^5*e^14*f^54 - 402653184*a^8*b^5*c^6*d*e^18*f^54 - 3338665984*a^9*b*c^9*d^3*e^16*f^54 + 1073741824*a^9*b^3*c^7*d*e^18*f^54 + 41943040*a^2*b^3*c^14*d^15*e^4*f^54 - 377487360*a^2*b^4*c^13*d^14*e^5*f^54 + 1428160512*a^2*b^5*c^12*d^13*e^6*f^54 - 2927624192*a^2*b^6*c^11*d^12*e^7*f^54 + 3435134976*a^2*b^7*c^10*d^11*e^8*f^54 - 2113929216*a^2*b^8*c^9*d^10*e^9*f^54 + 293601280*a^2*b^9*c^8*d^9*e^10*f^54 + 427819008*a^2*b^10*c^7*d^8*e^11*f^54 - 239075328*a^2*b^11*c^6*d^7*e^12*f^54 + 25165824*a^2*b^12*c^5*d^6*e^13*f^54 + 6291456*a^2*b^13*c^4*d^5*e^14*f^54 + 536870912*a^3*b^2*c^14*d^14*e^5*f^54 - 2231369728*a^3*b^3*c^13*d^13*e^6*f^54 + 4605345792*a^3*b^4*c^12*d^12*e^7*f^54 - 4530896896

*a^3*b^5*c^11*d^11*e^8*f^54 + 528482304*a^3*b^6*c^10*d^10*e^9*f^54 + 325897
4208*a^3*b^7*c^9*d^9*e^10*f^54 - 2993684480*a^3*b^8*c^8*d^8*e^11*f^54 + 812
646400*a^3*b^9*c^7*d^7*e^12*f^54 + 144703488*a^3*b^10*c^6*d^6*e^13*f^54 - 7
7594624*a^3*b^11*c^5*d^5*e^14*f^54 - 3145728*a^3*b^12*c^4*d^4*e^15*f^54 - 1
543503872*a^4*b^2*c^13*d^12*e^7*f^54 - 864026624*a^4*b^3*c^12*d^11*e^8*f^54
 + 7029653504*a^4*b^4*c^11*d^10*e^9*f^54 - 9953083392*a^4*b^5*c^10*d^9*e^10
*f^54 + 5167382528*a^4*b^6*c^9*d^8*e^11*f^54 + 592445440*a^4*b^7*c^8*d^7*e^
12*f^54 - 1488977920*a^4*b^8*c^7*d^6*e^13*f^54 + 304087040*a^4*b^9*c^6*d^5*
e^14*f^54 + 54525952*a^4*b^10*c^5*d^4*e^15*f^54 - 3145728*a^4*b^11*c^4*d^3*
e^16*f^54 - 6442450944*a^5*b^2*c^12*d^10*e^9*f^54 + 5872025600*a^5*b^3*c^11
*d^9*e^10*f^54 + 1459617792*a^5*b^4*c^10*d^8*e^11*f^54 - 6489636864*a^5*b^5
*c^9*d^7*e^12*f^54 + 3837788160*a^5*b^6*c^8*d^6*e^13*f^54 - 150994944*a^5*b
^7*c^7*d^5*e^14*f^54 - 396361728*a^5*b^8*c^6*d^4*e^15*f^54 + 38797312*a^5*b
^9*c^5*d^3*e^16*f^54 + 6291456*a^5*b^10*c^4*d^2*e^17*f^54 - 6576668672*a^6*
b^2*c^11*d^8*e^11*f^54 + 7642021888*a^6*b^3*c^10*d^7*e^12*f^54 - 2625634304
*a^6*b^4*c^9*d^6*e^13*f^54 - 1809842176*a^6*b^5*c^8*d^5*e^14*f^54 + 1501560
832*a^6*b^6*c^7*d^4*e^15*f^54 - 111149056*a^6*b^7*c^6*d^3*e^16*f^54 - 96468
992*a^6*b^8*c^5*d^2*e^17*f^54 - 1610612736*a^7*b^2*c^10*d^6*e^13*f^54 + 454
6625536*a^7*b^3*c^9*d^5*e^14*f^54 - 2810183680*a^7*b^4*c^8*d^4*e^15*f^54 -
376438784*a^7*b^5*c^7*d^3*e^16*f^54 + 536870912*a^7*b^6*c^6*d^2*e^17*f^54 +
 1409286144*a^8*b^2*c^9*d^4*e^15*f^54 + 2441084928*a^8*b^3*c^8*d^3*e^16*f^5
4 - 1207959552*a^8*b^4*c^7*d^2*e^17*f^54 + 536870912*a^9*b^2*c^8*d^2*e^17*f
^54)) + 8388608*a^7*c^9*e^16*f^53 - 131072*a^2*b^10*c^4*e^16*f^53 + 1966080
*a^3*b^8*c^5*e^16*f^53 - 11141120*a^4*b^6*c^6*e^16*f^53 + 28835840*a^5*b^4*
c^7*e^16*f^53 - 31457280*a^6*b^2*c^8*e^16*f^53 + 2097152*a^2*c^14*d^10*e^6*
f^53 + 3145728*a^3*c^13*d^8*e^8*f^53 - 14680064*a^4*c^12*d^6*e^10*f^53 - 24
641536*a^5*c^11*d^4*e^12*f^53 - 131072*b^2*c^14*d^12*e^4*f^53 + 655360*b^3*
c^13*d^11*e^5*f^53 - 1310720*b^4*c^12*d^10*e^6*f^53 + 1310720*b^5*c^11*d^9*
e^7*f^53 - 655360*b^6*c^10*d^8*e^8*f^53 + 262144*b^7*c^9*d^7*e^9*f^53 - 655
360*b^8*c^8*d^6*e^10*f^53 + 1310720*b^9*c^7*d^5*e^11*f^53 - 1310720*b^10*c^
6*d^4*e^12*f^53 + 655360*b^11*c^5*d^3*e^13*f^53 - 131072*b^12*c^4*d^2*e^14*
f^53 + 524288*a*c^15*d^12*e^4*f^53 - 2621440*a*b*c^14*d^11*e^5*f^53 + 26214
4*a*b^11*c^4*d*e^15*f^53 + 27262976*a^6*b*c^9*d*e^15*f^53 + 4718592*a*b^2*c
^13*d^10*e^6*f^53 - 3145728*a*b^3*c^12*d^9*e^7*f^53 - 524288*a*b^4*c^11*d^8
*e^8*f^53 + 131072*a*b^5*c^10*d^7*e^9*f^53 + 7208960*a*b^6*c^9*d^6*e^10*f^5
3 - 16252928*a*b^7*c^8*d^5*e^11*f^53 + 16515072*a*b^8*c^7*d^4*e^12*f^53 - 7
733248*a*b^9*c^6*d^3*e^13*f^53 + 917504*a*b^10*c^5*d^2*e^14*f^53 - 8388608*
a^2*b*c^13*d^9*e^7*f^53 - 3538944*a^2*b^9*c^5*d*e^15*f^53 - 15728640*a^3*b*
c^12*d^7*e^9*f^53 + 16908288*a^3*b^7*c^6*d*e^15*f^53 + 60817408*a^4*b*c^11*
d^5*e^11*f^53 - 30801920*a^4*b^5*c^7*d*e^15*f^53 + 98041856*a^5*b*c^10*d^3*
e^13*f^53 + 5242880*a^5*b^3*c^8*d*e^15*f^53 + 11796480*a^2*b^2*c^12*d^8*e^8
*f^53 - 786432*a^2*b^3*c^11*d^7*e^9*f^53 - 31719424*a^2*b^4*c^10*d^6*e^10*f
^53 + 71958528*a^2*b^5*c^9*d^5*e^11*f^53 - 73269248*a^2*b^6*c^8*d^4*e^12*f^
53 + 28835840*a^2*b^7*c^7*d^3*e^13*f^53 + 3145728*a^2*b^8*c^6*d^2*e^14*f^53
 + 57147392*a^3*b^2*c^11*d^6*e^10*f^53 - 126877696*a^3*b^3*c^10*d^5*e^11*f^
53 + 126877696*a^3*b^4*c^9*d^4*e^12*f^53 - 21102592*a^3*b^5*c^8*d^3*e^13*f^
53 - 42336256*a^3*b^6*c^7*d^2*e^14*f^53 - 50462720*a^4*b^2*c^10*d^4*e^12*f^
53 - 74317824*a^4*b^3*c^9*d^3*e^13*f^53 + 120586240*a^4*b^4*c^8*d^2*e^14*f^
53 - 106954752*a^5*b^2*c^9*d^2*e^14*f^53) + (f*x)^(1/2)*(131072*b^11*c^4*e^
15*f^52 + 131072*c^15*d^11*e^4*f^52 + 11272192*a^2*b^7*c^6*e^15*f^52 - 3027
7632*a^3*b^5*c^7*e^15*f^52 + 36700160*a^4*b^3*c^8*e^15*f^52 + 786432*a^2*c^
13*d^7*e^8*f^52 + 524288*a^3*c^12*d^5*e^10*f^52 - 16646144*a^4*c^11*d^3*e^1
2*f^52 + 786432*b^2*c^13*d^9*e^6*f^52 - 524288*b^3*c^12*d^8*e^7*f^52 + 1310
72*b^4*c^11*d^7*e^8*f^52 + 131072*b^7*c^8*d^4*e^11*f^52 - 524288*b^8*c^7*d^
3*e^12*f^52 + 786432*b^9*c^6*d^2*e^13*f^52 - 1966080*a*b^9*c^5*e^15*f^52 -
14680064*a^5*b*c^9*e^15*f^52 + 524288*a*c^14*d^9*e^6*f^52 + 16777216*a^5*c^
10*d*e^14*f^52 - 524288*b*c^14*d^10*e^5*f^52 - 524288*b^10*c^5*d*e^14*f^52
 - 1572864*a*b*c^13*d^8*e^7*f^52 + 7340032*a*b^8*c^6*d*e^14*f^52 + 1572864*a
*b^2*c^12*d^7*e^8*f^52 - 524288*a*b^3*c^11*d^6*e^9*f^52 - 1441792*a*b^5*c^9

*d^4*e^11*f^52 + 6291456*a*b^6*c^8*d^3*e^12*f^52 - 10223616*a*b^7*c^7*d^2*e
^13*f^52 - 1572864*a^2*b*c^12*d^6*e^9*f^52 - 38273024*a^2*b^6*c^7*d*e^14*f^
52 - 6815744*a^3*b*c^11*d^4*e^11*f^52 + 89128960*a^3*b^4*c^8*d*e^14*f^52 +
62914560*a^4*b*c^10*d^2*e^13*f^52 - 83886080*a^4*b^2*c^9*d*e^14*f^52 + 7864
32*a^2*b^2*c^11*d^5*e^10*f^52 + 5242880*a^2*b^3*c^10*d^4*e^11*f^52 - 262144
00*a^2*b^4*c^9*d^3*e^12*f^52 + 47972352*a^2*b^5*c^8*d^2*e^13*f^52 + 4194304
0*a^3*b^2*c^10*d^3*e^12*f^52 - 94371840*a^3*b^3*c^9*d^2*e^13*f^52)) + 8192*
b^3*c^9*e^12*f^51 + 8192*c^12*d^3*e^9*f^51 - 32768*a*b*c^10*e^12*f^51 + 409
60*a*c^11*d*e^11*f^51 - 8192*b*c^11*d^2*e^10*f^51 - 8192*b^2*c^10*d*e^11*f^
51) + 12288*c^11*e^11*f^50*(f*x)^(1/2)))*root(8388608*a^7*b*c^11*d^18*e*f^6
*h^12 - 513802240*a^10*b^2*c^7*d^11*e^8*f^6*h^12 - 381681664*a^11*b^2*c^6*d
^9*e^10*f^6*h^12 - 381681664*a^9*b^2*c^8*d^13*e^6*f^6*h^12 - 300941312*a^9*
b^5*c^5*d^10*e^9*f^6*h^12 - 300941312*a^8*b^5*c^6*d^12*e^7*f^6*h^12 + 29360
1280*a^10*b^3*c^6*d^10*e^9*f^6*h^12 + 293601280*a^9*b^3*c^7*d^12*e^7*f^6*h^
12 - 168820736*a^10*b^5*c^4*d^8*e^11*f^6*h^12 - 168820736*a^7*b^5*c^7*d^14*
e^5*f^6*h^12 + 166068224*a^8*b^6*c^5*d^11*e^8*f^6*h^12 - 146800640*a^12*b^2
*c^5*d^7*e^12*f^6*h^12 - 146800640*a^8*b^2*c^9*d^15*e^4*f^6*h^12 + 12478054
4*a^10*b^4*c^5*d^9*e^10*f^6*h^12 + 124780544*a^8*b^4*c^7*d^13*e^6*f^6*h^12
+ 119275520*a^9*b^4*c^6*d^11*e^8*f^6*h^12 + 117440512*a^11*b^3*c^5*d^8*e^11
*f^6*h^12 + 117440512*a^8*b^3*c^8*d^14*e^5*f^6*h^12 + 102760448*a^9*b^6*c^4
*d^9*e^10*f^6*h^12 + 102760448*a^7*b^6*c^6*d^13*e^6*f^6*h^12 + 91750400*a^1
1*b^4*c^4*d^7*e^12*f^6*h^12 + 91750400*a^7*b^4*c^8*d^15*e^4*f^6*h^12 - 7106
5600*a^7*b^8*c^4*d^11*e^8*f^6*h^12 - 53444608*a^8*b^8*c^3*d^9*e^10*f^6*h^12
 - 53444608*a^6*b^8*c^5*d^13*e^6*f^6*h^12 + 40370176*a^9*b^7*c^3*d^8*e^11*f
^6*h^12 + 40370176*a^6*b^7*c^6*d^14*e^5*f^6*h^12 - 36700160*a^11*b^5*c^3*d^
6*e^13*f^6*h^12 - 36700160*a^6*b^5*c^8*d^16*e^3*f^6*h^12 + 34078720*a^8*b^7
*c^4*d^10*e^9*f^6*h^12 + 34078720*a^7*b^7*c^5*d^12*e^7*f^6*h^12 + 26214400*
a^12*b^4*c^3*d^5*e^14*f^6*h^12 + 26214400*a^6*b^4*c^9*d^17*e^2*f^6*h^12 + 2
2118400*a^7*b^9*c^3*d^10*e^9*f^6*h^12 + 22118400*a^6*b^9*c^4*d^12*e^7*f^6*h
^12 - 20971520*a^13*b^2*c^4*d^5*e^14*f^6*h^12 - 20971520*a^7*b^2*c^10*d^17*
e^2*f^6*h^12 + 18350080*a^10*b^7*c^2*d^6*e^13*f^6*h^12 + 18350080*a^5*b^7*c
^7*d^16*e^3*f^6*h^12 - 16629760*a^9*b^8*c^2*d^7*e^12*f^6*h^12 - 16629760*a^
5*b^8*c^6*d^15*e^4*f^6*h^12 - 10485760*a^11*b^6*c^2*d^5*e^14*f^6*h^12 - 104
85760*a^5*b^6*c^8*d^17*e^2*f^6*h^12 + 9175040*a^10*b^6*c^3*d^7*e^12*f^6*h^1
2 + 9175040*a^6*b^6*c^7*d^15*e^4*f^6*h^12 - 8388608*a^13*b^3*c^3*d^4*e^15*f
^6*h^12 + 5619712*a^7*b^10*c^2*d^9*e^10*f^6*h^12 + 5619712*a^5*b^10*c^4*d^1
3*e^6*f^6*h^12 - 5570560*a^6*b^11*c^2*d^10*e^9*f^6*h^12 - 5570560*a^5*b^11*
c^3*d^12*e^7*f^6*h^12 + 4358144*a^8*b^9*c^2*d^8*e^11*f^6*h^12 + 4358144*a^5
*b^9*c^5*d^14*e^5*f^6*h^12 + 4259840*a^6*b^10*c^3*d^11*e^8*f^6*h^12 + 38993
92*a^4*b^10*c^5*d^15*e^4*f^6*h^12 - 3440640*a^4*b^9*c^6*d^16*e^3*f^6*h^12 +
 3145728*a^12*b^5*c^2*d^4*e^15*f^6*h^12 - 2523136*a^4*b^11*c^4*d^14*e^5*f^6
*h^12 + 1802240*a^4*b^8*c^7*d^17*e^2*f^6*h^12 + 1556480*a^5*b^12*c^2*d^11*e
^8*f^6*h^12 + 1048576*a^14*b^2*c^3*d^3*e^16*f^6*h^12 + 688128*a^4*b^12*c^3*
d^13*e^6*f^6*h^12 - 393216*a^13*b^4*c^2*d^3*e^16*f^6*h^12 - 286720*a^3*b^12
*c^4*d^15*e^4*f^6*h^12 + 229376*a^3*b^13*c^3*d^14*e^5*f^6*h^12 + 229376*a^3
*b^11*c^5*d^16*e^3*f^6*h^12 + 163840*a^4*b^13*c^2*d^12*e^7*f^6*h^12 - 11468
8*a^3*b^14*c^2*d^13*e^6*f^6*h^12 - 114688*a^3*b^10*c^6*d^17*e^2*f^6*h^12 +
293601280*a^11*b*c^7*d^10*e^9*f^6*h^12 + 293601280*a^10*b*c^8*d^12*e^7*f^6*
h^12 + 176160768*a^12*b*c^6*d^8*e^11*f^6*h^12 + 176160768*a^9*b*c^9*d^14*e^
5*f^6*h^12 + 58720256*a^13*b*c^5*d^6*e^13*f^6*h^12 + 58720256*a^8*b*c^10*d^
16*e^3*f^6*h^12 + 8388608*a^14*b*c^4*d^4*e^15*f^6*h^12 - 8388608*a^6*b^3*c^
10*d^18*e*f^6*h^12 + 3899392*a^8*b^10*c*d^7*e^12*f^6*h^12 - 3440640*a^9*b^9
*c*d^6*e^13*f^6*h^12 + 3145728*a^5*b^5*c^9*d^18*e*f^6*h^12 - 2523136*a^7*b^
11*c*d^8*e^11*f^6*h^12 + 1802240*a^10*b^8*c*d^5*e^14*f^6*h^12 + 688128*a^6*
b^12*c*d^9*e^10*f^6*h^12 - 524288*a^11*b^7*c*d^4*e^15*f^6*h^12 - 524288*a^4
*b^7*c^8*d^18*e*f^6*h^12 + 163840*a^5*b^13*c*d^10*e^9*f^6*h^12 - 163840*a^4
*b^14*c*d^11*e^8*f^6*h^12 + 65536*a^12*b^6*c*d^3*e^16*f^6*h^12 + 32768*a^3*
b^15*c*d^12*e^7*f^6*h^12 + 32768*a^3*b^9*c^7*d^18*e*f^6*h^12 - 73400320*a^1
1*c^8*d^11*e^8*f^6*h^12 - 58720256*a^12*c^7*d^9*e^10*f^6*h^12 - 58720256*a^

10*c^9*d^13*e^6*f^6*h^12 - 29360128*a^13*c^6*d^7*e^12*f^6*h^12 - 29360128*a
^9*c^10*d^15*e^4*f^6*h^12 - 8388608*a^14*c^5*d^5*e^14*f^6*h^12 - 8388608*a^
8*c^11*d^17*e^2*f^6*h^12 - 1048576*a^15*c^4*d^3*e^16*f^6*h^12 - 286720*a^7*
b^12*d^7*e^12*f^6*h^12 + 229376*a^8*b^11*d^6*e^13*f^6*h^12 + 229376*a^6*b^1
3*d^8*e^11*f^6*h^12 - 114688*a^9*b^10*d^5*e^14*f^6*h^12 - 114688*a^5*b^14*d
^9*e^10*f^6*h^12 + 32768*a^10*b^9*d^4*e^15*f^6*h^12 + 32768*a^4*b^15*d^10*e
^9*f^6*h^12 - 4096*a^11*b^8*d^3*e^16*f^6*h^12 - 4096*a^3*b^16*d^11*e^8*f^6*
h^12 + 1048576*a^6*b^2*c^11*d^19*f^6*h^12 - 393216*a^5*b^4*c^10*d^19*f^6*h^
12 + 65536*a^4*b^6*c^9*d^19*f^6*h^12 - 4096*a^3*b^8*c^8*d^19*f^6*h^12 - 104
8576*a^7*c^12*d^19*f^6*h^12 + 262144*a^10*b*c^4*d*e^14*f^4*h^8 - 23552*a*b^
6*c^8*d^14*e*f^4*h^8 - 16384*a^7*b^7*c*d*e^14*f^4*h^8 - 3328*a*b^13*c*d^7*e
^8*f^4*h^8 + 2429952*a^4*b^5*c^6*d^9*e^6*f^4*h^8 - 1865728*a^6*b^3*c^6*d^7*
e^8*f^4*h^8 - 1716224*a^4*b^4*c^7*d^10*e^5*f^4*h^8 + 1605632*a^6*b^2*c^7*d^
8*e^7*f^4*h^8 + 1584384*a^5*b^5*c^5*d^7*e^8*f^4*h^8 + 1572864*a^5*b^2*c^8*d
^10*e^5*f^4*h^8 - 1433600*a^5*b^3*c^7*d^9*e^6*f^4*h^8 - 1261568*a^4*b^6*c^5
*d^8*e^7*f^4*h^8 - 1124352*a^3*b^4*c^8*d^12*e^3*f^4*h^8 - 1110016*a^7*b^3*c
^5*d^5*e^10*f^4*h^8 + 1106176*a^3*b^5*c^7*d^11*e^4*f^4*h^8 - 936960*a^5*b^6
*c^4*d^6*e^9*f^4*h^8 - 838656*a^2*b^7*c^6*d^11*e^4*f^4*h^8 - 795648*a^3*b^7
*c^5*d^9*e^6*f^4*h^8 + 730880*a^3*b^8*c^4*d^8*e^7*f^4*h^8 + 714752*a^2*b^6*
c^7*d^12*e^3*f^4*h^8 + 686080*a^7*b^4*c^4*d^4*e^11*f^4*h^8 + 641024*a^6*b^4
*c^5*d^6*e^9*f^4*h^8 - 595968*a^8*b^3*c^4*d^3*e^12*f^4*h^8 + 544768*a^3*b^3
*c^9*d^13*e^2*f^4*h^8 + 516096*a^2*b^8*c^5*d^10*e^5*f^4*h^8 + 441856*a^6*b^
5*c^4*d^5*e^10*f^4*h^8 + 393216*a^7*b^2*c^6*d^6*e^9*f^4*h^8 + 376832*a^4*b^
2*c^9*d^12*e^3*f^4*h^8 - 366592*a^6*b^6*c^3*d^4*e^11*f^4*h^8 + 363520*a^4*b
^8*c^3*d^6*e^9*f^4*h^8 - 356352*a^5*b^4*c^6*d^8*e^7*f^4*h^8 - 348672*a^2*b^
5*c^8*d^13*e^2*f^4*h^8 - 344064*a^8*b^2*c^5*d^4*e^11*f^4*h^8 + 294912*a^8*b
^4*c^3*d^2*e^13*f^4*h^8 + 210944*a^4*b^3*c^8*d^11*e^4*f^4*h^8 - 198400*a^3*
b^9*c^3*d^7*e^8*f^4*h^8 - 144640*a^4*b^7*c^4*d^7*e^8*f^4*h^8 - 131072*a^9*b
^2*c^4*d^2*e^13*f^4*h^8 - 131072*a^7*b^6*c^2*d^2*e^13*f^4*h^8 - 129024*a^3*
b^6*c^6*d^10*e^5*f^4*h^8 - 104448*a^2*b^10*c^3*d^8*e^7*f^4*h^8 + 96768*a^5*
b^8*c^2*d^4*e^11*f^4*h^8 + 91904*a^7*b^5*c^3*d^3*e^12*f^4*h^8 - 74240*a^4*b
^9*c^2*d^5*e^10*f^4*h^8 - 71680*a^2*b^9*c^4*d^9*e^6*f^4*h^8 + 58368*a^2*b^1
1*c^2*d^7*e^8*f^4*h^8 + 36864*a^5*b^7*c^3*d^5*e^10*f^4*h^8 - 35328*a^3*b^10
*c^2*d^6*e^9*f^4*h^8 + 27136*a^6*b^7*c^2*d^3*e^12*f^4*h^8 + 909312*a^8*b*c^
6*d^5*e^10*f^4*h^8 + 815104*a^9*b*c^5*d^3*e^12*f^4*h^8 - 651264*a^5*b*c^9*d
^11*e^4*f^4*h^8 - 573440*a^6*b*c^8*d^9*e^6*f^4*h^8 - 262144*a^9*b^3*c^3*d*e
^14*f^4*h^8 + 217088*a^7*b*c^7*d^7*e^8*f^4*h^8 + 211456*a*b^9*c^5*d^11*e^4*
f^4*h^8 - 204800*a^4*b*c^10*d^13*e^2*f^4*h^8 - 172032*a*b^8*c^6*d^12*e^3*f^
4*h^8 - 157696*a*b^10*c^4*d^10*e^5*f^4*h^8 - 131072*a^3*b^2*c^10*d^14*e*f^4
*h^8 + 98304*a^8*b^5*c^2*d*e^14*f^4*h^8 + 92160*a^2*b^4*c^9*d^14*e*f^4*h^8
+ 84992*a*b^7*c^7*d^13*e^2*f^4*h^8 + 64512*a*b^11*c^3*d^9*e^6*f^4*h^8 + 235
52*a^6*b^8*c*d^2*e^13*f^4*h^8 + 18944*a^3*b^11*c*d^5*e^10*f^4*h^8 - 13312*a
^4*b^10*c*d^4*e^11*f^4*h^8 - 9472*a^5*b^9*c*d^3*e^12*f^4*h^8 - 8192*a*b^12*
c^2*d^8*e^7*f^4*h^8 - 6144*a^2*b^12*c*d^6*e^9*f^4*h^8 - 17920*b^11*c^4*d^11
*e^4*f^4*h^8 + 14336*b^12*c^3*d^10*e^5*f^4*h^8 + 14336*b^10*c^5*d^12*e^3*f^
4*h^8 - 7168*b^13*c^2*d^9*e^6*f^4*h^8 - 7168*b^9*c^6*d^13*e^2*f^4*h^8 - 425
984*a^9*c^6*d^4*e^11*f^4*h^8 - 360448*a^8*c^7*d^6*e^9*f^4*h^8 - 262144*a^10
*c^5*d^2*e^13*f^4*h^8 - 131072*a^7*c^8*d^8*e^7*f^4*h^8 + 98304*a^5*c^10*d^1
2*e^3*f^4*h^8 + 65536*a^6*c^9*d^10*e^5*f^4*h^8 - 1536*a^5*b^10*d^2*e^13*f^4
*h^8 - 1536*a^2*b^13*d^5*e^10*f^4*h^8 + 768*a^4*b^11*d^3*e^12*f^4*h^8 + 768
*a^3*b^12*d^4*e^11*f^4*h^8 + 65536*a^10*b^2*c^3*e^15*f^4*h^8 - 24576*a^9*b^
4*c^2*e^15*f^4*h^8 - 10240*a^2*b^3*c^10*d^15*f^4*h^8 + 2048*b^14*c*d^8*e^7*
f^4*h^8 + 2048*b^8*c^7*d^14*e*f^4*h^8 + 32768*a^4*c^11*d^14*e*f^4*h^8 + 102
4*a^6*b^9*d*e^14*f^4*h^8 + 1024*a*b^14*d^6*e^9*f^4*h^8 + 4096*a^8*b^6*c*e^1
5*f^4*h^8 + 12288*a^3*b*c^11*d^15*f^4*h^8 + 2816*a*b^5*c^9*d^15*f^4*h^8 - 2
56*b^15*d^7*e^8*f^4*h^8 - 65536*a^11*c^4*e^15*f^4*h^8 - 256*b^7*c^8*d^15*f^
4*h^8 - 256*a^7*b^8*e^15*f^4*h^8 - 896*a*b^8*c^2*d*e^10*f^2*h^4 + 192*a*b*c
^9*d^8*e^3*f^2*h^4 + 11520*a^3*b^3*c^5*d^2*e^9*f^2*h^4 - 5856*a^2*b^5*c^4*d
^2*e^9*f^2*h^4 - 5120*a^3*b^2*c^6*d^3*e^8*f^2*h^4 + 3200*a^2*b^4*c^5*d^3*e^

```
8*f^2*h^4 - 640*a^2*b^3*c^6*d^4*e^7*f^2*h^4 - 96*a^2*b^2*c^7*d^5*e^6*f^2*h^
4 - 10880*a^3*b^4*c^4*d*e^10*f^2*h^4 + 10240*a^4*b^2*c^5*d*e^10*f^2*h^4 - 7
680*a^4*b*c^6*d^2*e^9*f^2*h^4 + 4672*a^2*b^6*c^3*d*e^10*f^2*h^4 + 1248*a*b^
7*c^3*d^2*e^9*f^2*h^4 + 832*a^3*b*c^7*d^4*e^7*f^2*h^4 - 768*a*b^6*c^4*d^3*e
^8*f^2*h^4 + 192*a^2*b*c^8*d^6*e^5*f^2*h^4 - 192*a*b^2*c^8*d^7*e^4*f^2*h^4
+ 176*a*b^5*c^5*d^4*e^7*f^2*h^4 + 64*a*b^3*c^7*d^6*e^5*f^2*h^4 - 96*b^9*c^2
*d^2*e^9*f^2*h^4 - 96*b^2*c^9*d^9*e^2*f^2*h^4 + 64*b^8*c^3*d^3*e^8*f^2*h^4
+ 64*b^3*c^8*d^8*e^3*f^2*h^4 - 16*b^7*c^4*d^4*e^7*f^2*h^4 - 16*b^4*c^7*d^7*
e^4*f^2*h^4 + 2032*a^4*c^7*d^3*e^8*f^2*h^4 - 96*a^2*c^9*d^7*e^4*f^2*h^4 - 6
4*a^3*c^8*d^5*e^6*f^2*h^4 - 4480*a^4*b^3*c^4*e^11*f^2*h^4 + 3696*a^3*b^5*c^
3*e^11*f^2*h^4 - 1376*a^2*b^7*c^2*e^11*f^2*h^4 - 2048*a^5*c^6*d*e^10*f^2*h^
4 - 64*a*c^10*d^9*e^2*f^2*h^4 + 1792*a^5*b*c^5*e^11*f^2*h^4 + 64*b^10*c*d*e
^10*f^2*h^4 + 64*b*c^10*d^10*e*f^2*h^4 + 240*a*b^9*c*e^11*f^2*h^4 - 16*c^11
*d^11*f^2*h^4 - 16*b^11*e^11*f^2*h^4 - c^7*e^7, h, k), k, 1, 12)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{fx}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)/(f*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(f*x)*(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.233 $\int \frac{x^5 \sqrt{a+bx^2+cx^4}}{d+ex^2} \, dx$

Optimal. Leaf size=272

$$-\frac{\left(-8c^2de(bd-ae)-2bce^2(bd-2ae)-b^3e^3+16c^3d^3\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}e^4}+\frac{\sqrt{a+bx^2+cx^4}\left((2cd-be)(be+\right.}{16c^2e^3}$$

Rubi [A] time = 0.57, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1251, 1653, 814, 843, 621, 206, 724}

$$-\frac{\left(-8c^2de(bd-ae)-2bce^2(bd-2ae)-b^3e^3+16c^3d^3\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}e^4}+\frac{\sqrt{a+bx^2+cx^4}\left((2cd-be)(be+4cd)-2cex^2(be+2cd)\right)}{16c^2e^3}+\frac{d^2\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^4}+\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] (((2*c*d - b*e)*(4*c*d + b*e) - 2*c*e*(2*c*d + b*e)*x^2)*Sqrt[a + b*x^2 + c
*x^4])/(16*c^2*e^3) + (a + b*x^2 + c*x^4)^(3/2)/(6*c*e) - ((16*c^3*d^3 - b^
3*e^3 - 2*b*c*e^2*(b*d - 2*a*e) - 8*c^2*d*e*(b*d - a*e))*ArcTanh[(b + 2*c*x
^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(5/2)*e^4) + (d^2*Sqrt[c*d^
2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2
- b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)$$

$$= \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce} + \frac{\text{Subst}\left(\int \frac{\left(-\frac{3}{2}bde-\frac{3}{2}e(2cd+be)x\right)\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)}{6ce^2}$$

$$= \frac{\left((2cd-be)(4cd+be)-2ce(2cd+be)x^2\right)\sqrt{a+bx^2+cx^4}}{16c^2e^3} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce} -$$

$$= \frac{\left((2cd-be)(4cd+be)-2ce(2cd+be)x^2\right)\sqrt{a+bx^2+cx^4}}{16c^2e^3} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce} +$$

$$= \frac{\left((2cd-be)(4cd+be)-2ce(2cd+be)x^2\right)\sqrt{a+bx^2+cx^4}}{16c^2e^3} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce} -$$

$$= \frac{\left((2cd-be)(4cd+be)-2ce(2cd+be)x^2\right)\sqrt{a+bx^2+cx^4}}{16c^2e^3} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce} -$$

Mathematica [A] time = 0.43, size = 267, normalized size = 0.98

$$\frac{2\sqrt{c}\left(e\sqrt{a+bx^2+cx^4}\left(2ce\left(4ae-3bd+bex^2\right)-3b^2e^2+4c^2\left(6d^2-3dex^2+2e^2x^4\right)\right)+24c^2d^2\sqrt{e(ae-bd)+cd^2}\tanh^{-1}\left(\frac{-2ae+b\left(d-ex^2\right)+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)\right)-3\left(8c^2de(ae-bd)-2bce^2(bd-2ae)-b^3e^3+16c^3d^3\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{96c^{5/2}e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

[Out] (-3*(16*c^3*d^3 - b^3*e^3 - 2*b*c*e^2*(b*d - 2*a*e) + 8*c^2*d*e*(-(b*d) + a
*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])] + 2*Sqrt[c]
*(e*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2*e^2 + 2*c*e*(-3*b*d + 4*a*e + b*e*x^2)
+ 4*c^2*(6*d^2 - 3*d*e*x^2 + 2*e^2*x^4)) + 24*c^2*d^2*Sqrt[c*d^2 + e*(-(b*d
) + a*e)]*ArcTanh[(-2*a*e + 2*c*d*x^2 + b*(d - e*x^2))/(2*Sqrt[c*d^2 + e*(-
(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4])]))/(96*c^(5/2)*e^4)

IntegrateAlgebraic [A] time = 1.33, size = 316, normalized size = 1.16

$$\frac{\sqrt{a+bx^2+cx^4}\left(8ace^2-3b^2e^2-6bcde+2bce^2x^2+24c^2d^2-12c^2dex^2+8c^2e^2x^4\right)}{48c^2e^3}+\frac{\left(4abce^3+8ac^2de^2-b^3e^3-2b^2cde^2-8bc^2d^2e+16c^3d^3\right)\log\left(-2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{32c^{5/2}e^4}+\frac{d^2\sqrt{-ae^2+bde-cd^2}\tan^{-1}\left(\frac{\sqrt{c}ex^2}{\sqrt{-ae^2+bde-cd^2}}-\frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}d}{\sqrt{-ae^2+bde-cd^2}}\right)}{e^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]

[Out] (Sqrt[a + b*x^2 + c*x^4]*(24*c^2*d^2 - 6*b*c*d*e - 3*b^2*e^2 + 8*a*c*e^2 -
12*c^2*d*e*x^2 + 2*b*c*e^2*x^2 + 8*c^2*e^2*x^4))/(48*c^2*e^3) + (d^2*Sqrt[-
(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2]
+ (Sqrt[c]*e*x^2)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x^2 + c*x^
4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/e^4 + ((16*c^3*d^3 - 8*b*c^2*d^2*e - 2
*b^2*c*d*e^2 + 8*a*c^2*d*e^2 - b^3*e^3 + 4*a*b*c*e^3)*Log[b + 2*c*x^2 - 2*S
qrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(32*c^(5/2)*e^4)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.04, size = 1049, normalized size = 3.86

$$-\frac{a d^2\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2ae^2-2deb+2cd^2}{e^2}+2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\,e^3}-\frac{\sqrt{cx^4+bx^2+a}\,bx^2}{8ce}+\frac{b d^3\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2ae^2-2deb+2cd^2}{e^2}+2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\,e^4}-\frac{c d^4\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2ae^2-2deb+2cd^2}{e^2}+2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\,e^5}-\frac{\sqrt{cx^4+bx^2+a}\,dx^2}{4e^2}-\frac{ab\ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{8c^{\frac{3}{2}}e}-\frac{ad\ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{4\sqrt{c}\,e^2}+\frac{b^3\ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{32c^{\frac{5}{2}}e}+\frac{b^2d\ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{16c^{\frac{3}{2}}e^2}+\frac{bd^2\ln\left(\frac{\left(x^2+\frac{d}{e}\right)c+\frac{be-2cd}{2e}}{\sqrt{c}}+\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}\right)}{4\sqrt{c}\,e^3}-\frac{\sqrt{c}\,d^3\ln\left(\frac{\left(x^2+\frac{d}{e}\right)c+\frac{be-2cd}{2e}}{\sqrt{c}}+\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}\right)}{2e^4}-\frac{\sqrt{cx^4+bx^2+a}\,b^2}{16c^2e}-\frac{\sqrt{cx^4+bx^2+a}\,bd}{8ce^2}+\frac{\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}\,d^2}{2e^3}+\frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{6ce}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x)

[Out] 1/6*(c*x^4+b*x^2+a)^(3/2)/c/e-1/8/e*b/c*x^2*(c*x^4+b*x^2+a)^(1/2)-1/16/e*b^
2/c^2*(c*x^4+b*x^2+a)^(1/2)-1/8/e*b/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4
+b*x^2+a)^(1/2))*a+1/32/e*b^3/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2
+a)^(1/2))-1/4/e^2*d*(c*x^4+b*x^2+a)^(1/2)*x^2-1/8/e^2*d/c*(c*x^4+b*x^2+a)^
(1/2)*b-1/4/e^2*d/c^(1/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a
+1/16/e^2*d/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*b^2+1/2
d^2/e^3(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)+1/4*d^2/e^3*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+(c*(x^2+d/e)^2+(
b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b-1/2*d^3/e^
4*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(
x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)-1/2*d^2/e^3/((a*e^2-b*d*e+

```
c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^
2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a+1/2*d^3/e^4/((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c
*d^2)/e^2)^(1/2))/(x^2+d/e))*b-1/2*d^4/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))/(x^2+d/e))*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for mo
re details)Is b*e-2*c*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\sqrt{c\,x^4+b\,x^2+a}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2),x)
```

```
[Out] int((x^5*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
```

```
[Out] Integral(x**5*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)
```

3.234 $\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$

Optimal. Leaf size=208

$$\frac{\left(-4ce(bd-ae)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}e^3}-\frac{d\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^3}-\sqrt{}$$

Rubi [A] time = 0.31, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1251, 814, 843, 621, 206, 724}

$$\frac{\left(-4ce(bd-ae)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}e^3}-\frac{d\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^3}-\frac{\sqrt{a+bx^2+cx^4}\left(-be+4cd-2cex^2\right)}{8ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] -((4*c*d - b*e - 2*c*e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c*e^2) + ((8*c^2*d^
2 - b^2*e^2 - 4*c*e*(b*d - a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
b*x^2 + c*x^4])])/(16*c^(3/2)*e^3) - (d*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh
[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a +
b*x^2 + c*x^4])])/(2*e^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)$$

$$= -\frac{\left(4cd-be-2cex^2\right)\sqrt{a+bx^2+cx^4}}{8ce^2} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}d\left(4bcd-b^2e-4ace\right)-\frac{1}{2}\left(8c^2d^2-b^2e^2-4ce(bd\right.}{(d+ex)\sqrt{a+bx+cx^2}}\right.}{8ce^2}$$

$$= -\frac{\left(4cd-be-2cex^2\right)\sqrt{a+bx^2+cx^4}}{8ce^2} - \frac{\left(d\left(cd^2-bde+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+c}}\right.}{2e^3}$$

$$= -\frac{\left(4cd-be-2cex^2\right)\sqrt{a+bx^2+cx^4}}{8ce^2} + \frac{\left(d\left(cd^2-bde+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae}\right.}{e^3}$$

$$= -\frac{\left(4cd-be-2cex^2\right)\sqrt{a+bx^2+cx^4}}{8ce^2} + \frac{\left(8c^2d^2-b^2e^2-4ce(bd-ae)\right)\tanh^{-1}\left(\frac{b}{2\sqrt{c}\sqrt{}}\right.}{16c^{3/2}e^3}$$

Mathematica [A] time = 0.26, size = 205, normalized size = 0.99

$$\frac{\left(4ce(ae-bd)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)+2\sqrt{c}\left(4cd\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)+e\sqrt{a+bx^2+cx^4}\left(be-4cd+2cex^2\right)\right)}{16c^{3/2}e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] ((8*c^2*d^2 - b^2*e^2 + 4*c*e*(-(b*d) + a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt
[c]*Sqrt[a + b*x^2 + c*x^4])] + 2*Sqrt[c]*(e*(-4*c*d + b*e + 2*c*e*x^2)*Sqr
t[a + b*x^2 + c*x^4] + 4*c*d*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(-(b*d) +
2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2
+ c*x^4])]))/(16*c^(3/2)*e^3)
```

IntegrateAlgebraic [A] time = 0.84, size = 247, normalized size = 1.19

$$\frac{\left(-4ace^2+b^2e^2+4bcde-8c^2d^2\right)\log\left(-2c^{3/2}\sqrt{a+bx^2+cx^4}+bc+2c^2x^2\right)}{16c^{3/2}e^3}-\frac{d\sqrt{-ae^2+bde-cd^2}\tan^{-1}\left(\frac{\sqrt{c}ex^2}{\sqrt{-ae^2+bde-cd^2}}-\frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}d}{\sqrt{-ae^2+bde-cd^2}}\right)}{e^3}+\frac{\sqrt{a+bx^2+cx^4}\left(be-4cd+2cex^2\right)}{8ce^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] ((-4*c*d + b*e + 2*c*e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c*e^2) - (d*Sqrt[-(
c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] +
 (Sqrt[c]*e*x^2)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x^2 + c*x^4
```

```
])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/e^3 + ((-8*c^2*d^2 + 4*b*c*d*e + b^2*e^
2 - 4*a*c*e^2)*Log[b*c + 2*c^2*x^2 - 2*c^(3/2)*Sqrt[a + b*x^2 + c*x^4]])/(1
6*c^(3/2)*e^3)
```

fricas [A] time = 56.15, size = 1231, normalized size = 5.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/32*(8*sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d*log(-((8*c^2*d^2 - 8*b*c*d*e + (
b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*
c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sq
rt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d
*e*x^2 + d^2)) + (8*c^2*d^2 - 4*b*c*d*e - (b^2 - 4*a*c)*e^2)*sqrt(c)*log(-8
*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c
) - 4*a*c) + 4*(2*c^2*e^2*x^2 - 4*c^2*d*e + b*c*e^2)*sqrt(c*x^4 + b*x^2 + a
))/(c^2*e^3), -1/32*(16*sqrt(-c*d^2 + b*d*e - a*e^2)*c^2*d*arctan(-1/2*sqrt
(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d -
 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 +
(b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (8*c^2*d^2 - 4*b*c*d*e - (b^2 - 4*a*c
)*e^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)
*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*(2*c^2*e^2*x^2 - 4*c^2*d*e + b*c*e^2)*s
qrt(c*x^4 + b*x^2 + a))/(c^2*e^3), 1/16*(4*sqrt(c*d^2 - b*d*e + a*e^2)*c^2*
d*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2
*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*
x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*
x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - (8*c^2*d^2 - 4*b*c*d*e -
 (b^2 - 4*a*c)*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 +
b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(2*c^2*e^2*x^2 - 4*c^2*d*e + b*c
*e^2)*sqrt(c*x^4 + b*x^2 + a))/(c^2*e^3), -1/16*(8*sqrt(-c*d^2 + b*d*e - a*
e^2)*c^2*d*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)
*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c
*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) + (8*c^2*d^2
 - 4*b*c*d*e - (b^2 - 4*a*c)*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 +
a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*(2*c^2*e^2*x^2 - 4
*c^2*d*e + b*c*e^2)*sqrt(c*x^4 + b*x^2 + a))/(c^2*e^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 887, normalized size = 4.26

$$\frac{a d \ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2ae^2-2deb+2cd^2}{e^2}+2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\, e^2}-\frac{b d^2 \ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2ae^2-2deb+2cd^2}{e^2}+2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\, e^3}+\frac{c d^3 \ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2ae^2-2deb+2cd^2}{e^2}+2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\, e^4}+\frac{\sqrt{c x^4+b x^2+a}\, x^2}{4e}+\frac{a \ln\left(\frac{c x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{4\sqrt{c}\, e}-\frac{b^2 \ln\left(\frac{c x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{16 c^{\frac{3}{2}} e}-\frac{b d \ln\left(\frac{\left(x^2+\frac{d}{e}\right)c+\frac{be-2cd}{2e}}{\sqrt{c}}+\sqrt{\left(x^2+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}\right)}{4\sqrt{c}\, e^2}+\frac{\sqrt{c}\, d^2 \ln\left(\frac{\left(x^2+\frac{d}{e}\right)c+\frac{be-2cd}{2e}}{\sqrt{c}}+\sqrt{\left(x^2+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}\right)}{2e^3}+\frac{\sqrt{c x^4+b x^2+a}\, b}{8ce}-\frac{\sqrt{\left(x^2+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}\, d}{2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x)
```

```
[Out] 1/4/e*(c*x^4+b*x^2+a)^(1/2)*x^2+1/8/e/c*(c*x^4+b*x^2+a)^(1/2)*b+1/4/e/c^(1/
2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a-1/16/e/c^(3/2)*ln((c*x
^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*b^2-1/2*d/e^2*((x^2+d/e)^2*c+(b*e-
2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/4*d/e^2*ln(((x^2+d/e)*c
```

```
+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b+1/2*d^2/e^3*ln(((x^2+d/e)*c+1/2*(b*e-2*c*
d)/e)/c^(1/2)+((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2))*c^(1/2)+1/2*d/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)
*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((
x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d
/e))*a-1/2*d^2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e
)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^
2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b+1/
2*d^3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*
e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-
2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for mo
re details)Is b*e-2*c*d zero or nonzero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{c\,x^4+b\,x^2+a}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2),x)
```

```
[Out] int((x^3*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \sqrt{a + bx^{2} + cx^{4}}}{d + ex^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
```

```
[Out] Integral(x**3*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)
```

3.235 $\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$

Optimal. Leaf size=168

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\,e^2}+\frac{\sqrt{a+bx^2+cx^4}}{2e}$$

Rubi [A] time = 0.22, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1247, 734, 843, 621, 206, 724}

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\,e^2}+\frac{\sqrt{a+bx^2+cx^4}}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*e) - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x^2)/(2*Sqr
t[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*e^2) + (Sqrt[c*d^2 - b*d*e + a*e
^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2
]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{\operatorname{Subst}\left(\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4e}$$

$$= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{(2cd-be)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4e^2} + \frac{\left(cd^2-bde+ae^2\right)\operatorname{Sub}}{}$$

$$= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{(2cd-be)\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2e^2} - \frac{\left(cd^2-bde+ae^2\right)\operatorname{S}}{}$$

$$= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\,e^2} + \frac{\sqrt{cd^2-bde+ae^2}\,\tanh^{-1}\Big(}{2e^2}$$

Mathematica [A] time = 0.13, size = 167, normalized size = 0.99

$$\frac{2\sqrt{c}\left(e\sqrt{a+bx^2+cx^4}-\sqrt{ae^2-bde+cd^2}\,\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)\right)+(be-2cd)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\,e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] ((-2*c*d + b*e)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]
+ 2*Sqrt[c]*(e*Sqrt[a + b*x^2 + c*x^4] - Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTan
h[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqr
t[a + b*x^2 + c*x^4])]))/(4*Sqrt[c]*e^2)
```

IntegrateAlgebraic [A] time = 0.58, size = 203, normalized size = 1.21

$$\frac{\sqrt{-ae^2+bde-cd^2}\,\tan^{-1}\left(\frac{\sqrt{c}ex^2}{\sqrt{-ae^2+bde-cd^2}}-\frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}d}{\sqrt{-ae^2+bde-cd^2}}\right)}{e^2}+\frac{(2cd-be)\log\left(-2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{4\sqrt{c}\,e^2}+\frac{\sqrt{a+bx^2+cx^4}}{2e}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*e) + (Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqr
t[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x^2)/Sqrt[-(c*d^2) + b*
d*e - a*e^2] - (e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])
/e^2 + ((2*c*d - b*e)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])
/(4*Sqrt[c]*e^2)
```

fricas [A] time = 4.09, size = 1050, normalized size = 6.25

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(c*x^4 + b*x^2 + a)*c*e - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^4
- 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c
) + 2*sqrt(c*d^2 - b*d*e + a*e^2)*c*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4
*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 +
 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^
2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2
+ d^2)))/(c*e^2), 1/4*(2*sqrt(c*x^4 + b*x^2 + a)*c*e + (2*c*d - b*e)*sqrt(-
c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c
*x^2 + a*c)) + sqrt(c*d^2 - b*d*e + a*e^2)*c*log(-((8*c^2*d^2 - 8*b*c*d*e +
 (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*
b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*
sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2
*d*e*x^2 + d^2)))/(c*e^2), 1/8*(4*sqrt(c*x^4 + b*x^2 + a)*c*e + 4*sqrt(-c*d
^2 + b*d*e - a*e^2)*c*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d
*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2
)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) -
 (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*
x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c))/(c*e^2), 1/4*(2*sqrt(c*x^4 + b*x^2
 + a)*c*e + 2*sqrt(-c*d^2 + b*d*e - a*e^2)*c*arctan(-1/2*sqrt(c*x^4 + b*x^2
 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*
d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2
*d*e + a*b*e^2)*x^2)) + (2*c*d - b*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^
2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)))/(c*e^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.00, size = 757, normalized size = 4.51

$$-\frac{a \ln \left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2a e^2-2deb+2c d^2}{e^2}+2\sqrt{\frac{a e^2-deb+c d^2}{e^2}}\,\sqrt{\left(x^2+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-deb+c d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{a e^2-deb+c d^2}{e^2}}\, e}+\frac{bd \ln \left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2a e^2-2deb+2c d^2}{e^2}+2\sqrt{\frac{a e^2-deb+c d^2}{e^2}}\,\sqrt{\left(x^2+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-deb+c d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{a e^2-deb+c d^2}{e^2}}\, e^2}-\frac{c d^2 \ln \left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2a e^2-2deb+2c d^2}{e^2}+2\sqrt{\frac{a e^2-deb+c d^2}{e^2}}\,\sqrt{\left(x^2+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-deb+c d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{a e^2-deb+c d^2}{e^2}}\, e^3}+\frac{b \ln \left(\frac{\left(x^2+\frac{d}{e}\right)c+\frac{be-2cd}{2e}}{\sqrt{c}}+\sqrt{\left(x^2+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-deb+c d^2}{e^2}}\right)}{4\sqrt{c}\, e}-\frac{\sqrt{c}\, d \ln \left(\frac{\left(x^2+\frac{d}{e}\right)c+\frac{be-2cd}{2e}}{\sqrt{c}}+\sqrt{\left(x^2+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-deb+c d^2}{e^2}}\right)}{2e^2}+\frac{\sqrt{\left(x^2+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-deb+c d^2}{e^2}}}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x)
```

```
[Out] 1/2/e*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
+1/4/e*ln(((x^2+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x^2+d/e)^2*c+(b*e-2*c*d
)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b-1/2/e^2*ln(((x^2+d/
e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e
^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d-1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2))/(x^2+d/e))*a+1/2/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c
*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^
2+d/e))*d*b-1/2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/
e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)
^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c*d
^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` for
more details)Is a*e^2-b*d*e                              +c*d^2 zero or nonze
ro?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c\,x^4+b\,x^2+a}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2),x)
```

```
[Out] int((x*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
```

```
[Out] Integral(x*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)
```

3.236 $\int \frac{\sqrt{a+bx^2+cx^4}}{x(d+ex^2)}\,dx$

Optimal. Leaf size=186

$$-\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2de}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2e}$$

Rubi [A] time = 0.26, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1251, 895, 724, 206, 843, 621}

$$-\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2de}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/(x*(d + e*x^2)),x]
```

```
[Out] -(Sqrt[a]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d)
 + (Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*
e) - (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)
/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 895

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[(Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*(a + b*x + c*x^2)^(p -
1))/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
```

```
&& GtQ[p, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x\left(d+ex^2\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex)}\,dx, x, x^2\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{-bd+ae-cdx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2d} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2d}$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{d} + \frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2e} - \frac{1}{2}\left(-b+\frac{cd}{e}\right.$$
$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d} + \frac{c\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{e} - \left(b-\frac{cd}{e}-\frac{ae}{d}\right)$$
$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d} + \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2e} - \frac{\sqrt{cd^2-bde+ae^2}\tan}{}$$

Mathematica [A] time = 0.15, size = 179, normalized size = 0.96

$$-\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+bd-bex^2+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)-\sqrt{c}\,d\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)+\sqrt{a}\,e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2de}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/(x*(d + e*x^2)),x]
```

```
[Out] -1/2*(Sqrt[a]*e*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]
- Sqrt[c]*d*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])] + Sq
rt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + 2*c*d*x^2 - b*e*x^2)/(2*Sq
rt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(d*e)
```

IntegrateAlgebraic [A] time = 0.70, size = 227, normalized size = 1.22

$$-\frac{\sqrt{-ae^2+bde-cd^2}\tan^{-1}\left(\frac{\sqrt{c}ex^2}{\sqrt{-ae^2+bde-cd^2}}-\frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}d}{\sqrt{-ae^2+bde-cd^2}}\right)}{de}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}-\frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{d}-\frac{\sqrt{c}\log\left(-2\sqrt{c}e\sqrt{a+bx^2+cx^4}+be+2cex^2\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2 + c*x^4]/(x*(d + e*x^2)),x]
```

```
[Out] -((Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e
- a*e^2] + (Sqrt[c]*e*x^2)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x
^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(d*e)) + (Sqrt[a]*ArcTanh[(Sq
rt[c]*x^2)/Sqrt[a] - Sqrt[a + b*x^2 + c*x^4]/Sqrt[a]])/d - (Sqrt[c]*Log[b*e
 + 2*c*e*x^2 - 2*Sqrt[c]*e*Sqrt[a + b*x^2 + c*x^4]])/(2*e)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 851, normalized size = 4.58

$$\frac{a\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2ae^2-2deb+2cd^2}{e^2}+2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\,d}-\frac{b\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2ae^2-2deb+2cd^2}{e^2}+2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\,e}+\frac{cd\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2ae^2-2deb+2cd^2}{e^2}+2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\,e^2}-\frac{\sqrt{a}\,\ln\left(\frac{bx^2+2a+2\sqrt{cx^4+bx^2+a}\,\sqrt{a}}{x^2}\right)}{2d}+\frac{b\ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{4\sqrt{c}\,d}-\frac{b\ln\left(\frac{\left(x^2+\frac{d}{e}\right)c+\frac{be-2cd}{2e}}{\sqrt{c}}+\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}\right)}{4\sqrt{c}\,d}+\frac{\sqrt{c}\,\ln\left(\frac{\left(x^2+\frac{d}{e}\right)c+\frac{be-2cd}{2e}}{\sqrt{c}}+\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}\right)}{2e}+\frac{\sqrt{cx^4+bx^2+a}}{2d}-\frac{\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x)
```

```
[Out] 1/2/d*(c*x^4+b*x^2+a)^(1/2)+1/4/d*b*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a
)^(1/2))/c^(1/2)-1/2/d*a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2
))/x^2)-1/2/d*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)-1/4/d*ln(((x^2+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x^2+d/e)^2*c+(b
*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b+1/2/e*ln(((
x^2+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/
e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)+1/2/d/((a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+
c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2))/(x^2+d/e))*a-1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2
*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(
x^2+d/e))*b+1/2/e^2*d/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+
d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/
e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2+a}}{\left(ex^2+d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/((e*x^2 + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/(x*(d + e*x^2)),x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(1/2)/(x*(d + e*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x/(e*x**2+d),x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/(x*(d + e*x**2)), x)
```

3.237 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^3\left(d+ex^2\right)}\,dx$

Optimal. Leaf size=361

$$\frac{\sqrt{ae^2-bde+cd^2}\,\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\,\sqrt{ae^2-bde+cd^2}}\right)}{2d^2}+\frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\,\sqrt{a+bx^2+cx^4}}\right)}{2d^2}-\frac{be\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\,d^2}-\cdots$$

Rubi [A] time = 0.51, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1251, 960, 732, 843, 621, 206, 724, 734}

$$\frac{\sqrt{ae^2-bde+cd^2}\,\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\,\sqrt{ae^2-bde+cd^2}}\right)}{2d^2}+\frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\,\sqrt{a+bx^2+cx^4}}\right)}{2d^2}-\frac{be\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\,d^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\,d^2}-\frac{\sqrt{a+bx^2+cx^4}}{2dx^2}-\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\,\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\,d}+\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/(x^3*(d + e*x^2)),x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*d*x^2) - (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sq
rt[a + b*x^2 + c*x^4])])/(4*Sqrt[a]*d) + (Sqrt[a]*e*ArcTanh[(2*a + b*x^2)/(
2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2) + (Sqrt[c]*ArcTanh[(b + 2*c*x^
2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*d) - (b*e*ArcTanh[(b + 2*c*x^2)
/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*d^2) - ((2*c*d - b*e)*Arc
Tanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*d^2) +
(Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*S
qrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3\left(d+ex^2\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{\sqrt{a+bx+cx^2}}{dx^2} - \frac{e\sqrt{a+bx+cx^2}}{d^2x} + \frac{e^2\sqrt{a+bx+cx^2}}{d^2(d+ex)}\right) dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx, x, x^2\right)}{2d} - \frac{e\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx, x, x^2\right)}{2d^2} + \frac{e^2\,\text{Subst}\left(\int \frac{\sqrt{a+bx+}}{d+e}\right.}{2d^2}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{2dx^2} + \frac{\text{Subst}\left(\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4d} + \frac{e\,\text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right.}{4d^2}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{2dx^2} + \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4d} + \frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right.}{2d}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{2dx^2} - \frac{b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2d} + \frac{c\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+}{\sqrt{a+b}}\right.}{d}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{2dx^2} - \frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\,d} + \frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2} + \frac{\sqrt{c}}{}$$

Mathematica [A] time = 0.32, size = 165, normalized size = 0.46

$$\frac{2\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+bd-bex^2+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)+\frac{(2ae-bd)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{a}}-\frac{2d\sqrt{a+bx^2+cx^4}}{x^2}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/(x^3*(d + e*x^2)),x]
```

```
[Out] ((-2*d*Sqrt[a + b*x^2 + c*x^4])/x^2 + ((-(b*d) + 2*a*e)*ArcTanh[(2*a + b*x^
2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[a] + 2*Sqrt[c*d^2 - b*d*e + a
*e^2]*ArcTanh[(b*d - 2*a*e + 2*c*d*x^2 - b*e*x^2)/(2*Sqrt[c*d^2 - b*d*e + a
*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(4*d^2)
```

IntegrateAlgebraic [A] time = 0.78, size = 212, normalized size = 0.59

$$\frac{\sqrt{-ae^2+bde-cd^2}\tan^{-1}\left(\frac{\sqrt{c}ex^2}{\sqrt{-ae^2+bde-cd^2}}-\frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}d}{\sqrt{-ae^2+bde-cd^2}}\right)}{d^2}+\frac{(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}-\frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}d^2}-\frac{\sqrt{a+bx^2+cx^4}}{2dx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2 + c*x^4]/(x^3*(d + e*x^2)),x]
```

```
[Out] -1/2*Sqrt[a + b*x^2 + c*x^4]/(d*x^2) + (Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcT
an[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x^2)/Sqrt[-(c*d^
2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a
*e^2]])/d^2 + ((b*d - 2*a*e)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a] - Sqrt[a + b*x^2
 + c*x^4]/Sqrt[a]])/(2*Sqrt[a]*d^2)
```

fricas [A] time = 2.36, size = 1094, normalized size = 3.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(c*d^2 - b*d*e + a*e^2)*a*x^2*log(-((8*c^2*d^2 - 8*b*c*d*e + (b
^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c
*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqr
t(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*
e*x^2 + d^2)) - (b*d - 2*a*e)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x
^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*sqrt
(c*x^4 + b*x^2 + a)*a*d)/(a*d^2*x^2), 1/8*(4*sqrt(-c*d^2 + b*d*e - a*e^2)*a
*x^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c
*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 -
 a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (b*d - 2*a*e)*sq
rt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(
b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*sqrt(c*x^4 + b*x^2 + a)*a*d)/(a*d^2*
x^2), 1/4*((b*d - 2*a*e)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b
*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + sqrt(c*d^2 - b*d*e + a*e^
2)*a*x^2*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e
+ 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c
)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d
- b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 2*sqrt(c*x^4 + b*
x^2 + a)*a*d)/(a*d^2*x^2), 1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*a*x^2*arctan
(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x
^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e +
a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) + (b*d - 2*a*e)*sqrt(-a)*x^2*
arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^
2 + a^2)) - 2*sqrt(c*x^4 + b*x^2 + a)*a*d)/(a*d^2*x^2)]
```

giac [A] time = 0.52, size = 216, normalized size = 0.60

$$\frac{\left(cd^2-bde+ae^2\right)\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}\,d^2}+\frac{(bd-2ae)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}\,d^2}+\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)b+2a\sqrt{c}}{2\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x, algorithm="giac")
```

```
[Out] (c*d^2 - b*d*e + a*e^2)*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e
+ sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/(sqrt(-c*d^2 + b*d*e - a*e^2)*d^
2) + 1/2*(b*d - 2*a*e)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt
(-a))/(sqrt(-a)*d^2) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a
*sqrt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*d)
```

maple [B] time = 0.02, size = 1009, normalized size = 2.80

$$-\frac{ae\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2ae^2-2deb+2cd^2}{e^2}+2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\,d^2}+\frac{\sqrt{cx^4+bx^2+a}\,cx^2}{2ad}+\frac{b\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2ae^2-2deb+2cd^2}{e^2}+2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\,d}-\frac{c\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2ae^2-2deb+2cd^2}{e^2}+2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\,e}+\frac{\sqrt{a}\,e\ln\left(\frac{bx^2+2a+2\sqrt{cx^4+bx^2+a}\sqrt{a}}{x^2}\right)}{2d^2}-\frac{b\ln\left(\frac{bx^2+2a+2\sqrt{cx^4+bx^2+a}\sqrt{a}}{x^2}\right)}{4\sqrt{a}\,d}-\frac{be\ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{4\sqrt{c}\,d^2}+\frac{be\ln\left(\frac{\left(x^2+\frac{d}{e}\right)c+\frac{be-2cd}{2e}}{\sqrt{c}}+\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}\right)}{4\sqrt{c}\,d^2}+\frac{\sqrt{c}\ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{2d}-\frac{\sqrt{c}\ln\left(\frac{\left(x^2+\frac{d}{e}\right)c+\frac{be-2cd}{2e}}{\sqrt{c}}+\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}\right)}{2d}+\frac{\sqrt{cx^4+bx^2+a}\,b}{2ad}-\frac{\sqrt{cx^4+bx^2+a}\,e}{2d^2}+\frac{\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}\,e}{2d^2}-\frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{2adx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x)
```

```
[Out] -1/2/d^2*e*(c*x^4+b*x^2+a)^(1/2)-1/4/d^2*e*b*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^
4+b*x^2+a)^(1/2))/c^(1/2)+1/2/d^2*e*a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)
^(1/2)*a^(1/2))/x^2)+1/2*e/d^2*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^
2-b*d*e+c*d^2)/e^2)^(1/2)+1/4*e/d^2*ln(((x^2+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1
/2)+((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/
c^(1/2)*b-1/2/d*ln(((x^2+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x^2+d/e)^2*c+(
b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)-1/2*e/d^2/((
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*
d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+
d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a+1/2/d/((a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b-1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))/(x^2+d/e))*c-1/2/d/a/x^2*(c*x^4+b*x^2+a)^(3/2)+1/2/d*b/a*(c*x^4+b*
x^2+a)^(1/2)-1/4/d*b/a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))
/x^2)+1/2/d/a*c*(c*x^4+b*x^2+a)^(1/2)*x^2+1/2/d*c^(1/2)*ln((c*x^2+1/2*b)/c^
(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2+a}}{\left(ex^2+d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/((e*x^2 + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^3\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/(x^3*(d + e*x^2)),x)
[Out] int((a + b*x^2 + c*x^4)^(1/2)/(x^3*(d + e*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^3 \left(d + ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**3/(e*x**2+d),x)
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/(x**3*(d + e*x**2)), x)
```

3.238 $\int \frac{x^5(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx$

Optimal. Leaf size=482

$$-\frac{\left(16bc^2e^3\left(3a^2e^2-3abde+b^2d^2\right)+6b^3ce^4(bd-4ae)-384c^4d^3e(bd-ae)+96c^3de^2(bd-ae)^2+3b^5e^5+256c^5 \ldots}{512c^{7/2}e^6}$$

Rubi [A] time = 1.10, antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1251, 1653, 814, 843, 621, 206, 724}

$$-\frac{\left(16bc^2e^3\left(3a^2e^2-3abde+b^2d^2\right)+6b^3ce^4(bd-4ae)-384c^4d^3e(bd-ae)+96c^3de^2(bd-ae)^2+3b^5e^5+256c^5d^5\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}e^6}+\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(-3b^2e^2-6cex^2(be+2cd)-6bcde+16c^2d^2\right)}{96c^2e^3}+\frac{\sqrt{a+bx^2+cx^4}\left(-2cex^2\left(-8c^2de(2bd-3ae)-6bce^2(bd-2ae)-3b^3e^3+32c^3d^3\right)+6b^2ce^3(bd-2ae)-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+3b^4e^4+128c^4d^4\right)}{256c^3e^5}+\frac{d^2\left(ae^2-bde+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^6}+\frac{\left(a+bx^2+cx^4\right)^{5/2}}{10ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] ((128*c^4*d^4 + 3*b^4*e^4 - 32*c^3*d^2*e*(5*b*d - 4*a*e) + 8*b*c^2*d*e^2*(2
*b*d - 3*a*e) + 6*b^2*c*e^3*(b*d - 2*a*e) - 2*c*e*(32*c^3*d^3 - 3*b^3*e^3 -
 8*c^2*d*e*(2*b*d - 3*a*e) - 6*b*c*e^2*(b*d - 2*a*e))*x^2)*Sqrt[a + b*x^2 +
 c*x^4])/(256*c^3*e^5) + ((16*c^2*d^2 - 6*b*c*d*e - 3*b^2*e^2 - 6*c*e*(2*c*
d + b*e)*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(96*c^2*e^3) + (a + b*x^2 + c*x^4)
^(5/2)/(10*c*e) - ((256*c^5*d^5 + 3*b^5*e^5 + 6*b^3*c*e^4*(b*d - 4*a*e) - 3
84*c^4*d^3*e*(b*d - a*e) + 96*c^3*d*e^2*(b*d - a*e)^2 + 16*b*c^2*e^3*(b^2*d
^2 - 3*a*b*d*e + 3*a^2*e^2))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^
2 + c*x^4])])/(512*c^(7/2)*e^6) + (d^2*(c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTan
h[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a +
 b*x^2 + c*x^4])])/(2*e^6)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{x^5 \left(a+bx^2+cx^4\right)^{3/2}}{d+ex^2} \, dx = \frac{1}{2} \text{Subst}\left(\int \frac{x^2 \left(a+bx+cx^2\right)^{3/2}}{d+ex} \, dx, x, x^2\right)$$

$$= \frac{\left(a+bx^2+cx^4\right)^{5/2}}{10ce} + \frac{\text{Subst}\left(\int \frac{\left(-\frac{5}{2}bde-\frac{5}{2}e(2cd+be)x\right)\left(a+bx+cx^2\right)^{3/2}}{d+ex} dx, x, x^2\right)}{10ce^2}$$

$$= \frac{\left(16c^2d^2-6bcde-3b^2e^2-6ce(2cd+be)x^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{96c^2e^3} + \frac{\left(a+bx^2+cx^4\right.}{10ce}$$

$$= \frac{\left(128c^4d^4+3b^4e^4-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+6b^2ce^3(bd-2ae)\right.}{256}$$

$$= \frac{\left(128c^4d^4+3b^4e^4-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+6b^2ce^3(bd-2ae)\right.}{256}$$

$$= \frac{\left(128c^4d^4+3b^4e^4-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+6b^2ce^3(bd-2ae)\right.}{256}$$

$$= \frac{\left(128c^4d^4+3b^4e^4-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+6b^2ce^3(bd-2ae)\right.}{256}$$

Mathematica [A] time = 0.98, size = 545, normalized size = 1.13

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] (1280*d^2*(a + b*x^2 + c*x^4)^(3/2) - (480*d*e*(b + 2*c*x^2)*(a + b*x^2 + c
*x^4)^(3/2))/c + (768*e^2*(a + b*x^2 + c*x^4)^(5/2))/c - (90*(b^2 - 4*a*c)*
d*e*(-2*Sqrt[c]*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4] + (b^2 - 4*a*c)*ArcTa
nh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]))/c^(5/2) + (15*b*e^2
*(-16*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2) + 3*(b^2 - 4*a*c)*((2*(b + 2*
c*x^2)*Sqrt[a + b*x^2 + c*x^4])/c + ((-b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^2)/(
2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/c^(3/2))))/c^2 - (240*d^2*((2*c*d - b*
e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(b + 2*c*x^2)/(2*
Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])] + 2*Sqrt[c]*(e*Sqrt[a + b*x^2 + c*x^4]*(-
(b^2*e^2) + 4*c^2*d*(-2*d + e*x^2) - 2*c*e*(-5*b*d + 4*a*e + b*e*x^2)) + 8*
c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*
e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4])])))/(c^(3
/2)*e^3))/(7680*e^3)
```

IntegrateAlgebraic [B] time = 30.30, size = 13888, normalized size = 28.81

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.06, size = 2068, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x)
```

```
[Out] 11/80/e*b*x^6*(c*x^4+b*x^2+a)^(1/2)-5/8*d^3/e^4*b*(c*x^4+b*x^2+a)^(1/2)+1/2
*d^4/e^5*c*(c*x^4+b*x^2+a)^(1/2)+2/3*d^2/e^3*a*(c*x^4+b*x^2+a)^(1/2)-1/2*d^
5/e^6*c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/10/e*a^2/c*
(c*x^4+b*x^2+a)^(1/2)-3/512/e*b^5/c^(7/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b
*x^2+a)^(1/2))+1/5/e*a*x^4*(c*x^4+b*x^2+a)^(1/2)+3/256/e*b^4/c^3*(c*x^4+b*x
^2+a)^(1/2)+1/10/e*c*x^8*(c*x^4+b*x^2+a)^(1/2)-3/32/e*a^2*b/c^(3/2)*ln((c*x
^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/160/e*b^2*x^4/c*(c*x^4+b*x^2+a)^
(1/2)-1/128/e*b^3/c^2*x^2*(c*x^4+b*x^2+a)^(1/2)-5/64/e*a*b^2/c^2*(c*x^4+b*x
^2+a)^(1/2)+3/64/e*a*b^3/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(
1/2))-3/16/e^2*d*a^2*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2
)-5/16/e^2*d*a*x^2*(c*x^4+b*x^2+a)^(1/2)+3/128/e^2*d*b^3/c^2*(c*x^4+b*x^2+a
)^(1/2)-3/256/e^2*d*b^4/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1
/2))-3/16/e^2*d*b*x^4*(c*x^4+b*x^2+a)^(1/2)-1/8/e^2*d*c*x^6*(c*x^4+b*x^2+a)
^(1/2)+1/6*d^2/e^3*c*x^4*(c*x^4+b*x^2+a)^(1/2)+7/24*d^2/e^3*b*x^2*(c*x^4+b*
x^2+a)^(1/2)+1/16*d^2/e^3/c*b^2*(c*x^4+b*x^2+a)^(1/2)-1/32*d^2/e^3*b^3/c^(3
/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-3/4*d^3/e^4*a*c^(1/2)*l
n((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-3/16*d^3/e^4*b^2*ln((c*x^2+1
/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+3/4*d^4/e^5*b*c^(1/2)*ln((c*x^
2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/4*d^3/e^4*x^2*c*(c*x^4+b*x^2+a)^(
1/2)-1/2*d^2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/
e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*
c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a^2-1/
2*d^4/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*
e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-
2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b^2-1/2*d^6/e
^7/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d
*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*
(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c^2+7/160/e*a*b*x^2/
c*(c*x^4+b*x^2+a)^(1/2)-1/64/e^2*d*b^2*x^2/c*(c*x^4+b*x^2+a)^(1/2)-5/32/e^2
*d*a*b/c*(c*x^4+b*x^2+a)^(1/2)+3/32/e^2*d*a*b^2/c^(3/2)*ln((c*x^2+1/2*b)/c^
(1/2)+(c*x^4+b*x^2+a)^(1/2))+3/8*d^2/e^3*a*b*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^
4+b*x^2+a)^(1/2))/c^(1/2)+d^3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-
2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/
(x^2+d/e))*a*b-d^4/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2
+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d
/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*
a*c+d^5/e^6/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(
```

```
a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*
e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b*c
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for mo
re details)Is b*e-2*c*d zero or nonzero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{e\,x^2+d} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x)
```

```
[Out] int((x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5} \left(a + b x^{2} + c x^{4}\right)^{\frac{3}{2}}}{d + e x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
```

```
[Out] Integral(x**5*(a + b*x**2 + c*x**4)**(3/2)/(d + e*x**2), x)
```

3.239 $\int \frac{x^3\left(a+bx^2+cx^4\right)^{3/2}}{d+ex^2}\,dx$

Optimal. Leaf size=360

$$\frac{\left(8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2+3b^4e^4+128c^4d^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}e^5}-\sqrt{a+bx^2}$$

Rubi [A] time = 0.70, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1251, 814, 843, 621, 206, 724}

$$-\frac{\sqrt{a+bx^2+cx^4}\left(-2cex^2\left(-4ce(2bd-3ae)-3b^2e^2+16c^2d^2\right)-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)+3b^3e^3+64c^3d^3\right)}{128c^2e^4}+\frac{\left(8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2+3b^4e^4+128c^4d^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}e^5}-\frac{d\left(ae^2-bde+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^5}-\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(-3be+8cd-6cex^2\right)}{48ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] -((64*c^3*d^3 + 3*b^3*e^3 - 16*c^2*d*e*(5*b*d - 4*a*e) + 4*b*c*e^2*(2*b*d -
 3*a*e) - 2*c*e*(16*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*x^2)*Sqrt[
a + b*x^2 + c*x^4])/(128*c^2*e^4) - ((8*c*d - 3*b*e - 6*c*e*x^2)*(a + b*x^2
 + c*x^4)^(3/2))/(48*c*e^2) + ((128*c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e^3*(b*d
- 3*a*e) - 192*c^3*d^2*e*(b*d - a*e) + 48*c^2*e^2*(b*d - a*e)^2)*ArcTanh[(b
 + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(256*c^(5/2)*e^5) - (d*(c
*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sq
rt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^5)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3\left(a+bx^2+cx^4\right)^{3/2}}{d+ex^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx, x, x^2\right)$$

$$= -\frac{\left(8cd-3be-6cex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{48ce^2} - \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}d\left(4ace-2b\left(4cd-\frac{3be}{2}\right)\right)-\frac{1}{2}\left(16c^2d^2-\right.\right.}{d+}\right.}{16c}$$

$$= -\frac{\left(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)-2ce\left(16c^2d^2-3b^2e^2\right.\right.}{128c^2e^4}$$

$$= -\frac{\left(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)-2ce\left(16c^2d^2-3b^2e^2\right.\right.}{128c^2e^4}$$

$$= -\frac{\left(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)-2ce\left(16c^2d^2-3b^2e^2\right.\right.}{128c^2e^4}$$

$$= -\frac{\left(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)-2ce\left(16c^2d^2-3b^2e^2\right.\right.}{128c^2e^4}$$

Mathematica [A] time = 0.60, size = 344, normalized size = 0.96

$$\frac{2\sqrt{c}\left(e\sqrt{a+bx^2+cx^4}\left(8c^2e\left(ae\left(15ex^2-32d\right)+b\left(30d^2-14dex^2+9e^2x^4\right)\right)+6bce^2\left(10ae-4bd+bex^2\right)-9b^3e^3-16c^3\left(12d^3-6d^2ex^2+4de^2x^4-3e^3x^6\right)\right)+192c^2d\left(e(ae-bd)+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)\right)+3\left(8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2+3b^4e^4+128c^4d^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{768c^{5/2}e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] (3*(128*c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e^3*(b*d - 3*a*e) - 192*c^3*d^2*e*(b*
d - a*e) + 48*c^2*e^2*(b*d - a*e)^2)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[
a + b*x^2 + c*x^4])] + 2*Sqrt[c]*(e*Sqrt[a + b*x^2 + c*x^4]*(-9*b^3*e^3 + 6
*b*c*e^2*(-4*b*d + 10*a*e + b*e*x^2) - 16*c^3*(12*d^3 - 6*d^2*e*x^2 + 4*d*e
^2*x^4 - 3*e^3*x^6) + 8*c^2*e*(a*e*(-32*d + 15*e*x^2) + b*(30*d^2 - 14*d*e*
x^2 + 9*e^2*x^4))) + 192*c^2*d*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[(-(
b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[
a + b*x^2 + c*x^4])]))/(768*c^(5/2)*e^5)
```

IntegrateAlgebraic [A] time = 2.78, size = 410, normalized size = 1.14

$$\frac{\left(-48a^2c^2e^4+24ab^2ce^4+96abc^2de^3-192ac^3d^2e^2-3b^4e^4-8b^3cde^3-48b^2c^2d^2e^2+192bc^3d^3e-128c^4d^4\right)\log\left(-2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{256c^{5/2}e^5}+\frac{\sqrt{a+bx^2+cx^4}\left(60abce^3-256ac^2de^2+120ac^2e^3x^2-9b^3e^3-24b^2cde^2+6b^2ce^3x^2+240bc^2d^2e-112bc^2de^2x^2+72bc^2e^3x^4-192c^3d^3+96c^3d^2ex^2-64c^3de^2x^4+48c^3e^3x^6\right)}{384c^2e^4}-\frac{\sqrt{-ae^2+bde-cd^2}\left(ade^2-bd^2e+cd^3\right)\tan^{-1}\left(\frac{-e\sqrt{a+bx^2+cx^4}+\sqrt{c}d+\sqrt{c}ex^2}{\sqrt{-ae^2+bde-cd^2}}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-192*c^3*d^3 + 240*b*c^2*d^2*e - 24*b^2*c*d*e^2 -
 256*a*c^2*d*e^2 - 9*b^3*e^3 + 60*a*b*c*e^3 + 96*c^3*d^2*e*x^2 - 112*b*c^2*
d*e^2*x^2 + 6*b^2*c*e^3*x^2 + 120*a*c^2*e^3*x^2 - 64*c^3*d*e^2*x^4 + 72*b*c
^2*e^3*x^4 + 48*c^3*e^3*x^6))/(384*c^2*e^4) - (Sqrt[-(c*d^2) + b*d*e - a*e^
2]*(c*d^3 - b*d^2*e + a*d*e^2)*ArcTan[(Sqrt[c]*d + Sqrt[c]*e*x^2 - e*Sqrt[a
 + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/e^5 + ((-128*c^4*d^4 +
192*b*c^3*d^3*e - 48*b^2*c^2*d^2*e^2 - 192*a*c^3*d^2*e^2 - 8*b^3*c*d*e^3 +
96*a*b*c^2*d*e^3 - 3*b^4*e^4 + 24*a*b^2*c*e^4 - 48*a^2*c^2*e^4)*Log[b + 2*c
*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(256*c^(5/2)*e^5)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 1696, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x)
```

```
[Out] 1/64/e*b^2*x^2/c*(c*x^4+b*x^2+a)^(1/2)+5/32/e*a*b/c*(c*x^4+b*x^2+a)^(1/2)+1
/2*d^4/e^5*c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+5/8*d^2/
e^3*b*(c*x^4+b*x^2+a)^(1/2)-1/2*d^3/e^4*c*(c*x^4+b*x^2+a)^(1/2)-2/3*d/e^2*a
*(c*x^4+b*x^2+a)^(1/2)+3/16/e*a^2*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^
(1/2))/c^(1/2)+5/16/e*a*x^2*(c*x^4+b*x^2+a)^(1/2)-3/128/e*b^3/c^2*(c*x^4+b*
x^2+a)^(1/2)+3/256/e*b^4/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(
1/2))+3/16/e*b*x^4*(c*x^4+b*x^2+a)^(1/2)+1/8/e*c*x^6*(c*x^4+b*x^2+a)^(1/2)+
1/2*d^5/e^6/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(
a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*
e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c^2-1/6*d/e
^2*c*x^4*(c*x^4+b*x^2+a)^(1/2)-7/24*d/e^2*b*x^2*(c*x^4+b*x^2+a)^(1/2)-3/4*d
^3/e^4*b*c^(1/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/4*d^2/e^
3*x^2*c*(c*x^4+b*x^2+a)^(1/2)+1/2*d/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(
((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(
1/2))/(x^2+d/e))*a^2+1/2*d^3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2
```

```
*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(
x^2+d/e))*b^2-1/16*d/e^2/c*b^2*(c*x^4+b*x^2+a)^(1/2)+1/32*d/e^2*b^3/c^(3/2)
*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+3/4*d^2/e^3*a*c^(1/2)*ln((
c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+3/16*d^2/e^3*b^2*ln((c*x^2+1/2*
b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-3/32/e*a*b^2/c^(3/2)*ln((c*x^2+1/
2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-3/8*d/e^2*a*b*ln((c*x^2+1/2*b)/c^(1/2)+
(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-d^2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((
(b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1
/2))/(x^2+d/e))*a*b+d^3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)
*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((
x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d
/e))*a*c-d^4/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/
e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*
c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for mo
re details)Is b*e-2*c*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\left(c\,x^4+b\,x^2+a\right)^{3/2}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x)
```

```
[Out] int((x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\left(a+bx^{2}+cx^{4}\right)^{\frac{3}{2}}}{d+ex^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
```

```
[Out] Integral(x**3*(a + b*x**2 + c*x**4)**(3/2)/(d + e*x**2), x)
```

3.240 $\int \frac{x\left(a+bx^2+cx^4\right)^{3/2}}{d+ex^2}\,dx$

Optimal. Leaf size=269

$$\frac{\sqrt{a+bx^2+cx^4}\left(-2ce(5bd-4ae)+b^2e^2-2cex^2(2cd-be)+8c^2d^2\right)}{16ce^3}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)}{32c^{3/2}e^4}$$

Rubi [A] time = 0.46, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1247, 734, 814, 843, 621, 206, 724}

$$\frac{\sqrt{a+bx^2+cx^4}\left(-2ce(5bd-4ae)+b^2e^2-2cex^2(2cd-be)+8c^2d^2\right)}{16ce^3}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}e^4}+\frac{\left(ae^2-bde+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^4}+\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x^2)*Sq
rt[a + b*x^2 + c*x^4])/(16*c*e^3) + (a + b*x^2 + c*x^4)^(3/2)/(6*e) - ((2*c
*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x^
2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(3/2)*e^4) + ((c*d^2 - b*d*e
 + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b
*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
```

```
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x\left(a+bx^2+cx^4\right)^{3/2}}{d+ex^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx, x, x^2\right)$$

$$= \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6e} - \frac{\text{Subst}\left(\int \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)}{4e}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2\right)\sqrt{a+bx^2+cx^4}}{16ce^3} + \frac{\left(a+bx^2+c\right.}{6e}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2\right)\sqrt{a+bx^2+cx^4}}{16ce^3} + \frac{\left(a+bx^2+c\right.}{6e}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2\right)\sqrt{a+bx^2+cx^4}}{16ce^3} + \frac{\left(a+bx^2+c\right.}{6e}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2\right)\sqrt{a+bx^2+cx^4}}{16ce^3} + \frac{\left(a+bx^2+c\right.}{6e}$$

Mathematica [A] time = 0.35, size = 255, normalized size = 0.95

$$\frac{2\sqrt{c}\left(e\sqrt{a+bx^2+cx^4}\left(2ce\left(16ae-15bd+7bex^2\right)+3b^2e^2+4c^2\left(6d^2-3dex^2+2e^2x^4\right)\right)-24c\left(e(ae-bd)+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)\right)-3(2cd-be)\left(4ce(3ae-2bd)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{96c^{3/2}e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] (-3*(2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(b
 + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])] + 2*Sqrt[c]*(e*Sqrt[a + b*
x^2 + c*x^4]*(3*b^2*e^2 + 2*c*e*(-15*b*d + 16*a*e + 7*b*e*x^2) + 4*c^2*(6*d
```

```
^2 - 3*d*e*x^2 + 2*e^2*x^4)) - 24*c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTan
h[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*
Sqrt[a + b*x^2 + c*x^4])]))/(96*c^(3/2)*e^4)
```

IntegrateAlgebraic [A] time = 1.42, size = 291, normalized size = 1.08

$$\frac{\sqrt{a+bx^2+cx^4}\left(32ace^2+3b^2e^2-30bcde+14bce^2x^2+24c^2d^2-12c^2dex^2+8c^2e^2x^4\right)}{48ce^3}+\frac{\left(-12abce^3+24ac^2de^2+b^3e^3+6b^2cde^2-24bc^2d^2e+16c^3d^3\right)\log\left(-2c^{3/2}\sqrt{a+bx^2+cx^4}+bc+2c^2x^2\right)}{32c^{3/2}e^4}+\frac{\sqrt{-ae^2+bde-cd^2}\left(ae^2-bde+cd^2\right)\tan^{-1}\left(\frac{-e\sqrt{a+bx^2+cx^4}+\sqrt{c}d+\sqrt{c}ex^2}{\sqrt{-ae^2+bde-cd^2}}\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(24*c^2*d^2 - 30*b*c*d*e + 3*b^2*e^2 + 32*a*c*e^2
- 12*c^2*d*e*x^2 + 14*b*c*e^2*x^2 + 8*c^2*e^2*x^4))/(48*c*e^3) + (Sqrt[-(c*
d^2) + b*d*e - a*e^2]*(c*d^2 - b*d*e + a*e^2)*ArcTan[(Sqrt[c]*d + Sqrt[c]*e
*x^2 - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/e^4 + ((
16*c^3*d^3 - 24*b*c^2*d^2*e + 6*b^2*c*d*e^2 + 24*a*c^2*d*e^2 + b^3*e^3 - 12
*a*b*c*e^3)*Log[b*c + 2*c^2*x^2 - 2*c^(3/2)*Sqrt[a + b*x^2 + c*x^4]])/(32*c
^(3/2)*e^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 1411, normalized size = 5.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x)
```

```
[Out] 1/6/e*c*x^4*(c*x^4+b*x^2+a)^(1/2)+7/24/e*b*x^2*(c*x^4+b*x^2+a)^(1/2)+1/16/e
/c*b^2*(c*x^4+b*x^2+a)^(1/2)-5/8/e^2*b*(c*x^4+b*x^2+a)^(1/2)*d+1/2/e^3*c*(c
*x^4+b*x^2+a)^(1/2)*d^2-1/32/e*b^3/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+
b*x^2+a)^(1/2))-3/4/e^2*a*d*c^(1/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a
)^(1/2))-3/16/e^2*b^2*d*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(
1/2)+3/4/e^3*b*c^(1/2)*d^2*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+
2/3/e*a*(c*x^4+b*x^2+a)^(1/2)-1/4/e^2*x^2*c*(c*x^4+b*x^2+a)^(1/2)*d+3/8/e*a
*b*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-1/2/e^4*c^(3/2)*
d^3*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/2/e/((a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a^2+1/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2))/(x^2+d/e))*a*b*d-1/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-
```

```
2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/
(x^2+d/e))*a*c*d^2-1/2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*
(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x
^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/
e))*b^2*d^2+1/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)
/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2
*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b*c*d
^3-1/2/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a
*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e
-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c^2*d^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` for
more details)Is a*e^2-b*d*e                             +c*d^2 zero or nonze
ro?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(c\,x^4+b\,x^2+a\right)^{3/2}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x)
```

```
[Out] int((x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+bx^{2}+cx^{4}\right)^{\frac{3}{2}}}{d+ex^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
```

```
[Out] Integral(x*(a + b*x**2 + c*x**4)**(3/2)/(d + e*x**2), x)
```

3.241 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x\left(d+ex^2\right)}\,dx$

Optimal. Leaf size=350

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{\left(-12cde(bd-ae)+be^2(3bd-4ae)+8c^2d^3\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}\,de^3}-\sqrt{a+bx^2+}$$

Rubi [A] time = 0.57, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1251, 895, 734, 843, 621, 206, 724, 814}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{\left(-12cde(bd-ae)+be^2(3bd-4ae)+8c^2d^3\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}\,de^3}-\frac{\sqrt{a+bx^2+cx^4}\left(-e(5bd-4ae)+4cd^2-2cdex^2\right)}{8de^2}-\frac{\left(ae^2-bde+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2de^3}+\frac{a\sqrt{a+bx^2+cx^4}}{2d}+\frac{ab\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)),x]
```

```
[Out] (a*Sqrt[a + b*x^2 + c*x^4])/(2*d) - ((4*c*d^2 - e*(5*b*d - 4*a*e) - 2*c*d*e
*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*d*e^2) - (a^(3/2)*ArcTanh[(2*a + b*x^2)/(
2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d) + (a*b*ArcTanh[(b + 2*c*x^2)/(2*
Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*d) + ((8*c^2*d^3 + b*e^2*(3*b
*d - 4*a*e) - 12*c*d*e*(b*d - a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
 + b*x^2 + c*x^4])])/(16*Sqrt[c]*d*e^3) - ((c*d^2 - b*d*e + a*e^2)^(3/2)*Ar
cTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt
[a + b*x^2 + c*x^4])])/(2*d*e^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 895

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[(Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*(a + b*x + c*x^2)^(p -
1))/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x\left(d+ex^2\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x(d+ex)}\,dx, x, x^2\right)$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{(-bd+ae-cdx)\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)}{2d} + \frac{a\operatorname{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx, x, x^2\right)}{2d}$$

$$= \frac{a\sqrt{a+bx^2+cx^4}}{2d} - \frac{\left(4cd^2-e(5bd-4ae)-2cdex^2\right)\sqrt{a+bx^2+cx^4}}{8de^2} - \frac{a\operatorname{Subst}\left(\int \frac{-2}{x\sqrt{a-}}\right.}{4}$$

$$= \frac{a\sqrt{a+bx^2+cx^4}}{2d} - \frac{\left(4cd^2-e(5bd-4ae)-2cdex^2\right)\sqrt{a+bx^2+cx^4}}{8de^2} + \frac{a^2\operatorname{Subst}\left(\int \frac{}{x\sqrt{}}\right.}{}$$

$$= \frac{a\sqrt{a+bx^2+cx^4}}{2d} - \frac{\left(4cd^2-e(5bd-4ae)-2cdex^2\right)\sqrt{a+bx^2+cx^4}}{8de^2} - \frac{a^2\operatorname{Subst}\left(\int \frac{1}{4a-}\right.}{}$$

$$= \frac{a\sqrt{a+bx^2+cx^4}}{2d} - \frac{\left(4cd^2-e(5bd-4ae)-2cdex^2\right)\sqrt{a+bx^2+cx^4}}{8de^2} - \frac{a^{3/2}\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.}{2d}$$

Mathematica [A] time = 0.53, size = 251, normalized size = 0.72

$$\frac{1}{16}\left(-\frac{8a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{d} + \frac{\left(12ce(ae-bd)+3b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}\,e^3} + \frac{2\left(4\left(e(ae-bd)+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)+de\sqrt{a+bx^2+cx^4}\left(5be-4cd+2cex^2\right)\right)}{de^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)),x]
```

```
[Out] ((-8*a^(3/2)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/d
+ ((8*c^2*d^2 + 3*b^2*e^2 + 12*c*e*(-(b*d) + a*e))*ArcTanh[(b + 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[c]*e^3) + (2*(d*e*(-4*c*d + 5*b*e
 + 2*c*e*x^2)*Sqrt[a + b*x^2 + c*x^4] + 4*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*
ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) +
a*e)]*Sqrt[a + b*x^2 + c*x^4])]))/(d*e^3))/16
```

IntegrateAlgebraic [A] time = 1.28, size = 267, normalized size = 0.76

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}-\frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{d} + \frac{\left(-12ace^2-3b^2e^2+12bcde-8c^2d^2\right)\log\left(-2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{16\sqrt{c}\,e^3} - \frac{\sqrt{-ae^2+bde-cd^2}\left(ae^2-bde+cd^2\right)\tan^{-1}\left(\frac{-e\sqrt{a+bx^2+cx^4}+\sqrt{c}d+\sqrt{c}ex^2}{\sqrt{-ae^2+bde-cd^2}}\right)}{de^3} + \frac{\sqrt{a+bx^2+cx^4}\left(5be-4cd+2cex^2\right)}{8e^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)),x]
```

```
[Out] ((-4*c*d + 5*b*e + 2*c*e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*e^2) - (Sqrt[-(c*
d^2) + b*d*e - a*e^2]*(c*d^2 - b*d*e + a*e^2)*ArcTan[(Sqrt[c]*d + Sqrt[c]*e
*x^2 - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(d*e^3)
+ (a^(3/2)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a] - Sqrt[a + b*x^2 + c*x^4]/Sqrt[a]]
)/d + ((-8*c^2*d^2 + 12*b*c*d*e - 3*b^2*e^2 - 12*a*c*e^2)*Log[b + 2*c*x^2 -
 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(16*Sqrt[c]*e^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.04, size = 1270, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x)
```

```
[Out] 5/8/e*b*(c*x^4+b*x^2+a)^(1/2)-1/2/d*a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)
^(1/2)*a^(1/2))/x^2)-1/2/e^2*d*c*(c*x^4+b*x^2+a)^(1/2)+3/4/e*a*c^(1/2)*ln((
c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+3/16/e*b^2*ln((c*x^2+1/2*b)/c^(
1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+1/4/e*x^2*c*(c*x^4+b*x^2+a)^(1/2)+1/2/e
^3*d^2*c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/2/d/((a*e^
2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)
/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)
/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a^2+1/e^2*d/((a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a*c-1/e^3*d^2/((a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^
2)/e^2)^(1/2))/(x^2+d/e))*b*c-3/4/e^2*d*b*c^(1/2)*ln((c*x^2+1/2*b)/c^(1/2)+
(c*x^4+b*x^2+a)^(1/2))-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*
(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x
^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/
e))*a*b+1/2/e^2*d/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)
/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2
*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b^2+1
/2/e^4*d^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a
*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e
-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{\left(ex^2 + d\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4 + b\,x^2 + a\right)^{3/2}}{x\,\left(e\,x^2 + d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)),x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x\left(d + ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x/(e*x**2+d),x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/(x*(d + e*x**2)), x)
```

3.242 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)}\,dx$

Optimal. Leaf size=562

$$\frac{a^{3/2}e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2}+\frac{be\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}d^2}+\frac{\sqrt{a+bx^2+cx^4}\left(-2ce(5bd-4ae)+b^2\right.}{16cd^2e}$$

Rubi [A] time = 0.92, antiderivative size = 562, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {1251, 960, 732, 814, 843, 621, 206, 724, 734}

$$\frac{a^{3/2}e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2}+\frac{\sqrt{a+bx^2+cx^4}\left(-2ce(5bd-4ae)+b^2e^2-2cex^2(2cd-be)+8c^2d^2\right)}{16cd^2e}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}d^2e^2}+\frac{be\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}d^2}-\frac{e\left(8ac+b^2+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16cd^2}+\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}\,d}+\frac{\left(ae^2-bde+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2e^2}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{2dx^2}+\frac{3\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8d}-\frac{3\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)),x]
```

```
[Out] (3*(3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*d) - (e*(b^2 + 8*a*c + 2*b*c
*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c*d^2) + ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(
5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c*d^
2*e) - (a + b*x^2 + c*x^4)^(3/2)/(2*d*x^2) - (3*Sqrt[a]*b*ArcTanh[(2*a + b*
x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*d) + (a^(3/2)*e*ArcTanh[(2*a
+ b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2) + (3*(b^2 + 4*a*c)*A
rcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*Sqrt[c]*d) +
 (b*(b^2 - 12*a*c)*e*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(32*c^(3/2)*d^2) - ((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d
 - 3*a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*
c^(3/2)*d^2*e^2) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2
*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/
(2*d^2*e^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
```

```
b, c, d, e, m, p, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^3\left(d+ex^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^2(d+ex)}\,dx, x, x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{\left(a+bx+cx^2\right)^{3/2}}{dx^2} - \frac{e\left(a+bx+cx^2\right)^{3/2}}{d^2x} + \frac{e^2\left(a+bx+cx^2\right)^{3/2}}{d^2(d+ex)}\right) dx, x, \\
&= \frac{\operatorname{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^2}\,dx, x, x^2\right)}{2d} - \frac{e\operatorname{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x}\,dx, x, x^2\right)}{2d^2} + \frac{e^2\operatorname{Subst}\left(\int\right.}{} \\
&= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{2dx^2} + \frac{3\operatorname{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x}\,dx, x, x^2\right)}{4d} + \frac{e\operatorname{Subst}\left(\int \frac{(-2a-bx)\sqrt{}}{x}\right.}{4d} \\
&= \frac{3\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8d} - \frac{e\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16cd^2} + \frac{\left(8c^2d^2+\right.}{} \\
&= \frac{3\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8d} - \frac{e\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16cd^2} + \frac{\left(8c^2d^2+\right.}{} \\
&= \frac{3\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8d} - \frac{e\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16cd^2} + \frac{\left(8c^2d^2+\right.}{} \\
&= \frac{3\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8d} - \frac{e\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16cd^2} + \frac{\left(8c^2d^2+\right.}{}
\end{aligned}
$$

<u>Mathematica</u> [A] time = 0.50, size = 240, normalized size = 0.43

$$
\frac{1}{4}\left(-\frac{2\left(x^2\left(e(ae-bd)+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)+de\sqrt{a+bx^2+cx^4}\left(ae-cdx^2\right)\right)}{d^2e^2x^2}+\frac{\sqrt{a}(2ae-3bd)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{d^2}-\frac{\sqrt{c}(2cd-3be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{e^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)),x]
```

```
[Out] ((Sqrt[a]*(-3*b*d + 2*a*e)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2
+ c*x^4])])/d^2 - (Sqrt[c]*(2*c*d - 3*b*e)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]
*Sqrt[a + b*x^2 + c*x^4])])/e^2 - (2*(d*e*(a*e - c*d*x^2)*Sqrt[a + b*x^2 +
c*x^4] + (c*d^2 + e*(-(b*d) + a*e))^(3/2)*x^2*ArcTanh[(-(b*d) + 2*a*e - 2*c
*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4]
)]))/(d^2*e^2*x^2))/4
```

<u>IntegrateAlgebraic</u> [A] time = 1.29, size = 261, normalized size = 0.46

$$
\frac{\left(2a^{3/2}e-3\sqrt{a}\,bd\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2+cx^4}-\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2d^2}+\frac{\left(2c^{3/2}d-3b\sqrt{c}\,e\right)\log\left(-2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{4e^2}+\frac{\sqrt{-ae^2+bde-cd^2}\left(ae^2-bde+cd^2\right)\tan^{-1}\left(\frac{-e\sqrt{a+bx^2+cx^4}+\sqrt{c}\,d+\sqrt{c}\,ex^2}{\sqrt{-ae^2+bde-cd^2}}\right)}{d^2e^2}+\frac{\sqrt{a+bx^2+cx^4}\left(cdx^2-ae\right)}{2dex^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)),x]
```

```
[Out] ((-(a*e) + c*d*x^2)*Sqrt[a + b*x^2 + c*x^4])/(2*d*e*x^2) + (Sqrt[-(c*d^2) +
 b*d*e - a*e^2]*(c*d^2 - b*d*e + a*e^2)*ArcTan[(Sqrt[c]*d + Sqrt[c]*e*x^2 -
 e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(d^2*e^2) + ((
-3*Sqrt[a]*b*d + 2*a^(3/2)*e)*ArcTanh[(-(Sqrt[c]*x^2) + Sqrt[a + b*x^2 + c*
x^4])/Sqrt[a]])/(2*d^2) + ((2*c^(3/2)*d - 3*b*Sqrt[c]*e)*Log[b + 2*c*x^2 -
2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(4*e^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.03, size = 1207, normalized size = 2.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x)
```

```
[Out] 1/2/e*c*(c*x^4+b*x^2+a)^(1/2)-3/4/d*a^(1/2)*b*ln((b*x^2+2*a+2*(c*x^4+b*x^2+
a)^(1/2)*a^(1/2))/x^2)-1/2/d*a/x^2*(c*x^4+b*x^2+a)^(1/2)+3/4/e*b*c^(1/2)*ln
((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/2/e^2*d*c^(3/2)*ln((c*x^2+1
/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))/(x^2+d/e))*b^2+1/2/d^2*e*a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(
1/2)*a^(1/2))/x^2)-1/2*e/d^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d
)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(
(x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+
d/e))*a^2+1/d/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2
*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(
b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a*b-1/e/(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c
*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2
+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a*c-1/2/e^3*d^2/((a*e^2-
b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e
^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e
+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c^2+1/e^2*d/((a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^
2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{\left(ex^2 + d\right)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4+b\,x^2+a\right)^{3/2}}{x^3\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)),x)
[Out] int((a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{x^3\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**3/(e*x**2+d),x)
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/(x**3*(d + e*x**2)), x)
```

3.243 $\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=173

$$-\frac{(be+2cd)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2}+\frac{d^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2\sqrt{ae^2-bde+cd^2}}+\frac{\sqrt{a+bx^2+cx^4}}{2ce}$$

Rubi [A] time = 0.31, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1251, 1653, 843, 621, 206, 724}

$$-\frac{(be+2cd)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2}+\frac{d^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2\sqrt{ae^2-bde+cd^2}}+\frac{\sqrt{a+bx^2+cx^4}}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c*e) - ((2*c*d + b*e)*ArcTanh[(b + 2*c*x^2)/(2*S
qrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2)*e^2) + (d^2*ArcTanh[(b*d - 2*a
*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x
^4])])/(2*e^2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{x^5}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{a+bx^2+cx^4}}{2ce} + \frac{\operatorname{Subst}\left(\int \frac{-\frac{1}{2}bde-\frac{1}{2}e(2cd+be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2ce^2}$$

$$= \frac{\sqrt{a+bx^2+cx^4}}{2ce} + \frac{d^2\operatorname{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2e^2} - \frac{(2cd+be)\operatorname{Subs}}{}$$

$$= \frac{\sqrt{a+bx^2+cx^4}}{2ce} - \frac{d^2\operatorname{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)}{e^2} - \frac{(}{}$$

$$= \frac{\sqrt{a+bx^2+cx^4}}{2ce} - \frac{(2cd+be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2} + \frac{d^2\tanh^{-1}\left(\frac{bd-}{2\sqrt{cd^2-b}}\right)}{2e^2\sqrt{cd^2-}}$$

Mathematica [A] time = 0.37, size = 171, normalized size = 0.99

$$\frac{2\sqrt{c}\left(\frac{cd^2\tanh^{-1}\left(\frac{-2ae+bd-bex^2+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{\sqrt{ae^2-bde+cd^2}}+e\sqrt{a+bx^2+cx^4}\right)-(be+2cd)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-((2*c*d + b*e)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]
) + 2*Sqrt[c]*(e*Sqrt[a + b*x^2 + c*x^4] + (c*d^2*ArcTanh[(b*d - 2*a*e + 2*
c*d*x^2 - b*e*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])]
)/Sqrt[c*d^2 - b*d*e + a*e^2]))/(4*c^(3/2)*e^2)
```

IntegrateAlgebraic [A] time = 0.70, size = 230, normalized size = 1.33

$$\frac{(be+2cd)\log\left(-2c^{3/2}\sqrt{a+bx^2+cx^4}+bc+2c^2x^2\right)}{4c^{3/2}e^2}+\frac{d^2\sqrt{-ae^2+bde-cd^2}\,\tan^{-1}\left(\frac{\sqrt{c}ex^2}{\sqrt{-ae^2+bde-cd^2}}-\frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}d}{\sqrt{-ae^2+bde-cd^2}}\right)}{e^2\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{a+bx^2+cx^4}}{2ce}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c*e) + (d^2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTa
n[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x^2)/Sqrt[-(c*d^2
```

```
) + b*d*e - a*e^2] - (e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*
e^2]])/(e^2*(c*d^2 - b*d*e + a*e^2)) + ((2*c*d + b*e)*Log[b*c + 2*c^2*x^2 -
 2*c^(3/2)*Sqrt[a + b*x^2 + c*x^4]])/(4*c^(3/2)*e^2)
```

fricas [B] time = 58.18, size = 1364, normalized size = 7.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d^2*log(-((8*c^2*d^2 - 8*b*c*d*e +
(b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b
*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*s
qrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*
d*e*x^2 + d^2)) + (2*c^2*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c)*d*e^2)*s
qrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^
2 + b)*sqrt(c) - 4*a*c) + 4*(c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*sqrt(c*x^4 +
b*x^2 + a))/(c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2*e^4), 1/8*(4*sqrt(-c*d^2 + b
*d*e - a*e^2)*c^2*d^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d
*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2
)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) +
 (2*c^2*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c)*d*e^2)*sqrt(c)*log(-8*c^2
*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) -
4*a*c) + 4*(c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*sqrt(c*x^4 + b*x^2 + a))/(c^3*
d^2*e^2 - b*c^2*d*e^3 + a*c^2*e^4), 1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d^
2*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2
*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*
x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*
x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + (2*c^2*d^3 - b*c*d^2*e +
 a*b*e^3 - (b^2 - 2*a*c)*d*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)
*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(c^2*d^2*e - b*c*d*e
^2 + a*c*e^3)*sqrt(c*x^4 + b*x^2 + a))/(c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2*e
^4), 1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*c^2*d^2*arctan(-1/2*sqrt(c*x^4 + b
*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((
c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 -
 b^2*d*e + a*b*e^2)*x^2)) + (2*c^2*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c
)*d*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)
/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*sqrt(c*x^
4 + b*x^2 + a))/(c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2*e^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.02, size = 267, normalized size = 1.54

$$-\frac{d^{2} \ln \left(\frac{\frac{\left(b e-2 c d\right) \left(x^{2}+\frac{d}{e}\right)}{e}+\frac{2 a \,e^{2}-2 d e b+2 c \,d^{2}}{e^{2}}+2 \sqrt{\frac{a \,e^{2}-d e b+c \,d^{2}}{e^{2}}}\, \sqrt{\left(x^{2}+\frac{d}{e}\right)^{2} c+\frac{\left(b e-2 c d\right) \left(x^{2}+\frac{d}{e}\right)}{e}+\frac{a \,e^{2}-d e b+c \,d^{2}}{e^{2}}}}{x^{2}+\frac{d}{e}}\right)}{2 \sqrt{\frac{a \,e^{2}-d e b+c \,d^{2}}{e^{2}}}\, e^{3}}-\frac{b \ln \left(\frac{c \,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c \,x^{4}+b \,x^{2}+a}\right)}{4 c^{\frac{3}{2}} e}-\frac{d \ln \left(\frac{c \,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c \,x^{4}+b \,x^{2}+a}\right)}{2 \sqrt{c}\, e^{2}}+\frac{\sqrt{c \,x^{4}+b \,x^{2}+a}}{2 c e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*(c*x^4+b*x^2+a)^(1/2)/c/e-1/4/e*b/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x
^4+b*x^2+a)^(1/2))-1/2/e^2*d*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2)
)/c^(1/2)-1/2*d^2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+
d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/
e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.244 $\int \frac{x^3}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=137

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\,e}-\frac{d\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e\sqrt{ae^2-bde+cd^2}}$$

Rubi [A] time = 0.16, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\,e}-\frac{d\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[c]*e) -
(d*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]
*Sqrt[a + b*x^2 + c*x^4])])/(2*e*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)$$
$$=\frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2e}-\frac{d\,\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2e}$$
$$=\frac{\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{e}+\frac{d\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\frac{-bd+2a}{\sqrt{a}}\right.}{e}$$
$$=\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\,e}-\frac{d\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e\sqrt{cd^2-bde+ae^2}}$$

Mathematica [A] time = 0.12, size = 133, normalized size = 0.97

$$\frac{\frac{d\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/Sqrt[c] + (d*Ar
cTanh[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*
e)]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[c*d^2 + e*(-(b*d) + a*e)])/(2*e)
```

IntegrateAlgebraic [A] time = 0.54, size = 195, normalized size = 1.42

$$-\frac{d\sqrt{-ae^2+bde-cd^2}\,\tan^{-1}\left(\frac{\sqrt{c}\,ex^2}{\sqrt{-ae^2+bde-cd^2}}-\frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}\,d}{\sqrt{-ae^2+bde-cd^2}}\right)}{e\left(ae^2-bde+cd^2\right)}-\frac{\log\left(-2\sqrt{c}\,e\sqrt{a+bx^2+cx^4}+be+2cex^2\right)}{2\sqrt{c}\,e}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -((d*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*
e - a*e^2] + (Sqrt[c]*e*x^2)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b
*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(e*(c*d^2 - b*d*e + a*e^2))
) - Log[b*e + 2*c*e*x^2 - 2*Sqrt[c]*e*Sqrt[a + b*x^2 + c*x^4]]/(2*Sqrt[c]*e
)
```

fricas [B] time = 4.11, size = 1084, normalized size = 7.91

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*c*d*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 +
 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2
 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*
d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^
2 + d^2)) + (c*d^2 - b*d*e + a*e^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^
```

```
2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c))/(c^2*d^2*e -
b*c*d*e^2 + a*c*e^3), -1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*c*d*arctan(-1/2*
sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b
*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^
2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (c*d^2 - b*d*e + a*e^2)*sqrt(c)*l
og(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*s
qrt(c) - 4*a*c))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), 1/4*(sqrt(c*d^2 - b*d*e
 + a*e^2)*c*d*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b
*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 +
4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2
*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 2*(c*d^2 - b
*d*e + a*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqr
t(-c)/(c^2*x^4 + b*c*x^2 + a*c)))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), -1/2*(
sqrt(-c*d^2 + b*d*e - a*e^2)*c*d*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-
c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*
e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e
^2)*x^2)) + (c*d^2 - b*d*e + a*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2
+ a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)))/(c^2*d^2*e - b*c*d*
e^2 + a*c*e^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.01, size = 204, normalized size = 1.49

$$\frac{d \ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2a e^2-2deb+2c d^2}{e^2}+2\sqrt{\frac{a e^2-deb+c d^2}{e^2}}\,\sqrt{\left(x^2+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-deb+c d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{a e^2-deb+c d^2}{e^2}}\, e^2}+\frac{\ln\left(\frac{c x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{2\sqrt{c}\, e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2/e*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+1/2*d/e^2/((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d
^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d
/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(d + ex^2\right)\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.245 $\int \frac{x}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=86

$$\frac{\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\sqrt{ae^2-bde+cd^2}}$$

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1247, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sq
rt[a + b*x^2 + c*x^4])]/(2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= -\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{-bd+2ae-(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{cd^2-bde+ae^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 87, normalized size = 1.01

$$-\frac{\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)}{2\sqrt{e(ae-bd)+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*
d) + a*e)]*Sqrt[a + b*x^2 + c*x^4])]/Sqrt[c*d^2 + e*(-(b*d) + a*e)]
```

IntegrateAlgebraic [A] time = 0.38, size = 141, normalized size = 1.64

$$\frac{\sqrt{-ae^2+bde-cd^2}\tan^{-1}\left(\frac{\sqrt{c}ex^2}{\sqrt{-ae^2+bde-cd^2}}-\frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}d}{\sqrt{-ae^2+bde-cd^2}}\right)}{ae^2-bde+cd^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e -
a*e^2] + (Sqrt[c]*e*x^2)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x^2
 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(c*d^2 - b*d*e + a*e^2)
```

fricas [B] time = 1.19, size = 357, normalized size = 4.15

$$\left[\frac{\log\left(-\frac{\left(8c^2d^2-8bcde+\left(b^2+4ac\right)e^2\right)x^4-8abde+8a^2e^2+\left(b^2+4ac\right)d^2+2\left(4bcd^2+4abe^2-\left(3b^2+4ac\right)de\right)x^2+4\sqrt{cx^4+bx^2+a}\sqrt{cd^2-bde+ae^2}\left((2cd-be)x^2+bd-2ae\right)}{e^2x^4+2dex^2+d^2}\right)}{4\sqrt{cd^2-bde+ae^2}}, \frac{\sqrt{-cd^2+bde-ae^2}\arctan\left(-\frac{\sqrt{cx^4+bx^2+a}\sqrt{-cd^2+bde-ae^2}\left((2cd-be)x^2+bd-2ae\right)}{2\left(\left(c^2d^2-bcde+ace^2\right)x^4+acd^2-abde+a^2e^2+\left(bcd^2-b^2de+abe^2\right)x^2\right)}\right)}{2\left(cd^2-bde+ae^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*
a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*
e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*
e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2))/sqrt(c*d^2 - b*d*e + a*
e^2), 1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*
sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 -
b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e +
 a*b*e^2)*x^2))/(c*d^2 - b*d*e + a*e^2)]
```

giac [A] time = 0.49, size = 75, normalized size = 0.87

$$\frac{\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2
 + b*d*e - a*e^2))/sqrt(-c*d^2 + b*d*e - a*e^2)
```

maple [B] time = 0.01, size = 165, normalized size = 1.92

$$-\frac{\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2a\,e^2-2deb+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-deb+c\,d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2
-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c
*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.246 $\int \frac{1}{x(d+ex^2)\sqrt{a+bx^2+cx^4}} \, dx$

Optimal. Leaf size=138

$$-\frac{e \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d\sqrt{ae^2-bde+cd^2}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\,d}$$

Rubi [A] time = 0.19, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1251, 960, 724, 206}

$$-\frac{e \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d\sqrt{ae^2-bde+cd^2}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[a]*d) -
 (e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2
]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{dx\sqrt{a+bx+cx^2}} - \frac{e}{d(d+ex)\sqrt{a+bx+cx^2}}\right)dx, x, x^2\right)$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2d} - \frac{e\operatorname{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2d}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{d} + \frac{e\operatorname{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2a}{\sqrt{a}}\right.}{d}$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\,d} - \frac{e\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d\sqrt{cd^2-bde+ae^2}}$$

Mathematica [A] time = 0.14, size = 134, normalized size = 0.97

$$-\frac{\frac{e\tanh^{-1}\left(\frac{-2ae+b\left(d-ex^2\right)+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}} + \frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{a}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*(ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/Sqrt[a] +
(e*ArcTanh[(-2*a*e + 2*c*d*x^2 + b*(d - e*x^2))/(2*Sqrt[c*d^2 + e*(-(b*d) +
 a*e)]*Sqrt[a + b*x^2 + c*x^4])]])/Sqrt[c*d^2 + e*(-(b*d) + a*e)])/d
```

IntegrateAlgebraic [A] time = 0.52, size = 195, normalized size = 1.41

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}} - \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{e\sqrt{-ae^2+bde-cd^2}\,\tan^{-1}\left(\frac{\sqrt{c}\,ex^2}{\sqrt{-ae^2+bde-cd^2}} - \frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2+bde-cd^2}} + \frac{\sqrt{c}\,d}{\sqrt{-ae^2+bde-cd^2}}\right)}{d\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -((e*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*
e - a*e^2] + (Sqrt[c]*e*x^2)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b
*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(d*(c*d^2 - b*d*e + a*e^2))
) + ArcTanh[(Sqrt[c]*x^2)/Sqrt[a] - Sqrt[a + b*x^2 + c*x^4]/Sqrt[a]]/(Sqrt[
a]*d)
```

fricas [B] time = 1.74, size = 1097, normalized size = 7.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*a*e*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 +
 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2
 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*
d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^
```

```
2 + d^2)) + (c*d^2 - b*d*e + a*e^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b
*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4))/(a*c*
d^3 - a*b*d^2*e + a^2*d*e^2), -1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*a*e*arct
an(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)
*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e
+ a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (c*d^2 - b*d*e + a*e^2)*s
qrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x
^2 + 2*a)*sqrt(a) + 8*a^2)/x^4))/(a*c*d^3 - a*b*d^2*e + a^2*d*e^2), 1/4*(sq
rt(c*d^2 - b*d*e + a*e^2)*a*e*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*
e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b
*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*
d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)
) + 2*(c*d^2 - b*d*e + a*e^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(
b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)))/(a*c*d^3 - a*b*d^2*e + a^
2*d*e^2), -1/2*(sqrt(-c*d^2 + b*d*e - a*e^2)*a*e*arctan(-1/2*sqrt(c*x^4 + b
*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((
c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 -
 b^2*d*e + a*b*e^2)*x^2)) - (c*d^2 - b*d*e + a*e^2)*sqrt(-a)*arctan(1/2*sqr
t(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)))/(a*
c*d^3 - a*b*d^2*e + a^2*d*e^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.01, size = 207, normalized size = 1.50

$$\frac{\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2a\,e^2-2deb+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\,\sqrt{\left(x^2+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-deb+c\,d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\;d}-\frac{\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\;\sqrt{a}}{x^2}\right)}{2\sqrt{a}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2/d/a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)+1/2/d/((
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*
d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+
d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(1/(x*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.247 $\int \frac{1}{x^3(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=218

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}d}+\frac{e^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2\sqrt{ae^2-bde+cd^2}}+\frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\,d^2}-\frac{\sqrt{a+bx^2+cx^4}}{2adx^2}$$

Rubi [A] time = 0.27, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 960, 730, 724, 206}

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}d}+\frac{e^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2\sqrt{ae^2-bde+cd^2}}+\frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\,d^2}-\frac{\sqrt{a+bx^2+cx^4}}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*a*d*x^2) + (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*
Sqrt[a + b*x^2 + c*x^4])])/(4*a^(3/2)*d) + (e*ArcTanh[(2*a + b*x^2)/(2*Sqrt
[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a]*d^2) + (e^2*ArcTanh[(b*d - 2*a*e
+ (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]
)])/(2*d^2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
```

```
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{1}{dx^2\sqrt{a+bx+cx^2}}-\frac{e}{d^2x\sqrt{a+bx+cx^2}}+\frac{e^2}{d^2(d+ex)\sqrt{a+bx}}\right.\right.\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2d}-\frac{e\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2d^2}+\frac{e^2\operatorname{Sub}}{}\\ &= -\frac{\sqrt{a+bx^2+cx^4}}{2adx^2}-\frac{b\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4ad}+\frac{e\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx,\right.}{d^2}\\ &= -\frac{\sqrt{a+bx^2+cx^4}}{2adx^2}+\frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\,d^2}+\frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-}{2\sqrt{cd^2-bde+ae^2}\sqrt{a}}\right.}{2d^2\sqrt{cd^2-bde+a}}\\ &= -\frac{\sqrt{a+bx^2+cx^4}}{2adx^2}+\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}d}+\frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\,d^2}\end{aligned}$$

Mathematica [A] time = 0.37, size = 175, normalized size = 0.80

$$\frac{2\sqrt{a}\left(\frac{ae^2\tanh^{-1}\left(\frac{-2ae+bd-bex^2+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{\sqrt{ae^2-bde+cd^2}}-\frac{d\sqrt{a+bx^2+cx^4}}{x^2}\right)+(2ae+bd)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((b*d + 2*a*e)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])] +
 2*Sqrt[a]*(-((d*Sqrt[a + b*x^2 + c*x^4])/x^2) + (a*e^2*ArcTanh[(b*d - 2*a*
e + 2*c*d*x^2 - b*e*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*
x^4])])/Sqrt[c*d^2 - b*d*e + a*e^2]))/(4*a^(3/2)*d^2)
```

IntegrateAlgebraic [A] time = 0.82, size = 237, normalized size = 1.09

$$\frac{(-2ae-bd)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}-\frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{2a^{3/2}d^2}+\frac{e^2\sqrt{-ae^2+bde-cd^2}\,\tan^{-1}\left(\frac{\sqrt{c}ex^2}{\sqrt{-ae^2+bde-cd^2}}-\frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}d}{\sqrt{-ae^2+bde-cd^2}}\right)}{d^2\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{a+bx^2+cx^4}}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*Sqrt[a + b*x^2 + c*x^4]/(a*d*x^2) + (e^2*Sqrt[-(c*d^2) + b*d*e - a*e^2
]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x^2)/Sqrt[
-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d
*e - a*e^2]])/(d^2*(c*d^2 - b*d*e + a*e^2)) + ((-(b*d) - 2*a*e)*ArcTanh[(Sq
rt[c]*x^2)/Sqrt[a] - Sqrt[a + b*x^2 + c*x^4]/Sqrt[a]])/(2*a^(3/2)*d^2)
```

fricas [A] time = 2.78, size = 1414, normalized size = 6.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(c*d^2 - b*d*e + a*e^2)*a^2*e^2*x^2*log(-((8*c^2*d^2 - 8*b*c*d*
e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*
(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 +
a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4
+ 2*d*e*x^2 + d^2)) + (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)*d^2*
e)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 +
 a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(a*c*d^3 - a*b*d^2*e + a^2*d*e^
2)*sqrt(c*x^4 + b*x^2 + a))/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2)*x^2),
1/8*(4*sqrt(-c*d^2 + b*d*e - a*e^2)*a^2*e^2*x^2*arctan(-1/2*sqrt(c*x^4 + b*
x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c
^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 -
b^2*d*e + a*b*e^2)*x^2)) + (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)
*d^2*e)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*
x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(a*c*d^3 - a*b*d^2*e + a^2
*d*e^2)*sqrt(c*x^4 + b*x^2 + a))/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2)*x
^2), 1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*a^2*e^2*x^2*log(-((8*c^2*d^2 - 8*b*c*
d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 +
2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2
+ a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^
4 + 2*d*e*x^2 + d^2)) - (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)*d^
2*e)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)
/(a*c*x^4 + a*b*x^2 + a^2)) - 2*(a*c*d^3 - a*b*d^2*e + a^2*d*e^2)*sqrt(c*x^
4 + b*x^2 + a))/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2)*x^2), 1/4*(2*sqrt(
-c*d^2 + b*d*e - a*e^2)*a^2*e^2*x^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqr
t(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c
*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*
b*e^2)*x^2)) - (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)*d^2*e)*sqrt
(-a)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4
 + a*b*x^2 + a^2)) - 2*(a*c*d^3 - a*b*d^2*e + a^2*d*e^2)*sqrt(c*x^4 + b*x^2
 + a))/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2)*x^2)]
```

giac [A] time = 0.49, size = 208, normalized size = 0.95

$$\frac{\arctan\left(-\frac{\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2+bde-ae^2}}\right)e^2}{\sqrt{-cd^2+bde-ae^2}\,d^2}-\frac{(bd+2\,ae)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,ad^2}+\frac{\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)b+2\,a\sqrt{c}}{2\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2
 + b*d*e - a*e^2))*e^2/(sqrt(-c*d^2 + b*d*e - a*e^2)*d^2) - 1/2*(b*d + 2*a*
e)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a*d^
2) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a*sqrt(c))/(((sqrt(
c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*a*d)
```

maple [A] time = 0.01, size = 276, normalized size = 1.27

$$-\frac{e\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2a\,e^2-2deb+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\,\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-deb+c\,d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\,d^2}+\frac{e\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{2\sqrt{a}\,d^2}+\frac{b\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{4a^{\frac{3}{2}}d}-\frac{\sqrt{c\,x^4+b\,x^2+a}}{2ad\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2/d^2*e/a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-1/2*e
/d^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b
*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d
)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))-1/2*(c*x^4+b*x^2+a
)^(1/2)/a/d/x^2+1/4/d*b/a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/
2))/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}\,(ex^2+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.248 $\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=236

$$\frac{x^2\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)}{c\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}e}-\frac{d^3\tanh^{-1}\left(\frac{-2ae+x^2(2cd-\cdots}{2\sqrt{a+bx^2+cx^4}\sqrt{a\cdots}}\right)}{2e\left(ae^2-bde+cd^2\cdots\right.}$$

Rubi [A] time = 0.47, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1251, 1646, 843, 621, 206, 724}

$$\frac{x^2\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)}{c\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}e}-\frac{d^3\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e\left(ae^2-bde+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (a*(b^2*d - 2*a*c*d - a*b*e) + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*x^
2)/(c*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) + ArcT
anh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*c^(3/2)*e) - (d^3
*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*S
qrt[a + b*x^2 + c*x^4]])/(2*e*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
```

```
gerQ[(m - 1)/2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{x^7}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{a\left(b^2d-2acd-abe\right)+\left(b^3d-3abcd-ab^2e+2a^2ce\right)x^2}{c\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{\frac{\left(b^2-4ac\right)d(b}{2c\left(cd^2-bde-\right.}}{(d+ex)}\right.}{b^2}$$

$$= \frac{a\left(b^2d-2acd-abe\right)+\left(b^3d-3abcd-ab^2e+2a^2ce\right)x^2}{c\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+c}}\right.}{2ce}$$

$$= \frac{a\left(b^2d-2acd-abe\right)+\left(b^3d-3abcd-ab^2e+2a^2ce\right)x^2}{c\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} + \frac{\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx\right.}{ce}$$

$$= \frac{a\left(b^2d-2acd-abe\right)+\left(b^3d-3abcd-ab^2e+2a^2ce\right)x^2}{c\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} + \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx}}\right.}{2c^{3/2}e}$$

Mathematica [A] time = 0.82, size = 271, normalized size = 1.15

$$\frac{1}{2}\left(\frac{2\left(a^2\left(be+2c\left(d-ex^2\right)\right)+ab\left(-bd+bex^2+3cdx^2\right)+b^3(-d)x^2\right)}{c\left(4ac-b^2\right)\sqrt{a+bx^2+cx^4}\left(e(ae-bd)+cd^2\right)}+\frac{\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{c^{3/2}e}+\frac{d^3\log\left(2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}+2ae-bd+bex^2-2cdx^2\right)}{e\left(e(ae-bd)+cd^2\right)^{3/2}}-\frac{d^3\log\left(d+ex^2\right)}{e\left(e(ae-bd)+cd^2\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] ((2*(-(b^3*d*x^2) + a*b*(-(b*d) + 3*c*d*x^2 + b*e*x^2) + a^2*(b*e + 2*c*(d
- e*x^2))))/(c*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + b*x^2 + c
*x^4]) - (d^3*Log[d + e*x^2])/(e*(c*d^2 + e*(-(b*d) + a*e))^(3/2)) + Log[b
+ 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]/(c^(3/2)*e) + (d^3*Log[-(b*d
) + 2*a*e - 2*c*d*x^2 + b*e*x^2 + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*
x^2 + c*x^4]])/(e*(c*d^2 + e*(-(b*d) + a*e))^(3/2)))/2
```

IntegrateAlgebraic [A] time = 1.21, size = 308, normalized size = 1.31

$$\frac{a^2be+2a^2cd-2a^2cex^2-ab^2d+ab^2ex^2+3abcdx^2+b^3(-d)x^2}{c\left(4ac-b^2\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}-\frac{\log\left(-2c^{3/2}e\sqrt{a+bx^2+cx^4}+bce+2c^2ex^2\right)}{2c^{3/2}e}-\frac{d^3\sqrt{-ae^2+bde-cd^2}\tan^{-1}\left(\frac{\sqrt{c}ex^2}{\sqrt{-ae^2+bde-cd^2}}-\frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}d}{\sqrt{-ae^2+bde-cd^2}}\right)}{e\left(ae^2-bde+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-(a*b^2*d) + 2*a^2*c*d + a^2*b*e - b^3*d*x^2 + 3*a*b*c*d*x^2 + a*b^2*e*x^2
 - 2*a^2*c*e*x^2)/(c*(-b^2 + 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2
+ c*x^4]) - (d^3*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c
*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x^2)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (
e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(e*(c*d^2 - b*d
*e + a*e^2)^2) - Log[b*c*e + 2*c^2*e*x^2 - 2*c^(3/2)*e*Sqrt[a + b*x^2 + c*x
^4]]/(2*c^(3/2)*e)
```

fricas [B] time = 172.62, size = 4901, normalized size = 20.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b
^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^
3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d
^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2
)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2
*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*
(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(c)*log(-
8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(
c) - 4*a*c) + ((b^2*c^3 - 4*a*c^4)*d^3*x^4 + (b^3*c^2 - 4*a*b*c^3)*d^3*x^2
+ (a*b^2*c^2 - 4*a^2*c^3)*d^3)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2
 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*
c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4
 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)
)/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 4*(a^3*b*c*e^4 - (a*b^2*c^2 - 2*a^2*c^3)*d
^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 - 2*(a^2*b^2*c - a^3*c^2)*d*e^3 - ((b^
3*c^2 - 3*a*b*c^3)*d^3*e - (b^4*c - 2*a*b^2*c^2 - 2*a^2*c^3)*d^2*e^2 + (2*a
*b^3*c - 5*a^2*b*c^2)*d*e^3 - (a^2*b^2*c - 2*a^3*c^2)*e^4)*x^2)*sqrt(c*x^4
+ b*x^2 + a))/((a*b^2*c^4 - 4*a^2*c^5)*d^4*e - 2*(a*b^3*c^3 - 4*a^2*b*c^4)*
d^3*e^2 + (a*b^4*c^2 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*e^3 - 2*(a^2*b^3*c^2
- 4*a^3*b*c^3)*d*e^4 + (a^3*b^2*c^2 - 4*a^4*c^3)*e^5 + ((b^2*c^5 - 4*a*c^6)
*d^4*e - 2*(b^3*c^4 - 4*a*b*c^5)*d^3*e^2 + (b^4*c^3 - 2*a*b^2*c^4 - 8*a^2*c
^5)*d^2*e^3 - 2*(a*b^3*c^3 - 4*a^2*b*c^4)*d*e^4 + (a^2*b^2*c^3 - 4*a^3*c^4)
*e^5)*x^4 + ((b^3*c^4 - 4*a*b*c^5)*d^4*e - 2*(b^4*c^3 - 4*a*b^2*c^4)*d^3*e^
2 + (b^5*c^2 - 2*a*b^3*c^3 - 8*a^2*b*c^4)*d^2*e^3 - 2*(a*b^4*c^2 - 4*a^2*b^
2*c^3)*d*e^4 + (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^5)*x^2), -1/4*(2*((b^2*c^3 - 4
*a*c^4)*d^3*x^4 + (b^3*c^2 - 4*a*b*c^3)*d^3*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d
^3)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-
c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*
e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e
^2)*x^2)) - ((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e
+ (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3
 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*
c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2
*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d
^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^
2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(c)
*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)
*sqrt(c) - 4*a*c) + 4*(a^3*b*c*e^4 - (a*b^2*c^2 - 2*a^2*c^3)*d^3*e + (a*b^3
*c - a^2*b*c^2)*d^2*e^2 - 2*(a^2*b^2*c - a^3*c^2)*d*e^3 - ((b^3*c^2 - 3*a*b
*c^3)*d^3*e - (b^4*c - 2*a*b^2*c^2 - 2*a^2*c^3)*d^2*e^2 + (2*a*b^3*c - 5*a^
2*b*c^2)*d*e^3 - (a^2*b^2*c - 2*a^3*c^2)*e^4)*x^2)*sqrt(c*x^4 + b*x^2 + a))
/((a*b^2*c^4 - 4*a^2*c^5)*d^4*e - 2*(a*b^3*c^3 - 4*a^2*b*c^4)*d^3*e^2 + (a*
```

```
b^4*c^2 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*e^3 - 2*(a^2*b^3*c^2 - 4*a^3*b*c^3
)*d*e^4 + (a^3*b^2*c^2 - 4*a^4*c^3)*e^5 + ((b^2*c^5 - 4*a*c^6)*d^4*e - 2*(b
^3*c^4 - 4*a*b*c^5)*d^3*e^2 + (b^4*c^3 - 2*a*b^2*c^4 - 8*a^2*c^5)*d^2*e^3 -
 2*(a*b^3*c^3 - 4*a^2*b*c^4)*d*e^4 + (a^2*b^2*c^3 - 4*a^3*c^4)*e^5)*x^4 + (
(b^3*c^4 - 4*a*b*c^5)*d^4*e - 2*(b^4*c^3 - 4*a*b^2*c^4)*d^3*e^2 + (b^5*c^2
- 2*a*b^3*c^3 - 8*a^2*b*c^4)*d^2*e^3 - 2*(a*b^4*c^2 - 4*a^2*b^2*c^3)*d*e^4
+ (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^5)*x^2), -1/4*(2*((a*b^2*c^2 - 4*a^2*c^3)*d
^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^
2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3
 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*
a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2
)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (
b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a
^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(
2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - ((b^2*c^3 - 4*a*c^4)*d^3
*x^4 + (b^3*c^2 - 4*a*b*c^3)*d^3*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d^3)*sqrt(c*
d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4
- 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3
*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e
^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + 4*(a^
3*b*c*e^4 - (a*b^2*c^2 - 2*a^2*c^3)*d^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 -
 2*(a^2*b^2*c - a^3*c^2)*d*e^3 - ((b^3*c^2 - 3*a*b*c^3)*d^3*e - (b^4*c - 2*
a*b^2*c^2 - 2*a^2*c^3)*d^2*e^2 + (2*a*b^3*c - 5*a^2*b*c^2)*d*e^3 - (a^2*b^2
*c - 2*a^3*c^2)*e^4)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^4 - 4*a^2*c^5)
*d^4*e - 2*(a*b^3*c^3 - 4*a^2*b*c^4)*d^3*e^2 + (a*b^4*c^2 - 2*a^2*b^2*c^3 -
 8*a^3*c^4)*d^2*e^3 - 2*(a^2*b^3*c^2 - 4*a^3*b*c^3)*d*e^4 + (a^3*b^2*c^2 -
4*a^4*c^3)*e^5 + ((b^2*c^5 - 4*a*c^6)*d^4*e - 2*(b^3*c^4 - 4*a*b*c^5)*d^3*e
^2 + (b^4*c^3 - 2*a*b^2*c^4 - 8*a^2*c^5)*d^2*e^3 - 2*(a*b^3*c^3 - 4*a^2*b*c
^4)*d*e^4 + (a^2*b^2*c^3 - 4*a^3*c^4)*e^5)*x^4 + ((b^3*c^4 - 4*a*b*c^5)*d^4
*e - 2*(b^4*c^3 - 4*a*b^2*c^4)*d^3*e^2 + (b^5*c^2 - 2*a*b^3*c^3 - 8*a^2*b*c
^4)*d^2*e^3 - 2*(a*b^4*c^2 - 4*a^2*b^2*c^3)*d*e^4 + (a^2*b^3*c^2 - 4*a^3*b*
c^3)*e^5)*x^2), -1/2*(((b^2*c^3 - 4*a*c^4)*d^3*x^4 + (b^3*c^2 - 4*a*b*c^3)*
d^3*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d^3)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(
-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^
2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a
^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) + ((a*b^2*c^2 - 4*a^2*c^3)*d^4
 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*
e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 -
 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^
2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*
e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^
5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2
*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*
c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(a^3*b*c*e^4 - (a*b^2*c^
2 - 2*a^2*c^3)*d^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 - 2*(a^2*b^2*c - a^3*c
^2)*d*e^3 - ((b^3*c^2 - 3*a*b*c^3)*d^3*e - (b^4*c - 2*a*b^2*c^2 - 2*a^2*c^3
)*d^2*e^2 + (2*a*b^3*c - 5*a^2*b*c^2)*d*e^3 - (a^2*b^2*c - 2*a^3*c^2)*e^4)*
x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^4 - 4*a^2*c^5)*d^4*e - 2*(a*b^3*c^3
 - 4*a^2*b*c^4)*d^3*e^2 + (a*b^4*c^2 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*e^3 -
 2*(a^2*b^3*c^2 - 4*a^3*b*c^3)*d*e^4 + (a^3*b^2*c^2 - 4*a^4*c^3)*e^5 + ((b^
2*c^5 - 4*a*c^6)*d^4*e - 2*(b^3*c^4 - 4*a*b*c^5)*d^3*e^2 + (b^4*c^3 - 2*a*b
^2*c^4 - 8*a^2*c^5)*d^2*e^3 - 2*(a*b^3*c^3 - 4*a^2*b*c^4)*d*e^4 + (a^2*b^2*
c^3 - 4*a^3*c^4)*e^5)*x^4 + ((b^3*c^4 - 4*a*b*c^5)*d^4*e - 2*(b^4*c^3 - 4*a
*b^2*c^4)*d^3*e^2 + (b^5*c^2 - 2*a*b^3*c^3 - 8*a^2*b*c^4)*d^2*e^3 - 2*(a*b^
4*c^2 - 4*a^2*b^2*c^3)*d*e^4 + (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^5)*x^2)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.06, size = 720, normalized size = 3.05

$$-\frac{2 c d^{3} \ln \left(\frac{\frac{(b e-2 c d)\left(x^{2}+\frac{d}{e}\right)}{e}+\frac{2 a e^{2}-2 d e b+2 c d^{2}}{e^{2}}+2 \sqrt{\frac{a e^{2}-d e b+c d^{2}}{e^{2}}}\, \sqrt{\left(x^{2}+\frac{d}{e}\right)^{2} c+\frac{(b e-2 c d)\left(x^{2}+\frac{d}{e}\right)}{e}+\frac{a e^{2}-d e b+c d^{2}}{e^{2}}}}{x^{2}+\frac{d}{e}}\right)}{\left(-b e+2 c d+\sqrt{-4 a c+b^{2}}\, e\right)\left(b e-2 c d+\sqrt{-4 a c+b^{2}}\, e\right) \sqrt{\frac{a e^{2}-d e b+c d^{2}}{e^{2}}}\, e^{2}}+\frac{b^{2} x^{2}}{2\left(4 a c-b^{2}\right) \sqrt{c x^{4}+b x^{2}+a}\, c e}-\frac{2 \sqrt{\left(x^{2}+\frac{b+\sqrt{-4 a c+b^{2}}}{2 c}\right)^{2} c-\sqrt{-4 a c+b^{2}}\left(x^{2}+\frac{b+\sqrt{-4 a c+b^{2}}}{2 c}\right)}\, c d^{3}}{\left(b e-2 c d+\sqrt{-4 a c+b^{2}}\, e\right)\left(-4 a c+b^{2}\right)\left(x^{2}+\frac{b}{2 c}+\frac{\sqrt{-4 a c+b^{2}}}{2 c}\right) e^{3}}+\frac{2 \sqrt{\left(x^{2}-\frac{-b+\sqrt{-4 a c+b^{2}}}{2 c}\right)^{2} c+\sqrt{-4 a c+b^{2}}\left(x^{2}-\frac{-b+\sqrt{-4 a c+b^{2}}}{2 c}\right)}\, c d^{3}}{\left(-b e+2 c d+\sqrt{-4 a c+b^{2}}\, e\right)\left(-4 a c+b^{2}\right)\left(x^{2}+\frac{b}{2 c}-\frac{\sqrt{-4 a c+b^{2}}}{2 c}\right) e^{3}}+\frac{b^{3}}{4\left(4 a c-b^{2}\right) \sqrt{c x^{4}+b x^{2}+a}\, c^{2} e}-\frac{x^{2}}{2 \sqrt{c x^{4}+b x^{2}+a}\, c e}+\frac{\left(2 c x^{2}+b\right) d^{2}}{\left(4 a c-b^{2}\right) \sqrt{c x^{4}+b x^{2}+a}\, e^{3}}+\frac{\left(b x^{2}+2 a\right) d}{\sqrt{c x^{4}+b x^{2}+a}\,\left(4 a c-b^{2}\right) e^{2}}+\frac{\ln \left(\frac{c x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^{4}+b x^{2}+a}\right)}{2 c^{\frac{3}{2}} e}+\frac{b}{4 \sqrt{c x^{4}+b x^{2}+a}\, c^{2} e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/2/e*x^2/c/(c*x^4+b*x^2+a)^(1/2)+1/4/e*b/c^2/(c*x^4+b*x^2+a)^(1/2)+1/2/e*
b^2/c/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2+1/4/e*b^3/c^2/(4*a*c-b^2)/(c*x^
4+b*x^2+a)^(1/2)+1/2/e/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/
2))+d/e^2/(c*x^4+b*x^2+a)^(1/2)*(b*x^2+2*a)/(4*a*c-b^2)+d^2/e^3*(2*c*x^2+b)
/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)-2*d^3/e^3*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*
c*d)/(-4*a*c+b^2)/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)*(c*(x^2+1/2*(b+(-4
*a*c+b^2)^(1/2))/c)^2-(-4*a*c+b^2)^(1/2)*(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2)-2*d^3/e^2*c/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(e*(-4*a*c+b^2)^(1/2)+
b*e-2*c*d)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a
*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e
-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))+2*d^3/e^3*c/
(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(-4*a*c+b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)
^(1/2)/c)*(c*(x^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2+(-4*a*c+b^2)^(1/2)*(x^2-
1/2*(-b+(-4*a*c+b^2)^(1/2))/c))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{\left(c x^{4}+b x^{2}+a\right)^{\frac{3}{2}}\left(e x^{2}+d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{\left(e\,x^2+d\right)\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{\left(d + e x^{2}\right) \left(a + b x^{2} + c x^{4}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**7/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.249 $\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=167

$$\frac{d^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{x^2\left(-abe-2acd+b^2d\right)+a(bd-2ae)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}$$

Rubi [A] time = 0.29, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 1646, 12, 724, 206}

$$\frac{d^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{x^2\left(-abe-2acd+b^2d\right)+a(bd-2ae)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -((a*(b*d - 2*a*e) + (b^2*d - 2*a*c*d - a*b*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 -
 b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])) + (d^2*ArcTanh[(b*d - 2*a*e + (2*
c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(
2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
```

```
- ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{x^5}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{a(bd-2ae)+\left(b^2d-2acd-abe\right)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int -\frac{\left(b^2-\right.}{2\left(cd^2-bde+ae^2\right)(} \ldots\right.}{b^2-\ldots}$$

$$= -\frac{a(bd-2ae)+\left(b^2d-2acd-abe\right)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+\ldots}} \ldots\right.}{2\left(cd^2-bde+\ldots\right.}$$

$$= -\frac{a(bd-2ae)+\left(b^2d-2acd-abe\right)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} - \frac{d^2\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae\ldots} \ldots\right.}{cd^2-\ldots}$$

$$= -\frac{a(bd-2ae)+\left(b^2d-2acd-abe\right)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} + \frac{d^2\tanh^{-1}\left(\frac{bd-2ae+(2c\ldots}{2\sqrt{cd^2-bde+ae^2}\ldots}\right.}{2\left(cd^2-bde+ae\ldots\right.}$$

Mathematica [A] time = 0.62, size = 204, normalized size = 1.22

$$\frac{1}{2}\left(\frac{2\left(-2a^2e+ab\left(d-ex^2\right)-2acdx^2+b^2dx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(e(bd-ae)-cd^2\right)} - \frac{d^2\log\left(2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}+2ae-bd+bex^2-2cdx^2\right)}{\left(e(ae-bd)+cd^2\right)^{3/2}} + \frac{d^2\log\left(d+ex^2\right)}{\left(e(ae-bd)+cd^2\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] ((2*(-2*a^2*e + b^2*d*x^2 - 2*a*c*d*x^2 + a*b*(d - e*x^2)))/((b^2 - 4*a*c)*
(-(c*d^2) + e*(b*d - a*e))*Sqrt[a + b*x^2 + c*x^4]) + (d^2*Log[d + e*x^2])/
(c*d^2 + e*(-(b*d) + a*e))^(3/2) - (d^2*Log[-(b*d) + 2*a*e - 2*c*d*x^2 + b*
e*x^2 + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]])/(c*d^2 + e*
(-(b*d) + a*e))^(3/2))/2
```

IntegrateAlgebraic [A] time = 0.78, size = 226, normalized size = 1.35

$$\frac{-2a^2e+abd-abex^2-2acdx^2+b^2dx^2}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(-ae^2+bde-cd^2\right)} + \frac{d^2\sqrt{-ae^2+bde-cd^2}\,\tan^{-1}\left(\frac{\sqrt{c}ex^2}{\sqrt{-ae^2+bde-cd^2}} - \frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2+bde-cd^2}} + \frac{\sqrt{c}d}{\sqrt{-ae^2+bde-cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (a*b*d - 2*a^2*e + b^2*d*x^2 - 2*a*c*d*x^2 - a*b*e*x^2)/((b^2 - 4*a*c)*(-(c
*d^2) + b*d*e - a*e^2)*Sqrt[a + b*x^2 + c*x^4]) + (d^2*Sqrt[-(c*d^2) + b*d*
e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x
^2)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d
^2) + b*d*e - a*e^2]])/(c*d^2 - b*d*e + a*e^2)^2
```

fricas [B] time = 2.20, size = 1381, normalized size = 8.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*d^2*x^4 + (b^3 - 4*a*b*c)*d^2*x^2 + (a*b^2 - 4*a^2
*c)*d^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 +
4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2
+ 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d
^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2
 + d^2)) - 4*(a*b*c*d^3 + 3*a^2*b*d*e^2 - 2*a^3*e^3 - (a*b^2 + 2*a^2*c)*d^2
*e - (a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (b^3 - a*b*c)*d^2*e - 2*(a*b^2 -
a^2*c)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 -
2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2
 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*
a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c
^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4
)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 -
 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^
3 - 4*a^3*b*c)*e^4)*x^2), 1/2*(((b^2*c - 4*a*c^2)*d^2*x^4 + (b^3 - 4*a*b*c)
*d^2*x^2 + (a*b^2 - 4*a^2*c)*d^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*
sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b
*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^
2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - 2*(a*b*c*d^3 + 3*a^2*b*d*e^2 - 2*
a^3*e^3 - (a*b^2 + 2*a^2*c)*d^2*e - (a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (b
^3 - a*b*c)*d^2*e - 2*(a*b^2 - a^2*c)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/
((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2
*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2
- 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e +
 (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^
3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*
c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4
 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)]
```

giac [B] time = 0.72, size = 458, normalized size = 2.74

$$\frac{d^2\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\left(cd^2-bde+ae^2\right)\sqrt{-cd^2+bde-ae^2}}-\frac{\frac{\left(b^2cd^3-2ac^2d^3-b^3d^2e+abcd^2e+2ab^2de^2-2a^2cde^2-a^2be^3\right)x^2}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2b^2e^4-4a^3ce^4}+\frac{abcd^3-ab^2d^2e-2a^2cd^2e+3a^2bde^2-2a^3e^3}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2b^2e^4-4a^3ce^4}}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] d^2*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c
*d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2
)) - ((b^2*c*d^3 - 2*a*c^2*d^3 - b^3*d^2*e + a*b*c*d^2*e + 2*a*b^2*d*e^2 -
2*a^2*c*d*e^2 - a^2*b*e^3)*x^2/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e +
 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*
a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (a*b*c*d^3 - a
*b^2*d^2*e - 2*a^2*c*d^2*e + 3*a^2*b*d*e^2 - 2*a^3*e^3)/(b^2*c^2*d^4 - 4*a*
c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2
 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^
3*c*e^4))/sqrt(c*x^4 + b*x^2 + a)
```

maple [B] time = 0.02, size = 613, normalized size = 3.67

$$\frac{2cd^2\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2ae^2-2deb+2cd^2}{e^2}+2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{\left(-be+2cd+\sqrt{-4ac+b^2}\,e\right)\left(be-2cd+\sqrt{-4ac+b^2}\,e\right)\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\,e}-\frac{bx^2}{\sqrt{cx^4+bx^2+a}\,\left(4ac-b^2\right)e}-\frac{2cdx^2}{\left(4ac-b^2\right)\sqrt{cx^4+bx^2+a}\,e^2}+\frac{2\sqrt{\left(x^2+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)^2c-\sqrt{-4ac+b^2}\left(x^2+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)}\,cd^2}{\left(be-2cd+\sqrt{-4ac+b^2}\,e\right)\left(-4ac+b^2\right)\left(x^2+\frac{b}{2c}+\frac{\sqrt{-4ac+b^2}}{2c}\right)e^2}-\frac{2\sqrt{\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)^2c+\sqrt{-4ac+b^2}\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)}\,cd^2}{\left(-be+2cd+\sqrt{-4ac+b^2}\,e\right)\left(-4ac+b^2\right)\left(x^2+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}\right)e^2}-\frac{2a}{\sqrt{cx^4+bx^2+a}\,\left(4ac-b^2\right)e}-\frac{bd}{\left(4ac-b^2\right)\sqrt{cx^4+bx^2+a}\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/e/(c*x^4+b*x^2+a)^(1/2)/(4*a*c-b^2)*x^2*b-2/e/(c*x^4+b*x^2+a)^(1/2)/(4*a
*c-b^2)*a-2/e^2*d/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*c*x^2-1/e^2*d/(4*a*c-b^
2)/(c*x^4+b*x^2+a)^(1/2)*b+2*d^2/e^2*c/(b*e-2*c*d+(-4*a*c+b^2)^(1/2)*e)/(-4
*a*c+b^2)/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)*((x^2+1/2*(b+(-4*a*c+b^2)^
(1/2))/c)^2*c-(-4*a*c+b^2)^(1/2)*(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)+
2*d^2/e*c/(-b*e+2*c*d+(-4*a*c+b^2)^(1/2)*e)/(b*e-2*c*d+(-4*a*c+b^2)^(1/2)*e
)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*
e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(
x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))-2*d^2/e^2*c/(-b*e+2*c
*d+(-4*a*c+b^2)^(1/2)*e)/(-4*a*c+b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c
)*((x^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2*c+(-4*a*c+b^2)^(1/2)*(x^2-1/2*(-b+
(-4*a*c+b^2)^(1/2))/c))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x^5/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(e\,x^2+d\right)\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
[Out] int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral(x**5/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.250 $\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=159

$$\frac{cx^2(bd-2ae)+a(2cd-be)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}\,(ae^2-bde+cd^2)}-\frac{de\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2(ae^2-bde+cd^2)^{3/2}}$$

Rubi [A] time = 0.19, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 822, 12, 724, 206}

$$\frac{cx^2(bd-2ae)+a(2cd-be)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}\,(ae^2-bde+cd^2)}-\frac{de\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2(ae^2-bde+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (a*(2*c*d - b*e) + c*(b*d - 2*a*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)*Sqrt[a + b*x^2 + c*x^4]) - (d*e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^
2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*(c*d^2 - b*
d*e + a*e^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{a(2cd-be)+c(bd-2ae)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{(b^2-4ac)de}{2(d+ex)\sqrt{a+bx+cx^2}}\right.}{(b^2-4ac)(cd^2-bde}$$

$$= \frac{a(2cd-be)+c(bd-2ae)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} - \frac{(de)\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+}}\right.}{2(cd^2-bde+a}$$

$$= \frac{a(2cd-be)+c(bd-2ae)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} + \frac{(de)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4a}\right.}{cd^2-}$$

$$= \frac{a(2cd-be)+c(bd-2ae)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} - \frac{de\tanh^{-1}\left(\frac{bd-2ae+(2cd}{2\sqrt{cd^2-bde+ae^2}\sqrt{}}\right.}{2(cd^2-bde+ae^2)}$$

Mathematica [A] time = 0.17, size = 162, normalized size = 1.02

$$\frac{a(be-2cd+2cex^2)-bcdx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}\,(e(bd-ae)-cd^2)} + \frac{de\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)}{2(e(ae-bd)+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-(b*c*d*x^2) + a*(-2*c*d + b*e + 2*c*e*x^2))/((b^2 - 4*a*c)*(-(c*d^2) + e*
(b*d - a*e))*Sqrt[a + b*x^2 + c*x^4]) + (d*e*ArcTanh[(-(b*d) + 2*a*e - 2*c*
d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4])
])/(2*(c*d^2 + e*(-(b*d) + a*e))^(3/2))
```

IntegrateAlgebraic [A] time = 0.75, size = 218, normalized size = 1.37

$$\frac{abe-2acd+2acex^2-bcdx^2}{(4ac-b^2)\sqrt{a+bx^2+cx^4}\,(ae^2-bde+cd^2)} - \frac{de\sqrt{-ae^2+bde-cd^2}\,\tan^{-1}\left(\frac{\sqrt{c}\,ex^2}{\sqrt{-ae^2+bde-cd^2}} - \frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2+bde-cd^2}} + \frac{\sqrt{c}\,d}{\sqrt{-ae^2+bde-cd^2}}\right)}{(ae^2-bde+cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*a*c*d + a*b*e - b*c*d*x^2 + 2*a*c*e*x^2)/((-b^2 + 4*a*c)*(c*d^2 - b*d*e
 + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - (d*e*Sqrt[-(c*d^2) + b*d*e - a*e^2]*Ar
cTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x^2)/Sqrt[-(c*
d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e -
 a*e^2]])/(c*d^2 - b*d*e + a*e^2)^2
```

fricas [B] time = 2.25, size = 1349, normalized size = 8.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*d*e*x^4 + (b^3 - 4*a*b*c)*d*e*x^2 + (a*b^2 - 4*a^2
*c)*d*e)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 +
4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2
+ 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d
^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2
 + d^2)) + 4*(2*a*c^2*d^3 - 3*a*b*c*d^2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d
*e^2 + (b*c^2*d^3 + 3*a*b*c*d*e^2 - 2*a^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*
x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4
*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3
- 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2
*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 -
2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*
c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8
*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)
*e^4)*x^2), -1/2*(((b^2*c - 4*a*c^2)*d*e*x^4 + (b^3 - 4*a*b*c)*d*e*x^2 + (a
*b^2 - 4*a^2*c)*d*e)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 +
b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/(
(c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2
- b^2*d*e + a*b*e^2)*x^2)) - 2*(2*a*c^2*d^3 - 3*a*b*c*d^2*e - a^2*b*e^3 + (
a*b^2 + 2*a^2*c)*d*e^2 + (b*c^2*d^3 + 3*a*b*c*d*e^2 - 2*a^2*c*e^3 - (b^2*c
+ 2*a*c^2)*d^2*e)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^
4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2
*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3
- 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a
^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)
*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b
^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^
2*b^3 - 4*a^3*b*c)*e^4)*x^2)]
```

giac [B]   time = 0.64, size = 441, normalized size = 2.77

$$-\frac{d \arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)e}{\left(cd^2-bde+ae^2\right)\sqrt{-cd^2+bde-ae^2}}+\frac{\frac{\left(bc^2d^3-b^2cd^2e-2\,ac^2d^2e+3\,abcde^2-2\,a^2ce^3\right)x^2}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4}+\frac{2\,ac^2d^3-3\,abcd^2e+ab^2de^2+2\,a^2cde^2-a^2be^3}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4}}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -d*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*
d^2 + b*d*e - a*e^2))*e/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^
2)) + ((b*c^2*d^3 - b^2*c*d^2*e - 2*a*c^2*d^2*e + 3*a*b*c*d*e^2 - 2*a^2*c*e
^3)*x^2/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*
d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c
*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (2*a*c^2*d^3 - 3*a*b*c*d^2*e + a*b^2*
d*e^2 + 2*a^2*c*d*e^2 - a^2*b*e^3)/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3
*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2
- 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4))/sqrt(c*x^4
+ b*x^2 + a)
```

maple [B]   time = 0.01, size = 506, normalized size = 3.18

$$-\frac{2cd\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2a\,e^2-2deb+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\,\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-deb+c\,d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{\left(-be+2cd+\sqrt{-4ac+b^2}\,e\right)\left(be-2cd+\sqrt{-4ac+b^2}\,e\right)\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}}-\frac{2\sqrt{\left(x^2+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)^2c-\sqrt{-4ac+b^2}\,\left(x^2+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)}\,cd}{\left(be-2cd+\sqrt{-4ac+b^2}\,e\right)\left(-4ac+b^2\right)\left(x^2+\frac{b}{2c}+\frac{\sqrt{-4ac+b^2}}{2c}\right)e}+\frac{2\sqrt{\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)^2c+\sqrt{-4ac+b^2}\,\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)}\,cd}{\left(-be+2cd+\sqrt{-4ac+b^2}\,e\right)\left(-4ac+b^2\right)\left(x^2+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}\right)e}+\frac{2c\,x^2+b}{\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/e*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)-2*d/e*c/(b*e-2*c*d+(-4*a*
c+b^2)^(1/2)*e)/(-4*a*c+b^2)/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)*((x^2+1
/2*(b+(-4*a*c+b^2)^(1/2))/c)^2*c-(-4*a*c+b^2)^(1/2)*(x^2+1/2*(b+(-4*a*c+b^2
)^(1/2))/c))^(1/2)-2*d*c/(-b*e+2*c*d+(-4*a*c+b^2)^(1/2)*e)/(b*e-2*c*d+(-4*a
*c+b^2)^(1/2)*e)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/
e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*
c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))+2*d/e*
c/(-b*e+2*c*d+(-4*a*c+b^2)^(1/2)*e)/(-4*a*c+b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b
^2)^(1/2)/c)*((x^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2*c+(-4*a*c+b^2)^(1/2)*(x
^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(e\,x^2+d\right)\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**3/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.251 $\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=166

$$\frac{e^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2ace+b^2(-e)+cx^2(2cd-be)+bcd}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}$$

Rubi [A] time = 0.17, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1247, 740, 12, 724, 206}

$$\frac{e^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2ace+b^2(-e)+cx^2(2cd-be)+bcd}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -((b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 - b
*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])) + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*
d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*
(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int -\frac{\left(b^2-4ac\right)e^2}{2(d+ex)\sqrt{a+bx+\ldots}}\right.}{\left(b^2-4ac\right)\left(cd^2-bd\ldots\right.}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} + \frac{e^2\,\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+\ldots}}\right.}{2\left(cd^2-bde+\ldots\right.}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} - \frac{e^2\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae\ldots}\right.}{cd^2-\ldots}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} + \frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2c\ldots}{2\sqrt{cd^2-bde+ae^2}\ldots}\right.}{2\left(cd^2-bde+ae\ldots\right.}$$

Mathematica [A] time = 0.15, size = 167, normalized size = 1.01

$$-\frac{2ace+b^2(-e)+cx^2(2cd-be)+bcd}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(e(ae-bd)+cd^2\right)} - \frac{e^2\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)}{2\left(e(ae-bd)+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -((b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 + e
*(-(b*d) + a*e))*Sqrt[a + b*x^2 + c*x^4])) - (e^2*ArcTanh[(-(b*d) + 2*a*e -
 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + b*x^2 + c*
x^4])])/(2*(c*d^2 + e*(-(b*d) + a*e))^(3/2))
```

IntegrateAlgebraic [A] time = 0.75, size = 224, normalized size = 1.35

$$\frac{2ace+b^2(-e)+bcd-bcex^2+2c^2dx^2}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(-ae^2+bde-cd^2\right)} + \frac{e^2\sqrt{-ae^2+bde-cd^2}\,\tan^{-1}\left(\frac{\sqrt{c}ex^2}{\sqrt{-ae^2+bde-cd^2}} - \frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2+bde-cd^2}} + \frac{\sqrt{c}d}{\sqrt{-ae^2+bde-cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b*c*d - b^2*e + 2*a*c*e + 2*c^2*d*x^2 - b*c*e*x^2)/((b^2 - 4*a*c)*(-(c*d^2
) + b*d*e - a*e^2)*Sqrt[a + b*x^2 + c*x^4]) + (e^2*Sqrt[-(c*d^2) + b*d*e -
a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x^2)/
Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2)
+ b*d*e - a*e^2]])/(c*d^2 - b*d*e + a*e^2)^2
```

fricas [B] time = 2.49, size = 1379, normalized size = 8.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*e^2*x^4 + (b^3 - 4*a*b*c)*e^2*x^2 + (a*b^2 - 4*a^2
*c)*e^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 +
4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2
+ 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d
^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2
 + d^2)) - 4*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 - (
a*b^2 - 2*a^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2*c + 2*
a*c^2)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 -
2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2
 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*
a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c
^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4
)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 -
 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^
3 - 4*a^3*b*c)*e^4)*x^2), 1/2*(((b^2*c - 4*a*c^2)*e^2*x^4 + (b^3 - 4*a*b*c)
*e^2*x^2 + (a*b^2 - 4*a^2*c)*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*
sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b
*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^
2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - 2*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*
d^2*e + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*
d^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/
((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2
*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2
- 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e +
 (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^
3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*
c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4
 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)]
```

giac [B] time = 0.62, size = 454, normalized size = 2.73

$$-\frac{\frac{\left(2c^3d^3-3bc^2d^2e+b^2cde^2+2ac^2de^2-abce^3\right)x^2}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2b^2e^4-4a^3ce^4}+\frac{bc^2d^3-2b^2cd^2e+2ac^2d^2e+b^3de^2-abcde^2-ab^2e^3+2a^2ce^3}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2b^2e^4-4a^3ce^4}}{\sqrt{cx^4+bx^2+a}}+\frac{\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)e^2}{\left(cd^2-bde+ae^2\right)\sqrt{-cd^2+bde-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -((2*c^3*d^3 - 3*b*c^2*d^2*e + b^2*c*d*e^2 + 2*a*c^2*d*e^2 - a*b*c*e^3)*x^2
/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2
 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3
+ a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3 - 2*b^2*c*d^2*e + 2*a*c^2*d^2*e +
 b^3*d*e^2 - a*b*c*d*e^2 - a*b^2*e^3 + 2*a^2*c*e^3)/(b^2*c^2*d^4 - 4*a*c^3*
d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8
*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*
e^4))/sqrt(c*x^4 + b*x^2 + a) + arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))*e^2/((c*d^2 - b*d*e + a*
e^2)*sqrt(-c*d^2 + b*d*e - a*e^2))
```

maple [B] time = 0.01, size = 454, normalized size = 2.73

$$\frac{2ce\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2ae^2-2deb+2cd^2}{e^2}+2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{\left(-be+2cd+\sqrt{-4ac+b^2}\,e\right)\left(be-2cd+\sqrt{-4ac+b^2}\,e\right)\sqrt{\frac{ae^2-deb+cd^2}{e^2}}}+\frac{2\sqrt{\left(x^2+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)^2c-\sqrt{-4ac+b^2}\left(x^2+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}}{\left(be-2cd+\sqrt{-4ac+b^2}\,e\right)\left(-4ac+b^2\right)\left(x^2+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)}-\frac{2\sqrt{\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)^2c+\sqrt{-4ac+b^2}\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)c}}{\left(-be+2cd+\sqrt{-4ac+b^2}\,e\right)\left(-4ac+b^2\right)\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 2*c/(b*e-2*c*d+(-4*a*c+b^2)^(1/2)*e)/(-4*a*c+b^2)/(x^2+1/2*(b+(-4*a*c+b^2)^
(1/2))/c)*((x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2*c-(-4*a*c+b^2)^(1/2)*(x^2+1
```

```
/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)+2*c*e/(-b*e+2*c*d+(-4*a*c+b^2)^(1/2)*e)
/(b*e-2*c*d+(-4*a*c+b^2)^(1/2)*e)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-
2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/
(x^2+d/e))-2*c/(-b*e+2*c*d+(-4*a*c+b^2)^(1/2)*e)/(-4*a*c+b^2)/(x^2-1/2*(-b+
(-4*a*c+b^2)^(1/2))/c)*((x^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2*c+(-4*a*c+b^2
)^(1/2)*(x^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}\left(ex^2 + d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(e\,x^2 + d\right)\,\left(c\,x^4 + b\,x^2 + a\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
[Out] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(d + ex^2\right)\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral(x/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.252 $\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=266

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d}+\frac{e\left(2ace+b^2(-e)+cx^2(2cd-be)+bcd\right)}{d\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}+\frac{-2ac+b^2+bcx^2}{ad\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-e^3\tan$$

Rubi [A] time = 0.39, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1251, 960, 740, 12, 724, 206}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d}+\frac{e\left(2ace+b^2(-e)+cx^2(2cd-be)+bcd\right)}{d\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}+\frac{-2ac+b^2+bcx^2}{ad\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{e^3\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d\left(ae^2-bde+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x^2 + c*x^4]) + (e*(b
*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2))/((b^2 - 4*a*c)*d*(c*d^2 - b*
d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sq
rt[a + b*x^2 + c*x^4])]/(2*a^(3/2)*d) - (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d
- b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*
(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}}-\frac{e}{d(d+ex)\left(a+bx+cx^2\right)^{3/2}}\right)dx,x,$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)}{2d} - \frac{e\,\text{Subst}\left(\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)}{2d}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)d\sqrt{a+bx^2+cx^4}} + \frac{e\left(bcd-b^2e+2ace+c(2cd-be)x\right.}{\left(b^2-4ac\right)d\left(cd^2-bde+ae^2\right)\sqrt{a+bx}}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)d\sqrt{a+bx^2+cx^4}} + \frac{e\left(bcd-b^2e+2ace+c(2cd-be)x\right.}{\left(b^2-4ac\right)d\left(cd^2-bde+ae^2\right)\sqrt{a+bx}}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)d\sqrt{a+bx^2+cx^4}} + \frac{e\left(bcd-b^2e+2ace+c(2cd-be)x\right.}{\left(b^2-4ac\right)d\left(cd^2-bde+ae^2\right)\sqrt{a+bx}}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)d\sqrt{a+bx^2+cx^4}} + \frac{e\left(bcd-b^2e+2ace+c(2cd-be)x\right.}{\left(b^2-4ac\right)d\left(cd^2-bde+ae^2\right)\sqrt{a+bx}}$$

Mathematica [A] time = 0.73, size = 236, normalized size = 0.89

$$-\frac{\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{a^{3/2}}+\frac{2d\left(bc\left(3ae+cdx^2\right)-2ac^2\left(d-ex^2\right)+b^3(-e)+b^2c\left(d-ex^2\right)\right)}{a\left(4ac-b^2\right)\sqrt{a+bx^2+cx^4}\left(e(ae-bd)+cd^2\right)}+\frac{e^3\tanh^{-1}\left(\frac{-2ae+b\left(d-ex^2\right)+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)}{\left(e(ae-bd)+cd^2\right)^{3/2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -1/2*((2*d*(-(b^3*e) + b*c*(3*a*e + c*d*x^2) + b^2*c*(d - e*x^2) - 2*a*c^2*
(d - e*x^2)))/(a*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + b*x^2 +
 c*x^4]) + ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/a^(3/
2) + (e^3*ArcTanh[(-2*a*e + 2*c*d*x^2 + b*(d - e*x^2))/(2*Sqrt[c*d^2 + e*(-
```

```
(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4])])/(c*d^2 + e*(-(b*d) + a*e))^(3/2))/
d
```

IntegrateAlgebraic [A] time = 1.49, size = 324, normalized size = 1.22

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}-\frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{-3abce+2ac^2d-2ac^2ex^2+b^3e-b^2cd+b^2cex^2-bc^2dx^2}{a\sqrt{a+bx^2+cx^4}\left(4a^2ce^2-ab^2e^2-4abcde+4ac^2d^2+b^3de-b^2cd^2\right)}-\frac{e^3\sqrt{-ae^2+bde-cd^2}\tan^{-1}\left(\frac{\sqrt{c}ex^2}{\sqrt{-ae^2+bde-cd^2}}-\frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}d}{\sqrt{-ae^2+bde-cd^2}}\right)}{d\left(ae^2-bde+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-(b^2*c*d) + 2*a*c^2*d + b^3*e - 3*a*b*c*e - b*c^2*d*x^2 + b^2*c*e*x^2 - 2
*a*c^2*e*x^2)/(a*(-(b^2*c*d^2) + 4*a*c^2*d^2 + b^3*d*e - 4*a*b*c*d*e - a*b^
2*e^2 + 4*a^2*c*e^2)*Sqrt[a + b*x^2 + c*x^4]) - (e^3*Sqrt[-(c*d^2) + b*d*e
- a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x^2
)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2
) + b*d*e - a*e^2]])/(d*(c*d^2 - b*d*e + a*e^2)^2) + ArcTanh[(Sqrt[c]*x^2)/
Sqrt[a] - Sqrt[a + b*x^2 + c*x^4]/Sqrt[a]]/(a^(3/2)*d)
```

fricas [B] time = 10.49, size = 4909, normalized size = 18.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^2*b^2*c - 4*a^3*c^2)*e^3*x^4 + (a^2*b^3 - 4*a^3*b*c)*e^3*x^2 + (a
^3*b^2 - 4*a^4*c)*e^3)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c
*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 +
 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2
 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x
^4 + 2*d*e*x^2 + d^2)) + ((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*
b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a
^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3
*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*
b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 -
 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*
b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)
*x^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 +
a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*((a*b^2*c^2 - 2*a^2*c^3)*d^4 - (
2*a*b^3*c - 5*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 2*a^3*c^2)*d^2*e^2
- (a^2*b^3 - 3*a^3*b*c)*d*e^3 + (a*b*c^3*d^4 - 2*(a*b^2*c^2 - a^2*c^3)*d^3*
e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 - (a^2*b^2*c - 2*a^3*c^2)*d*e^3)*x^2)*sqr
t(c*x^4 + b*x^2 + a))/((a^3*b^2*c^2 - 4*a^4*c^3)*d^5 - 2*(a^3*b^3*c - 4*a^4
*b*c^2)*d^4*e + (a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2)*d^3*e^2 - 2*(a^4*b^3 -
4*a^5*b*c)*d^2*e^3 + (a^5*b^2 - 4*a^6*c)*d*e^4 + ((a^2*b^2*c^3 - 4*a^3*c^4)
*d^5 - 2*(a^2*b^3*c^2 - 4*a^3*b*c^3)*d^4*e + (a^2*b^4*c - 2*a^3*b^2*c^2 - 8
*a^4*c^3)*d^3*e^2 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^2*e^3 + (a^4*b^2*c - 4*a^
5*c^2)*d*e^4)*x^4 + ((a^2*b^3*c^2 - 4*a^3*b*c^3)*d^5 - 2*(a^2*b^4*c - 4*a^3
*b^2*c^2)*d^4*e + (a^2*b^5 - 2*a^3*b^3*c - 8*a^4*b*c^2)*d^3*e^2 - 2*(a^3*b^
4 - 4*a^4*b^2*c)*d^2*e^3 + (a^4*b^3 - 4*a^5*b*c)*d*e^4)*x^2), -1/4*(2*((a^2
*b^2*c - 4*a^3*c^2)*e^3*x^4 + (a^2*b^3 - 4*a^3*b*c)*e^3*x^2 + (a^3*b^2 - 4*
a^4*c)*e^3)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a
)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2
- b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e
 + a*b*e^2)*x^2)) - ((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2
)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*
c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2
- 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c
 - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*
```

```
b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2
)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)
*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b
*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*((a*b^2*c^2 - 2*a^2*c^3)*d^4 - (2*a*b
^3*c - 5*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 2*a^3*c^2)*d^2*e^2 - (a^
2*b^3 - 3*a^3*b*c)*d*e^3 + (a*b*c^3*d^4 - 2*(a*b^2*c^2 - a^2*c^3)*d^3*e + (
a*b^3*c - a^2*b*c^2)*d^2*e^2 - (a^2*b^2*c - 2*a^3*c^2)*d*e^3)*x^2)*sqrt(c*x
^4 + b*x^2 + a))/((a^3*b^2*c^2 - 4*a^4*c^3)*d^5 - 2*(a^3*b^3*c - 4*a^4*b*c^
2)*d^4*e + (a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2)*d^3*e^2 - 2*(a^4*b^3 - 4*a^5
*b*c)*d^2*e^3 + (a^5*b^2 - 4*a^6*c)*d*e^4 + ((a^2*b^2*c^3 - 4*a^3*c^4)*d^5
- 2*(a^2*b^3*c^2 - 4*a^3*b*c^3)*d^4*e + (a^2*b^4*c - 2*a^3*b^2*c^2 - 8*a^4*
c^3)*d^3*e^2 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^2*e^3 + (a^4*b^2*c - 4*a^5*c^2
)*d*e^4)*x^4 + ((a^2*b^3*c^2 - 4*a^3*b*c^3)*d^5 - 2*(a^2*b^4*c - 4*a^3*b^2*
c^2)*d^4*e + (a^2*b^5 - 2*a^3*b^3*c - 8*a^4*b*c^2)*d^3*e^2 - 2*(a^3*b^4 - 4
*a^4*b^2*c)*d^2*e^3 + (a^4*b^3 - 4*a^5*b*c)*d*e^4)*x^2), 1/4*(2*((a*b^2*c^2
 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c
- 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*
e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2
*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^
2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2
*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^
2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4
 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + ((a^2*b^2
*c - 4*a^3*c^2)*e^3*x^4 + (a^2*b^3 - 4*a^3*b*c)*e^3*x^2 + (a^3*b^2 - 4*a^4*
c)*e^3)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4
*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 +
 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^
2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2
+ d^2)) + 4*((a*b^2*c^2 - 2*a^2*c^3)*d^4 - (2*a*b^3*c - 5*a^2*b*c^2)*d^3*e
+ (a*b^4 - 2*a^2*b^2*c - 2*a^3*c^2)*d^2*e^2 - (a^2*b^3 - 3*a^3*b*c)*d*e^3 +
 (a*b*c^3*d^4 - 2*(a*b^2*c^2 - a^2*c^3)*d^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e
^2 - (a^2*b^2*c - 2*a^3*c^2)*d*e^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^3*b^2
*c^2 - 4*a^4*c^3)*d^5 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^4*e + (a^3*b^4 - 2*a^
4*b^2*c - 8*a^5*c^2)*d^3*e^2 - 2*(a^4*b^3 - 4*a^5*b*c)*d^2*e^3 + (a^5*b^2 -
 4*a^6*c)*d*e^4 + ((a^2*b^2*c^3 - 4*a^3*c^4)*d^5 - 2*(a^2*b^3*c^2 - 4*a^3*b
*c^3)*d^4*e + (a^2*b^4*c - 2*a^3*b^2*c^2 - 8*a^4*c^3)*d^3*e^2 - 2*(a^3*b^3*
c - 4*a^4*b*c^2)*d^2*e^3 + (a^4*b^2*c - 4*a^5*c^2)*d*e^4)*x^4 + ((a^2*b^3*c
^2 - 4*a^3*b*c^3)*d^5 - 2*(a^2*b^4*c - 4*a^3*b^2*c^2)*d^4*e + (a^2*b^5 - 2*
a^3*b^3*c - 8*a^4*b*c^2)*d^3*e^2 - 2*(a^3*b^4 - 4*a^4*b^2*c)*d^2*e^3 + (a^4
*b^3 - 4*a^5*b*c)*d*e^4)*x^2), -1/2*(((a^2*b^2*c - 4*a^3*c^2)*e^3*x^4 + (a^
2*b^3 - 4*a^3*b*c)*e^3*x^2 + (a^3*b^2 - 4*a^4*c)*e^3)*sqrt(-c*d^2 + b*d*e -
 a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((
2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^
2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - ((a*b^2*c^2 -
 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c -
8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^
4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a
*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*
c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c
^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*
c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 +
 b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*((a*b^2*c
^2 - 2*a^2*c^3)*d^4 - (2*a*b^3*c - 5*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*
c - 2*a^3*c^2)*d^2*e^2 - (a^2*b^3 - 3*a^3*b*c)*d*e^3 + (a*b*c^3*d^4 - 2*(a*
b^2*c^2 - a^2*c^3)*d^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 - (a^2*b^2*c - 2*a
^3*c^2)*d*e^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^3*b^2*c^2 - 4*a^4*c^3)*d^5
 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^4*e + (a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2)*
d^3*e^2 - 2*(a^4*b^3 - 4*a^5*b*c)*d^2*e^3 + (a^5*b^2 - 4*a^6*c)*d*e^4 + ((a
^2*b^2*c^3 - 4*a^3*c^4)*d^5 - 2*(a^2*b^3*c^2 - 4*a^3*b*c^3)*d^4*e + (a^2*b^
```

```
4*c - 2*a^3*b^2*c^2 - 8*a^4*c^3)*d^3*e^2 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^2*
e^3 + (a^4*b^2*c - 4*a^5*c^2)*d*e^4)*x^4 + ((a^2*b^3*c^2 - 4*a^3*b*c^3)*d^5
 - 2*(a^2*b^4*c - 4*a^3*b^2*c^2)*d^4*e + (a^2*b^5 - 2*a^3*b^3*c - 8*a^4*b*c
^2)*d^3*e^2 - 2*(a^3*b^4 - 4*a^4*b^2*c)*d^2*e^3 + (a^4*b^3 - 4*a^5*b*c)*d*e
^4)*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 612, normalized size = 2.30

$$-\frac{2 c e^{2} \ln\left(\frac{\frac{(be-2cd)\left(x^{2}+\frac{d}{e}\right)}{e}+\frac{2a e^{2}-2deb+2c d^{2}}{e^{2}}+2\sqrt{\frac{a e^{2}-deb+c d^{2}}{e^{2}}}\,\sqrt{\left(x^{2}+\frac{d}{e}\right)^{2} c+\frac{(be-2cd)\left(x^{2}+\frac{d}{e}\right)}{e}+\frac{a e^{2}-deb+c d^{2}}{e^{2}}}}{x^{2}+\frac{d}{e}}\right)}{\left(-be+2cd+\sqrt{-4ac+b^{2}}\,e\right)\left(be-2cd+\sqrt{-4ac+b^{2}}\,e\right)\sqrt{\frac{a e^{2}-deb+c d^{2}}{e^{2}}}\,d}-\frac{bc x^{2}}{\left(4ac-b^{2}\right)\sqrt{c x^{4}+b x^{2}+a}\,ad}-\frac{2\sqrt{\left(x^{2}+\frac{b+\sqrt{-4ac+b^{2}}}{2c}\right)^{2} c-\sqrt{-4ac+b^{2}}\,\left(x^{2}+\frac{b+\sqrt{-4ac+b^{2}}}{2c}\right)}\,ce}{\left(be-2cd+\sqrt{-4ac+b^{2}}\,e\right)\left(-4ac+b^{2}\right)\left(x^{2}+\frac{b}{2c}+\frac{\sqrt{-4ac+b^{2}}}{2c}\right)d}+\frac{2\sqrt{\left(x^{2}-\frac{-b+\sqrt{-4ac+b^{2}}}{2c}\right)^{2} c+\sqrt{-4ac+b^{2}}\,\left(x^{2}-\frac{-b+\sqrt{-4ac+b^{2}}}{2c}\right)}\,ce}{\left(-be+2cd+\sqrt{-4ac+b^{2}}\,e\right)\left(-4ac+b^{2}\right)\left(x^{2}+\frac{b}{2c}-\frac{\sqrt{-4ac+b^{2}}}{2c}\right)d}-\frac{b^{2}}{2\left(4ac-b^{2}\right)\sqrt{c x^{4}+b x^{2}+a}\,ad}-\frac{\ln\left(\frac{b x^{2}+2a+2\sqrt{c x^{4}+b x^{2}+a}\,\sqrt{a}}{x^{2}}\right)}{2a^{\frac{3}{2}} d}+\frac{1}{2\sqrt{c x^{4}+b x^{2}+a}\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/2/d/a/(c*x^4+b*x^2+a)^(1/2)-1/d*b/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*c*x
^2-1/2/d*b^2/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)-1/2/d/a^(3/2)*ln((b*x^2+2*
a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-2*e/d*c/(b*e-2*c*d+(-4*a*c+b^2)^(1/
2)*e)/(-4*a*c+b^2)/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)*((x^2+1/2*(b+(-4*
a*c+b^2)^(1/2))/c)^2*c-(-4*a*c+b^2)^(1/2)*(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c
))^(1/2)-2*e^2/d*c/(-b*e+2*c*d+(-4*a*c+b^2)^(1/2)*e)/(b*e-2*c*d+(-4*a*c+b^2
)^(1/2)*e)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a
*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e
-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))+2*e/d*c/(-b*
e+2*c*d+(-4*a*c+b^2)^(1/2)*e)/(-4*a*c+b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1
/2)/c)*((x^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2*c+(-4*a*c+b^2)^(1/2)*(x^2-1/2
*(-b+(-4*a*c+b^2)^(1/2))/c))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c x^{4}+b x^{2}+a\right)^{\frac{3}{2}}\left(e x^{2}+d\right) x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\left(e\,x^2+d\right)\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.253 $\int \frac{1}{x^3(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=419

$$\frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d^2}+\frac{3b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}d}-\frac{\left(3b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{2a^2dx^2\left(b^2-4ac\right)}-\frac{e^2\left(2ace+b^2(-e)+cx^2(\right.}{d^2\left(b^2-4ac\right)\sqrt{a+bx^2+c}}$$

Rubi [A] time = 0.56, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1251, 960, 740, 806, 724, 206, 12}

$$-\frac{\left(3b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{2a^2dx^2\left(b^2-4ac\right)}+\frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d^2}+\frac{3b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}d}-\frac{e^2\left(2ace+b^2(-e)+cx^2(2cd-be)+bcd\right)}{d^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}-\frac{e\left(-2ac+b^2+bcx^2\right)}{ad^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}+\frac{-2ac+b^2+bcx^2}{adx^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}+\frac{e^4\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2\left(ae^2-bde+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -((e*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*d^2*Sqrt[a + b*x^2 + c*x^4])
) + (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*d*x^2*Sqrt[a + b*x^2 + c*x^4])
 - (e^2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2))/((b^2 - 4*a*c)*d^2
*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - ((3*b^2 - 8*a*c)*Sqrt[a
 + b*x^2 + c*x^4])/(2*a^2*(b^2 - 4*a*c)*d*x^2) + (3*b*ArcTanh[(2*a + b*x^2)
/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(5/2)*d) + (e*ArcTanh[(2*a + b*
x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*a^(3/2)*d^2) + (e^4*ArcTanh[(
b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*
x^2 + c*x^4])])/(2*d^2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{dx^2\left(a+bx+cx^2\right)^{3/2}} - \frac{e}{d^2x\left(a+bx+cx^2\right)^{3/2}} + \frac{\ }{d^2(d+e\ldots}\right.\right.$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)}{2d} - \frac{e\,\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)}{2d^2} - \ldots$$

$$= -\frac{e\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)d^2\sqrt{a+bx^2+cx^4}} + \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)dx^2\sqrt{a+bx^2+cx^4}} - \ldots$$

$$= -\frac{e\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)d^2\sqrt{a+bx^2+cx^4}} + \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)dx^2\sqrt{a+bx^2+cx^4}} - \ldots$$

$$= -\frac{e\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)d^2\sqrt{a+bx^2+cx^4}} + \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)dx^2\sqrt{a+bx^2+cx^4}} - \ldots$$

$$= -\frac{e\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)d^2\sqrt{a+bx^2+cx^4}} + \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)dx^2\sqrt{a+bx^2+cx^4}} - \ldots$$

Mathematica [A] time = 1.48, size = 350, normalized size = 0.84

$$\frac{\frac{(2ae+3bd)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{a^{5/2}} + \frac{2d\left(-4a^3ce^2+a^2\left(b^2e^2+4bce\left(d-ex^2\right)-4c^2\left(d^2+dex^2+e^2x^4\right)\right)+a\left(b^3e\left(ex^2-d\right)+b^2c\left(d^2+12dex^2+e^2x^4\right)-10bc^2dx^2\left(d-ex^2\right)-8c^3d^2x^4\right)+3b^2dx^2\left(b+cx^2\right)(cd-be)\right)}{a^2x^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(e(bd-ae)-cd^2\right)} + \frac{2e^4\tanh^{-1}\left(\frac{-2ae+b\left(d-ex^2\right)+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)}{\left(e(ae-bd)+cd^2\right)^{3/2}}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] ((2*d*(-4*a^3*c*e^2 + 3*b^2*d*(c*d - b*e)*x^2*(b + c*x^2) + a^2*(b^2*e^2 +
4*b*c*e*(d - e*x^2) - 4*c^2*(d^2 + d*e*x^2 + e^2*x^4)) + a*(-8*c^3*d^2*x^4
- 10*b*c^2*d*x^2*(d - e*x^2) + b^3*e*(-d + e*x^2) + b^2*c*(d^2 + 12*d*e*x^2
 + e^2*x^4))))/(a^2*(b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*x^2*Sqrt[a + b
*x^2 + c*x^4]) + ((3*b*d + 2*a*e)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a +
 b*x^2 + c*x^4])])/a^(5/2) + (2*e^4*ArcTanh[(-2*a*e + 2*c*d*x^2 + b*(d - e*
x^2))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4])])/(c*d^2 +
 e*(-(b*d) + a*e))^(3/2))/(4*d^2)
```

IntegrateAlgebraic [A] time = 2.07, size = 479, normalized size = 1.14

$$\frac{(-2ae-3bd)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{2a^{5/2}d^2}+\frac{-4a^3ce^2+a^2b^2e^2+4a^2bcde-4a^2bce^2x^2-4a^2c^2d^2-4a^2c^2dex^2-4a^2c^2e^2x^4-ab^3de+ab^3e^2x^2+ab^2cd^2+12ab^2cdex^2+ab^2ce^2x^4-10abc^2d^2x^2+10abc^2dex^4-8ac^3d^2x^4-3b^4dex^2+3b^3cd^2x^2-3b^3cdex^4+3b^2c^2d^2x^4}{2a^2dx^2\left(4ac-b^2\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}+\frac{e^4\sqrt{-ae^2+bde-cd^2}\tan^{-1}\left(\frac{\sqrt{c}\,ex^2}{\sqrt{-ae^2+bde-cd^2}}-\frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}\,d}{\sqrt{-ae^2+bde-cd^2}}\right)}{d^2\left(ae^2-bde+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (a*b^2*c*d^2 - 4*a^2*c^2*d^2 - a*b^3*d*e + 4*a^2*b*c*d*e + a^2*b^2*e^2 - 4*
a^3*c*e^2 + 3*b^3*c*d^2*x^2 - 10*a*b*c^2*d^2*x^2 - 3*b^4*d*e*x^2 + 12*a*b^2
*c*d*e*x^2 - 4*a^2*c^2*d*e*x^2 + a*b^3*e^2*x^2 - 4*a^2*b*c*e^2*x^2 + 3*b^2*
c^2*d^2*x^4 - 8*a*c^3*d^2*x^4 - 3*b^3*c*d*e*x^4 + 10*a*b*c^2*d*e*x^4 + a*b^
2*c*e^2*x^4 - 4*a^2*c^2*e^2*x^4)/(2*a^2*(-b^2 + 4*a*c)*d*(c*d^2 - b*d*e + a
*e^2)*x^2*Sqrt[a + b*x^2 + c*x^4]) + (e^4*Sqrt[-(c*d^2) + b*d*e - a*e^2]*Ar
cTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x^2)/Sqrt[-(c*
d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e -
 a*e^2]])/(d^2*(c*d^2 - b*d*e + a*e^2)^2) + ((-3*b*d - 2*a*e)*ArcTanh[(Sqrt
[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(2*a^(5/2)*d^2)
```

fricas [B] time = 22.94, size = 6486, normalized size = 15.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*((a^3*b^2*c - 4*a^4*c^2)*e^4*x^6 + (a^3*b^3 - 4*a^4*b*c)*e^4*x^4 +
(a^4*b^2 - 4*a^5*c)*e^4*x^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 -
 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)
*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 +
 b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/
(e^2*x^4 + 2*d*e*x^2 + d^2)) + ((3*(b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(3*b^4*c^2
 - 13*a*b^2*c^3 + 4*a^2*c^4)*d^4*e + (3*b^5*c - 10*a*b^3*c^2 - 8*a^2*b*c^3)
*d^3*e^2 - 4*(a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e^3 - (a^2*b^3*c - 4
*a^3*b*c^2)*d*e^4 + 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*x^6 + (3*(b^4*c^2 - 4*a*
b^2*c^3)*d^5 - 2*(3*b^5*c - 13*a*b^3*c^2 + 4*a^2*b*c^3)*d^4*e + (3*b^6 - 10
*a*b^4*c - 8*a^2*b^2*c^2)*d^3*e^2 - 4*(a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d
^2*e^3 - (a^2*b^4 - 4*a^3*b^2*c)*d*e^4 + 2*(a^3*b^3 - 4*a^4*b*c)*e^5)*x^4 +
 (3*(a*b^3*c^2 - 4*a^2*b*c^3)*d^5 - 2*(3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^3*c
^3)*d^4*e + (3*a*b^5 - 10*a^2*b^3*c - 8*a^3*b*c^2)*d^3*e^2 - 4*(a^2*b^4 - 5
*a^3*b^2*c + 4*a^4*c^2)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + 2*(a^4*b^2
- 4*a^5*c)*e^5)*x^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c
*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*((a^2*b^2*c^2 - 4
*a^3*c^3)*d^5 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^4*e + (a^2*b^4 - 2*a^3*b^2*c
- 8*a^4*c^2)*d^3*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d^2*e^3 + (a^4*b^2 - 4*a^5*c
)*d*e^4 + ((3*a*b^2*c^3 - 8*a^2*c^4)*d^5 - 6*(a*b^3*c^2 - 3*a^2*b*c^3)*d^4*
e + 3*(a*b^4*c - 2*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^2 - 2*(2*a^2*b^3*c - 7*a^
3*b*c^2)*d^2*e^3 + (a^3*b^2*c - 4*a^4*c^2)*d*e^4)*x^4 + ((3*a*b^3*c^2 - 10*
a^2*b*c^3)*d^5 - 2*(3*a*b^4*c - 11*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e + (3*a*b^
```

```
5 - 8*a^2*b^3*c - 10*a^3*b*c^2)*d^3*e^2 - 4*(a^2*b^4 - 4*a^3*b^2*c + a^4*c^
2)*d^2*e^3 + (a^3*b^3 - 4*a^4*b*c)*d*e^4)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(((
a^3*b^2*c^3 - 4*a^4*c^4)*d^6 - 2*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d^5*e + (a^3*b
^4*c - 2*a^4*b^2*c^2 - 8*a^5*c^3)*d^4*e^2 - 2*(a^4*b^3*c - 4*a^5*b*c^2)*d^3
*e^3 + (a^5*b^2*c - 4*a^6*c^2)*d^2*e^4)*x^6 + ((a^3*b^3*c^2 - 4*a^4*b*c^3)*
d^6 - 2*(a^3*b^4*c - 4*a^4*b^2*c^2)*d^5*e + (a^3*b^5 - 2*a^4*b^3*c - 8*a^5*
b*c^2)*d^4*e^2 - 2*(a^4*b^4 - 4*a^5*b^2*c)*d^3*e^3 + (a^5*b^3 - 4*a^6*b*c)*
d^2*e^4)*x^4 + ((a^4*b^2*c^2 - 4*a^5*c^3)*d^6 - 2*(a^4*b^3*c - 4*a^5*b*c^2)
*d^5*e + (a^4*b^4 - 2*a^5*b^2*c - 8*a^6*c^2)*d^4*e^2 - 2*(a^5*b^3 - 4*a^6*b
*c)*d^3*e^3 + (a^6*b^2 - 4*a^7*c)*d^2*e^4)*x^2), 1/8*(4*((a^3*b^2*c - 4*a^4
*c^2)*e^4*x^6 + (a^3*b^3 - 4*a^4*b*c)*e^4*x^4 + (a^4*b^2 - 4*a^5*c)*e^4*x^2
)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*
d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e
 + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2
)*x^2)) + ((3*(b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(3*b^4*c^2 - 13*a*b^2*c^3 + 4*a
^2*c^4)*d^4*e + (3*b^5*c - 10*a*b^3*c^2 - 8*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c
 - 5*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e^3 - (a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 + 2
*(a^3*b^2*c - 4*a^4*c^2)*e^5)*x^6 + (3*(b^4*c^2 - 4*a*b^2*c^3)*d^5 - 2*(3*b
^5*c - 13*a*b^3*c^2 + 4*a^2*b*c^3)*d^4*e + (3*b^6 - 10*a*b^4*c - 8*a^2*b^2*
c^2)*d^3*e^2 - 4*(a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^2*e^3 - (a^2*b^4 - 4
*a^3*b^2*c)*d*e^4 + 2*(a^3*b^3 - 4*a^4*b*c)*e^5)*x^4 + (3*(a*b^3*c^2 - 4*a^
2*b*c^3)*d^5 - 2*(3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^3*c^3)*d^4*e + (3*a*b^5
 - 10*a^2*b^3*c - 8*a^3*b*c^2)*d^3*e^2 - 4*(a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^
2)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + 2*(a^4*b^2 - 4*a^5*c)*e^5)*x^2)*
sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*
x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*((a^2*b^2*c^2 - 4*a^3*c^3)*d^5 - 2*(a^
2*b^3*c - 4*a^3*b*c^2)*d^4*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^3*e^2
 - 2*(a^3*b^3 - 4*a^4*b*c)*d^2*e^3 + (a^4*b^2 - 4*a^5*c)*d*e^4 + ((3*a*b^2*c
^3 - 8*a^2*c^4)*d^5 - 6*(a*b^3*c^2 - 3*a^2*b*c^3)*d^4*e + 3*(a*b^4*c - 2*a^
2*b^2*c^2 - 4*a^3*c^3)*d^3*e^2 - 2*(2*a^2*b^3*c - 7*a^3*b*c^2)*d^2*e^3 + (a
^3*b^2*c - 4*a^4*c^2)*d*e^4)*x^4 + ((3*a*b^3*c^2 - 10*a^2*b*c^3)*d^5 - 2*(3
*a*b^4*c - 11*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e + (3*a*b^5 - 8*a^2*b^3*c - 10*
a^3*b*c^2)*d^3*e^2 - 4*(a^2*b^4 - 4*a^3*b^2*c + a^4*c^2)*d^2*e^3 + (a^3*b^3
 - 4*a^4*b*c)*d*e^4)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(((a^3*b^2*c^3 - 4*a^4*c
^4)*d^6 - 2*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d^5*e + (a^3*b^4*c - 2*a^4*b^2*c^2
 - 8*a^5*c^3)*d^4*e^2 - 2*(a^4*b^3*c - 4*a^5*b*c^2)*d^3*e^3 + (a^5*b^2*c - 4
*a^6*c^2)*d^2*e^4)*x^6 + ((a^3*b^3*c^2 - 4*a^4*b*c^3)*d^6 - 2*(a^3*b^4*c -
4*a^4*b^2*c^2)*d^5*e + (a^3*b^5 - 2*a^4*b^3*c - 8*a^5*b*c^2)*d^4*e^2 - 2*(a
^4*b^4 - 4*a^5*b^2*c)*d^3*e^3 + (a^5*b^3 - 4*a^6*b*c)*d^2*e^4)*x^4 + ((a^4*
b^2*c^2 - 4*a^5*c^3)*d^6 - 2*(a^4*b^3*c - 4*a^5*b*c^2)*d^5*e + (a^4*b^4 - 2
*a^5*b^2*c - 8*a^6*c^2)*d^4*e^2 - 2*(a^5*b^3 - 4*a^6*b*c)*d^3*e^3 + (a^6*b^
2 - 4*a^7*c)*d^2*e^4)*x^2), -1/4*(((3*(b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(3*b^4*
c^2 - 13*a*b^2*c^3 + 4*a^2*c^4)*d^4*e + (3*b^5*c - 10*a*b^3*c^2 - 8*a^2*b*c
^3)*d^3*e^2 - 4*(a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e^3 - (a^2*b^3*c
 - 4*a^3*b*c^2)*d*e^4 + 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*x^6 + (3*(b^4*c^2 - 4
*a*b^2*c^3)*d^5 - 2*(3*b^5*c - 13*a*b^3*c^2 + 4*a^2*b*c^3)*d^4*e + (3*b^6 -
 10*a*b^4*c - 8*a^2*b^2*c^2)*d^3*e^2 - 4*(a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2
)*d^2*e^3 - (a^2*b^4 - 4*a^3*b^2*c)*d*e^4 + 2*(a^3*b^3 - 4*a^4*b*c)*e^5)*x^
4 + (3*(a*b^3*c^2 - 4*a^2*b*c^3)*d^5 - 2*(3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^
3*c^3)*d^4*e + (3*a*b^5 - 10*a^2*b^3*c - 8*a^3*b*c^2)*d^3*e^2 - 4*(a^2*b^4
 - 5*a^3*b^2*c + 4*a^4*c^2)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + 2*(a^4*b
^2 - 4*a^5*c)*e^5)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
+ 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - ((a^3*b^2*c - 4*a^4*c^2)*e^4*x
^6 + (a^3*b^3 - 4*a^4*b*c)*e^4*x^4 + (a^4*b^2 - 4*a^5*c)*e^4*x^2)*sqrt(c*d^
2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 -
8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b
^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2
)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + 2*((a^2
*b^2*c^2 - 4*a^3*c^3)*d^5 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^4*e + (a^2*b^4 -
```

```
2*a^3*b^2*c - 8*a^4*c^2)*d^3*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d^2*e^3 + (a^4*b
^2 - 4*a^5*c)*d*e^4 + ((3*a*b^2*c^3 - 8*a^2*c^4)*d^5 - 6*(a*b^3*c^2 - 3*a^2
*b*c^3)*d^4*e + 3*(a*b^4*c - 2*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^2 - 2*(2*a^2*
b^3*c - 7*a^3*b*c^2)*d^2*e^3 + (a^3*b^2*c - 4*a^4*c^2)*d*e^4)*x^4 + ((3*a*b
^3*c^2 - 10*a^2*b*c^3)*d^5 - 2*(3*a*b^4*c - 11*a^2*b^2*c^2 + 2*a^3*c^3)*d^4
*e + (3*a*b^5 - 8*a^2*b^3*c - 10*a^3*b*c^2)*d^3*e^2 - 4*(a^2*b^4 - 4*a^3*b^
2*c + a^4*c^2)*d^2*e^3 + (a^3*b^3 - 4*a^4*b*c)*d*e^4)*x^2)*sqrt(c*x^4 + b*x
^2 + a))/(((a^3*b^2*c^3 - 4*a^4*c^4)*d^6 - 2*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d^
5*e + (a^3*b^4*c - 2*a^4*b^2*c^2 - 8*a^5*c^3)*d^4*e^2 - 2*(a^4*b^3*c - 4*a^
5*b*c^2)*d^3*e^3 + (a^5*b^2*c - 4*a^6*c^2)*d^2*e^4)*x^6 + ((a^3*b^3*c^2 - 4
*a^4*b*c^3)*d^6 - 2*(a^3*b^4*c - 4*a^4*b^2*c^2)*d^5*e + (a^3*b^5 - 2*a^4*b^
3*c - 8*a^5*b*c^2)*d^4*e^2 - 2*(a^4*b^4 - 4*a^5*b^2*c)*d^3*e^3 + (a^5*b^3 -
 4*a^6*b*c)*d^2*e^4)*x^4 + ((a^4*b^2*c^2 - 4*a^5*c^3)*d^6 - 2*(a^4*b^3*c -
4*a^5*b*c^2)*d^5*e + (a^4*b^4 - 2*a^5*b^2*c - 8*a^6*c^2)*d^4*e^2 - 2*(a^5*b
^3 - 4*a^6*b*c)*d^3*e^3 + (a^6*b^2 - 4*a^7*c)*d^2*e^4)*x^2), 1/4*(2*((a^3*b
^2*c - 4*a^4*c^2)*e^4*x^6 + (a^3*b^3 - 4*a^4*b*c)*e^4*x^4 + (a^4*b^2 - 4*a^
5*c)*e^4*x^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 +
 a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^
2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d
*e + a*b*e^2)*x^2)) - ((3*(b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(3*b^4*c^2 - 13*a*b
^2*c^3 + 4*a^2*c^4)*d^4*e + (3*b^5*c - 10*a*b^3*c^2 - 8*a^2*b*c^3)*d^3*e^2
- 4*(a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e^3 - (a^2*b^3*c - 4*a^3*b*c^
2)*d*e^4 + 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*x^6 + (3*(b^4*c^2 - 4*a*b^2*c^3)*
d^5 - 2*(3*b^5*c - 13*a*b^3*c^2 + 4*a^2*b*c^3)*d^4*e + (3*b^6 - 10*a*b^4*c
- 8*a^2*b^2*c^2)*d^3*e^2 - 4*(a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^2*e^3 -
(a^2*b^4 - 4*a^3*b^2*c)*d*e^4 + 2*(a^3*b^3 - 4*a^4*b*c)*e^5)*x^4 + (3*(a*b^
3*c^2 - 4*a^2*b*c^3)*d^5 - 2*(3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^3*c^3)*d^4*e
 + (3*a*b^5 - 10*a^2*b^3*c - 8*a^3*b*c^2)*d^3*e^2 - 4*(a^2*b^4 - 5*a^3*b^2*
c + 4*a^4*c^2)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + 2*(a^4*b^2 - 4*a^5*c
)*e^5)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(
-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*((a^2*b^2*c^2 - 4*a^3*c^3)*d^5 - 2*(a^2*
b^3*c - 4*a^3*b*c^2)*d^4*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^3*e^2 -
2*(a^3*b^3 - 4*a^4*b*c)*d^2*e^3 + (a^4*b^2 - 4*a^5*c)*d*e^4 + ((3*a*b^2*c^3
 - 8*a^2*c^4)*d^5 - 6*(a*b^3*c^2 - 3*a^2*b*c^3)*d^4*e + 3*(a*b^4*c - 2*a^2*
b^2*c^2 - 4*a^3*c^3)*d^3*e^2 - 2*(2*a^2*b^3*c - 7*a^3*b*c^2)*d^2*e^3 + (a^3
*b^2*c - 4*a^4*c^2)*d*e^4)*x^4 + ((3*a*b^3*c^2 - 10*a^2*b*c^3)*d^5 - 2*(3*a
*b^4*c - 11*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e + (3*a*b^5 - 8*a^2*b^3*c - 10*a^
3*b*c^2)*d^3*e^2 - 4*(a^2*b^4 - 4*a^3*b^2*c + a^4*c^2)*d^2*e^3 + (a^3*b^3 -
 4*a^4*b*c)*d*e^4)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(((a^3*b^2*c^3 - 4*a^4*c^4
)*d^6 - 2*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d^5*e + (a^3*b^4*c - 2*a^4*b^2*c^2 -
8*a^5*c^3)*d^4*e^2 - 2*(a^4*b^3*c - 4*a^5*b*c^2)*d^3*e^3 + (a^5*b^2*c - 4*a
^6*c^2)*d^2*e^4)*x^6 + ((a^3*b^3*c^2 - 4*a^4*b*c^3)*d^6 - 2*(a^3*b^4*c - 4*
a^4*b^2*c^2)*d^5*e + (a^3*b^5 - 2*a^4*b^3*c - 8*a^5*b*c^2)*d^4*e^2 - 2*(a^4
*b^4 - 4*a^5*b^2*c)*d^3*e^3 + (a^5*b^3 - 4*a^6*b*c)*d^2*e^4)*x^4 + ((a^4*b^
2*c^2 - 4*a^5*c^3)*d^6 - 2*(a^4*b^3*c - 4*a^5*b*c^2)*d^5*e + (a^4*b^4 - 2*a
^5*b^2*c - 8*a^6*c^2)*d^4*e^2 - 2*(a^5*b^3 - 4*a^6*b*c)*d^3*e^3 + (a^6*b^2
- 4*a^7*c)*d^2*e^4)*x^2)]
```

giac [B] time = 2.74, size = 762, normalized size = 1.82

$$-\frac{\frac{\left(a^2b^2c^3d^3-2\,a^3c^4d^3-2\,a^2b^3c^2d^2e+5\,a^3bc^3d^2e+a^2b^4cde^2-2\,a^3b^2c^2de^2-2\,a^4c^3de^2-a^3b^3ce^3+3\,a^4bc^2e^3\right)x^2}{a^4b^2c^2d^4-4\,a^5c^3d^4-2\,a^4b^3cd^3e+8\,a^5bc^2d^3e+a^4b^4d^2e^2-2\,a^5b^2cd^2e^2-8\,a^6c^2d^2e^2-2\,a^5b^3de^3+8\,a^6bcde^3+a^6b^2e^4-4\,a^7ce^4}+\frac{a^2b^3c^2d^3-3\,a^3bc^3d^3-2\,a^2b^4cd^2e+7\,a^3b^2c^2d^2e-2\,a^4c^3d^2e+a^2b^5de^2-3\,a^3b^3cde^2-a^4bc^2de^2-a^3b^4e^3+4\,a^4b^2ce^3-2\,a^5c^2e^3}{a^4b^2c^2d^4-4\,a^5c^3d^4-2\,a^4b^3cd^3e+8\,a^5bc^2d^3e+a^4b^4d^2e^2-2\,a^5b^2cd^2e^2-8\,a^6c^2d^2e^2-2\,a^5b^3de^3+8\,a^6bcde^3+a^6b^2e^4-4\,a^7ce^4}}{\sqrt{cx^4+bx^2+a}}+\frac{\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)e^4}{\left(cd^4-bd^3e+ad^2e^2\right)\sqrt{-cd^2+bde-ae^2}}-\frac{(3\,bd+2\,ae)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}\,a^2d^2}+\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)b+2\,a\sqrt{c}}{2\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -((a^2*b^2*c^3*d^3 - 2*a^3*c^4*d^3 - 2*a^2*b^3*c^2*d^2*e + 5*a^3*b*c^3*d^2*
e + a^2*b^4*c*d*e^2 - 2*a^3*b^2*c^2*d*e^2 - 2*a^4*c^3*d*e^2 - a^3*b^3*c*e^3
 + 3*a^4*b*c^2*e^3)*x^2/(a^4*b^2*c^2*d^4 - 4*a^5*c^3*d^4 - 2*a^4*b^3*c*d^3*
e + 8*a^5*b*c^2*d^3*e + a^4*b^4*d^2*e^2 - 2*a^5*b^2*c*d^2*e^2 - 8*a^6*c^2*d
```

```
^2*e^2 - 2*a^5*b^3*d*e^3 + 8*a^6*b*c*d*e^3 + a^6*b^2*e^4 - 4*a^7*c*e^4) + (
a^2*b^3*c^2*d^3 - 3*a^3*b*c^3*d^3 - 2*a^2*b^4*c*d^2*e + 7*a^3*b^2*c^2*d^2*e
 - 2*a^4*c^3*d^2*e + a^2*b^5*d*e^2 - 3*a^3*b^3*c*d*e^2 - a^4*b*c^2*d*e^2 -
a^3*b^4*e^3 + 4*a^4*b^2*c*e^3 - 2*a^5*c^2*e^3)/(a^4*b^2*c^2*d^4 - 4*a^5*c^3
*d^4 - 2*a^4*b^3*c*d^3*e + 8*a^5*b*c^2*d^3*e + a^4*b^4*d^2*e^2 - 2*a^5*b^2*
c*d^2*e^2 - 8*a^6*c^2*d^2*e^2 - 2*a^5*b^3*d*e^3 + 8*a^6*b*c*d*e^3 + a^6*b^2
*e^4 - 4*a^7*c*e^4))/sqrt(c*x^4 + b*x^2 + a) + arctan(-((sqrt(c)*x^2 - sqrt
(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))*e^4/((c*d
^4 - b*d^3*e + a*d^2*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2)) - 1/2*(3*b*d + 2*a*
e)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2*
d^2) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a*sqrt(c))/(((sqr
t(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*a^2*d)
```

maple [B]   time = 0.03, size = 863, normalized size = 2.06

$$\frac{2ce^{3}\ln\left(\frac{\frac{(be-2cd)\left(x^{2}+\frac{d}{e}\right)}{e}+\frac{2ae^{2}-2deb+2cd^{2}}{e^{2}}+2\sqrt{\frac{ae^{2}-deb+cd^{2}}{e^{2}}}\sqrt{\left(x^{2}+\frac{d}{e}\right)^{2}c+\frac{(be-2cd)\left(x^{2}+\frac{d}{e}\right)}{e}+\frac{ae^{2}-deb+cd^{2}}{e^{2}}}}{x^{2}+\frac{d}{e}}\right)}{\left(-be+2cd+\sqrt{-4ac+b^{2}}\,e\right)\left(be-2cd+\sqrt{-4ac+b^{2}}\,e\right)\sqrt{\frac{ae^{2}-deb+cd^{2}}{e^{2}}}\,d^{2}}+\frac{bcex^{2}}{\left(4ac-b^{2}\right)\sqrt{cx^{4}+bx^{2}+a}\,ad^{2}}-\frac{4c^{2}x^{2}}{\left(4ac-b^{2}\right)\sqrt{cx^{4}+bx^{2}+a}\,ad}+\frac{3b^{2}cx^{2}}{2\left(4ac-b^{2}\right)\sqrt{cx^{4}+bx^{2}+a}\,a^{2}d}+\frac{2\sqrt{\left(x^{2}+\frac{b+\sqrt{-4ac+b^{2}}}{2c}\right)^{2}c-\sqrt{-4ac+b^{2}}\left(x^{2}+\frac{b+\sqrt{-4ac+b^{2}}}{2c}\right)}\,ce^{2}}{\left(be-2cd+\sqrt{-4ac+b^{2}}\,e\right)\left(-4ac+b^{2}\right)\left(x^{2}+\frac{b}{2c}+\frac{\sqrt{-4ac+b^{2}}}{2c}\right)d^{2}}-\frac{2\sqrt{\left(x^{2}-\frac{-b+\sqrt{-4ac+b^{2}}}{2c}\right)^{2}c+\sqrt{-4ac+b^{2}}\left(x^{2}-\frac{-b+\sqrt{-4ac+b^{2}}}{2c}\right)}\,ce^{2}}{\left(-be+2cd+\sqrt{-4ac+b^{2}}\,e\right)\left(-4ac+b^{2}\right)\left(x^{2}+\frac{b}{2c}-\frac{\sqrt{-4ac+b^{2}}}{2c}\right)d^{2}}+\frac{b^{2}e}{2\left(4ac-b^{2}\right)\sqrt{cx^{4}+bx^{2}+a}\,ad^{2}}-\frac{2bc}{\left(4ac-b^{2}\right)\sqrt{cx^{4}+bx^{2}+a}\,ad}+\frac{3b^{3}}{4\left(4ac-b^{2}\right)\sqrt{cx^{4}+bx^{2}+a}\,a^{2}d}+\frac{e\ln\left(\frac{bx^{2}+2a+2\sqrt{cx^{4}+bx^{2}+a}\,\sqrt{a}}{x^{2}}\right)}{2a^{\frac{3}{2}}d^{2}}+\frac{3b\ln\left(\frac{bx^{2}+2a+2\sqrt{cx^{4}+bx^{2}+a}\,\sqrt{a}}{x^{2}}\right)}{4a^{\frac{5}{2}}d}-\frac{e}{2\sqrt{cx^{4}+bx^{2}+a}\,ad^{2}}-\frac{3b}{4\sqrt{cx^{4}+bx^{2}+a}\,a^{2}d}-\frac{1}{2\sqrt{cx^{4}+bx^{2}+a}\,adx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/2/d^2*e/a/(c*x^4+b*x^2+a)^(1/2)+1/d^2*e*b/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^
(1/2)*c*x^2+1/2/d^2*e*b^2/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)+1/2/d^2*e/a^(
3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)+2*e^2/d^2*c/(b*e-2
*c*d+(-4*a*c+b^2)^(1/2)*e)/(-4*a*c+b^2)/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)
/c)*((x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2*c-(-4*a*c+b^2)^(1/2)*(x^2+1/2*(b+
(-4*a*c+b^2)^(1/2))/c))^(1/2)+2*e^3/d^2*c/(-b*e+2*c*d+(-4*a*c+b^2)^(1/2)*e)
/(b*e-2*c*d+(-4*a*c+b^2)^(1/2)*e)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-
2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/
(x^2+d/e))-2*e^2/d^2*c/(-b*e+2*c*d+(-4*a*c+b^2)^(1/2)*e)/(-4*a*c+b^2)/(x^2+
1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*((x^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2*c+
(-4*a*c+b^2)^(1/2)*(x^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c))^(1/2)-1/2/d/a/x^2/(
c*x^4+b*x^2+a)^(1/2)-3/4/d*b/a^2/(c*x^4+b*x^2+a)^(1/2)+3/2/d*b^2/a^2/(4*a*c
-b^2)/(c*x^4+b*x^2+a)^(1/2)*c*x^2+3/4/d*b^3/a^2/(4*a*c-b^2)/(c*x^4+b*x^2+a)
^(1/2)+3/4/d*b/a^(5/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-
4/d*c^2/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2-2/d*c/a/(4*a*c-b^2)/(c*x^4+
b*x^2+a)^(1/2)*b
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{4}+bx^{2}+a\right)^{\frac{3}{2}}\left(ex^{2}+d\right)x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*x^3), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{3}\,\left(e\,x^{2}+d\right)\,{\left(c\,x^{4}+b\,x^{2}+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(d + ex^2\right) \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.254 $\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=406

$$-\frac{\left(-\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^3(-e)+b^2cd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{-2a^2c^2e+4ab^2ce\cdots}{\sqrt{\cdots}}\right.}{\cdots}$$

Rubi [A] time = 8.59, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 897, 1287, 1166, 208}

$$-\frac{\left(-\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^3cd+b^4(-e)}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^2cd+b^3(-e)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^3cd+b^4(-e)}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^2cd+b^3(-e)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\,c^{7/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\left(b^2-ac\right)\sqrt{d+ex^2}}{c^3}-\frac{\left(d+ex^2\right)^{3/2}(be+cd)}{3c^2e^2}+\frac{\left(d+ex^2\right)^{5/2}}{5ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((b^2 - a*c)*Sqrt[d + e*x^2])/c^3 - ((c*d + b*e)*(d + e*x^2)^(3/2))/(3*c^2*
e^2) + (d + e*x^2)^(5/2)/(5*c*e^2) - ((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*
e - (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 -
4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])
- ((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e + (b^3*c*d - 3*a*b*c^2*d - b^4*e
+ 4*a*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sq
rt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/2)*S
qrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
```

```
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(b^2-ac)e}{c^3}-\frac{(cd+be)x^2}{c^2e}+\frac{x^4}{ce}-\frac{(b^2-ac)(cd^2-bde+ae^2)-(b^2cd-ac^2d-b^3e+2abce)x^2}{c^3e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right)dx, x, \sqrt{d+ex\ldots}\right.}{e}$$

$$= \frac{(b^2-ac)\sqrt{d+ex^2}}{c^3}-\frac{(cd+be)(d+ex^2)^{3/2}}{3c^2e^2}+\frac{(d+ex^2)^{5/2}}{5ce^2}-\frac{\text{Subst}\left(\int \frac{(b^2-ac)(cd^2-bde+ae^2)+(\ldots}{\frac{cd^2-bde+ae^2}{e^2}-\ldots}\right.}{\ldots}$$

$$= \frac{(b^2-ac)\sqrt{d+ex^2}}{c^3}-\frac{(cd+be)(d+ex^2)^{3/2}}{3c^2e^2}+\frac{(d+ex^2)^{5/2}}{5ce^2}+\frac{(b^2cd-ac^2d-b^3e+2abce-\ldots}{\ldots}$$

$$= \frac{(b^2-ac)\sqrt{d+ex^2}}{c^3}-\frac{(cd+be)(d+ex^2)^{3/2}}{3c^2e^2}+\frac{(d+ex^2)^{5/2}}{5ce^2}-\frac{(b^2cd-ac^2d-b^3e+2abce-\ldots}{\sqrt{2}\ldots}$$

Mathematica [B] time = 10.84, size = 943, normalized size = 2.32

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^7*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*(d + e*x^2)^(9/2)*(Sqrt[2]*Sqrt[(c*(d + e*x^2))/(2*c*d - b*e + Sqrt[b^2
 - 4*a*c]*e)]*(105*b^3*e^3 - 35*b^2*e^2*(3*Sqrt[b^2 - 4*a*c]*e + c*(d + e*x
^2)) + 7*b*c*e*(-45*a*e^2 + (d + e*x^2)*(-5*c*d + 5*Sqrt[b^2 - 4*a*c]*e + 3
*c*(d + e*x^2))) + c*(35*a*e^2*(3*Sqrt[b^2 - 4*a*c]*e + 2*c*(d + e*x^2)) +
c*(d + e*x^2)*(7*Sqrt[b^2 - 4*a*c]*e*(5*d - 3*(d + e*x^2)) + c*(-70*d^2 + 8
4*d*(d + e*x^2) - 30*(d + e*x^2)^2)))) + 105*(-b^3 + 3*a*b*c + b^2*Sqrt[b^2
 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*e^3*ArcTanh[Sqrt[2]*Sqrt[(c*(d + e*x^2))
/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)]]))/(210*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e^4
*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)^3*(-(Sqrt[b^2 - 4*a*c]/e) - (2*c*d -
b*e)/e^2)*((c*(d + e*x^2))/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e))^(9/2)) + (2
```

```
*c*d^3*(d + e*x^2)^(3/2)*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(-105*b^3*e^
3 + 35*b^2*e^2*(-3*Sqrt[b^2 - 4*a*c]*e + c*(d + e*x^2)) - 7*b*c*e*(-45*a*e^
2 + (d + e*x^2)*(-5*c*d - 5*Sqrt[b^2 - 4*a*c]*e + 3*c*(d + e*x^2))) + c*(35
*a*e^2*(3*Sqrt[b^2 - 4*a*c]*e - 2*c*(d + e*x^2)) + c*(d + e*x^2)*(7*Sqrt[b^
2 - 4*a*c]*e*(5*d - 3*(d + e*x^2)) + c*(70*d^2 - 84*d*(d + e*x^2) + 30*(d +
 e*x^2)^2)))))/(140*c^4*d^3*(d + e*x^2)) + (3*(b^3 - 3*a*b*c + b^2*Sqrt[b^2
 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*e^3*(d + e*x^2)^3*ArcTanh[Sqrt[2]*Sqrt[(
c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]])/(4*Sqrt[2]*d^3*(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)^3*((c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e))^(9/2))))/(3*Sqrt[b^2 - 4*a*c]*e^4*(Sqrt[b^2 - 4*a*c]/e - (2*c*d
- b*e)/e^2)))/e
```

IntegrateAlgebraic [A] time = 1.65, size = 502, normalized size = 1.24

$$-\frac{\left(-2a^2c^2e+ac^2d\sqrt{b^2-4ac}-b^2cd\sqrt{b^2-4ac}+4ab^2ce-2abce\sqrt{b^2-4ac}+b^3e\sqrt{b^2-4ac}-3abc^2d+b^4(-e)+b^3cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}\right)}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}-\frac{\left(2a^2c^2e+ac^2d\sqrt{b^2-4ac}-b^2cd\sqrt{b^2-4ac}-4ab^2ce-2abce\sqrt{b^2-4ac}+b^3e\sqrt{b^2-4ac}+3abc^2d+b^4e-b^3cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}+be-2cd}}\right)}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}\sqrt{e\sqrt{b^2-4ac}+be-2cd}}+\frac{\sqrt{d+ex^2}\left(-15ace^2+15b^2e^2-5bcde-5bce^2x^2-2c^2d^2+c^2dex^2+3c^2e^2x^4\right)}{15c^3e^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^7*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[d + e*x^2]*(-2*c^2*d^2 - 5*b*c*d*e + 15*b^2*e^2 - 15*a*c*e^2 + c^2*d*
e*x^2 - 5*b*c*e^2*x^2 + 3*c^2*e^2*x^4))/(15*c^3*e^2) - ((b^3*c*d - 3*a*b*c^
2*d - b^2*c*Sqrt[b^2 - 4*a*c]*d + a*c^2*Sqrt[b^2 - 4*a*c]*d - b^4*e + 4*a*b
^2*c*e - 2*a^2*c^2*e + b^3*Sqrt[b^2 - 4*a*c]*e - 2*a*b*c*Sqrt[b^2 - 4*a*c]*
e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - Sqrt[b^2 -
4*a*c]*e]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + b*e - Sqrt[b^2
 - 4*a*c]*e]) - ((-(b^3*c*d) + 3*a*b*c^2*d - b^2*c*Sqrt[b^2 - 4*a*c]*d + a*
c^2*Sqrt[b^2 - 4*a*c]*d + b^4*e - 4*a*b^2*c*e + 2*a^2*c^2*e + b^3*Sqrt[b^2
- 4*a*c]*e - 2*a*b*c*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d +
e*x^2])/Sqrt[-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[2]*c^(7/2)*Sqrt[b^
2 - 4*a*c]*Sqrt[-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.65, size = 928, normalized size = 2.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -((((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*
e^2)*c^2 + 2*(b^3*c^4 - 3*a*b*c^5)*d^2 - (3*b^4*c^3 - 11*a*b^2*c^4 + 4*a^2*
c^5)*d*e - 2*((b^2*c^3 - a*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^3*c^2 - a*b*c^3)
*sqrt(b^2 - 4*a*c)*d*e + (a*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^2)*abs(c
) + (b^5*c^2 - 4*a*b^3*c^3 + 2*a^2*b*c^4)*e^2)*arctan(2*sqrt(1/2)*sqrt(x^2*
e + d)/sqrt(-(2*c^6*d*e^12 - b*c^5*e^13 + sqrt(-4*(c^6*d^2*e^12 - b*c^5*d*e
^13 + a*c^5*e^14)*c^6*e^12 + (2*c^6*d*e^12 - b*c^5*e^13)^2))*e^(-12)/c^6))/
((2*sqrt(b^2 - 4*a*c)*c^4*d + (b^2*c^3 - 4*a*c^4 - sqrt(b^2 - 4*a*c)*b*c^3)
*e)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2) + (((b^4*c - 5*a*
b^2*c^2 + 4*a^2*c^3)*d*e - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^2)*c^2 + 2*(b^
3*c^4 - 3*a*b*c^5)*d^2 - (3*b^4*c^3 - 11*a*b^2*c^4 + 4*a^2*c^5)*d*e + 2*((b
^2*c^3 - a*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^3*c^2 - a*b*c^3)*sqrt(b^2 - 4*a*
```

```
c)*d*e + (a*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^2)*abs(c) + (b^5*c^2 - 4
*a*b^3*c^3 + 2*a^2*b*c^4)*e^2)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*
c^6*d*e^12 - b*c^5*e^13 - sqrt(-4*(c^6*d^2*e^12 - b*c^5*d*e^13 + a*c^5*e^14
)*c^6*e^12 + (2*c^6*d*e^12 - b*c^5*e^13)^2))*e^(-12)/c^6))/((2*sqrt(b^2 - 4
*a*c)*c^4*d - (b^2*c^3 - 4*a*c^4 + sqrt(b^2 - 4*a*c)*b*c^3)*e)*sqrt(-4*c^2*
d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2) + 1/15*(3*(x^2*e + d)^(5/2)*c^4*e
^8 - 5*(x^2*e + d)^(3/2)*c^4*d*e^8 - 5*(x^2*e + d)^(3/2)*b*c^3*e^9 + 15*sqr
t(x^2*e + d)*b^2*c^2*e^10 - 15*sqrt(x^2*e + d)*a*c^3*e^10)*e^(-10)/c^5
```

maple [C] time = 0.06, size = 496, normalized size = 1.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/5/c*x^2*(e*x^2+d)^(3/2)/e-2/15/c*d/e^2*(e*x^2+d)^(3/2)-1/3/c^2*b*(e*x^2+d
)^(3/2)/e+1/2/c^2*e^(1/2)*x*a-1/2/c^3*e^(1/2)*x*b^2-1/2/c^2*(e*x^2+d)^(1/2)
*a+1/2/c^3*(e*x^2+d)^(1/2)*b^2-1/4/c^3*sum(((-2*a*b*c*e+a*c^2*d+b^3*e-b^2*c
*d)*_R^6+(-4*a^2*c*e^2+4*a*b^2*e^2+2*a*b*c*d*e-3*a*c^2*d^2-3*b^3*d*e+3*b^2*
c*d^2)*_R^4+d*(4*a^2*c*e^2-4*a*b^2*e^2-2*a*b*c*d*e+3*a*c^2*d^2+3*b^3*d*e-3*
b^2*c*d^2)*_R^2+2*a*b*c*d^3*e-a*c^2*d^4-b^3*d^3*e+b^2*c*d^4)/(_R^7*c+3*_R^5
*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)
*ln((e*x^2+d)^(1/2)-e^(1/2)*x-_R),_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^6+(16*a
*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2+c*d^4))-1/2/c^2*d/((e*x
^2+d)^(1/2)-e^(1/2)*x)*a+1/2/c^3*d/((e*x^2+d)^(1/2)-e^(1/2)*x)*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d}\, x^7}{cx^4 + bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*x^7/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 2.48, size = 11195, normalized size = 27.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (d + e*x^2)^(1/2)*((3*d^2)/(c*e^2) - (a*e^4 + c*d^2*e^2 - b*d*e^3)/(c^2*e^4
) + (((3*d)/(c*e^2) + (b*e^3 - 2*c*d*e^2)/(c^2*e^4))*(b*e^3 - 2*c*d*e^2))/(
c*e^2)) - (d + e*x^2)^(3/2)*(d/(c*e^2) + (b*e^3 - 2*c*d*e^2)/(3*c^2*e^4)) +
 atan(((((16*a^3*c^6*e^4 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b^2*c
^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 - 16*a
^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d^2*e^2)/c^5 - (2*(d + e*x^2)^(1/2)*(-(b^9*e
- 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d
 + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*
a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*
c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b
^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2)
- 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*
b^2*c^8)))^(1/2)*(4*b^3*c^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 32*a*c
^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b
^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^
3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*
d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*
```

c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i - (((16*a^3*c^6*e^4 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d^2*e^2)/c^5 + (2*(d + e*x^2)^(1/2)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(4*b^3*c^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 32*a*c^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i)/((((16*a^3*c^6*e^4 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d^2*e^2)/c^5 - (2*(d + e*x^2)^(1/2)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(4*b^3*c^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 32*a*c^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e

^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (((16*a^3*c^6*e^4 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d^2*e^2)/c^5 + (2*(d + e*x^2)^(1/2)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(4*b^3*c^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 32*a*c^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (2*(a^4*b^3*e^5 - a^3*b^4*d*e^4 + a^5*c^2*d*e^4 + a^4*c^3*d^3*e^2 - 2*a^5*b*c*e^5 - a^3*b^2*c^2*d^3*e^2 + a^4*b^2*c*d*e^4 + 2*a^3*b^3*c*d^2*e^3 - 3*a^4*b*c^2*d^2*e^3))/c^5))*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*2i + atan(((((16*a^3*c^6*e^4 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d^2*e^2)/c^5 - (2*(d + e*x^2)^(1/2)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(4*b^3*c^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 32*a*c^9*d*e^2))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^8*e

^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^
2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2
*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a
^2*b^3*c^3*d*e^3))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1
/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63
*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b
^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*
(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b
*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i - (((16*a^3*c^6*e^4 + 4
*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d^2*e^2
+ 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a*b^2*c^
6*d^2*e^2)/c^5 + (2*(d + e*x^2)^(1/2)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*
c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*
b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^
7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2)
+ 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(
1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(4*b^3*c^7*e
^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 32*a*c^9*d*e^2))/c^5)*((8*a^4*c^5*d
 - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38
*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c -
 b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(
-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^
2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a
^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c
^8)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^
2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*
c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3
*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*((8*a^4*c^
5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d -
 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*
c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*
d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3
*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) -
6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^
2*c^8)))^(1/2)*1i)/((((16*a^3*c^6*e^4 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 -
 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5
*d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d^2*e^2)/c^5 - (2*(d + e*x^2)^(1
/2)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a
^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3
*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*
c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(
1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^
2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^
4*c^7 - 8*a*b^2*c^8)))^(1/2)*(4*b^3*c^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*
e^3 + 32*a*c^9*d*e^2))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3
)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e
+ 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10
*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*
c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a
^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1
/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (2*(d + e*x^2)^(1/2)
*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3
*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*
c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2
 - 28*a^2*b^3*c^3*d*e^3))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2
)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2
*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e -

```
10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b
^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) +
3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (((16*a^3*c^6*e^4
 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d^2*
e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a*b^
2*c^6*d^2*e^2)/c^5 + (2*(d + e*x^2)^(1/2)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(
4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*
a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*
a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1
/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^
3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(4*b^3*c
^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 32*a*c^9*d*e^2))/c^5)*((8*a^4*c
^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d
- 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a
*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c
*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^
3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) -
 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b
^2*c^8)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^
4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*
b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14
*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*((8*a^
4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3
*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(
4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^
5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a
*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2
) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*
a*b^2*c^8)))^(1/2) - (2*(a^4*b^3*e^5 - a^3*b^4*d*e^4 + a^5*c^2*d*e^4 + a^4*
c^3*d^3*e^2 - 2*a^5*b*c*e^5 - a^3*b^2*c^2*d^3*e^2 + a^4*b^2*c*d*e^4 + 2*a^3
*b^3*c*d^2*e^3 - 3*a^4*b*c^2*d^2*e^3))/c^5))*((8*a^4*c^5*d - b^9*e - b^6*e*
(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d -
42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) +
11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)
^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^
2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4
*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*2i +
(d + e*x^2)^(5/2)/(5*c*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.255 $\int \frac{x^5 \sqrt{d+ex^2}}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=324

$$\frac{\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-\right.}{\sqrt{2}c^{5/2}\sqrt{2cd-}}$$

Rubi [A] time = 3.53, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 897, 1287, 1166, 208}

$$\frac{\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{b\sqrt{d+ex^2}}{c^2}+\frac{\left(d+ex^2\right)^{3/2}}{3ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*Sqrt[d + e*x^2])/c^2) + (d + e*x^2)^(3/2)/(3*c*e) + ((b*c*d - b^2*e +
a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
h[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]
])/(Sqrt[2]*c^(5/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((b*c*d - b^
2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*
ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{be}{c^2}+\frac{x^2}{c}+\frac{b\left(cd^2-bde+ae^2\right)-\left(bcd-b^2e+ace\right)x^2}{c^2e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right)dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= -\frac{b\sqrt{d+ex^2}}{c^2}+\frac{\left(d+ex^2\right)^{3/2}}{3ce}+\frac{\text{Subst}\left(\int \frac{b\left(cd^2-bde+ae^2\right)+\left(-bcd+b^2e-ace\right)x^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex^2}\right)}{c^2e^2}$$

$$= -\frac{b\sqrt{d+ex^2}}{c^2}+\frac{\left(d+ex^2\right)^{3/2}}{3ce}-\frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{-\frac{\sqrt{b^2-4ac}}{2e}-\frac{2cd\cdots}{2e\cdots}}\cdots\right)}{2c^2e^2}$$

$$= -\frac{b\sqrt{d+ex^2}}{c^2}+\frac{\left(d+ex^2\right)^{3/2}}{3ce}+\frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+e\cdots}}{\sqrt{2cd-\left(b-\sqrt{b^2-\cdots}\right.}}\cdots\right)}{\sqrt{2}\,c^{5/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] time = 7.25, size = 591, normalized size = 1.82

$$\frac{c\left(d+ex^2\right)^{7/2}\left(\frac{e^2\left(\sqrt{2}\sqrt{\frac{c(d+ex^2)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}\left(5be\left(3e\sqrt{b^2-4ac}+c(d+ex^2)\right)+c(d+ex^2)\left(-5e\sqrt{b^2-4ac}+4cd-6cex^2\right)+30ace^2-15b^2e^2\right)-15e^2\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tanh^{-1}\left(\sqrt{2}\sqrt{\frac{c(d+ex^2)}{e\sqrt{b^2-4ac}-be+2cd}}\right)\right)}{\left(e\left(b-\sqrt{b^2-4ac}\right)-2cd\right)\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)^2\left(\frac{c(d+ex^2)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}\right)^{7/2}}-\frac{e^2\left(\sqrt{2}\sqrt{\frac{c(d+ex^2)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(-5be\left(3e\sqrt{b^2-4ac}-c(d+ex^2)\right)+c(d+ex^2)\left(5e\sqrt{b^2-4ac}+4cd-6cex^2\right)+30ace^2-15b^2e^2\right)+15e^2\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tanh^{-1}\left(\sqrt{2}\sqrt{\frac{c(d+ex^2)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)\right)}{\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)^3\left(\frac{c(d+ex^2)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{7/2}}\right)}{30\sqrt{2}\,c^4\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (c*(d + e*x^2)^(7/2)*((e^2*(Sqrt[2]*Sqrt[(c*(d + e*x^2))/(2*c*d + (-b + Sqr
t[b^2 - 4*a*c])*e)]*(-15*b^2*e^2 + 30*a*c*e^2 + c*(d + e*x^2)*(4*c*d - 5*Sq
rt[b^2 - 4*a*c]*e - 6*c*e*x^2) + 5*b*e*(3*Sqrt[b^2 - 4*a*c]*e + c*(d + e*x^
2))) - 15*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*e^2*ArcTanh[Sqrt[2]*Sqrt[(c*
(d + e*x^2))/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)]]))/((-2*c*d + (b - Sqrt[b
^2 - 4*a*c])*e)*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)^2*((c*(d + e*x^2))/(2*
c*d + (-b + Sqrt[b^2 - 4*a*c])*e))^(7/2)) - (e^2*(Sqrt[2]*Sqrt[(c*(d + e*x^
2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(-15*b^2*e^2 + 30*a*c*e^2 + c*(d +
 e*x^2)*(4*c*d + 5*Sqrt[b^2 - 4*a*c]*e - 6*c*e*x^2) - 5*b*e*(3*Sqrt[b^2 - 4
*a*c]*e - c*(d + e*x^2))) + 15*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*e^2*ArcT
anh[Sqrt[2]*Sqrt[(c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]]))/((
-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)^3*((c*(d + e*x^2))/(2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e))^(7/2))))/(30*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e^4)
```

IntegrateAlgebraic [A] time = 1.23, size = 454, normalized size = 1.40

$$\frac{\left(-\sqrt{2}\,bcd\sqrt{b^2-4ac}+\sqrt{2}\,b^2e\sqrt{b^2-4ac}-\sqrt{2}\,ace\sqrt{b^2-4ac}+3\sqrt{2}\,abce-2\sqrt{2}\,ac^2d-\sqrt{2}\,b^3e+\sqrt{2}\,b^2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}\right)}{2c^{5/2}\sqrt{b^2-4ac}\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}+\frac{\left(-\sqrt{2}\,bcd\sqrt{b^2-4ac}+\sqrt{2}\,b^2e\sqrt{b^2-4ac}-\sqrt{2}\,ace\sqrt{b^2-4ac}-3\sqrt{2}\,abce+2\sqrt{2}\,ac^2d+\sqrt{2}\,b^3e-\sqrt{2}\,b^2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}+be-2cd}}\right)}{2c^{5/2}\sqrt{b^2-4ac}\sqrt{e\sqrt{b^2-4ac}+be-2cd}}+\frac{\sqrt{d+ex^2}\left(-3be+cd+cex^2\right)}{3c^2e}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[d + e*x^2]*(c*d - 3*b*e + c*e*x^2))/(3*c^2*e) + ((Sqrt[2]*b^2*c*d - 2
*Sqrt[2]*a*c^2*d - Sqrt[2]*b*c*Sqrt[b^2 - 4*a*c]*d - Sqrt[2]*b^3*e + 3*Sqrt
[2]*a*b*c*e + Sqrt[2]*b^2*Sqrt[b^2 - 4*a*c]*e - Sqrt[2]*a*c*Sqrt[b^2 - 4*a*
c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - Sqrt[b^2
 - 4*a*c]*e]])/(2*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + b*e - Sqrt[b^2 -
4*a*c]*e]) + ((-(Sqrt[2]*b^2*c*d) + 2*Sqrt[2]*a*c^2*d - Sqrt[2]*b*c*Sqrt[b^
2 - 4*a*c]*d + Sqrt[2]*b^3*e - 3*Sqrt[2]*a*b*c*e + Sqrt[2]*b^2*Sqrt[b^2 - 4
*a*c]*e - Sqrt[2]*a*c*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d +
 e*x^2])/Sqrt[-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e]])/(2*c^(5/2)*Sqrt[b^2 - 4
*a*c]*Sqrt[-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.82, size = 745, normalized size = 2.30

$$\frac{\left((x^2e+d)^{\frac{3}{2}}c^2e^2-3\sqrt{x^2e+d}\,bce^3\right)e^{(-3)}}{3c^3}+\frac{\left(\left(\left(b^3c-4abc^2\right)de-\left(b^4-5ab^2c+4a^2c^2\right)e^2\right)c^2+2\left(b^2c^4-2ac^5\right)d^2-\left(3b^3c^3-8abc^4\right)de-2\left(\sqrt{b^2-4ac}\,bc^3d^2-\sqrt{b^2-4ac}\,b^2c^2de+\sqrt{b^2-4ac}\,abc^2e^2\right)|c|+\left(b^4c^2-3ab^2c^3\right)e^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{\left(2c^4de^4-bc^3e^5+\sqrt{-4\left(c^4d^2e^4-bc^3de^5+ac^3e^6\right)c^4e^4+\left(2c^4de^4-bc^3e^5\right)^2}\right)e^{(-4)}}{c^4}}}\right)}{\left(2\sqrt{b^2-4ac}\,c^3d+\left(b^2c^2-4ac^3-\sqrt{b^2-4ac}\,bc^2\right)e\right)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\,c^2}-\frac{\left(\left(\left(b^3c-4abc^2\right)de-\left(b^4-5ab^2c+4a^2c^2\right)e^2\right)c^2+2\left(b^2c^4-2ac^5\right)d^2-\left(3b^3c^3-8abc^4\right)de+2\left(\sqrt{b^2-4ac}\,bc^3d^2-\sqrt{b^2-4ac}\,b^2c^2de+\sqrt{b^2-4ac}\,abc^2e^2\right)|c|+\left(b^4c^2-3ab^2c^3\right)e^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{\left(2c^4de^4-bc^3e^5-\sqrt{-4\left(c^4d^2e^4-bc^3de^5+ac^3e^6\right)c^4e^4+\left(2c^4de^4-bc^3e^5\right)^2}\right)e^{(-4)}}{c^4}}}\right)}{\left(2\sqrt{b^2-4ac}\,c^3d-\left(b^2c^2-4ac^3+\sqrt{b^2-4ac}\,bc^2\right)e\right)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/3*((x^2*e + d)^(3/2)*c^2*e^2 - 3*sqrt(x^2*e + d)*b*c*e^3)*e^(-3)/c^3 + ((
(b^3*c - 4*a*b*c^2)*d*e - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^2)*c^2 + 2*(b^2*c
^4 - 2*a*c^5)*d^2 - (3*b^3*c^3 - 8*a*b*c^4)*d*e - 2*(sqrt(b^2 - 4*a*c)*b*c^
3*d^2 - sqrt(b^2 - 4*a*c)*b^2*c^2*d*e + sqrt(b^2 - 4*a*c)*a*b*c^2*e^2)*abs(
c) + (b^4*c^2 - 3*a*b^2*c^3)*e^2)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-
(2*c^4*d*e^4 - b*c^3*e^5 + sqrt(-4*(c^4*d^2*e^4 - b*c^3*d*e^5 + a*c^3*e^6)*
c^4*e^4 + (2*c^4*d*e^4 - b*c^3*e^5)^2))*e^(-4)/c^4))/((2*sqrt(b^2 - 4*a*c)*
c^3*d + (b^2*c^2 - 4*a*c^3 - sqrt(b^2 - 4*a*c)*b*c^2)*e)*sqrt(-4*c^2*d + 2*
(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2) - (((b^3*c - 4*a*b*c^2)*d*e - (b^4 - 5*
a*b^2*c + 4*a^2*c^2)*e^2)*c^2 + 2*(b^2*c^4 - 2*a*c^5)*d^2 - (3*b^3*c^3 - 8*
a*b*c^4)*d*e + 2*(sqrt(b^2 - 4*a*c)*b*c^3*d^2 - sqrt(b^2 - 4*a*c)*b^2*c^2*d
*e + sqrt(b^2 - 4*a*c)*a*b*c^2*e^2)*abs(c) + (b^4*c^2 - 3*a*b^2*c^3)*e^2)*a
rctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*c^4*d*e^4 - b*c^3*e^5 - sqrt(-4*
(c^4*d^2*e^4 - b*c^3*d*e^5 + a*c^3*e^6)*c^4*e^4 + (2*c^4*d*e^4 - b*c^3*e^5)
^2))*e^(-4)/c^4))/((2*sqrt(b^2 - 4*a*c)*c^3*d - (b^2*c^2 - 4*a*c^3 + sqrt(b
^2 - 4*a*c)*b*c^2)*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2)
```

maple [C] time = 0.03, size = 332, normalized size = 1.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3*(e*x^2+d)^(3/2)/c/e+1/2/c^2*b*e^(1/2)*x-1/2*b*(e*x^2+d)^(1/2)/c^2-1/4/c
^2*sum(((a*c*e-b^2*e+b*c*d)*_R^6+(-4*a*b*e^2+a*c*d*e+3*b^2*d*e-3*b*c*d^2)*_
R^4+d*(4*a*b*e^2-a*c*d*e-3*b^2*d*e+3*b*c*d^2)*_R^2-a*c*d^3*e+b^2*d^3*e-c*d^
4*b)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_
R*b*d^2*e-_R*c*d^3)*ln(-e^(1/2)*x-_R+(e*x^2+d)^(1/2)),_R=RootOf(_Z^8*c+(4*b
*e-4*c*d)*_Z^6+c*d^4+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z
^2))-1/2/c^2*b*d/(-e^(1/2)*x+(e*x^2+d)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}\,x^5}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*x^5/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 1.99, size = 8222, normalized size = 25.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (d + e*x^2)^(3/2)/(3*c*e) - atan(((((4*a*b^3*c^3*e^4 - 16*a^2*b*c^4*e^4 - 4
*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2*c^4*d*e^
3)/c^3 - (2*(d + e*x^2)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2
)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c
*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^
2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(
1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c
^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*
a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a
*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2
) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(
1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*(d + e*x^2)^(1/2
)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^
2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c
^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a
*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^
2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e
 - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2)
- 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*
c^6)))^(1/2)*1i - (((4*a*b^3*c^3*e^4 - 16*a^2*b*c^4*e^4 - 4*b^4*c^3*d*e^3 +
 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2*c^4*d*e^3)/c^3 + (2*(d +
 e*x^2)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6
*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(
1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^
2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c -
 b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*
e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*(-(b^7*e + 8
*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d +
25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b
^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*
(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2
*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^6*e^4 - 2*a
^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*
b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b
```

```
^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/
2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b
^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*
a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)
/((2*(a^4*c*e^5 - a^3*b^2*e^5 + a^2*b^3*d*e^4 + a^3*c^2*d^2*e^3 + a^2*b*c^2
*d^3*e^2 - 2*a^2*b^2*c*d^2*e^3))/c^3 + (((4*a*b^3*c^3*e^4 - 16*a^2*b*c^4*e^
4 - 4*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2*c^4
*d*e^3)/c^3 - (2*(d + e*x^2)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c
- b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e
*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e -
b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3
*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6
)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^
2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d
- 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2)
- 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)
^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*(d + e*x^2)
^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b
^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^
2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(
-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a
^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*
c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(
1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a
*b^2*c^6)))^(1/2) + (((4*a*b^3*c^3*e^4 - 16*a^2*b*c^4*e^4 - 4*b^4*c^3*d*e^3
 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2*c^4*d*e^3)/c^3 + (2*(d
 + e*x^2)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b
^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)
^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c -
b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^
5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*(-(b^7*e +
 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d
+ 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a
*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*
d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^6*e^4 - 2
*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*
a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a
*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(
1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c -
 b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(
4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)))
*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2
*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^
5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) +
 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2
))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i - atan(((((4*a*b^3*c^
3*e^4 - 16*a^2*b*c^4*e^4 - 4*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5
*d^2*e^2 + 16*a*b^2*c^4*d*e^3)/c^3 - (2*(d + e*x^2)^(1/2)*((b^4*e*(-(4*a*c
- b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2
*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2
*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a
*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 +
 b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c
^6*e^3 + 32*a*c^7*d*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4
```

*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4
*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c
d(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^
2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(
1/2) - (2*(d + e*x^2)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 +
2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*
b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*((b^4*e*(-(
4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d -
25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b
^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*
(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2
*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((4*a*b^3*c^3*e^4 - 16*a^2*b*c^
4*e^4 - 4*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2
*c^4*d*e^3)/c^3 + (2*(d + e*x^2)^(1/2)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8
*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^
2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e
 - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2)
- 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*
c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d
e^2))/c^3)((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*
d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2
) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^
3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^
2)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 +
 b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*
a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/
2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e +
a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b
*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^
(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*
a*b^2*c^6)))^(1/2)*1i)/((2*(a^4*c*e^5 - a^3*b^2*e^5 + a^2*b^3*d*e^4 + a^3*c
^2*d^2*e^3 + a^2*b*c^2*d^3*e^2 - 2*a^2*b^2*c*d^2*e^3))/c^3 + (((4*a*b^3*c^3
*e^4 - 16*a^2*b*c^4*e^4 - 4*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*
d^2*e^2 + 16*a*b^2*c^4*d*e^3)/c^3 - (2*(d + e*x^2)^(1/2)*((b^4*e*(-(4*a*c -
 b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*
b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*
d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*
c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 +
b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^
6*e^3 + 32*a*c^7*d*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*
d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*
a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*
d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2
*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1
/2) - (2*(d + e*x^2)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2
*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b
^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*((b^4*e*(-(4
*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 2
5*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^
4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(
-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*
c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((4*a*b^3*c^3*e^4 - 16*a^2*b*c^4*e^
4 - 4*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2*c^4
*d*e^3)/c^3 + (2*(d + e*x^2)^(1/2)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3
*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*
(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b
^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*
a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)

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))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2
))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d +
18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) +
9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(
1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3
)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^2)^(
1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4
*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*
b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) -
 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*
c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3
*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2
) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^
2*c^6)))^(1/2)))*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b
^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)
^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c -
b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i - ((2*
d)/(c*e) + (b*e^2 - 2*c*d*e)/(c^2*e^2))*(d + e*x^2)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.256 $\int \frac{x^3 \sqrt{d+ex^2}}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=292

$$\frac{\left(-\sqrt{b^2-4ac}\,(cd-be)+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\sqrt{b^2-4ac}\,(cd-be)+2ace+b^2(-e)\right.}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd}}$$

Rubi [A] time = 3.60, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1251, 824, 826, 1166, 208}

$$\frac{\left(-\sqrt{b^2-4ac}\,(cd-be)+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\sqrt{b^2-4ac}\,(cd-be)+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{d+ex^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Sqrt[d + e*x^2]/c + ((b*c*d - b^2*e + 2*a*c*e - Sqrt[b^2 - 4*a*c]*(c*d - b*
e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 -
4*a*c])*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]) - ((b*c*d - b^2*e + 2*a*c*e + Sqrt[b^2 - 4*a*c]*(c*d - b*e))*
ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= \frac{\sqrt{d+ex^2}}{c} + \frac{\text{Subst}\left(\int \frac{-ae+(cd-be)x}{\sqrt{d+ex}\,(a+bx+cx^2)}\,dx, x, x^2\right)}{2c}\\
&= \frac{\sqrt{d+ex^2}}{c} + \frac{\text{Subst}\left(\int \frac{-ae^2-d(cd-be)+(cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{c}\\
&= \frac{\sqrt{d+ex^2}}{c} - \frac{\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}\,(cd-be)\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be)+cx^2}\right.}{2c\sqrt{b^2-4ac}}\\
&= \frac{\sqrt{d+ex^2}}{c} + \frac{\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}\,(cd-be)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{(bc}{}
\end{aligned}$$

Mathematica [A] time = 0.55, size = 308, normalized size = 1.05

$$\frac{\frac{\left(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{\left(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}-2ace+b^2e-bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\sqrt{d+ex^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[d + e*x^2] + ((b*c*d - c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e + b*Sq
rt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b
*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d +
 (-b + Sqrt[b^2 - 4*a*c])*e]) + ((-(b*c*d) - c*Sqrt[b^2 - 4*a*c]*d + b^2*e
- 2*a*c*e + b*Sqrt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2]
)/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a
*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/c
```

IntegrateAlgebraic [C] time = 1.11, size = 352, normalized size = 1.21

$$\frac{\left(cd\sqrt{4ac-b^2}-be\sqrt{4ac-b^2}+2iace-ib^2e+ibcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{4ac-b^2}\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}+\frac{\left(cd\sqrt{4ac-b^2}-be\sqrt{4ac-b^2}-2iace+ib^2e-ibcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{4ac-b^2}\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}+\frac{\sqrt{d+ex^2}}{c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Sqrt[d + e*x^2]/c + ((I*b*c*d + c*Sqrt[-b^2 + 4*a*c]*d - I*b^2*e + (2*I)*a*
c*e - b*Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt
```

```
[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[2]*c^(3/2)*Sqrt[-b^2 + 4*a*
c]*Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]) + (((-I)*b*c*d + c*Sqrt[-b^
2 + 4*a*c]*d + I*b^2*e - (2*I)*a*c*e - b*Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt
[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/
(Sqrt[2]*c^(3/2)*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c
]*e])
```

fricas [B] time = 138.37, size = 2435, normalized size = 8.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(1/2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 -
 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c +
a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a*b^2*c*d^2
 - 2*a*b^3*d*e + 2*(a^2*b^2 - a^3*c)*e^2 + (a*b^2*c*d*e - (a*b^3 - a^2*b*c)
*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c - 4*a*b^2*c^2)*d - (b^5 - 5
*a*b^3*c + 4*a^2*b*c^2)*e - (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt((b^2*c
^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^
6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4
*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^
2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - ((a*b^2*c^3 - 4*a^
2*c^4)*e*x^2 + 2*(a*b^2*c^3 - 4*a^2*c^4)*d)*sqrt((b^2*c^2*d^2 - 2*(b^3*c -
a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/x^2)
 - sqrt(1/2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*
a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2
*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a*b^2*c*d^2 -
2*a*b^3*d*e + 2*(a^2*b^2 - a^3*c)*e^2 + (a*b^2*c*d*e - (a*b^3 - a^2*b*c)*e^
2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c - 4*a*b^2*c^2)*d - (b^5 - 5*a*
b^3*c + 4*a^2*b*c^2)*e - (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt((b^2*c^2*
d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 -
 4*a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*a*
c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c
^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - ((a*b^2*c^3 - 4*a^2*c
^4)*e*x^2 + 2*(a*b^2*c^3 - 4*a^2*c^4)*d)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b
*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/x^2) + s
qrt(1/2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c
^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^
2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a*b^2*c*d^2 - 2*a
*b^3*d*e + 2*(a^2*b^2 - a^3*c)*e^2 + (a*b^2*c*d*e - (a*b^3 - a^2*b*c)*e^2)*
x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c - 4*a*b^2*c^2)*d - (b^5 - 5*a*b^3
*c + 4*a^2*b*c^2)*e + (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt((b^2*c^2*d^2
 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*
a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4
)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)
*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + ((a*b^2*c^3 - 4*a^2*c^4)
*e*x^2 + 2*(a*b^2*c^3 - 4*a^2*c^4)*d)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^
2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/x^2) - sqrt
(1/2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)
*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*
e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a*b^2*c*d^2 - 2*a*b^
3*d*e + 2*(a^2*b^2 - a^3*c)*e^2 + (a*b^2*c*d*e - (a*b^3 - a^2*b*c)*e^2)*x^2
 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c - 4*a*b^2*c^2)*d - (b^5 - 5*a*b^3*c
+ 4*a^2*b*c^2)*e + (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt((b^2*c^2*d^2 -
2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c
^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)*s
qrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^
2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + ((a*b^2*c^3 - 4*a^2*c^4)*e*
```

```
x^2 + 2*(a*b^2*c^3 - 4*a^2*c^4)*d)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*
d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/x^2) + 4*sqrt(
e*x^2 + d))/c
```

giac [B] time = 0.78, size = 619, normalized size = 2.12

$$\frac{\sqrt{x^2e+d}}{c} - \frac{\left(2\,bc^4d^2 + \left(\left(b^2c - 4\,ac^2\right)de - \left(b^3 - 4\,abc\right)e^2\right)c^2 - \left(3\,b^2c^3 - 4\,ac^4\right)de - 2\left(\sqrt{b^2-4\,ac}\,c^3d^2 - \sqrt{b^2-4\,ac}\,bc^2de + \sqrt{b^2-4\,ac}\,ac^2e^2\right)|c| + \left(b^3c^2 - 2\,abc^3\right)e^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{2\,c^2d-bce+\sqrt{-4\left(c^2d^2-bcde+ace^2\right)c^2+\left(2\,c^2d-bce\right)^2}}{c^2}}}\right)}{\left(2\sqrt{b^2-4\,ac}\,c^2d - \left(b^2c - 4\,ac^2 + \sqrt{b^2-4\,ac}\,bc\right)e\right)\sqrt{-4\,c^2d + 2\left(bc - \sqrt{b^2-4\,ac}\,c\right)e}\,c^2} + \frac{\left(2\,bc^4d^2 + \left(\left(b^2c - 4\,ac^2\right)de - \left(b^3 - 4\,abc\right)e^2\right)c^2 - \left(3\,b^2c^3 - 4\,ac^4\right)de + 2\left(\sqrt{b^2-4\,ac}\,c^3d^2 - \sqrt{b^2-4\,ac}\,bc^2de + \sqrt{b^2-4\,ac}\,ac^2e^2\right)|c| + \left(b^3c^2 - 2\,abc^3\right)e^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{2\,c^2d-bce-\sqrt{-4\left(c^2d^2-bcde+ace^2\right)c^2+\left(2\,c^2d-bce\right)^2}}{c^2}}}\right)}{\left(2\sqrt{b^2-4\,ac}\,c^2d + \left(b^2c - 4\,ac^2 - \sqrt{b^2-4\,ac}\,bc\right)e\right)\sqrt{-4\,c^2d + 2\left(bc + \sqrt{b^2-4\,ac}\,c\right)e}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] sqrt(x^2*e + d)/c - (2*b*c^4*d^2 + ((b^2*c - 4*a*c^2)*d*e - (b^3 - 4*a*b*c)
*e^2)*c^2 - (3*b^2*c^3 - 4*a*c^4)*d*e - 2*(sqrt(b^2 - 4*a*c)*c^3*d^2 - sqrt
(b^2 - 4*a*c)*b*c^2*d*e + sqrt(b^2 - 4*a*c)*a*c^2*e^2)*abs(c) + (b^3*c^2 -
2*a*b*c^3)*e^2)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*c^2*d - b*c*e +
 sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/((
2*sqrt(b^2 - 4*a*c)*c^2*d - (b^2*c - 4*a*c^2 + sqrt(b^2 - 4*a*c)*b*c)*e)*sq
rt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2) + (2*b*c^4*d^2 + ((b^2*
c - 4*a*c^2)*d*e - (b^3 - 4*a*b*c)*e^2)*c^2 - (3*b^2*c^3 - 4*a*c^4)*d*e + 2
*(sqrt(b^2 - 4*a*c)*c^3*d^2 - sqrt(b^2 - 4*a*c)*b*c^2*d*e + sqrt(b^2 - 4*a*
c)*a*c^2*e^2)*abs(c) + (b^3*c^2 - 2*a*b*c^3)*e^2)*arctan(2*sqrt(1/2)*sqrt(x
^2*e + d)/sqrt(-(2*c^2*d - b*c*e - sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^
2 + (2*c^2*d - b*c*e)^2))/c^2))/((2*sqrt(b^2 - 4*a*c)*c^2*d + (b^2*c - 4*a*
c^2 - sqrt(b^2 - 4*a*c)*b*c)*e)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*
c)*e)*c^2)
```

maple [C] time = 0.03, size = 275, normalized size = 0.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/c*e^(1/2)*x+1/2*(e*x^2+d)^(1/2)/c+1/4/c*sum(((-b*e+c*d)*_R^6+(-4*a*e^2
+3*b*d*e-3*c*d^2)*_R^4+d*(4*a*e^2-3*b*d*e+3*c*d^2)*_R^2+b*d^3*e-c*d^4)/(_R^
7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e
-_R*c*d^3)*ln(-e^(1/2)*x-_R+(e*x^2+d)^(1/2)),_R=RootOf(_Z^8*c+(4*b*e-4*c*d)
*_Z^6+c*d^4+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2))+1/2/
c*d/(-e^(1/2)*x+(e*x^2+d)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}\,x^3}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*x^3/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 2.34, size = 5705, normalized size = 19.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (d + e*x^2)^(1/2)/c - atan(((((16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*
d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c - (2*(d
 + e*x^2)^(1/2)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^
```

```
4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2
)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a
*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*
c^5*d*e^2))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4
*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)
^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*
d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3
))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*
a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2)
 - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))
)^(1/2)*1i - (((16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3
*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c + (2*(d + e*x^2)^(1/2)
*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3
*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*
a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/
2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*
((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*
c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a
*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2
) + (2*(d + e*x^2)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c
^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a^2*c
^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c
*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*
d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/(((
(16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*
b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c - (2*(d + e*x^2)^(1/2)*((8*a^2*c^3*d
 - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-
(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 1
2*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e
^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*((8*a^2*c^3*d -
 b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(
4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12
*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(d + e*x
^2)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*
a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a^2*c^3*d - b^5*e -
b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b
^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^
2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(a*c^2*d^3*e^2 -
a^2*b*e^5 + a*b^2*d*e^4 + a^2*c*d*e^4 - 2*a*b*c*d^2*e^3))/c + (((16*a^2*c^3
*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2
*e^2 - 16*a*b*c^3*d*e^3)/c + (2*(d + e*x^2)^(1/2)*((8*a^2*c^3*d - b^5*e - b
^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^
2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2
*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*
c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*((8*a^2*c^3*d - b^5*e - b^
2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2
)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*
e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2)*(
b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4
 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*
a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2
) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16
*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)))*((8*a^2*c^3*d - b^5*e - b^2*e*(
-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^
(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8
*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i - atan(((((16*a^2*c^3*e^4
- 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2
- 16*a*b*c^3*d*e^3)/c - (2*(d + e*x^2)^(1/2)*(-(b^5*e - 8*a^2*c^3*d - b^2*e
```

```
*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3
)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/
(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*
d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e
*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3
)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/
(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^4
*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 -
2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*
c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2)
+ b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a
^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3*e^4 - 4*a*b^2*c^
2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3
*d*e^3)/c + (2*(d + e*x^2)^(1/2)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c -
b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*
c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c
^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a
*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c -
b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*
c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c
^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^4*e^4 + 2*a^2
*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^
3 + 6*a*b*c^2*d*e^3))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^
(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4
*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4
*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a
*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c -
(2*(d + e*x^2)^(1/2)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2
) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c
 - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3
 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 +
 32*a*c^5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2
) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c
 - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3
 - 8*a*b^2*c^4)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2
*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^
2*d*e^3))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*
c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^
3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b
^2*c^4)))^(1/2) - (2*(a*c^2*d^3*e^2 - a^2*b*e^5 + a*b^2*d*e^4 + a^2*c*d*e^4
 - 2*a*b*c*d^2*e^3))/c + (((16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2
*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c + (2*(d +
e*x^2)^(1/2)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*
c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^
3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b
^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^
5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*
c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^
3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b
^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d
^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3)
)/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*
a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2)
 + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))
)^(1/2)))*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d
 - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^
(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*
c^4)))^(1/2)*2i
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**3*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

3.257 $\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4} dx$

Optimal. Leaf size=202

$$\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.36, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1247, 699, 1130, 208}

$$\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
+ e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b
^2 - 4*a*c])) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*S
qrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*
Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 699

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx, x, x^2\right)$$
$$= e\,\text{Subst}\left(\int \frac{x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)$$
$$= -\left(\frac{1}{2}\left(-e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d+ex^2}\right)\right)$$
$$= -\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\ \tan}{\sqrt{2}\sqrt{c}\sqrt{b^2}}$$

Mathematica [A] time = 0.33, size = 179, normalized size = 0.89

$$\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)-\sqrt{e\sqrt{b^2-4ac}-be+2cd}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
+ e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]]) + Sqrt[2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

IntegrateAlgebraic [C] time = 0.73, size = 279, normalized size = 1.38

$$\frac{\left(e\sqrt{4ac-b^2}+ibe-2icd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}\right)}{\sqrt{2}\sqrt{c}\sqrt{4ac-b^2}\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}+\frac{\left(e\sqrt{4ac-b^2}-ibe+2icd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}\right)}{\sqrt{2}\sqrt{c}\sqrt{4ac-b^2}\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (((-2*I)*c*d + I*b*e + Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d
 + e*x^2])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[2]*Sqrt[c]*S
qrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]) + (((2*I)*c*
d - I*b*e + Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/
Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[-b^2 +
4*a*c]*Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e])
```

fricas [B] time = 21.45, size = 1085, normalized size = 5.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(1/2)*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*
a*c^3)))/(b^2*c - 4*a*c^2))*log((b*e^2*x^2 + 2*b*d*e - 2*a*e^2 + 2*sqrt(1/2
)*sqrt(e*x^2 + d)*((b^2 - 4*a*c)*e + (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2*c^2
```

```
- 4*a*c^3)))*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*
c^3)))/(b^2*c - 4*a*c^2)) + ((b^2*c - 4*a*c^2)*e*x^2 + 2*(b^2*c - 4*a*c^2)*
d)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/x^2) + 1/4*sqrt(1/2)*sqrt((2*c*d - b*e +
(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log((b*
e^2*x^2 + 2*b*d*e - 2*a*e^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^2 - 4*a*c)*e
+ (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))*sqrt((2*c*d - b*e + (b
^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) + ((b^2*c
 - 4*a*c^2)*e*x^2 + 2*(b^2*c - 4*a*c^2)*d)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/x
^2) - 1/4*sqrt(1/2)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2
 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log((b*e^2*x^2 + 2*b*d*e - 2*a*e^2 + 2*sqr
t(1/2)*sqrt(e*x^2 + d)*((b^2 - 4*a*c)*e - (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2
*c^2 - 4*a*c^3)))*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 -
 4*a*c^3)))/(b^2*c - 4*a*c^2)) - ((b^2*c - 4*a*c^2)*e*x^2 + 2*(b^2*c - 4*a*
c^2)*d)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/x^2) + 1/4*sqrt(1/2)*sqrt((2*c*d - b
*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*lo
g((b*e^2*x^2 + 2*b*d*e - 2*a*e^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^2 - 4*a*
c)*e - (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))*sqrt((2*c*d - b*e
 - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) - ((
b^2*c - 4*a*c^2)*e*x^2 + 2*(b^2*c - 4*a*c^2)*d)*sqrt(e^2/(b^2*c^2 - 4*a*c^3
)))/x^2)
```

giac [A] time = 0.55, size = 228, normalized size = 1.13

$$-\frac{\sqrt{-4\,c^{2}d+2\left(bc-\sqrt{b^{2}-4\,ac}\,c\right)e}\,\arctan\left(\frac{2\,\sqrt{\frac{1}{2}}\,\sqrt{x^{2}e+d}}{\sqrt{-\frac{2\,cd-be+\sqrt{(2\,cd-be)^{2}-4\,(cd^{2}-bde+ae^{2})c}}{c}}}\right)}{2\,\sqrt{b^{2}-4\,ac}\,|c|}+\frac{\sqrt{-4\,c^{2}d+2\left(bc+\sqrt{b^{2}-4\,ac}\,c\right)e}\,\arctan\left(\frac{2\,\sqrt{\frac{1}{2}}\,\sqrt{x^{2}e+d}}{\sqrt{-\frac{2\,cd-be-\sqrt{(2\,cd-be)^{2}-4\,(cd^{2}-bde+ae^{2})c}}{c}}}\right)}{2\,\sqrt{b^{2}-4\,ac}\,|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*arctan(2*sqrt(1/2)*sq
rt(x^2*e + d)/sqrt(-(2*c*d - b*e + sqrt((2*c*d - b*e)^2 - 4*(c*d^2 - b*d*e
+ a*e^2)*c))/c))/(sqrt(b^2 - 4*a*c)*abs(c)) + 1/2*sqrt(-4*c^2*d + 2*(b*c +
sqrt(b^2 - 4*a*c)*c)*e)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*c*d - b
*e - sqrt((2*c*d - b*e)^2 - 4*(c*d^2 - b*d*e + a*e^2)*c))/c))/(sqrt(b^2 - 4
*a*c)*abs(c))
```

maple [C] time = 0.02, size = 177, normalized size = 0.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*e*sum((_R^6+_R^4*d-_R^2*d^2-d^3)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a
*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln(-e^(1/2)*x-_R+(e*x^2
+d)^(1/2)),_R=RootOf(_Z^8*c+(4*b*e-4*c*d)*_Z^6+c*d^4+(16*a*e^2-8*b*d*e+6*c*
d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^{2}+d}\,x}{cx^{4}+bx^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*x/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 1.72, size = 717, normalized size = 3.55

$$-2\,\mathrm{atanh}\left(\frac{2\left(\sqrt{e\,x^2+d}\,\left(-2\,b^2\,c\,e^4+4\,b\,c^2\,d\,e^3-4\,c^3\,d^2\,e^2+4\,a\,c^2\,e^4\right)+\frac{\sqrt{e\,x^2+d}\,\left(8\,b^3\,c^2\,e^3-16\,d\,b^2\,c^3\,e^2-32\,a\,b\,c^3\,e^3+64\,a\,d\,c^4\,e^2\right)\left(b^3\,e+e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+8\,a\,c^2\,d-2\,b^2\,c\,d-4\,a\,b\,c\,e\right)}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}\right)\sqrt{-\frac{b^3\,e+e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+8\,a\,c^2\,d-2\,b^2\,c\,d-4\,a\,b\,c\,e}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}}{2\,c^2\,d^2\,e^3-2\,b\,c\,d\,e^4+2\,a\,c\,e^5}\right)\sqrt{-\frac{b^3\,e+e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+8\,a\,c^2\,d-2\,b^2\,c\,d-4\,a\,b\,c\,e}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}-2\,\mathrm{atanh}\left(\frac{2\left(\sqrt{e\,x^2+d}\,\left(-2\,b^2\,c\,e^4+4\,b\,c^2\,d\,e^3-4\,c^3\,d^2\,e^2+4\,a\,c^2\,e^4\right)-\frac{\sqrt{e\,x^2+d}\,\left(8\,b^3\,c^2\,e^3-16\,d\,b^2\,c^3\,e^2-32\,a\,b\,c^3\,e^3+64\,a\,d\,c^4\,e^2\right)\left(e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3\,e-8\,a\,c^2\,d+2\,b^2\,c\,d+4\,a\,b\,c\,e\right)}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}\right)\sqrt{\frac{e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3\,e-8\,a\,c^2\,d+2\,b^2\,c\,d+4\,a\,b\,c\,e}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}}{2\,c^2\,d^2\,e^3-2\,b\,c\,d\,e^4+2\,a\,c\,e^5}\right)\sqrt{\frac{e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3\,e-8\,a\,c^2\,d+2\,b^2\,c\,d+4\,a\,b\,c\,e}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] - 2*atanh((2*((d + e*x^2)^(1/2)*(4*a*c^2*e^4 - 2*b^2*c*e^4 - 4*c^3*d^2*e^2
+ 4*b*c^2*d*e^3) + ((d + e*x^2)^(1/2)*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 - 3
2*a*b*c^3*e^3 + 64*a*c^4*d*e^2)*(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c
^2*d - 2*b^2*c*d - 4*a*b*c*e))/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b
^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^2*d - 2*b^2*c*d - 4*a*b*c*e)/(8*(
b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(2*c^2*d^2*e^3 + 2*a*c*e^5 - 2*b
*c*d*e^4))*(-(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^2*d - 2*b^2*c*d -
4*a*b*c*e)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2) - 2*atanh((2*((d +
 e*x^2)^(1/2)*(4*a*c^2*e^4 - 2*b^2*c*e^4 - 4*c^3*d^2*e^2 + 4*b*c^2*d*e^3) -
 ((d + e*x^2)^(1/2)*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 - 32*a*b*c^3*e^3 + 64
*a*c^4*d*e^2)*(e*(-(4*a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d +
 4*a*b*c*e))/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((e*(-(4*a*c - b^2)^3)
^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4*a*b*c*e)/(8*(b^4*c + 16*a^2*c^3
- 8*a*b^2*c^2)))^(1/2))/(2*c^2*d^2*e^3 + 2*a*c*e^5 - 2*b*c*d*e^4))*((e*(-(4
*a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4*a*b*c*e)/(8*(b^4*c
 + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

3.258 $\int \frac{\sqrt{d+ex^2}}{x\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=281

$$\frac{\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{c}\left(-d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

Rubi [A] time = 1.35, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1251, 897, 1287, 206, 1166, 208}

$$\frac{\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{c}\left(-d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a) + (Sqrt[c]*(b*d + Sqrt[b^2
- 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d -
(b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[c]*(b*d - Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcT
anh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{x\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{\operatorname{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\operatorname{Subst}\left(\int \left(-\frac{de}{a\left(d-x^2\right)}+\frac{e\left(cd^2-bde+ae^2-cdx^2\right)}{a\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{cd^2-bde+ae^2-cdx^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{a} - \frac{d\operatorname{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex^2}\right)}{a} \ldots$$

$$= -\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a} + \frac{\left(c\left(bd-\sqrt{b^2-4ac}\,d-2ae\right)\right)\operatorname{Subst}\left(\int \frac{1}{\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be)+cx \ldots}\ldots\right)}{2a\sqrt{b^2-4ac}} \ldots$$

$$= -\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a} + \frac{\sqrt{c}\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \ldots$$

Mathematica [A] time = 0.81, size = 241, normalized size = 0.86

$$\frac{\frac{\sqrt{2}\left(\left(\sqrt{b^2-4ac}+b\right)\sqrt{e\sqrt{b^2-4ac}-be+2cd}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)+\left(\sqrt{b^2-4ac}-b\right)\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)\right)}{\sqrt{c}\sqrt{b^2-4ac}}-4\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-4*Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]] + (Sqrt[2]*((b + Sqrt[b^2 - 4*
a*c])*Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt
[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]] + (-b + Sqrt[b^2 - 4*
a*c])*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt
[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]]))/(Sqrt[c]*Sqrt[b^2 -
 4*a*c]))/(4*a)
```

IntegrateAlgebraic [A] time = 0.73, size = 280, normalized size = 1.00

$$-\frac{\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}-\frac{\sqrt{c}\left(d\sqrt{b^2-4ac}+2ae-bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}+be-2cd}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{e\sqrt{b^2-4ac}+be-2cd}}-\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[c]*(b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt
[d + e*x^2])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[2]*a*Sqrt[b^2
 - 4*a*c]*Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e])) - (Sqrt[c]*(-(b*d) + S
qrt[b^2 - 4*a*c]*d + 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-
2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c
*d + b*e + Sqrt[b^2 - 4*a*c]*e]) - (Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]
])/a
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.68, size = 717, normalized size = 2.55

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] d*arctan(sqrt(x^2*e + d)/sqrt(-d))/(a*sqrt(-d)) - 1/8*(sqrt(-4*c^2*d + 2*(b
*c - sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*a^2*d*e - 2*(sqrt(b^2 - 4*a*c)*a
*c*d^2 - sqrt(b^2 - 4*a*c)*a*b*d*e + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-4*c^2
*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(a) - (2*a^2*b*c*d^2 + 2*a^3*b*e^2
 - (a^2*b^2 + 4*a^3*c)*d*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e
))*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*a*c*d - a*b*e + sqrt(-4*(a*c
*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 -
4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)
*abs(a)*abs(c)) + 1/8*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*(b^
2 - 4*a*c)*a^2*d*e + 2*(sqrt(b^2 - 4*a*c)*a*c*d^2 - sqrt(b^2 - 4*a*c)*a*b*d
*e + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*
c)*e)*abs(a) - (2*a^2*b*c*d^2 + 2*a^3*b*e^2 - (a^2*b^2 + 4*a^3*c)*d*e)*sqrt
(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x^2*e
 + d)/sqrt(-(2*a*c*d - a*b*e - sqrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c +
(2*a*c*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*
a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(a)*abs(c))
```

maple [C] time = 0.03, size = 294, normalized size = 1.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/a*d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)+1/2/a*(e*x^2+d)^(1/2)+1/
2/a*e^(1/2)*x-1/4/a*sum((_R^6*c*d+(-4*a*e^2+4*b*d*e-3*c*d^2)*_R^4+d*(4*a*e^
```

```
2-4*b*d*e+3*c*d^2)*_R^2-c*d^4)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4
*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln(-e^(1/2)*x-_R+(e*x^2+d)^(1
/2)),_R=RootOf(_Z^8*c+(4*b*e-4*c*d)*_Z^6+c*d^4+(16*a*e^2-8*b*d*e+6*c*d^2)*_
Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2))-1/2/a*d/(-e^(1/2)*x+(e*x^2+d)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}}{\left(cx^4+bx^2+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x), x)
```

mupad [B] time = 6.87, size = 10964, normalized size = 39.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9
+ 4*b^2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) + ((b^4*d + 8*a^2*c^2*d - a*b^3*e
+ a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d
 + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d +
 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)
^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2
*c)))^(1/2)*((d + e*x^2)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a
*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c
*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*
a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*
c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^
2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 192*a^4*c^4*d*e^10 - 192*a^3*c^5*d^3*e^8
 + 48*a^2*b^2*c^4*d^3*e^8 - 48*a^2*b^3*c^3*d^2*e^9 + 192*a^3*b*c^4*d^2*e^9
+ 48*a^3*b^2*c^3*d*e^10) - (d + e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^
4*d*e^10 - 8*a^2*b^3*c^2*e^11 + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 -
16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*
c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a
^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^
(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))
)^(1/2) + 12*a*c^5*d^4*e^8 + 12*a^2*c^4*d^2*e^10 - 4*b^2*c^4*d^4*e^8 + 4*b^
4*c^2*d^2*e^10 + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e^11 + 20*a^2*b*c^3*d*e^
11 - 24*a*b^2*c^3*d^2*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c
 - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e
)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*1i + ((d + e*x^2)^(1/2)*(
2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^10 - 4*a
*b*c^3*d*e^11) - ((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(
1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^
4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(12*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*
c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/
2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(
1/2)*((d + e*x^2)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^
2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*
(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4
*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2
*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9
 + 480*a^3*b^3*c^3*d*e^9) + 192*a^4*c^4*d*e^10 + 192*a^3*c^5*d^3*e^8 - 48*a
^2*b^2*c^4*d^3*e^8 + 48*a^2*b^3*c^3*d^2*e^9 - 192*a^3*b*c^4*d^2*e^9 - 48*a^
3*b^2*c^3*d*e^10) - (d + e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^1
```

0 - 8*a^2*b^3*c^2*e^11 + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*
c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2
*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*
d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) -
 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)
 + 12*a^2*c^4*d^2*e^10 - 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^10 + 8*a*b*c^4
*d^3*e^9 - 4*a*b^3*c^2*d*e^11 + 20*a^2*b*c^3*d*e^11 - 24*a*b^2*c^3*d^2*e^10
))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(
4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2
 - 8*a^3*b^2*c)))^(1/2)*1i)/(((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^4
*e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) - ((b^4*d +
 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)
^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2
c)))^(1/2)(12*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4
*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b
*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x^2)^(1/2)*((
b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c
 - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*
a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^
3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2
*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9)
+ 192*a^4*c^4*d*e^10 + 192*a^3*c^5*d^3*e^8 - 48*a^2*b^2*c^4*d^3*e^8 + 48*a^
2*b^3*c^3*d^2*e^9 - 192*a^3*b*c^4*d^2*e^9 - 48*a^3*b^2*c^3*d*e^10) - (d + e
x^2)^(1/2)(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^10 - 8*a^2*b^3*c^2*e^11 + 1
44*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2
*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*e
^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c
 - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e
)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 12*a^2*c^4*d^2*e^10 - 4
*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^10 + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e
^11 + 20*a^2*b*c^3*d*e^11 - 24*a*b^2*c^3*d^2*e^10))*((b^4*d + 8*a^2*c^2*d -
 a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*
a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) -
((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 + 4*b^
2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) + ((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*
(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a
^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2
*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1
/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^
(1/2)*((d + e*x^2)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b
^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8
*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^
4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^
2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^
9 + 480*a^3*b^3*c^3*d*e^9) - 192*a^4*c^4*d*e^10 - 192*a^3*c^5*d^3*e^8 + 48*
a^2*b^2*c^4*d^3*e^8 - 48*a^2*b^3*c^3*d^2*e^9 + 192*a^3*b*c^4*d^2*e^9 + 48*a
^3*b^2*c^3*d*e^10) - (d + e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^
10 - 8*a^2*b^3*c^2*e^11 + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5
*c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^
2*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2
*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2)
- 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2
) + 12*a*c^5*d^4*e^8 + 12*a^2*c^4*d^2*e^10 - 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*
d^2*e^10 + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e^11 + 20*a^2*b*c^3*d*e^11 - 2
4*a*b^2*c^3*d^2*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2
)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(
a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 2*c^4*d^3*e^10 - 2*b*c^3*d^2*
e^11 + 2*a*c^3*d*e^12))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^
2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*

```
(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*2i + atan((((d + e*x^2)^(1/2)*
(2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^10 - 4*
a*b*c^3*d*e^11) + ((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^
(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b
^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2*c^2*d - a*b^3*e -
a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d +
 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x^2)^
(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*
(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*
c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^
4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2
*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3
*d*e^9) - 192*a^4*c^4*d*e^10 - 192*a^3*c^5*d^3*e^8 + 48*a^2*b^2*c^4*d^3*e^8
 - 48*a^2*b^3*c^3*d^2*e^9 + 192*a^3*b*c^4*d^2*e^9 + 48*a^3*b^2*c^3*d*e^10)
- (d + e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^10 - 8*a^2*b^3*c^2*
e^11 + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a
*b^4*c^2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c
^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*
(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a
^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 12*a*c^5*d^4*e^
8 + 12*a^2*c^4*d^2*e^10 - 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^10 + 8*a*b*c^
4*d^3*e^9 - 4*a*b^3*c^2*d*e^11 + 20*a^2*b*c^3*d*e^11 - 24*a*b^2*c^3*d^2*e^1
0))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-
(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^
2 - 8*a^3*b^2*c)))^(1/2)*1i + ((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^
4*e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) - ((b^4*d
+ 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2
)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^
2*c)))^(1/2)*(12*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(
4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*
b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x^2)^(1/2)*(
(b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*
c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8
*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c
^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^
2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9)
 + 192*a^4*c^4*d*e^10 + 192*a^3*c^5*d^3*e^8 - 48*a^2*b^2*c^4*d^3*e^8 + 48*a
^2*b^3*c^3*d^2*e^9 - 192*a^3*b*c^4*d^2*e^9 - 48*a^3*b^2*c^3*d*e^10) - (d +
e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^10 - 8*a^2*b^3*c^2*e^11 +
144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^
2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*
e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*
c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*
e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 12*a^2*c^4*d^2*e^10 -
4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^10 + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*
e^11 + 20*a^2*b*c^3*d*e^11 - 24*a*b^2*c^3*d^2*e^10))*((b^4*d + 8*a^2*c^2*d
- a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6
*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*1
i)/(((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 +
4*b^2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) - ((b^4*d + 8*a^2*c^2*d - a*b^3*e -
a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d +
 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(12*a*c^5*d^4
*e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*
d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^
4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x^2)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b
^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^
2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a
^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*
e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^
```

9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 192*a^4*c^4*d*e^10 + 192*a^3*c^5*d^3*e^8 - 48*a^2*b^2*c^4*d^3*e^8 + 48*a^2*b^3*c^3*d^2*e^9 - 192*a^3*b*c^4*d^2*e^9 - 48*a^3*b^2*c^3*d*e^10) - (d + e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^10 - 8*a^2*b^3*c^2*e^11 + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 12*a^2*c^4*d^2*e^10 - 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^10 + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e^11 + 20*a^2*b*c^3*d*e^11 - 24*a*b^2*c^3*d^2*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) + ((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x^2)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 192*a^4*c^4*d*e^10 - 192*a^3*c^5*d^3*e^8 + 48*a^2*b^2*c^4*d^3*e^8 - 48*a^2*b^3*c^3*d^2*e^9 + 192*a^3*b*c^4*d^2*e^9 + 48*a^3*b^2*c^3*d*e^10) - (d + e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^10 - 8*a^2*b^3*c^2*e^11 + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 12*a*c^5*d^4*e^8 + 12*a^2*c^4*d^2*e^10 - 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^10 + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e^11 + 20*a^2*b*c^3*d*e^11 - 24*a*b^2*c^3*d^2*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 2*c^4*d^3*e^10 - 2*b*c^3*d^2*e^11 + 2*a*c^3*d*e^12))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*2i - (d^(1/2)*atanh((20*c^4*d^(5/2)*e^10*(d + e*x^2)^(1/2))/(20*c^4*d^3*e^10 - 12*b*c^3*d^2*e^11 + (18*c^5*d^5*e^8)/a + 2*a*c^3*d*e^12 + (6*b^2*c^3*d^3*e^10)/a + (2*b^3*c^2*d^2*e^11)/a - (4*b^2*c^4*d^5*e^8)/a^2 + (6*b^3*c^3*d^4*e^9)/a^2 - (2*b^4*c^2*d^3*e^10)/a^2 - (28*b*c^4*d^4*e^9)/a) + (18*c^5*d^(9/2)*e^8*(d + e*x^2)^(1/2))/(18*c^5*d^5*e^8 + 20*a*c^4*d^3*e^10 + 2*a^2*c^3*d*e^12 - 28*b*c^4*d^4*e^9 + 6*b^2*c^3*d^3*e^10 + 2*b^3*c^2*d^2*e^11 - (4*b^2*c^4*d^5*e^8)/a + (6*b^3*c^3*d^4*e^9)/a - (2*b^4*c^2*d^3*e^10)/a - 12*a*b*c^3*d^2*e^11) - (28*b*c^4*d^(7/2)*e^9*(d + e*x^2)^(1/2))/(18*c^5*d^5*e^8 + 20*a*c^4*d^3*e^10 + 2*a^2*c^3*d*e^12 - 28*b*c^4*d^4*e^9 + 6*b^2*c^3*d^3*e^10 + 2*b^3*c^2*d^2*e^11 - (4*b^2*c^4*d^5*e^8)/a + (6*b^3*c^3*d^4*e^9)/a - (2*b^4*c^2*d^3*e^10)/a - 12*a*b*c^3*d^2*e^11) + (2*b^3*c^2*d^(3/2)*e^11*(d + e*x^2)^(1/2))/(18*c^5*d^5*e^8 + 20*a*c^4*d^3*e^10 + 2*a^2*c^3*d*e^12 - 28*b*c^4*d^4*e^9 + 6*b^2*c^3*d^3*e^10 + 2*b^3*c^2*d^2*e^11 - (4*b^2*c^4*d^5*e^8)/a + (6*b^3*c^3*d^4*e^9)/a - (2*b^4*c^2*d^3*e^10)/a - 12*a*b*c^3*d^2*e^11) + (6*b^2*c^3*d^(5/2)*e^10*(d + e*x^2)^(1/2))/(18*c^5*d^5*e^8 + 20*a*c^4*d^3*e^10 + 2*a^2*c^3*d*e^12 - 28*b*c^4*d^4*e^9 + 6*b^2*c^3*d^3*e^10 + 2*b^3*c^2*d^2*e^11 - (4*b^2*c^4*d^5*e^8)/a + (6*b^3*c^3*d^4*e^9)/a - (2*b^4*c^2*d^3*e^10)/a - 12*a*b*c^3*d^2*e^11) - (2*b^4*c^2*d^(5/2)*e^10*(d + e*x^2)^(1/2))/(18*a*c^5*d^5*e^8 + 2*a^3*c^3*d*e^12 + 20*a^2*c^4*d^3*e^10 - 4*b^2*c^4*d^5*e^8 + 6*b^3*c^3*d^4*e^9 - 2*b^4*c^2*d^3*e^10 - 28*

```
a*b*c^4*d^4*e^9 + 6*a*b^2*c^3*d^3*e^10 + 2*a*b^3*c^2*d^2*e^11 - 12*a^2*b*c^
3*d^2*e^11) + (6*b^3*c^3*d^(7/2)*e^9*(d + e*x^2)^(1/2))/(18*a*c^5*d^5*e^8 +
 2*a^3*c^3*d*e^12 + 20*a^2*c^4*d^3*e^10 - 4*b^2*c^4*d^5*e^8 + 6*b^3*c^3*d^4
*e^9 - 2*b^4*c^2*d^3*e^10 - 28*a*b*c^4*d^4*e^9 + 6*a*b^2*c^3*d^3*e^10 + 2*a
*b^3*c^2*d^2*e^11 - 12*a^2*b*c^3*d^2*e^11) - (4*b^2*c^4*d^(9/2)*e^8*(d + e*
x^2)^(1/2))/(18*a*c^5*d^5*e^8 + 2*a^3*c^3*d*e^12 + 20*a^2*c^4*d^3*e^10 - 4*
b^2*c^4*d^5*e^8 + 6*b^3*c^3*d^4*e^9 - 2*b^4*c^2*d^3*e^10 - 28*a*b*c^4*d^4*e
^9 + 6*a*b^2*c^3*d^3*e^10 + 2*a*b^3*c^2*d^2*e^11 - 12*a^2*b*c^3*d^2*e^11) +
 (2*a*c^3*d^(1/2)*e^12*(d + e*x^2)^(1/2))/(20*c^4*d^3*e^10 - 12*b*c^3*d^2*e
^11 + (18*c^5*d^5*e^8)/a + 2*a*c^3*d*e^12 + (6*b^2*c^3*d^3*e^10)/a + (2*b^3
*c^2*d^2*e^11)/a - (4*b^2*c^4*d^5*e^8)/a^2 + (6*b^3*c^3*d^4*e^9)/a^2 - (2*b
^4*c^2*d^3*e^10)/a^2 - (28*b*c^4*d^4*e^9)/a) - (12*b*c^3*d^(3/2)*e^11*(d +
e*x^2)^(1/2))/(20*c^4*d^3*e^10 - 12*b*c^3*d^2*e^11 + (18*c^5*d^5*e^8)/a + 2
*a*c^3*d*e^12 + (6*b^2*c^3*d^3*e^10)/a + (2*b^3*c^2*d^2*e^11)/a - (4*b^2*c^
4*d^5*e^8)/a^2 + (6*b^3*c^3*d^4*e^9)/a^2 - (2*b^4*c^2*d^3*e^10)/a^2 - (28*b
*c^4*d^4*e^9)/a)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{x\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(x*(a + b*x**2 + c*x**4)), x)
```

3.259 $\int \frac{\sqrt{d+ex^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=382

$$-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}\,(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}\left(-b\left(d\sqrt{b^2-4ac}+ae\right)-a\left(2\right.\right.}{\sqrt{2}\,a^2\sqrt{b^2-}}$$

Rubi [A] time = 4.13, antiderivative size = 370, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1251, 897, 1287, 199, 206, 1166, 208}

$$-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}\,(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}\left(-\sqrt{b^2-4ac}\,(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2\sqrt{d}}-\frac{\sqrt{d+ex^2}}{2ax^2}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -Sqrt[d + e*x^2]/(2*a*x^2) + (e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*a*Sqrt
[d]) + ((b*d - a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(a^2*Sqrt[d]) - (Sqrt
[c]*(b^2*d - 2*a*c*d - a*b*e + Sqrt[b^2 - 4*a*c]*(b*d - a*e))*ArcTanh[(Sqrt
[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqr
t[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt
[c]*(b^2*d - 2*a*c*d - a*b*e - Sqrt[b^2 - 4*a*c]*(b*d - a*e))*ArcTanh[(Sqrt
[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqr
t[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{de^2}{a\left(d-x^2\right)^2}-\frac{e(-bd+ae)}{a^2\left(d-x^2\right)}+\frac{e\left(-b\left(cd^2-bde+ae^2\right)+c(bd-ae)x^2\right)}{a^2\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{-b\left(cd^2-bde+ae^2\right)+c(bd-ae)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{a^2} + \frac{(de)\,\text{Subst}\left(\int \frac{1}{\left(d-x^2\right)^2}\,dx, x, \sqrt{d \cdots\right.}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{2ax^2} + \frac{(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2\sqrt{d}} + \frac{e\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex^2}\right)}{2a} - \frac{\left(c\left(b^2 \cdots\right.\right.}{}$$

$$= -\frac{\sqrt{d+ex^2}}{2ax^2} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a\sqrt{d}} + \frac{(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2\sqrt{d}} - \frac{\sqrt{c}\left(b^2d-2acd-ab \cdots\right.}{\sqrt{2}\,a^2 \cdots}$$

Mathematica [A] time = 1.39, size = 349, normalized size = 0.91

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(\frac{\left(-bd\sqrt{b^2-4ac}+ae\sqrt{b^2-4ac}+abe+2acd+b^2(-d)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{\left(bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+abe+2acd+b^2(-d)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}}+\frac{(2bd-ae)\log\left(\sqrt{d}\sqrt{d+ex^2}+d\right)}{\sqrt{d}}+\frac{\log(x)(ae-2bd)}{\sqrt{d}}-\frac{a\sqrt{d+ex^2}}{x^2}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-((a*Sqrt[d + e*x^2])/x^2) + (Sqrt[2]*Sqrt[c]*(((-(b^2*d) + 2*a*c*d - b*Sq
rt[b^2 - 4*a*c]*d + a*b*e + a*Sqrt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]
*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/Sqrt[2*c*d + (-
b + Sqrt[b^2 - 4*a*c])*e] - ((-(b^2*d) + 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d +
a*b*e - a*Sqrt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]))/Sqrt[b^2 - 4*a*c] + ((-2*b*d + a*e)*Log[x])/Sqrt[d] + ((2*b*d - a*e)
*Log[d + Sqrt[d]*Sqrt[d + e*x^2]])/Sqrt[d])/(2*a^2)
```

IntegrateAlgebraic [A] time = 1.50, size = 398, normalized size = 1.04

$$-\frac{\left(-b\sqrt{c}\,d\sqrt{b^2-4ac}+a\sqrt{c}\,e\sqrt{b^2-4ac}+ab\sqrt{c}\,e+2ac^{3/2}d+b^2\left(-\sqrt{c}\right)d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}+\frac{\left(b\sqrt{c}\,d\sqrt{b^2-4ac}-a\sqrt{c}\,e\sqrt{b^2-4ac}+ab\sqrt{c}\,e+2ac^{3/2}d+b^2\left(-\sqrt{c}\right)d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}+be-2cd}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\sqrt{e\sqrt{b^2-4ac}+be-2cd}}+\frac{(2bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a^2\sqrt{d}}-\frac{\sqrt{d+ex^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*Sqrt[d + e*x^2]/(a*x^2) - ((-(b^2*Sqrt[c]*d) + 2*a*c^(3/2)*d - b*Sqrt[
c]*Sqrt[b^2 - 4*a*c]*d + a*b*Sqrt[c]*e + a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)*Arc
Tan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]
*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*
e]) + ((-(b^2*Sqrt[c]*d) + 2*a*c^(3/2)*d + b*Sqrt[c]*Sqrt[b^2 - 4*a*c]*d +
a*b*Sqrt[c]*e - a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt
[d + e*x^2])/Sqrt[-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[2]*a^2*Sqrt[b
^2 - 4*a*c]*Sqrt[-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e]) + ((2*b*d - a*e)*ArcT
anh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*a^2*Sqrt[d])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

maple [C] time = 0.03, size = 401, normalized size = 1.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/a^2*b*d^(1/2)*ln((2*d+2*(e*x^2+d)^(1/2)*d^(1/2))/x)-1/2/a^2*b*(e*x^2+d)^(
1/2)-1/2/a^2*e^(1/2)*x*b+1/4/a^2*sum((c*(-a*e+b*d)*_R^6+(-4*a*b*e^2-a*c*d*e
+4*b^2*d*e-3*b*c*d^2)*_R^4+d*(4*a*b*e^2+a*c*d*e-4*b^2*d*e+3*b*c*d^2)*_R^2+a
*c*d^3*e-b*c*d^4)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3
```

```
*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln(-e^(1/2)*x-_R+(e*x^2+d)^(1/2)),_R=RootO
f(_Z^8*c+(4*b*e-4*c*d)*_Z^6+c*d^4+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*
e-4*c*d^3)*_Z^2))+1/2/a^2*b*d/(-e^(1/2)*x+(e*x^2+d)^(1/2))-1/2/a/d/x^2*(e*x
^2+d)^(3/2)-1/2/a*e/d^(1/2)*ln((2*d+2*(e*x^2+d)^(1/2)*d^(1/2))/x)+1/2/a*e/d
*(e*x^2+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d}}{\left(cx^4 + bx^2 + a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^3), x)
```

mupad [B] time = 5.46, size = 19959, normalized size = 52.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (atan((((a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e
^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*
a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4) - (
((16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*
e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9
 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^
10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a
^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b
*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11)/a^4 - ((a*e - 2*b*d)*(((d + e*x^2)^(
1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^
5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*
c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b
^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*
b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^10))/(2*a^4) -
 ((a*e - 2*b*d)*((128*a^8*c^4*e^11 + 8*a^6*b^4*c^2*e^11 - 64*a^7*b^2*c^3*e^
11 + 128*a^7*c^5*d^2*e^9 + 32*a^5*b^3*c^4*d^3*e^8 - 24*a^5*b^4*c^3*d^2*e^9
+ 64*a^6*b^2*c^4*d^2*e^9 - 256*a^7*b*c^4*d*e^10 - 8*a^5*b^5*c^2*d*e^10 - 12
8*a^6*b*c^5*d^3*e^8 + 96*a^6*b^3*c^3*d*e^10)/a^4 - ((d + e*x^2)^(1/2)*(a*e
- 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 +
1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 -
1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a
^6*d^(1/2))))/(4*a^2*d^(1/2))))/(4*a^2*d^(1/2)))*(a*e - 2*b*d))/(4*a^2*d^(1
/2)))*1i)/(4*a^2*d^(1/2)) + ((a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(6*a^4*c^5*e
^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b
^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*
d^3*e^9))/(2*a^4) + (((16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*
e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 2
0*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 +
84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b
^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*
c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11)/a^4 + ((a*e - 2
*b*d)*(((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*
b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*
d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^
3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*
c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c
^4*d*e^10))/(2*a^4) + ((a*e - 2*b*d)*((128*a^8*c^4*e^11 + 8*a^6*b^4*c^2*e^1
```

1 - 64*a^7*b^2*c^3*e^11 + 128*a^7*c^5*d^2*e^9 + 32*a^5*b^3*c^4*d^3*e^8 - 24
*a^5*b^4*c^3*d^2*e^9 + 64*a^6*b^2*c^4*d^2*e^9 - 256*a^7*b*c^4*d*e^10 - 8*a^
5*b^5*c^2*d*e^10 - 128*a^6*b*c^5*d^3*e^8 + 96*a^6*b^3*c^3*d*e^10)/a^4 + ((d
 + e*x^2)^(1/2)*(a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 51
2*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a
^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7
*b^3*c^3*d*e^9))/(8*a^6*d^(1/2))))/(4*a^2*d^(1/2))))/(4*a^2*d^(1/2)))*(a*e
 - 2*b*d))/(4*a^2*d^(1/2)))*1i)/(4*a^2*d^(1/2)))/(((a^3*c^5*e^13)/2 + a*c^7*
d^4*e^9 - 2*b*c^7*d^5*e^8 + (3*a^2*c^6*d^2*e^11)/2 + 2*b^2*c^6*d^4*e^9 - 4*
a*b*c^6*d^3*e^10 - (3*a^2*b*c^5*d*e^12)/2 + a*b^2*c^5*d^2*e^11)/a^4 - ((a*e
 - 2*b*d)*(((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c
^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*
e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4) - (((16*a^5*b*c
^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^
4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^
5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5
*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d
*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^1
0 - 68*a^4*b^2*c^4*d*e^11)/a^4 - ((a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(240*a^
6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^
11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9
 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e
^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^
10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^10))/(2*a^4) - ((a*e - 2*b
d)((128*a^8*c^4*e^11 + 8*a^6*b^4*c^2*e^11 - 64*a^7*b^2*c^3*e^11 + 128*a^7
*c^5*d^2*e^9 + 32*a^5*b^3*c^4*d^3*e^8 - 24*a^5*b^4*c^3*d^2*e^9 + 64*a^6*b^2
*c^4*d^2*e^9 - 256*a^7*b*c^4*d*e^10 - 8*a^5*b^5*c^2*d*e^10 - 128*a^6*b*c^5*
d^3*e^8 + 96*a^6*b^3*c^3*d*e^10)/a^4 - ((d + e*x^2)^(1/2)*(a*e - 2*b*d)*(10
24*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5
*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c
^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a^6*d^(1/2)))
)/(4*a^2*d^(1/2))))/(4*a^2*d^(1/2)))*(a*e - 2*b*d))/(4*a^2*d^(1/2))))/(4*a^
2*d^(1/2)) + ((a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7
*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10
 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^
4) + (((16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^
3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d
^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*
d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10
 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36
*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11)/a^4 + ((a*e - 2*b*d)*(((d + e*
x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 -
140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^
2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432
*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 34
8*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^10))/(2*
a^4) + ((a*e - 2*b*d)*((128*a^8*c^4*e^11 + 8*a^6*b^4*c^2*e^11 - 64*a^7*b^2*
c^3*e^11 + 128*a^7*c^5*d^2*e^9 + 32*a^5*b^3*c^4*d^3*e^8 - 24*a^5*b^4*c^3*d^
2*e^9 + 64*a^6*b^2*c^4*d^2*e^9 - 256*a^7*b*c^4*d*e^10 - 8*a^5*b^5*c^2*d*e^1
0 - 128*a^6*b*c^5*d^3*e^8 + 96*a^6*b^3*c^3*d*e^10)/a^4 + ((d + e*x^2)^(1/2)
*(a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e
^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*
e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9)
)/(8*a^6*d^(1/2))))/(4*a^2*d^(1/2))))/(4*a^2*d^(1/2)))*(a*e - 2*b*d))/(4*a^
2*d^(1/2))))/(4*a^2*d^(1/2))))*(a*e - 2*b*d)*1i)/(2*a^2*d^(1/2)) - atan((((
(64*a^5*b*c^4*e^12 + 80*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^
3*e^12 + 80*a^4*c^6*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*
e^9 - 108*a^2*b^5*c^3*d^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d
^2*e^10 - 32*a*b^5*c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^1

0 - 12*a^2*b^6*c^2*d*e^11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144*a^4*b*c^5*d^2*e^10 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4 *e^11 + 32*a^6*b^4*c^2*e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a^5*b^3*c^4*d^3*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^7*b*c^4*d*e^10 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 3 84*a^6*b^3*c^3*d*e^10)/(4*a^4) - ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2* c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4* b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2 *e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e ^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4* a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b ^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c ^5*d*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^ 8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e ^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^ 2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2* e^9 - 648*a^5*b^2*c^4*d*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4 *a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6 *c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2) ^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b ^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^ (1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^ 5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3* b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a ^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^ 2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a ^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i - (((64*a^5*b*c ^4*e^12 + 80*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^12 + 80 *a^4*c^6*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 - 108*a ^2*b^5*c^3*d^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^10 - 3 2*a*b^5*c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 12*a^2* b^6*c^2*d*e^11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144*a^4*b *c^5*d^2*e^10 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^11 + 32* a^6*b^4*c^2*e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a^5*b^3 *c^4*d^3*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^7* b*c^4*d*e^10 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a^6*b^3* c^3*d*e^10)/(4*a^4) + ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e* (-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a* c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^ 6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512 *a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^ 7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7* b^3*c^3*d*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^ (1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^ 3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^ 2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2* b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^

3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*
a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a
^5*b^2*c^4*d*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)
^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b
^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^
(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^
5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) +
a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2
) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a
*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^
(1/2) + ((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*
d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^1
1 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a^3*c^3*d -
b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b
^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e
- a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8
*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i)/((a^3*c^5*e^13 + 2*a*c^7*
d^4*e^9 - 4*b*c^7*d^5*e^8 + 3*a^2*c^6*d^2*e^11 + 4*b^2*c^6*d^4*e^9 - 8*a*b*
c^6*d^3*e^10 - 3*a^2*b*c^5*d*e^12 + 2*a*b^2*c^5*d^2*e^11)/(2*a^4) + (((64*a
^5*b*c^4*e^12 + 80*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^1
2 + 80*a^4*c^6*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 -
108*a^2*b^5*c^3*d^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^
10 - 32*a*b^5*c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 1
2*a^2*b^6*c^2*d*e^11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144
*a^4*b*c^5*d^2*e^10 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^11
+ 32*a^6*b^4*c^2*e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a
^5*b^3*c^4*d^3*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 102
4*a^7*b*c^4*d*e^10 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a^
6*b^3*c^3*d*e^10)/(4*a^4) - ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3
d(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*
b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(
-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 +
16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10
- 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 -
896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 96
0*a^7*b^3*c^3*d*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^
2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c -
b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3
)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*
a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*
e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 3
2*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 -
208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9
- 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 -
648*a^5*b^2*c^4*d*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c
- b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a
*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^
2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2
- 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(
1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3
)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2)
+ 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2
*c)))^(1/2) - ((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^
3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5
*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a^3*c^
3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d +
8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*
c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/
2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (((64*a^5*b*c^4*e^12

+ 80*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^12 + 80*a^4*c^6 *d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 - 108*a^2*b^5*c ^3*d^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^10 - 32*a*b^5* c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 12*a^2*b^6*c^2* d*e^11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144*a^4*b*c^5*d^2 *e^10 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^11 + 32*a^6*b^4* c^2*e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a^5*b^3*c^4*d^3 *e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^7*b*c^4*d* e^10 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a^6*b^3*c^3*d*e^ 10)/(4*a^4) + ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2) ^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2 *c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^ 4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3* d*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a* b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4* b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3* d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^ 3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6* c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c ^4*d*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^ 2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a* b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c* e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^ 4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4 *a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6 *c^2 - 8*a^5*b^2*c)))^(1/2)*2i - (d + e*x^2)^(1/2)/(2*a*x^2) - atan(((((64* a^5*b*c^4*e^12 + 80*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^ 12 + 80*a^4*c^6*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 - 108*a^2*b^5*c^3*d^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e ^10 - 32*a*b^5*c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 12*a^2*b^6*c^2*d*e^11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 14 4*a^4*b*c^5*d^2*e^10 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^1 1 + 32*a^6*b^4*c^2*e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128* a^5*b^3*c^4*d^3*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 10 24*a^7*b*c^4*d*e^10 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a ^6*b^3*c^3*d*e^10)/(4*a^4) - ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^ 3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a *b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e* (-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^1 0 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 -

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 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 9
60*a^7*b^3*c^3*d*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b
^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c
- b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^
3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8
*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d
*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 -
32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 -
 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^
9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9
- 648*a^5*b^2*c^4*d*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c
 - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*
a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b
^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2
 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^
(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^
3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2
) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^
2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a
^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^
5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a^3*c
^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d
+ 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b
*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1
/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i - (((64*a^5*b*c^4*e
^12 + 80*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^12 + 80*a^4
*c^6*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 - 108*a^2*b
^5*c^3*d^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^10 - 32*a*
b^5*c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 12*a^2*b^6*
c^2*d*e^11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144*a^4*b*c^5
*d^2*e^10 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^11 + 32*a^6*
b^4*c^2*e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a^5*b^3*c^4
*d^3*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^7*b*c^
4*d*e^10 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a^6*b^3*c^3*
d*e^10)/(4*a^4) + ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*
c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c -
b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^
2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8
*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^
2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*
c^3*d*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2
) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(
1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) -
2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)
))^(1/2) + ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*
a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*
c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^
5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*
b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b
^2*c^4*d*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^
(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^
3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2
) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^
2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b
^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) -
7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c
*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2
) + ((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*
```

```
e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 -
8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*
d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c
*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^
2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^
4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i)/((a^3*c^5*e^13 + 2*a*c^7*d^4*
e^9 - 4*b*c^7*d^5*e^8 + 3*a^2*c^6*d^2*e^11 + 4*b^2*c^6*d^4*e^9 - 8*a*b*c^6*
d^3*e^10 - 3*a^2*b*c^5*d*e^12 + 2*a*b^2*c^5*d^2*e^11)/(2*a^4) + (((64*a^5*b
*c^4*e^12 + 80*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^12 +
80*a^4*c^6*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 - 108
*a^2*b^5*c^3*d^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^10 -
 32*a*b^5*c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 12*a^
2*b^6*c^2*d*e^11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144*a^4
*b*c^5*d^2*e^10 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^11 + 3
2*a^6*b^4*c^2*e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a^5*b
^3*c^4*d^3*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^
7*b*c^4*d*e^10 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a^6*b^
3*c^3*d*e^10)/(4*a^4) - ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(
-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*
e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*
a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*
a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 5
12*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*
a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^
7*b^3*c^3*d*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3
)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2
)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1
/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*
b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10
 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^
2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*
a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 4
8*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648
*a^5*b^2*c^4*d*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^
2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c -
 b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3
)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*
a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2)
 + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1
/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2
*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))
)^(1/2) - ((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^
6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e
^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a^3*c^3*d
- b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a
*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*
e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (((64*a^5*b*c^4*e^12 + 80
*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^12 + 80*a^4*c^6*d^3
*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 - 108*a^2*b^5*c^3*d
^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^10 - 32*a*b^5*c^4*
d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 12*a^2*b^6*c^2*d*e^
11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144*a^4*b*c^5*d^2*e^1
0 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^11 + 32*a^6*b^4*c^2*
e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a^5*b^3*c^4*d^3*e^8
 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^7*b*c^4*d*e^10
 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a^6*b^3*c^3*d*e^10)/
(4*a^4) + ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)
^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b
```

```
^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^
(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^
5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3
*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^
2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^
9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^
5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7
*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*
d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)
 + ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*
c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*
e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^
2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*
d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d
*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) +
a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2)
 - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*
b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(
1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 1
8*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^
3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d
+ e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4
*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*
c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*
d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b
^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-
(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 +
16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c
 - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*
a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b
^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2
 - 8*a^5*b^2*c)))^(1/2)*2i
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(x**3*(a + b*x**2 + c*x**4)), x)
```

3.260 $\int \frac{\sqrt{d+ex^2}}{x^5(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=552

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\left(-abe-acd+b^2d\right)}{a^3\sqrt{d}}+\frac{\sqrt{c}\left(b^2\left(d\sqrt{b^2-4ac}-ae\right)-ab\left(e\sqrt{b^2-4ac}+3cd\right)-ac\left(d\sqrt{b^2-4ac}-2\right.\right.}{\sqrt{2}\,a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4\right.}}}$$

Rubi [A] time = 4.24, antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1251, 897, 1287, 199, 206, 1166, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\left(-abe-acd+b^2d\right)}{a^3\sqrt{d}}+\frac{\sqrt{c}\left(b^2\left(d\sqrt{b^2-4ac}-ae\right)-ab\left(e\sqrt{b^2-4ac}+3cd\right)-ac\left(d\sqrt{b^2-4ac}-2ae\right)+b^3d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{c}\left(-b^2\left(d\sqrt{b^2-4ac}+ae\right)-ab\left(3cd-e\sqrt{b^2-4ac}\right)+ac\left(d\sqrt{b^2-4ac}+2ae\right)+b^3d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\,a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{e(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a^2d^{3/2}}+\frac{\sqrt{d+ex^2}(bd-ae)}{2a^2dx^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8ad^{3/2}}+\frac{3e\sqrt{d+ex^2}}{8adx^2}-\frac{\sqrt{d+ex^2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -Sqrt[d + e*x^2]/(4*a*x^4) + (3*e*Sqrt[d + e*x^2])/(8*a*d*x^2) + ((b*d - a*
e)*Sqrt[d + e*x^2])/(2*a^2*d*x^2) - (3*e^2*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]]
)/(8*a*d^(3/2)) - (e*(b*d - a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*a^2*d
^(3/2)) - ((b^2*d - a*c*d - a*b*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(a^3*S
qrt[d]) + (Sqrt[c]*(b^3*d - a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(Sqrt[b
^2 - 4*a*c]*d - a*e) - a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*
Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]
*a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[c]*
(b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e
) - a*b*(3*c*d - Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*
x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^3*Sqrt[b^2 - 4*a
*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
```

```
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{x^5\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{de^3}{a\left(d-x^2\right)^3}+\frac{e^2(-bd+ae)}{a^2\left(d-x^2\right)^2}+\frac{e\left(-b^2d+acd+abe\right)}{a^3\left(d-x^2\right)}+\frac{e\left(\left(b^2-ac\right)\left(cd^2-bde+ae^2\right)-c\left(b^2d-acd-abe\right)x^2\right)}{a^3\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right.\right.}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(b^2-ac\right)\left(cd^2-bde+ae^2\right)-c\left(b^2d-acd-abe\right)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{a^3} - \frac{\left(de^2\right)\text{Subst}\left(\int \frac{1}{\left(d-x^2\right)^3}\,d\right.}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{4ax^4}+\frac{(bd-ae)\sqrt{d+ex^2}}{2a^2dx^2}-\frac{\left(b^2d-acd-abe\right)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^3\sqrt{d}}-\frac{\left(3e^2\right)\text{Subst}}{}$$

$$= -\frac{\sqrt{d+ex^2}}{4ax^4}+\frac{3e\sqrt{d+ex^2}}{8adx^2}+\frac{(bd-ae)\sqrt{d+ex^2}}{2a^2dx^2}-\frac{e(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a^2d^{3/2}}-\frac{\left(b^2d\right.}{}$$

$$= -\frac{\sqrt{d+ex^2}}{4ax^4}+\frac{3e\sqrt{d+ex^2}}{8adx^2}+\frac{(bd-ae)\sqrt{d+ex^2}}{2a^2dx^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8ad^{3/2}}-\frac{e(bd-ae)\,\text{t}}{2}$$

Mathematica [A] time = 1.97, size = 466, normalized size = 0.84

$$\frac{\frac{\log\left(\sqrt{d}\sqrt{d+ex^2}+d\right)\left(4abde+a\left(ae^2+8cd^2\right)-8b^2d^2\right)}{d^{3/2}}-\frac{\log(x)\left(4abde+a\left(ae^2+8cd^2\right)-8b^2d^2\right)}{d^{3/2}}-\frac{4\sqrt{2}\sqrt{c}\left(\frac{\left(b^2\left(ae-d\sqrt{b^2-4ac}\right)+ab\left(e\sqrt{b^2-4ac}+3cd\right)+ac\left(d\sqrt{b^2-4ac}-2ae\right)+b^3(-d)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{\left(-b^2\left(d\sqrt{b^2-4ac}+ae\right)+ab\left(e\sqrt{b^2-4ac}-3cd\right)+ac\left(d\sqrt{b^2-4ac}+2ae\right)+b^3d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}}+\frac{a\sqrt{d+ex^2}\left(4bdx^2-a\left(2d+ex^2\right)\right)}{dx^4}}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((a*Sqrt[d + e*x^2]*(4*b*d*x^2 - a*(2*d + e*x^2)))/(d*x^4) - (4*Sqrt[2]*Sqr
t[c]*(((-(b^3*d) + a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(-(Sqrt[b^2 - 4*
a*c]*d) + a*e) + a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c
]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/Sqrt[2*c*d + (
-b + Sqrt[b^2 - 4*a*c])*e] + ((b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*
c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) + a*b*(-3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTa
nh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/Sqrt[b^2 - 4*a*c] - ((-8*b^2*
d^2 + 4*a*b*d*e + a*(8*c*d^2 + a*e^2))*Log[x])/d^(3/2) + ((-8*b^2*d^2 + 4*a
*b*d*e + a*(8*c*d^2 + a*e^2))*Log[d + Sqrt[d]*Sqrt[d + e*x^2]])/d^(3/2))/(8
*a^3)
```

IntegrateAlgebraic [A] time = 2.39, size = 579, normalized size = 1.05

$$\frac{\sqrt{d+ex^2}\left(-2ad-aex^2+4bdx^2\right)}{8a^2dx^4}+\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\left(a^2e^2+4abde+8acd^2-8b^2d^2\right)}{8a^3d^{3/2}}+\frac{\left(-2\sqrt{2}a^2c^{3/2}e+\sqrt{2}ac^{3/2}d\sqrt{b^2-4ac}-\sqrt{2}b^2\sqrt{c}d\sqrt{b^2-4ac}+\sqrt{2}ab^2\sqrt{c}e+\sqrt{2}ab\sqrt{c}e\sqrt{b^2-4ac}+3\sqrt{2}abc^{3/2}d-\sqrt{2}b^3\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}\right)}{2a^3\sqrt{b^2-4ac}\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}+\frac{\left(2\sqrt{2}a^2c^{3/2}e+\sqrt{2}ac^{3/2}d\sqrt{b^2-4ac}-\sqrt{2}b^2\sqrt{c}d\sqrt{b^2-4ac}-\sqrt{2}ab^2\sqrt{c}e+\sqrt{2}ab\sqrt{c}e\sqrt{b^2-4ac}-3\sqrt{2}abc^{3/2}d+\sqrt{2}b^3\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}+be-2cd}}\right)}{2a^3\sqrt{b^2-4ac}\sqrt{e\sqrt{b^2-4ac}+be-2cd}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x^2]/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[d + e*x^2]*(-2*a*d + 4*b*d*x^2 - a*e*x^2))/(8*a^2*d*x^4) + ((-(Sqrt[2
]*b^3*Sqrt[c]*d) + 3*Sqrt[2]*a*b*c^(3/2)*d - Sqrt[2]*b^2*Sqrt[c]*Sqrt[b^2 -
 4*a*c]*d + Sqrt[2]*a*c^(3/2)*Sqrt[b^2 - 4*a*c]*d + Sqrt[2]*a*b^2*Sqrt[c]*e
 - 2*Sqrt[2]*a^2*c^(3/2)*e + Sqrt[2]*a*b*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)*ArcTa
n[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e
]])/(2*a^3*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]) + ((
Sqrt[2]*b^3*Sqrt[c]*d - 3*Sqrt[2]*a*b*c^(3/2)*d - Sqrt[2]*b^2*Sqrt[c]*Sqrt[
b^2 - 4*a*c]*d + Sqrt[2]*a*c^(3/2)*Sqrt[b^2 - 4*a*c]*d - Sqrt[2]*a*b^2*Sqrt
[c]*e + 2*Sqrt[2]*a^2*c^(3/2)*e + Sqrt[2]*a*b*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)*
ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + Sqrt[b^2 - 4*a
*c]*e]])/(2*a^3*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e])
 + ((-8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e + a^2*e^2)*ArcTanh[Sqrt[d + e*x^2]/
Sqrt[d]])/(8*a^3*d^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.90, size = 1055, normalized size = 1.91

$$-\frac{\left(\sqrt{-4\,c^2 d + 2\left(bc - \sqrt{b^2-4\,ac}\,c\right)e}\left((b^4 - 5\,ab^2c + 4\,a^2c^2)de - (ab^3 - 4\,a^2bc)e^2\right)a^2 - 2\left((ab^2c - a^2c^2)\sqrt{b^2-4\,ac}\,d^2 - (ab^3 - a^2bc)\sqrt{b^2-4\,ac}\,de + (a^2b^2 - a^3c)\sqrt{b^2-4\,ac}\,e^2\right)\sqrt{-4\,c^2d + 2\left(bc - \sqrt{b^2-4\,ac}\,c\right)e}\,|a| - \sqrt{-4\,c^2d + 2\left(bc - \sqrt{b^2-4\,ac}\,c\right)e}\left(2\left(a^2b^3c - 3\,a^3bc^2\right)d^2 - \left(a^2b^4 - a^3b^2c - 4\,a^4c^2\right)de + \left(a^3b^3 - 2\,a^4bc\right)e^2\right)\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{2\,a^3cd - a^3be + \sqrt{-4\left(a^3cd^2 - a^3bde + a^4e^2\right)a^3c + \left(2\,a^3cd - a^3be\right)^2}}{a^3c}}}\right)}{8\left(\sqrt{b^2-4\,ac}\,a^4cd^2 - \sqrt{b^2-4\,ac}\,a^4bde + \sqrt{b^2-4\,ac}\,a^5e^2\right)|a||c|} + \frac{\left(\sqrt{-4\,c^2 d + 2\left(bc + \sqrt{b^2-4\,ac}\,c\right)e}\left((b^4 - 5\,ab^2c + 4\,a^2c^2)de - (ab^3 - 4\,a^2bc)e^2\right)a^2 + 2\left((ab^2c - a^2c^2)\sqrt{b^2-4\,ac}\,d^2 - (ab^3 - a^2bc)\sqrt{b^2-4\,ac}\,de + (a^2b^2 - a^3c)\sqrt{b^2-4\,ac}\,e^2\right)\sqrt{-4\,c^2d + 2\left(bc + \sqrt{b^2-4\,ac}\,c\right)e}\,|a| - \sqrt{-4\,c^2d + 2\left(bc + \sqrt{b^2-4\,ac}\,c\right)e}\left(2\left(a^2b^3c - 3\,a^3bc^2\right)d^2 - \left(a^2b^4 - a^3b^2c - 4\,a^4c^2\right)de + \left(a^3b^3 - 2\,a^4bc\right)e^2\right)\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{2\,a^3cd - a^3be - \sqrt{-4\left(a^3cd^2 - a^3bde + a^4e^2\right)a^3c + \left(2\,a^3cd - a^3be\right)^2}}{a^3c}}}\right)}{8\left(\sqrt{b^2-4\,ac}\,a^4cd^2 - \sqrt{b^2-4\,ac}\,a^4bde + \sqrt{b^2-4\,ac}\,a^5e^2\right)|a||c|} + \frac{\left(8\,b^2d^2 - 8\,acd^2 - 4\,abde - a^2e^2\right)\arctan\left(\frac{\sqrt{x^2e+d}}{\sqrt{-d}}\right)}{8\,a^3\sqrt{-d}\,d} + \frac{4\left(x^2e+d\right)^{\frac{3}{2}}bde - 4\sqrt{x^2e+d}\,bd^2e - \left(x^2e+d\right)^{\frac{3}{2}}ae^2 - \sqrt{x^2e+d}\,ade^2}{8\,a^2de^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^4 - 5*a*b^2*c +
4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*a^2 - 2*((a*b^2*c - a^2*c^2)*sqrt
(b^2 - 4*a*c)*d^2 - (a*b^3 - a^2*b*c)*sqrt(b^2 - 4*a*c)*d*e + (a^2*b^2 - a^
3*c)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e
)*abs(a) - sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*(2*(a^2*b^3*c -
 3*a^3*b*c^2)*d^2 - (a^2*b^4 - a^3*b^2*c - 4*a^4*c^2)*d*e + (a^3*b^3 - 2*a^
4*b*c)*e^2))*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*a^3*c*d - a^3*b*e
+ sqrt(-4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*a^3*c + (2*a^3*c*d - a^3*b*e)^2
))/(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4*c*d^2 - sqrt(b^2 - 4*a*c)*a^4*b*d*e +
sqrt(b^2 - 4*a*c)*a^5*e^2)*abs(a)*abs(c)) + 1/8*(sqrt(-4*c^2*d + 2*(b*c + s
qrt(b^2 - 4*a*c)*c)*e)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*
b*c)*e^2)*a^2 + 2*((a*b^2*c - a^2*c^2)*sqrt(b^2 - 4*a*c)*d^2 - (a*b^3 - a^2
*b*c)*sqrt(b^2 - 4*a*c)*d*e + (a^2*b^2 - a^3*c)*sqrt(b^2 - 4*a*c)*e^2)*sqrt
(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(a) - sqrt(-4*c^2*d + 2*(b*
c + sqrt(b^2 - 4*a*c)*c)*e)*(2*(a^2*b^3*c - 3*a^3*b*c^2)*d^2 - (a^2*b^4 - a
^3*b^2*c - 4*a^4*c^2)*d*e + (a^3*b^3 - 2*a^4*b*c)*e^2))*arctan(2*sqrt(1/2)*
sqrt(x^2*e + d)/sqrt(-(2*a^3*c*d - a^3*b*e - sqrt(-4*(a^3*c*d^2 - a^3*b*d*e
 + a^4*e^2)*a^3*c + (2*a^3*c*d - a^3*b*e)^2))/(a^3*c)))/((sqrt(b^2 - 4*a*c)
*a^4*c*d^2 - sqrt(b^2 - 4*a*c)*a^4*b*d*e + sqrt(b^2 - 4*a*c)*a^5*e^2)*abs(a
)*abs(c)) + 1/8*(8*b^2*d^2 - 8*a*c*d^2 - 4*a*b*d*e - a^2*e^2)*arctan(sqrt(x
^2*e + d)/sqrt(-d))/(a^3*sqrt(-d)*d) + 1/8*(4*(x^2*e + d)^(3/2)*b*d*e - 4*s
qrt(x^2*e + d)*b*d^2*e - (x^2*e + d)^(3/2)*a*e^2 - sqrt(x^2*e + d)*a*d*e^2)
*e^(-2)/(a^2*d*x^4)
```

maple [C] time = 0.04, size = 655, normalized size = 1.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/a^2*d^(1/2)*ln((2*d+2*(e*x^2+d)^(1/2)*d^(1/2))/x)*c-1/a^3*d^(1/2)*ln((2*d
+2*(e*x^2+d)^(1/2)*d^(1/2))/x)*b^2-1/2/a^2*(e*x^2+d)^(1/2)*c+1/2/a^3*(e*x^2
+d)^(1/2)*b^2-1/2/a^2*e^(1/2)*x*c+1/2/a^3*e^(1/2)*x*b^2+1/4/a^3*sum((c*(a*b
*e+a*c*d-b^2*d)*_R^6+(-4*a^2*c*e^2+4*a*b^2*e^2+5*a*b*c*d*e-3*a*c^2*d^2-4*b^
3*d*e+3*b^2*c*d^2)*_R^4+d*(4*a^2*c*e^2-4*a*b^2*e^2-5*a*b*c*d*e+3*a*c^2*d^2+
4*b^3*d*e-3*b^2*c*d^2)*_R^2-a*b*c*d^3*e-a*c^2*d^4+b^2*c*d^4)/(_R^7*c+3*_R^5
*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)
*ln(-e^(1/2)*x-_R+(e*x^2+d)^(1/2)),_R=RootOf(_Z^8*c+(4*b*e-4*c*d)*_Z^6+c*d^
4+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2))+1/2/a^2*d/(-e^
(1/2)*x+(e*x^2+d)^(1/2))*c-1/2/a^3*d/(-e^(1/2)*x+(e*x^2+d)^(1/2))*b^2-1/4/a
/d/x^4*(e*x^2+d)^(3/2)+1/8/a*e/d^2/x^2*(e*x^2+d)^(3/2)+1/8/a*e^2/d^(3/2)*ln
((2*d+2*(e*x^2+d)^(1/2)*d^(1/2))/x)-1/8/a*e^2/d^2*(e*x^2+d)^(1/2)+1/2/a^2*b
/d/x^2*(e*x^2+d)^(3/2)+1/2/a^2*b*e/d^(1/2)*ln((2*d+2*(e*x^2+d)^(1/2)*d^(1/2
))/x)-1/2/a^2*b*e/d*(e*x^2+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}}{\left(cx^4+bx^2+a\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^5), x)
```

mupad [B] time = 7.30, size = 33925, normalized size = 61.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(x^5*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((((((2048*a^12*c^4*d*e^12 + 12288*a^10*c^6*d^5*e^8 + 14336*a^11*c^5*d
^3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b^6
*c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8 + 7168*a^9*b^3*c^4*d^4*e^9 + 6272
*a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d^2*e^11 - 20480*a^10*b^2*c^4*d^3*e
^10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*a^10*b*c^5*d^4*e^9 + 128*a^10*b^4*c
^2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1024*a^11*b^2*c^3*d*e^12)/(64*a^8*d^
2) - ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1
/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^
3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3
)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(
1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^
2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(24576*a^12*c^
5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10
*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9
+ 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^
4*d^3*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^
(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e +
a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)
^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)
^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^
2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28
*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*
b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352
*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8
 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*
```

d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7
*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10
- 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^
9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^
12))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) -
a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2
e(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/
2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2)
- 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (16*a^9*c^5*e^14
- 4*a^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^14 - 52*a^8*b^2*c^4*e^14 - 768*a^6*
c^8*d^6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a^8*c^6*d^2*e^12 - 512*a^2*b^8*c^4*
d^6*e^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a^2*b^10*c^2*d^4*e^10 + 3840*a^3*b^
6*c^5*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9 - 2208*a^3*b^8*c^3*d^4*e^10 - 224*
a^3*b^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d^6*e^8 + 896*a^4*b^5*c^5*d^5*e^9 +
10752*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^7*c^3*d^3*e^11 + 96*a^4*b^8*c^2*d^
2*e^12 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*a^5*b^3*c^6*d^5*e^9 - 18144*a^5*b^
4*c^5*d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^11 - 836*a^5*b^6*c^3*d^2*e^12 + 93
44*a^6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c^5*d^3*e^11 + 2236*a^6*b^4*c^4*d^2
*e^12 - 1716*a^7*b^2*c^5*d^2*e^12 - 528*a^8*b*c^5*d*e^13 + 4*a^5*b^7*c^2*d*
e^13 - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d*e^13 - 5632*a^7*b*c^6*d^3*
e^11 + 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*
(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d -
25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^
4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d
*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*
b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))
^(1/2) - ((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^
9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8
+ 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e
^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*
d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3
*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*
b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^
8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2))*(
(b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^
4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3
)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e +
20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4
*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4
+ 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i - (((((2048*a^12*c^4*d*e^12 + 12288*
a^10*c^6*d^5*e^8 + 14336*a^11*c^5*d^3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 153
6*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b^6*c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^
8 + 7168*a^9*b^3*c^4*d^4*e^9 + 6272*a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*
d^2*e^11 - 20480*a^10*b^2*c^4*d^3*e^10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*
a^10*b*c^5*d^4*e^9 + 128*a^10*b^4*c^2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1
024*a^11*b^2*c^3*d*e^12)/(64*a^8*d^2) + ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*
c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^
3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*
b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e
+ 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^
(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 -
8*a^7*b^2*c)))^(1/2)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 20
48*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^
4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12
*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^
4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*
a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*
a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3

*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b *c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^ 9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d ^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6* b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7 168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2 *e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6 *c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b* c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2 *c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c *d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4* a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7 *b^2*c)))^(1/2) + (16*a^9*c^5*e^14 - 4*a^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^ 14 - 52*a^8*b^2*c^4*e^14 - 768*a^6*c^8*d^6*e^8 - 768*a^7*c^7*d^4*e^10 + 16* a^8*c^6*d^2*e^12 - 512*a^2*b^8*c^4*d^6*e^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128* a^2*b^10*c^2*d^4*e^10 + 3840*a^3*b^6*c^5*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9 - 2208*a^3*b^8*c^3*d^4*e^10 - 224*a^3*b^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6* d^6*e^8 + 896*a^4*b^5*c^5*d^5*e^9 + 10752*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b ^7*c^3*d^3*e^11 + 96*a^4*b^8*c^2*d^2*e^12 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632 *a^5*b^3*c^6*d^5*e^9 - 18144*a^5*b^4*c^5*d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e ^11 - 836*a^5*b^6*c^3*d^2*e^12 + 9344*a^6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3* c^5*d^3*e^11 + 2236*a^6*b^4*c^4*d^2*e^12 - 1716*a^7*b^2*c^5*d^2*e^12 - 528* a^8*b*c^5*d*e^13 + 4*a^5*b^7*c^2*d*e^13 - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b ^5*c^3*d*e^13 - 5632*a^7*b*c^6*d^3*e^11 + 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d ^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33* a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5 *c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2* d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^ 6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(a^6*b^2*c^5 *e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6 *c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5* c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3 *b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704 *a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 1 92*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a ^5*b^3*c^5*d*e^13))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3* c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a* c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i)/ ((((((2048*a^12*c^4*d*e^12 + 12288*a^10*c^6*d^5*e^8 + 14336*a^11*c^5*d^3*e^ 10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b^6*c^2* d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8 + 7168*a^9*b^3*c^4*d^4*e^9 + 6272*a^9* b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d^2*e^11 - 20480*a^10*b^2*c^4*d^3*e^10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*a^10*b*c^5*d^4*e^9 + 128*a^10*b^4*c^2*d* e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1024*a^11*b^2*c^3*d*e^12)/(64*a^8*d^2) - ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2 *e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/ 2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)

^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(24576*a^12*c^5*d^4 *e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5* c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 102 4*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3 *e^9))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c ^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^ 3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2)^(1 /2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9* b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c ^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5* b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11 264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e ^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4* c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 614 4*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 2 28*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/ (32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^ 7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(- (4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a ^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (16*a^9*c^5*e^14 - 4*a ^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^14 - 52*a^8*b^2*c^4*e^14 - 768*a^6*c^8*d ^6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a^8*c^6*d^2*e^12 - 512*a^2*b^8*c^4*d^6*e ^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a^2*b^10*c^2*d^4*e^10 + 3840*a^3*b^6*c^5 *d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9 - 2208*a^3*b^8*c^3*d^4*e^10 - 224*a^3*b ^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d^6*e^8 + 896*a^4*b^5*c^5*d^5*e^9 + 1075 2*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^7*c^3*d^3*e^11 + 96*a^4*b^8*c^2*d^2*e^1 2 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*a^5*b^3*c^6*d^5*e^9 - 18144*a^5*b^4*c^5 *d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^11 - 836*a^5*b^6*c^3*d^2*e^12 + 9344*a^ 6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c^5*d^3*e^11 + 2236*a^6*b^4*c^4*d^2*e^12 - 1716*a^7*b^2*c^5*d^2*e^12 - 528*a^8*b*c^5*d*e^13 + 4*a^5*b^7*c^2*d*e^13 - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d*e^13 - 5632*a^7*b*c^6*d^3*e^11 + 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4* a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^ 3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4 *a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c *e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6 *e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704 *a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e ^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6* d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6 *d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5 *e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2))*((b^8* d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2 *d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/ 2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a ^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16* a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (((((2048*a^12*c^4*d*e^12 + 12288*a^10*c^6 *d^5*e^8 + 14336*a^11*c^5*d^3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^ 5*c^3*d^4*e^9 - 512*a^8*b^6*c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8 + 7168 *a^9*b^3*c^4*d^4*e^9 + 6272*a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d^2*e^11

 - 20480*a^10*b^2*c^4*d^3*e^10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*a^10*b*c
^5*d^4*e^9 + 128*a^10*b^4*c^2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1024*a^11
*b^2*c^3*d*e^12)/(64*a^8*d^2) + ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*c^4*d -
b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^
3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d
+ a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b
^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) -
3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^
2*c)))^(1/2)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*
b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 +
15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3
*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d
- b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*
c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*
d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a
*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2)
- 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*
b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^1
3 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*
a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608
*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10
+ 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*
d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*
b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 -
18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e
^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*
e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d
*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d -
 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b
^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*
d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2
*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c))
)^(1/2) + (16*a^9*c^5*e^14 - 4*a^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^14 - 52*
a^8*b^2*c^4*e^14 - 768*a^6*c^8*d^6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a^8*c^6*
d^2*e^12 - 512*a^2*b^8*c^4*d^6*e^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a^2*b^10
*c^2*d^4*e^10 + 3840*a^3*b^6*c^5*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9 - 2208*
a^3*b^8*c^3*d^4*e^10 - 224*a^3*b^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d^6*e^8
+ 896*a^4*b^5*c^5*d^5*e^9 + 10752*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^7*c^3*d
^3*e^11 + 96*a^4*b^8*c^2*d^2*e^12 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*a^5*b^3
*c^6*d^5*e^9 - 18144*a^5*b^4*c^5*d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^11 - 83
6*a^5*b^6*c^3*d^2*e^12 + 9344*a^6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c^5*d^3*
e^11 + 2236*a^6*b^4*c^4*d^2*e^12 - 1716*a^7*b^2*c^5*d^2*e^12 - 528*a^8*b*c^
5*d*e^13 + 4*a^5*b^7*c^2*d*e^13 - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d
*e^13 - 5632*a^7*b*c^6*d^3*e^11 + 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^2))*((b
^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*
c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^
(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 2
0*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a
*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 +
16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 -
2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2
*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*
e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7
*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2
*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5
*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7
*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c
^5*d*e^13))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^
(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e +

a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)
^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)
^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (7*a^5*c^7
*d*e^14 + 56*a^3*c^9*d^5*e^10 + 63*a^4*c^8*d^3*e^12 - 64*b^4*c^8*d^7*e^8 +
64*b^5*c^7*d^6*e^9 + 64*a^2*b^2*c^8*d^5*e^10 + 224*a^2*b^3*c^7*d^4*e^11 - 1
12*a^3*b^2*c^7*d^3*e^12 + 64*a*b^2*c^9*d^7*e^8 + 64*a*b^3*c^8*d^6*e^9 - 192
*a*b^4*c^7*d^5*e^10 - 96*a^2*b*c^9*d^6*e^9 - 136*a^3*b*c^8*d^4*e^11 + 9*a^4
*b*c^7*d^2*e^13)/(32*a^8*d^2)))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^
2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^
2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c
- b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b
^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*2i + a
tan(((((((2048*a^12*c^4*d*e^12 + 12288*a^10*c^6*d^5*e^8 + 14336*a^11*c^5*d^
3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b^6*
c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8 + 7168*a^9*b^3*c^4*d^4*e^9 + 6272*
a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d^2*e^11 - 20480*a^10*b^2*c^4*d^3*e^
10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*a^10*b*c^5*d^4*e^9 + 128*a^10*b^4*c^
2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1024*a^11*b^2*c^3*d*e^12)/(64*a^8*d^2
) - ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/
2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3
*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)
^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1
/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(24576*a^12*c^5
*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*
b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 +
 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4
*d^3*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(
1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a
^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^
3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^
(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b
^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2
)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*
a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b
^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*
a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8
- 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d
^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*
b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 -
 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9
 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^1
2))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) -
a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*
e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2
) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) +
 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^
(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (16*a^9*c^5*e^14 -
 4*a^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^14 - 52*a^8*b^2*c^4*e^14 - 768*a^6*c
^8*d^6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a^8*c^6*d^2*e^12 - 512*a^2*b^8*c^4*d
^6*e^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a^2*b^10*c^2*d^4*e^10 + 3840*a^3*b^6
*c^5*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9 - 2208*a^3*b^8*c^3*d^4*e^10 - 224*a
^3*b^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d^6*e^8 + 896*a^4*b^5*c^5*d^5*e^9 +
10752*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^7*c^3*d^3*e^11 + 96*a^4*b^8*c^2*d^2
*e^12 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*a^5*b^3*c^6*d^5*e^9 - 18144*a^5*b^4
*c^5*d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^11 - 836*a^5*b^6*c^3*d^2*e^12 + 934
4*a^6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c^5*d^3*e^11 + 2236*a^6*b^4*c^4*d^2*

e^12 - 1716*a^7*b^2*c^5*d^2*e^12 - 528*a^8*b*c^5*d*e^13 + 4*a^5*b^7*c^2*d*e
^13 - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d*e^13 - 5632*a^7*b*c^6*d^3*e
^11 + 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(
-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 2
5*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4
e(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*
(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b
^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^
(1/2) - ((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9
*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 +
704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^
10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d
^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*
c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b
*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8
*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2))*((
b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4
*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)
^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e +
20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*
a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 +
16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i - (((((2048*a^12*c^4*d*e^12 + 12288*a
^10*c^6*d^5*e^8 + 14336*a^11*c^5*d^3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536
*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b^6*c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8
+ 7168*a^9*b^3*c^4*d^4*e^9 + 6272*a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d
^2*e^11 - 20480*a^10*b^2*c^4*d^3*e^10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*a
^10*b*c^5*d^4*e^9 + 128*a^10*b^4*c^2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 10
24*a^11*b^2*c^3*d*e^12)/(64*a^8*d^2) + ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*c
^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3
*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b
^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e
- 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(
1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8
*a^7*b^2*c)))^(1/2)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 204
8*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4
*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*
b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^4
*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a
^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a
*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*
e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)
^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 -
8*a^7*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*
c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8
+ 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9
- 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^
3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b
^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 71
68*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*
e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*
c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c
^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d
+ b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*
c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*
d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a
*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2)
+ 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*
b^2*c)))^(1/2) + (16*a^9*c^5*e^14 - 4*a^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^1
4 - 52*a^8*b^2*c^4*e^14 - 768*a^6*c^8*d^6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a

^8*c^6*d^2*e^12 - 512*a^2*b^8*c^4*d^6*e^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a
^2*b^10*c^2*d^4*e^10 + 3840*a^3*b^6*c^5*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9
- 2208*a^3*b^8*c^3*d^4*e^10 - 224*a^3*b^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d
^6*e^8 + 896*a^4*b^5*c^5*d^5*e^9 + 10752*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^
7*c^3*d^3*e^11 + 96*a^4*b^8*c^2*d^2*e^12 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*
a^5*b^3*c^6*d^5*e^9 - 18144*a^5*b^4*c^5*d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^
11 - 836*a^5*b^6*c^3*d^2*e^12 + 9344*a^6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c
^5*d^3*e^11 + 2236*a^6*b^4*c^4*d^2*e^12 - 1716*a^7*b^2*c^5*d^2*e^12 - 528*a
^8*b*c^5*d*e^13 + 4*a^5*b^7*c^2*d*e^13 - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^
5*c^3*d*e^13 - 5632*a^7*b*c^6*d^3*e^11 + 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^
2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a
^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b
^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*
c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d
*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6
*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(a^6*b^2*c^5*
e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*
c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c
^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*
b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*
a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 +
60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 19
2*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^
5*b^3*c^5*d*e^13))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b
^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c
^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c
- b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c -
b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4
*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i)/(
(((((2048*a^12*c^4*d*e^12 + 12288*a^10*c^6*d^5*e^8 + 14336*a^11*c^5*d^3*e^1
0 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b^6*c^2*d
^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8 + 7168*a^9*b^3*c^4*d^4*e^9 + 6272*a^9*b
^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d^2*e^11 - 20480*a^10*b^2*c^4*d^3*e^10 -
3072*a^10*b^3*c^3*d^2*e^11 - 4096*a^10*b*c^5*d^4*e^9 + 128*a^10*b^4*c^2*d*e
^12 + 6144*a^11*b*c^4*d^2*e^11 - 1024*a^11*b^2*c^3*d*e^12)/(64*a^8*d^2) - (
(d + e*x^2)^(1/2)*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) -
a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*
e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2
) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) +
3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^
(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(24576*a^12*c^5*d^4*
e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c
^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024
*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3*
e^9))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2)
- a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^
2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1
/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2)
+ 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3
)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2)^(1/
2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b
^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^
3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b
^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 112
64*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^
11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c
^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144
*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 22
8*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(

```
32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7
*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(
4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9
*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^
2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (16*a^9*c^5*e^14 - 4*a^
6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^14 - 52*a^8*b^2*c^4*e^14 - 768*a^6*c^8*d^
6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a^8*c^6*d^2*e^12 - 512*a^2*b^8*c^4*d^6*e^
8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a^2*b^10*c^2*d^4*e^10 + 3840*a^3*b^6*c^5*
d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9 - 2208*a^3*b^8*c^3*d^4*e^10 - 224*a^3*b^
9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d^6*e^8 + 896*a^4*b^5*c^5*d^5*e^9 + 10752
*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^7*c^3*d^3*e^11 + 96*a^4*b^8*c^2*d^2*e^12
 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*a^5*b^3*c^6*d^5*e^9 - 18144*a^5*b^4*c^5*
d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^11 - 836*a^5*b^6*c^3*d^2*e^12 + 9344*a^6
*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c^5*d^3*e^11 + 2236*a^6*b^4*c^4*d^2*e^12
- 1716*a^7*b^2*c^5*d^2*e^12 - 528*a^8*b*c^5*d*e^13 + 4*a^5*b^7*c^2*d*e^13 -
 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d*e^13 - 5632*a^7*b*c^6*d^3*e^11 +
 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a
*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3
*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-
(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*
a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*
e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)
 - ((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*
e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*
a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 -
512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^
10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d
^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*
d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*
e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2))*((b^8*d
 + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*
d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2
) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^
4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c -
 b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a
^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (((((2048*a^12*c^4*d*e^12 + 12288*a^10*c^6*
d^5*e^8 + 14336*a^11*c^5*d^3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^5
*c^3*d^4*e^9 - 512*a^8*b^6*c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8 + 7168*
a^9*b^3*c^4*d^4*e^9 + 6272*a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d^2*e^11
- 20480*a^10*b^2*c^4*d^3*e^10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*a^10*b*c^
5*d^4*e^9 + 128*a^10*b^4*c^2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1024*a^11*
b^2*c^3*d*e^12)/(64*a^8*d^2) + ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*c^4*d + b
^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3
*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d -
 a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^
3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3
*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2
*c)))^(1/2)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b
^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 1
5360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*
d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d +
 b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c
^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d
 - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*
b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) +
 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b
^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13
 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a
```

^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*
a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 +
 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d
^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b
^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 1
8048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^
12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e
^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*
(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d -
25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^
4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d
*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*
b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))
^(1/2) + (16*a^9*c^5*e^14 - 4*a^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^14 - 52*a
^8*b^2*c^4*e^14 - 768*a^6*c^8*d^6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a^8*c^6*d
^2*e^12 - 512*a^2*b^8*c^4*d^6*e^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a^2*b^10*
c^2*d^4*e^10 + 3840*a^3*b^6*c^5*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9 - 2208*a
^3*b^8*c^3*d^4*e^10 - 224*a^3*b^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d^6*e^8 +
 896*a^4*b^5*c^5*d^5*e^9 + 10752*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^7*c^3*d^
3*e^11 + 96*a^4*b^8*c^2*d^2*e^12 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*a^5*b^3*
c^6*d^5*e^9 - 18144*a^5*b^4*c^5*d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^11 - 836
*a^5*b^6*c^3*d^2*e^12 + 9344*a^6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c^5*d^3*e
^11 + 2236*a^6*b^4*c^4*d^2*e^12 - 1716*a^7*b^2*c^5*d^2*e^12 - 528*a^8*b*c^5
*d*e^13 + 4*a^5*b^7*c^2*d*e^13 - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d*
e^13 - 5632*a^7*b*c^6*d^3*e^11 + 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^2))*((b^
8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c
^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(
1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20
*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*
c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 1
6*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2
*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*
e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e
^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*
d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*
c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*
b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*
c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^
5*d*e^13))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(
1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a
^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^
3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^
(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b
^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (7*a^5*c^7*
d*e^14 + 56*a^3*c^9*d^5*e^10 + 63*a^4*c^8*d^3*e^12 - 64*b^4*c^8*d^7*e^8 + 6
4*b^5*c^7*d^6*e^9 + 64*a^2*b^2*c^8*d^5*e^10 + 224*a^2*b^3*c^7*d^4*e^11 - 11
2*a^3*b^2*c^7*d^3*e^12 + 64*a*b^2*c^9*d^7*e^8 + 64*a*b^3*c^8*d^6*e^9 - 192*
a*b^4*c^7*d^5*e^10 - 96*a^2*b*c^9*d^6*e^9 - 136*a^3*b*c^8*d^4*e^11 + 9*a^4*
b*c^7*d^2*e^13)/(32*a^8*d^2)))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2
)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2
*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c -
 b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^
2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a
c - b^2)^3)^(1/2))/(8(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*2i - ((
(d + e*x^2)^(1/2)*(a*e^2 + 4*b*d*e))/(8*a^2) + ((d + e*x^2)^(3/2)*(a*e^2 -
4*b*d*e))/(8*a^2*d))/((d + e*x^2)^2 - 2*d*(d + e*x^2) + d^2) + (atan(-(((((
d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 +
 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b
^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a

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^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 -
56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^
11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^1
3 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 -
 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2) - ((((a^9*c^5
*e^14)/4 - (a^6*b^6*c^2*e^14)/16 + (7*a^7*b^4*c^3*e^14)/16 - (13*a^8*b^2*c^
4*e^14)/16 - 12*a^6*c^8*d^6*e^8 - 12*a^7*c^7*d^4*e^10 + (a^8*c^6*d^2*e^12)/
4 - 8*a^2*b^8*c^4*d^6*e^8 + 6*a^2*b^9*c^3*d^5*e^9 + 2*a^2*b^10*c^2*d^4*e^10
 + 60*a^3*b^6*c^5*d^6*e^8 - 32*a^3*b^7*c^4*d^5*e^9 - (69*a^3*b^8*c^3*d^4*e^
10)/2 - (7*a^3*b^9*c^2*d^3*e^11)/2 - 136*a^4*b^4*c^6*d^6*e^8 + 14*a^4*b^5*c
^5*d^5*e^9 + 168*a^4*b^6*c^4*d^4*e^10 + 42*a^4*b^7*c^3*d^3*e^11 + (3*a^4*b^
8*c^2*d^2*e^12)/2 + 100*a^5*b^2*c^7*d^6*e^8 + 88*a^5*b^3*c^6*d^5*e^9 - (567
*a^5*b^4*c^5*d^4*e^10)/2 - (327*a^5*b^5*c^4*d^3*e^11)/2 - (209*a^5*b^6*c^3*
d^2*e^12)/16 + 146*a^6*b^2*c^6*d^4*e^10 + 228*a^6*b^3*c^5*d^3*e^11 + (559*a
^6*b^4*c^4*d^2*e^12)/16 - (429*a^7*b^2*c^5*d^2*e^12)/16 - (33*a^8*b*c^5*d*e
^13)/4 + (a^5*b^7*c^2*d*e^13)/16 - 68*a^6*b*c^7*d^5*e^9 - (23*a^6*b^5*c^3*d
*e^13)/16 - 88*a^7*b*c^6*d^3*e^11 + (109*a^7*b^3*c^4*d*e^13)/16)/(a^8*d^2)
+ (((((32*a^12*c^4*d*e^12 + 192*a^10*c^6*d^5*e^8 + 224*a^11*c^5*d^3*e^10 +
32*a^8*b^4*c^4*d^5*e^8 - 24*a^8*b^5*c^3*d^4*e^9 - 8*a^8*b^6*c^2*d^3*e^10 -
176*a^9*b^2*c^5*d^5*e^8 + 112*a^9*b^3*c^4*d^4*e^9 + 98*a^9*b^4*c^3*d^3*e^10
 + 6*a^9*b^5*c^2*d^2*e^11 - 320*a^10*b^2*c^4*d^3*e^10 - 48*a^10*b^3*c^3*d^2
*e^11 - 64*a^10*b*c^5*d^4*e^9 + 2*a^10*b^4*c^2*d*e^12 + 96*a^11*b*c^4*d^2*e
^11 - 16*a^11*b^2*c^3*d*e^12)/(a^8*d^2) - ((d + e*x^2)^(1/2)*(a^2*e^2 - 8*b
^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^
2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11
*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10
- 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(512*a^11*d^2*(d^
3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/
2)) + ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b
^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3
*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^
4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6
*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 -
256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4
*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b
^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*
a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*
a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b
*d*e))/(16*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))
/(16*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*1i)/(1
6*a^3*(d^3)^(1/2)) + ((((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^1
4 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^
8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2
*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 75
2*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10
 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*
e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9
+ 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(
32*a^8*d^2) + ((((a^9*c^5*e^14)/4 - (a^6*b^6*c^2*e^14)/16 + (7*a^7*b^4*c^3*
e^14)/16 - (13*a^8*b^2*c^4*e^14)/16 - 12*a^6*c^8*d^6*e^8 - 12*a^7*c^7*d^4*e
^10 + (a^8*c^6*d^2*e^12)/4 - 8*a^2*b^8*c^4*d^6*e^8 + 6*a^2*b^9*c^3*d^5*e^9
+ 2*a^2*b^10*c^2*d^4*e^10 + 60*a^3*b^6*c^5*d^6*e^8 - 32*a^3*b^7*c^4*d^5*e^9
 - (69*a^3*b^8*c^3*d^4*e^10)/2 - (7*a^3*b^9*c^2*d^3*e^11)/2 - 136*a^4*b^4*c
^6*d^6*e^8 + 14*a^4*b^5*c^5*d^5*e^9 + 168*a^4*b^6*c^4*d^4*e^10 + 42*a^4*b^7
*c^3*d^3*e^11 + (3*a^4*b^8*c^2*d^2*e^12)/2 + 100*a^5*b^2*c^7*d^6*e^8 + 88*a
^5*b^3*c^6*d^5*e^9 - (567*a^5*b^4*c^5*d^4*e^10)/2 - (327*a^5*b^5*c^4*d^3*e^
11)/2 - (209*a^5*b^6*c^3*d^2*e^12)/16 + 146*a^6*b^2*c^6*d^4*e^10 + 228*a^6*
b^3*c^5*d^3*e^11 + (559*a^6*b^4*c^4*d^2*e^12)/16 - (429*a^7*b^2*c^5*d^2*e^1
2)/16 - (33*a^8*b*c^5*d*e^13)/4 + (a^5*b^7*c^2*d*e^13)/16 - 68*a^6*b*c^7*d^
```

5*e^9 - (23*a^6*b^5*c^3*d*e^13)/16 - 88*a^7*b*c^6*d^3*e^11 + (109*a^7*b^3*c^4*d*e^13)/16)/(a^8*d^2) + (((((32*a^12*c^4*d*e^12 + 192*a^10*c^6*d^5*e^8 + 224*a^11*c^5*d^3*e^10 + 32*a^8*b^4*c^4*d^5*e^8 - 24*a^8*b^5*c^3*d^4*e^9 - 8*a^8*b^6*c^2*d^3*e^10 - 176*a^9*b^2*c^5*d^5*e^8 + 112*a^9*b^3*c^4*d^4*e^9 + 98*a^9*b^4*c^3*d^3*e^10 + 6*a^9*b^5*c^2*d^2*e^11 - 320*a^10*b^2*c^4*d^3*e^10 - 48*a^10*b^3*c^3*d^2*e^11 - 64*a^10*b*c^5*d^4*e^9 + 2*a^10*b^4*c^2*d*e^12 + 96*a^11*b*c^4*d^2*e^11 - 16*a^11*b^2*c^3*d*e^12)/(a^8*d^2) + ((d + e*x^2)^(1/2)*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(512*a^11*d^2*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)) - ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*1i)/(16*a^3*(d^3)^(1/2)))/(((7*a^5*c^7*d*e^14)/32 + (7*a^3*c^9*d^5*e^10)/4 + (63*a^4*c^8*d^3*e^12)/32 - 2*b^4*c^8*d^7*e^8 + 2*b^5*c^7*d^6*e^9 + 2*a^2*b^2*c^8*d^5*e^10 + 7*a^2*b^3*c^7*d^4*e^11 - (7*a^3*b^2*c^7*d^3*e^12)/2 + 2*a*b^2*c^9*d^7*e^8 + 2*a*b^3*c^8*d^6*e^9 - 6*a*b^4*c^7*d^5*e^10 - 3*a^2*b*c^9*d^6*e^9 - (17*a^3*b*c^8*d^4*e^11)/4 + (9*a^4*b*c^7*d^2*e^13)/32)/(a^8*d^2) - ((((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2) - ((((a^9*c^5*e^14)/4 - (a^6*b^6*c^2*e^14)/16 + (7*a^7*b^4*c^3*e^14)/16 - (13*a^8*b^2*c^4*e^14)/16 - 12*a^6*c^8*d^6*e^8 - 12*a^7*c^7*d^4*e^10 + (a^8*c^6*d^2*e^12)/4 - 8*a^2*b^8*c^4*d^6*e^8 + 6*a^2*b^9*c^3*d^5*e^9 + 2*a^2*b^10*c^2*d^4*e^10 + 60*a^3*b^6*c^5*d^6*e^8 - 32*a^3*b^7*c^4*d^5*e^9 - (69*a^3*b^8*c^3*d^4*e^10)/2 - (7*a^3*b^9*c^2*d^3*e^11)/2 - 136*a^4*b^4*c^6*d^6*e^8 + 14*a^4*b^5*c^5*d^5*e^9 + 168*a^4*b^6*c^4*d^4*e^10 + 42*a^4*b^7*c^3*d^3*e^11 + (3*a^4*b^8*c^2*d^2*e^12)/2 + 100*a^5*b^2*c^7*d^6*e^8 + 88*a^5*b^3*c^6*d^5*e^9 - (567*a^5*b^4*c^5*d^4*e^10)/2 - (327*a^5*b^5*c^4*d^3*e^11)/2 - (209*a^5*b^6*c^3*d^2*e^12)/16 + 146*a^6*b^2*c^6*d^4*e^10 + 228*a^6*b^3*c^5*d^3*e^11 + (559*a^6*b^4*c^4*d^2*e^12)/16 - (429*a^7*b^2*c^5*d^2*e^12)/16 - (33*a^8*b*c^5*d*e^13)/4 + (a^5*b^7*c^2*d*e^13)/16 - 68*a^6*b*c^7*d^5*e^9 - (23*a^6*b^5*c^3*d*e^13)/16 - 88*a^7*b*c^6*d^3*e^11 + (109*a^7*b^3*c^4*d*e^13)/16)/(a^8*d^2) + (((((32*a^12*c^4*d*e^12 + 192*a^10*c^6*d^5*e^8 + 224*a^11*c^5*d^3*e^10 + 32*a^8*b^4*c^4*d^5*e^8 - 24*a^8*b^5*c^3*d^4*e^9 - 8*a^8*b^6*c^2*d^3*e^10 - 176*a^9*b^2*c^5*d^5*e^8 + 112*a^9*b^3*c^4*d^4*e^9 + 98*a^9*b^4*c^3*d^3*e^10 + 6*a^9*b^5*c^2*d^2*e^11 - 320*a^10*b^2*c^4*d^3*e^10 - 48*a^10*b^3*c^3*d^2*e^11 - 64*a^10*b*c^5*d^4*e^9 + 2*a^10*b^4*c^2*d*e^12 + 96*a^11*b*c^4*d^2*e^11 - 16*a^11*b^2*c^3*d*e^12)/(a^8*d^2) - ((d + e*x^2)^(1/2)*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*

```
d^3*e^9))/(512*a^11*d^2*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*
a*b*d*e))/(16*a^3*(d^3)^(1/2)) + ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 4
8*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7
*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^
2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b
^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6
912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5
*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5
*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32
*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 460
8*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*(a^2*e^2 - 8*
b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^
2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*
a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)) + ((((d + e*x^2)^(1/2)*(a^6*b^2*
c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*
a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b
^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*
a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 +
704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^
12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8
- 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 +
6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2) + ((((a^9*c^5*e^14)/4 - (a^6*b^6*c^2*e^
14)/16 + (7*a^7*b^4*c^3*e^14)/16 - (13*a^8*b^2*c^4*e^14)/16 - 12*a^6*c^8*d^
6*e^8 - 12*a^7*c^7*d^4*e^10 + (a^8*c^6*d^2*e^12)/4 - 8*a^2*b^8*c^4*d^6*e^8
+ 6*a^2*b^9*c^3*d^5*e^9 + 2*a^2*b^10*c^2*d^4*e^10 + 60*a^3*b^6*c^5*d^6*e^8
- 32*a^3*b^7*c^4*d^5*e^9 - (69*a^3*b^8*c^3*d^4*e^10)/2 - (7*a^3*b^9*c^2*d^3
*e^11)/2 - 136*a^4*b^4*c^6*d^6*e^8 + 14*a^4*b^5*c^5*d^5*e^9 + 168*a^4*b^6*c
^4*d^4*e^10 + 42*a^4*b^7*c^3*d^3*e^11 + (3*a^4*b^8*c^2*d^2*e^12)/2 + 100*a^
5*b^2*c^7*d^6*e^8 + 88*a^5*b^3*c^6*d^5*e^9 - (567*a^5*b^4*c^5*d^4*e^10)/2 -
 (327*a^5*b^5*c^4*d^3*e^11)/2 - (209*a^5*b^6*c^3*d^2*e^12)/16 + 146*a^6*b^2
*c^6*d^4*e^10 + 228*a^6*b^3*c^5*d^3*e^11 + (559*a^6*b^4*c^4*d^2*e^12)/16 -
(429*a^7*b^2*c^5*d^2*e^12)/16 - (33*a^8*b*c^5*d*e^13)/4 + (a^5*b^7*c^2*d*e^
13)/16 - 68*a^6*b*c^7*d^5*e^9 - (23*a^6*b^5*c^3*d*e^13)/16 - 88*a^7*b*c^6*d
^3*e^11 + (109*a^7*b^3*c^4*d*e^13)/16)/(a^8*d^2) + (((((32*a^12*c^4*d*e^12
+ 192*a^10*c^6*d^5*e^8 + 224*a^11*c^5*d^3*e^10 + 32*a^8*b^4*c^4*d^5*e^8 - 2
4*a^8*b^5*c^3*d^4*e^9 - 8*a^8*b^6*c^2*d^3*e^10 - 176*a^9*b^2*c^5*d^5*e^8 +
112*a^9*b^3*c^4*d^4*e^9 + 98*a^9*b^4*c^3*d^3*e^10 + 6*a^9*b^5*c^2*d^2*e^11
- 320*a^10*b^2*c^4*d^3*e^10 - 48*a^10*b^3*c^3*d^2*e^11 - 64*a^10*b*c^5*d^4*
e^9 + 2*a^10*b^4*c^2*d*e^12 + 96*a^11*b*c^4*d^2*e^11 - 16*a^11*b^2*c^3*d*e^
12)/(a^8*d^2) + ((d + e*x^2)^(1/2)*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b
*d*e)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3
*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a
^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^
10 - 28672*a^12*b*c^4*d^3*e^9))/(512*a^11*d^2*(d^3)^(1/2)))*(a^2*e^2 - 8*b^
2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)) - ((d + e*x^2)^(1/2)*(
32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c
^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^
5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c
^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a
^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 -
 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d
^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8
*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^
8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a
^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)
))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)))*(a^
2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2))))*(a^2*e^2
 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*1i)/(8*a^3*(d^3)^(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.261 $\int \frac{x^4 \sqrt{d+ex^2}}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=390

$$-\frac{\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2c}}$$

Rubi [A] time = 2.92, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1291, 388, 217, 206, 1692, 377, 205}

$$-\frac{\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{(cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}}+\frac{x\sqrt{d+ex^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(2*c) - ((b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d
- b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b
 - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((b*c*d -
b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4
*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e]) + ((c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^
2]])/(2*c^2*Sqrt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1291

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[f^4/c^2, Int[(f*x)^(m - 4)*(c*d - b*e + c
*e*x^2)*(d + e*x^2)^(q - 1), x], x] - Dist[f^4/c^2, Int[((f*x)^(m - 4)*(d +
 e*x^2)^(q - 1)*Simp[a*(c*d - b*e) + (b*c*d - b^2*e + a*c*e)*x^2, x])/(a +
b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{\int \frac{cd-be+cex^2}{\sqrt{d+ex^2}}\,dx}{c^2} - \frac{\int \frac{a(cd-be)+\left(bcd-b^2e+ace\right)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{c^2}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} - \frac{\int \left(\frac{bcd-b^2e+ace+\frac{-b^2cd+2ac^2d+b^3e-3abce}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{bcd-b^2e+ace-\frac{-b^2cd+2ac^2d+b^3e-3abce}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{c^2} + \frac{(cd-\ldots}{\ldots}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(cd-2be)\,\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c^2} - \frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3\ldots}{\sqrt{b^2-4a\ldots}}\right.\ldots}{\ldots}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}} - \frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\text{Subs}\ldots}{\ldots}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} - \frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\left(bcd\ldots\right.}{\ldots}$$

Mathematica [B] time = 6.40, size = 10915, normalized size = 27.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [C] time = 91.94, size = 566, normalized size = 1.45

$$-\frac{\sqrt{e}\,\text{RootSum}\left[\#1^8c+4\#1^6be-4\#1^6cd+16\#1^4ae^2-8\#1^4bde+6\#1^4cd^2+4\#1^2bd^2e-4\#1^2cd^3+cd^4\&, \frac{-\#1^4ace\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+\#1^4b^2e\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+\#1^4(-b)cd\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+4\#1^2abe^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2\#1^2acde\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2\#1^2b^2de\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+2\#1^2bcd^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-acd^2e\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+b^2d^2e\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-bcd^3\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)}{\#1^6c+3\#1^4be-3\#1^4cd+8\#1^2ae^2-4\#1^2bde+3\#1^2cd^2+bd^2e-cd^3}\&\right]}{2c^2}+\frac{(2be-cd)\log\left(\sqrt{d+ex^2}-\sqrt{e}x\right)}{2c^2\sqrt{e}}+\frac{x\sqrt{d+ex^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(2*c) + ((-(c*d) + 2*b*e)*Log[-(Sqrt[e]*x) + Sqrt[d + e
*x^2]])/(2*c^2*Sqrt[e]) - (Sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e
*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1
^6 + c*#1^8 & , (-(b*c*d^3*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]) + b^2*
d^2*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - a*c*d^2*e*Log[-(Sqrt[e]*x)
 + Sqrt[d + e*x^2] - #1] + 2*b*c*d^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #
1]*#1^2 - 2*b^2*d*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 - 2*a*c*d
*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 4*a*b*e^2*Log[-(Sqrt[e]*
x) + Sqrt[d + e*x^2] - #1]*#1^2 - b*c*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]
- #1]*#1^4 + b^2*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4 - a*c*e*Lo
g[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4)/(-(c*d^3) + b*d^2*e + 3*c*d^2*
#1^2 - 4*b*d*e*#1^2 + 8*a*e^2*#1^2 - 3*c*d*#1^4 + 3*b*e*#1^4 + c*#1^6) & ])
/(2*c^2)
```

fricas [B] time = 104.32, size = 6534, normalized size = 16.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(1/2)*c^2*e*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a
^2*c^2)*e + (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2
 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2
*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(((a*b^2*c^4 - 4*a
^2*c^5)*d*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*
b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^
8 - 4*a*c^9)) + 2*(a^2*b^2*c - a^3*c^2)*d^2 - 2*(a^2*b^3 - 2*a^3*b*c)*d*e -
 ((a*b^3*c - a^2*b*c^2)*d^2 - (a*b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^
2*b^3 - 2*a^3*b*c)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c^4 - 6*a*b
^2*c^5 + 8*a^2*c^6)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*
c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)
/(b^2*c^8 - 4*a*c^9)) - ((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a
*b^4*c + 8*a^2*b^2*c^2)*e)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2
*c + 2*a^2*c^2)*e + (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*
c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4
*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)))/x^2) - sqrt(
1/2)*c^2*e*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e +
 (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c
 - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/
(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(((a*b^2*c^4 - 4*a^2*c^5)*d*x
^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2
*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9
)) + 2*(a^2*b^2*c - a^3*c^2)*d^2 - 2*(a^2*b^3 - 2*a^3*b*c)*d*e - ((a*b^3*c
- a^2*b*c^2)*d^2 - (a*b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b^3 - 2*a
^3*b*c)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c^4 - 6*a*b^2*c^5 + 8*
a^2*c^6)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3
*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 -
 4*a*c^9)) - ((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c + 8*
a^2*b^2*c^2)*e)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*
c^2)*e + (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 -
2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^
2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)))/x^2) + sqrt(1/2)*c^2*e*
sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (b^2*c^4 -
 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*
c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 -
4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(-((a*b^2*c^4 - 4*a^2*c^5)*d*x^2*sqrt(((
b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3
)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) - 2*(a^
```

```
2*b^2*c - a^3*c^2)*d^2 + 2*(a^2*b^3 - 2*a^3*b*c)*d*e + ((a*b^3*c - a^2*b*c^
2)*d^2 - (a*b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b^3 - 2*a^3*b*c)*e^
2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x
*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a
^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9))
 + ((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c + 8*a^2*b^2*c^
2)*e)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (
b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c -
 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b
^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)))/x^2) - sqrt(1/2)*c^2*e*sqrt(-((b^
3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (b^2*c^4 - 4*a*c^5)*
sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^
2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)))
/(b^2*c^4 - 4*a*c^5))*log(-((a*b^2*c^4 - 4*a^2*c^5)*d*x^2*sqrt(((b^4*c^2 -
2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b
^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) - 2*(a^2*b^2*c -
a^3*c^2)*d^2 + 2*(a^2*b^3 - 2*a^3*b*c)*d*e + ((a*b^3*c - a^2*b*c^2)*d^2 - (
a*b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b^3 - 2*a^3*b*c)*e^2)*x^2 - 2
*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x*sqrt(((b^
4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*
d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) + ((b^5*c
 - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*e)*x)*s
qrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (b^2*c^4 -
4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c
^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4
*a*c^9)))/(b^2*c^4 - 4*a*c^5)))/x^2) + 2*sqrt(e*x^2 + d)*c*e*x - (c*d - 2*b
*e)*sqrt(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d))/(c^2*e), 1/4*(
sqrt(1/2)*c^2*e*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2
)*e + (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(
b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*
e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(((a*b^2*c^4 - 4*a^2*c^5
)*d*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^
2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*
a*c^9)) + 2*(a^2*b^2*c - a^3*c^2)*d^2 - 2*(a^2*b^3 - 2*a^3*b*c)*d*e - ((a*b
^3*c - a^2*b*c^2)*d^2 - (a*b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b^3
- 2*a^3*b*c)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c^4 - 6*a*b^2*c^5
 + 8*a^2*c^6)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*
a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*
c^8 - 4*a*c^9)) - ((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c
 + 8*a^2*b^2*c^2)*e)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2
*a^2*c^2)*e + (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d
^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b
^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)))/x^2) - sqrt(1/2)*c
^2*e*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e + (b^2*
c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a
*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c
^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(((a*b^2*c^4 - 4*a^2*c^5)*d*x^2*sqr
t(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b
*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) + 2
*(a^2*b^2*c - a^3*c^2)*d^2 - 2*(a^2*b^3 - 2*a^3*b*c)*d*e - ((a*b^3*c - a^2*
b*c^2)*d^2 - (a*b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b^3 - 2*a^3*b*c
)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^
6)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 +
 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c
^9)) - ((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c + 8*a^2*b^
2*c^2)*e)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e
 + (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5
*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2
)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)))/x^2) + sqrt(1/2)*c^2*e*sqrt(-
```

```
((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (b^2*c^4 - 4*a*c
^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 +
2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^
9)))/(b^2*c^4 - 4*a*c^5))*log(-((a*b^2*c^4 - 4*a^2*c^5)*d*x^2*sqrt(((b^4*c^
2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e
+ (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) - 2*(a^2*b^2*
c - a^3*c^2)*d^2 + 2*(a^2*b^3 - 2*a^3*b*c)*d*e + ((a*b^3*c - a^2*b*c^2)*d^2
 - (a*b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b^3 - 2*a^3*b*c)*e^2)*x^2
 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x*sqrt(
((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c
^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) + ((b
^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*e)*
x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (b^2*c^
4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b
^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8
 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)))/x^2) - sqrt(1/2)*c^2*e*sqrt(-((b^3*c -
3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (b^2*c^4 - 4*a*c^5)*sqrt((
(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^
3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*
c^4 - 4*a*c^5))*log(-((a*b^2*c^4 - 4*a^2*c^5)*d*x^2*sqrt(((b^4*c^2 - 2*a*b^
2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4
*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) - 2*(a^2*b^2*c - a^3*c^
2)*d^2 + 2*(a^2*b^3 - 2*a^3*b*c)*d*e + ((a*b^3*c - a^2*b*c^2)*d^2 - (a*b^4
+ 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b^3 - 2*a^3*b*c)*e^2)*x^2 - 2*sqrt(
1/2)*sqrt(e*x^2 + d)*((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x*sqrt(((b^4*c^2
- 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e +
(b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) + ((b^5*c - 5*a
*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*e)*x)*sqrt(-(
(b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (b^2*c^4 - 4*a*c^
5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2
*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9
)))/(b^2*c^4 - 4*a*c^5)))/x^2) + 2*sqrt(e*x^2 + d)*c*e*x - 2*(c*d - 2*b*e)*
sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/(c^2*e)]
```

giac [A] time = 2.04, size = 53, normalized size = 0.14

$$-\frac{(cd - 2\,be)e^{\left(-\frac{1}{2}\right)}\log\left(\left(xe^{\frac{1}{2}} - \sqrt{x^2e + d}\right)^2\right)}{4\,c^2} + \frac{\sqrt{x^2e + d}\,x}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(c*d - 2*b*e)*e^(-1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c^2 + 1/2*
sqrt(x^2*e + d)*x/c
```

maple [C] time = 0.04, size = 290, normalized size = 0.74

$$\frac{b\sqrt{e}\,\ln\left(-\sqrt{e}\,x+\sqrt{e x^2+d}\right)}{c^2}+\frac{d\ln\left(\sqrt{e}\,x+\sqrt{e x^2+d}\right)}{2c\sqrt{e}}+\frac{\sqrt{e x^2+d}\,x}{2c}+\frac{\sqrt{e}\left(a c d^2 e-b^2 d^2 e+b c d^3+\left(a c e-b^2 e+b c d\right)\mathrm{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)^2+2\left(-2abe^2+acde+b^2de-bcd^2\right)\mathrm{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)+\left(-\sqrt{e}\,x+\sqrt{e x^2+d}\right)^2\right)}{2c^2\left(\mathrm{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)^3 c+3\,\mathrm{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)^2 be-3\,\mathrm{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)^2 cd+8\,\mathrm{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)ae^2-4\,\mathrm{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)bde+3\,\mathrm{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)cd^2+bd^2e-cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*x*(e*x^2+d)^(1/2)/c+1/2/c*d/e^(1/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))+1/2/c
^2*e^(1/2)*sum(((a*c*e-b^2*e+b*c*d)*_R^2+2*(-2*a*b*e^2+a*c*d*e+b^2*d*e-b*c*
d^2)*_R+e*c*d^2*a-b^2*d^2*e+b*c*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e
^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln((-e^(1/2)*x+(e*x^2+d)^(1/2))^2-_
R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b
*d^2*e-4*c*d^3)*_Z+c*d^4))+1/c^2*e^(1/2)*b*ln(-e^(1/2)*x+(e*x^2+d)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}\,x^4}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x^2 + d)*x^4/(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sqrt{e\,x^2+d}}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
[Out] int((x^4*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x**4*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

3.262 $\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=324

$$\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{\ }}{}$$

Rubi [A] time = 1.52, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1293, 217, 206, 1692, 377, 205}

$$\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c
*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*
x^2])])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e]) + ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt
[d + e*x^2])])/(c*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]) + (Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/c
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1293

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a
*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d,
```

```
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = -\frac{\int \frac{ae-(cd-be)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{c}+\frac{e\int \frac{1}{\sqrt{d+ex^2}}\,dx}{c}$$

$$= -\frac{\int \left(\frac{-cd+be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}+\frac{-cd+be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{c}+\frac{e\,\mathrm{Subst}\left(\int \frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+e}}\right.}{c}$$

$$= \frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c}+\frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{c}-\frac{\left(-cd+be\right.}{}$$

$$= \frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c}+\frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\mathrm{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\right.}{c}$$

$$= \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2c}}{\sqrt{b}}\right.}{c\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\right.}}$$

Mathematica [B] time = 6.17, size = 7768, normalized size = 23.98

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [C] time = 57.34, size = 425, normalized size = 1.31

$$\frac{\sqrt{e}\,\mathrm{RootSum}\left[\#1^8c+4\#1^6be-4\#1^6cd+16\#1^4ae^2-8\#1^4bde+6\#1^4cd^2+4\#1^2bd^2e-4\#1^2cd^3+cd^4\&,\frac{-\#1^4be\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+\#1^4cd\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-4\#1^2ae^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+2\#1^2bde\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2\#1^2cd^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-bd^2e\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+cd^3\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)}{\#1^6(-c)-3\#1^4be+3\#1^4cd-8\#1^2ae^2+4\#1^2bde-3\#1^2cd^2-bd^2e+cd^3}\&\right]}{2c}-\frac{\sqrt{e}\log\left(\sqrt{d+ex^2}-\sqrt{e}x\right)}{c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((Sqrt[e]*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/c) + (Sqrt[e]*RootSum[c*d^4
 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#
1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (c*d^3*Log[-(Sqrt[e]*x) + Sqrt[d
 + e*x^2] - #1] - b*d^2*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - 2*c*d^
2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 2*b*d*e*Log[-(Sqrt[e]*x)
+ Sqrt[d + e*x^2] - #1]*#1^2 - 4*a*e^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] -
```

```
#1]*#1^2 + c*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4 - b*e*Log[-(S
qrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4)/(c*d^3 - b*d^2*e - 3*c*d^2*#1^2 + 4
*b*d*e*#1^2 - 8*a*e^2*#1^2 + 3*c*d*#1^4 - 3*b*e*#1^4 - c*#1^6) & ])/(2*c)
```

fricas [B] time = 13.84, size = 3260, normalized size = 10.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 - 4*a*c^3)*sqr
t((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)
)*log(-((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2
*c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (b*c*d^2 + 4*a*b*e^2 - (b^2 + 4*
a*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^2 - 4*a*b*c^3)*x*sqrt((
c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - ((b^2*c - 4*a*c^2)*d
- (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 - 4*a*c^3
)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a
*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 - 4*a*
c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 -
4*a*c^3))*log(-((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e
^2)/(b^2*c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (b*c*d^2 + 4*a*b*e^2 - (
b^2 + 4*a*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^2 - 4*a*b*c^3)*
x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - ((b^2*c - 4*a
*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 -
 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c
^2 - 4*a*c^3)))/x^2) + sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - (b^2*c^
2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^
2*c^2 - 4*a*c^3))*log(((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b*c*d*e
+ b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 + 2*a*b*d*e + (b*c*d^2 + 4*a*b*
e^2 - (b^2 + 4*a*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^2 - 4*a*
b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) + ((b^2*
c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - (b^
2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))
/(b^2*c^2 - 4*a*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e -
(b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5
)))/(b^2*c^2 - 4*a*c^3))*log(((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b
*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 + 2*a*b*d*e + (b*c*d^2 +
 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^2
 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) +
 ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*
e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a
*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) - 2*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2
+ d)*sqrt(e)*x - d))/c, -1/4*(sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e +
(b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5
)))/(b^2*c^2 - 4*a*c^3))*log(-((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*
b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (b*c*d^2
+ 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^
2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5))
- ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)
*e + (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*
a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a
*c)*e + (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 -
 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(-((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*
d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (
b*c*d^2 + 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*
((b^3*c^2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*
a*c^5)) - ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2
- 2*a*c)*e + (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*
```

```
c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) + sqrt(1/2)*c*sqrt(-(b*c*d - (b
^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b
^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(((b^2*c^2 - 4*a*c^3)*d*x^2*sqr
t((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 + 2*a*b*
d*e + (b*c*d^2 + 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^
2 + d)*((b^3*c^2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c
^4 - 4*a*c^5)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d
- (b^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2
)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c
*d - (b^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*
e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(((b^2*c^2 - 4*a*c^3)*d*
x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 +
 2*a*b*d*e + (b*c*d^2 + 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 - 2*sqrt(1/2)*sq
rt(e*x^2 + d)*((b^3*c^2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)
/(b^2*c^4 - 4*a*c^5)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-
(b*c*d - (b^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e +
b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) + 4*sqrt(-e)*arct
an(sqrt(-e)*x/sqrt(e*x^2 + d)))/c]
```

giac [A] time = 1.83, size = 27, normalized size = 0.08

$$-\frac{e^{\frac{1}{2}} \log\left(\left(xe^{\frac{1}{2}} - \sqrt{x^2e + d}\right)^2\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*e^(1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c
```

maple [C] time = 0.03, size = 224, normalized size = 0.69

$$-\frac{\sqrt{e}\,\ln\left(-\sqrt{e}\,x+\sqrt{e x^2+d}\right)}{c}+\frac{\sqrt{e}\left(b d^2 e-c d^3+(be-cd)\,\mathrm{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)^2+2\left(2ae^2-deb+cd^2\right)\mathrm{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)+\left(-\sqrt{e}\,x+\sqrt{e x^2+d}\right)^2\right)}{2c\left(\mathrm{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)^3 c+3\,\mathrm{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)^2 be-3\,\mathrm{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)^2 cd+8\,\mathrm{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right) ae^2-4\,\mathrm{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right) bde+3\,\mathrm{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right) cd^2+bd^2e-cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*e^(1/2)/c*sum(((b*e-c*d)*_R^2+2*(2*a*e^2-b*d*e+c*d^2)*_R+b*d^2*e-c*d^3)
/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d
^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*
c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))-e^(1/2)/
c*ln(-e^(1/2)*x+(e*x^2+d)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d}\,x^2}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*x^2/(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sqrt{e\,x^2+d}}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
[Out] int((x^2*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x**2*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

3.263 $\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=240

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.32, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1174, 402, 217, 206, 377, 205}

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2
 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2
- 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2
 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]
])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 1174

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{(2c)\int \frac{\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{\sqrt{b^2-4ac}} + \frac{\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)\int \frac{1}{(b-}}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}} + \frac{\left(-2cd\right.}{}$$

$$= \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,\tan^{-1}}{\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b}}}$$

Mathematica [B] time = 5.12, size = 2585, normalized size = 10.77

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*Sq
rt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[-(Sqrt[(-b + Sqrt[b^2 - 4*a*c])
/c]/Sqrt[2]) + x] - Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*(2*c*d + (-b + Sqrt[
b^2 - 4*a*c])*e)*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[Sqrt[(-b + S
qrt[b^2 - 4*a*c])/c]/Sqrt[2] + x] - 2*c*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*d*
Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*
a*c])/c)]/Sqrt[2]) + x] + b*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d
- b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sq
rt[2]) + x] + Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*
c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)
]/Sqrt[2]) + x] + 2*c*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*d*Sqrt[(2*c*d - b*e
+ Sqrt[b^2 - 4*a*c]*e)/c]*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2] +
x] - b*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*
a*c]*e)/c]*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2] + x] - Sqrt[b^2 -
 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4
*a*c]*e)/c]*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2] + x] + 2*c*Sqrt[
-((b + Sqrt[b^2 - 4*a*c])/c)]*d*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*L
og[2*d - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e
 + 2*Sqrt[b^2 - 4*a*c]*e)/c]*Sqrt[d + e*x^2]] - b*Sqrt[-((b + Sqrt[b^2 - 4*
a*c])/c)]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d - Sqrt[2]*Sqr
t[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*
c]*e)/c]*Sqrt[d + e*x^2]] + Sqrt[b^2 - 4*a*c]*Sqrt[-((b + Sqrt[b^2 - 4*a*c]
```

```
)/c)]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d - Sqrt[2]*Sqrt[(-
b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c]*e
)/c]*Sqrt[d + e*x^2]] - 2*c*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*d*Sqrt[2*d -
 ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d + Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a
*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c]*e)/c]*Sqrt[d + e*x^
2]] + b*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*
a*c])*e)/c]*Log[2*d + Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(
4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c]*e)/c]*Sqrt[d + e*x^2]] - Sqrt[b^2 - 4*a
*c]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c]
)*e)/c]*Log[2*d + Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*
d - 2*b*e + 2*Sqrt[b^2 - 4*a*c]*e)/c]*Sqrt[d + e*x^2]] - 2*c*Sqrt[(-b + Sqr
t[b^2 - 4*a*c])/c]*d*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[2*d -
Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*x + Sqrt[4*d - (2*(b + Sqrt[b^
2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] + b*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*S
qrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[2*d - Sqrt[2]*Sqrt[-((b + Sq
rt[b^2 - 4*a*c])/c)]*e*x + Sqrt[4*d - (2*(b + Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt
[d + e*x^2]] + Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2
*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[2*d - Sqrt[2]*Sqrt[-((b + Sqrt[b^2
 - 4*a*c])/c)]*e*x + Sqrt[4*d - (2*(b + Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e
*x^2]] + 2*c*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*d*Sqrt[(2*c*d - b*e + Sqrt[b^
2 - 4*a*c]*e)/c]*Log[2*d + Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*x +
 Sqrt[4*d - (2*(b + Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] - b*Sqrt[(-b
+ Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[2
*d + Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*x + Sqrt[4*d - (2*(b + Sq
rt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] - Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt
[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[2*d + S
qrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*x + Sqrt[4*d - (2*(b + Sqrt[b^2
 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]])/(2*c*Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b
^2 - 4*a*c])/c]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*Sqrt[(2*c*d - b*e + Sqrt
[b^2 - 4*a*c]*e)/c]*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c])
```

IntegrateAlgebraic [C] time = 17.05, size = 250, normalized size = 1.04

$$\frac{1}{2}e^{3/2}\text{RootSum}\left[\#1^8c+4\#1^6be-4\#1^6cd+16\#1^4ae^2-8\#1^4bde+6\#1^4cd^2+4\#1^2bd^2e-4\#1^2cd^3+cd^4\&,\frac{\#1^4\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+2\#1^2d\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+d^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)}{\#1^6(-c)-3\#1^4be+3\#1^4cd-8\#1^2ae^2+4\#1^2bde-3\#1^2cd^2-bd^2e+cd^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b
*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (d^2*Log[-
(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] + 2*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2
] - #1]*#1^2 + Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4)/(c*d^3 - b*d^
2*e - 3*c*d^2*#1^2 + 4*b*d*e*#1^2 - 8*a*e^2*#1^2 + 3*c*d*#1^4 - 3*b*e*#1^4
- c*#1^6) & ])/2
```

fricas [B] time = 3.34, size = 985, normalized size = 4.10

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 - 4*
a^3*c)))/(a*b^2 - 4*a^2*c))*log(-((a*b^2 - 4*a^2*c)*d*sqrt(d^2/(a^2*b^2 - 4
*a^3*c))*x^2 + 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^2 + d)*sqrt(d^2/(a^
2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^
2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) - 2*a*d^2 + (b*d^2 - 4*a*d*e)*x^2)/x^2) -
 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 - 4
*a^3*c)))/(a*b^2 - 4*a^2*c))*log(-((a*b^2 - 4*a^2*c)*d*sqrt(d^2/(a^2*b^2 -
```

```
4*a^3*c))*x^2 - 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^2 + d)*sqrt(d^2/(a
^2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b
^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) - 2*a*d^2 + (b*d^2 - 4*a*d*e)*x^2)/x^2)
+ 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 -
4*a^3*c)))/(a*b^2 - 4*a^2*c))*log(((a*b^2 - 4*a^2*c)*d*sqrt(d^2/(a^2*b^2 -
4*a^3*c))*x^2 + 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^2 + d)*sqrt(d^2/(a
^2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b
^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) + 2*a*d^2 - (b*d^2 - 4*a*d*e)*x^2)/x^2)
- 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 -
4*a^3*c)))/(a*b^2 - 4*a^2*c))*log(((a*b^2 - 4*a^2*c)*d*sqrt(d^2/(a^2*b^2 -
4*a^3*c))*x^2 - 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^2 + d)*sqrt(d^2/(a
^2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b
^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) + 2*a*d^2 - (b*d^2 - 4*a*d*e)*x^2)/x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b,c]=[44,93,-37]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a,b,c]=[-72,-7,6]Evaluation time: 0.44Unable to divide, perhaps due
to rounding error%%%{18446744069414584320,[4,7,8,2,3,14,2]%%%}+%%%{-2147483
648,[3,8,10,8,3,10,1]%%%}+%%%{-12884901888,[3,8,10,7,2,12,2]%%%}+%%%{463856
467968,[3,8,10,6,1,14,3]%%%}+%%%{1924145348608,[3,8,10,5,0,16,4]%%%}+%%%{53
6870912,[3,8,9,8,5,10,0]%%%}+%%%{20401094656,[3,8,9,7,4,12,1]%%%}+%%%{-1503
23855360,[3,8,9,6,3,14,2]%%%}+%%%{-3135326126080,[3,8,9,5,2,16,3]%%%}+%%%{-
4672924418048,[3,8,9,4,1,18,4]%%%}+%%%{6047313952768,[3,8,9,3,0,20,5]%%%}+%
%%{-4294967296,[3,8,8,7,6,12,0]%%%}+%%%{-42412802048,[3,8,8,6,5,14,1]%%%}+%
%%{1046898278400,[3,8,8,5,4,16,2]%%%}+%%%{6210522710016,[3,8,8,4,3,18,3]%%%
}+%%%{-1786706395136,[3,8,8,3,2,20,4]%%%}+%%%{-11544872091648,[3,8,8,2,1,22
,5]%%%}+%%%{4398046511104,[3,8,8,1,0,24,6]%%%}+%%%{12750684160,[3,8,7,6,7,1
4,0]%%%}+%%%{-23890755584,[3,8,7,5,6,16,1]%%%}+%%%{-2103460233216,[3,8,7,4,
5,18,2]%%%}+%%%{-3324304687104,[3,8,7,3,4,20,3]%%%}+%%%{9758165696512,[3,8,
7,2,3,22,4]%%%}+%%%{1649267441664,[3,8,7,1,2,24,5]%%%}+%%%{-4398046511104,[
3,8,7,0,1,26,6]%%%}+%%%{-17985175552,[3,8,6,5,8,16,0]%%%}+%%%{161866579968,
[3,8,6,4,7,18,1]%%%}+%%%{1586990415872,[3,8,6,3,6,20,2]%%%}+%%%{-1795296329
728,[3,8,6,2,5,22,3]%%%}+%%%{-4123168604160,[3,8,6,1,4,24,4]%%%}+%%%{384829
0697216,[3,8,6,0,3,26,5]%%%}+%%%{12213813248,[3,8,5,4,9,18,0]%%%}+%%%{-1717
98691840,[3,8,5,3,8,20,1]%%%}+%%%{-212600881152,[3,8,5,2,7,22,2]%%%}+%%%{14
77468749824,[3,8,5,1,6,24,3]%%%}+%%%{-1099511627776,[3,8,5,0,5,26,4]%%%}+%%
%{-3221225472,[3,8,4,3,10,20,0]%%%}+%%%{57982058496,[3,8,4,2,9,22,1]%%%}+%%
%{-154618822656,[3,8,4,1,8,24,2]%%%}+%%%{103079215104,[3,8,4,0,7,26,3]%%%}+
%%%{1048576,[3,6,10,4,2,4,0]%%%}+%%%{8388608,[3,6,10,3,1,6,1]%%%}+%%%{16777
216,[3,6,10,2,0,8,2]%%%}+%%%{-5242880,[3,6,9,3,3,6,0]%%%}+%%%{-29360128,[3,
6,9,2,2,8,1]%%%}+%%%{-33554432,[3,6,9,1,1,10,2]%%%}+%%%{9699328,[3,6,8,2,4,
8,0]%%%}+%%%{33554432,[3,6,8,1,3,10,1]%%%}+%%%{16777216,[3,6,8,0,2,12,2]%%%
}+%%%{-7864320,[3,6,7,1,5,10,0]%%%}+%%%{-12582912,[3,6,7,0,4,12,1]%%%}+%%%{
2359296,[3,6,6,0,6,12,0]%%%}+%%%{536870912,[2,7,10,6,2,8,1]%%%}+%%%{6710886
400,[2,7,10,5,1,10,2]%%%}+%%%{18253611008,[2,7,10,4,0,12,3]%%%}+%%%{-134217
728,[2,7,9,6,4,8,0]%%%}+%%%{-5502926848,[2,7,9,5,3,10,1]%%%}+%%%{-369098752
00,[2,7,9,4,2,12,2]%%%}+%%%{-42949672960,[2,7,9,3,1,14,3]%%%}+%%%{429496729
60,[2,7,9,2,0,16,4]%%%}+%%%{956301312,[2,7,8,5,5,10,0]%%%}+%%%{18656264192,
[2,7,8,4,4,12,1]%%%}+%%%{64961380352,[2,7,8,3,3,14,2]%%%}+%%%{-8589934592,[
2,7,8,2,2,16,3]%%%}+%%%{-85899345920,[2,7,8,1,1,18,4]%%%}+%%%{-2642411520,[
```

```
2,7,7,4,6,12,0]%%%}+%%%{-27783069696,[2,7,7,3,5,14,1]%%%}+%%%{-33957085184,
[2,7,7,2,4,16,2]%%%}+%%%{73014444032,[2,7,7,1,3,18,3]%%%}+%%%{42949672960,[
2,7,7,0,2,20,4]%%%}+%%%{3556769792,[2,7,6,3,7,14,0]%%%}+%%%{17716740096,[2,
7,6,2,6,16,1]%%%}+%%%{-12884901888,[2,7,6,1,5,18,2]%%%}+%%%{-39728447488,[2
,7,6,0,4,20,3]%%%}+%%%{-2340421632,[2,7,5,2,8,16,0]%%%}+%%%{-2415919104,[2,
7,5,1,7,18,1]%%%}+%%%{12079595520,[2,7,5,0,6,20,2]%%%}+%%%{603979776,[2,7,4
,1,9,18,0]%%%}+%%%{-1207959552,[2,7,4,0,8,20,1]%%%}+%%%{2147483648,[1,8,10,
9,3,10,1]%%%}+%%%{38654705664,[1,8,10,8,2,12,2]%%%}+%%%{51539607552,[1,8,10
,7,1,14,3]%%%}+%%%{-274877906944,[1,8,10,6,0,16,4]%%%}+%%%{-536870912,[1,8,
9,9,5,10,0]%%%}+%%%{-26843545600,[1,8,9,8,4,12,1]%%%}+%%%{-188978561024,[1,
8,9,7,3,14,2]%%%}+%%%{146028888064,[1,8,9,6,2,16,3]%%%}+%%%{962072674304,[1
,8,9,5,1,18,4]%%%}+%%%{-549755813888,[1,8,9,4,0,20,5]%%%}+%%%{4294967296,[1
,8,8,8,6,12,0]%%%}+%%%{95026151424,[1,8,8,7,5,14,1]%%%}+%%%{239444426752,[1
,8,8,6,4,16,2]%%%}+%%%{-858993459200,[1,8,8,5,3,18,3]%%%}+%%%{-618475290624
,[1,8,8,4,2,20,4]%%%}+%%%{1099511627776,[1,8,8,3,1,22,5]%%%}+%%%{-127506841
60,[1,8,7,7,7,14,0]%%%}+%%%{-136633647104,[1,8,7,6,6,16,1]%%%}+%%%{62277025
792,[1,8,7,5,5,18,2]%%%}+%%%{936302870528,[1,8,7,4,4,20,3]%%%}+%%%{-5497558
13888,[1,8,7,3,3,22,4]%%%}+%%%{-549755813888,[1,8,7,2,2,24,5]%%%}+%%%{17985
175552,[1,8,6,6,8,16,0]%%%}+%%%{71940702208,[1,8,6,5,7,18,1]%%%}+%%%{-26736
1714176,[1,8,6,4,6,20,2]%%%}+%%%{-137438953472,[1,8,6,3,5,22,3]%%%}+%%%{481
036337152,[1,8,6,2,4,24,4]%%%}+%%%{-12213813248,[1,8,5,5,9,18,0]%%%}+%%%{72
47757312,[1,8,5,4,8,20,1]%%%}+%%%{103079215104,[1,8,5,3,7,22,2]%%%}+%%%{-13
7438953472,[1,8,5,2,6,24,3]%%%}+%%%{3221225472,[1,8,4,4,10,20,0]%%%}+%%%{-1
2884901888,[1,8,4,3,9,22,1]%%%}+%%%{12884901888,[1,8,4,2,8,24,2]%%%}+%%%{-1
048576,[1,6,10,5,2,4,0]%%%}+%%%{-8388608,[1,6,10,4,1,6,1]%%%}+%%%{-16777216
,[1,6,10,3,0,8,2]%%%}+%%%{8388608,[1,6,9,4,3,6,0]%%%}+%%%{62914560,[1,6,9,3
,2,8,1]%%%}+%%%{150994944,[1,6,9,2,1,10,2]%%%}+%%%{134217728,[1,6,9,1,0,12,
3]%%%}+%%%{-26476544,[1,6,8,3,4,8,0]%%%}+%%%{-163577856,[1,6,8,2,3,10,1]%%%
}+%%%{-301989888,[1,6,8,1,2,12,2]%%%}+%%%{-134217728,[1,6,8,0,1,14,3]%%%}+%
%%{41156608,[1,6,7,2,5,10,0]%%%}+%%%{178257920,[1,6,7,1,4,12,1]%%%}+%%%{167
772160,[1,6,7,0,3,14,2]%%%}+%%%{-31457280,[1,6,6,1,6,12,0]%%%}+%%%{-6920601
6,[1,6,6,0,5,14,1]%%%}+%%%{9437184,[1,6,5,0,7,14,0]%%%}+%%%{-402653184,[0,7
,10,7,2,8,1]%%%}+%%%{-5637144576,[0,7,10,6,1,10,2]%%%}+%%%{-16106127360,[0,
7,10,5,0,12,3]%%%}+%%%{100663296,[0,7,9,7,4,8,0]%%%}+%%%{4160749568,[0,7,9,
6,3,10,1]%%%}+%%%{30198988800,[0,7,9,5,2,12,2]%%%}+%%%{28991029248,[0,7,9,4
,1,14,3]%%%}+%%%{-68719476736,[0,7,9,3,0,16,4]%%%}+%%%{-687865856,[0,7,8,6,
5,10,0]%%%}+%%%{-13925089280,[0,7,8,5,4,12,1]%%%}+%%%{-48184164352,[0,7,8,4
,3,14,2]%%%}+%%%{49392123904,[0,7,8,3,2,16,3]%%%}+%%%{120259084288,[0,7,8,2
,1,18,4]%%%}+%%%{-68719476736,[0,7,8,1,0,20,5]%%%}+%%%{1845493760,[0,7,7,5,
6,12,0]%%%}+%%%{19964887040,[0,7,7,4,5,14,1]%%%}+%%%{11542724608,[0,7,7,3,4
,16,2]%%%}+%%%{-113816633344,[0,7,7,2,3,18,3]%%%}+%%%{8589934592,[0,7,7,1,2
,20,4]%%%}+%%%{68719476736,[0,7,7,0,1,22,5]%%%}+%%%{-2432696320,[0,7,6,4,7,
14,0]%%%}+%%%{-11207180288,[0,7,6,3,6,16,1]%%%}+%%%{28185722880,[0,7,6,2,5,
18,2]%%%}+%%%{34359738368,[0,7,6,1,4,20,3]%%%}+%%%{-60129542144,[0,7,6,0,3,
22,4]%%%}+%%%{1577058304,[0,7,5,3,8,16,0]%%%}+%%%{-201326592,[0,7,5,2,7,18,
1]%%%}+%%%{-14495514624,[0,7,5,1,6,20,2]%%%}+%%%{17179869184,[0,7,5,0,5,22,
3]%%%}+%%%{-402653184,[0,7,4,2,9,18,0]%%%}+%%%{1610612736,[0,7,4,1,8,20,1]%
%%}+%%%{-1610612736,[0,7,4,0,7,22,2]%%%} / %%%{-1024,[0,3,4,2,1,2,0]%%%}+%%
%{-4096,[0,3,4,1,0,4,1]%%%}+%%%{2560,[0,3,3,1,2,4,0]%%%}+%%%{4096,[0,3,3,0,
1,6,1]%%%}+%%%{-1536,[0,3,2,0,3,6,0]%%%} Error: Bad Argument Value
```

maple [C] time = 0.02, size = 161, normalized size = 0.67

$$-\frac{e^{\frac{3}{2}}\left(\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)^2+2\,\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)d+d^2\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)+\left(-\sqrt{e}\,x+\sqrt{e\,x^2+d}\right)^2\right)}{2\left(\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)^3c+3\,\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)^2be-3\,\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)^2cd+8\,\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)a\,e^2-4\,\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)bde+3\,\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)c\,d^2+b\,d^2e-c\,d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*e^(3/2)*sum((_R^2+2*_R*d+d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2
-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2
```

```
),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2
+(4*b*d^2*e-4*c*d^3)*_Z))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x^2+d}}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] int((d + e*x^2)^(1/2)/(a + b*x^2 + c*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

3.264 $\int \frac{\sqrt{d+ex^2}}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=291

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d+ex^2}}{ax}$$

Rubi [A] time = 0.67, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1295, 264, 1692, 377, 205}

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d+ex^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[d + e*x^2]/(a*x)) - (c*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sq
rt[d + e*x^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]) - (c*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^
2])])/(a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1295

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[((f*x)^(m + 2)*(d + e*x^2)^(q - 1)*Simp[b*d - a*e
+ c*d*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx = -\frac{\int \frac{bd-ae+cdx^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{a} + \frac{d\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{ax} - \frac{\int \left(\frac{cd+\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{cd-\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{ax} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{a} - \frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{}{\left(b-\right.}}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{ax} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right.}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{ax} - \frac{c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}}{a\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{2c}}$$

Mathematica [B] time = 6.32, size = 4644, normalized size = 15.96

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d + e*x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[d + e*x^2]/(a*x)) - (-1/2*(b*d*(Log[Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/
c]/Sqrt[2] + x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d -
Sqrt[2]*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) - Sqrt
[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*
c]/c)*e)/2]))/(Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c)
- Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2
])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) + Sqrt[b^2
- 4*a*c]/c]/Sqrt[2])) + (Sqrt[b^2 - 4*a*c]*d*(Log[Sqrt[-(b/c) - Sqrt[b^2 -
4*a*c]/c]/Sqrt[2] + x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log
[2*d - Sqrt[2]*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c)
 - Sqrt[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2
 - 4*a*c]/c)*e)/2]))/(2*Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(-(Sqr
t[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/
c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) +
Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])) + (a*e*(Log[Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/
c]/Sqrt[2] + x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d -
Sqrt[2]*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) - Sqrt
[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*
c]/c)*e)/2]))/(Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c)
- Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2
])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) + Sqrt[b^2
```

```
- 4*a*c]/c]/Sqrt[2])) + (b*d*(Log[-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt
[2]) + x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d + Sqrt[2
]*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) - Sqrt[b^2 -
 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*
e)/2]))/(2*Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(Sqrt[-(b/c) - Sqrt
[b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqr
t[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c
]/Sqrt[2])) - (Sqrt[b^2 - 4*a*c]*d*(Log[-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c
]/Sqrt[2]) + x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d +
Sqrt[2]*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) - Sqrt
[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*
c]/c)*e)/2]))/(2*Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(Sqrt[-(b/c)
- Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]
)*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*
a*c]/c]/Sqrt[2])) - (a*e*(Log[-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])
 + x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d + Sqrt[2]*Sq
rt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a
*c]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2
]))/(Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(Sqrt[-(b/c) - Sqrt[b^2 -
 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/
c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt
[2])) - (b*d*(Log[Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + x]/Sqrt[d +
((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d - Sqrt[2]*Sqrt[-(b/c) + Sqr
t[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]*Sq
rt[d + e*x^2]]/Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]))/(2*Sqrt[2]*
c*Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/
Sqrt[2]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) - Sqrt[
b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])) - (S
qrt[b^2 - 4*a*c]*d*(Log[Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + x]/Sqr
t[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d - Sqrt[2]*Sqrt[-(b/c)
 + Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)
/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]))/(2*Sq
rt[2]*c*Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*
c]/c]/Sqrt[2]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) -
 Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])
) + (a*e*(Log[Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + x]/Sqrt[d + ((-(
b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d - Sqrt[2]*Sqrt[-(b/c) + Sqrt[b^
2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d
 + e*x^2]]/Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]))/(Sqrt[2]*c*Sqrt
[-(b/c) + Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2
]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) - Sqrt[b^2 -
4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])) + (b*d*(Lo
g[-(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x]/Sqrt[d + ((-(b/c) + Sq
rt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d + Sqrt[2]*Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c
]/c]*e*x + 2*Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]
]/Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]))/(2*Sqrt[2]*c*Sqrt[-(b/c)
 + Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + Sq
rt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/
c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])) + (Sqrt[b^2 - 4*a
*c]*d*(Log[-(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x]/Sqrt[d + ((-(
b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d + Sqrt[2]*Sqrt[-(b/c) + Sqrt[b^
2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d
 + e*x^2]]/Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]))/(2*Sqrt[2]*c*Sq
rt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt
[2]) + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) - Sqrt[b^2
- 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])) - (a*e*(
Log[-(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x]/Sqrt[d + ((-(b/c) +
Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d + Sqrt[2]*Sqrt[-(b/c) + Sqrt[b^2 - 4*a
*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d + e*x^
```

```
2]]/Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]))/(Sqrt[2]*c*Sqrt[-(b/c)
 + Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + Sq
rt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/
c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])))/a
```

IntegrateAlgebraic [C]   time = 70.51, size = 348, normalized size = 1.20

$$-\frac{\sqrt{e}\,\text{RootSum}\left[\#1^8c+4\#1^6be-4\#1^6cd+16\#1^4ae^2-8\#1^4bde+6\#1^4cd^2+4\#1^2bd^2e-4\#1^2cd^3+cd^4\&,\frac{\#1^4cd\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-4\#1^2ae^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+4\#1^2bde\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2\#1^2cd^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+cd^3\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)}{\#1^6(-c)-3\#1^4be+3\#1^4cd-8\#1^2ae^2+4\#1^2bde-3\#1^2cd^2-bd^2e+cd^3}\&\right]}{2a}-\frac{\sqrt{d+ex^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[d + e*x^2]/(a*x)) - (Sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*
e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#
1^6 + c*#1^8 & , (c*d^3*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - 2*c*d^2*
Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 4*b*d*e*Log[-(Sqrt[e]*x) +
Sqrt[d + e*x^2] - #1]*#1^2 - 4*a*e^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #
1]*#1^2 + c*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4)/(c*d^3 - b*d^2
*e - 3*c*d^2*#1^2 + 4*b*d*e*#1^2 - 8*a*e^2*#1^2 + 3*c*d*#1^4 - 3*b*e*#1^4 -
 c*#1^6) & ])/(2*a)
```

fricas [B]   time = 7.22, size = 2402, normalized size = 8.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*a*x*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + (a^3*b
^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^
3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a^2*b*c
*d*e + (a^3*b^2*c - 4*a^4*c^2)*d*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c +
 a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) - 2*(a*b^2*c
- a^2*c^2)*d^2 + (4*a^2*b*c*e^2 + (b^3*c - a*b*c^2)*d^2 - (5*a*b^2*c - 4*a^
2*c^2)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^4*b^3 - 4*a^5*b*c)*x*sqrt
((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/
(a^6*b^2 - 4*a^7*c)) - ((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*
a^3*b*c)*e)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + (a^3*b^2 -
4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a
^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) - sqrt(1/2)*a
*x*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + (a^3*b^2 - 4*a^4*c)*sqr
t((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)
/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a^2*b*c*d*e + (a^3*b^2*c
 - 4*a^4*c^2)*d*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2
*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) - 2*(a*b^2*c - a^2*c^2)*d^2 +
(4*a^2*b*c*e^2 + (b^3*c - a*b*c^2)*d^2 - (5*a*b^2*c - 4*a^2*c^2)*d*e)*x^2 -
 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^4*b^3 - 4*a^5*b*c)*x*sqrt((a^2*b^2*e^2 + (
b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*
c)) - ((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e)*x)*sq
rt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + (a^3*b^2 - 4*a^4*c)*sqrt((a^
2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6
*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) - sqrt(1/2)*a*x*sqrt(-((b^3 -
3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 +
(b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7
*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a^2*b*c*d*e - (a^3*b^2*c - 4*a^4*c^2)*d*x
^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c
)*d*e)/(a^6*b^2 - 4*a^7*c)) - 2*(a*b^2*c - a^2*c^2)*d^2 + (4*a^2*b*c*e^2 +
(b^3*c - a*b*c^2)*d^2 - (5*a*b^2*c - 4*a^2*c^2)*d*e)*x^2 + 2*sqrt(1/2)*sqrt
(e*x^2 + d)*((a^4*b^3 - 4*a^5*b*c)*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c +
 a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) + ((a*b^4 - 5
```

```
*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e)*x)*sqrt(-((b^3 - 3*a*b
*c)*d - (a*b^2 - 2*a^2*c)*e - (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4
- 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))
/(a^3*b^2 - 4*a^4*c)))/x^2) + sqrt(1/2)*a*x*sqrt(-((b^3 - 3*a*b*c)*d - (a*b
^2 - 2*a^2*c)*e - (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c
+ a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 -
4*a^4*c))*log((2*a^2*b*c*d*e - (a^3*b^2*c - 4*a^4*c^2)*d*x^2*sqrt((a^2*b^2*
e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 -
 4*a^7*c)) - 2*(a*b^2*c - a^2*c^2)*d^2 + (4*a^2*b*c*e^2 + (b^3*c - a*b*c^2)
*d^2 - (5*a*b^2*c - 4*a^2*c^2)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^4
*b^3 - 4*a^5*b*c)*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2
*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) + ((a*b^4 - 5*a^2*b^2*c + 4*a^
3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 -
2*a^2*c)*e - (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2
*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4
*c)))/x^2) + 4*sqrt(e*x^2 + d))/(a*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.03, size = 272, normalized size = 0.93

$$\frac{\sqrt{e}\,\ln\left(-\sqrt{e}\,x+\sqrt{e x^2+d}\right)}{a}+\frac{\sqrt{e}\,\ln\left(\sqrt{e}\,x+\sqrt{e x^2+d}\right)}{a}+\frac{\sqrt{e}\left(\operatorname{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^2 cd+cd^3+2\left(-2ae^2+2deb-cd^2\right)\operatorname{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)\right)\ln\left(-\operatorname{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)+\left(-\sqrt{e}\,x+\sqrt{e x^2+d}\right)^2\right)}{2a\left(\operatorname{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^3 c+3\operatorname{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^2 be-3\operatorname{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^2 cd+8\operatorname{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)ae^2-4\operatorname{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)bde+3\operatorname{RootOf}\left(\_Z^4 c+c d^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)cd^2+bd^2e-cd^3\right)}+\frac{\sqrt{e x^2+d}\,ex}{ad}-\frac{\left(e x^2+d\right)^{\frac{3}{2}}}{adx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/a/d/x*(e*x^2+d)^(3/2)+1/a*e/d*x*(e*x^2+d)^(1/2)+1/a*e^(1/2)*ln(e^(1/2)*x
+(e*x^2+d)^(1/2))+1/2/a*e^(1/2)*sum((_R^2*c*d+2*(-2*a*e^2+2*b*d*e-c*d^2)*_R
+c*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^
2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4
*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))+1
/a*e^(1/2)*ln(-e^(1/2)*x+(e*x^2+d)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e x^2 + d}}{\left(c x^4 + b x^2 + a\right) x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x^2+d}}{x^2\,\left(c\,x^4+b\,x^2+a\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(x^2*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int((d + e*x^2)^(1/2)/(x^2*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(x**2*(a + b*x**2 + c*x**4)), x)
```

3.265 $\int \frac{\sqrt{d+ex^2}}{x^4\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=373

$$\frac{c\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{c\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

Rubi [A] time = 2.53, antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1295, 271, 264, 6728, 1692, 377, 205}

$$\frac{c\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{c\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{d+ex^2}(bd-ae)}{a^2dx}+\frac{2e\sqrt{d+ex^2}}{3adx}-\frac{\sqrt{d+ex^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -Sqrt[d + e*x^2]/(3*a*x^3) + (2*e*Sqrt[d + e*x^2])/(3*a*d*x) + ((b*d - a*e)
*Sqrt[d + e*x^2])/(a^2*d*x) + (c*(b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b
- Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sq
rt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (c*(b*d - a*e - (b^2*d - 2*a*c*d -
 a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*
x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1295

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
```

```
x] - Dist[1/(a*f^2), Int[((f*x)^(m + 2)*(d + e*x^2)^(q - 1)*Simp[b*d - a*e
+ c*d*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{x^4\left(a+bx^2+cx^4\right)}dx = -\frac{\int \frac{bd-ae+cdx^2}{x^2\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}dx}{a}+\frac{d\int \frac{1}{x^4\sqrt{d+ex^2}}dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3}-\frac{\int \left(\frac{bd-ae}{ax^2\sqrt{d+ex^2}}+\frac{-b^2d+acd+abe-c(bd-ae)x^2}{a\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right)dx}{a}-\frac{(2e)\int \frac{1}{x^2\sqrt{d+ex^2}}dx}{3a}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3}+\frac{2e\sqrt{d+ex^2}}{3adx}-\frac{\int \frac{-b^2d+acd+abe-c(bd-ae)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}dx}{a^2}-\frac{(bd-ae)\int \frac{1}{x^2\sqrt{d+ex^2}}dx}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3}+\frac{2e\sqrt{d+ex^2}}{3adx}+\frac{(bd-ae)\sqrt{d+ex^2}}{a^2dx}-\frac{\int \left(\frac{-c(bd-ae)-\frac{c\left(b^2d-2acd-abe\right)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}+\frac{-c(bd-}{\left(b+\sqrt{b^2\right.}}\right.}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3}+\frac{2e\sqrt{d+ex^2}}{3adx}+\frac{(bd-ae)\sqrt{d+ex^2}}{a^2dx}+\frac{\left(c\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\right)\int \frac{}{\left(b+\sqrt{\right.}}}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3}+\frac{2e\sqrt{d+ex^2}}{3adx}+\frac{(bd-ae)\sqrt{d+ex^2}}{a^2dx}+\frac{\left(c\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3}+\frac{2e\sqrt{d+ex^2}}{3adx}+\frac{(bd-ae)\sqrt{d+ex^2}}{a^2dx}+\frac{c\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\right.}}$$

Mathematica [B] time = 6.39, size = 7777, normalized size = 20.85

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d + e*x^2]/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [C] time = 111.74, size = 479, normalized size = 1.28

$$\frac{\sqrt{d+ex^2}\left(-ad-aex^2+3bdx^2\right)}{3a^2dx^3}-\frac{\sqrt{e}\,\text{RootSum}\left[\#1^8c+4\#1^6be-4\#1^6cd+16\#1^4ae^2-8\#1^4bde+6\#1^4cd^2+4\#1^2bd^2e-4\#1^2cd^3+cd^4\&,\frac{-\#1^4ace\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+\#1^4bcd\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-4\#1^2abe^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2\#1^2acde\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+4\#1^2b^2de\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2\#1^2bcd^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-acd^2e\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+bcd^3\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)}{\#1^6c+3\#1^4be-3\#1^4cd+8\#1^2ae^2-4\#1^2bde+3\#1^2cd^2+bd^2e-cd^3}\&\right]}{2a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x^2]/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[d + e*x^2]*(-(a*d) + 3*b*d*x^2 - a*e*x^2))/(3*a^2*d*x^3) - (Sqrt[e]*R
ootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4
+ 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (b*c*d^3*Log[-(Sqrt[
e]*x) + Sqrt[d + e*x^2] - #1] - a*c*d^2*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2
] - #1] - 2*b*c*d^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 4*b^2*d
*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 - 2*a*c*d*e*Log[-(Sqrt[e]*
x) + Sqrt[d + e*x^2] - #1]*#1^2 - 4*a*b*e^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x
^2] - #1]*#1^2 + b*c*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4 - a*c*
e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4)/(-(c*d^3) + b*d^2*e + 3*c*
d^2*#1^2 - 4*b*d*e*#1^2 + 8*a*e^2*#1^2 - 3*c*d*#1^4 + 3*b*e*#1^4 + c*#1^6)
& ])/(2*a^2)
```

fricas [B] time = 31.16, size = 4095, normalized size = 10.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*sqrt(1/2)*a^2*d*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b
^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*
c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c
+ 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2
)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(((a^5*b^2*c^2 - 4*a
^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^
2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) + 2*(a*b^4
*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^2 - 2*(a^2*b^3*c^2 - 2*a^3*b*c^3)*d*e - (
(b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2 - (5*a*b^4*c^2 - 14*a^2*b^2*c^3 + 4
*a^3*c^4)*d*e + 4*(a^2*b^3*c^2 - 2*a^3*b*c^3)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e
*x^2 + d)*((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*x*sqrt(((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a
^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2
)/(a^10*b^2 - 4*a^11*c)) + ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b
*c^3)*d - (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e)*x)*sqrt(-((b^5 - 5*a*b
^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b^2 - 4*
a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d
^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 -
 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6
*c)))/x^2) - 3*sqrt(1/2)*a^2*d*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d
 - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6
*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2
*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*
b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(((a^5*b^2*c^
2 - 4*a^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^
3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*
e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) + 2
*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^2 - 2*(a^2*b^3*c^2 - 2*a^3*b*c^3)*
d*e - ((b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2 - (5*a*b^4*c^2 - 14*a^2*b^2*
c^3 + 4*a^3*c^4)*d*e + 4*(a^2*b^3*c^2 - 2*a^3*b*c^3)*e^2)*x^2 - 2*sqrt(1/2)
*sqrt(e*x^2 + d)*((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*x*sqrt(((b^8 - 6*a*b^
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*
c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c
```

```
^2)*e^2)/(a^10*b^2 - 4*a^11*c)) + ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^3*c^2 -
4*a^4*b*c^3)*d - (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e)*x)*sqrt(-((b^5
- 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b
^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^
2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2
- 4*a^6*c)))/x^2) + 3*sqrt(1/2)*a^2*d*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b
*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e + (a^5*b^2 - 4*a^6*c)*sqrt(((
b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7
- 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c +
 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(-((a^
5*b^2*c^2 - 4*a^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b
*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11
*c)) - 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^2 + 2*(a^2*b^3*c^2 - 2*a^3
*b*c^3)*d*e + ((b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2 - (5*a*b^4*c^2 - 14*
a^2*b^2*c^3 + 4*a^3*c^4)*d*e + 4*(a^2*b^3*c^2 - 2*a^3*b*c^3)*e^2)*x^2 + 2*s
qrt(1/2)*sqrt(e*x^2 + d)*((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*x*sqrt(((b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*
a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a
^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) - ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^
3*c^2 - 4*a^4*b*c^3)*d - (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e)*x)*sqrt
(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e
+ (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d
*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(
a^5*b^2 - 4*a^6*c)))/x^2) - 3*sqrt(1/2)*a^2*d*x^3*sqrt(-((b^5 - 5*a*b^3*c +
 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e + (a^5*b^2 - 4*a^6*c)
*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2
*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3
*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*l
og(-((a^5*b^2*c^2 - 4*a^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^
2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 -
 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2
- 4*a^11*c)) - 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^2 + 2*(a^2*b^3*c^2
 - 2*a^3*b*c^3)*d*e + ((b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2 - (5*a*b^4*c
^2 - 14*a^2*b^2*c^3 + 4*a^3*c^4)*d*e + 4*(a^2*b^3*c^2 - 2*a^3*b*c^3)*e^2)*x
^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*x*sqr
t(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*
b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4
*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) - ((a*b^7 - 7*a^2*b^5*c + 1
3*a^3*b^3*c^2 - 4*a^4*b*c^3)*d - (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e)
*x)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3
*c^2)*e + (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a
^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*
b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^1
1*c)))/(a^5*b^2 - 4*a^6*c)))/x^2) + 4*((3*b*d - a*e)*x^2 - a*d)*sqrt(e*x^2
+ d))/(a^2*d*x^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.04, size = 322, normalized size = 0.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/a^2*b/d/x*(e*x^2+d)^(3/2)-1/a^2*b*e/d*x*(e*x^2+d)^(1/2)-1/a^2*b*e^(1/2)*l
n(e^(1/2)*x+(e*x^2+d)^(1/2))+1/2/a^2*e^(1/2)*sum((c*(a*e-b*d)*_R^2+2*(2*a*b
*e^2+a*c*d*e-2*b^2*d*e+b*c*d^2)*_R+a*c*d^2*e-b*c*d^3)/(_R^3*c+3*_R^2*b*e-3*
_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x
+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*
b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))-1/a^2*e^(1/2)*b*ln(-e^(1/2)*x+
(e*x^2+d)^(1/2))-1/3/a/d/x^3*(e*x^2+d)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d}}{\left(cx^4 + bx^2 + a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x^2 + d}}{x^4\,\left(c\,x^4 + b\,x^2 + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(x^4*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int((d + e*x^2)^(1/2)/(x^4*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + ex^2}}{x^4\left(a + bx^2 + cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(x**4*(a + b*x**2 + c*x**4)), x)
```

3.266 $\int \frac{\sqrt{d+ex^2}}{x^6\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=512

$$-\frac{\sqrt{d+ex^2}\left(-abe-acd+b^2d\right)}{a^3dx}-\frac{2e\sqrt{d+ex^2}\,(bd-ae)}{3a^2d^2x}+\frac{\sqrt{d+ex^2}\,(bd-ae)}{3a^2dx^3}-\frac{c\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2\right.}{a^3\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-}}$$

Rubi [A] time = 4.94, antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1295, 271, 264, 6728, 1692, 377, 205}

$$-\frac{\sqrt{d+ex^2}\left(-abe-acd+b^2d\right)}{a^3dx}-\frac{c\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{a^3\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2e\sqrt{d+ex^2}\,(bd-ae)}{3a^2d^2x}+\frac{\sqrt{d+ex^2}\,(bd-ae)}{3a^2dx^3}-\frac{8e^2\sqrt{d+ex^2}}{15ad^2x}+\frac{4e\sqrt{d+ex^2}}{15adx^3}-\frac{\sqrt{d+ex^2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x^6*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -Sqrt[d + e*x^2]/(5*a*x^5) + (4*e*Sqrt[d + e*x^2])/(15*a*d*x^3) + ((b*d - a
*e)*Sqrt[d + e*x^2])/(3*a^2*d*x^3) - (8*e^2*Sqrt[d + e*x^2])/(15*a*d^2*x) -
 (2*e*(b*d - a*e)*Sqrt[d + e*x^2])/(3*a^2*d^2*x) - ((b^2*d - a*c*d - a*b*e)
*Sqrt[d + e*x^2])/(a^3*d*x) - (c*(b^2*d - a*c*d - a*b*e + (b^3*d - 3*a*b*c*
d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^3*Sq
rt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (c*(b^
2*d - a*c*d - a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b
^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1295

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[((f*x)^(m + 2)*(d + e*x^2)^(q - 1)*Simp[b*d - a*e
+ c*d*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{x^6\left(a+bx^2+cx^4\right)}\,dx = -\frac{\int \frac{bd-ae+cdx^2}{x^4\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{a}+\frac{d\int \frac{1}{x^6\sqrt{d+ex^2}}\,dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5}-\frac{\int \left(\frac{bd-ae}{ax^4\sqrt{d+ex^2}}+\frac{-b^2d+acd+abe}{a^2x^2\sqrt{d+ex^2}}+\frac{b^3d-2abcd-ab^2e+a^2ce+c\left(b^2d-acd-abe\right)x^2}{a^2\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right)dx}{a}-$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5}+\frac{4e\sqrt{d+ex^2}}{15adx^3}-\frac{\int \frac{b^3d-2abcd-ab^2e+a^2ce+c\left(b^2d-acd-abe\right)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{a^3}+\frac{\left(8e^2\right)\int \frac{1}{x^2\sqrt{d}}}{15ad}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5}+\frac{4e\sqrt{d+ex^2}}{15adx^3}+\frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3}-\frac{8e^2\sqrt{d+ex^2}}{15ad^2x}-\frac{\left(b^2d-acd-ab\right.}{a^3d}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5}+\frac{4e\sqrt{d+ex^2}}{15adx^3}+\frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3}-\frac{8e^2\sqrt{d+ex^2}}{15ad^2x}-\frac{2e(bd-ae)\sqrt{d}}{3a^2d^2x}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5}+\frac{4e\sqrt{d+ex^2}}{15adx^3}+\frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3}-\frac{8e^2\sqrt{d+ex^2}}{15ad^2x}-\frac{2e(bd-ae)\sqrt{d}}{3a^2d^2x}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5}+\frac{4e\sqrt{d+ex^2}}{15adx^3}+\frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3}-\frac{8e^2\sqrt{d+ex^2}}{15ad^2x}-\frac{2e(bd-ae)\sqrt{d}}{3a^2d^2x}$$

Mathematica [B] time = 6.59, size = 10933, normalized size = 21.35

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d + e*x^2]/(x^6*(a + b*x^2 + c*x^4)),x]
```

[Out] Result too large to show

IntegrateAlgebraic [C]   time = 160.94, size = 687, normalized size = 1.34

$$\frac{\sqrt{e}\,\text{RootSum}\left[\#1^8c+4\#1^6be-4\#1^6cd+16\#1^4ae^2-8\#1^4bde+6\#1^4cd^2+4\#1^2bd^2e-4\#1^2cd^3+cd^4\&,\frac{-\#1^4abce\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+\#1^4(-a)c^2d\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+\#1^4b^2cd\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+4\#1^2a^2ce^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-4\#1^2ab^2e^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-6\#1^2abcde\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+2\#1^2ac^2d^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+4\#1^2b^3de\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2\#1^2b^2cd^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-abcd^2e\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-ac^2d^3\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+b^2cd^3\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)}{\#1^6c+3\#1^4be-3\#1^4cd+8\#1^2ae^2-4\#1^2bde+3\#1^2cd^2+bd^2e-cd^3}\&\right]}{2a^3}+\frac{\sqrt{d+ex^2}\left(-3a^2d^2-a^2dex^2+2a^2e^2x^4+5abd^2x^2+5abdex^4+15acd^2x^4-15b^2d^2x^4\right)}{15a^3d^2x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x^2]/(x^6*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[d + e*x^2]*(-3*a^2*d^2 + 5*a*b*d^2*x^2 - a^2*d*e*x^2 - 15*b^2*d^2*x^4
 + 15*a*c*d^2*x^4 + 5*a*b*d*e*x^4 + 2*a^2*e^2*x^4))/(15*a^3*d^2*x^5) + (Sqr
t[e]*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e
*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (b^2*c*d^3*Log
[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - a*c^2*d^3*Log[-(Sqrt[e]*x) + Sqrt[d
 + e*x^2] - #1] - a*b*c*d^2*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - 2*
b^2*c*d^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 2*a*c^2*d^2*Log[-
(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 4*b^3*d*e*Log[-(Sqrt[e]*x) + Sqr
t[d + e*x^2] - #1]*#1^2 - 6*a*b*c*d*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] -
#1]*#1^2 - 4*a*b^2*e^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 4*a^
2*c*e^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + b^2*c*d*Log[-(Sqrt[
e]*x) + Sqrt[d + e*x^2] - #1]*#1^4 - a*c^2*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*
x^2] - #1]*#1^4 - a*b*c*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4)/(-
(c*d^3) + b*d^2*e + 3*c*d^2*#1^2 - 4*b*d*e*#1^2 + 8*a*e^2*#1^2 - 3*c*d*#1^4
 + 3*b*e*#1^4 + c*#1^6) & ])/(2*a^3)
```

fricas [B]   time = 77.19, size = 5773, normalized size = 11.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/60*(15*sqrt(1/2)*a^3*d^2*x^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 -
7*a^3*b*c^3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e - (a^7
*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c
 + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a
^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^
2)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))*log(-((a^7*b^2*c^3 - 4*a^8*
c^4)*d*x^2*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46
*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29
*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^
10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a
^14*b^2 - 4*a^15*c)) + 2*(a*b^6*c^3 - 5*a^2*b^4*c^4 + 6*a^3*b^2*c^5 - a^4*c
^6)*d^2 - 2*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e - ((b^7*c^3 - 5
*a*b^5*c^4 + 6*a^2*b^3*c^5 - a^3*b*c^6)*d^2 - (5*a*b^6*c^3 - 24*a^2*b^4*c^4
 + 27*a^3*b^2*c^5 - 4*a^4*c^6)*d*e + 4*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4
*b*c^5)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^8*b^5 - 7*a^9*b^3*c + 12
*a^10*b*c^2)*x*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
+ 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c
+ 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^
2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2
)/(a^14*b^2 - 4*a^15*c)) + ((a*b^10 - 10*a^2*b^8*c + 35*a^3*b^6*c^2 - 51*a^
4*b^4*c^3 + 29*a^5*b^2*c^4 - 4*a^6*c^5)*d - (a^2*b^9 - 9*a^3*b^7*c + 27*a^4
*b^5*c^2 - 31*a^5*b^3*c^3 + 12*a^6*b*c^4)*e)*x)*sqrt(-((b^7 - 7*a*b^5*c + 1
4*a^2*b^3*c^2 - 7*a^3*b*c^3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a
^4*c^3)*e - (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2
- 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^
11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6
*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9
```

```
*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c)))/x^2) - 15*
sqrt(1/2)*a^3*d^2*x^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^
3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e - (a^7*b^2 - 4*a
^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*
b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*
b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 -
8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b
^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))*log(-((a^7*b^2*c^3 - 4*a^8*c^4)*d*x^2
*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c
^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c
^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3
*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 -
4*a^15*c)) + 2*(a*b^6*c^3 - 5*a^2*b^4*c^4 + 6*a^3*b^2*c^5 - a^4*c^6)*d^2 -
2*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e - ((b^7*c^3 - 5*a*b^5*c^4
 + 6*a^2*b^3*c^5 - a^3*b*c^6)*d^2 - (5*a*b^6*c^3 - 24*a^2*b^4*c^4 + 27*a^3*
b^2*c^5 - 4*a^4*c^6)*d*e + 4*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*e^
2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^
2)*x*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b
^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b
^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8
*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^
2 - 4*a^15*c)) + ((a*b^10 - 10*a^2*b^8*c + 35*a^3*b^6*c^2 - 51*a^4*b^4*c^3
+ 29*a^5*b^2*c^4 - 4*a^6*c^5)*d - (a^2*b^9 - 9*a^3*b^7*c + 27*a^4*b^5*c^2 -
 31*a^5*b^3*c^3 + 12*a^6*b*c^4)*e)*x)*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*
c^2 - 7*a^3*b*c^3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e
- (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b
^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2
*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*
e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c
^4)*e^2)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c)))/x^2) + 15*sqrt(1/2)*
a^3*d^2*x^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d - (a*
b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e + (a^7*b^2 - 4*a^8*c)*sqrt
(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 -
12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 -
40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*
c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^1
5*c)))/(a^7*b^2 - 4*a^8*c))*log(((a^7*b^2*c^3 - 4*a^8*c^4)*d*x^2*sqrt(((b^1
2 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5
*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4
*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22
*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c))
- 2*(a*b^6*c^3 - 5*a^2*b^4*c^4 + 6*a^3*b^2*c^5 - a^4*c^6)*d^2 + 2*(a^2*b^5*
c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e + ((b^7*c^3 - 5*a*b^5*c^4 + 6*a^2*b^
3*c^5 - a^3*b*c^6)*d^2 - (5*a*b^6*c^3 - 24*a^2*b^4*c^4 + 27*a^3*b^2*c^5 - 4
*a^4*c^6)*d*e + 4*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*e^2)*x^2 + 2*
sqrt(1/2)*sqrt(e*x^2 + d)*((a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^2)*x*sqrt((
(b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12
*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40
*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c
+ 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*
c)) - ((a*b^10 - 10*a^2*b^8*c + 35*a^3*b^6*c^2 - 51*a^4*b^4*c^3 + 29*a^5*b^
2*c^4 - 4*a^6*c^5)*d - (a^2*b^9 - 9*a^3*b^7*c + 27*a^4*b^5*c^2 - 31*a^5*b^3
*c^3 + 12*a^6*b*c^4)*e)*x)*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3
*b*c^3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e + (a^7*b^2
- 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46
*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29
*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^
10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a
^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c)))/x^2) - 15*sqrt(1/2)*a^3*d^2*x^5
```

```
*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d - (a*b^6 - 6*a^2
*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e + (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 1
0*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*
c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*
c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*
b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)))/(a^7
*b^2 - 4*a^8*c))*log(((a^7*b^2*c^3 - 4*a^8*c^4)*d*x^2*sqrt(((b^12 - 10*a*b^
10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 +
a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 +
22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^
2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)) - 2*(a*b^6*
c^3 - 5*a^2*b^4*c^4 + 6*a^3*b^2*c^5 - a^4*c^6)*d^2 + 2*(a^2*b^5*c^3 - 4*a^3
*b^3*c^4 + 3*a^4*b*c^5)*d*e + ((b^7*c^3 - 5*a*b^5*c^4 + 6*a^2*b^3*c^5 - a^3
*b*c^6)*d^2 - (5*a*b^6*c^3 - 24*a^2*b^4*c^4 + 27*a^3*b^2*c^5 - 4*a^4*c^6)*d
*e + 4*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*e^2)*x^2 - 2*sqrt(1/2)*s
qrt(e*x^2 + d)*((a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^2)*x*sqrt(((b^12 - 10*
a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^
5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^
3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^
6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)) - ((a*b
^10 - 10*a^2*b^8*c + 35*a^3*b^6*c^2 - 51*a^4*b^4*c^3 + 29*a^5*b^2*c^4 - 4*a
^6*c^5)*d - (a^2*b^9 - 9*a^3*b^7*c + 27*a^4*b^5*c^2 - 31*a^5*b^3*c^3 + 12*a
^6*b*c^4)*e)*x)*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d -
 (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e + (a^7*b^2 - 4*a^8*c)*
sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^
4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^
2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*
b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4
*a^15*c)))/(a^7*b^2 - 4*a^8*c)))/x^2) - 4*((5*a*b*d*e + 2*a^2*e^2 - 15*(b^2
 - a*c)*d^2)*x^4 - 3*a^2*d^2 + (5*a*b*d^2 - a^2*d*e)*x^2)*sqrt(e*x^2 + d))/
(a^3*d^2*x^5)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.04, size = 503, normalized size = 0.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/a^2/d/x*(e*x^2+d)^(3/2)*c-1/a^3/d/x*(e*x^2+d)^(3/2)*b^2-1/a^2*e/d*x*(e*x^
2+d)^(1/2)*c+1/a^3*e/d*x*(e*x^2+d)^(1/2)*b^2-1/a^2*e^(1/2)*ln(e^(1/2)*x+(e*
x^2+d)^(1/2))*c+1/a^3*e^(1/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))*b^2-1/2/a^3*e^(
1/2)*sum((c*(a*b*e+a*c*d-b^2*d)*_R^2+2*(-2*a^2*c*e^2+2*a*b^2*e^2+3*a*b*c*d*
e-a*c^2*d^2-2*b^3*d*e+b^2*c*d^2)*_R+a*b*c*d^2*e+a*c^2*d^3-b^2*c*d^3)/(_R^3*
c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(
-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z
^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))-1/a^2*e^(1/2)*l
n(-e^(1/2)*x+(e*x^2+d)^(1/2))*c+1/a^3*e^(1/2)*ln(-e^(1/2)*x+(e*x^2+d)^(1/2)
)*b^2+1/3/a^2*b/d/x^3*(e*x^2+d)^(3/2)-1/5/a/d/x^5*(e*x^2+d)^(3/2)+2/15/a*e/
d^2/x^3*(e*x^2+d)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}}{(cx^4+bx^2+a)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x^2+d}}{x^6\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(x^6*(a + b*x^2 + c*x^4)),x)
[Out] int((d + e*x^2)^(1/2)/(x^6*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{x^6\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x**6/(c*x**4+b*x**2+a),x)
[Out] Integral(sqrt(d + e*x**2)/(x**6*(a + b*x**2 + c*x**4)), x)
```

3.267 $\int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=460

$$\frac{\left(bc\left(e\left(2d\sqrt{b^2-4ac}-3ae\right)+cd^2\right)+c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)\right)-b^2e\left(e\sqrt{b^2-4ac}+2cd\right)+b^3e^2\right.}{\sqrt{2}\,c^{5/2}\sqrt{b^2-4ac}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Rubi [A] time = 5.08, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 824, 826, 1166, 208}

$$\frac{\left(bc\left(e\left(2d\sqrt{b^2-4ac}-3ae\right)+cd^2\right)+c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)\right)-b^2e\left(e\sqrt{b^2-4ac}+2cd\right)+b^3e^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b^2-4ac}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(bc\left(cd^2-e\left(2d\sqrt{b^2-4ac}+3ae\right)\right)-c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}+4ae\right)\right)-b^2e\left(2cd-e\sqrt{b^2-4ac}\right)+b^3e^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b^2-4ac}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{d+ex^2}\,(cd-be)}{c^2}+\frac{\left(d+ex^2\right)^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d - b*e)*Sqrt[d + e*x^2])/c^2 + (d + e*x^2)^(3/2)/(3*c) + ((b^3*e^2 - b
^2*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + c*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt
[b^2 - 4*a*c]*d - 4*a*e)) + b*c*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - 3*a*e))
)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*
a*c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 -
4*a*c])*e]) - ((b^3*e^2 - b^2*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b*c*(c*d^2
- e*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e)) - c*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sq
rt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3\left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{\left(d+ex^2\right)^{3/2}}{3c} + \frac{\mathrm{Subst}\left(\int \frac{\sqrt{d+ex}\,(-ae+(cd-be)x)}{a+bx+cx^2}\,dx, x, x^2\right)}{2c}$$

$$= \frac{(cd-be)\sqrt{d+ex^2}}{c^2} + \frac{\left(d+ex^2\right)^{3/2}}{3c} + \frac{\mathrm{Subst}\left(\int \frac{-ae(2cd-be)+\left(c^2d^2+b^2e^2-ce(2bd+ae)\right)x}{\sqrt{d+ex}\,\left(a+bx+cx^2\right)}\,dx, x, x^2\right)}{2c^2}$$

$$= \frac{(cd-be)\sqrt{d+ex^2}}{c^2} + \frac{\left(d+ex^2\right)^{3/2}}{3c} + \frac{\mathrm{Subst}\left(\int \frac{-ae^2(2cd-be)-d\left(c^2d^2+b^2e^2-ce(2bd+ae)\right)+\left(c^2d^2+b^2e^2 \cdots\right.}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4} \cdots\right.}{c^2}$$

$$= \frac{(cd-be)\sqrt{d+ex^2}}{c^2} + \frac{\left(d+ex^2\right)^{3/2}}{3c} - \frac{\left(b^3e^2-b^2e\left(2cd+\sqrt{b^2-4ac}\,e\right)+c\left(a\sqrt{b^2-4ac}\,e^{\cdots}\right.\right.}{\cdots}$$

$$= \frac{(cd-be)\sqrt{d+ex^2}}{c^2} + \frac{\left(d+ex^2\right)^{3/2}}{3c} + \frac{\left(b^3e^2-b^2e\left(2cd+\sqrt{b^2-4ac}\,e\right)+c\left(a\sqrt{b^2-4ac}\,e \cdots\right.\right.}{\cdots}$$

Mathematica [A] time = 0.97, size = 457, normalized size = 0.99

$$-\frac{\left(-bc\left(e\left(2d\sqrt{b^2-4ac}-3ae\right)+cd^2\right)+c\left(cd\left(d\sqrt{b^2-4ac}-4ae\right)-ae^2\sqrt{b^2-4ac}\right)+b^2e\left(e\sqrt{b^2-4ac}+2cd\right)+b^3\left(-e^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{\left(bc\left(cd^2-e\left(2d\sqrt{b^2-4ac}+3ae\right)\right)+c\left(cd\left(d\sqrt{b^2-4ac}+4ae\right)-ae^2\sqrt{b^2-4ac}\right)+b^2e\left(e\sqrt{b^2-4ac}-2cd\right)+b^3e^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{d+ex^2}\,(cd-be)}{c^2}+\frac{\left(d+ex^2\right)^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d - b*e)*Sqrt[d + e*x^2])/c^2 + (d + e*x^2)^(3/2)/(3*c) - ((-(b^3*e^2)
+ b^2*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + c*(-(a*Sqrt[b^2 - 4*a*c]*e^2) + c*d
*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) - b*c*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - 3
*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b
^2 - 4*a*c]*e]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt
[b^2 - 4*a*c])*e]) - ((b^3*e^2 + b^2*e*(-2*c*d + Sqrt[b^2 - 4*a*c]*e) + b*c
*(c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e)) + c*(-(a*Sqrt[b^2 - 4*a*c]*e^2
) + c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e
*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2
- 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

IntegrateAlgebraic [C] time = 2.12, size = 524, normalized size = 1.14

$$\frac{\left(c^2d^2\sqrt{4ac-b^2}-2bcde\sqrt{4ac-b^2}+b^2e^2\sqrt{4ac-b^2}-ace^2\sqrt{4ac-b^2}-3iabce^2+4iac^2de+ib^3e^2-2ib^2cde+ibc^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}\right)}{\sqrt{2}c^{5/2}\sqrt{4ac-b^2}\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}+\frac{\left(c^2d^2\sqrt{4ac-b^2}-2bcde\sqrt{4ac-b^2}+b^2e^2\sqrt{4ac-b^2}-ace^2\sqrt{4ac-b^2}+3iabce^2-4iac^2de-ib^3e^2+2ib^2cde-ibc^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}\right)}{\sqrt{2}c^{5/2}\sqrt{4ac-b^2}\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}+\frac{\sqrt{d+ex^2}\left(-3be+4cd+cex^2\right)}{3c^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[d + e*x^2]*(4*c*d - 3*b*e + c*e*x^2))/(3*c^2) + ((I*b*c^2*d^2 + c^2*S
qrt[-b^2 + 4*a*c]*d^2 - (2*I)*b^2*c*d*e + (4*I)*a*c^2*d*e - 2*b*c*Sqrt[-b^2
 + 4*a*c]*d*e + I*b^3*e^2 - (3*I)*a*b*c*e^2 + b^2*Sqrt[-b^2 + 4*a*c]*e^2 -
a*c*Sqrt[-b^2 + 4*a*c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-
2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[2]*c^(5/2)*Sqrt[-b^2 + 4*a*c]
*Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]) + (((-I)*b*c^2*d^2 + c^2*Sqrt
[-b^2 + 4*a*c]*d^2 + (2*I)*b^2*c*d*e - (4*I)*a*c^2*d*e - 2*b*c*Sqrt[-b^2 +
4*a*c]*d*e - I*b^3*e^2 + (3*I)*a*b*c*e^2 + b^2*Sqrt[-b^2 + 4*a*c]*e^2 - a*c
*Sqrt[-b^2 + 4*a*c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c
*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[2]*c^(5/2)*Sqrt[-b^2 + 4*a*c]*Sq
rt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.25, size = 857, normalized size = 1.86

$$-\frac{\left(2 b c^{5} d^{3} - \left(5 b^{2} c^{4} - 8 a c^{5}\right) d^{2} e + \left(\left(b^{2} c^{2} - 4 a c^{3}\right) d^{2} e - 2 \left(b^{3} c - 4 a b c^{2}\right) d e^{2} + \left(b^{4} - 5 a b^{2} c + 4 a^{2} c^{2}\right) e^{3}\right) c^{2} + 2 \left(2 b^{3} c^{3} - 5 a b c^{4}\right) d e^{2} - 2 \left(\sqrt{b^{2} - 4 a c} c^{4} d^{3} - 2 \sqrt{b^{2} - 4 a c} b c^{3} d^{2} e - \sqrt{b^{2} - 4 a c} a b c^{2} e^{3} + \left(b^{2} c^{2} + a c^{3}\right) \sqrt{b^{2} - 4 a c} d e^{2}\right) |c| - \left(b^{4} c^{2} - 3 a b^{2} c^{3}\right) e^{3}\right) \arctan\left(\frac{2 \sqrt{\frac{1}{2}} \sqrt{x^{2} e + d}}{\sqrt{-\frac{2 c^{4} d - b c^{3} e + \sqrt{-4 \left(c^{4} d^{2} - b c^{3} d e + a c^{3} e^{2}\right) c^{4} + \left(2 c^{4} d - b c^{3} e\right)^{2}}}{c^{4}}}}\right)}{\left(2 \sqrt{b^{2} - 4 a c} c^{3} d - \left(b^{2} c^{2} - 4 a c^{3} + \sqrt{b^{2} - 4 a c} b c^{2}\right) e\right) \sqrt{-4 c^{2} d + 2 \left(b c - \sqrt{b^{2} - 4 a c} c\right) e} c^{2}} + \frac{\left(2 b c^{5} d^{3} - \left(5 b^{2} c^{4} - 8 a c^{5}\right) d^{2} e + \left(\left(b^{2} c^{2} - 4 a c^{3}\right) d^{2} e - 2 \left(b^{3} c - 4 a b c^{2}\right) d e^{2} + \left(b^{4} - 5 a b^{2} c + 4 a^{2} c^{2}\right) e^{3}\right) c^{2} + 2 \left(2 b^{3} c^{3} - 5 a b c^{4}\right) d e^{2} + 2 \left(\sqrt{b^{2} - 4 a c} c^{4} d^{3} - 2 \sqrt{b^{2} - 4 a c} b c^{3} d^{2} e - \sqrt{b^{2} - 4 a c} a b c^{2} e^{3} + \left(b^{2} c^{2} + a c^{3}\right) \sqrt{b^{2} - 4 a c} d e^{2}\right) |c| - \left(b^{4} c^{2} - 3 a b^{2} c^{3}\right) e^{3}\right) \arctan\left(\frac{2 \sqrt{\frac{1}{2}} \sqrt{x^{2} e + d}}{\sqrt{-\frac{2 c^{4} d - b c^{3} e - \sqrt{-4 \left(c^{4} d^{2} - b c^{3} d e + a c^{3} e^{2}\right) c^{4} + \left(2 c^{4} d - b c^{3} e\right)^{2}}}{c^{4}}}}\right)}{\left(2 \sqrt{b^{2} - 4 a c} c^{3} d + \left(b^{2} c^{2} - 4 a c^{3} - \sqrt{b^{2} - 4 a c} b c^{2}\right) e\right) \sqrt{-4 c^{2} d + 2 \left(b c + \sqrt{b^{2} - 4 a c} c\right) e} c^{2}} + \frac{{\left(x^{2} e + d\right)}^{\frac{3}{2}} c^{2} + 3 \sqrt{x^{2} e + d} c^{2} d - 3 \sqrt{x^{2} e + d} b c e}{3 c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -(2*b*c^5*d^3 - (5*b^2*c^4 - 8*a*c^5)*d^2*e + ((b^2*c^2 - 4*a*c^3)*d^2*e -
2*(b^3*c - 4*a*b*c^2)*d*e^2 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^3)*c^2 + 2*(2
*b^3*c^3 - 5*a*b*c^4)*d*e^2 - 2*(sqrt(b^2 - 4*a*c)*c^4*d^3 - 2*sqrt(b^2 - 4
*a*c)*b*c^3*d^2*e - sqrt(b^2 - 4*a*c)*a*b*c^2*e^3 + (b^2*c^2 + a*c^3)*sqrt(
b^2 - 4*a*c)*d*e^2)*abs(c) - (b^4*c^2 - 3*a*b^2*c^3)*e^3)*arctan(2*sqrt(1/2
)*sqrt(x^2*e + d)/sqrt(-(2*c^4*d - b*c^3*e + sqrt(-4*(c^4*d^2 - b*c^3*d*e +
 a*c^3*e^2)*c^4 + (2*c^4*d - b*c^3*e)^2))/c^4))/((2*sqrt(b^2 - 4*a*c)*c^3*d
 - (b^2*c^2 - 4*a*c^3 + sqrt(b^2 - 4*a*c)*b*c^2)*e)*sqrt(-4*c^2*d + 2*(b*c
- sqrt(b^2 - 4*a*c)*c)*e)*c^2) + (2*b*c^5*d^3 - (5*b^2*c^4 - 8*a*c^5)*d^2*e
 + ((b^2*c^2 - 4*a*c^3)*d^2*e - 2*(b^3*c - 4*a*b*c^2)*d*e^2 + (b^4 - 5*a*b^
2*c + 4*a^2*c^2)*e^3)*c^2 + 2*(2*b^3*c^3 - 5*a*b*c^4)*d*e^2 + 2*(sqrt(b^2 -
 4*a*c)*c^4*d^3 - 2*sqrt(b^2 - 4*a*c)*b*c^3*d^2*e - sqrt(b^2 - 4*a*c)*a*b*c
^2*e^3 + (b^2*c^2 + a*c^3)*sqrt(b^2 - 4*a*c)*d*e^2)*abs(c) - (b^4*c^2 - 3*a
*b^2*c^3)*e^3)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*c^4*d - b*c^3*e
- sqrt(-4*(c^4*d^2 - b*c^3*d*e + a*c^3*e^2)*c^4 + (2*c^4*d - b*c^3*e)^2))/c
^4))/((2*sqrt(b^2 - 4*a*c)*c^3*d + (b^2*c^2 - 4*a*c^3 - sqrt(b^2 - 4*a*c)*b
*c^2)*e)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2) + 1/3*((x^2*
e + d)^(3/2)*c^2 + 3*sqrt(x^2*e + d)*c^2*d - 3*sqrt(x^2*e + d)*b*c*e)/c^3
```

maple [C] time = 0.03, size = 490, normalized size = 1.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/6/c*e^(3/2)*x^3+1/8/c*e*(e*x^2+d)^(1/2)*x^2-3/4/c*e^(1/2)*x*d+1/24*(e*x^
2+d)^(3/2)/c+1/2/c^2*e^(3/2)*x*b-1/2/c^2*(e*x^2+d)^(1/2)*b*e+5/8/c*(e*x^2+d
)^(1/2)*d+1/4/c^2*sum(((-a*c*e^2+b^2*e^2-2*b*c*d*e+c^2*d^2)*_R^6+(4*a*b*e^3
```

```
-5*a*c*d*e^2-3*b^2*d*e^2+6*b*c*d^2*e-3*c^2*d^3)*_R^4+d*(-4*a*b*e^3+5*a*c*d*
e^2+3*b^2*d*e^2-6*b*c*d^2*e+3*c^2*d^3)*_R^2+a*c*d^3*e^2-b^2*d^3*e^2+2*b*c*d
^4*e-c^2*d^5)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^
3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln(-e^(1/2)*x-_R+(e*x^2+d)^(1/2)),_R=RootOf(_Z
^8*c+(4*b*e-4*c*d)*_Z^6+c*d^4+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*
c*d^3)*_Z^2))-1/2/c^2*d/(-e^(1/2)*x+(e*x^2+d)^(1/2))*b*e+5/8/c*d^2/(-e^(1/2
)*x+(e*x^2+d)^(1/2))+1/24/c*d^3/(-e^(1/2)*x+(e*x^2+d)^(1/2))^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}x^3}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^(3/2)*x^3/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 3.41, size = 16951, normalized size = 36.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x)
[Out] (d + e*x^2)^(3/2)/(3*c) - atan(((((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*
a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*
b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b^2*c^4*d*e^4)/c^3 - (2*(d + e*
x^2)^(1/2)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d
^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c
^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^
2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 +
 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2
*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^
2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3
 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 4
8*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6
*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e
 - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*
(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*(
-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*
b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*
a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 14
4*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5
- 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3
 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*
e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^
5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*
e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2
 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b
^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*(d + e*
x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 +
 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e
^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^
5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(((4*b^7*e^3 - 32*a^
2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^
4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*
c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 2
16*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5
*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4
```

```
+ 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3
 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3
 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e
 + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e
 + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16
*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((4*a*b^3*c^3*e^5 - 16*a^2*
b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*d*e^4 - 4*b^2*c
^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b^2*c^4*d*e^4)/c
^3 + (2*(d + e*x^2)^(1/2)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 +
 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e
+ 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*
e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 -
 (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*
c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4
*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1
/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^
3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*
a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*
a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^
2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7
*d*e^2))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^
4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^
3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4
*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^
7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 -
a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3
*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*
e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3
+ 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3
- 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^
2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/
2) + (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^
2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 +
6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20
*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(((4*
b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*
e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*
c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*
b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*
a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a
^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*
a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3
- 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 +
6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*
a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3
*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((((4*a*b^3*c^
3*e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*
d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b
^2*c^4*d*e^4)/c^3 - (2*(d + e*x^2)^(1/2)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 -
4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12
*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84
*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c
^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c
^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*
d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2
*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*
c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^
3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*
c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b
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^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6
*e^3 + 32*a*c^7*d*e^2))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^
3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2
*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d
^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/
4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a
^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*
a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))
^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40
*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 -
18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 +
72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a
*b^2*c^6)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5
*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3
*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c
^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4
))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3
- 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*
e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d
*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16
*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^
3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b
*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 -
16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a
^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^
6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e -
108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (
((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4
- 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*
e^3 + 12*a*b^2*c^4*d*e^4)/c^3 + (2*(d + e*x^2)^(1/2)*(-(((4*b^7*e^3 - 32*a^
2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^
4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*
c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 2
16*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5
*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4
+ 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3
 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3
 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e
 + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e
 + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16
*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2
- 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 -
4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12
*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84
*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c
^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c
^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*
d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2
*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*
c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^
3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*
c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b
^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e
^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4
*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^
5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2
*c^3*d^2*e^4))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*
b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*
a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96
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*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*
a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*
e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e
^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) +
2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^
3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*
c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*
c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)
))^(1/2) + (2*(a^4*c*e^8 - a^3*b^2*e^8 - a*b^4*d^2*e^6 + 2*a^2*b^3*d*e^7 -
a*c^4*d^6*e^2 - a^2*c^3*d^4*e^4 + a^3*c^2*d^2*e^6 + 4*a*b*c^3*d^5*e^3 + 4*a
*b^3*c*d^3*e^5 - 6*a*b^2*c^2*d^4*e^4 + 4*a^2*b*c^2*d^3*e^5 - 5*a^2*b^2*c*d^
2*e^6))/c^3))*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^
4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^
3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4
*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^
7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 -
a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3
*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*
e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3
+ 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3
- 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^
2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/
2)*2i - (d + e*x^2)^(1/2)*(d/c + (b*e - 2*c*d)/c^2) - atan(((((4*a*b^3*c^3*
e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*d*
e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b^2
*c^4*d*e^4)/c^3 - (2*(d + e*x^2)^(1/2)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b
^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^
5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*
b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*
d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*
d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4
*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*
d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4
*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c
^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2
*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*
c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^
3 + 32*a*c^7*d*e^2))/c^3)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 +
24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e +
 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e
 + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 -
(256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c
*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*
b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/
2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3
*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a
*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a
^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2
*c^6)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4
*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3
*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d
^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c
^3)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*
a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 -
 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2
+ 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*
c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3
*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*
d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^
```

2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^
4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d
*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a
^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((
4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 -
4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^
3 + 12*a*b^2*c^4*d*e^4)/c^3 + (2*(d + e*x^2)^(1/2)*((((4*b^7*e^3 - 32*a^2*c
^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d
*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d
*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*
a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^
6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3
*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 +
3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 -
12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e -
50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e -
48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^
2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 1
6*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^
4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5
*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b
^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d
*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d
^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*
e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d
^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*
d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^
2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*
d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c
^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 -
2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2
- 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 +
12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3
*d^2*e^4))/c^3)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c
^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b
^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^
4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c
^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 -
a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 -
3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7
*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3
- 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3
+ 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d
^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1
/2)*1i)/((((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*
c^5*d*e^4 - 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*
b*c^5*d^2*e^3 + 12*a*b^2*c^4*d*e^4)/c^3 - (2*(d + e*x^2)^(1/2)*((((4*b^7*e^
3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 +
96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3
- 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*
d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*
c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2
*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*
c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^
4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*
c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*
c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2
)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c
^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*((((4*b^7*e^3 - 32*a^2*c^

5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b^2*c^4*d*e^4)/c^3 + (2*(d + e*x^2)^(1/2)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^

```
6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e
^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^
4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d
^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2
*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d
*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^
5 - 8*a*b^2*c^6)))^(1/2) + (2*(a^4*c*e^8 - a^3*b^2*e^8 - a*b^4*d^2*e^6 + 2*
a^2*b^3*d*e^7 - a*c^4*d^6*e^2 - a^2*c^3*d^4*e^4 + a^3*c^2*d^2*e^6 + 4*a*b*c
^3*d^5*e^3 + 4*a*b^3*c*d^3*e^5 - 6*a*b^2*c^2*d^4*e^4 + 4*a^2*b*c^2*d^3*e^5
- 5*a^2*b^2*c*d^2*e^6))/c^3))*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^
3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2
*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d
^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/
4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a
^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*
a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))
^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40
*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 +
18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 -
72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a
*b^2*c^6)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

3.268 $\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=327

$$-\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(-2ce\left(d\sqrt{b^2-4a\right.\right.}{}$$

Rubi [A] time = 1.46, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1247, 703, 826, 1166, 208}

$$-\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{e\sqrt{d+ex^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e*Sqrt[d + e*x^2])/c - ((2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e
*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2
])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*
a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((2*c^2*d^2 + b*(b + Sqrt[b
^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[
2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= \frac{e\sqrt{d+ex^2}}{c} + \frac{\operatorname{Subst}\left(\int \frac{cd^2-ae^2+e(2cd-be)x}{\sqrt{d+ex}\left(a+bx+cx^2\right)}\,dx, x, x^2\right)}{2c}\\
&= \frac{e\sqrt{d+ex^2}}{c} + \frac{\operatorname{Subst}\left(\int \frac{-de(2cd-be)+e\left(cd^2-ae^2\right)+e(2cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{c}\\
&= \frac{e\sqrt{d+ex^2}}{c} + \frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\operatorname{Subst}\left(\int - \cdots\right.}{2c\sqrt{b^2-4ac}}\\
&= \frac{e\sqrt{d+ex^2}}{c} - \frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\tanh^{-1}\left(\frac{\cdots}{\sqrt{2}\cdots}\right.}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}
\end{aligned}$$

Mathematica [A] time = 0.57, size = 324, normalized size = 0.99

$$\frac{\left(2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}-b\right)-2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{e\sqrt{d+ex^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e*Sqrt[d + e*x^2])/c + ((-2*c^2*d^2 + b*(-b + Sqrt[b^2 - 4*a*c])*e^2 + 2*c
*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x
^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 -
4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + ((2*c^2*d^2 + b*(b + Sqr
t[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqr
t[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sq
rt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

IntegrateAlgebraic [C] time = 1.55, size = 447, normalized size = 1.37

$$\frac{\left(2\sqrt{2}\,cde\sqrt{4ac-b^2}-\sqrt{2}\,be^2\sqrt{4ac-b^2}+2i\sqrt{2}\,ace^2-i\sqrt{2}\,b^2e^2+2i\sqrt{2}\,bcde-2i\sqrt{2}\,c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}\right)}{2c^{3/2}\sqrt{4ac-b^2}\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}+\frac{\left(2\sqrt{2}\,cde\sqrt{4ac-b^2}-\sqrt{2}\,be^2\sqrt{4ac-b^2}-2i\sqrt{2}\,ace^2+i\sqrt{2}\,b^2e^2-2i\sqrt{2}\,bcde+2i\sqrt{2}\,c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}\right)}{2c^{3/2}\sqrt{4ac-b^2}\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}+\frac{e\sqrt{d+ex^2}}{c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e*Sqrt[d + e*x^2])/c + (((-2*I)*Sqrt[2]*c^2*d^2 + (2*I)*Sqrt[2]*b*c*d*e +
2*Sqrt[2]*c*Sqrt[-b^2 + 4*a*c]*d*e - I*Sqrt[2]*b^2*e^2 + (2*I)*Sqrt[2]*a*c*
e^2 - Sqrt[2]*b*Sqrt[-b^2 + 4*a*c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*
x^2])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(2*c^(3/2)*Sqrt[-b^2 +
```

```
4*a*c]*Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]) + (((2*I)*Sqrt[2]*c^2*d
^2 - (2*I)*Sqrt[2]*b*c*d*e + 2*Sqrt[2]*c*Sqrt[-b^2 + 4*a*c]*d*e + I*Sqrt[2]
*b^2*e^2 - (2*I)*Sqrt[2]*a*c*e^2 - Sqrt[2]*b*Sqrt[-b^2 + 4*a*c]*e^2)*ArcTan
[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]
*e]])/(2*c^(3/2)*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c
]*e])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.09, size = 649, normalized size = 1.98

$$\frac{\sqrt{x^2e+d}\,e}{c}+\frac{\left(4c^5d^3-6bc^4d^2e-\left(2\left(b^2c-4ac^2\right)de^2-\left(b^3-4abc\right)e^3\right)c^2+4\left(b^2c^3-ac^4\right)de^2+2\left(\sqrt{b^2-4ac}\,c^3d^2e-\sqrt{b^2-4ac}\,bc^2de^2+\sqrt{b^2-4ac}\,ac^2e^3\right)|c|-\left(b^3c^2-2abc^3\right)e^3\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{2c^2d-bce+\sqrt{-4\left(c^2d^2-bcde+ace^2\right)c^2+\left(2c^2d-bce\right)^2}}{c^2}}}\right)}{\left(2\sqrt{b^2-4ac}\,c^2d-\left(b^2c-4ac^2+\sqrt{b^2-4ac}\,bc\right)e\right)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\,c^2}-\frac{\left(4c^5d^3-6bc^4d^2e-\left(2\left(b^2c-4ac^2\right)de^2-\left(b^3-4abc\right)e^3\right)c^2+4\left(b^2c^3-ac^4\right)de^2-2\left(\sqrt{b^2-4ac}\,c^3d^2e-\sqrt{b^2-4ac}\,bc^2de^2+\sqrt{b^2-4ac}\,ac^2e^3\right)|c|-\left(b^3c^2-2abc^3\right)e^3\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{2c^2d-bce-\sqrt{-4\left(c^2d^2-bcde+ace^2\right)c^2+\left(2c^2d-bce\right)^2}}{c^2}}}\right)}{\left(2\sqrt{b^2-4ac}\,c^2d+\left(b^2c-4ac^2-\sqrt{b^2-4ac}\,bc\right)e\right)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] sqrt(x^2*e + d)*e/c + (4*c^5*d^3 - 6*b*c^4*d^2*e - (2*(b^2*c - 4*a*c^2)*d*e
^2 - (b^3 - 4*a*b*c)*e^3)*c^2 + 4*(b^2*c^3 - a*c^4)*d*e^2 + 2*(sqrt(b^2 - 4
*a*c)*c^3*d^2*e - sqrt(b^2 - 4*a*c)*b*c^2*d*e^2 + sqrt(b^2 - 4*a*c)*a*c^2*e
^3)*abs(c) - (b^3*c^2 - 2*a*b*c^3)*e^3)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/
sqrt(-(2*c^2*d - b*c*e + sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2
*d - b*c*e)^2))/c^2))/((2*sqrt(b^2 - 4*a*c)*c^2*d - (b^2*c - 4*a*c^2 + sqrt
(b^2 - 4*a*c)*b*c)*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2)
 - (4*c^5*d^3 - 6*b*c^4*d^2*e - (2*(b^2*c - 4*a*c^2)*d*e^2 - (b^3 - 4*a*b*c
)*e^3)*c^2 + 4*(b^2*c^3 - a*c^4)*d*e^2 - 2*(sqrt(b^2 - 4*a*c)*c^3*d^2*e - s
qrt(b^2 - 4*a*c)*b*c^2*d*e^2 + sqrt(b^2 - 4*a*c)*a*c^2*e^3)*abs(c) - (b^3*c
^2 - 2*a*b*c^3)*e^3)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*c^2*d - b*
c*e - sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2
))/((2*sqrt(b^2 - 4*a*c)*c^2*d + (b^2*c - 4*a*c^2 - sqrt(b^2 - 4*a*c)*b*c)*
e)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2)
```

maple [C] time = 0.02, size = 279, normalized size = 0.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*e^(3/2)/c*x+1/2*e*(e*x^2+d)^(1/2)/c+1/4*e/c*sum(((-b*e+2*c*d)*_R^6+(-4
*a*e^2+3*b*d*e-2*c*d^2)*_R^4+d*(4*a*e^2-3*b*d*e+2*c*d^2)*_R^2+b*d^3*e-2*c*d
^4)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R
*b*d^2*e-_R*c*d^3)*ln(-e^(1/2)*x-_R+(e*x^2+d)^(1/2)),_R=RootOf(_Z^8*c+(4*b*
e-4*c*d)*_Z^6+c*d^4+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^
2))+1/2*e/c*d/(-e^(1/2)*x+(e*x^2+d)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}x}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*x/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 4.13, size = 12392, normalized size = 37.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (e*(d + e*x^2)^(1/2))/c - atan(((((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 + 16*a*
c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4)/c - (
2*(d + e*x^2)^(1/2)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*
b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c
*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4
*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 +
3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c
^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d
^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3
 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c
^4*e^3 + 32*a*c^5*d*e^2))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3
+ 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 -
 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5
 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d
^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^
5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*
b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*
b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*
a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^4*e^6 +
2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*
c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(((4*
b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^
2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e +
 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*
e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d
^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3)
)^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*
a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c
^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(
1/2)*1i - (((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 + 16*a*c^4*d^2*e^3 + 4*b^3*c^
2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4)/c + (2*(d + e*x^2)^(1/2)*(-
(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3
*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^
2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*
(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^
2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3
*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3
 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*
a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4
)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^
2))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 9
6*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a
*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*
b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e
^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*
a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b
*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*
e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*
a*b^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*
d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*
```

c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 + 16*a*c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4)/c - (2*(d + e*x^2)^(1/2)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 + 16*a*c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4)/c + (2*(d + e*x^2)^(1/2)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 +

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6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(
-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^
3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d
^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)
*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a
^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^
3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3
 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24
*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^
4)))^(1/2) - (2*(2*c^3*d^5*e^3 - b^3*d^2*e^6 - a^2*b*e^8 + 4*a*c^2*d^3*e^5
- 5*b*c^2*d^4*e^4 + 4*b^2*c*d^3*e^5 + 2*a*b^2*d*e^7 + 2*a^2*c*d*e^7 - 6*a*b
*c*d^2*e^6))/c))*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c
^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*
e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^
3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a
*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*
d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3
+ 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 -
6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^
4*c^3 - 8*a*b^2*c^4))))^(1/2)*2i - atan(((((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5
 + 16*a*c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^
4)/c - (2*(d + e*x^2)^(1/2)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 +
48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 1
2*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 +
 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2
*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5
- 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^
2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^
3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^
2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 1
6*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^
3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c
*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a
^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a
*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*
b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^
3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e +
 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(1
6*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^4*
e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 +
6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(
(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3
*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^
2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*
(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^
2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3
*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3
+ 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*
a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4
)))^(1/2)*1i - (((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 + 16*a*c^4*d^2*e^3 + 4*b
^3*c^2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4)/c + (2*(d + e*x^2)^(1/
2)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2
*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^
3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c
^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 +
3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c
*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*
e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 +
 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2
```

c^4)))^(1/2)(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*
d*e^2))/c)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3
- 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 4
8*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128
*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^
4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e -
6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^
2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c
*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 -
8*a*b^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c
^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b
^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*((((4*b^5*e^3 + 32*a*c^4*d
^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e
- 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)
^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*
d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4
*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3
- 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^
3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*
c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*
c^3*e^5 - 4*a*b^2*c^2*e^5 + 16*a*c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*
d^2*e^3 - 16*a*b*c^3*d*e^4)/c - (2*(d + e*x^2)^(1/2)*((((4*b^5*e^3 + 32*a*c
^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d
^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*
e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 -
b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c
*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*
e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 -
6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*
b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*
e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*((((4*b^5*e^3
+ 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b
^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^
2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^
3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 +
3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2)
- 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*
d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e
- 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (
2*(d + e*x^2)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2
*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5
+ 12*a*b*c^2*d*e^5))/c)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a
^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^
4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*
b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4
+ 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*
b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^
3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*
e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^
5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 +
16*a*c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4)/
c + (2*(d + e*x^2)^(1/2)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*
a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b
^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16
*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^
4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3
*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c
^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c
*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c

```
^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a
*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d
^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^
3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*
c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^
2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d
*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 +
 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14
*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(
16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^4*e^6
 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b
^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*((((
4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*
e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e
 + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^
3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c
*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^
3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 4
8*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b
*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))
^(1/2) - (2*(2*c^3*d^5*e^3 - b^3*d^2*e^6 - a^2*b*e^8 + 4*a*c^2*d^3*e^5 - 5*
b*c^2*d^4*e^4 + 4*b^2*c*d^3*e^5 + 2*a*b^2*d*e^7 + 2*a^2*c*d*e^7 - 6*a*b*c*d
^2*e^6))/c))*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^
3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 -
 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 1
28*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*
d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e
 - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*
a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4
*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3
 - 8*a*b^2*c^4)))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.269 $\int \frac{\left(d+ex^2\right)^{3/2}}{x\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=346

$$-\frac{\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(-cd\left(d\sqrt{b^2-4ac}+4a\right.\right.}{\sqrt{2}}$$

Rubi [A] time = 1.74, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1251, 897, 1287, 206, 1166, 208}

$$-\frac{\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(-cd\left(d\sqrt{b^2-4ac}+4ae\right)+ae^2\sqrt{b^2-4ac}+b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\,a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d^(3/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a) - ((a*Sqrt[b^2 - 4*a*c]*e^2
 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) - b*(c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*
Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]
*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((a
*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(c*d^2 + a*e
^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}}{x\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(d+ex)^{3/2}}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{\operatorname{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\operatorname{Subst}\left(\int \left(-\frac{d^2e}{a\left(d-x^2\right)}+\frac{e\left(d\left(cd^2-bde+ae^2\right)-\left(cd^2-ae^2\right)x^2\right)}{a\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{d\left(cd^2-bde+ae^2\right)+\left(-cd^2+ae^2\right)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{a} - \frac{d^2\operatorname{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+\ldots}\right.}{a}$$

$$= -\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a} + \frac{\left(a\sqrt{b^2-4ac}\,e^2-cd\left(\sqrt{b^2-4ac}\,d-4ae\right)-b\left(cd^2+ae^2\right)\right)\ldots}{2a\sqrt{b^2-4\ldots}}$$

$$= -\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a} - \frac{\left(a\sqrt{b^2-4ac}\,e^2-cd\left(\sqrt{b^2-4ac}\,d-4ae\right)-b\left(cd^2+ae^2\right)\right)\ldots}{\sqrt{2}\,a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-\ldots}\right.}}$$

Mathematica [A] time = 1.38, size = 333, normalized size = 0.96

$$-\frac{\frac{\left(-cd\left(d\sqrt{b^2-4ac}+4ae\right)+ae^2\sqrt{b^2-4ac}+b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\left(cd\left(d\sqrt{b^2-4ac}-4ae\right)-ae^2\sqrt{b^2-4ac}+b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}}{\sqrt{2}\,a\sqrt{c}\sqrt{b^2-4ac}}-\frac{d^{3/2}\log\left(\sqrt{d}\sqrt{d+ex^2}+d\right)}{a}+\frac{d^{3/2}\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((-(((-(a*Sqrt[b^2 - 4*a*c]*e^2) + c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) + b*(
c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e
+ Sqrt[b^2 - 4*a*c]*e]])/Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + ((a*Sq
rt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(c*d^2 + a*e^2)
```

```
)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])/(Sqrt[2]*a*Sqrt[c]*Sqrt
[b^2 - 4*a*c])) + (d^(3/2)*Log[x])/a - (d^(3/2)*Log[d + Sqrt[d]*Sqrt[d + e*
x^2]])/a
```

IntegrateAlgebraic [C] time = 1.53, size = 389, normalized size = 1.12

$$-\frac{\left(cd^2\sqrt{4ac-b^2}-ae^2\sqrt{4ac-b^2}-iabe^2+4iacde-ibcd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{4ac-b^2}\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}-\frac{\left(cd^2\sqrt{4ac-b^2}-ae^2\sqrt{4ac-b^2}+iabe^2-4iacde+ibcd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{4ac-b^2}\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}-\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((((-I)*b*c*d^2 + c*Sqrt[-b^2 + 4*a*c]*d^2 + (4*I)*a*c*d*e - I*a*b*e^2 - a
*Sqrt[-b^2 + 4*a*c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c
*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[-b^2 + 4*a*c]*
Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e])) - ((I*b*c*d^2 + c*Sqrt[-b^2 +
 4*a*c]*d^2 - (4*I)*a*c*d*e + I*a*b*e^2 - a*Sqrt[-b^2 + 4*a*c]*e^2)*ArcTan[
(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*
e]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e + I*Sqrt[-b^2
+ 4*a*c]*e]) - (d^(3/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.02, size = 827, normalized size = 2.39

$$\frac{d^2\arctan\left(\frac{\sqrt{x^2e+d}}{\sqrt{-d}}\right)}{a\sqrt{-d}}-\frac{\left(\left((b^2c-4ac^2)d^2e-(ab^2-4a^2c)e^3\right)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\,a^2-2\left(\sqrt{b^2-4ac}\,ac^2d^3-\sqrt{b^2-4ac}\,abcd^2e+\sqrt{b^2-4ac}\,a^2cde^2\right)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\,|a|-\left(2a^2bc^2d^3+6a^3bcde^2-a^3b^2e^3-(a^2b^2c+8a^3c^2)d^2e\right)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{2acd-abe+\sqrt{-4(acd^2-abde+a^2e^2)ac+(2acd-abe)^2}}{ac}}}\right)}{8\left(\sqrt{b^2-4ac}\,a^2c^2d^2-\sqrt{b^2-4ac}\,a^2bcde+\sqrt{b^2-4ac}\,a^3ce^2\right)|a||c|}+\frac{\left(\left((b^2c-4ac^2)d^2e-(ab^2-4a^2c)e^3\right)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\,a^2+2\left(\sqrt{b^2-4ac}\,ac^2d^3-\sqrt{b^2-4ac}\,abcd^2e+\sqrt{b^2-4ac}\,a^2cde^2\right)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\,|a|-\left(2a^2bc^2d^3+6a^3bcde^2-a^3b^2e^3-(a^2b^2c+8a^3c^2)d^2e\right)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{2acd-abe-\sqrt{-4(acd^2-abde+a^2e^2)ac+(2acd-abe)^2}}{ac}}}\right)}{8\left(\sqrt{b^2-4ac}\,a^2c^2d^2-\sqrt{b^2-4ac}\,a^2bcde+\sqrt{b^2-4ac}\,a^3ce^2\right)|a||c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] d^2*arctan(sqrt(x^2*e + d)/sqrt(-d))/(a*sqrt(-d)) - 1/8*(((b^2*c - 4*a*c^2)
*d^2*e - (a*b^2 - 4*a^2*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*
c)*e)*a^2 - 2*(sqrt(b^2 - 4*a*c)*a*c^2*d^3 - sqrt(b^2 - 4*a*c)*a*b*c*d^2*e
+ sqrt(b^2 - 4*a*c)*a^2*c*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)
*c)*e)*abs(a) - (2*a^2*b*c^2*d^3 + 6*a^3*b*c*d*e^2 - a^3*b^2*e^3 - (a^2*b^2
*c + 8*a^3*c^2)*d^2*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*ar
ctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*a*c*d - a*b*e + sqrt(-4*(a*c*d^2
- a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*a*c
)*a^2*c^2*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*c*d*e + sqrt(b^2 - 4*a*c)*a^3*c*e^2
)*abs(a)*abs(c)) + 1/8*(((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3)*s
qrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*a^2 + 2*(sqrt(b^2 - 4*a*c)*
a*c^2*d^3 - sqrt(b^2 - 4*a*c)*a*b*c*d^2*e + sqrt(b^2 - 4*a*c)*a^2*c*d*e^2)*
sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(a) - (2*a^2*b*c^2*d^3
+ 6*a^3*b*c*d*e^2 - a^3*b^2*e^3 - (a^2*b^2*c + 8*a^3*c^2)*d^2*e)*sqrt(-4*c^
2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/
sqrt(-(2*a*c*d - a*b*e - sqrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c
*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*a*c)*a^2*c^2*d^2 - sqrt(b^2 - 4*a*c)
*a^2*b*c*d*e + sqrt(b^2 - 4*a*c)*a^3*c*e^2)*abs(a)*abs(c))
```

maple [C] time = 0.03, size = 388, normalized size = 1.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] 7/24/a*(e*x^2+d)^(3/2)-1/a*d^(3/2)*ln((2*d+2*(e*x^2+d)^(1/2)*d^(1/2))/x)+3/
8/a*(e*x^2+d)^(1/2)*d+1/6/a*e^(3/2)*x^3-1/8/a*e*(e*x^2+d)^(1/2)*x^2+3/4/a*e
^(1/2)*x*d-1/4/a*sum(((-a*e^2+c*d^2)*_R^6+d*(-5*a*e^2+4*b*d*e-3*c*d^2)*_R^4
+d^2*(5*a*e^2-4*b*d*e+3*c*d^2)*_R^2+a*d^3*e^2-c*d^5)/(_R^7*c+3*_R^5*b*e-3*_
R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln(-e^(
1/2)*x-_R+(e*x^2+d)^(1/2)),_R=RootOf(_Z^8*c+(4*b*e-4*c*d)*_Z^6+c*d^4+(16*a*
e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2))-5/8/a*d^2/(-e^(1/2)*x+
(e*x^2+d)^(1/2))-1/24/a*d^3/(-e^(1/2)*x+(e*x^2+d)^(1/2))^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2 + d\right)^{\frac{3}{2}}}{\left(cx^4 + bx^2 + a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x), x)
```

mupad [B] time = 7.67, size = 28434, normalized size = 82.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((d + e*x^2)^(1/2)*(2*a^4*c*e^16 + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10
 - 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*e^12 + 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2
*e^14 + 24*b^2*c^3*d^6*e^10 - 16*b^3*c^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*
b^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4*e^12 - 24*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2
*c*d^2*e^14) + (-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2
*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*
c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3
*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*
e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6
*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*
a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2
*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*
c^2)))^(1/2)*(((-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2
*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*
c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3
*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*
e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6
*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*
a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2
*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*
c^2)))^(1/2)*((d + e*x^2)^(1/2)*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c
^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*
d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^
2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e
^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 -
3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*
a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3
*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b
^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256
*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*
```

b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*
c^3*d*e^9) - 192*a^3*c^5*d^4*e^8 - 192*a^4*c^4*d^2*e^10 + 48*a^2*b^2*c^4*d^
4*e^8 - 64*a^2*b^3*c^3*d^3*e^9 + 16*a^2*b^4*c^2*d^2*e^10 - 16*a^3*b^2*c^3*d
^2*e^10 + 64*a^4*b*c^3*d*e^11 + 256*a^3*b*c^4*d^3*e^9 - 16*a^3*b^3*c^2*d*e^
11) + (d + e*x^2)^(1/2)*(8*a^3*b^3*c*e^13 - 32*a^4*b*c^2*e^13 + 176*a^4*c^3
*d*e^12 - 144*a^2*c^5*d^5*e^8 + 224*a^3*c^4*d^3*e^10 - 16*b^4*c^3*d^5*e^8 +
16*b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*e^10 + 112*a^2*b^3*c^2*d^2*e^11 -
16*a^2*b^4*c*d*e^12 + 96*a*b^2*c^4*d^5*e^8 - 80*a*b^3*c^3*d^4*e^9 - 32*a*b^
4*c^2*d^3*e^10 + 96*a^2*b*c^4*d^4*e^9 - 416*a^3*b*c^3*d^2*e^11 + 16*a^3*b^2
*c^2*d*e^12))*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*
c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c
^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*
b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e
^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*
a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a
*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*
b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c
^2)))^(1/2) + 12*a*c^5*d^7*e^8 + 4*a^4*c^2*d*e^14 - 84*a^2*c^4*d^5*e^10 - 9
2*a^3*c^3*d^3*e^12 - 4*b^2*c^4*d^7*e^8 - 4*b^3*c^3*d^6*e^9 + 8*b^4*c^2*d^5*
e^10 - 12*a^2*b^2*c^2*d^3*e^12 + 32*a*b*c^4*d^6*e^9 - 4*a^3*b^2*c*d*e^14 -
36*a*b^2*c^3*d^5*e^10 - 20*a*b^3*c^2*d^4*e^11 + 160*a^2*b*c^3*d^4*e^11 + 4*
a^2*b^3*c*d^2*e^13 + 16*a^3*b*c^2*d^2*e^13))*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e
^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3
- 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^
4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 +
3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a
^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*
b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*
e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^
4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*1i + ((d + e*x^2)^(1/2)*(2*a^4*c
*e^16 + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10 - 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*
e^12 + 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2*e^14 + 24*b^2*c^3*d^6*e^10 - 16*
b^3*c^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*b^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4
*e^12 - 24*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2*c*d^2*e^14) + (-(((4*b^4*c*d^3 -
4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*
a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2
/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^
3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d
^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*
d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2
- 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2
)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(((-(((4*b^4*c*d^3 -
4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*
a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2
/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^
3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d
^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*
d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2
- 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2
)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x^2)^(1/2)*(
-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^
3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a
^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e
^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^
2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))
^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 +
48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 1
2*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(51
2*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d

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^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d
*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 192*a^3*c^5*d^4*e^8
+ 192*a^4*c^4*d^2*e^10 - 48*a^2*b^2*c^4*d^4*e^8 + 64*a^2*b^3*c^3*d^3*e^9 -
16*a^2*b^4*c^2*d^2*e^10 + 16*a^3*b^2*c^3*d^2*e^10 - 64*a^4*b*c^3*d*e^11 - 2
56*a^3*b*c^4*d^3*e^9 + 16*a^3*b^3*c^2*d*e^11) + (d + e*x^2)^(1/2)*(8*a^3*b^
3*c*e^13 - 32*a^4*b*c^2*e^13 + 176*a^4*c^3*d*e^12 - 144*a^2*c^5*d^5*e^8 + 2
24*a^3*c^4*d^3*e^10 - 16*b^4*c^3*d^5*e^8 + 16*b^5*c^2*d^4*e^9 + 48*a^2*b^2*
c^3*d^3*e^10 + 112*a^2*b^3*c^2*d^2*e^11 - 16*a^2*b^4*c*d*e^12 + 96*a*b^2*c^
4*d^5*e^8 - 80*a*b^3*c^3*d^4*e^9 - 32*a*b^4*c^2*d^3*e^10 + 96*a^2*b*c^4*d^4
*e^9 - 416*a^3*b*c^3*d^2*e^11 + 16*a^3*b^2*c^2*d*e^12))*(-(((4*b^4*c*d^3 -
4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a
^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/
4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3
*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^
4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d
^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 -
 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)
/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - 12*a*c^5*d^7*e^8 -
4*a^4*c^2*d*e^14 + 84*a^2*c^4*d^5*e^10 + 92*a^3*c^3*d^3*e^12 + 4*b^2*c^4*d^
7*e^8 + 4*b^3*c^3*d^6*e^9 - 8*b^4*c^2*d^5*e^10 + 12*a^2*b^2*c^2*d^3*e^12 -
32*a*b*c^4*d^6*e^9 + 4*a^3*b^2*c*d*e^14 + 36*a*b^2*c^3*d^5*e^10 + 20*a*b^3*
c^2*d^4*e^11 - 160*a^2*b*c^3*d^4*e^11 - 4*a^2*b^3*c*d^2*e^13 - 16*a^3*b*c^2
*d^2*e^13))*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^
2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2
*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^
2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2
 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*
b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b
^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*
c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2
)))^(1/2)*1i)/(((d + e*x^2)^(1/2)*(2*a^4*c*e^16 + 6*c^5*d^8*e^8 - 16*a*c^4*
d^6*e^10 - 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*e^12 + 16*a^2*c^3*d^4*e^12 + 8*a^
3*c^2*d^2*e^14 + 24*b^2*c^3*d^6*e^10 - 16*b^3*c^2*d^5*e^11 - 8*a^3*b*c*d*e^
15 - 8*a*b^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4*e^12 - 24*a^2*b*c^2*d^3*e^13 + 1
2*a^2*b^2*c*d^2*e^14) + (-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 -
 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e +
48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c
- 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a
*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*
d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*
d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e
 - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8
*a^3*b^2*c^2)))^(1/2)*(((-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 -
 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e +
48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c
- 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a
*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*
d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*
d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e
 - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8
*a^3*b^2*c^2)))^(1/2)*((d + e*x^2)^(1/2)*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 +
 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12
*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^
3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*
b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b
*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*
e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3
+ 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^
3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e
```

^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480 *a^3*b^3*c^3*d*e^9) + 192*a^3*c^5*d^4*e^8 + 192*a^4*c^4*d^2*e^10 - 48*a^2*b ^2*c^4*d^4*e^8 + 64*a^2*b^3*c^3*d^3*e^9 - 16*a^2*b^4*c^2*d^2*e^10 + 16*a^3* b^2*c^3*d^2*e^10 - 64*a^4*b*c^3*d*e^11 - 256*a^3*b*c^4*d^3*e^9 + 16*a^3*b^3 *c^2*d*e^11) + (d + e*x^2)^(1/2)*(8*a^3*b^3*c*e^13 - 32*a^4*b*c^2*e^13 + 17 6*a^4*c^3*d*e^12 - 144*a^2*c^5*d^5*e^8 + 224*a^3*c^4*d^3*e^10 - 16*b^4*c^3* d^5*e^8 + 16*b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*e^10 + 112*a^2*b^3*c^2*d^ 2*e^11 - 16*a^2*b^4*c*d*e^12 + 96*a*b^2*c^4*d^5*e^8 - 80*a*b^3*c^3*d^4*e^9 - 32*a*b^4*c^2*d^3*e^10 + 96*a^2*b*c^4*d^4*e^9 - 416*a^3*b*c^3*d^2*e^11 + 1 6*a^3*b^2*c^2*d*e^12))*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 4 8*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a* c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d ^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d ^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8* a^3*b^2*c^2)))^(1/2) - 12*a*c^5*d^7*e^8 - 4*a^4*c^2*d*e^14 + 84*a^2*c^4*d^5 *e^10 + 92*a^3*c^3*d^3*e^12 + 4*b^2*c^4*d^7*e^8 + 4*b^3*c^3*d^6*e^9 - 8*b^4 *c^2*d^5*e^10 + 12*a^2*b^2*c^2*d^3*e^12 - 32*a*b*c^4*d^6*e^9 + 4*a^3*b^2*c* d*e^14 + 36*a*b^2*c^3*d^5*e^10 + 20*a*b^3*c^2*d^4*e^11 - 160*a^2*b*c^3*d^4* e^11 - 4*a^2*b^3*c*d^2*e^13 - 16*a^3*b*c^2*d^2*e^13))*(-(((4*b^4*c*d^3 - 4* a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3 *b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d ^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4* e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8 *a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - ((d + e*x^2)^(1/2)*(2 *a^4*c*e^16 + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10 - 16*b*c^4*d^7*e^9 + 4*b^4* c*d^4*e^12 + 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2*e^14 + 24*b^2*c^3*d^6*e^10 - 16*b^3*c^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*b^3*c*d^3*e^13 + 16*a*b^2*c ^2*d^4*e^12 - 24*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2*c*d^2*e^14) + (-(((4*b^4*c *d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d* e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^ 6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b ^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2* b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2* d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c *d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(((-(((4*b^4*c *d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d* e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^ 6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b ^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2* b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2* d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c *d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x^2)^(1/2)*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)* (a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^ 2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3 *e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2* d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2 *e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/

2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4
*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b
*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 192*a^3*c^5*d^
4*e^8 - 192*a^4*c^4*d^2*e^10 + 48*a^2*b^2*c^4*d^4*e^8 - 64*a^2*b^3*c^3*d^3*
e^9 + 16*a^2*b^4*c^2*d^2*e^10 - 16*a^3*b^2*c^3*d^2*e^10 + 64*a^4*b*c^3*d*e^
11 + 256*a^3*b*c^4*d^3*e^9 - 16*a^3*b^3*c^2*d*e^11) + (d + e*x^2)^(1/2)*(8*
a^3*b^3*c*e^13 - 32*a^4*b*c^2*e^13 + 176*a^4*c^3*d*e^12 - 144*a^2*c^5*d^5*e
^8 + 224*a^3*c^4*d^3*e^10 - 16*b^4*c^3*d^5*e^8 + 16*b^5*c^2*d^4*e^9 + 48*a^
2*b^2*c^3*d^3*e^10 + 112*a^2*b^3*c^2*d^2*e^11 - 16*a^2*b^4*c*d*e^12 + 96*a*
b^2*c^4*d^5*e^8 - 80*a*b^3*c^3*d^4*e^9 - 32*a*b^4*c^2*d^3*e^10 + 96*a^2*b*c
^4*d^4*e^9 - 416*a^3*b*c^3*d^2*e^11 + 16*a^3*b^2*c^2*d*e^12))*(-(((4*b^4*c*
d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2
+ 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e
^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6
 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^
2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b
^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d
*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*
d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) + 12*a*c^5*d^7*
e^8 + 4*a^4*c^2*d*e^14 - 84*a^2*c^4*d^5*e^10 - 92*a^3*c^3*d^3*e^12 - 4*b^2*
c^4*d^7*e^8 - 4*b^3*c^3*d^6*e^9 + 8*b^4*c^2*d^5*e^10 - 12*a^2*b^2*c^2*d^3*e
^12 + 32*a*b*c^4*d^6*e^9 - 4*a^3*b^2*c*d*e^14 - 36*a*b^2*c^3*d^5*e^10 - 20*
a*b^3*c^2*d^4*e^11 + 160*a^2*b*c^3*d^4*e^11 + 4*a^2*b^3*c*d^2*e^13 + 16*a^3
*b*c^2*d^2*e^13))*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*
b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2
*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*
a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d
^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e
- 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 +
12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*
a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b
^2*c^2)))^(1/2) + 6*c^4*d^8*e^10 + 14*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 1
6*b*c^3*d^7*e^11 - 4*b^3*c*d^5*e^13 + 10*a^2*c^2*d^4*e^14 + 14*b^2*c^2*d^6*
e^12 - 24*a*b*c^2*d^5*e^13 + 10*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15))*(-(
((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*
c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2
*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6
 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*
e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(
1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48
*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*
a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*2i +
atan((((d + e*x^2)^(1/2)*(2*a^4*c*e^16 + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10
- 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*e^12 + 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2
*e^14 + 24*b^2*c^3*d^6*e^10 - 16*b^3*c^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*
b^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4*e^12 - 24*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2
*c*d^2*e^14) + ((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*
c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c
^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*
b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e
^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*
a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a
*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*
b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c
^2)))^(1/2)*((((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c
^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^
2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b
^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^
2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a

```
*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*
b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b
*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^
2)))^(1/2)*((d + e*x^2)^(1/2)*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*
d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2
*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b
^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4
+ 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b
*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2
*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*
d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*
c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^
4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2
*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3
*d*e^9) - 192*a^3*c^5*d^4*e^8 - 192*a^4*c^4*d^2*e^10 + 48*a^2*b^2*c^4*d^4*e
^8 - 64*a^2*b^3*c^3*d^3*e^9 + 16*a^2*b^4*c^2*d^2*e^10 - 16*a^3*b^2*c^3*d^2*
e^10 + 64*a^4*b*c^3*d*e^11 + 256*a^3*b*c^4*d^3*e^9 - 16*a^3*b^3*c^2*d*e^11)
 + (d + e*x^2)^(1/2)*(8*a^3*b^3*c*e^13 - 32*a^4*b*c^2*e^13 + 176*a^4*c^3*d*
e^12 - 144*a^2*c^5*d^5*e^8 + 224*a^3*c^4*d^3*e^10 - 16*b^4*c^3*d^5*e^8 + 16
*b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*e^10 + 112*a^2*b^3*c^2*d^2*e^11 - 16*
a^2*b^4*c*d*e^12 + 96*a*b^2*c^4*d^5*e^8 - 80*a*b^3*c^3*d^4*e^9 - 32*a*b^4*c
^2*d^3*e^10 + 96*a^2*b*c^4*d^4*e^9 - 416*a^3*b*c^3*d^2*e^11 + 16*a^3*b^2*c^
2*d*e^12))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*
d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d
^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*
c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 +
 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*
c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2
*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^
2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2))
)^(1/2) + 12*a*c^5*d^7*e^8 + 4*a^4*c^2*d*e^14 - 84*a^2*c^4*d^5*e^10 - 92*a^
3*c^3*d^3*e^12 - 4*b^2*c^4*d^7*e^8 - 4*b^3*c^3*d^6*e^9 + 8*b^4*c^2*d^5*e^10
 - 12*a^2*b^2*c^2*d^3*e^12 + 32*a*b*c^4*d^6*e^9 - 4*a^3*b^2*c*d*e^14 - 36*a
*b^2*c^3*d^5*e^10 - 20*a*b^3*c^2*d^4*e^11 + 160*a^2*b*c^3*d^4*e^11 + 4*a^2*
b^3*c*d^2*e^13 + 16*a^3*b*c^2*d^2*e^13))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 +
32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*
a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3
 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b
^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*
d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e
^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 -
 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3
 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*1i + ((d + e*x^2)^(1/2)*(2*a^4*c*e^16
 + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10 - 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*e^12
+ 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2*e^14 + 24*b^2*c^3*d^6*e^10 - 16*b^3*c
^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*b^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4*e^12
 - 24*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2*c*d^2*e^14) + ((((4*b^4*c*d^3 - 4*a^2
*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*
c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (
256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*
e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2
 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 -
2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^
3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*
(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((((4*b^4*c*d^3 - 4*a^2*
b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c
*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (2
56*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e
^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2
```

- 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2
*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3
*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(
16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x^2)^(1/2)*((((4*b^
4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*
e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c
*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3
*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 +
3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) +
2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c
^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^
2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^
4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 +
64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 6
4*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 192*a^3*c^5*d^4*e^8 + 192*a^
4*c^4*d^2*e^10 - 48*a^2*b^2*c^4*d^4*e^8 + 64*a^2*b^3*c^3*d^3*e^9 - 16*a^2*b
^4*c^2*d^2*e^10 + 16*a^3*b^2*c^3*d^2*e^10 - 64*a^4*b*c^3*d*e^11 - 256*a^3*b
*c^4*d^3*e^9 + 16*a^3*b^3*c^2*d*e^11) + (d + e*x^2)^(1/2)*(8*a^3*b^3*c*e^13
- 32*a^4*b*c^2*e^13 + 176*a^4*c^3*d*e^12 - 144*a^2*c^5*d^5*e^8 + 224*a^3*c
^4*d^3*e^10 - 16*b^4*c^3*d^5*e^8 + 16*b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*
e^10 + 112*a^2*b^3*c^2*d^2*e^11 - 16*a^2*b^4*c*d*e^12 + 96*a*b^2*c^4*d^5*e^
8 - 80*a*b^3*c^3*d^4*e^9 - 32*a*b^4*c^2*d^3*e^10 + 96*a^2*b*c^4*d^4*e^9 - 4
16*a^3*b*c^3*d^2*e^11 + 16*a^3*b^2*c^2*d*e^12))*((((4*b^4*c*d^3 - 4*a^2*b^3
*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^
3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*
a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3
+ 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3
*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^
2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*
c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*
a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - 12*a*c^5*d^7*e^8 - 4*a^4*c^2
*d*e^14 + 84*a^2*c^4*d^5*e^10 + 92*a^3*c^3*d^3*e^12 + 4*b^2*c^4*d^7*e^8 + 4
*b^3*c^3*d^6*e^9 - 8*b^4*c^2*d^5*e^10 + 12*a^2*b^2*c^2*d^3*e^12 - 32*a*b*c^
4*d^6*e^9 + 4*a^3*b^2*c*d*e^14 + 36*a*b^2*c^3*d^5*e^10 + 20*a*b^3*c^2*d^4*e
^11 - 160*a^2*b*c^3*d^4*e^11 - 4*a^2*b^3*c*d^2*e^13 - 16*a^3*b*c^2*d^2*e^13
))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96
*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 2
4*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^
3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c
*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^
3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3
- 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e
+ 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*
1i)/(((d + e*x^2)^(1/2)*(2*a^4*c*e^16 + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10 -
16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*e^12 + 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2*
e^14 + 24*b^2*c^3*d^6*e^10 - 16*b^3*c^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*b
^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4*e^12 - 24*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2*
c*d^2*e^14) + ((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c
^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^
2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b
^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^
2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a
*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*
b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b
*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^
2)))^(1/2)*((((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^
2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2
*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^
2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2

+ 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*
b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b
^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*
c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2
)))^(1/2)*((d + e*x^2)^(1/2)*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d
^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*
e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^
4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 +
3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*
c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*
c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d
^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c
- 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4
*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*
c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*
d*e^9) + 192*a^3*c^5*d^4*e^8 + 192*a^4*c^4*d^2*e^10 - 48*a^2*b^2*c^4*d^4*e^
8 + 64*a^2*b^3*c^3*d^3*e^9 - 16*a^2*b^4*c^2*d^2*e^10 + 16*a^3*b^2*c^3*d^2*e
^10 - 64*a^4*b*c^3*d*e^11 - 256*a^3*b*c^4*d^3*e^9 + 16*a^3*b^3*c^2*d*e^11)
+ (d + e*x^2)^(1/2)*(8*a^3*b^3*c*e^13 - 32*a^4*b*c^2*e^13 + 176*a^4*c^3*d*e
^12 - 144*a^2*c^5*d^5*e^8 + 224*a^3*c^4*d^3*e^10 - 16*b^4*c^3*d^5*e^8 + 16*
b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*e^10 + 112*a^2*b^3*c^2*d^2*e^11 - 16*a
^2*b^4*c*d*e^12 + 96*a*b^2*c^4*d^5*e^8 - 80*a*b^3*c^3*d^4*e^9 - 32*a*b^4*c^
2*d^3*e^10 + 96*a^2*b*c^4*d^4*e^9 - 416*a^3*b*c^3*d^2*e^11 + 16*a^3*b^2*c^2
*d*e^12))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d
^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^
2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c
^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 +
3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c
*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*
c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2
*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))
^(1/2) - 12*a*c^5*d^7*e^8 - 4*a^4*c^2*d*e^14 + 84*a^2*c^4*d^5*e^10 + 92*a^3
*c^3*d^3*e^12 + 4*b^2*c^4*d^7*e^8 + 4*b^3*c^3*d^6*e^9 - 8*b^4*c^2*d^5*e^10
+ 12*a^2*b^2*c^2*d^3*e^12 - 32*a*b*c^4*d^6*e^9 + 4*a^3*b^2*c*d*e^14 + 36*a*
b^2*c^3*d^5*e^10 + 20*a*b^3*c^2*d^4*e^11 - 160*a^2*b*c^3*d^4*e^11 - 4*a^2*b
^3*c*d^2*e^13 - 16*a^3*b*c^2*d^2*e^13))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 3
2*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a
*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3
+ 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^
2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d
*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^
3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 -
6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3
+ a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - ((d + e*x^2)^(1/2)*(2*a^4*c*e^16 + 6
*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10 - 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*e^12 + 16
*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2*e^14 + 24*b^2*c^3*d^6*e^10 - 16*b^3*c^2*d
^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*b^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4*e^12 - 2
4*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2*c*d^2*e^14) + ((((4*b^4*c*d^3 - 4*a^2*b^3
*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^
3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*
a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3
+ 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3
*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^
2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*
c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*
a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((((4*b^4*c*d^3 - 4*a^2*b^3*
e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3
- 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a
^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 +

3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x^2)^(1/2)*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 192*a^3*c^5*d^4*e^8 - 192*a^4*c^4*d^2*e^10 + 48*a^2*b^2*c^4*d^4*e^8 - 64*a^2*b^3*c^3*d^3*e^9 + 16*a^2*b^4*c^2*d^2*e^10 - 16*a^3*b^2*c^3*d^2*e^10 + 64*a^4*b*c^3*d*e^11 + 256*a^3*b*c^4*d^3*e^9 - 16*a^3*b^3*c^2*d*e^11) + (d + e*x^2)^(1/2)*(8*a^3*b^3*c*e^13 - 32*a^4*b*c^2*e^13 + 176*a^4*c^3*d*e^12 - 144*a^2*c^5*d^5*e^8 + 224*a^3*c^4*d^3*e^10 - 16*b^4*c^3*d^5*e^8 + 16*b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*e^10 + 112*a^2*b^3*c^2*d^2*e^11 - 16*a^2*b^4*c*d*e^12 + 96*a*b^2*c^4*d^5*e^8 - 80*a*b^3*c^3*d^4*e^9 - 32*a*b^4*c^2*d^3*e^10 + 96*a^2*b*c^4*d^4*e^9 - 416*a^3*b*c^3*d^2*e^11 + 16*a^3*b^2*c^2*d*e^12))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) + 12*a*c^5*d^7*e^8 + 4*a^4*c^2*d*e^14 - 84*a^2*c^4*d^5*e^10 - 92*a^3*c^3*d^3*e^12 - 4*b^2*c^4*d^7*e^8 - 4*b^3*c^3*d^6*e^9 + 8*b^4*c^2*d^5*e^10 - 12*a^2*b^2*c^2*d^3*e^12 + 32*a*b*c^4*d^6*e^9 - 4*a^3*b^2*c*d*e^14 - 36*a*b^2*c^3*d^5*e^10 - 20*a*b^3*c^2*d^4*e^11 + 160*a^2*b*c^3*d^4*e^11 + 4*a^2*b^3*c*d^2*e^13 + 16*a^3*b*c^2*d^2*e^13))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) + 6*c^4*d^8*e^10 + 14*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 16*b*c^3*d^7*e^11 - 4*b^3*c*d^5*e^13 + 10*a^2*c^2*d^4*e^14 + 14*b^2*c^2*d^6*e^12 - 24*a*b*c^2*d^5*e^13 + 10*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*2i - (atanh((72*c^4*d^6*e^10*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^4*d^8*e^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*b^3*c*d^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*e^12 + (20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2

- (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2*d^5*e^13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a) + (2*a^3*c*e^16*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^4*d^8*e^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*b^3*c*d^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*e^12 + (20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2*d^5*e^13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a) + (18*c^5*d^8*e^8*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(18*c^5*d^10*e^8 + 72*a*c^4*d^8*e^10 + 2*a^4*c*d^2*e^16 - 48*b*c^4*d^9*e^9 + 60*a^2*c^3*d^6*e^12 + 8*a^3*c^2*d^4*e^14 + 20*b^2*c^3*d^8*e^10 + 12*b^3*c^2*d^7*e^11 - (4*b^2*c^4*d^10*e^8)/a + (10*b^3*c^3*d^9*e^9)/a - (6*b^4*c^2*d^8*e^10)/a - 104*a*b*c^3*d^7*e^11 - 6*a*b^3*c*d^5*e^13 - 8*a^3*b*c*d^3*e^15 + 20*a*b^2*c^2*d^6*e^12 - 32*a^2*b*c^2*d^5*e^13 + 12*a^2*b^2*c*d^4*e^14) + (8*a^2*c^2*d^2*e^14*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^4*d^8*e^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*b^3*c*d^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*e^12 + (20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2*d^5*e^13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a) + (20*b^2*c^2*d^4*e^12*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^4*d^8*e^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*b^3*c*d^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*e^12 + (20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2*d^5*e^13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a) - (48*b*c^4*d^7*e^9*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(18*c^5*d^10*e^8 + 72*a*c^4*d^8*e^10 + 2*a^4*c*d^2*e^16 - 48*b*c^4*d^9*e^9 + 60*a^2*c^3*d^6*e^12 + 8*a^3*c^2*d^4*e^14 + 20*b^2*c^3*d^8*e^10 + 12*b^3*c^2*d^7*e^11 - (4*b^2*c^4*d^10*e^8)/a + (10*b^3*c^3*d^9*e^9)/a - (6*b^4*c^2*d^8*e^10)/a - 104*a*b*c^3*d^7*e^11 - 6*a*b^3*c*d^5*e^13 - 8*a^3*b*c*d^3*e^15 + 20*a*b^2*c^2*d^6*e^12 - 32*a^2*b*c^2*d^5*e^13 + 12*a^2*b^2*c*d^4*e^14) - (4*b^2*c^4*d^8*e^8*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(18*a*c^5*d^10*e^8 + 2*a^5*c*d^2*e^16 + 72*a^2*c^4*d^8*e^10 + 60*a^3*c^3*d^6*e^12 + 8*a^4*c^2*d^4*e^14 - 4*b^2*c^4*d^10*e^8 + 10*b^3*c^3*d^9*e^9 - 6*b^4*c^2*d^8*e^10 + 20*a^2*b^2*c^2*d^6*e^12 - 48*a*b*c^4*d^9*e^9 - 8*a^4*b*c*d^3*e^15 + 20*a*b^2*c^3*d^8*e^10 + 12*a*b^3*c^2*d^7*e^11 - 104*a^2*b*c^3*d^7*e^11 - 6*a^2*b^3*c*d^5*e^13 - 32*a^3*b*c^2*d^5*e^13 + 12*a^3*b^2*c*d^4*e^14) + (10*b^3*c^3*d^7*e^9*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(18*a*c^5*d^10*e^8 + 2*a^5*c*d^2*e^16 + 72*a^2*c^4*d^8*e^10 + 60*a^3*c^3*d^6*e^12 + 8*a^4*c^2*d^4*e^14 - 4*b^2*c^4*d^10*e^8 + 10*b^3*c^3*d^9*e^9 - 6*b^4*c^2*d^8*e^10 + 20*a^2*b^2*c^2*d^6*e^12 - 48*a*b*c^4*d^9*e^9 - 8*a^4*b*c*d^3*e^15 + 20*a*b^2*c^3*d^8*e^10 + 12*a*b^3*c^2*d^7*e^11 - 104*a^2*b*c^3*d^7*e^11 - 6*a^2*b^3*c*d^5*e^13 - 32*a^3*b*c^2*d^5*e^13 + 12*a^3*b^2*c*d^4*e^14) - (6*b^4*c^2*d^6*e^10*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(18*a*c^5*d^10*e^8 + 2*a^5*c*d^2*e^16 + 72*a^2*c^4*d^8*e^10 + 60*a^3*c^3*d^6*e^12 + 8*a^4*c^2*d^4*e^14 - 4*b^2*c^4*d^10*e^8 + 10*b^3*c^3*d^9*e^9 - 6*b^4*c^2*d^8*e^10 + 20*a^2*b^2*c^2*d^6*e^12 - 48*a*b*c^4*d^9*e^9 - 8*a^4*b*c*d^3*e^15 + 20*a*b^2*c^3*d^8*e^10 + 12*a*b^3*c^2*d^7*e^11 - 104*a^2*b*c^3*d^7*e^11 - 6*a^2*b^3*c*d^5*e^13 - 32*a^3*b*c^2*d^5*e^13 + 12*a^3*b^2*c*d^4*e^14) + (60*a*c^3*d^4*e^12*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^4*d^8*e^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*b^3*c*d^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*e^12 + (20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2*d^5*e^13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a) - (104*b*c^3*d^5*e^11*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^4*d^8*e^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*b^3*c*d^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*e^12 + (20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2*d^5*e^13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d

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^9*e^9)/a) - (6*b^3*c*d^3*e^13*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^4*d^8*e
^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*b^3*c*d
^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*e^12 +
(20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*e^8)/a^
2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2*d^5*e^
13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a) + (20
*b^2*c^3*d^6*e^10*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(18*c^5*d^10*e^8 + 72*a*c^
4*d^8*e^10 + 2*a^4*c*d^2*e^16 - 48*b*c^4*d^9*e^9 + 60*a^2*c^3*d^6*e^12 + 8*
a^3*c^2*d^4*e^14 + 20*b^2*c^3*d^8*e^10 + 12*b^3*c^2*d^7*e^11 - (4*b^2*c^4*d
^10*e^8)/a + (10*b^3*c^3*d^9*e^9)/a - (6*b^4*c^2*d^8*e^10)/a - 104*a*b*c^3*
d^7*e^11 - 6*a*b^3*c*d^5*e^13 - 8*a^3*b*c*d^3*e^15 + 20*a*b^2*c^2*d^6*e^12
- 32*a^2*b*c^2*d^5*e^13 + 12*a^2*b^2*c*d^4*e^14) + (12*b^3*c^2*d^5*e^11*(d
+ e*x^2)^(1/2)*(d^3)^(1/2))/(18*c^5*d^10*e^8 + 72*a*c^4*d^8*e^10 + 2*a^4*c*
d^2*e^16 - 48*b*c^4*d^9*e^9 + 60*a^2*c^3*d^6*e^12 + 8*a^3*c^2*d^4*e^14 + 20
*b^2*c^3*d^8*e^10 + 12*b^3*c^2*d^7*e^11 - (4*b^2*c^4*d^10*e^8)/a + (10*b^3*
c^3*d^9*e^9)/a - (6*b^4*c^2*d^8*e^10)/a - 104*a*b*c^3*d^7*e^11 - 6*a*b^3*c*
d^5*e^13 - 8*a^3*b*c*d^3*e^15 + 20*a*b^2*c^2*d^6*e^12 - 32*a^2*b*c^2*d^5*e^
13 + 12*a^2*b^2*c*d^4*e^14) - (32*a*b*c^2*d^3*e^13*(d + e*x^2)^(1/2)*(d^3)^
(1/2))/(72*c^4*d^8*e^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*
d^7*e^11 - 6*b^3*c*d^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20
*b^2*c^2*d^6*e^12 + (20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*
b^2*c^4*d^10*e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2
 - 32*a*b*c^2*d^5*e^13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c
^4*d^9*e^9)/a) + (12*a*b^2*c*d^2*e^14*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^
4*d^8*e^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*
b^3*c*d^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*
e^12 + (20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*
e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2
*d^5*e^13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a
) - (8*a^2*b*c*d*e^15*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^4*d^8*e^10 + 60*
a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*b^3*c*d^5*e^13 +
 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*e^12 + (20*b^2*c
^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*e^8)/a^2 + (10*b
^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2*d^5*e^13 + 12*a
*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a))*(d^3)^(1/2))/
a
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.270 $\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=417

$$-\frac{\sqrt{c}\left(-2a\left(e\left(d\sqrt{b^2-4ac}-ae\right)+cd^2\right)+bd\left(d\sqrt{b^2-4ac}-2ae\right)+b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}\left(-2a\left(\right.\right.}{}$$

Rubi [A] time = 3.24, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1251, 897, 1287, 199, 206, 1166, 208}

$$-\frac{\sqrt{c}\left(bd\left(d\sqrt{b^2-4ac}-2ae\right)-2ae\left(d\sqrt{b^2-4ac}-ae\right)-2acd^2+b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}\left(-bd\left(d\sqrt{b^2-4ac}+2ae\right)+2ae\left(d\sqrt{b^2-4ac}+ae\right)-2acd^2+b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{d}(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2}-\frac{d\sqrt{d+ex^2}}{2ax^2}+\frac{\sqrt{d}\,e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(d*Sqrt[d + e*x^2])/(2*a*x^2) + (Sqrt[d]*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]
])/(2*a) + (Sqrt[d]*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a^2 - (
Sqrt[c]*(b^2*d^2 - 2*a*c*d^2 + b*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) - 2*a*e*(S
qrt[b^2 - 4*a*c]*d - a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c
*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[c]*(b^2*d^2 - 2*a*c*d^2 + 2*a*e*(S
qrt[b^2 - 4*a*c]*d + a*e) - b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e))*ArcTanh[(Sqr
t[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sq
rt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
```

```
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}}{x^3\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(d+ex)^{3/2}}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{\operatorname{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{d^2e^2}{a\left(d-x^2\right)^2}-\frac{de(-bd+2ae)}{a^2\left(d-x^2\right)}+\frac{e\left(-\left((bd-ae)\left(cd^2-bde+ae^2\right)\right)+cd(bd-2ae)x^2\right)}{a^2\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx, x, \sqrt{d+ex^2}\cdots}{e}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{-\left((bd-ae)\left(cd^2-bde+ae^2\right)\right)+cd(bd-2ae)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{a^2}+\frac{\left(d^2e\right)\operatorname{Subst}\left(\int \frac{1}{\left(d-x^2\right)^2}\cdots\right.}{a}$$

$$= -\frac{d\sqrt{d+ex^2}}{2ax^2}+\frac{\sqrt{d}\,(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2}+\frac{(de)\operatorname{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex^2}\cdots\right.}{2a}$$

$$= -\frac{d\sqrt{d+ex^2}}{2ax^2}+\frac{\sqrt{d}\,e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a}+\frac{\sqrt{d}\,(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2}-\frac{\sqrt{c}\,\left(b^2d\cdots\right.}{\cdots}$$

Mathematica [A] time = 1.60, size = 380, normalized size = 0.91

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(\frac{\left(2a\left(e\left(d\sqrt{b^2-4ac}-ae\right)+cd^2\right)+bd\left(2ae-d\sqrt{b^2-4ac}\right)-b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{\left(bd\left(d\sqrt{b^2-4ac}+2ae\right)-2ae\left(d\sqrt{b^2-4ac}+ae\right)+2acd^2-b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}}+\sqrt{d}\,(2bd-3ae)\log\left(\sqrt{d}\sqrt{d+ex^2}+d\right)-\sqrt{d}\log(x)(2bd-3ae)-\frac{ad\sqrt{d+ex^2}}{x^2}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-((a*d*Sqrt[d + e*x^2])/x^2) + (Sqrt[2]*Sqrt[c]*(((-(b^2*d^2) + b*d*(-(Sqr
t[b^2 - 4*a*c]*d) + 2*a*e) + 2*a*(c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d - a*e)))*A
rcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*
c]*e]])/Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e] - ((-(b^2*d^2) + 2*a*c*d^2
 - 2*a*e*(Sqrt[b^2 - 4*a*c]*d + a*e) + b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e))*A
rcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/Sqrt[b^2 - 4*a*c] - Sqrt[
d]*(2*b*d - 3*a*e)*Log[x] + Sqrt[d]*(2*b*d - 3*a*e)*Log[d + Sqrt[d]*Sqrt[d
+ e*x^2]])/(2*a^2)
```

IntegrateAlgebraic [C] time = 2.12, size = 548, normalized size = 1.31

$$\frac{\left(-2i\sqrt{2}\,a^2\sqrt{c}\,e^2+\sqrt{2}\,b\sqrt{c}\,d^2\sqrt{4ac-b^2}-2\sqrt{2}\,a\sqrt{c}\,de\sqrt{4ac-b^2}+2i\sqrt{2}\,ab\sqrt{c}\,de+2i\sqrt{2}\,ac^{3/2}d^2-i\sqrt{2}\,b^2\sqrt{c}\,d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}\right)}{2a^2\sqrt{4ac-b^2}\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}+\frac{\left(2i\sqrt{2}\,a^2\sqrt{c}\,e^2+\sqrt{2}\,b\sqrt{c}\,d^2\sqrt{4ac-b^2}-2\sqrt{2}\,a\sqrt{c}\,de\sqrt{4ac-b^2}-2i\sqrt{2}\,ab\sqrt{c}\,de-2i\sqrt{2}\,ac^{3/2}d^2+i\sqrt{2}\,b^2\sqrt{c}\,d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}\right)}{2a^2\sqrt{4ac-b^2}\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}+\frac{\left(2bd^{3/2}-3a\sqrt{d}\,e\right)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a^2}-\frac{d\sqrt{d+ex^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*(d*Sqrt[d + e*x^2])/(a*x^2) + (((-I)*Sqrt[2]*b^2*Sqrt[c]*d^2 + (2*I)*S
qrt[2]*a*c^(3/2)*d^2 + Sqrt[2]*b*Sqrt[c]*Sqrt[-b^2 + 4*a*c]*d^2 + (2*I)*Sqr
t[2]*a*b*Sqrt[c]*d*e - 2*Sqrt[2]*a*Sqrt[c]*Sqrt[-b^2 + 4*a*c]*d*e - (2*I)*S
qrt[2]*a^2*Sqrt[c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*
d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(2*a^2*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d +
 b*e - I*Sqrt[-b^2 + 4*a*c]*e]) + ((I*Sqrt[2]*b^2*Sqrt[c]*d^2 - (2*I)*Sqrt[
2]*a*c^(3/2)*d^2 + Sqrt[2]*b*Sqrt[c]*Sqrt[-b^2 + 4*a*c]*d^2 - (2*I)*Sqrt[2]
*a*b*Sqrt[c]*d*e - 2*Sqrt[2]*a*Sqrt[c]*Sqrt[-b^2 + 4*a*c]*d*e + (2*I)*Sqrt[
2]*a^2*Sqrt[c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d +
b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(2*a^2*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e
 + I*Sqrt[-b^2 + 4*a*c]*e]) + ((2*b*d^(3/2) - 3*a*Sqrt[d]*e)*ArcTanh[Sqrt[d
 + e*x^2]/Sqrt[d]])/(2*a^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.72, size = 433, normalized size = 1.04

$$-\frac{\left(2\,bd^2-3\,ade\right)\arctan\left(\frac{\sqrt{x^2e+d}}{\sqrt{-d}}\right)}{2\,a^2\sqrt{-d}}-\frac{\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}\,c\right)e}\left(\left(b^2-2\,ac+\sqrt{b^2-4\,ac}\,b\right)d-\left(ab+\sqrt{b^2-4\,ac}\,a\right)e\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{2\,a^2cd-a^2be+\sqrt{-4\left(a^2cd^2-a^2bde+a^3e^2\right)a^2c+\left(2\,a^2cd-a^2be\right)^2}}{a^2c}}}\right)}{4\sqrt{b^2-4\,ac}\,a^2|c|}+\frac{\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}\,c\right)e}\left(\left(b^2-2\,ac-\sqrt{b^2-4\,ac}\,b\right)d-\left(ab-\sqrt{b^2-4\,ac}\,a\right)e\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{2\,a^2cd-a^2be-\sqrt{-4\left(a^2cd^2-a^2bde+a^3e^2\right)a^2c+\left(2\,a^2cd-a^2be\right)^2}}{a^2c}}}\right)}{4\sqrt{b^2-4\,ac}\,a^2|c|}-\frac{\sqrt{x^2e+d}\,d}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*d^2 - 3*a*d*e)*arctan(sqrt(x^2*e + d)/sqrt(-d))/(a^2*sqrt(-d)) -
1/4*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^2 - 2*a*c + sqrt(b
^2 - 4*a*c)*b)*d - (a*b + sqrt(b^2 - 4*a*c)*a)*e)*arctan(2*sqrt(1/2)*sqrt(x
^2*e + d)/sqrt(-(2*a^2*c*d - a^2*b*e + sqrt(-4*(a^2*c*d^2 - a^2*b*d*e + a^3
*e^2)*a^2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/(sqrt(b^2 - 4*a*c)*a^2*ab
s(c)) + 1/4*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^2 - 2*a*c
- sqrt(b^2 - 4*a*c)*b)*d - (a*b - sqrt(b^2 - 4*a*c)*a)*e)*arctan(2*sqrt(1/2
)*sqrt(x^2*e + d)/sqrt(-(2*a^2*c*d - a^2*b*e - sqrt(-4*(a^2*c*d^2 - a^2*b*d
```

```
*e + a^3*e^2)*a^2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/(sqrt(b^2 - 4*a*c
)*a^2*abs(c)) - 1/2*sqrt(x^2*e + d)*d/(a*x^2)
```

maple [C] time = 0.04, size = 555, normalized size = 1.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] -7/24/a^2*b*(e*x^2+d)^(3/2)+1/a^2*b*d^(3/2)*ln((2*d+2*(e*x^2+d)^(1/2)*d^(1/
2))/x)-3/8/a^2*b*(e*x^2+d)^(1/2)*d-1/6/a^2*e^(3/2)*x^3*b+1/8/a^2*e*(e*x^2+d
)^(1/2)*x^2*b-3/4/a^2*e^(1/2)*x*b*d+1/2/a*e^(3/2)*x+1/a*(e*x^2+d)^(1/2)*e+1
/4/a^2*sum((c*d*(-2*a*e+b*d)*_R^6+(4*a^2*e^3-8*a*b*d*e^2+2*a*c*d^2*e+4*b^2*
d^2*e-3*b*c*d^3)*_R^4+d*(-4*a^2*e^3+8*a*b*d*e^2-2*a*c*d^2*e-4*b^2*d^2*e+3*b
*c*d^3)*_R^2+2*a*c*d^4*e-b*c*d^5)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^
2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln(-e^(1/2)*x-_R+(e*x^2+d)
^(1/2)),_R=RootOf(_Z^8*c+(4*b*e-4*c*d)*_Z^6+c*d^4+(16*a*e^2-8*b*d*e+6*c*d^2
)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2))-1/2/a*d/(-e^(1/2)*x+(e*x^2+d)^(1/2))*e+5/
8/a^2*d^2/(-e^(1/2)*x+(e*x^2+d)^(1/2))*b+1/24/a^2*b*d^3/(-e^(1/2)*x+(e*x^2+
d)^(1/2))^3-1/2/a/d/x^2*(e*x^2+d)^(5/2)+1/2/a*e/d*(e*x^2+d)^(3/2)-3/2/a*e*d
^(1/2)*ln((2*d+2*(e*x^2+d)^(1/2)*d^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}}{\left(cx^4+bx^2+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^3), x)
```

mupad [B] time = 6.10, size = 35855, normalized size = 85.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (d^(1/2)*atan(((d^(1/2)*(3*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16
 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^
4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*
d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^
3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^
8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11
 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4) - (d^(1/2)*((56*a^4*c^6*d^6*e^9 - 44*a^5
*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*
c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b
^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*
a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 +
 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a
*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c
^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14)/a^4 + (d^(1/2)
*(3*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^1
2 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a
^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144
*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 +
 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e
^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d
```

^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*
d*e^12))/(2*a^4) + (d^(1/2)*((320*a^8*c^4*d*e^11 + 320*a^7*c^5*d^3*e^9 + 32
*a^5*b^3*c^4*d^4*e^8 - 24*a^5*b^4*c^3*d^3*e^9 - 8*a^5*b^5*c^2*d^2*e^10 + 16
*a^6*b^2*c^4*d^3*e^9 + 144*a^6*b^3*c^3*d^2*e^10 - 128*a^6*b*c^5*d^4*e^8 + 8
*a^6*b^4*c^2*d*e^11 - 448*a^7*b*c^4*d^2*e^10 - 112*a^7*b^2*c^3*d*e^11)/a^4
- (d^(1/2)*(d + e*x^2)^(1/2)*(3*a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^
4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*
d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*
d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a^6))*(3*a*e - 2*b*d))/(4*a^2)))/(4*a^2)
)*(3*a*e - 2*b*d))/(4*a^2))*1i)/(4*a^2) + (d^(1/2)*(3*a*e - 2*b*d)*(((d + e
x^2)^(1/2)(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*
a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5
*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c
^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*
c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e
^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4) + (d^(1/2)*((
56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^
3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3
*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*
a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12
- 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14
- 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3
*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3
*c^2*d*e^14)/a^4 - (d^(1/2)*(3*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(64*a^7*b*c
^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 +
736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 22
4*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10
+ 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e
^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3
*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*
d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4) - (d^(1/2)*((320*a^8*c^4*d*e^11
 + 320*a^7*c^5*d^3*e^9 + 32*a^5*b^3*c^4*d^4*e^8 - 24*a^5*b^4*c^3*d^3*e^9 -
8*a^5*b^5*c^2*d^2*e^10 + 16*a^6*b^2*c^4*d^3*e^9 + 144*a^6*b^3*c^3*d^2*e^10
- 128*a^6*b*c^5*d^4*e^8 + 8*a^6*b^4*c^2*d*e^11 - 448*a^7*b*c^4*d^2*e^10 - 1
12*a^7*b^2*c^3*d*e^11)/a^4 + (d^(1/2)*(d + e*x^2)^(1/2)*(3*a*e - 2*b*d)*(10
24*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5
*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c
^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a^6))*(3*a*e
- 2*b*d))/(4*a^2)))/(4*a^2))*(3*a*e - 2*b*d))/(4*a^2))*1i)/(4*a^2))/((3*a*c
^7*d^9*e^9 + 3*a^5*c^3*d*e^17 - 2*b*c^7*d^10*e^8 + 3*a^2*c^6*d^7*e^11 + 3*a
^4*c^4*d^3*e^15 + 4*b^2*c^6*d^9*e^9 - 2*b^3*c^5*d^8*e^10 + 2*a^2*b^2*c^4*d^
5*e^13 - (11*a^2*b^3*c^3*d^4*e^14)/2 + 11*a^3*b^2*c^3*d^3*e^15 - 8*a*b*c^6*
d^8*e^10 + 4*a*b^2*c^5*d^7*e^11 + a*b^4*c^3*d^5*e^13 - (3*a^2*b*c^5*d^6*e^1
2)/2 - 5*a^3*b*c^4*d^4*e^14 - (19*a^4*b*c^3*d^2*e^16)/2)/a^4 - (d^(1/2)*(3*
a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^
3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*
e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^
4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3
*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^
9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/
(2*a^4) - (d^(1/2)*((56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4
*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c
^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b
^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*
a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 -
 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*
c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4
*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14)/a^4 + (d^(1/2)*(3*a*e - 2*b*d)*(((d + e*x
^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 1

60*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2
*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*
a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 -
716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e
^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e
^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4) + (d^(1/2)
*((320*a^8*c^4*d*e^11 + 320*a^7*c^5*d^3*e^9 + 32*a^5*b^3*c^4*d^4*e^8 - 24*a
^5*b^4*c^3*d^3*e^9 - 8*a^5*b^5*c^2*d^2*e^10 + 16*a^6*b^2*c^4*d^3*e^9 + 144*
a^6*b^3*c^3*d^2*e^10 - 128*a^6*b*c^5*d^4*e^8 + 8*a^6*b^4*c^2*d*e^11 - 448*a
^7*b*c^4*d^2*e^10 - 112*a^7*b^2*c^3*d*e^11)/a^4 - (d^(1/2)*(d + e*x^2)^(1/2
)*(3*a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^
3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d
^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e
^9))/(8*a^6))*(3*a*e - 2*b*d))/(4*a^2)))/(4*a^2))*(3*a*e - 2*b*d))/(4*a^2))
)/(4*a^2) + (d^(1/2)*(3*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16 +
4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d
^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5
*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d
^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e
^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 -
60*a^4*b*c^4*d^3*e^13))/(2*a^4) + (d^(1/2)*((56*a^4*c^6*d^6*e^9 - 44*a^5*c^
5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3
*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*
c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4
*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 11
1*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^
6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*
d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14)/a^4 - (d^(1/2)*(3
*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 -
16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*
b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^
3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 14
4*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11
 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*
e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e
^12))/(2*a^4) - (d^(1/2)*((320*a^8*c^4*d*e^11 + 320*a^7*c^5*d^3*e^9 + 32*a^
5*b^3*c^4*d^4*e^8 - 24*a^5*b^4*c^3*d^3*e^9 - 8*a^5*b^5*c^2*d^2*e^10 + 16*a^
6*b^2*c^4*d^3*e^9 + 144*a^6*b^3*c^3*d^2*e^10 - 128*a^6*b*c^5*d^4*e^8 + 8*a^
6*b^4*c^2*d*e^11 - 448*a^7*b*c^4*d^2*e^10 - 112*a^7*b^2*c^3*d*e^11)/a^4 + (
d^(1/2)*(d + e*x^2)^(1/2)*(3*a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c
^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2
*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e
^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a^6))*(3*a*e - 2*b*d))/(4*a^2)))/(4*a^2))*(
3*a*e - 2*b*d))/(4*a^2)))/(4*a^2)))*(3*a*e - 2*b*d)*1i)/(2*a^2) - atan(((((
224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d^4*e^11 - 400*a^6*c^4*d^2*e^13 + 160*a^2
*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*d^5*e^10 - 44*a^2*b^6*c^2*d^4*e^11 - 432
*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*c^4*d^5*e^10 + 444*a^3*b^4*c^3*d^4*e^11
+ 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4*b^2*c^4*d^4*e^11 - 644*a^4*b^3*c^3*d^3*
e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 444*a^5*b^2*c^3*d^2*e^13 - 112*a^6*b*c^3*d
*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a*b^6*c^3*d^6*e^9 + 8*a*b^7*c^2*d^5*e^10
- 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b*c^5*d^5*e^10 + 1008*a^5*b*c^4*d^3*e^12
+ 24*a^5*b^3*c^2*d*e^14)/(4*a^4) + (((1280*a^8*c^4*d*e^11 + 1280*a^7*c^5*d^
3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a^5*b^4*c^3*d^3*e^9 - 32*a^5*b^5*c^2*d
^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 + 576*a^6*b^3*c^3*d^2*e^10 - 512*a^6*b*c^5
*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 - 1792*a^7*b*c^4*d^2*e^10 - 448*a^7*b^2*c^
3*d*e^11)/(4*a^4) - ((d + e*x^2)^(1/2)*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a
^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*
a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^
3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a

```
^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^
2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2
*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c
^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4
*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*
d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2
)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^
8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^
8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*((
((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^
2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d
^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 -
 (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*
d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d
^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*
b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e
^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e +
42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4
+ 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13
 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6
*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b
^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a
^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 1
32*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^
11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^1
1 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^
3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a
*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3
*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^
5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2
*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*
d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^
3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*
c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d
^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)
)*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^
4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b
^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2
/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*
c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c
^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2)
+ 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2
*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*
e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*
b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(4*a^6*c^3*e^1
6 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c
^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4
*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c
^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d
^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^1
1 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3
*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*
b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*
b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5
*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*
d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d
^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3
*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c
*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^
```

2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*
1i - (((224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d^4*e^11 - 400*a^6*c^4*d^2*e^13 +
160*a^2*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*d^5*e^10 - 44*a^2*b^6*c^2*d^4*e^
11 - 432*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*c^4*d^5*e^10 + 444*a^3*b^4*c^3*d
^4*e^11 + 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4*b^2*c^4*d^4*e^11 - 644*a^4*b^3*
c^3*d^3*e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 444*a^5*b^2*c^3*d^2*e^13 - 112*a^6
*b*c^3*d*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a*b^6*c^3*d^6*e^9 + 8*a*b^7*c^2*d
^5*e^10 - 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b*c^5*d^5*e^10 + 1008*a^5*b*c^4*d
^3*e^12 + 24*a^5*b^3*c^2*d*e^14)/(4*a^4) + (((1280*a^8*c^4*d*e^11 + 1280*a^
7*c^5*d^3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a^5*b^4*c^3*d^3*e^9 - 32*a^5*b
^5*c^2*d^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 + 576*a^6*b^3*c^3*d^2*e^10 - 512*a
^6*b*c^5*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 - 1792*a^7*b*c^4*d^2*e^10 - 448*a^
7*b^2*c^3*d*e^11)/(4*a^4) + ((d + e*x^2)^(1/2)*((((4*b^6*d^3 - 4*a^3*b^3*e^
3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d
^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e
- 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2
- 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 +
3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6
*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 -
16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 -
16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^
3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c
)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 +
1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 -
1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*
a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 9
6*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12
*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^
2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 +
3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3
*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1
/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4
*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*
d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(
a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(64*a^7*b*
c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 +
736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 2
24*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10
+ 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*
e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^
3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4
*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3
- 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^
3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e -
144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2
- 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3
*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*
a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 1
6*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 1
6*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3
*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)
))^(1/2))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2
+ 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3
- 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*
d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^
6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2
- 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4)
)^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48
*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*

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b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(
16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(4*a^6
*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 -
 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2
*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a
^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b
^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5
*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3
- 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3
 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e -
144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 -
 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*
a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a
*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16
*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16
*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*
b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))
)^(1/2)*1i)/((6*a*c^7*d^9*e^9 + 6*a^5*c^3*d*e^17 - 4*b*c^7*d^10*e^8 + 6*a^2
*c^6*d^7*e^11 + 6*a^4*c^4*d^3*e^15 + 8*b^2*c^6*d^9*e^9 - 4*b^3*c^5*d^8*e^10
 + 4*a^2*b^2*c^4*d^5*e^13 - 11*a^2*b^3*c^3*d^4*e^14 + 22*a^3*b^2*c^3*d^3*e^
15 - 16*a*b*c^6*d^8*e^10 + 8*a*b^2*c^5*d^7*e^11 + 2*a*b^4*c^3*d^5*e^13 - 3*
a^2*b*c^5*d^6*e^12 - 10*a^3*b*c^4*d^4*e^14 - 19*a^4*b*c^3*d^2*e^16)/(2*a^4)
 + (((224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d^4*e^11 - 400*a^6*c^4*d^2*e^13 + 1
60*a^2*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*d^5*e^10 - 44*a^2*b^6*c^2*d^4*e^11
 - 432*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*c^4*d^5*e^10 + 444*a^3*b^4*c^3*d^4
*e^11 + 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4*b^2*c^4*d^4*e^11 - 644*a^4*b^3*c^
3*d^3*e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 444*a^5*b^2*c^3*d^2*e^13 - 112*a^6*b
*c^3*d*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a*b^6*c^3*d^6*e^9 + 8*a*b^7*c^2*d^5
*e^10 - 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b*c^5*d^5*e^10 + 1008*a^5*b*c^4*d^3
*e^12 + 24*a^5*b^3*c^2*d*e^14)/(4*a^4) + (((1280*a^8*c^4*d*e^11 + 1280*a^7*
c^5*d^3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a^5*b^4*c^3*d^3*e^9 - 32*a^5*b^5
*c^2*d^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 + 576*a^6*b^3*c^3*d^2*e^10 - 512*a^6
*b*c^5*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 - 1792*a^7*b*c^4*d^2*e^10 - 448*a^7*
b^2*c^3*d*e^11)/(4*a^4) - ((d + e*x^2)^(1/2)*((((4*b^6*d^3 - 4*a^3*b^3*e^3
- 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3
 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e -
144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 -
 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*
a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a
*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16
*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16
*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*
b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))
)^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1
536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1
792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*a^
4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*
a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a
*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)
^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*
a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b
*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2
) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c
^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^
2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^
4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(64*a^7*b*c^
3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 7
36*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224
*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 +
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432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^
10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*
d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d
^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 -
32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3
- 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 1
44*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 -
128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a
^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*
b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*
a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*
a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b
*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))
^(1/2))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 +
96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 -
12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*
e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6
+ 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 -
3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^
(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a
^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^
5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16
*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(4*a^6*c
^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2
*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b
^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3
*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2
*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d
^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 -
32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 -
32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 14
4*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 1
28*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^
2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b
*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a
^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a
*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*
c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^
(1/2) + (((224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d^4*e^11 - 400*a^6*c^4*d^2*e^1
3 + 160*a^2*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*d^5*e^10 - 44*a^2*b^6*c^2*d^4
*e^11 - 432*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*c^4*d^5*e^10 + 444*a^3*b^4*c^
3*d^4*e^11 + 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4*b^2*c^4*d^4*e^11 - 644*a^4*b
^3*c^3*d^3*e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 444*a^5*b^2*c^3*d^2*e^13 - 112*
a^6*b*c^3*d*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a*b^6*c^3*d^6*e^9 + 8*a*b^7*c^
2*d^5*e^10 - 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b*c^5*d^5*e^10 + 1008*a^5*b*c^
4*d^3*e^12 + 24*a^5*b^3*c^2*d*e^14)/(4*a^4) + (((1280*a^8*c^4*d*e^11 + 1280
*a^7*c^5*d^3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a^5*b^4*c^3*d^3*e^9 - 32*a^
5*b^5*c^2*d^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 + 576*a^6*b^3*c^3*d^2*e^10 - 51
2*a^6*b*c^5*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 - 1792*a^7*b*c^4*d^2*e^10 - 448
*a^7*b^2*c^3*d*e^11)/(4*a^4) + ((d + e*x^2)^(1/2)*((((4*b^6*d^3 - 4*a^3*b^3
*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^
2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2
*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*
c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3
+ 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5
- 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3
- 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3
- 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72
*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^

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2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^1
0 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^
8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/
(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2
+ 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 -
 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d
*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6
 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2
- 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))
^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*
a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b
^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(1
6*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(64*a^7
*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^
8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9
- 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e
^10 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d
^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3
*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*
c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*
e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2
*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*
e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c
^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3
+ 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 -
 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3
- 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3
- 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*
a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2
*c)))^(1/2))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*
e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e
^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2
*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c
*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*
e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e
^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 +
 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6
*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2
)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(4*
a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^1
2 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*
a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 2
8*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*
a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*
c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e
^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*
d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e
 - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^
2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 +
 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 -
6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 -
 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 -
 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a
^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*
c)))^(1/2)))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*
e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e
^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2
*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c
*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*
```

e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e
^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 +
48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6
*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2
)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*2i - (d*(d + e*x^2)^(1/2
))/(2*a*x^2) - atan(((((224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d^4*e^11 - 400*a^
6*c^4*d^2*e^13 + 160*a^2*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*d^5*e^10 - 44*a^
2*b^6*c^2*d^4*e^11 - 432*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*c^4*d^5*e^10 + 4
44*a^3*b^4*c^3*d^4*e^11 + 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4*b^2*c^4*d^4*e^1
1 - 644*a^4*b^3*c^3*d^3*e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 444*a^5*b^2*c^3*d^
2*e^13 - 112*a^6*b*c^3*d*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a*b^6*c^3*d^6*e^9
+ 8*a*b^7*c^2*d^5*e^10 - 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b*c^5*d^5*e^10 +
1008*a^5*b*c^4*d^3*e^12 + 24*a^5*b^3*c^2*d*e^14)/(4*a^4) + (((1280*a^8*c^4*
d*e^11 + 1280*a^7*c^5*d^3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a^5*b^4*c^3*d^
3*e^9 - 32*a^5*b^5*c^2*d^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 + 576*a^6*b^3*c^3*
d^2*e^10 - 512*a^6*b*c^5*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 - 1792*a^7*b*c^4*d
^2*e^10 - 448*a^7*b^2*c^3*d*e^11)/(4*a^4) - ((d + e*x^2)^(1/2)*(-(((4*b^6*d
^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 +
72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84
*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*
b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 -
b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*
a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 +
2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a
^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^
3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*
c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a
^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*
b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^
3*c^3*d*e^9))/(2*a^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12
*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 1
6*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e -
72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*
d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b
^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*
b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*
b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*
b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*
b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2
)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160
*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b
^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^
3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 71
6*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^1
0 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^1
2 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4))*(-(((4*b^6*
d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2
+ 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 8
4*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4
*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2
- b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3
*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3
+ 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*
a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b
^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6
*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^
3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d
^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d
^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)

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*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4
 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3
+ 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 -
6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 -
8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 3
6*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d +
 e*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 13
2*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c
^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2
*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*
b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7
*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4))*(-(((4*b^6
*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2
 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e +
84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^
4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2
 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e -
3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3
 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36
*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*
b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^
6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i - (((224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d^4*
e^11 - 400*a^6*c^4*d^2*e^13 + 160*a^2*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*d^5
*e^10 - 44*a^2*b^6*c^2*d^4*e^11 - 432*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*c^4
*d^5*e^10 + 444*a^3*b^4*c^3*d^4*e^11 + 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4*b^
2*c^4*d^4*e^11 - 644*a^4*b^3*c^3*d^3*e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 444*a
^5*b^2*c^3*d^2*e^13 - 112*a^6*b*c^3*d*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a*b^
6*c^3*d^6*e^9 + 8*a*b^7*c^2*d^5*e^10 - 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b*c^
5*d^5*e^10 + 1008*a^5*b*c^4*d^3*e^12 + 24*a^5*b^3*c^2*d*e^14)/(4*a^4) + (((
1280*a^8*c^4*d*e^11 + 1280*a^7*c^5*d^3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a
^5*b^4*c^3*d^3*e^9 - 32*a^5*b^5*c^2*d^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 + 576
*a^6*b^3*c^3*d^2*e^10 - 512*a^6*b*c^5*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 - 179
2*a^7*b*c^4*d^2*e^10 - 448*a^7*b^2*c^3*d*e^11)/(4*a^4) + ((d + e*x^2)^(1/2)
*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^
4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b
^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2
/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*
c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c
^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2)
- 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2
*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*
e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*
b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2
*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e
^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9
 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3
*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*
b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*
b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5
*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*
d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d
^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3
*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c
*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^
2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)
- ((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c
^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e
^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4
*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4
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*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i)/((6*a*c^7*d^9*e^9 + 6*a^5*c^3*d*e^17 - 4*b*c^7*d^10*e^8 + 6*a^2*c^6*d^7*e^11 + 6*a^4*c^4*d^3*e^15 + 8*b^2*c^6*d^9*e^9 - 4*b^3*c^5*d^8*e^10 + 4*a^2*b^2*c^4*d^5*e^13 - 11*a^2*b^3*c^3*d^4*e^14 + 22*a^3*b^2*c^3*d^3*e^15 - 16*a*b*c^6*d^8*e^10 + 8*a*b^2*c^5*d^7*e^11 + 2*a*b^4*c^3*d^5*e^13 - 3*a^2*b*c^5*d^6*e^12 - 10*a^3*b*c^4*d^4*e^14 - 19*a^4*b*c^3*d^2*e^16)/(2*a^4) + (((224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d^4*e^11 - 400*a^6*c^4*d^2*e^13 + 160*a^2*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*d^5*e^10 - 44*a^2*b^6*c^2*d^4*e^11 - 432*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*c^4*d^5*e^10 + 444*a^3*b^4*c^3*d^4*e^11 + 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4*b^2*c^4*d^4*e^11 - 644*a^4*b^3*c^3*d^3*e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 444*a^5*b^2*c^3*d^2*e^13 - 112*a^6*b*c^3*d*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a*b^6*c^3*d^6*e^9 + 8*a*b^7*c^2*d^5*e^10 - 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b*c^5*d^5*e^10 + 1008*a^5*b*c^4*d^3*e^12 + 24*a^5*b^3*c^2*d*e^14)/(4*a^4) + (((1280*a^8*c^4*d*e^11 + 1280*a^7*c^5*d^3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a^5*b^4*c^3*d^3*e^9 - 32*a^5*b^5*c^2*d^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 + 576*a^6*b^3*c^3*d^2*e^10 - 512*a^6*b*c^5*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 - 1792*a^7*b*c^4*d^2*e^10 - 448*a^7*b^2*c^3*d*e^11))/(4*a^4) - ((d + e*x^2)^(1/2)*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*

b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/
2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*
c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d
^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a
^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*
c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^
2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*
e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*
a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32
*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a
^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*
a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c
^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^
2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*
c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^
4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2
*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/
2) + ((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^
3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^
5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*
d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*
c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5
*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a
^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a
^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 9
6*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12
*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^
2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 +
3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3
*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1
/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4
*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*
d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(
a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3
 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^
3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e -
 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2
- 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3
*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*
a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 1
6*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 1
6*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3
*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)
))^(1/2) - ((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c
^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8
 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e
^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^
2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 +
 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*
a^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 +
96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 1
2*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e
^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 +
 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 -
3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(
1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^
4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5
*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*
(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (((224*a^4*c^6*d^6*e^9 - 176

*a^5*c^5*d^4*e^11 - 400*a^6*c^4*d^2*e^13 + 160*a^2*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*d^5*e^10 - 44*a^2*b^6*c^2*d^4*e^11 - 432*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*c^4*d^5*e^10 + 444*a^3*b^4*c^3*d^4*e^11 + 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4*b^2*c^4*d^4*e^11 - 644*a^4*b^3*c^3*d^3*e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 444*a^5*b^2*c^3*d^2*e^13 - 112*a^6*b*c^3*d*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a*b^6*c^3*d^6*e^9 + 8*a*b^7*c^2*d^5*e^10 - 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b*c^5*d^5*e^10 + 1008*a^5*b*c^4*d^3*e^12 + 24*a^5*b^3*c^2*d*e^14)/(4*a^4) + (((1280*a^8*c^4*d*e^11 + 1280*a^7*c^5*d^3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a^5*b^4*c^3*d^3*e^9 - 32*a^5*b^5*c^2*d^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 + 576*a^6*b^3*c^3*d^2*e^10 - 512*a^6*b*c^5*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 - 1792*a^7*b*c^4*d^2*e^10 - 448*a^7*b^2*c^3*d*e^11)/(4*a^4) + ((d + e*x^2)^(1/2)*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*

```
a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8
 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c
^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4
*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*
b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5
*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*
e^13))/(2*a^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^
4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b
*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3
*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a
^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*
d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d
^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e
^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3
 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d
*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)))*(-(((4*b^6*d^3 - 4
*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^
2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b
^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 +
256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c
*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*
c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3
*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2
*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^
2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 -
8*a^5*b^2*c)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/x**3/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

3.271 $\int \frac{x^4(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=595

$$-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^3\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd}\cdots}{}$$

Rubi [A] time = 3.28, antiderivative size = 595, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {1291, 388, 195, 217, 206, 1692, 402, 377, 205}

$$-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^3\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{2c^3\sqrt{\sqrt{b^2-4ac}+b}}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)}{2c^3}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)}{2c^3}+\frac{x\sqrt{d+ex^2}(3cd-4be)}{8c^2}+\frac{d(3cd-4be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8c^2\sqrt{e}}+\frac{x\left(d+ex^2\right)^{3/2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((3*c*d - 4*b*e)*x*Sqrt[d + e*x^2])/(8*c^2) + (x*(d + e*x^2)^(3/2))/(4*c) -
 (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(b*c*d - b^2*e + a*c*e - (b^2*c*d
 - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (
b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]
)/(2*c^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b
 + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*c^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]
) + (d*(3*c*d - 4*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*c^2*Sqrt[e]
) - (Sqrt[e]*(b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*
c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^3) - (Sq
rt[e]*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sq
rt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 1291

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[f^4/c^2, Int[(f*x)^(m - 4)*(c*d - b*e + c
*e*x^2)*(d + e*x^2)^(q - 1), x], x] - Dist[f^4/c^2, Int[((f*x)^(m - 4)*(d +
 e*x^2)^(q - 1)*Simp[a*(c*d - b*e) + (b*c*d - b^2*e + a*c*e)*x^2, x])/(a +
b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4\left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{\int \sqrt{d+ex^2}\left(cd-be+cex^2\right)dx}{c^2} - \frac{\int \frac{\sqrt{d+ex^2}\left(a(cd-be)+\left(bcd-b^2e+ace\right)x^2\right)}{a+bx^2+cx^4}\,dx}{c^2}$$

$$= \frac{x\left(d+ex^2\right)^{3/2}}{4c} - \frac{\int \left(\frac{\left(bcd-b^2e+ace+\frac{-b^2cd+2ac^2d+b^3e-3abce}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(bcd-b^2e+ace-\frac{-b^2cd+2ac^2d+b^3e-3abce}{\sqrt{b^2-4ac}}\right.}{b+\sqrt{b^2-4ac}+2cx^2}\right.}{c^2}$$

$$= \frac{(3cd-4be)x\sqrt{d+ex^2}}{8c^2} + \frac{x\left(d+ex^2\right)^{3/2}}{4c} + \frac{(d(3cd-4be))\int \frac{1}{\sqrt{d+ex^2}}\,dx}{8c^2} - \frac{\left(bcd-b^2e+a\right.}{}$$

$$= \frac{(3cd-4be)x\sqrt{d+ex^2}}{8c^2} + \frac{x\left(d+ex^2\right)^{3/2}}{4c} + \frac{(d(3cd-4be))\,\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{8c^2}$$

$$= \frac{(3cd-4be)x\sqrt{d+ex^2}}{8c^2} + \frac{x\left(d+ex^2\right)^{3/2}}{4c} + \frac{d(3cd-4be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8c^2\sqrt{e}} - \frac{\left(e\left(bcd-b^2\right.\right.}{}$$

$$= \frac{(3cd-4be)x\sqrt{d+ex^2}}{8c^2} + \frac{x\left(d+ex^2\right)^{3/2}}{4c} - \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(bcd-b^2e+ace\right.}{2c^3\sqrt{b-}}$$

Mathematica [B] time = 6.49, size = 18689, normalized size = 31.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
[Out] Result too large to show
```

IntegrateAlgebraic [F] time = 180.17, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
[Out] $Aborted
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 1.87, size = 104, normalized size = 0.17

$$\frac{1}{8}\sqrt{x^2e+d}\left(\frac{2\,x^2e}{c} + \frac{\left(5\,c^5de^2-4\,bc^4e^3\right)e^{(-2)}}{c^6}\right)x - \frac{\left(3\,c^2d^2-12\,bcde+8\,b^2e^2-8\,ace^2\right)e^{\left(-\frac{1}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{16\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(x^2*e + d)*(2*x^2*e/c + (5*c^5*d*e^2 - 4*b*c^4*e^3)*e^(-2)/c^6)*x
- 1/16*(3*c^2*d^2 - 12*b*c*d*e + 8*b^2*e^2 - 8*a*c*e^2)*e^(-1/2)*log((x*e^(
1/2) - sqrt(x^2*e + d))^2)/c^3
```

maple [C] time = 0.04, size = 516, normalized size = 0.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*x*(e*x^2+d)^(3/2)/c+3/8/c*d*x*(e*x^2+d)^(1/2)+3/8/c*d^2/e^(1/2)*ln(e^(1
/2)*x+(e*x^2+d)^(1/2))+1/4/c^2*e^(3/2)*b*x^2-1/4/c^2*e*b*(e*x^2+d)^(1/2)*x+
1/8/c^2*e^(1/2)*b*d-1/2/c^3*e^(1/2)*sum(((2*a*b*c*e^2-2*a*c^2*d*e-b^3*e^2+2
*b^2*c*d*e-b*c^2*d^2)*_R^2+2*(2*a^2*c*e^3-2*a*b^2*e^3+2*a*b*c*d*e^2+b^3*d*e
^2-2*b^2*c*d^2*e+b*c^2*d^3)*_R+2*a*b*c*d^2*e^2-2*a*c^2*d^3*e-b^3*d^2*e^2+2*
b^2*c*d^3*e-b*c^2*d^4)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+
3*_R*c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(
_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-
4*c*d^3)*_Z))+1/c^2*e^(3/2)*ln(-e^(1/2)*x+(e*x^2+d)^(1/2))*a-1/c^3*e^(3/2)*
ln(-e^(1/2)*x+(e*x^2+d)^(1/2))*b^2+3/2/c^2*e^(1/2)*ln(-e^(1/2)*x+(e*x^2+d)^
(1/2))*b*d-1/8/c^2*e^(1/2)*b*d^2/(-e^(1/2)*x+(e*x^2+d)^(1/2))^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}x^4}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*x^4/(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\left(e\,x^2+d\right)^{3/2}}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] int((x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(d+ex^2\right)^{\frac{3}{2}}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**4*(d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

3.272 $\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=491

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{}$$

Rubi [A] time = 1.80, antiderivative size = 491, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1293, 195, 217, 206, 1692, 402, 377, 205}

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2c^2}+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2c^2}+\frac{ex\sqrt{d+ex^2}}{2c}+\frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e*x*Sqrt[d + e*x^2])/(2*c) + (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d
 - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d -
(b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])
])/(2*c^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[
d + e*x^2])])/(2*c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (d*Sqrt[e]*ArcTanh[(Sqr
t[e]*x)/Sqrt[d + e*x^2]])/(2*c) + (Sqrt[e]*(c*d - b*e - (b*c*d - b^2*e + 2*
a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^2) + (
Sqrt[e]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(
Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^2)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 1293

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a
*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx = -\frac{\int \frac{\sqrt{d+ex^2}\left(ae-(cd-be)x^2\right)}{a+bx^2+cx^4}\,dx}{c}+\frac{e\int \sqrt{d+ex^2}\,dx}{c}$$

$$= \frac{ex\sqrt{d+ex^2}}{2c}-\frac{\int \left(\frac{\left(-cd+be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2}+\frac{\left(-cd+be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}\right)dx}{c}+\frac{(de)\int \frac{1}{\sqrt{d+ex^2}}

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [C] time = 154.73, size = 692, normalized size = 1.41

$$\frac{\sqrt{e}\,\text{RootSum}\left[\#1^8c+4\#1^6be-4\#1^6cd+16\#1^4ae^2-8\#1^4bde+6\#1^4cd^2+4\#1^2bd^2e-4\#1^2cd^3+cd^4\&,\frac{-\#1^4ace^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+\#1^4b^2e^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2\#1^4bcde\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+\#1^4c^2d^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+4\#1^2abe^3\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-6\#1^2acde^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2\#1^2b^2de^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+4\#1^2bcd^2e\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2\#1^2c^2d^3\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-acd^2e^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+b^2d^2e^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2bcd^3e\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+c^2d^4\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)}{\#1^6(-c)-3\#1^4be+3\#1^4cd-8\#1^2ae^2+4\#1^2bde-3\#1^2cd^2-bd^2e+cd^3}\&\right]}{2c^2}+\frac{\left(2be^{3/2}-3cd\sqrt{e}\right)\log\left(\sqrt{d+ex^2}-\sqrt{e}x\right)}{2c^2}+\frac{ex\sqrt{d+ex^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e*x*Sqrt[d + e*x^2])/(2*c) + ((-3*c*d*Sqrt[e] + 2*b*e^(3/2))*Log[-(Sqrt[e]
*x) + Sqrt[d + e*x^2]])/(2*c^2) + (Sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4
*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 +
4*b*e*#1^6 + c*#1^8 & , (c^2*d^4*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] -
 2*b*c*d^3*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] + b^2*d^2*e^2*Log[-(S
qrt[e]*x) + Sqrt[d + e*x^2] - #1] - a*c*d^2*e^2*Log[-(Sqrt[e]*x) + Sqrt[d +
 e*x^2] - #1] - 2*c^2*d^3*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 4
*b*c*d^2*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 - 2*b^2*d*e^2*Log[
-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 - 6*a*c*d*e^2*Log[-(Sqrt[e]*x) +
Sqrt[d + e*x^2] - #1]*#1^2 + 4*a*b*e^3*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] -
 #1]*#1^2 + c^2*d^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4 - 2*b*c*d
*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4 + b^2*e^2*Log[-(Sqrt[e]*x)
 + Sqrt[d + e*x^2] - #1]*#1^4 - a*c*e^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]
- #1]*#1^4)/(c*d^3 - b*d^2*e - 3*c*d^2*#1^2 + 4*b*d*e*#1^2 - 8*a*e^2*#1^2 +
 3*c*d*#1^4 - 3*b*e*#1^4 - c*#1^6) & ])/(2*c^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.97, size = 58, normalized size = 0.12

$$\frac{\sqrt{x^2e+d}\,xe}{2\,c}-\frac{\left(3\,cde-2\,be^2\right)e^{\left(-\frac{1}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2*e + d)*x*e/c - 1/4*(3*c*d*e - 2*b*e^2)*e^(-1/2)*log((x*e^(1/2)
 - sqrt(x^2*e + d))^2)/c^2
```

maple [C] time = 0.03, size = 382, normalized size = 0.78

$$-\frac{e^{\frac{3}{2}}x^2}{4c}+\frac{be^{\frac{3}{2}}\ln\left(-\sqrt{e}\,x+\sqrt{ex^2+d}\right)}{c^2}+\frac{d^2\sqrt{e}}{8\left(-\sqrt{e}\,x+\sqrt{ex^2+d}\right)^2c}-\frac{3d\sqrt{e}\,\ln\left(-\sqrt{e}\,x+\sqrt{ex^2+d}\right)}{2c}+\frac{\sqrt{ex^2+d}\,ex}{4c}-\frac{d\sqrt{e}}{8c}+\frac{\sqrt{e}\left(acd^2e^2-b^2d^2e^2+2bcd^3e-c^2d^4+\left(ace^2-b^2e^2+2bcde-c^2d^2\right)\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^2+2\left(-2abe^3+3ade^2c+b^2de^2-2bcd^2e+c^2d^3\right)\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)+\left(-\sqrt{e}\,x+\sqrt{ex^2+d}\right)^2\right)}{2c^2\left(\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^3c+3\,\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^2be-3\,\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^2cd+8\,\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)ae^2-4\,\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)bde+3\,\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)cd^2+bd^2e-cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4*e^(3/2)/c*x^2+1/4*e*x*(e*x^2+d)^(1/2)/c-1/8*e^(1/2)/c*d+1/2*e^(1/2)/c^
2*sum(((a*c*e^2-b^2*e^2+2*b*c*d*e-c^2*d^2)*_R^2+2*(-2*a*b*e^3+3*a*c*d*e^2+b
```

```
^2*d*e^2-2*b*c*d^2*e+c^2*d^3)*_R+a*c*d^2*e^2-b^2*d^2*e^2+2*b*c*d^3*e-c^2*d^
4)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c
*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-
4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))+1/8*e^
(1/2)/c*d^2/(-e^(1/2)*x+(e*x^2+d)^(1/2))^2+e^(3/2)/c^2*ln(-e^(1/2)*x+(e*x^2
+d)^(1/2))*b-3/2*e^(1/2)/c*ln(-e^(1/2)*x+(e*x^2+d)^(1/2))*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}x^2}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^(3/2)*x^2/(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\left(e\,x^2+d\right)^{3/2}}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x)
[Out] int((x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(d+ex^2\right)^{\frac{3}{2}}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x**2*(d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

3.273 $\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=487

$$\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}} - \frac{\left(-2ce\left(d\sqrt{b^2-4\right.\right.}{}$$

Rubi [A] time = 1.57, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1174, 416, 523, 217, 206, 377, 205}

$$\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(3cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}{2c\sqrt{b^2-4ac}}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(3cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}{2c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c
]*d + a*e))*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sq
rt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2
- 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((2*c^2*d^2 + b*(b + S
qrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[
d + e*x^2])])/(c*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[e]*(3*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c*Sqrt[b^2 - 4*a*c]) - (Sqrt[e]*(
3*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2
*c*Sqrt[b^2 - 4*a*c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
```

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1174

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{(2c)\int \frac{\left(d+ex^2\right)^{3/2}}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{\left(d+ex^2\right)^{3/2}}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\int \frac{d\left(4cd-\left(b-\sqrt{b^2-4ac}\right)e\right)+2e\left(3cd-\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{2\sqrt{b^2-4ac}} - \frac{\int \frac{d\left(4cd-\left(b+\sqrt{b^2-4ac}\right)e\right)+2e\left(3cd-\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,d}{2\sqrt{b^2-4ac}}$$

$$= \frac{\left(e\left(3cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\right)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2c\sqrt{b^2-4ac}} - \frac{\left(e\left(3cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\right)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2c\sqrt{b^2-4ac}} +$$

$$= \frac{\left(e\left(3cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\right)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c\sqrt{b^2-4ac}} - \frac{\left(e\left(3cd-\left(b+\sqrt{b^2-4ac}\right)e\right.\right.}{2c\sqrt{b}}$$

$$= \frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} -$$

Mathematica [B] time = 6.16, size = 9290, normalized size = 19.08

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [C] time = 121.93, size = 392, normalized size = 0.80

$$\frac{e^{3/2}\text{RootSum}\left[\#1^8c+4\#1^6be-4\#1^6cd+16\#1^4ae^2-8\#1^4bde+6\#1^4cd^2+4\#1^2bd^2e-4\#1^2cd^3+cd^4\&, \frac{-\#1^4be\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+2\#1^4cd\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-4\#1^2ae^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+2\#1^2bde\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-bd^2e\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+2cd^3\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)}{\#1^6(-c)-3\#1^4be+3\#1^4cd-8\#1^2ae^2+4\#1^2bde-3\#1^2cd^2-bd^2e+cd^3}\&\right]}{2c} - \frac{e^{3/2}\log\left(\sqrt{d+ex^2}-\sqrt{e}x\right)}{c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((e^(3/2)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/c) + (e^(3/2)*RootSum[c*d^4
 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#
1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (2*c*d^3*Log[-(Sqrt[e]*x) + Sqrt
[d + e*x^2] - #1] - b*d^2*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] + 2*b*
d*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 - 4*a*e^2*Log[-(Sqrt[e]*x
) + Sqrt[d + e*x^2] - #1]*#1^2 + 2*c*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] -
 #1]*#1^4 - b*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4)/(c*d^3 - b*d
^2*e - 3*c*d^2*#1^2 + 4*b*d*e*#1^2 - 8*a*e^2*#1^2 + 3*c*d*#1^4 - 3*b*e*#1^4
 - c*#1^6) & ])/(2*c)
```

fricas [B] time = 99.05, size = 7721, normalized size = 15.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(1/2)*c*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2
 - 2*a^2*c)*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 +
 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^
2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))*log((2*a*c^3*d^6 - 2*a*b*
c^2*d^5*e - 4*a^2*c^2*d^4*e^2 + 8*a^2*b*c*d^3*e^3 + 2*a^3*b*d*e^5 - 2*(a^2*
b^2 + 3*a^3*c)*d^2*e^4 + ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*
b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((c^4*d^6 - 6*a*c^3
*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^
2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) - (b*c^3*d^6 + 2*a*b*c^2*d^4*e^2 - 4*a^3*
b*e^6 - (b^2*c^2 + 4*a*c^3)*d^5*e + 4*(a*b^2*c + 2*a^2*c^2)*d^3*e^3 - (a*b^
3 + 19*a^2*b*c)*d^2*e^4 + (5*a^2*b^2 + 12*a^3*c)*d*e^5)*x^2 + 2*sqrt(1/2)*s
qrt(e*x^2 + d)*((2*(a^2*b^2*c^3 - 4*a^3*c^4)*d - (a^2*b^3*c^2 - 4*a^3*b*c^3
)*e)*x*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*
e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) + ((a*b^2*c
^2 - 4*a^2*c^3)*d^3*e - 3*(a^2*b^2*c - 4*a^3*c^2)*d*e^3 + (a^2*b^3 - 4*a^3*
b*c)*e^4)*x)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*
a^2*c)*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*
b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2
*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c
^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 - (a*b^2*c^2
 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c
^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*
b^2*c^2 - 4*a^2*c^3))*log((2*a*c^3*d^6 - 2*a*b*c^2*d^5*e - 4*a^2*c^2*d^4*e^
2 + 8*a^2*b*c*d^3*e^3 + 2*a^3*b*d*e^5 - 2*(a^2*b^2 + 3*a^3*c)*d^2*e^4 + ((a
*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2
- 4*a^3*c^3)*d*e^2)*x^2*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3
 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*
c^5)) - (b*c^3*d^6 + 2*a*b*c^2*d^4*e^2 - 4*a^3*b*e^6 - (b^2*c^2 + 4*a*c^3)*
d^5*e + 4*(a*b^2*c + 2*a^2*c^2)*d^3*e^3 - (a*b^3 + 19*a^2*b*c)*d^2*e^4 + (5
*a^2*b^2 + 12*a^3*c)*d*e^5)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*
c^3 - 4*a^3*c^4)*d - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e)*x*sqrt((c^4*d^6 - 6*a*c
^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*
b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) + ((a*b^2*c^2 - 4*a^2*c^3)*d^3*e - 3*(a
^2*b^2*c - 4*a^3*c^2)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)*sqrt(-(b*c^2*d^
3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 - (a*b^2*c^2 - 4*
a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^
2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c
^2 - 4*a^2*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a
*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6
- 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5
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 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))*log((2
*a*c^3*d^6 - 2*a*b*c^2*d^5*e - 4*a^2*c^2*d^4*e^2 + 8*a^2*b*c*d^3*e^3 + 2*a^
3*b*d*e^5 - 2*(a^2*b^2 + 3*a^3*c)*d^2*e^4 - ((a*b^2*c^3 - 4*a^2*c^4)*d^3 -
(a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt
((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2
*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) - (b*c^3*d^6 + 2*a*b*c
^2*d^4*e^2 - 4*a^3*b*e^6 - (b^2*c^2 + 4*a*c^3)*d^5*e + 4*(a*b^2*c + 2*a^2*c
^2)*d^3*e^3 - (a*b^3 + 19*a^2*b*c)*d^2*e^4 + (5*a^2*b^2 + 12*a^3*c)*d*e^5)*
x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^3 - 4*a^3*c^4)*d - (a^2*b^
3*c^2 - 4*a^3*b*c^3)*e)*x*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e
^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^
3*c^5)) - ((a*b^2*c^2 - 4*a^2*c^3)*d^3*e - 3*(a^2*b^2*c - 4*a^3*c^2)*d*e^3
+ (a^2*b^3 - 4*a^3*b*c)*e^4)*x)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*
d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a
*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^
2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3)))/x^2) + sqr
t(1/2)*c*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*
c)*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^
2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4
 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))*log((2*a*c^3*d^6 - 2*a*b*c^2*d^5*e
 - 4*a^2*c^2*d^4*e^2 + 8*a^2*b*c*d^3*e^3 + 2*a^3*b*d*e^5 - 2*(a^2*b^2 + 3*a
^3*c)*d^2*e^4 - ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^
2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2
+ 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a
^2*b^2*c^4 - 4*a^3*c^5)) - (b*c^3*d^6 + 2*a*b*c^2*d^4*e^2 - 4*a^3*b*e^6 - (
b^2*c^2 + 4*a*c^3)*d^5*e + 4*(a*b^2*c + 2*a^2*c^2)*d^3*e^3 - (a*b^3 + 19*a^
2*b*c)*d^2*e^4 + (5*a^2*b^2 + 12*a^3*c)*d*e^5)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2
 + d)*((2*(a^2*b^2*c^3 - 4*a^3*c^4)*d - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e)*x*sq
rt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a
^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) - ((a*b^2*c^2 - 4*a^
2*c^3)*d^3*e - 3*(a^2*b^2*c - 4*a^3*c^2)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)
*x)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^
3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3
*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*
a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3)))/x^2) + 2*e^(3/2)*log(-2*e*x^2 - 2*sqrt
(e*x^2 + d)*sqrt(e)*x - d))/c, 1/4*(sqrt(1/2)*c*sqrt(-(b*c^2*d^3 - 6*a*c^2*
d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqr
t((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^
2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c
^3))*log((2*a*c^3*d^6 - 2*a*b*c^2*d^5*e - 4*a^2*c^2*d^4*e^2 + 8*a^2*b*c*d^3
*e^3 + 2*a^3*b*d*e^5 - 2*(a^2*b^2 + 3*a^3*c)*d^2*e^4 + ((a*b^2*c^3 - 4*a^2*
c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^
2)*x^2*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*
e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) - (b*c^3*d^
6 + 2*a*b*c^2*d^4*e^2 - 4*a^3*b*e^6 - (b^2*c^2 + 4*a*c^3)*d^5*e + 4*(a*b^2*
c + 2*a^2*c^2)*d^3*e^3 - (a*b^3 + 19*a^2*b*c)*d^2*e^4 + (5*a^2*b^2 + 12*a^3
*c)*d*e^5)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^3 - 4*a^3*c^4)*
d - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e)*x*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*
b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2
*c^4 - 4*a^3*c^5)) + ((a*b^2*c^2 - 4*a^2*c^3)*d^3*e - 3*(a^2*b^2*c - 4*a^3*
c^2)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e
 + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^
4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c
*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3)))
/x^2) - sqrt(1/2)*c*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b
^2 - 2*a^2*c)*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2
 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(
a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))*log((2*a*c^3*d^6 - 2*a*
b*c^2*d^5*e - 4*a^2*c^2*d^4*e^2 + 8*a^2*b*c*d^3*e^3 + 2*a^3*b*d*e^5 - 2*(a^
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2*b^2 + 3*a^3*c)*d^2*e^4 + ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^
2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((c^4*d^6 - 6*a*c
^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*
b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) - (b*c^3*d^6 + 2*a*b*c^2*d^4*e^2 - 4*a^
3*b*e^6 - (b^2*c^2 + 4*a*c^3)*d^5*e + 4*(a*b^2*c + 2*a^2*c^2)*d^3*e^3 - (a*
b^3 + 19*a^2*b*c)*d^2*e^4 + (5*a^2*b^2 + 12*a^3*c)*d*e^5)*x^2 - 2*sqrt(1/2)
*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^3 - 4*a^3*c^4)*d - (a^2*b^3*c^2 - 4*a^3*b*c
^3)*e)*x*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^
2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) + ((a*b^2
*c^2 - 4*a^2*c^3)*d^3*e - 3*(a^2*b^2*c - 4*a^3*c^2)*d*e^3 + (a^2*b^3 - 4*a^
3*b*c)*e^4)*x)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 -
2*a^2*c)*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*
a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b
^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b
*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (a*b^2*c
^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2
*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(
a*b^2*c^2 - 4*a^2*c^3))*log((2*a*c^3*d^6 - 2*a*b*c^2*d^5*e - 4*a^2*c^2*d^4*
e^2 + 8*a^2*b*c*d^3*e^3 + 2*a^3*b*d*e^5 - 2*(a^2*b^2 + 3*a^3*c)*d^2*e^4 - (
(a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^
2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e
^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^
3*c^5)) - (b*c^3*d^6 + 2*a*b*c^2*d^4*e^2 - 4*a^3*b*e^6 - (b^2*c^2 + 4*a*c^3
)*d^5*e + 4*(a*b^2*c + 2*a^2*c^2)*d^3*e^3 - (a*b^3 + 19*a^2*b*c)*d^2*e^4 +
(5*a^2*b^2 + 12*a^3*c)*d*e^5)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^
2*c^3 - 4*a^3*c^4)*d - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e)*x*sqrt((c^4*d^6 - 6*a
*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^
2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) - ((a*b^2*c^2 - 4*a^2*c^3)*d^3*e - 3*
(a^2*b^2*c - 4*a^3*c^2)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)*sqrt(-(b*c^2*
d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (a*b^2*c^2 -
4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*
d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2
*c^2 - 4*a^2*c^3)))/x^2) + sqrt(1/2)*c*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3
*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^
6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e
^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))*log(
(2*a*c^3*d^6 - 2*a*b*c^2*d^5*e - 4*a^2*c^2*d^4*e^2 + 8*a^2*b*c*d^3*e^3 + 2*
a^3*b*d*e^5 - 2*(a^2*b^2 + 3*a^3*c)*d^2*e^4 - ((a*b^2*c^3 - 4*a^2*c^4)*d^3
- (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sq
rt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a
^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) - (b*c^3*d^6 + 2*a*b
*c^2*d^4*e^2 - 4*a^3*b*e^6 - (b^2*c^2 + 4*a*c^3)*d^5*e + 4*(a*b^2*c + 2*a^2
*c^2)*d^3*e^3 - (a*b^3 + 19*a^2*b*c)*d^2*e^4 + (5*a^2*b^2 + 12*a^3*c)*d*e^5
)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^3 - 4*a^3*c^4)*d - (a^2*
b^3*c^2 - 4*a^3*b*c^3)*e)*x*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3
*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*
a^3*c^5)) - ((a*b^2*c^2 - 4*a^2*c^3)*d^3*e - 3*(a^2*b^2*c - 4*a^3*c^2)*d*e^
3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*
c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6
*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 +
a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3)))/x^2) - 4
*sqrt(-e)*e*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/c]
```

giac [A] time = 1.97, size = 27, normalized size = 0.06

$$-\frac{e^{\frac{3}{2}}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^{2}e+d}\right)^{2}\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*e^(3/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c
```

maple [C] time = 0.03, size = 217, normalized size = 0.45

$$-\frac{e^{\frac{3}{2}}\ln\left(-\sqrt{e}\,x+\sqrt{e x^{2}+d}\right)}{c}+\frac{e^{\frac{3}{2}}\left(b d^{2} e-2 c d^{3}+\left(b e-2 c d\right)\mathrm{RootOf}\left(\_Z^{4} c+c d^{4}+\left(4 b e-4 c d\right)\_Z^{3}+\left(16 a e^{2}-8 d e b+6 c d^{2}\right)\_Z^{2}+\left(4 b d^{2} e-4 c d^{3}\right)\_Z\right)^{2}+2\left(2 a e-b d\right)\mathrm{RootOf}\left(\_Z^{4} c+c d^{4}+\left(4 b e-4 c d\right)\_Z^{3}+\left(16 a e^{2}-8 d e b+6 c d^{2}\right)\_Z^{2}+\left(4 b d^{2} e-4 c d^{3}\right)\_Z\right) e\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{4} c+c d^{4}+\left(4 b e-4 c d\right)\_Z^{3}+\left(16 a e^{2}-8 d e b+6 c d^{2}\right)\_Z^{2}+\left(4 b d^{2} e-4 c d^{3}\right)\_Z\right)+\left(-\sqrt{e}\,x+\sqrt{e x^{2}+d}\right)^{2}\right)}{2 c\left(\mathrm{RootOf}\left(\_Z^{4} c+c d^{4}+\left(4 b e-4 c d\right)\_Z^{3}+\left(16 a e^{2}-8 d e b+6 c d^{2}\right)\_Z^{2}+\left(4 b d^{2} e-4 c d^{3}\right)\_Z\right)^{3} c+3\,\mathrm{RootOf}\left(\_Z^{4} c+c d^{4}+\left(4 b e-4 c d\right)\_Z^{3}+\left(16 a e^{2}-8 d e b+6 c d^{2}\right)\_Z^{2}+\left(4 b d^{2} e-4 c d^{3}\right)\_Z\right)^{2} b e-3\,\mathrm{RootOf}\left(\_Z^{4} c+c d^{4}+\left(4 b e-4 c d\right)\_Z^{3}+\left(16 a e^{2}-8 d e b+6 c d^{2}\right)\_Z^{2}+\left(4 b d^{2} e-4 c d^{3}\right)\_Z\right)^{2} c d+8\,\mathrm{RootOf}\left(\_Z^{4} c+c d^{4}+\left(4 b e-4 c d\right)\_Z^{3}+\left(16 a e^{2}-8 d e b+6 c d^{2}\right)\_Z^{2}+\left(4 b d^{2} e-4 c d^{3}\right)\_Z\right) a e^{2}-4\,\mathrm{RootOf}\left(\_Z^{4} c+c d^{4}+\left(4 b e-4 c d\right)\_Z^{3}+\left(16 a e^{2}-8 d e b+6 c d^{2}\right)\_Z^{2}+\left(4 b d^{2} e-4 c d^{3}\right)\_Z\right) b d e+3\,\mathrm{RootOf}\left(\_Z^{4} c+c d^{4}+\left(4 b e-4 c d\right)\_Z^{3}+\left(16 a e^{2}-8 d e b+6 c d^{2}\right)\_Z^{2}+\left(4 b d^{2} e-4 c d^{3}\right)\_Z\right) c d^{2}+b d^{2} e-c d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*e^(3/2)/c*sum(((b*e-2*c*d)*_R^2+2*e*(2*a*e-b*d)*_R+b*d^2*e-2*c*d^3)/(_R
^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*
ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)
*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))-e^(3/2)/c*ln
(-e^(1/2)*x+(e*x^2+d)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e x^{2}+d\right)^{\frac{3}{2}}}{c x^{4}+b x^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^{3/2}}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] int((d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+e x^{2}\right)^{\frac{3}{2}}}{a+b x^{2}+c x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral((d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

3.274 $\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=260

$$-\frac{\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{3/2}}+\frac{\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{3/2}}$$

Rubi [A] time = 0.85, antiderivative size = 432, normalized size of antiderivative = 1.66, number of steps used = 16, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1295, 277, 217, 206, 1692, 402, 377, 205}

$$-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{2a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{2a}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{2a}-\frac{d\sqrt{d+ex^2}}{ax}+\frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d*Sqrt[d + e*x^2])/(a*x)) - (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(d
+ (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a
*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(2*a*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(d - (b*d - 2*
a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/
(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(2*a*Sqrt[b + Sqrt[b^2 - 4*
a*c]]) + (d*Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/a - (Sqrt[e]*(d -
 (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*
a) - (Sqrt[e]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqr
t[d + e*x^2]])/(2*a)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 1295

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[((f*x)^(m + 2)*(d + e*x^2)^(q - 1)*Simp[b*d - a*e
+ c*d*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}}{x^2\left(a+bx^2+cx^4\right)}\,dx = -\frac{\int \frac{\left(bd-ae+cdx^2\right)\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx}{a}+\frac{d\int \frac{\sqrt{d+ex^2}}{x^2}\,dx}{a}$$

$$= -\frac{d\sqrt{d+ex^2}}{ax}-\frac{\int\left(\frac{\left(cd+\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2}+\frac{\left(cd-\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}\right)dx}{a}+\frac{(de)\int\frac{1}{\sqrt{d+ex^2}}\,dx}{a}$$

$$= -\frac{d\sqrt{d+ex^2}}{ax}+\frac{(de)\,\mathrm{Subst}\left(\int\frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{a}-\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int\frac{\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}}{a}$$

$$= -\frac{d\sqrt{d+ex^2}}{ax}+\frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a}-\frac{\left(e\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int\frac{1}{\sqrt{d+ex^2}}\,dx}{2a}-\frac{\left(\left(2cd-\left(b+\right.\right.\right.}{}$$

$$= -\frac{d\sqrt{d+ex^2}}{ax}+\frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a}-\frac{\left(e\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\mathrm{Subst}\left(\int\frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right.}{2a}$$

$$= -\frac{d\sqrt{d+ex^2}}{ax}-\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a\sqrt{b-\sqrt{b^2-4ac}}}-$$

Mathematica [B] time = 6.30, size = 7789, normalized size = 29.96

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [C] time = 121.56, size = 423, normalized size = 1.63

$$-\frac{\sqrt{e}\,\text{RootSum}\left[\#1^8c+4\#1^6be-4\#1^6cd+16\#1^4ae^2-8\#1^4bde+6\#1^4cd^2+4\#1^2bd^2e-4\#1^2cd^3+cd^4\&,\frac{-\#1^4ae^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+\#1^4cd^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-6\#1^2ade^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+4\#1^2bd^2e\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2\#1^2cd^3\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-ad^2e^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+cd^4\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)}{\#1^6(-c)-3\#1^4be+3\#1^4cd-8\#1^2ae^2+4\#1^2bde-3\#1^2cd^2-bd^2e+cd^3}\&\right]}{2a}-\frac{d\sqrt{d+ex^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d*Sqrt[d + e*x^2])/(a*x)) - (Sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*
d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b
*e*#1^6 + c*#1^8 & , (c*d^4*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - a*d^
2*e^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - 2*c*d^3*Log[-(Sqrt[e]*x) +
 Sqrt[d + e*x^2] - #1]*#1^2 + 4*b*d^2*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]
- #1]*#1^2 - 6*a*d*e^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + c*d^
2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4 - a*e^2*Log[-(Sqrt[e]*x) +
Sqrt[d + e*x^2] - #1]*#1^4)/(c*d^3 - b*d^2*e - 3*c*d^2*#1^2 + 4*b*d*e*#1^2
- 8*a*e^2*#1^2 + 3*c*d*#1^4 - 3*b*e*#1^4 - c*#1^6) & ])/(2*a)
```

fricas [B] time = 37.77, size = 4059, normalized size = 15.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*a*x*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3
- 3*(a*b^2 - 2*a^2*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 -
 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*
e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4
*c))*log(-(12*a^3*b*d^3*e^3 - 6*a^4*d^2*e^4 - 2*(a*b^2*c - a^2*c^2)*d^6 + 2
*(a*b^3 + 2*a^2*b*c)*d^5*e - 4*(2*a^2*b^2 + a^3*c)*d^4*e^2 + ((a^3*b^2*c -
4*a^4*c^2)*d^3 - (a^3*b^3 - 4*a^4*b*c)*d^2*e + (a^4*b^2 - 4*a^5*c)*d*e^2)*x
^2*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^
6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 -
 4*a^7*c)) + (27*a^3*b*d^2*e^4 - 12*a^4*d*e^5 + (b^3*c - a*b*c^2)*d^6 - (b^
4 + 6*a*b^2*c - 4*a^2*c^2)*d^5*e + 2*(4*a*b^3 + 5*a^2*b*c)*d^4*e^2 - 2*(11*
a^2*b^2 + 4*a^3*c)*d^3*e^3)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3 -
4*a^5*b*c)*d - 2*(a^5*b^2 - 4*a^6*c)*e)*x*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d
^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5
*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) - ((a*b^4 - 5*a^2*b^2*c +
 4*a^3*c^2)*d^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d^3*e + 3*(a^3*b^2 - 4*a^4*c)*d^2
*e^2)*x)*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2
- 2*a^2*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*
e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^
2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2)
- sqrt(1/2)*a*x*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*
(a*b^2 - 2*a^2*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a
^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e -
3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))
*log(-(12*a^3*b*d^3*e^3 - 6*a^4*d^2*e^4 - 2*(a*b^2*c - a^2*c^2)*d^6 + 2*(a*
b^3 + 2*a^2*b*c)*d^5*e - 4*(2*a^2*b^2 + a^3*c)*d^4*e^2 + ((a^3*b^2*c - 4*a^
4*c^2)*d^3 - (a^3*b^3 - 4*a^4*b*c)*d^2*e + (a^4*b^2 - 4*a^5*c)*d*e^2)*x^2*s
qrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 +
6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a
^7*c)) + (27*a^3*b*d^2*e^4 - 12*a^4*d*e^5 + (b^3*c - a*b*c^2)*d^6 - (b^4 +
6*a*b^2*c - 4*a^2*c^2)*d^5*e + 2*(4*a*b^3 + 5*a^2*b*c)*d^4*e^2 - 2*(11*a^2*
b^2 + 4*a^3*c)*d^3*e^3)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3 - 4*a^
5*b*c)*d - 2*(a^5*b^2 - 4*a^6*c)*e)*x*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e
```

```
^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2
*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) - ((a*b^4 - 5*a^2*b^2*c + 4*a
^3*c^2)*d^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d^3*e + 3*(a^3*b^2 - 4*a^4*c)*d^2*e^2
)*x)*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 - 2*
a^2*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4
- (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^
2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) - sq
rt(1/2)*a*x*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b
^2 - 2*a^2*c)*d^2*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d
^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5
*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log
(-(12*a^3*b*d^3*e^3 - 6*a^4*d^2*e^4 - 2*(a*b^2*c - a^2*c^2)*d^6 + 2*(a*b^3
+ 2*a^2*b*c)*d^5*e - 4*(2*a^2*b^2 + a^3*c)*d^4*e^2 - ((a^3*b^2*c - 4*a^4*c^
2)*d^3 - (a^3*b^3 - 4*a^4*b*c)*d^2*e + (a^4*b^2 - 4*a^5*c)*d*e^2)*x^2*sqrt(
-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a
*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c
)) + (27*a^3*b*d^2*e^4 - 12*a^4*d*e^5 + (b^3*c - a*b*c^2)*d^6 - (b^4 + 6*a*
b^2*c - 4*a^2*c^2)*d^5*e + 2*(4*a*b^3 + 5*a^2*b*c)*d^4*e^2 - 2*(11*a^2*b^2
+ 4*a^3*c)*d^3*e^3)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3 - 4*a^5*b*
c)*d - 2*(a^5*b^2 - 4*a^6*c)*e)*x*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 -
 (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2
 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) + ((a*b^4 - 5*a^2*b^2*c + 4*a^3*c
^2)*d^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d^3*e + 3*(a^3*b^2 - 4*a^4*c)*d^2*e^2)*x)
*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 - 2*a^2*
c)*d^2*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b
^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 -
2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) + sqrt(1
/2)*a*x*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 -
 2*a^2*c)*d^2*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e
^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2
*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(1
2*a^3*b*d^3*e^3 - 6*a^4*d^2*e^4 - 2*(a*b^2*c - a^2*c^2)*d^6 + 2*(a*b^3 + 2*
a^2*b*c)*d^5*e - 4*(2*a^2*b^2 + a^3*c)*d^4*e^2 - ((a^3*b^2*c - 4*a^4*c^2)*d
^3 - (a^3*b^3 - 4*a^4*b*c)*d^2*e + (a^4*b^2 - 4*a^5*c)*d*e^2)*x^2*sqrt(-(18
*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3
 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) +
 (27*a^3*b*d^2*e^4 - 12*a^4*d*e^5 + (b^3*c - a*b*c^2)*d^6 - (b^4 + 6*a*b^2*
c - 4*a^2*c^2)*d^5*e + 2*(4*a*b^3 + 5*a^2*b*c)*d^4*e^2 - 2*(11*a^2*b^2 + 4*
a^3*c)*d^3*e^3)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3 - 4*a^5*b*c)*d
 - 2*(a^5*b^2 - 4*a^6*c)*e)*x*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^
4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2
*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) + ((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*
d^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d^3*e + 3*(a^3*b^2 - 4*a^4*c)*d^2*e^2)*x)*sqr
t(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 - 2*a^2*c)*d
^2*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 -
 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^
3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) + 4*sqrt(e*x
^2 + d)*d)/(a*x)
```

giac **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple **[C]** time = 0.04, size = 360, normalized size = 1.38

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/a/d/x*(e*x^2+d)^(5/2)+1/a*e/d*x*(e*x^2+d)^(3/2)+5/4/a*e*x*(e*x^2+d)^(1/2
)+3/2/a*e^(1/2)*d*ln(e^(1/2)*x+(e*x^2+d)^(1/2))+1/4/a*e^(3/2)*x^2+1/8/a*e^(
1/2)*d-1/2/a*e^(1/2)*sum(((a*e^2-c*d^2)*_R^2+2*d*(3*a*e^2-2*b*d*e+c*d^2)*_R
+a*d^2*e^2-c*d^4)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*
c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*
c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d
^3)*_Z))-1/8/a*e^(1/2)*d^2/(-e^(1/2)*x+(e*x^2+d)^(1/2))^2+3/2/a*e^(1/2)*d*l
n(-e^(1/2)*x+(e*x^2+d)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}}{\left(cx^4+bx^2+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^{3/2}}{x^2\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int((d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^{\frac{3}{2}}}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral((d + e*x**2)**(3/2)/(x**2*(a + b*x**2 + c*x**4)), x)
```

3.275 $\int \frac{(d+ex^2)^{3/2}}{x^4(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=523

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{2a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(-\cdots\right.}{2\cdots}$$

Rubi [A] time = 2.62, antiderivative size = 523, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {1295, 264, 6728, 277, 217, 206, 1692, 402, 377, 205}

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{2a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{2a^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{2a^2}+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{2a^2}+\frac{\sqrt{d+ex^2}(bd-ae)}{a^2x}-\frac{\sqrt{e}(bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a^2}-\frac{(d+ex^2)^{3/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*d - a*e)*Sqrt[d + e*x^2])/(a^2*x) - (d + e*x^2)^(3/2)/(3*a*x^3) + (Sqrt
[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt
[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(b*d - a*e - (b^2*d - 2*a*c*
d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a^2*Sqrt[b + Sqrt[
b^2 - 4*a*c]]) - (Sqrt[e]*(b*d - a*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])
/a^2 + (Sqrt[e]*(b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*A
rcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*a^2) + (Sqrt[e]*(b*d - a*e + (b^2*d
 - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
)/(2*a^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 1295

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[((f*x)^(m + 2)*(d + e*x^2)^(q - 1)*Simp[b*d - a*e
+ c*d*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}}{x^4\left(a+bx^2+cx^4\right)}\,dx = -\frac{\int \frac{\left(bd-ae+cdx^2\right)\sqrt{d+ex^2}}{x^2\left(a+bx^2+cx^4\right)}dx}{a} + \frac{d\int \frac{\sqrt{d+ex^2}}{x^4}dx}{a}$$

$$= -\frac{\left(d+ex^2\right)^{3/2}}{3ax^3} - \frac{\int \left(\frac{(bd-ae)\sqrt{d+ex^2}}{ax^2} + \frac{\sqrt{d+ex^2}\left(-b^2d+acd+abe-c(bd-ae)x^2\right)}{a\left(a+bx^2+cx^4\right)}\right)dx}{a}$$

$$= -\frac{\left(d+ex^2\right)^{3/2}}{3ax^3} - \frac{\int \frac{\sqrt{d+ex^2}\left(-b^2d+acd+abe-c(bd-ae)x^2\right)}{a+bx^2+cx^4}dx}{a^2} - \frac{(bd-ae)\int \frac{\sqrt{d+ex^2}}{x^2}dx}{a^2}$$

$$= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{\left(d+ex^2\right)^{3/2}}{3ax^3} - \frac{\int \left(\frac{\left(-c(bd-ae)-\frac{c\left(b^2d-2acd-abe\right)}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(-c(bd-ae)+\frac{c\left(b^2d-\cdots\right.}{\sqrt{b\cdots}}\right.}{b+\sqrt{b^2-4\cdots}}\right.}{a^2}$$

$$= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{\left(d+ex^2\right)^{3/2}}{3ax^3} - \frac{(e(bd-ae))\,\text{Subst}\left(\int \frac{1}{1-ex^2}dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{a^2} + \frac{\left(c\left(b\cdots\right.\right.}{\cdots}$$

$$= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{\left(d+ex^2\right)^{3/2}}{3ax^3} - \frac{\sqrt{e}\,(bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a^2} + \frac{\left(e\left(bd-ae-\frac{b^2d\cdots}{\cdots}\right.\right.}{\cdots}$$

$$= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{\left(d+ex^2\right)^{3/2}}{3ax^3} - \frac{\sqrt{e}\,(bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a^2} + \frac{\left(e\left(bd-ae-\frac{b^2d\cdots}{\cdots}\right.\right.}{\cdots}$$

$$= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{\left(d+ex^2\right)^{3/2}}{3ax^3} + \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(bd-ae+\frac{b^2d-2acd-ab\cdots}{\sqrt{b^2-4ac}}\right.}{2a^2\sqrt{b-\sqrt{b^2-4ac}\cdots}}$$

Mathematica [B] time = 6.41, size = 9321, normalized size = 17.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [C] time = 179.82, size = 484, normalized size = 0.93

$$\frac{\sqrt{d+ex^2}\left(-ad-4aex^2+3bdx^2\right)}{3a^2x^3} - \frac{\sqrt{e}\,\text{RootSum}\left[\#1^8c+4\#1^6be-4\#1^6cd+16\#1^4ae^2-8\#1^4bde+6\#1^4cd^2+4\#1^2bd^2e-4\#1^2cd^3+cd^4\&, \frac{-2\#1^4acde\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+\#1^4bcd^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+4\#1^2a^2e^3\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-8\#1^2abde^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+4\#1^2b^2d^2e\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2\#1^2bcd^3\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2acd^3e\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+bcd^4\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)}{\#1^6c+3\#1^4be-3\#1^4cd+8\#1^2ae^2-4\#1^2bde+3\#1^2cd^2+bd^2e-cd^3}\&\right]}{2a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[d + e*x^2]*(-(a*d) + 3*b*d*x^2 - 4*a*e*x^2))/(3*a^2*x^3) - (Sqrt[e]*R
ootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4
+ 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (b*c*d^4*Log[-(Sqrt[
e]*x) + Sqrt[d + e*x^2] - #1] - 2*a*c*d^3*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x
^2] - #1] - 2*b*c*d^3*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 4*b^2
*d^2*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 - 8*a*b*d*e^2*Log[-(Sq
rt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 4*a^2*e^3*Log[-(Sqrt[e]*x) + Sqrt[d
 + e*x^2] - #1]*#1^2 + b*c*d^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^
4 - 2*a*c*d*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4)/(-(c*d^3) + b*
```

```
d^2*e + 3*c*d^2*#1^2 - 4*b*d*e*#1^2 + 8*a*e^2*#1^2 - 3*c*d*#1^4 + 3*b*e*#1^
4 + c*#1^6) & ])/(2*a^2)
```

fricas [B] time = 149.20, size = 7830, normalized size = 14.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*sqrt(1/2)*a^2*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a
*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^
3*b^2 - 2*a^4*c)*e^3 + (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5
*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*
a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*
c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^
3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log((2*a^5
*b*c*d*e^5 - 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^6 + 2*(a*b^5*c - 5*a
^3*b*c^3)*d^5*e - 4*(2*a^2*b^4*c - 3*a^3*b^2*c^2 - a^4*c^3)*d^4*e^2 + 4*(3*
a^3*b^3*c - 4*a^4*b*c^2)*d^3*e^3 - 2*(4*a^4*b^2*c - 3*a^5*c^2)*d^2*e^4 + ((
a^5*b^2*c^2 - 4*a^6*c^3)*d^3 - (a^5*b^3*c - 4*a^6*b*c^2)*d^2*e + (a^6*b^2*c
 - 4*a^7*c^2)*d*e^2)*x^2*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*
c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2
 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^
5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(
5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5
)/(a^10*b^2 - 4*a^11*c)) + (4*a^5*b*c*e^6 + (b^5*c^2 - 3*a*b^3*c^3 + a^2*b*
c^4)*d^6 - (b^6*c + 4*a*b^4*c^2 - 17*a^2*b^2*c^3 + 4*a^3*c^4)*d^5*e + 2*(4*
a*b^5*c - 3*a^2*b^3*c^2 - 11*a^3*b*c^3)*d^4*e^2 - 2*(11*a^2*b^4*c - 16*a^3*
b^2*c^2 - 4*a^4*c^3)*d^3*e^3 + 7*(4*a^3*b^3*c - 5*a^4*b*c^2)*d^2*e^4 - (17*
a^4*b^2*c - 12*a^5*c^2)*d*e^5)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^6*b^4
 - 6*a^7*b^2*c + 8*a^8*c^2)*d - (a^7*b^3 - 4*a^8*b*c)*e)*x*sqrt((a^6*b^2*e^
6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a
*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20
*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b
^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2
*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)) - ((a*b^7 - 7*a^
2*b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b*c^3)*d^4 - (4*a^2*b^6 - 25*a^3*b^4*c + 3
7*a^4*b^2*c^2 - 4*a^5*c^3)*d^3*e + 3*(2*a^3*b^5 - 11*a^4*b^3*c + 12*a^5*b*c
^2)*d^2*e^2 - (4*a^4*b^4 - 19*a^5*b^2*c + 12*a^6*c^2)*d*e^3 + (a^5*b^3 - 4*
a^6*b*c)*e^4)*x)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*
a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2
*a^4*c)*e^3 + (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11
*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^
3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c
^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*
e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b
*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c)))/x^2) - 3*sqrt(1/2)
*a^2*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*
c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*
e^3 + (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4
*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^
2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a
^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*
(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^
5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log((2*a^5*b*c*d*e^5 - 2*(a
*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^6 + 2*(a*b^5*c - 5*a^3*b*c^3)*d^5*e -
 4*(2*a^2*b^4*c - 3*a^3*b^2*c^2 - a^4*c^3)*d^4*e^2 + 4*(3*a^3*b^3*c - 4*a^4
*b*c^2)*d^3*e^3 - 2*(4*a^4*b^2*c - 3*a^5*c^2)*d^2*e^4 + ((a^5*b^2*c^2 - 4*a
```

```
^6*c^3)*d^3 - (a^5*b^3*c - 4*a^6*b*c^2)*d^2*e + (a^6*b^2*c - 4*a^7*c^2)*d*e
^2)*x^2*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d
^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 -
2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^
5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a
^11*c)) + (4*a^5*b*c*e^6 + (b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^6 - (b^6*c
 + 4*a*b^4*c^2 - 17*a^2*b^2*c^3 + 4*a^3*c^4)*d^5*e + 2*(4*a*b^5*c - 3*a^2*b
^3*c^2 - 11*a^3*b*c^3)*d^4*e^2 - 2*(11*a^2*b^4*c - 16*a^3*b^2*c^2 - 4*a^4*c
^3)*d^3*e^3 + 7*(4*a^3*b^3*c - 5*a^4*b*c^2)*d^2*e^4 - (17*a^4*b^2*c - 12*a^
5*c^2)*d*e^5)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^6*b^4 - 6*a^7*b^2*c +
8*a^8*c^2)*d - (a^7*b^3 - 4*a^8*b*c)*e)*x*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*
c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a
^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c
^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3
 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)) - ((a*b^7 - 7*a^2*b^5*c + 13*a^3*
b^3*c^2 - 4*a^4*b*c^3)*d^4 - (4*a^2*b^6 - 25*a^3*b^4*c + 37*a^4*b^2*c^2 - 4
*a^5*c^3)*d^3*e + 3*(2*a^3*b^5 - 11*a^4*b^3*c + 12*a^5*b*c^2)*d^2*e^2 - (4*
a^4*b^4 - 19*a^5*b^2*c + 12*a^6*c^2)*d*e^3 + (a^5*b^3 - 4*a^6*b*c)*e^4)*x)*
sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3
*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 + (a^
5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4
*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d
^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^
4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*
b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c)))/x^2) - 3*sqrt(1/2)*a^2*x^3*sqrt(-((
b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^
2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 - (a^5*b^2 -
4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*
c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*
d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 -
 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a
^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*
a^11*c)))/(a^5*b^2 - 4*a^6*c))*log((2*a^5*b*c*d*e^5 - 2*(a*b^4*c^2 - 3*a^2*
b^2*c^3 + a^3*c^4)*d^6 + 2*(a*b^5*c - 5*a^3*b*c^3)*d^5*e - 4*(2*a^2*b^4*c -
 3*a^3*b^2*c^2 - a^4*c^3)*d^4*e^2 + 4*(3*a^3*b^3*c - 4*a^4*b*c^2)*d^3*e^3 -
 2*(4*a^4*b^2*c - 3*a^5*c^2)*d^2*e^4 - ((a^5*b^2*c^2 - 4*a^6*c^3)*d^3 - (a^
5*b^3*c - 4*a^6*b*c^2)*d^2*e + (a^6*b^2*c - 4*a^7*c^2)*d*e^2)*x^2*sqrt((a^6
*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6
 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b
^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 3
0*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c
^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)) + (4*a^5*
b*c*e^6 + (b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^6 - (b^6*c + 4*a*b^4*c^2 -
17*a^2*b^2*c^3 + 4*a^3*c^4)*d^5*e + 2*(4*a*b^5*c - 3*a^2*b^3*c^2 - 11*a^3*b
*c^3)*d^4*e^2 - 2*(11*a^2*b^4*c - 16*a^3*b^2*c^2 - 4*a^4*c^3)*d^3*e^3 + 7*(
4*a^3*b^3*c - 5*a^4*b*c^2)*d^2*e^4 - (17*a^4*b^2*c - 12*a^5*c^2)*d*e^5)*x^2
 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*d - (a
^7*b^3 - 4*a^8*b*c)*e)*x*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*
c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2
 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^
5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(
5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5
)/(a^10*b^2 - 4*a^11*c)) + ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b
*c^3)*d^4 - (4*a^2*b^6 - 25*a^3*b^4*c + 37*a^4*b^2*c^2 - 4*a^5*c^3)*d^3*e +
 3*(2*a^3*b^5 - 11*a^4*b^3*c + 12*a^5*b*c^2)*d^2*e^2 - (4*a^4*b^4 - 19*a^5*
b^2*c + 12*a^6*c^2)*d*e^3 + (a^5*b^3 - 4*a^6*b*c)*e^4)*x)*sqrt(-((b^5 - 5*a
```

```
*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(
a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 - (a^5*b^2 - 4*a^6*c)*
sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3
*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^
3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c
+ 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))
/(a^5*b^2 - 4*a^6*c)))/x^2) + 3*sqrt(1/2)*a^2*x^3*sqrt(-((b^5 - 5*a*b^3*c +
 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3
- 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 - (a^5*b^2 - 4*a^6*c)*sqrt((a^
6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^
6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*
b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 -
30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*
c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^
2 - 4*a^6*c))*log((2*a^5*b*c*d*e^5 - 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4
)*d^6 + 2*(a*b^5*c - 5*a^3*b*c^3)*d^5*e - 4*(2*a^2*b^4*c - 3*a^3*b^2*c^2 -
a^4*c^3)*d^4*e^2 + 4*(3*a^3*b^3*c - 4*a^4*b*c^2)*d^3*e^3 - 2*(4*a^4*b^2*c -
 3*a^5*c^2)*d^2*e^4 - ((a^5*b^2*c^2 - 4*a^6*c^3)*d^3 - (a^5*b^3*c - 4*a^6*b
*c^2)*d^2*e + (a^6*b^2*c - 4*a^7*c^2)*d*e^2)*x^2*sqrt((a^6*b^2*e^6 + (b^8 -
 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a
^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c
 + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*
a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(
a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)) + (4*a^5*b*c*e^6 + (b^5*c^
2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^6 - (b^6*c + 4*a*b^4*c^2 - 17*a^2*b^2*c^3 +
4*a^3*c^4)*d^5*e + 2*(4*a*b^5*c - 3*a^2*b^3*c^2 - 11*a^3*b*c^3)*d^4*e^2 - 2
*(11*a^2*b^4*c - 16*a^3*b^2*c^2 - 4*a^4*c^3)*d^3*e^3 + 7*(4*a^3*b^3*c - 5*a
^4*b*c^2)*d^2*e^4 - (17*a^4*b^2*c - 12*a^5*c^2)*d*e^5)*x^2 - 2*sqrt(1/2)*sq
rt(e*x^2 + d)*(((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*d - (a^7*b^3 - 4*a^8*b*
c)*e)*x*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d
^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 -
2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^
5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a
^11*c)) + ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b*c^3)*d^4 - (4*a^
2*b^6 - 25*a^3*b^4*c + 37*a^4*b^2*c^2 - 4*a^5*c^3)*d^3*e + 3*(2*a^3*b^5 - 1
1*a^4*b^3*c + 12*a^5*b*c^2)*d^2*e^2 - (4*a^4*b^4 - 19*a^5*b^2*c + 12*a^6*c^
2)*d*e^3 + (a^5*b^3 - 4*a^6*b*c)*e^4)*x)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*
c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b
*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 - (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6
 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*
b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*
a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^
3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*
e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6
*c)))/x^2) + 4*((3*b*d - 4*a*e)*x^2 - a*d)*sqrt(e*x^2 + d))/(a^2*x^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.04, size = 511, normalized size = 0.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/a^2*b/d/x*(e*x^2+d)^(5/2)-1/a^2*b*e/d*x*(e*x^2+d)^(3/2)-5/4/a^2*b*e*x*(e*
x^2+d)^(1/2)-3/2/a^2*b*e^(1/2)*d*ln(e^(1/2)*x+(e*x^2+d)^(1/2))-1/4/a^2*e^(3
/2)*x^2*b-1/8/a^2*e^(1/2)*b*d+1/2/a^2*e^(1/2)*sum((c*d*(2*a*e-b*d)*_R^2+2*(
-2*a^2*e^3+4*a*b*d*e^2-2*b^2*d^2*e+b*c*d^3)*_R+2*a*c*d^3*e-b*c*d^4)/(_R^3*c
+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(-
_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^
3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))+1/a*e^(3/2)*ln(-
e^(1/2)*x+(e*x^2+d)^(1/2))-3/2/a^2*e^(1/2)*ln(-e^(1/2)*x+(e*x^2+d)^(1/2))*b
*d+1/8/a^2*e^(1/2)*b*d^2/(-e^(1/2)*x+(e*x^2+d)^(1/2))^2-1/3/a/d/x^3*(e*x^2+
d)^(5/2)-2/3/a*e/d^2/x*(e*x^2+d)^(5/2)+2/3/a*e^2/d^2*x*(e*x^2+d)^(3/2)+1/a*
e^2/d*x*(e*x^2+d)^(1/2)+1/a*e^(3/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}}{\left(cx^4+bx^2+a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^{3/2}}{x^4\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int((d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.276 $\int \frac{x^5 \sqrt{1-x^2}}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=281

$$\frac{\left(-\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{\left(\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{}}{\sqrt{\sqrt{b^2-}}}\right.}{\sqrt{2}\,c^{5/2}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

Rubi [A] time = 7.34, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1251, 897, 1287, 1166, 208}

$$\frac{\left(-\frac{-3abc-2ac^2+b^2c+b^3}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{\left(\frac{-3abc-2ac^2+b^2c+b^3}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{b\sqrt{1-x^2}}{c^2}-\frac{\left(1-x^2\right)^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*Sqrt[1 - x^2])/c^2) - (1 - x^2)^(3/2)/(3*c) + ((b^2 - a*c + b*c - (b^3
 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*S
qrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b +
 2*c - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c + b*c + (b^3 - 3*a*b*c + b^2*c - 2
*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b +
 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*
c]])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{1-x}\,x^2}{a+bx+cx^2}\,dx, x, x^2\right)$$
$$= -\,\text{Subst}\left(\int \frac{x^2\left(1-x^2\right)^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)$$
$$= -\,\text{Subst}\left(\int \left(\frac{b}{c^2}+\frac{x^2}{c}-\frac{b(a+b+c)-\left(b^2-ac+bc\right)x^2}{c^2\left(a+b+c+(-b-2c)x^2+cx^4\right)}\right)dx, x, \sqrt{1-x^2}\right)$$
$$= -\frac{b\sqrt{1-x^2}}{c^2}-\frac{\left(1-x^2\right)^{3/2}}{3c}+\frac{\text{Subst}\left(\int \frac{b(a+b+c)+\left(-b^2+ac-bc\right)x^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)}{c^2}$$
$$= -\frac{b\sqrt{1-x^2}}{c^2}-\frac{\left(1-x^2\right)^{3/2}}{3c}-\frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\right.}{2c^2}$$
$$= -\frac{b\sqrt{1-x^2}}{c^2}-\frac{\left(1-x^2\right)^{3/2}}{3c}+\frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b+2c-\sqrt{b^2-4ac}}}+\frac{\left(b^2\right.}{}$$

Mathematica [A] time = 0.54, size = 354, normalized size = 1.26

$$\frac{-\frac{3\sqrt{2}\left(b^2\left(\sqrt{b^2-4ac}+c\right)+bc\left(\sqrt{b^2-4ac}-3a\right)-ac\left(\sqrt{b^2-4ac}+2c\right)+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}-b-2c}}\right)}{\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}-b-2c}}-\frac{3\sqrt{2}\left(b^2\left(\sqrt{b^2-4ac}-c\right)+bc\left(\sqrt{b^2-4ac}+3a\right)+ac\left(2c-\sqrt{b^2-4ac}\right)-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}-b-2c}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}-b-2c}}-6b\sqrt{c}\sqrt{1-x^2}-2c^{3/2}\left(1-x^2\right)^{3/2}}{6c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-6*b*Sqrt[c]*Sqrt[1 - x^2] - 2*c^(3/2)*(1 - x^2)^(3/2) - (3*Sqrt[2]*(b^3 +
 b*c*(-3*a + Sqrt[b^2 - 4*a*c]) + b^2*(c + Sqrt[b^2 - 4*a*c]) - a*c*(2*c +
Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]])
 - (3*Sqrt[2]*(-b^3 + a*c*(2*c - Sqrt[b^2 - 4*a*c]) + b*c*(3*a + Sqrt[b^2 -
 4*a*c]) + b^2*(-c + Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x
^2])/Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c
+ Sqrt[b^2 - 4*a*c]]))/(6*c^(5/2))
```

IntegrateAlgebraic [A] time = 0.98, size = 425, normalized size = 1.51

$$\frac{\left(-\sqrt{2}b^2\sqrt{b^2-4ac}-\sqrt{2}bc\sqrt{b^2-4ac}+\sqrt{2}ac\sqrt{b^2-4ac}+3\sqrt{2}abc+2\sqrt{2}ac^2-\sqrt{2}b^3-\sqrt{2}b^2c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}-b-2c}}\right)}{2c^{5/2}\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}-b-2c}}+\frac{\left(-\sqrt{2}b^2\sqrt{b^2-4ac}-\sqrt{2}bc\sqrt{b^2-4ac}+\sqrt{2}ac\sqrt{b^2-4ac}-3\sqrt{2}abc-2\sqrt{2}ac^2+\sqrt{2}b^3+\sqrt{2}b^2c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}-b-2c}}\right)}{2c^{5/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}-b-2c}}+\frac{\sqrt{1-x^2}\left(-3b+cx^2-c\right)}{3c^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[1 - x^2]*(-3*b - c + c*x^2))/(3*c^2) + ((-(Sqrt[2]*b^3) + 3*Sqrt[2]*a
*b*c - Sqrt[2]*b^2*c + 2*Sqrt[2]*a*c^2 - Sqrt[2]*b^2*Sqrt[b^2 - 4*a*c] + Sq
```

```
rt[2]*a*c*Sqrt[b^2 - 4*a*c] - Sqrt[2]*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]])/(2*c^(5/2)*Sq
rt[b^2 - 4*a*c]*Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]) + ((Sqrt[2]*b^3 - 3*Sqr
t[2]*a*b*c + Sqrt[2]*b^2*c - 2*Sqrt[2]*a*c^2 - Sqrt[2]*b^2*Sqrt[b^2 - 4*a*c
] + Sqrt[2]*a*c*Sqrt[b^2 - 4*a*c] - Sqrt[2]*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]])/(2*c^(5
/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]])
```

fricas [B] time = 16.82, size = 3615, normalized size = 12.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(1/2)*c^2*sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*
a*b^3 - b^4)*c - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2
)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 +
b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)
)*log(-(2*a^3*b^4 + (a^2*b^2*c^5 - 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*
b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4
- 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^
5 - 2*a^4*b)*c^2 + (a^2*b^5 + (a^4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a^2*b^
4)*c)*x^2 - 2*(3*a^4*b^2 - a^3*b^3)*c + sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 +
 12*a^2*b*c^7)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b
^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*
b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + (b^8 + 4*(a^4 - 2*a^3*b)*c^4 - (17*a
^3*b^2 - 14*a^2*b^3)*c^3 + (20*a^2*b^4 - 7*a*b^5)*c^2 - (8*a*b^6 - b^7)*c)*
x^2)*sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c -
(b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*
b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a
*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 2*(a^3*b^4 +
(a^5 - 2*a^4*b)*c^2 - (3*a^4*b^2 - a^3*b^3)*c)*sqrt(-x^2 + 1))/x^2) - 3*sqr
t(1/2)*c^2*sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4
)*c - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(
3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 -
2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-(2*a
^3*b^4 + (a^2*b^2*c^5 - 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b
^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5
+ b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^5 - 2*a^4*b
)*c^2 + (a^2*b^5 + (a^4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a^2*b^4)*c)*x^2 -
 2*(3*a^4*b^2 - a^3*b^3)*c - sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c
^7)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*
b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*
c)/(b^2*c^10 - 4*a*c^11)) + (b^8 + 4*(a^4 - 2*a^3*b)*c^4 - (17*a^3*b^2 - 14
*a^2*b^3)*c^3 + (20*a^2*b^4 - 7*a*b^5)*c^2 - (8*a*b^6 - b^7)*c)*x^2)*sqrt((
b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c - (b^2*c^5 -
4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2
*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)
*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 2*(a^3*b^4 + (a^5 - 2*a^
4*b)*c^2 - (3*a^4*b^2 - a^3*b^3)*c)*sqrt(-x^2 + 1))/x^2) - 3*sqrt(1/2)*c^2*
sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c + (b^2*
c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 -
 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6
- b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-(2*a^3*b^4 - (a
^2*b^2*c^5 - 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2
*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2
- 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^5 - 2*a^4*b)*c^2 + (a^
2*b^5 + (a^4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a^2*b^4)*c)*x^2 - 2*(3*a^4*b
^2 - a^3*b^3)*c + sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c^7)*x^2*sqr
```

```
t((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b
^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^1
0 - 4*a*c^11)) - (b^8 + 4*(a^4 - 2*a^3*b)*c^4 - (17*a^3*b^2 - 14*a^2*b^3)*c
^3 + (20*a^2*b^4 - 7*a*b^5)*c^2 - (8*a*b^6 - b^7)*c)*x^2)*sqrt((b^5 + 2*a^2
*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c + (b^2*c^5 - 4*a*c^6)*sq
rt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*
b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^
10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 2*(a^3*b^4 + (a^5 - 2*a^4*b)*c^2 -
(3*a^4*b^2 - a^3*b^3)*c)*sqrt(-x^2 + 1))/x^2) + 3*sqrt(1/2)*c^2*sqrt((b^5 +
 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c + (b^2*c^5 - 4*a*c
^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3
+ 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(
b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-(2*a^3*b^4 - (a^2*b^2*c^5
- 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2
 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^
6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^5 - 2*a^4*b)*c^2 + (a^2*b^5 + (a^
4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a^2*b^4)*c)*x^2 - 2*(3*a^4*b^2 - a^3*b^
3)*c - sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c^7)*x^2*sqrt((b^8 + (a
^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (
11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^1
1)) - (b^8 + 4*(a^4 - 2*a^3*b)*c^4 - (17*a^3*b^2 - 14*a^2*b^3)*c^3 + (20*a^
2*b^4 - 7*a*b^5)*c^2 - (8*a*b^6 - b^7)*c)*x^2)*sqrt((b^5 + 2*a^2*c^3 + (5*a
^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (
a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 +
(11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^
11)))/(b^2*c^5 - 4*a*c^6)) - 2*(a^3*b^4 + (a^5 - 2*a^4*b)*c^2 - (3*a^4*b^2
- a^3*b^3)*c)*sqrt(-x^2 + 1))/x^2) - 2*(c*x^2 - 3*b - c)*sqrt(-x^2 + 1))/c^
2
```

giac [B] time = 4.48, size = 4637, normalized size = 16.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 6*b^5*c^5 + 24*a^2*b^2*c^6 - 40*a*b^3*c^6
+ 4*b^4*c^6 + 64*a^2*b*c^7 - 24*a*b^2*c^7 + 32*a^2*c^8 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 5*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4
 + 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*
b^3*c^4 - 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c
)*c)*b^4*c^4 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 -
4*a*c)*c)*a^2*b*c^5 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt
(b^2 - 4*a*c)*c)*a*b^2*c^5 - 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2
 - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c -
2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^6 + 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b
*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^2*c^6 + 20*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*c^7 - 2*(b^2 - 4*a*c)*b^4*c^4
 + 6*(b^2 - 4*a*c)*a*b^2*c^5 - 6*(b^2 - 4*a*c)*b^3*c^5 + 16*(b^2 - 4*a*c)*a
*b*c^6 - 4*(b^2 - 4*a*c)*b^2*c^6 + 8*(b^2 - 4*a*c)*a*c^7 - (2*b^6*c^2 - 18*
a*b^4*c^3 + 2*b^5*c^3 + 48*a^2*b^2*c^4 - 16*a*b^3*c^4 - 32*a^3*c^5 + 32*a^2
*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)
*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)
*a*b^4*c - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c
)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*
a*c)*c)*a^2*b^2*c^2 + 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt
```

(b^2 - 4*a*c)*c)*a*b^3*c^2 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 33*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4 + 8*(b^2 - 4*a*c)*a*b*c^4)*c^2 - 2*(sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 6*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 3*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 + 2*a*b^5*c^3 + 2*b^6*c^3 + 16*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 11*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 + 7*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 16*a*b^4*c^4 + 2*b^5*c^4 - 4*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 28*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + 32*a^2*b^2*c^5 + 5*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 - 16*a*b^3*c^5 - 20*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^6 + 32*a^2*b*c^6 - 2*(b^2 - 4*a*c)*a*b^3*c^3 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4 + 8*(b^2 - 4*a*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)*abs(c))*arctan(2*sqrt(1/2)*sqrt(-x^2 + 1)/sqrt(-(b*c^3 + 2*c^4 + sqrt(-4*(a*c^3 + b*c^3 + c^4)*c^4 + (b*c^3 + 2*c^4)^2))/c^4))/((a*b^4*c^4 + b^5*c^4 - 8*a^2*b^2*c^5 - 6*a*b^3*c^5 + 3*b^4*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 - 11*a*b^2*c^6 + 7*b^3*c^6 - 4*a^2*c^7 - 28*a*b*c^7 + 5*b^2*c^7 - 20*a*c^8)*c^2) + 1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 6*b^5*c^5 + 24*a^2*b^2*c^6 - 40*a*b^3*c^6 + 4*b^4*c^6 + 64*a^2*b*c^7 - 24*a*b^2*c^7 + 32*a^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c^5 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 + 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^2*c^6 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*c^7 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5 - 6*(b^2 - 4*a*c)*b^3*c^5 + 16*(b^2 - 4*a*c)*a*b*c^6 - 4*(b^2 - 4*a*c)*b^2*c^6 + 8*(b^2 - 4*a*c)*a*c^7 - (2*b^6*c^2 - 18*a*b^4*c^3 + 2*b^5*c^3 + 48*a^2*b^2*c^4 - 16*a*b^3*c^4 - 32*a^3*c^5 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 33*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^

```
2 - 4*a*c)*c)*a^2*c^4 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sq
rt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b
^2*c^3 - 2*(b^2 - 4*a*c)*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4 + 8*(b^2 - 4*a*c
)*a*b*c^4)*c^2 + 2*(sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^5*
c^2 + sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 - 8*sqrt(2)*
sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 6*sqrt(2)*sqrt(-b*c
- 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 3*sqrt(2)*sqrt(-b*c - 2*c^2 + sq
rt(b^2 - 4*a*c)*c)*b^5*c^3 - 2*a*b^5*c^3 - 2*b^6*c^3 + 16*sqrt(2)*sqrt(-b*c
 - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(-b*c - 2*c^2 + s
qrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 11*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 -
 4*a*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 + 7*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(
b^2 - 4*a*c)*c)*b^4*c^4 + 16*a*b^4*c^4 - 2*b^5*c^4 - 4*sqrt(2)*sqrt(-b*c -
2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*c^5 - 28*sqrt(2)*sqrt(-b*
c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 32*a^2*b^2*c^5 + 5*sqrt(2)*sqr
t(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 16*a*b^3*c^5 - 20*sqrt(2)*s
qrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 32*a^2*b*c^6 + 2*(b^2 - 4
*a*c)*a*b^3*c^3 + 2*(b^2 - 4*a*c)*b^4*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4 - 8*(
b^2 - 4*a*c)*a*b^2*c^4 + 2*(b^2 - 4*a*c)*b^3*c^4 - 8*(b^2 - 4*a*c)*a*b*c^5)
*abs(c))*arctan(2*sqrt(1/2)*sqrt(-x^2 + 1)/sqrt(-(b*c^3 + 2*c^4 - sqrt(-4*(
a*c^3 + b*c^3 + c^4)*c^4 + (b*c^3 + 2*c^4)^2))/c^4))/((a*b^4*c^4 + b^5*c^4
- 8*a^2*b^2*c^5 - 6*a*b^3*c^5 + 3*b^4*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 - 11*a
*b^2*c^6 + 7*b^3*c^6 - 4*a^2*c^7 - 28*a*b*c^7 + 5*b^2*c^7 - 20*a*c^8)*c^2)
- 1/3*((-x^2 + 1)^(3/2)*c^2 + 3*sqrt(-x^2 + 1)*b*c)/c^3
```

maple [B] time = 0.10, size = 2134, normalized size = 7.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/3*(-x^2+1)^(3/2)/c+4/c*a^2/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/
2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^
2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2
*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)-2/c^2*a/(8*a*c-2*b^2
)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*a
rctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a
*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2*(-
4*a*c+b^2)^(1/2)-2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*
b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a
+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*
a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)*b+8/c*a^2/(8*a*c-2*b^2)/(4*
a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan
(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*
b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b+8*a^2/(8*
a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b
)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-
2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2
))-2/c^2*a/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^
2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b
^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/
2)-2*a*b)^(1/2))*b^3-2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*
a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x
^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*
(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2+4/c*a^2/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(
-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^
2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+
b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)-2/c^2*
a/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-
2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+
```

```
2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^
(1/2))*b^2*(-4*a*c+b^2)^(1/2)-2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^
2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2
)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2
)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)*b-8/c*a^2/(8*a*
c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^
(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b
)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*
b-8*a^2/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^
(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^
(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2
*a*b)^(1/2))+2/c^2*a/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*
(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*
(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c
+b^2)^(1/2)-2*a*b)^(1/2))*b^3+2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^
2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2
)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2
)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2-2/c^2*b/(2/x^2-2/x^2*(-x^2+1)^
(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}\,x^5}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^5/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 1.45, size = 917, normalized size = 3.26

$$\sqrt{1-x^2}\left(\frac{2}{3c}-\frac{\frac{b}{c}+1}{c}+\frac{x^2}{3c}\right)-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(b^3c+b^4-b^3\sqrt{b^2-4ac}+4a^2c^2+2ac^2\sqrt{b^2-4ac}-b^2c\sqrt{b^2-4ac}-4abc^2-5ab^2c+3abc\sqrt{b^2-4ac}\right)}{4c^3\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,\left(4ac-b^2\right)}-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+1\right)1i}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(b^3c+b^4+b^3\sqrt{b^2-4ac}+4a^2c^2-2ac^2\sqrt{b^2-4ac}+b^2c\sqrt{b^2-4ac}-4abc^2-5ab^2c-3abc\sqrt{b^2-4ac}\right)}{4c^3\left(4ac-b^2\right)\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(b^3c+b^4+b^3\sqrt{b^2-4ac}+4a^2c^2-2ac^2\sqrt{b^2-4ac}+b^2c\sqrt{b^2-4ac}-4abc^2-5ab^2c-3abc\sqrt{b^2-4ac}\right)}{4c^3\left(4ac-b^2\right)\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(b^3c+b^4-b^3\sqrt{b^2-4ac}+4a^2c^2+2ac^2\sqrt{b^2-4ac}-b^2c\sqrt{b^2-4ac}-4abc^2-5ab^2c+3abc\sqrt{b^2-4ac}\right)}{4c^3\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,\left(4ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (1 - x^2)^(1/2)*(2/(3*c) - (b/c + 1)/c + x^2/(3*c)) - (log((((x*(-(b - (b^2
 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1
)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)
))*(b^3*c + b^4 - b^3*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2 + 2*a*c^2*(b^2 - 4*a*
c)^(1/2) - b^2*c*(b^2 - 4*a*c)^(1/2) - 4*a*b*c^2 - 5*a*b^2*c + 3*a*b*c*(b^2
 - 4*a*c)^(1/2)))/(4*c^3*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c
 - b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b
+ (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b + (
b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^3*c + b^4 + b^3*(b^2 - 4*a*c)^(1/2) +
 4*a^2*c^2 - 2*a*c^2*(b^2 - 4*a*c)^(1/2) + b^2*c*(b^2 - 4*a*c)^(1/2) - 4*a*
b*c^2 - 5*a*b^2*c - 3*a*b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^3*(4*a*c - b^2)*((b
+ (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(
1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (
1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^3*c +
 b^4 + b^3*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2 - 2*a*c^2*(b^2 - 4*a*c)^(1/2) +
b^2*c*(b^2 - 4*a*c)^(1/2) - 4*a*b*c^2 - 5*a*b^2*c - 3*a*b*c*(b^2 - 4*a*c)^(
1/2)))/(4*c^3*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) -
(log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c
)^(1/2))/(2*c))^(1/2)))*(b^3*c + b^4 - b^3*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2
+ 2*a*c^2*(b^2 - 4*a*c)^(1/2) - b^2*c*(b^2 - 4*a*c)^(1/2) - 4*a*b*c^2 - 5*a
*b^2*c + 3*a*b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^3*((b - (b^2 - 4*a*c)^(1/2))/(2
*c) + 1)^(1/2)*(4*a*c - b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**5*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

3.277 $\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=229

$$-\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{3/2}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{\sqrt{1-x^2}}{c}$$

Rubi [A] time = 1.75, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 824, 826, 1166, 208}

$$-\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{3/2}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{\sqrt{1-x^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Sqrt[1 - x^2]/c - ((b + c - (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[
(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*c^(3/2)*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - ((b + c + (b^2 - 2*a*c + b*c
)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c +
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{1-x}\,x}{a+bx+cx^2}\,dx,x,x^2\right)\\ &= \frac{\sqrt{1-x^2}}{c}+\frac{\operatorname{Subst}\left(\int \frac{a+(b+c)x}{\sqrt{1-x}\,(a+bx+cx^2)}\,dx,x,x^2\right)}{2c}\\ &= \frac{\sqrt{1-x^2}}{c}+\frac{\operatorname{Subst}\left(\int \frac{-a-b-c+(b+c)x^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx,x,\sqrt{1-x^2}\right)}{c}\\ &= \frac{\sqrt{1-x^2}}{c}+\frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,\sqrt{1-x^2}\right)}{2c}+\frac{\left(b+c+\frac{b}{}\right.}{}\\ &= \frac{\sqrt{1-x^2}}{c}-\frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b+2c-\sqrt{b^2-4ac}}}-\frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}}{\sqrt{b+2}}\right.}{\sqrt{2}\,c^{3/2}\sqrt{b+2c+\sqrt{b^2-4}}}\end{aligned}$$

Mathematica [A] time = 0.39, size = 276, normalized size = 1.21

$$\frac{\frac{\left(b\left(c-\sqrt{b^2-4ac}\right)-c\left(\sqrt{b^2-4ac}+2a\right)+b^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(b\left(\sqrt{b^2-4ac}+c\right)+c\left(\sqrt{b^2-4ac}-2a\right)+b^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b+2c}}+\sqrt{1-x^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[1 - x^2] + ((b^2 + b*(c - Sqrt[b^2 - 4*a*c]) - c*(2*a + Sqrt[b^2 - 4*
a*c]))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*
a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]
]) - ((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))*ArcT
anh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sq
rt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]))/c
```

IntegrateAlgebraic [A] time = 0.68, size = 285, normalized size = 1.24

$$\frac{\left(b\sqrt{b^2-4ac}+c\sqrt{b^2-4ac}-2ac+b^2+bc\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}-b-2c}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}-b-2c}}+\frac{\left(b\sqrt{b^2-4ac}+c\sqrt{b^2-4ac}+2ac-b^2-bc\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}-b-2c}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}-b-2c}}+\frac{\sqrt{1-x^2}}{c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Sqrt[1 - x^2]/c + ((b^2 - 2*a*c + b*c + b*Sqrt[b^2 - 4*a*c] + c*Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*
c]]) + ((-b^2 + 2*a*c - b*c + b*Sqrt[b^2 - 4*a*c] + c*Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]])
```

fricas [B] time = 6.46, size = 2053, normalized size = 8.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*c*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c^4
)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c
^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*b^2 + (a*b^2*c^3 - 4*a^2*c^4)*x^2*sqr
t((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7))
+ (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c + sqrt(1/2)*((b^4*c^3
 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*
b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (b^5 + 4*(a^2*b - a*b^2)*c^2 - (5*a*b^
3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c^4
)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c
^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-x^2 + 1))/
x^2) - sqrt(1/2)*c*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c
^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a
*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*b^2 + (a*b^2*c^3 - 4*a^2*c^4)*x^2*s
qrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)
) + (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c - sqrt(1/2)*((b^4*c
^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(
a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (b^5 + 4*(a^2*b - a*b^2)*c^2 - (5*a*
b^3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c
^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a
*c^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-x^2 + 1)
)/x^2) - sqrt(1/2)*c*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4*a
*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4
*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*b^2 - (a*b^2*c^3 - 4*a^2*c^4)*x^2
*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^
7)) + (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c + sqrt(1/2)*((b^4
*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2
*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) - (b^5 + 4*(a^2*b - a*b^2)*c^2 - (5*
a*b^3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4*a
*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4
*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-x^2 +
1))/x^2) + sqrt(1/2)*c*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4
*a*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*b^2 - (a*b^2*c^3 - 4*a^2*c^4)*x
^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*
c^7)) + (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c - sqrt(1/2)*((b
^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 -
 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) - (b^5 + 4*(a^2*b - a*b^2)*c^2 - (
5*a*b^3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4
*a*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-x^2
+ 1))/x^2) + 2*sqrt(-x^2 + 1))/c
```

giac [B] time = 4.10, size = 4060, normalized size = 17.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] sqrt(-x^2 + 1)/c + 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 6*b^4*c^5 + 16*a^2*b*c^6
 - 32*a*b^2*c^6 + 4*b^3*c^6 + 32*a^2*c^7 - 16*a*b*c^7 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 5*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(
```

```
2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 2
0*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^4 - 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)
*b^3*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*
c)*c)*a^2*c^5 + 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 -
 4*a*c)*c)*a*b*c^5 - 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(
b^2 - 4*a*c)*c)*b^2*c^5 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 -
sqrt(b^2 - 4*a*c)*c)*a*c^6 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2
 - sqrt(b^2 - 4*a*c)*c)*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a
*b*c^5 - 6*(b^2 - 4*a*c)*b^2*c^5 + 8*(b^2 - 4*a*c)*a*c^6 - 4*(b^2 - 4*a*c)*
b*c^6 - (2*b^5*c^2 - 16*a*b^3*c^3 + 2*b^4*c^3 + 32*a^2*b*c^4 - 16*a*b^2*c^4
 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*
a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*
a*c)*c)*a*b^3*c - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2
- 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b
^2 - 4*a*c)*c)*a^2*b*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 -
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2
*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*
c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 20*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^3*
c^2 + 8*(b^2 - 4*a*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a
*c^4)*c^2 - 2*(sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 +
 sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 8*sqrt(2)*sqrt(
-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 6*sqrt(2)*sqrt(-b*c - 2*c
^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 3*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^
2 - 4*a*c)*c)*b^4*c^3 + 2*a*b^4*c^3 + 2*b^5*c^3 + 16*sqrt(2)*sqrt(-b*c - 2*
c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2
 - 4*a*c)*c)*a^2*b*c^4 - 11*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^4 - 16*a^2*b^2*c^4 + 7*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a
*c)*c)*b^3*c^4 - 16*a*b^3*c^4 + 2*b^4*c^4 - 4*sqrt(2)*sqrt(-b*c - 2*c^2 - s
qrt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*c^5 - 28*sqrt(2)*sqrt(-b*c - 2*c^2 - s
qrt(b^2 - 4*a*c)*c)*a*b*c^5 + 32*a^2*b*c^5 + 5*sqrt(2)*sqrt(-b*c - 2*c^2 -
sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 16*a*b^2*c^5 - 20*sqrt(2)*sqrt(-b*c - 2*c^2
- sqrt(b^2 - 4*a*c)*c)*a*c^6 + 32*a^2*c^6 - 2*(b^2 - 4*a*c)*a*b^2*c^3 - 2*(
b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4 + 8*(b^2 - 4*a*c)*a*b*c^4 -
2*(b^2 - 4*a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*abs(c))*arctan(2*sqrt(1/2)
*sqrt(-x^2 + 1)/sqrt(-(b*c + 2*c^2 + sqrt(-4*(a*c + b*c + c^2)*c^2 + (b*c +
 2*c^2)^2))/c^2))/((a*b^4*c^3 + b^5*c^3 - 8*a^2*b^2*c^4 - 6*a*b^3*c^4 + 3*b
^4*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 - 11*a*b^2*c^5 + 7*b^3*c^5 - 4*a^2*c^6 -
28*a*b*c^6 + 5*b^2*c^6 - 20*a*c^7)*c^2) - 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 6
*b^4*c^5 + 16*a^2*b*c^6 - 32*a*b^2*c^6 + 4*b^3*c^6 + 32*a^2*c^7 - 16*a*b*c^
7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^5*
c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*
a*b^3*c^3 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*
c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 -
4*a*c)*c)*a^2*b*c^4 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt
(b^2 - 4*a*c)*c)*a*b^2*c^4 - 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2
 + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c -
2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b
*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 + 20*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*c^6 - 10*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b*c^6 - 2*(b^2 - 4*a*c)*b^3*
c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - 6*(b^2 - 4*a*c)*b^2*c^5 + 8*(b^2 - 4*a*c)*a
*c^6 - 4*(b^2 - 4*a*c)*b*c^6 - (2*b^5*c^2 - 16*a*b^3*c^3 + 2*b^4*c^3 + 32*a
^2*b*c^4 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c
- 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c
```

```
- 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-
b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 7*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 28*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 5*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 +
20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*c^4
 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^2*
c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2 + 2*(sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2
- 4*a*c)*c)*a*b^4*c^2 + sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^
5*c^2 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 6*
sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 3*sqrt(2)*sqrt
(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 2*a*b^4*c^3 - 2*b^5*c^3 + 16
*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(
-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 11*sqrt(2)*sqrt(-b*c - 2*c^
2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 + 7*sqrt(2)*sqrt(-b*c -
 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 16*a*b^3*c^4 - 2*b^4*c^4 - 4*sqrt(2
)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 - 28*sqrt(2
)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 32*a^2*b*c^5 + 5*sqrt(
2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 + 16*a*b^2*c^5 - 20*sqr
t(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*c^6 - 32*a^2*c^6 + 2*(b^2 -
 4*a*c)*a*b^2*c^3 + 2*(b^2 - 4*a*c)*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4 - 8*(
b^2 - 4*a*c)*a*b*c^4 + 2*(b^2 - 4*a*c)*b^2*c^4 - 8*(b^2 - 4*a*c)*a*c^5)*abs
(c))*arctan(2*sqrt(1/2)*sqrt(-x^2 + 1)/sqrt(-(b*c + 2*c^2 - sqrt(-4*(a*c +
b*c + c^2)*c^2 + (b*c + 2*c^2)^2))/c^2))/((a*b^4*c^3 + b^5*c^3 - 8*a^2*b^2*
c^4 - 6*a*b^3*c^4 + 3*b^4*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 - 11*a*b^2*c^5 + 7
*b^3*c^5 - 4*a^2*c^6 - 28*a*b*c^6 + 5*b^2*c^6 - 20*a*c^7)*c^2)
```

<u>**maple**</u> **[B]** time = 0.06, size = 1223, normalized size = 5.34

$$-\frac{8a^{2}\arctan\left(\frac{-2a-2b-\frac{2\left(\sqrt{-x^{2}+1}-1\right)^{2}a}{x^{2}}+2\sqrt{-4ac+b^{2}}}{2\sqrt{-2ab+4ac-2b^{2}+2\sqrt{-4ac+b^{2}}\,a+2\sqrt{-4ac+b^{2}}\,b}}\right)}{\left(8ac-2b^{2}\right)\sqrt{-2ab+4ac-2b^{2}+2\sqrt{-4ac+b^{2}}\,a+2\sqrt{-4ac+b^{2}}\,b}}+\frac{8a^{2}\arctan\left(\frac{2a+2b+\frac{2\left(\sqrt{-x^{2}+1}-1\right)^{2}a}{x^{2}}+2\sqrt{-4ac+b^{2}}}{2\sqrt{-2ab+4ac-2b^{2}-2\sqrt{-4ac+b^{2}}\,a-2\sqrt{-4ac+b^{2}}\,b}}\right)}{\left(8ac-2b^{2}\right)\sqrt{-2ab+4ac-2b^{2}-2\sqrt{-4ac+b^{2}}\,a-2\sqrt{-4ac+b^{2}}\,b}}+\frac{2ab^{2}\arctan\left(\frac{-2a-2b-\frac{2\left(\sqrt{-x^{2}+1}-1\right)^{2}a}{x^{2}}+2\sqrt{-4ac+b^{2}}}{2\sqrt{-2ab+4ac-2b^{2}+2\sqrt{-4ac+b^{2}}\,a+2\sqrt{-4ac+b^{2}}\,b}}\right)}{\left(8ac-2b^{2}\right)\sqrt{-2ab+4ac-2b^{2}+2\sqrt{-4ac+b^{2}}\,a+2\sqrt{-4ac+b^{2}}\,b}\,c}-\frac{2ab^{2}\arctan\left(\frac{2a+2b+\frac{2\left(\sqrt{-x^{2}+1}-1\right)^{2}a}{x^{2}}+2\sqrt{-4ac+b^{2}}}{2\sqrt{-2ab+4ac-2b^{2}-2\sqrt{-4ac+b^{2}}\,a-2\sqrt{-4ac+b^{2}}\,b}}\right)}{\left(8ac-2b^{2}\right)\sqrt{-2ab+4ac-2b^{2}-2\sqrt{-4ac+b^{2}}\,a-2\sqrt{-4ac+b^{2}}\,b}\,c}+\frac{2\sqrt{-4ac+b^{2}}\,ab\arctan\left(\frac{-2a-2b-\frac{2\left(\sqrt{-x^{2}+1}-1\right)^{2}a}{x^{2}}+2\sqrt{-4ac+b^{2}}}{2\sqrt{-2ab+4ac-2b^{2}+2\sqrt{-4ac+b^{2}}\,a+2\sqrt{-4ac+b^{2}}\,b}}\right)}{\left(8ac-2b^{2}\right)\sqrt{-2ab+4ac-2b^{2}+2\sqrt{-4ac+b^{2}}\,a+2\sqrt{-4ac+b^{2}}\,b}\,c}+\frac{2\sqrt{-4ac+b^{2}}\,ab\arctan\left(\frac{2a+2b+\frac{2\left(\sqrt{-x^{2}+1}-1\right)^{2}a}{x^{2}}+2\sqrt{-4ac+b^{2}}}{2\sqrt{-2ab+4ac-2b^{2}-2\sqrt{-4ac+b^{2}}\,a-2\sqrt{-4ac+b^{2}}\,b}}\right)}{\left(8ac-2b^{2}\right)\sqrt{-2ab+4ac-2b^{2}-2\sqrt{-4ac+b^{2}}\,a-2\sqrt{-4ac+b^{2}}\,b}\,c}+\frac{4\sqrt{-4ac+b^{2}}\,a\arctan\left(\frac{-2a-2b-\frac{2\left(\sqrt{-x^{2}+1}-1\right)^{2}a}{x^{2}}+2\sqrt{-4ac+b^{2}}}{2\sqrt{-2ab+4ac-2b^{2}+2\sqrt{-4ac+b^{2}}\,a+2\sqrt{-4ac+b^{2}}\,b}}\right)}{\left(8ac-2b^{2}\right)\sqrt{-2ab+4ac-2b^{2}+2\sqrt{-4ac+b^{2}}\,a+2\sqrt{-4ac+b^{2}}\,b}}+\frac{4\sqrt{-4ac+b^{2}}\,a\arctan\left(\frac{2a+2b+\frac{2\left(\sqrt{-x^{2}+1}-1\right)^{2}a}{x^{2}}+2\sqrt{-4ac+b^{2}}}{2\sqrt{-2ab+4ac-2b^{2}-2\sqrt{-4ac+b^{2}}\,a-2\sqrt{-4ac+b^{2}}\,b}}\right)}{\left(8ac-2b^{2}\right)\sqrt{-2ab+4ac-2b^{2}-2\sqrt{-4ac+b^{2}}\,a-2\sqrt{-4ac+b^{2}}\,b}}+\frac{2}{\left(\frac{-x^{2}+1}{x^{2}}-\frac{2\sqrt{-x^{2}+1}}{x^{2}}+\frac{1}{x^{2}}+1\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 2/(8*a*c-2*b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(
1/2)*b)^(1/2)*(-4*a*c+b^2)^(1/2)*a*b/c*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/
2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2
)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2))+4*a/(8*a*c-2*b^2)/(-2*a*b+4*a*c-2*b^2+2*
(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(-2*a-2*b-2*(
(-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a
*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*(-4*a*c+b^2)^(1/2)-8/(8*a*c-
2*b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(
1/2)*a^2*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(
1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1
/2))+2/(8*a*c-2*b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b
^2)^(1/2)*b)^(1/2)*a*b^2/c*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^
2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*
c+b^2)^(1/2)*b)^(1/2))+2/(8*a*c-2*b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(
1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*(-4*a*c+b^2)^(1/2)*a*b/c*arctan(1/2*(2
*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b
^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))+4*a/(8*a*c-2*b^2)/
(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*ar
ctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*
b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*(-4*a*c
+b^2)^(1/2)+8/(8*a*c-2*b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-
4*a*c+b^2)^(1/2)*b)^(1/2)*a^2*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/
x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*
```

```
a*c+b^2)^(1/2)*b)^(1/2))-2/(8*a*c-2*b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)
^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*a*b^2/c*arctan(1/2*(2*a+2*b+2*((-x^2
+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^
2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))+2/c/(1/x^2*(-x^2+1)-2*(-x^2+1)^(1
/2)/x^2+1/x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}\,x^3}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^3/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 1.31, size = 776, normalized size = 3.39

$$\frac{\sqrt{1-x^2}}{c}-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(4ac^2-b^2c-b^3+b^2\sqrt{b^2-4ac}+4abc-2ac\sqrt{b^2-4ac}+bc\sqrt{b^2-4ac}\right)}{4c^2\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,\left(4ac-b^2\right)}+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(b^2c-4ac^2+b^3+b^2\sqrt{b^2-4ac}-4abc-2ac\sqrt{b^2-4ac}+bc\sqrt{b^2-4ac}\right)}{4c^2\left(4ac-b^2\right)\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(4ac^2-b^2c-b^3+b^2\sqrt{b^2-4ac}+4abc-2ac\sqrt{b^2-4ac}+bc\sqrt{b^2-4ac}\right)}{4c^2\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,\left(4ac-b^2\right)}+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+1\right)1i}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(b^2c-4ac^2+b^3+b^2\sqrt{b^2-4ac}-4abc-2ac\sqrt{b^2-4ac}+bc\sqrt{b^2-4ac}\right)}{4c^2\left(4ac-b^2\right)\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (1 - x^2)^(1/2)/c - (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)
*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x -
 (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(4*a*c^2 - b^2*c - b^3 + b^2*(b
^2 - 4*a*c)^(1/2) + 4*a*b*c - 2*a*c*(b^2 - 4*a*c)^(1/2) + b*c*(b^2 - 4*a*c)
^(1/2)))/(4*c^2*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2))
+ (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 -
4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2)))*(b^2*c - 4*a*c^2 + b^3 + b^2*(b^2 - 4*a*c)^(1/2)
- 4*a*b*c - 2*a*c*(b^2 - 4*a*c)^(1/2) + b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^2*(4
*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b -
 (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c
) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^
(1/2)))*(4*a*c^2 - b^2*c - b^3 + b^2*(b^2 - 4*a*c)^(1/2) + 4*a*b*c - 2*a*c*
(b^2 - 4*a*c)^(1/2) + b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^2*((b - (b^2 - 4*a*c)^
(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2)) + (log((((x*(-(b + (b^2 - 4*a*c)^(1/
2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1
- x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2*c - 4
*a*c^2 + b^3 + b^2*(b^2 - 4*a*c)^(1/2) - 4*a*b*c - 2*a*c*(b^2 - 4*a*c)^(1/2
) + b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2
))/(2*c) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**3*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

3.278 $\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=182

$$\frac{\sqrt{\sqrt{b^2-4ac}+b+2c}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{-\sqrt{b^2-4ac}+b+2c}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.27, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1247, 699, 1130, 208}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b+2c}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{-\sqrt{b^2-4ac}+b+2c}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2]
)/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]))
+ (Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2]
)/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 699

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{1-x}}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= -\,\text{Subst}\left(\int \frac{x^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)$$

$$= \frac{1}{2}\left(-1-\frac{b+2c}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, \sqrt{1-x^2}\right) - \frac{1}{2}\left(1-\dots\right.$$

$$= -\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\dots}{\sqrt{b+2c+\dots}}\right.}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.24, size = 169, normalized size = 0.93

$$\frac{\sqrt{-\sqrt{b^2-4ac}-b-2c}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}-b-2c}}\right)-\sqrt{\sqrt{b^2-4ac}-b-2c}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}-b-2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/
Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]] - Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]*Ar
cTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

IntegrateAlgebraic [A] time = 0.31, size = 190, normalized size = 1.04

$$\frac{\sqrt{-\sqrt{b^2-4ac}-b-2c}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}-b-2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{\sqrt{b^2-4ac}-b-2c}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}-b-2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/
Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]) -
(Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/
Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

fricas [B] time = 3.12, size = 871, normalized size = 4.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt((b + 2*c - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(
b^2*c - 4*a*c^2))*log((b*x^2 + (b^2*c - 4*a*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3
) + sqrt(1/2)*((b^2 - 4*a*c)*x^2 + (b^3*c - 4*a*b*c^2)*x^2/sqrt(b^2*c^2 - 4
*a*c^3))*sqrt((b + 2*c - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2)) - 2*sqrt(-x^2 + 1)*a + 2*a)/x^2) + 1/2*sqrt(1/2)*sqrt((b + 2*c
- (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log((b*x^2
```

```
+ (b^2*c - 4*a*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3) - sqrt(1/2)*((b^2 - 4*a*c)*
x^2 + (b^3*c - 4*a*b*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3))*sqrt((b + 2*c - (b^2
*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2)) - 2*sqrt(-x^2 + 1
)*a + 2*a)/x^2) - 1/2*sqrt(1/2)*sqrt((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*
c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log((b*x^2 - (b^2*c - 4*a*c^2)*x^2/sqrt(
b^2*c^2 - 4*a*c^3) + sqrt(1/2)*((b^2 - 4*a*c)*x^2 - (b^3*c - 4*a*b*c^2)*x^2
/sqrt(b^2*c^2 - 4*a*c^3))*sqrt((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 -
4*a*c^3))/(b^2*c - 4*a*c^2)) - 2*sqrt(-x^2 + 1)*a + 2*a)/x^2) + 1/2*sqrt(1/
2)*sqrt((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*
c^2))*log((b*x^2 - (b^2*c - 4*a*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3) - sqrt(1/2
)*((b^2 - 4*a*c)*x^2 - (b^3*c - 4*a*b*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3))*sqr
t((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))
- 2*sqrt(-x^2 + 1)*a + 2*a)/x^2)
```

giac [B] time = 4.24, size = 591, normalized size = 3.25

$$-\frac{\left(2 b^2 c^2 - 8 a c^3 - \sqrt{2} \sqrt{b^2-4ac} \sqrt{-bc-2c^2-\sqrt{b^2-4ac}\, c}\, b^2 + 4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bc-2c^2-\sqrt{b^2-4ac}\, c}\, ac - 2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bc-2c^2-\sqrt{b^2-4ac}\, c}\, bc - 5\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bc-2c^2-\sqrt{b^2-4ac}\, c}\, c^2 - 2\left(b^2-4ac\right)c^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{-x^2+1}}{\sqrt{-\frac{b+2c+\sqrt{(b+2c)^2-4(a+b+c)c}}{c}}}\right)}{2\left(b^4-8ab^2c+2b^3c+16a^2c^2-8abc^2+5b^2c^2-20ac^3\right)|c|} + \frac{\left(2 b^2 c^2 - 8 a c^3 - \sqrt{2} \sqrt{b^2-4ac} \sqrt{-bc-2c^2+\sqrt{b^2-4ac}\, c}\, b^2 + 4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bc-2c^2+\sqrt{b^2-4ac}\, c}\, ac - 2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bc-2c^2+\sqrt{b^2-4ac}\, c}\, bc - 5\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bc-2c^2+\sqrt{b^2-4ac}\, c}\, c^2 - 2\left(b^2-4ac\right)c^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{-x^2+1}}{\sqrt{-\frac{b+2c-\sqrt{(b+2c)^2-4(a+b+c)c}}{c}}}\right)}{2\left(b^4-8ab^2c+2b^3c+16a^2c^2-8abc^2+5b^2c^2-20ac^3\right)|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - s
qrt(b^2 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - s
qrt(b^2 - 4*a*c)*c)*a*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - s
qrt(b^2 - 4*a*c)*c)*b*c - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - s
qrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*arctan(2*sqrt(1/2)*sqrt(-x^2
 + 1)/sqrt(-(b + 2*c + sqrt((b + 2*c)^2 - 4*(a + b + c)*c))/c))/((b^4 - 8*a
*b^2*c + 2*b^3*c + 16*a^2*c^2 - 8*a*b*c^2 + 5*b^2*c^2 - 20*a*c^3)*abs(c)) +
 1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + s
qrt(b^2 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + s
qrt(b^2 - 4*a*c)*c)*a*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + s
qrt(b^2 - 4*a*c)*c)*b*c - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + s
qrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*arctan(2*sqrt(1/2)*sqrt(-x^2
 + 1)/sqrt(-(b + 2*c - sqrt((b + 2*c)^2 - 4*(a + b + c)*c))/c))/((b^4 - 8*a
*b^2*c + 2*b^3*c + 16*a^2*c^2 - 8*a*b*c^2 + 5*b^2*c^2 - 20*a*c^3)*abs(c))
```

maple [B] time = 0.05, size = 1167, normalized size = 6.41

$$-\frac{4ac\arctan\left(\frac{-2a-2b-\frac{2\left(\sqrt{-x^2+1}-1\right)^2 a}{x^2}+2\sqrt{-4ac+b^2}}{2\sqrt{-2ab+4ac-2b^2+2\sqrt{-4ac+b^2}\,a+2\sqrt{-4ac+b^2}\,b}}\right)}{\left(4ac-b^2\right)\sqrt{-2ab+4ac-2b^2+2\sqrt{-4ac+b^2}\,a+2\sqrt{-4ac+b^2}\,b}}+\frac{4ac\arctan\left(\frac{2a+2b+\frac{2\left(\sqrt{-x^2+1}-1\right)^2 a}{x^2}+2\sqrt{-4ac+b^2}}{2\sqrt{-2ab+4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2\sqrt{-4ac+b^2}\,b}}\right)}{\left(4ac-b^2\right)\sqrt{-2ab+4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2\sqrt{-4ac+b^2}\,b}}+\frac{b^2\arctan\left(\frac{-2a-2b-\frac{2\left(\sqrt{-x^2+1}-1\right)^2 a}{x^2}+2\sqrt{-4ac+b^2}}{2\sqrt{-2ab+4ac-2b^2+2\sqrt{-4ac+b^2}\,a+2\sqrt{-4ac+b^2}\,b}}\right)}{\left(4ac-b^2\right)\sqrt{-2ab+4ac-2b^2+2\sqrt{-4ac+b^2}\,a+2\sqrt{-4ac+b^2}\,b}}-\frac{b^2\arctan\left(\frac{2a+2b+\frac{2\left(\sqrt{-x^2+1}-1\right)^2 a}{x^2}+2\sqrt{-4ac+b^2}}{2\sqrt{-2ab+4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2\sqrt{-4ac+b^2}\,b}}\right)}{\left(4ac-b^2\right)\sqrt{-2ab+4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2\sqrt{-4ac+b^2}\,b}}-\frac{2\sqrt{-4ac+b^2}\,a\arctan\left(\frac{-2a-2b-\frac{2\left(\sqrt{-x^2+1}-1\right)^2 a}{x^2}+2\sqrt{-4ac+b^2}}{2\sqrt{-2ab+4ac-2b^2+2\sqrt{-4ac+b^2}\,a+2\sqrt{-4ac+b^2}\,b}}\right)}{\left(4ac-b^2\right)\sqrt{-2ab+4ac-2b^2+2\sqrt{-4ac+b^2}\,a+2\sqrt{-4ac+b^2}\,b}}-\frac{2\sqrt{-4ac+b^2}\,a\arctan\left(\frac{2a+2b+\frac{2\left(\sqrt{-x^2+1}-1\right)^2 a}{x^2}+2\sqrt{-4ac+b^2}}{2\sqrt{-2ab+4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2\sqrt{-4ac+b^2}\,b}}\right)}{\left(4ac-b^2\right)\sqrt{-2ab+4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2\sqrt{-4ac+b^2}\,b}}-\frac{\sqrt{-4ac+b^2}\,b\arctan\left(\frac{-2a-2b-\frac{2\left(\sqrt{-x^2+1}-1\right)^2 a}{x^2}+2\sqrt{-4ac+b^2}}{2\sqrt{-2ab+4ac-2b^2+2\sqrt{-4ac+b^2}\,a+2\sqrt{-4ac+b^2}\,b}}\right)}{\left(4ac-b^2\right)\sqrt{-2ab+4ac-2b^2+2\sqrt{-4ac+b^2}\,a+2\sqrt{-4ac+b^2}\,b}}-\frac{\sqrt{-4ac+b^2}\,b\arctan\left(\frac{2a+2b+\frac{2\left(\sqrt{-x^2+1}-1\right)^2 a}{x^2}+2\sqrt{-4ac+b^2}}{2\sqrt{-2ab+4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2\sqrt{-4ac+b^2}\,b}}\right)}{\left(4ac-b^2\right)\sqrt{-2ab+4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2\sqrt{-4ac+b^2}\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -2*a/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^
(1/2)*b)^(1/2)*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+
b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)
*b)^(1/2))*(-4*a*c+b^2)^(1/2)-1/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b
^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^
(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(
1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*(-4*a*c+b^2)^(1/2)*b-4*a/(4*a*c-b^2)/
(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*ar
ctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a
*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*c+1/(4
*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b
)^(1/2)*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1
/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/
2))*b^2-2*a/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*
c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-
4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)
```

```
^(1/2)*b)^(1/2))*(-4*a*c+b^2)^(1/2)-1/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4
*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(2*a+2*b+2*((-x^
2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b
^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*(-4*a*c+b^2)^(1/2)*b+4*a/(4*a*c-
b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/
2)*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(
-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*c-
1/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/
2)*b)^(1/2)*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)
^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^
(1/2))*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}\,x}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 1.29, size = 649, normalized size = 3.57

$$\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(4ac+b\sqrt{b^2-4ac}+2c\sqrt{b^2-4ac}-b^2\right)}{4c\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,\left(4ac-b^2\right)}-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(b\sqrt{b^2-4ac}-4ac+2c\sqrt{b^2-4ac}+b^2\right)}{4c\left(4ac-b^2\right)\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(4ac+b\sqrt{b^2-4ac}+2c\sqrt{b^2-4ac}-b^2\right)}{4c\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,\left(4ac-b^2\right)}-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+1\right)1i}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(b\sqrt{b^2-4ac}-4ac+2c\sqrt{b^2-4ac}+b^2\right)}{4c\left(4ac-b^2\right)\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c
)^(1/2))/(2*c))^(1/2)))*(4*a*c + b*(b^2 - 4*a*c)^(1/2) + 2*c*(b^2 - 4*a*c)^
(1/2) - b^2))/(4*c*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2
)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2
 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 -
4*a*c)^(1/2))/(2*c))^(1/2)))*(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + 2*c*(b^2 - 4*
a*c)^(1/2) + b^2))/(4*c*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)
^(1/2)) + (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b -
 (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b
^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(4*a*c + b*(b^2 - 4*a*c)^(1/2) + 2*c*(b^2
 - 4*a*c)^(1/2) - b^2))/(4*c*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4
*a*c - b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/
((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b
 + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + 2*c
*(b^2 - 4*a*c)^(1/2) + b^2))/(4*c*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/
(2*c) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

3.279 $\int \frac{\sqrt{1-x^2}}{x\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=241

$$\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{c}\left(-\sqrt{b^2-4ac}+2a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\tan}{}$$

Rubi [A] time = 1.65, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1251, 897, 1287, 207, 1166, 208}

$$\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{c}\left(-\sqrt{b^2-4ac}+2a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\tanh^{-1}\left(\sqrt{1-x^2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(ArcTanh[Sqrt[1 - x^2]]/a) + (Sqrt[c]*(2*a + b + Sqrt[b^2 - 4*a*c])*ArcTan
h[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*a
+ b - Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2
*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c + Sqrt[
b^2 - 4*a*c]])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-x^2}}{x\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right) \\
&= -\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)\left(a+b+c+(-b-2c)x^2+cx^4\right)}\,dx, x, \sqrt{1-x^2}\right) \\
&= -\text{Subst}\left(\int \left(-\frac{1}{a\left(-1+x^2\right)}+\frac{-a-b-c+cx^2}{a\left(a+b+c-(b+2c)x^2+cx^4\right)}\right)dx, x, \sqrt{1-x^2}\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1-x^2}\right)}{a} - \frac{\text{Subst}\left(\int \frac{-a-b-c+cx^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)}{a} \\
&= -\frac{\tanh^{-1}\left(\sqrt{1-x^2}\right)}{a} + \frac{\left(c\left(2a+b-\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, \ldots\right.}{2a\sqrt{b^2-4ac}} \\
&= -\frac{\tanh^{-1}\left(\sqrt{1-x^2}\right)}{a} + \frac{\sqrt{c}\left(2a+b+\sqrt{b^2-4ac}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(2a+\ldots\right.}{\sqrt{2}\,\ldots}
\end{aligned}$$

Mathematica [A] time = 0.43, size = 212, normalized size = 0.88

$$\frac{\frac{\sqrt{2}\left(\sqrt{-\sqrt{b^2-4ac}+b+2c}\left(\sqrt{b^2-4ac}+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)+\left(\sqrt{b^2-4ac}-b\right)\sqrt{\sqrt{b^2-4ac}+b+2c}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)\right)}{\sqrt{c}\sqrt{b^2-4ac}}-4\tanh^{-1}\left(\sqrt{1-x^2}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-4*ArcTanh[Sqrt[1 - x^2]] + (Sqrt[2]*(Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*(b
 + Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c
- Sqrt[b^2 - 4*a*c]]] + (-b + Sqrt[b^2 - 4*a*c])*Sqrt[b + 2*c + Sqrt[b^2 -
4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4
*a*c]]]))/(Sqrt[c]*Sqrt[b^2 - 4*a*c]))/(4*a)
```

IntegrateAlgebraic [A] time = 0.65, size = 277, normalized size = 1.15

$$-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}-2a-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}-b-2c}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}-b-2c}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2a+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}-b-2c}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}-b-2c}}+\frac{\log\left(\sqrt{1-x^2}-1\right)}{2a}-\frac{\log\left(a\sqrt{1-x^2}+a\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[c]*(-2*a - b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 -
x^2])/Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqr
t[-b - 2*c - Sqrt[b^2 - 4*a*c]])) - (Sqrt[c]*(2*a + b + Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]) + Log[-1
+ Sqrt[1 - x^2]]/(2*a) - Log[a + a*Sqrt[1 - x^2]]/(2*a)
```

fricas [B] time = 11.65, size = 1232, normalized size = 5.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*a*sqrt((a*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3*c)*sqrt((a^2 +
2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log((2*sqrt(1/2)*(a
^3*b^2 - 4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))*sqrt((a
*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 -
4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) + (a^2*b^2 - 4*a^3*c)*x^2*sqrt((a^2 + 2*a*b
 + b^2)/(a^4*b^2 - 4*a^5*c)) + (a*b + b^2)*x^2 + 2*a^2 + 2*a*b - 2*(a^2 + a
*b)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*a*sqrt((a*b + b^2 - 2*a*c + (a^2*b^2 -
 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c
))*log(-(2*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*
b^2 - 4*a^5*c))*sqrt((a*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2
*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - (a^2*b^2 - 4*a^3*c
)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)) - (a*b + b^2)*x^2 - 2*a
^2 - 2*a*b + 2*(a^2 + a*b)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*a*sqrt((a*b + b
^2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*
c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*x^2*sqrt((a
^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))*sqrt((a*b + b^2 - 2*a*c - (a^2*b^2 -
 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c
)) + (a^2*b^2 - 4*a^3*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))
- (a*b + b^2)*x^2 - 2*a^2 - 2*a*b + 2*(a^2 + a*b)*sqrt(-x^2 + 1))/x^2) - sq
rt(1/2)*a*sqrt((a*b + b^2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b +
 b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log((2*sqrt(1/2)*(a^3*b^2
- 4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))*sqrt((a*b + b^
2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c
)))/(a^2*b^2 - 4*a^3*c)) - (a^2*b^2 - 4*a^3*c)*x^2*sqrt((a^2 + 2*a*b + b^2)
/(a^4*b^2 - 4*a^5*c)) + (a*b + b^2)*x^2 + 2*a^2 + 2*a*b - 2*(a^2 + a*b)*sqr
t(-x^2 + 1))/x^2) + 2*log((sqrt(-x^2 + 1) - 1)/x))/a
```

giac [B] time = 3.67, size = 3639, normalized size = 15.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*log(sqrt(-x^2 + 1) + 1)/a + 1/2*log(-sqrt(-x^2 + 1) + 1)/a + 1/8*(4*a^
3*b^3*c^2 + 2*a^2*b^4*c^2 - 16*a^4*b*c^3 + 4*a^2*b^3*c^3 - 32*a^4*c^4 - 16*
a^3*b*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*
c)*c)*a^3*b^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*
a*c)*c)*a^2*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2
- 4*a*c)*c)*a^4*b*c - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(
b^2 - 4*a*c)*c)*a^3*b^2*c - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c -
2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^4*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b
*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 9*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 20*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 10*sqrt(2)*sqrt(
```

b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 4*(b^2 - 4*a*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3 - 4*(b^2 - 4*a*c)*a^2*b*c^3 + (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 + 2*(sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - 6*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 3*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a^2*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 11*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 16*a^3*b^2*c^2 + 7*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 - 2*a*b^4*c^2 - 4*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 32*a^4*c^3 - 28*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 5*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 20*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a^2*b^2*c + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^3*c^2 - 8*(b^2 - 4*a*c)*a^2*b*c^2 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*abs(a))*arctan(2*sqrt(1/2)*sqrt(-x^2 + 1)/sqrt(-(a*b + 2*a*c + sqrt(-4*(a^2 + a*b + a*c)*a*c + (a*b + 2*a*c)^2))/(a*c)))/((a^3*b^4 + a^2*b^5 - 8*a^4*b^2*c - 6*a^3*b^3*c + 3*a^2*b^4*c + 16*a^5*c^2 + 8*a^4*b*c^2 - 11*a^3*b^2*c^2 + 7*a^2*b^3*c^2 - 4*a^4*c^3 - 28*a^3*b*c^3 + 5*a^2*b^2*c^3 - 20*a^3*c^4)*abs(a)*abs(c)) - 1/8*(4*a^3*b^3*c^2 + 2*a^2*b^4*c^2 - 16*a^4*b*c^3 + 4*a^2*b^3*c^3 - 32*a^4*c^4 - 16*a^3*b*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*b^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^4*b*c - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^4*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 4*(b^2 - 4*a*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3 - 4*(b^2 - 4*a*c)*a^2*b*c^3 + (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 - 2*(sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - 6*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 3*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*a^2*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 11*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 16*a^3*b^2

```
*c^2 + 7*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 16*a^
2*b^3*c^2 + 2*a*b^4*c^2 - 4*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c
)*a^3*c^3 + 32*a^4*c^3 - 28*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^3 + 32*a^3*b*c^3 + 5*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c
)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 20*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 -
 4*a*c)*c)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a^2*b^2*c - 2*(b^2 - 4*a*
c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^2 - 2*(b^2 -
 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*abs(a))*arctan(2*sqrt(1/2)*sqr
t(-x^2 + 1)/sqrt(-(a*b + 2*a*c - sqrt(-4*(a^2 + a*b + a*c)*a*c + (a*b + 2*a
*c)^2))/(a*c)))/((a^3*b^4 + a^2*b^5 - 8*a^4*b^2*c - 6*a^3*b^3*c + 3*a^2*b^4
*c + 16*a^5*c^2 + 8*a^4*b*c^2 - 11*a^3*b^2*c^2 + 7*a^2*b^3*c^2 - 4*a^4*c^3
- 28*a^3*b*c^3 + 5*a^2*b^2*c^3 - 20*a^3*c^4)*abs(a)*abs(c))
```

maple [B] time = 0.06, size = 2099, normalized size = 8.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/a*(-x^2+1)^(1/2)-1/a*arctanh(1/(-x^2+1)^(1/2))+1/(4*a*c-b^2)/(-2*a*b+4*a*
c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*(-4*a*c+b^2)^(
1/2)*b*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/
2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2
))-2/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^
(1/2)*b)^(1/2)*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+
b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)
*b)^(1/2))*c*(-4*a*c+b^2)^(1/2)+1/a/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a
*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(-2*a-2*b-2*((-x^2
+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^
2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*b^2*(-4*a*c+b^2)^(1/2)+4/(4*a*c-b
^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2
)*a*c*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2
))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)
)-1/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(
1/2)*b)^(1/2)*b^2*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a
*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1
/2)*b)^(1/2))+4/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-
4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2
+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c
+b^2)^(1/2)*b)^(1/2))*b*c-1/a/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2
)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1
/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/
2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*b^3+1/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2
*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*(-4*a*c+b^2)^(1/2)*b*ar
ctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*
b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))-2/(4*a*
c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(
1/2)*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))
/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*
c*(-4*a*c+b^2)^(1/2)+1/a/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/
2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)
^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2
*(-4*a*c+b^2)^(1/2)*b)^(1/2))*b^2*(-4*a*c+b^2)^(1/2)-4/(4*a*c-b^2)/(-2*a*b+
4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*a*c*arctan
(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*
a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))+1/(4*a*c-b^
2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)
*b^2*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))
/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))-
```

```
4/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/
2)*b)^(1/2)*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)
^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^
(1/2))*b*c+1/a/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4
*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2
*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b
^2)^(1/2)*b)^(1/2))*b^3-2/a/(2/x^2-2*(-x^2+1)^(1/2)/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}}{\left(cx^4+bx^2+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x), x)
```

mupad [B] time = 1.30, size = 669, normalized size = 2.78

$$\frac{\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)}{a}+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+1\right)1i}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(4ac+2a\sqrt{b^2-4ac}+b\sqrt{b^2-4ac}-b^2\right)}{4a\left(4ac-b^2\right)\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(2a\sqrt{b^2-4ac}-4ac+b\sqrt{b^2-4ac}+b^2\right)}{4a\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\left(4ac-b^2\right)}+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(4ac+2a\sqrt{b^2-4ac}+b\sqrt{b^2-4ac}-b^2\right)}{4a\left(4ac-b^2\right)\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(2a\sqrt{b^2-4ac}-4ac+b\sqrt{b^2-4ac}+b^2\right)}{4a\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\left(4ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2))/a + (log((((x*(-(b + (b^2 - 4*a*c)^(
1/2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (
1 - x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(4*a*c +
 2*a*(b^2 - 4*a*c)^(1/2) + b*(b^2 - 4*a*c)^(1/2) - b^2))/(4*a*(4*a*c - b^2)
*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b - (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2
) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(2*
a*(b^2 - 4*a*c)^(1/2) - 4*a*c + b*(b^2 - 4*a*c)^(1/2) + b^2))/(4*a*((b - (b
^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2)) + (log((((x*(-(b + (b^2
- 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)
^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2))
)*(4*a*c + 2*a*(b^2 - 4*a*c)^(1/2) + b*(b^2 - 4*a*c)^(1/2) - b^2))/(4*a*(4*
a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b -
(b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c)
 + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(
1/2)))*(2*a*(b^2 - 4*a*c)^(1/2) - 4*a*c + b*(b^2 - 4*a*c)^(1/2) + b^2))/(4*
a*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)}}{x\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(x*(a + b*x**2 + c*x**4)), x)
```

3.280 $\int \frac{\sqrt{1-x^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=290

$$-\frac{\sqrt{c}\left(\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}}+a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\,a^2\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{c}\left(-\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}}+a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\,a^2\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{(a+2b}{}$$

Rubi [A] time = 2.36, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1251, 897, 1287, 207, 1166, 208}

$$-\frac{\sqrt{c}\left(\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}}+a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\,a^2\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{c}\left(-\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}}+a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\,a^2\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{(a+2b)\tanh^{-1}\left(\sqrt{1-x^2}\right)}{2a^2}-\frac{1}{4a\left(1-\sqrt{1-x^2}\right)}+\frac{1}{4a\left(\sqrt{1-x^2}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(4*a*(1 - Sqrt[1 - x^2])) + 1/(4*a*(1 + Sqrt[1 - x^2])) + ((a + 2*b)*Arc
Tanh[Sqrt[1 - x^2]])/(2*a^2) - (Sqrt[c]*(a + b + (b^2 + a*(b - 2*c))/Sqrt[b
^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(a
 + b - (b^2 + a*(b - 2*c))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt
[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + 2*c +
Sqrt[b^2 - 4*a*c]])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{1-x^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2\left(a+bx+cx^2\right)}\,dx,x,x^2\right)$$

$$= -\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^2\left(a+b+c+(-b-2c)x^2+cx^4\right)}\,dx,x,\sqrt{1-x^2}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{1}{4a(-1+x)^2}+\frac{1}{4a(1+x)^2}+\frac{a+2b}{2a^2\left(-1+x^2\right)}+\frac{b(a+b+c)-(a+b)c\ldots}{a^2\left(a+b+c-(b+2c)x^2\ldots\right.}\right.\right.$$

$$= -\frac{1}{4a\left(1-\sqrt{1-x^2}\right)}+\frac{1}{4a\left(1+\sqrt{1-x^2}\right)}-\frac{\text{Subst}\left(\int \frac{b(a+b+c)-(a+b)cx^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx,x,\sqrt{1-x^2}\right)}{a^2}\ldots$$

$$= -\frac{1}{4a\left(1-\sqrt{1-x^2}\right)}+\frac{1}{4a\left(1+\sqrt{1-x^2}\right)}+\frac{(a+2b)\tanh^{-1}\left(\sqrt{1-x^2}\right)}{2a^2}+\frac{\left(c\left(a+b-\frac{b^2}{\ldots}\right.\right.}{\ldots}$$

$$= -\frac{1}{4a\left(1-\sqrt{1-x^2}\right)}+\frac{1}{4a\left(1+\sqrt{1-x^2}\right)}+\frac{(a+2b)\tanh^{-1}\left(\sqrt{1-x^2}\right)}{2a^2}-\frac{\sqrt{c}\left(a+b+\ldots\right.}{\sqrt{2}\ldots}$$

Mathematica [A] time = 0.76, size = 292, normalized size = 1.01

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(-\frac{\left(b\left(\sqrt{b^2-4ac}+b\right)+a\left(\sqrt{b^2-4ac}+b-2c\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(b\left(\sqrt{b^2-4ac}-b\right)+a\left(\sqrt{b^2-4ac}-b+2c\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{b^2-4ac}}+(a+2b)\log\left(\sqrt{1-x^2}+1\right)-(a+2b)\log(x)-\frac{a\sqrt{1-x^2}}{x^2}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-((a*Sqrt[1 - x^2])/x^2) + (Sqrt[2]*Sqrt[c]*(-(((b*(b + Sqrt[b^2 - 4*a*c])
 + a*(b - 2*c + Sqrt[b^2 - 4*a*c]))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])
/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (
(b*(-b + Sqrt[b^2 - 4*a*c]) + a*(-b + 2*c + Sqrt[b^2 - 4*a*c]))*ArcTanh[(Sq
rt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + 2
*c + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c] - (a + 2*b)*Log[x] + (a + 2*b)*
Log[1 + Sqrt[1 - x^2]])/(2*a^2)
```

IntegrateAlgebraic [A] time = 2.09, size = 360, normalized size = 1.24

$$\frac{\left(b\sqrt{c}\sqrt{b^2-4ac}+a\sqrt{c}\sqrt{b^2-4ac}-ab\sqrt{c}+2ac^{3/2}+b^2\left(-\sqrt{c}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}-b-2c}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}-b-2c}}+\frac{\left(b\sqrt{c}\sqrt{b^2-4ac}+a\sqrt{c}\sqrt{b^2-4ac}+ab\sqrt{c}-2ac^{3/2}+b^2\sqrt{c}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}-b-2c}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}-b-2c}}+\frac{(a+2b)\tanh^{-1}\left(\sqrt{1-x^2}\right)}{2a^2}-\frac{\sqrt{1-x^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*Sqrt[1 - x^2]/(a*x^2) + ((-(a*b*Sqrt[c]) - b^2*Sqrt[c] + 2*a*c^(3/2) +
 a*Sqrt[c]*Sqrt[b^2 - 4*a*c] + b*Sqrt[c]*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*S
qrt[b^2 - 4*a*c]*Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]) + ((a*b*Sqrt[c] + b^2*
Sqrt[c] - 2*a*c^(3/2) + a*Sqrt[c]*Sqrt[b^2 - 4*a*c] + b*Sqrt[c]*Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c + Sqrt[b^2 - 4
*a*c]]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]])
 + ((a + 2*b)*ArcTanh[Sqrt[1 - x^2]])/(2*a^2)
```

fricas [B] time = 32.22, size = 2799, normalized size = 9.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*a^2*x^2*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)
*c - (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4
*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/
(a^4*b^2 - 4*a^5*c))*log(((a^4*b^2*c - 4*a^5*c^2)*x^2*sqrt((a^2*b^4 + 2*a*b
^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b
^4)*c)/(a^8*b^2 - 4*a^9*c)) + 2*(a^3 + 2*a^2*b)*c^2 + ((a^2*b + 2*a*b^2)*c^
2 - (a*b^3 + b^4)*c)*x^2 - 2*(a^2*b^2 + a*b^3)*c + sqrt(1/2)*((a^5*b^3 - 4*
a^6*b*c)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^
2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) + (a^2*b^4 +
a*b^5 + 4*(a^4 + 2*a^3*b)*c^2 - (5*a^3*b^2 + 6*a^2*b^3)*c)*x^2)*sqrt((a*b^3
 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c - (a^4*b^2 - 4*a^5*c)*sqrt((a^2*
b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*
b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 2*((a^3 + 2*
a^2*b)*c^2 - (a^2*b^2 + a*b^3)*c)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*a^2*x^2*
sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c - (a^4*b^2 - 4*a^5*c)
*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b
^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log
(((a^4*b^2*c - 4*a^5*c^2)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*
b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*
c)) + 2*(a^3 + 2*a^2*b)*c^2 + ((a^2*b + 2*a*b^2)*c^2 - (a*b^3 + b^4)*c)*x^2
 - 2*(a^2*b^2 + a*b^3)*c - sqrt(1/2)*((a^5*b^3 - 4*a^6*b*c)*x^2*sqrt((a^2*b
^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b
^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) + (a^2*b^4 + a*b^5 + 4*(a^4 + 2*a^3*b
)*c^2 - (5*a^3*b^2 + 6*a^2*b^3)*c)*x^2)*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*
a^2*b + 4*a*b^2)*c - (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a
^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b
^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 2*((a^3 + 2*a^2*b)*c^2 - (a^2*b^2 +
a*b^3)*c)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*a^2*x^2*sqrt((a*b^3 + b^4 + 2*a^
2*c^2 - (3*a^2*b + 4*a*b^2)*c + (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5
 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4
)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-((a^4*b^2*c - 4*a^5*c^
2)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*
(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) - 2*(a^3 + 2*a^2*b)
*c^2 - ((a^2*b + 2*a*b^2)*c^2 - (a*b^3 + b^4)*c)*x^2 + 2*(a^2*b^2 + a*b^3)*
c + sqrt(1/2)*((a^5*b^3 - 4*a^6*b*c)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a
^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b
^2 - 4*a^9*c)) - (a^2*b^4 + a*b^5 + 4*(a^4 + 2*a^3*b)*c^2 - (5*a^3*b^2 + 6*
a^2*b^3)*c)*x^2)*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c + (a
^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^
2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^
2 - 4*a^5*c)) + 2*((a^3 + 2*a^2*b)*c^2 - (a^2*b^2 + a*b^3)*c)*sqrt(-x^2 + 1
```

```
))/x^2) - sqrt(1/2)*a^2*x^2*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*
b^2)*c + (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b
 + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c
)))/(a^4*b^2 - 4*a^5*c))*log(-((a^4*b^2*c - 4*a^5*c^2)*x^2*sqrt((a^2*b^4 +
2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 +
2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) - 2*(a^3 + 2*a^2*b)*c^2 - ((a^2*b + 2*a*b^
2)*c^2 - (a*b^3 + b^4)*c)*x^2 + 2*(a^2*b^2 + a*b^3)*c - sqrt(1/2)*((a^5*b^3
 - 4*a^6*b*c)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^
2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) - (a^2*b
^4 + a*b^5 + 4*(a^4 + 2*a^3*b)*c^2 - (5*a^3*b^2 + 6*a^2*b^3)*c)*x^2)*sqrt((
a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c + (a^4*b^2 - 4*a^5*c)*sqrt(
(a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3
*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 2*((a^3
 + 2*a^2*b)*c^2 - (a^2*b^2 + a*b^3)*c)*sqrt(-x^2 + 1))/x^2) + (a + 2*b)*x^2
*log((sqrt(-x^2 + 1) - 1)/x) + sqrt(-x^2 + 1)*a)/(a^2*x^2)
```

giac [B] time = 7.12, size = 1675, normalized size = 5.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt
(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(-b*c - 2*c^2
- sqrt(b^2 - 4*a*c)*c)*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt(-b*c - 2*c^2 - sqr
t(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^2 + 5*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c^
2 - 16*a*b^3*c^2 + 2*b^4*c^2 - 20*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*
a*c)*c)*a*b*c^3 + 32*a^2*b*c^3 - 12*a*b^2*c^3 + 16*a^2*c^4 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4 + 6*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c - 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 5*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^2*c^2
+ 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*c
^3 - 2*(b^2 - 4*a*c)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*b^2*
c^2 + 4*(b^2 - 4*a*c)*a*c^3)*arctan(2*sqrt(1/2)*sqrt(-x^2 + 1)/sqrt(-(a^2*b
 + 2*a^2*c + sqrt(-4*(a^3 + a^2*b + a^2*c)*a^2*c + (a^2*b + 2*a^2*c)^2))/(a
^2*c)))/((a^2*b^4 - 8*a^3*b^2*c + 2*a^2*b^3*c + 16*a^4*c^2 - 8*a^3*b*c^2 +
5*a^2*b^2*c^2 - 20*a^3*c^3)*abs(c)) - 1/4*(sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt
(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c + 2*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4*c - 2*b^5*c
 + 16*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2
)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 5*sqrt(2)*sqrt(-b*c
- 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - 2*b^4*c^2 - 20*sqrt
(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 32*a^2*b*c^3 + 12*a*
b^2*c^3 - 16*a^2*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b
^2 - 4*a*c)*c)*b^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b
^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sq
rt(b^2 - 4*a*c)*c)*b^3*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 +
sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^
2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c -
2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b
*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 -
4*a*c)*a*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c^2 - 4*(b^2 - 4*a*c)*a*c^3)*arctan(2*
sqrt(1/2)*sqrt(-x^2 + 1)/sqrt(-(a^2*b + 2*a^2*c - sqrt(-4*(a^3 + a^2*b + a^
2*c)*a^2*c + (a^2*b + 2*a^2*c)^2))/(a^2*c)))/((a^2*b^4 - 8*a^3*b^2*c + 2*a^
2*b^3*c + 16*a^4*c^2 - 8*a^3*b*c^2 + 5*a^2*b^2*c^2 - 20*a^3*c^3)*abs(c)) +
```

```
1/4*(a + 2*b)*log(sqrt(-x^2 + 1) + 1)/a^2 - 1/4*(a + 2*b)*log(-sqrt(-x^2 +
1) + 1)/a^2 - 1/2*sqrt(-x^2 + 1)/(a*x^2)
```

maple [B] time = 0.08, size = 2770, normalized size = 9.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/a^2*b*(-x^2+1)^(1/2)+1/a^2*b*arctanh(1/(-x^2+1)^(1/2))+2/(4*a*c-b^2)/(-2
*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*(-4*a
*c+b^2)^(1/2)*c*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c
+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2
)*b)^(1/2))-1/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*
a*c+b^2)^(1/2)*b)^(1/2)*(-4*a*c+b^2)^(1/2)/a*b^2*arctan(1/2*(-2*a-2*b-2*((-
x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c
+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2))+3/a/(4*a*c-b^2)/(-2*a*b+4*a*c-
2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(-2*a
-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2
+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*b*c*(-4*a*c+b^2)^(1/
2)-1/a^2/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b
^2)^(1/2)*b)^(1/2)*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*
a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(
1/2)*b)^(1/2))*b^3*(-4*a*c+b^2)^(1/2)-4/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(
-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*b*c*arctan(1/2*(-2*a-2*b-
2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-
4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2))+4/(4*a*c-b^2)/(-2*a*b+4*a
*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(-
2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*
b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*c^2+1/(4*a*c-b^2)
/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)/a
*b^3*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2)
)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2))
-5/a/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^
(1/2)*b)^(1/2)*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+
b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)
*b)^(1/2))*b^2*c+1/a^2/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)
*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^
2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*
(-4*a*c+b^2)^(1/2)*b)^(1/2))*b^4+2/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*
c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*(-4*a*c+b^2)^(1/2)*c*arctan(1/
2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c
-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))-1/(4*a*c-b^2)/
(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*(-
4*a*c+b^2)^(1/2)/a*b^2*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(
-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2
)^(1/2)*b)^(1/2))+3/a/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*
a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*
a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-
4*a*c+b^2)^(1/2)*b)^(1/2))*b*c*(-4*a*c+b^2)^(1/2)-1/a^2/(4*a*c-b^2)/(-2*a*b
+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/
2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c
-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*b^3*(-4*a*c+b^
2)^(1/2)+4/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c
+b^2)^(1/2)*b)^(1/2)*b*c*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2
*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b
^2)^(1/2)*b)^(1/2))-4/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*
a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*
a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-
```

```
4*a*c+b^2)^(1/2)*b)^(1/2))*c^2-1/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+
b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)/a*b^3*arctan(1/2*(2*a+2*b+2*((-x
^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+
b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))+5/a/(4*a*c-b^2)/(-2*a*b+4*a*c-2
*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(2*a+2
*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2
*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*b^2*c-1/a^2/(4*a*c-b^2
)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*
arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*
a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*b^4+2
/a^2*b/(2/x^2-2*(-x^2+1)^(1/2)/x^2)-1/2/a/x^2*(-x^2+1)^(3/2)-1/2*(-x^2+1)^(
1/2)/a+1/2/a*arctanh(1/(-x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}}{\left(cx^4+bx^2+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x^3), x)
```

mupad [B] time = 1.41, size = 825, normalized size = 2.84

$$\frac{\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)}{2a}-\frac{\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)(a+b)}{a^2}-\frac{\sqrt{1-x^2}}{2ax^2}-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+1\right)1i}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(4a^2c-ab^2-b^3+b^2\sqrt{b^2-4ac}+4abc+ab\sqrt{b^2-4ac}-2ac\sqrt{b^2-4ac}\right)}{4a^2\left(4ac-b^2\right)\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(ab^2-4a^2c+b^3+b^2\sqrt{b^2-4ac}-4abc+ab\sqrt{b^2-4ac}-2ac\sqrt{b^2-4ac}\right)}{4a^2\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\left(4ac-b^2\right)}-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(4a^2c-ab^2-b^3+b^2\sqrt{b^2-4ac}+4abc+ab\sqrt{b^2-4ac}-2ac\sqrt{b^2-4ac}\right)}{4a^2\left(4ac-b^2\right)\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(ab^2-4a^2c+b^3+b^2\sqrt{b^2-4ac}-4abc+ab\sqrt{b^2-4ac}-2ac\sqrt{b^2-4ac}\right)}{4a^2\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\left(4ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2))/(2*a) - (log((1/x^2 - 1)^(1/2) - (1/
x^2)^(1/2))*(a + b))/a^2 - (1 - x^2)^(1/2)/(2*a*x^2) - (log((((x*(-(b + (b^
2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) +
1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2
)))*(4*a^2*c - a*b^2 - b^3 + b^2*(b^2 - 4*a*c)^(1/2) + 4*a*b*c + a*b*(b^2 -
 4*a*c)^(1/2) - 2*a*c*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4*a*c - b^2)*((b + (b^2
 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) + (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/
(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^
2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(a*b^2 - 4*a^2
*c + b^3 + b^2*(b^2 - 4*a*c)^(1/2) - 4*a*b*c + a*b*(b^2 - 4*a*c)^(1/2) - 2*
a*c*(b^2 - 4*a*c)^(1/2)))/(4*a^2*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2
)*(4*a*c - b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*
1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x -
(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(4*a^2*c - a*b^2 - b^3 + b^2*(b^
2 - 4*a*c)^(1/2) + 4*a*b*c + a*b*(b^2 - 4*a*c)^(1/2) - 2*a*c*(b^2 - 4*a*c)^
(1/2)))/(4*a^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) +
 (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4
*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*
c)^(1/2))/(2*c))^(1/2)))*(a*b^2 - 4*a^2*c + b^3 + b^2*(b^2 - 4*a*c)^(1/2) -
 4*a*b*c + a*b*(b^2 - 4*a*c)^(1/2) - 2*a*c*(b^2 - 4*a*c)^(1/2)))/(4*a^2*((b
 - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.281 $\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=325

$$-\frac{\left(-\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b}}}{\sqrt{1-x^2}}\right.}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+}}$$

Rubi [A] time = 5.39, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1291, 388, 216, 1692, 377, 205}

$$-\frac{\left(-\frac{-3abc-2ac^2+b^2c+b^3}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(\frac{-3abc-2ac^2+b^2c+b^3}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{(2b+c)\sin^{-1}(x)}{2c^2}+\frac{\sqrt{1-x^2}\,x}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (x*Sqrt[1 - x^2])/(2*c) + ((2*b + c)*ArcSin[x])/(2*c^2) - ((b^2 - a*c + b*c
 - (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*
c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(c^
2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - ((b^2 -
a*c + b*c + (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[b + 2*c + Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^
2])])/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1291

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[f^4/c^2, Int[(f*x)^(m - 4)*(c*d - b*e + c
*e*x^2)*(d + e*x^2)^(q - 1), x], x] - Dist[f^4/c^2, Int[((f*x)^(m - 4)*(d +
 e*x^2)^(q - 1)*Simp[a*(c*d - b*e) + (b*c*d - b^2*e + a*c*e)*x^2, x])/(a +
b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
```

```
0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{\int \frac{b+c-cx^2}{\sqrt{1-x^2}}\,dx}{c^2} - \frac{\int \frac{a(b+c)+\left(b^2-ac+bc\right)x^2}{\sqrt{1-x^2}\left(a+bx^2+cx^4\right)}\,dx}{c^2}$$

$$= \frac{x\sqrt{1-x^2}}{2c} - \frac{\int \left(\frac{b^2-ac+bc+\frac{-b^3+3abc-b^2c+2ac^2}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)} + \frac{b^2-ac+bc-\frac{-b^3+3abc-b^2c+2ac^2}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\right)dx}{c^2} + \frac{(2b+c)\int \frac{1}{\sqrt{1-x^2}}\,dx}{2c^2}$$

$$= \frac{x\sqrt{1-x^2}}{2c} + \frac{(2b+c)\sin^{-1}(x)}{2c^2} - \frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)}\,dx}{c^2}$$

$$= \frac{x\sqrt{1-x^2}}{2c} + \frac{(2b+c)\sin^{-1}(x)}{2c^2} - \frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-b-2c\right.}\right.}{c^2}$$

$$= \frac{x\sqrt{1-x^2}}{2c} + \frac{(2b+c)\sin^{-1}(x)}{2c^2} - \frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{1-x^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{b+2c-\sqrt{b^2-4ac}}}$$

Mathematica [B] time = 6.31, size = 10606, normalized size = 32.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [C] time = 0.97, size = 594, normalized size = 1.83

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (x*Sqrt[1 - x^2])/(2*c) + ((2*b + c)*ArcTan[x/(-1 + Sqrt[1 - x^2])])/c^2 +
RootSum[a + 4*a*#1^2 + 4*b*#1^2 + 6*a*#1^4 + 8*b*#1^4 + 16*c*#1^4 + 4*a*#1^
6 + 4*b*#1^6 + a*#1^8 & , (-(a*b*Log[x]) - a*c*Log[x] + a*b*Log[-1 + Sqrt[1
 - x^2] - x*#1] + a*c*Log[-1 + Sqrt[1 - x^2] - x*#1] - 3*a*b*Log[x]*#1^2 -
4*b^2*Log[x]*#1^2 + a*c*Log[x]*#1^2 - 4*b*c*Log[x]*#1^2 + 3*a*b*Log[-1 + Sq
rt[1 - x^2] - x*#1]*#1^2 + 4*b^2*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^2 - a*c*
Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^2 + 4*b*c*Log[-1 + Sqrt[1 - x^2] - x*#1]*
#1^2 - 3*a*b*Log[x]*#1^4 - 4*b^2*Log[x]*#1^4 + a*c*Log[x]*#1^4 - 4*b*c*Log[
```

```
x]*#1^4 + 3*a*b*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^4 + 4*b^2*Log[-1 + Sqrt[1
 - x^2] - x*#1]*#1^4 - a*c*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^4 + 4*b*c*Log[
-1 + Sqrt[1 - x^2] - x*#1]*#1^4 - a*b*Log[x]*#1^6 - a*c*Log[x]*#1^6 + a*b*L
og[-1 + Sqrt[1 - x^2] - x*#1]*#1^6 + a*c*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^
6)/(a*#1 + b*#1 + 3*a*#1^3 + 4*b*#1^3 + 8*c*#1^3 + 3*a*#1^5 + 3*b*#1^5 + a*
#1^7) & ]/(4*c^2)
```

fricas [B] time = 4.25, size = 2860, normalized size = 8.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c^2*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c +
(b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*
b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*
c^4 - 4*a*c^5))*log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a^2*b^3 - a^3*c^2 - (2*a^3
*b - a^2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^2)*c + sqrt(1/2)*((b^6 + 4*a^2*b*
c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*sqrt(-x^2 + 1)*x - (b^
6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*x - ((b^4*
c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*sqrt(-x^2 + 1)*x - (b^4*c^4 - 6*a*b^2*c^5 +
8*a^2*c^6)*x)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 -
6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))*sqrt(-(b^4
+ (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^6 +
 a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2
*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - 2*(a^2*b^3 -
a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*c^2*sqrt(
-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c + (b^2*c^4 - 4*a*c^5)*sqrt(
(b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2
- 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(-(2*a
^2*b^3 - 2*a^3*c^2 - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*x^2 - 2*
(2*a^3*b - a^2*b^2)*c - sqrt(1/2)*((b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^
3)*c^2 - (6*a*b^4 - b^5)*c)*sqrt(-x^2 + 1)*x - (b^6 + 4*a^2*b*c^3 + (8*a^2*
b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*x - ((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2
*c^6)*sqrt(-x^2 + 1)*x - (b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x)*sqrt((b^6 +
 a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2
*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (
4*a*b^2 - b^3)*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a
*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8
 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*
b^2)*c)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*c^2*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c
^2 - (4*a*b^2 - b^3)*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2
*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b
^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a
^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^2)*c + s
qrt(1/2)*((b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*
c)*sqrt(-x^2 + 1)*x - (b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a
*b^4 - b^5)*c)*x + ((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*sqrt(-x^2 + 1)*x -
(b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a
*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8
 - 4*a*c^9)))*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c - (b^2*c
^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 -
6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 -
4*a*c^5)) - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*sqrt(-x^2 + 1))/x
^2) - sqrt(1/2)*c^2*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c -
(b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*
b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*
c^4 - 4*a*c^5))*log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a^2*b^3 - a^3*c^2 - (2*a^3
*b - a^2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^2)*c - sqrt(1/2)*((b^6 + 4*a^2*b*
```

```
c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*sqrt(-x^2 + 1)*x - (b^
6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*x + ((b^4*
c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*sqrt(-x^2 + 1)*x - (b^4*c^4 - 6*a*b^2*c^5 +
8*a^2*c^6)*x)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 -
6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))*sqrt(-(b^4
+ (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^6 +
 a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2
*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - 2*(a^2*b^3 -
a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*sqrt(-x^2 + 1))/x^2) - sqrt(-x^2 + 1)*c*x
+ 2*(2*b + c)*arctan((sqrt(-x^2 + 1) - 1)/x))/c^2
```

giac [B] time = 6.69, size = 1710, normalized size = 5.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(3*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*b^3 + 2*sqrt(2)*
sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a*b^4 - 2*a^2*b^4 - sqrt(2)*sqrt(2*
a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*b^5 + 2*a*b^5 - 12*sqrt(2)*sqrt(2*a^2 + a*
b + sqrt(b^2 - 4*a*c)*a)*a^3*b*c - 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 -
4*a*c)*a)*a^2*b^2*c + 12*a^3*b^2*c + 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2
- 4*a*c)*a)*a*b^3*c - 16*a^2*b^3*c - 16*a^4*c^2 - 16*sqrt(2)*sqrt(2*a^2 + a
*b + sqrt(b^2 - 4*a*c)*a)*a^2*b*c^2 + 32*a^3*b*c^2 - 3*sqrt(2)*sqrt(2*a^2 +
 a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^2 - 2*sqrt(2)*sqrt(2*a^
2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^3 + sqrt(2)*sqrt(2*a^2
 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^4 + 6*sqrt(2)*sqrt(2*a^2
+ a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c + 4*sqrt(2)*sqrt(2*a^2
 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c - 6*sqrt(2)*sqrt(2*
a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^2*c + 8*sqrt(2)*sqrt
(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*c^2 + 2*(b^2 - 4*
a*c)*a^2*b^2 - 2*(b^2 - 4*a*c)*a*b^3 - 4*(b^2 - 4*a*c)*a^3*c + 8*(b^2 - 4*a
*c)*a^2*b*c)*abs(a)*arctan(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^
2 + 1) - 1)/x)/sqrt((2*a*c^2 + b*c^2 + sqrt(-4*(a*c^2 + b*c^2 + c^3)*a*c^2
+ (2*a*c^2 + b*c^2)^2))/(a*c^2)))/(3*a^4*b^2*c^2 + 2*a^3*b^3*c^2 - a^2*b^4*
c^2 - 12*a^5*c^3 - 8*a^4*b*c^3 + 8*a^3*b^2*c^3 - 16*a^4*c^4) + 1/4*(3*sqrt(
2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^2*b^3 + 2*sqrt(2)*sqrt(2*a^2 +
 a*b - sqrt(b^2 - 4*a*c)*a)*a*b^4 + 2*a^2*b^4 - sqrt(2)*sqrt(2*a^2 + a*b -
sqrt(b^2 - 4*a*c)*a)*b^5 - 2*a*b^5 - 12*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2
 - 4*a*c)*a)*a^3*b*c - 8*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^
2*b^2*c - 12*a^3*b^2*c + 8*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*
a*b^3*c + 16*a^2*b^3*c + 16*a^4*c^2 - 16*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^
2 - 4*a*c)*a)*a^2*b*c^2 - 32*a^3*b*c^2 + 3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(
b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^2 + 2*sqrt(2)*sqrt(2*a^2 + a*b - sq
rt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^3 - sqrt(2)*sqrt(2*a^2 + a*b - sqr
t(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^4 - 6*sqrt(2)*sqrt(2*a^2 + a*b - sqrt
(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c - 4*sqrt(2)*sqrt(2*a^2 + a*b - sqr
t(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c + 6*sqrt(2)*sqrt(2*a^2 + a*b -
sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^2*c - 8*sqrt(2)*sqrt(2*a^2 + a*b
 - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2
 + 2*(b^2 - 4*a*c)*a*b^3 + 4*(b^2 - 4*a*c)*a^3*c - 8*(b^2 - 4*a*c)*a^2*b*c)
*abs(a)*arctan(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/
x)/sqrt((2*a*c^2 + b*c^2 - sqrt(-4*(a*c^2 + b*c^2 + c^3)*a*c^2 + (2*a*c^2 +
 b*c^2)^2))/(a*c^2)))/(3*a^4*b^2*c^2 + 2*a^3*b^3*c^2 - a^2*b^4*c^2 - 12*a^5
*c^3 - 8*a^4*b*c^3 + 8*a^3*b^2*c^3 - 16*a^4*c^4) + 1/2*sqrt(-x^2 + 1)*x/c +
 1/4*(pi*sgn(x) + 2*arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x
^2 + 1) - 1)))*(2*b + c)/c^2
```

maple [C] time = 0.04, size = 222, normalized size = 0.68

$$\frac{x\sqrt{-x^2+1}}{2c}+\frac{\arcsin(x)}{2c}+\frac{b\arcsin(x)}{c^2}-\frac{\left((b+c)\operatorname{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)^6a+(3ab-ac+4b^2+4bc)\operatorname{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)^4+(3ab-ac+4b^2+4bc)\operatorname{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)^2+ab+ac\right)\ln\left(-\operatorname{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)+\frac{\sqrt{-x^2+1}-1}{x}\right)}{4c^2\left(\operatorname{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)^7a+3\operatorname{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)^5a+3\operatorname{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)^5b+3\operatorname{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)^3a+4\operatorname{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)^3b+8\operatorname{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)^3c+\operatorname{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)a+\operatorname{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*x*(-x^2+1)^(1/2)/c+1/2*arcsin(x)/c+1/4/c^2*sum((a*(b+c)*_R^6+(3*a*b-a*c
+4*b^2+4*b*c)*_R^4+(3*a*b-a*c+4*b^2+4*b*c)*_R^2+a*b+a*c)/(_R^7*a+3*_R^5*a+3
*_R^5*b+3*_R^3*a+4*_R^3*b+8*_R^3*c+_R*a+_R*b)*ln(((-x^2+1)^(1/2)-1)/x-_R),_
R=RootOf(a*_Z^8+(4*a+4*b)*_Z^6+(6*a+8*b+16*c)*_Z^4+(4*a+4*b)*_Z^2+a))-2/c^2
*b*arctan(((-x^2+1)^(1/2)-1)/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}\,x^4}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^4/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 1.30, size = 1024, normalized size = 3.15

$$\operatorname{asin}(x)\left(\frac{\frac{b}{c}+1}{c}-\frac{1}{2c}\right)+\frac{x\sqrt{1-x^2}}{2c}-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(b^2\left(-\frac{b-\sqrt{b^2-4ac}}{2c}\right)^{3/2}+ab\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+2ac\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-2ac\left(-\frac{b-\sqrt{b^2-4ac}}{2c}\right)^{3/2}+bc\left(-\frac{b-\sqrt{b^2-4ac}}{2c}\right)^{3/2}\right)}{2c\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,(4ac-b^2)}+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+1\right)1i}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(b^2\left(-\frac{b+\sqrt{b^2-4ac}}{2c}\right)^{3/2}+ab\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+2ac\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-2ac\left(-\frac{b+\sqrt{b^2-4ac}}{2c}\right)^{3/2}+bc\left(-\frac{b+\sqrt{b^2-4ac}}{2c}\right)^{3/2}\right)}{2c\,(4ac-b^2)\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(b^2\left(-\frac{b-\sqrt{b^2-4ac}}{2c}\right)^{3/2}+ab\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+2ac\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-2ac\left(-\frac{b-\sqrt{b^2-4ac}}{2c}\right)^{3/2}+bc\left(-\frac{b-\sqrt{b^2-4ac}}{2c}\right)^{3/2}\right)}{2c\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,(4ac-b^2)}-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(b^2\left(-\frac{b+\sqrt{b^2-4ac}}{2c}\right)^{3/2}+ab\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+2ac\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-2ac\left(-\frac{b+\sqrt{b^2-4ac}}{2c}\right)^{3/2}+bc\left(-\frac{b+\sqrt{b^2-4ac}}{2c}\right)^{3/2}\right)}{2c\,(4ac-b^2)\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] asin(x)*((b/c + 1)/c - 1/(2*c)) + (x*(1 - x^2)^(1/2))/(2*c) - (log((((x*(-(
b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(
2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c)^(1/2))/(2*c
))^(1/2)))*(b^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + a*b*(-(b - (b^2
- 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/
2) - 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-(b - (b^2 - 4*a
*c)^(1/2))/(2*c))^(3/2)))/(2*c*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*
(4*a*c - b^2)) + (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i
)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-
(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(
2*c))^(3/2) + a*b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b + (
b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))
^(3/2) + b*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*c*(4*a*c - b^2)*
((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) + (log((((x*(-(b - (b^2 - 4*a*
c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)
 + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2
*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + a*b*(-(b - (b^2 - 4*a*c)^(1/2))
/(2*c))^(1/2) + 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b
 - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c
))^(3/2)))/(2*c*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2))
- (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 -
4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + a
*b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b + (b^2 - 4*a*c)^(1
/2))/(2*c))^(1/2) - 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-
(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*c*(4*a*c - b^2)*((b + (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**4*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

3.282 $\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=263

$$\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{\left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\sin^{-1}(x)}{c}$$

Rubi [A] time = 2.13, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1293, 216, 1692, 377, 205}

$$\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{\left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\sin^{-1}(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(ArcSin[x]/c) + ((b + c - (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x
^2])])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) +
((b + c + (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*c + Sqr
t[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(c*Sqrt[b
+ Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1293

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a
*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
```

```
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = -\frac{\int \frac{1}{\sqrt{1-x^2}}\,dx}{c} - \frac{\int \frac{-a-(b+c)x^2}{\sqrt{1-x^2}\left(a+bx^2+cx^4\right)}\,dx}{c}$$

$$= -\frac{\sin^{-1}(x)}{c} - \frac{\int \left(\frac{-b-c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)} + \frac{-b-c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\right)dx}{c}$$

$$= -\frac{\sin^{-1}(x)}{c} + \frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)}\,dx}{c} + \frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\int \frac{}{\sqrt{1-x^2}\left(b\right.} \cdots}{c}$$

$$= -\frac{\sin^{-1}(x)}{c} + \frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-b-2c+\sqrt{b^2-4ac}\right)x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)}{c} + \frac{\left(b+\right.\cdots}{}$$

$$= -\frac{\sin^{-1}(x)}{c} + \frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{1-x^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{b+2c-\sqrt{b^2-4ac}}} + \frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b}\cdots}{\sqrt{b+}\cdots}\right)}{c\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{b+2c}\cdots}$$

Mathematica [B]   time = 6.14, size = 7543, normalized size = 28.68

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [C]   time = 0.63, size = 415, normalized size = 1.58

$$-\frac{\text{RootSum}\left[\#1^8a+4\#1^6a+4\#1^6b+6\#1^4a+8\#1^4b+16\#1^4c+4\#1^2a+4\#1^2b+a\&, \frac{\#1^6a\log\left(\#1(-x)+\sqrt{1-x^2}-1\right)+\#1^6(-a)\log(x)+3\#1^4a\log\left(\#1(-x)+\sqrt{1-x^2}-1\right)-3\#1^4a\log(x)+4\#1^4b\log\left(\#1(-x)+\sqrt{1-x^2}-1\right)-4\#1^4b\log(x)+4\#1^4c\log\left(\#1(-x)+\sqrt{1-x^2}-1\right)-4\#1^4c\log(x)+3\#1^2a\log\left(\#1(-x)+\sqrt{1-x^2}-1\right)-3\#1^2a\log(x)+4\#1^2b\log\left(\#1(-x)+\sqrt{1-x^2}-1\right)-4\#1^2b\log(x)+4\#1^2c\log\left(\#1(-x)+\sqrt{1-x^2}-1\right)-4\#1^2c\log(x)+a\log\left(\#1(-x)+\sqrt{1-x^2}-1\right)-a\log(x)}{\#1^7a+3\#1^5a+3\#1^5b+3\#1^3a+4\#1^3b+8\#1^3c+\#1a+\#1b}\&\right]}{4c}-\frac{2\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}-1}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-2*ArcTan[x/(-1 + Sqrt[1 - x^2])])/c - RootSum[a + 4*a*#1^2 + 4*b*#1^2 + 6
*a*#1^4 + 8*b*#1^4 + 16*c*#1^4 + 4*a*#1^6 + 4*b*#1^6 + a*#1^8 & , (-(a*Log[
x]) + a*Log[-1 + Sqrt[1 - x^2] - x*#1] - 3*a*Log[x]*#1^2 - 4*b*Log[x]*#1^2
- 4*c*Log[x]*#1^2 + 3*a*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^2 + 4*b*Log[-1 +
Sqrt[1 - x^2] - x*#1]*#1^2 + 4*c*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^2 - 3*a*
Log[x]*#1^4 - 4*b*Log[x]*#1^4 - 4*c*Log[x]*#1^4 + 3*a*Log[-1 + Sqrt[1 - x^2
] - x*#1]*#1^4 + 4*b*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^4 + 4*c*Log[-1 + Sqr
t[1 - x^2] - x*#1]*#1^4 - a*Log[x]*#1^6 + a*Log[-1 + Sqrt[1 - x^2] - x*#1]*
#1^6)/(a*#1 + b*#1 + 3*a*#1^3 + 4*b*#1^3 + 8*c*#1^3 + 3*a*#1^5 + 3*b*#1^5 +
 a*#1^7) & ]/(4*c)
```

fricas [B]   time = 1.98, size = 1430, normalized size = 5.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c^3)*sqrt((b^2
+ 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(-(2*(a*b + a*
c)*x^2 - 2*a*b - 2*a*c + sqrt(1/2)*((b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*sqrt(
-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*x - ((b^3*c^2 - 4*a*b*c^3)*
sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a*b*c^3)*x)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c
^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c^3)*sqrt((b^2 +
2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)) + 2*(a*b + a*c)*sqr
t(-x^2 + 1))/x^2) - sqrt(1/2)*c*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c
^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log
(-(2*(a*b + a*c)*x^2 - 2*a*b - 2*a*c - sqrt(1/2)*((b^3 - 4*a*c^2 - (4*a*b -
 b^2)*c)*sqrt(-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*x - ((b^3*c^2
 - 4*a*b*c^3)*sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a*b*c^3)*x)*sqrt((b^2 + 2*b*c
 + c^2)/(b^2*c^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c^3
)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)) + 2*(
a*b + a*c)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*c*sqrt(-(b^2 - (2*a - b)*c - (b
^2*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 -
 4*a*c^3))*log(-(2*(a*b + a*c)*x^2 - 2*a*b - 2*a*c + sqrt(1/2)*((b^3 - 4*a*
c^2 - (4*a*b - b^2)*c)*sqrt(-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4*a*b - b^2)*c)
*x + ((b^3*c^2 - 4*a*b*c^3)*sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a*b*c^3)*x)*sqr
t((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a - b)*c - (b^2
*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4
*a*c^3)) + 2*(a*b + a*c)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*c*sqrt(-(b^2 - (2
*a - b)*c - (b^2*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5
)))/(b^2*c^2 - 4*a*c^3))*log(-(2*(a*b + a*c)*x^2 - 2*a*b - 2*a*c - sqrt(1/2
)*((b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*sqrt(-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4
*a*b - b^2)*c)*x + ((b^3*c^2 - 4*a*b*c^3)*sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a
*b*c^3)*x)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a
 - b)*c - (b^2*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5))
)/(b^2*c^2 - 4*a*c^3)) + 2*(a*b + a*c)*sqrt(-x^2 + 1))/x^2) - 4*arctan((sqr
t(-x^2 + 1) - 1)/x))/c
```

giac [B] time = 4.73, size = 3580, normalized size = 13.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(pi*sgn(x) + 2*arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x
^2 + 1) - 1)))/c + 1/8*((2*a^2*b^4 - 16*a^3*b^2*c + 32*a^4*c^2 + 3*sqrt(2)*
sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^2 + 2*sqrt(
2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^3 - sqrt(2
)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^4 - 12*sqrt(2
)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c - 8*sqrt(
2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c + 8*sq
rt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^2*c - 1
6*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*c^2
 - 2*(b^2 - 4*a*c)*a^2*b^2 + 8*(b^2 - 4*a*c)*a^3*c)*c^2*abs(a) - 2*(3*sqrt(
2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^3*b^2*c + 5*sqrt(2)*sqrt(2*a^2
 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*b^3*c + sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b
^2 - 4*a*c)*a)*a*b^4*c + 2*a^2*b^4*c - sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2
- 4*a*c)*a)*b^5*c + 2*a*b^5*c - 12*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*
a*c)*a)*a^4*c^2 - 20*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^3*b*
c^2 + 3*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*b^2*c^2 - 16*a^
3*b^2*c^2 + 10*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a*b^3*c^2 -
16*a^2*b^3*c^2 - sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*b^4*c^2 +
2*a*b^4*c^2 - 28*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^3*c^3 +
32*a^4*c^3 - 24*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*b*c^3 +
 32*a^3*b*c^3 + 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a*b^2*c^3
 - 16*a^2*b^2*c^3 - 16*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*
```

c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a^2*b^2*c - 2*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*abs(a)*abs(c) + (4*a^3*b^3*c^2 + 2*a^2*b^4*c^2 - 16*a^4*b*c^3 + 4*a^2*b^3*c^3 - 32*a^4*c^4 - 16*a^3*b*c^4 + 6*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b*c^2 + 7*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^4*c^2 + 12*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c^3 + 22*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c^3 + 4*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^2*c^3 - 2*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^3*c^3 + 16*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*c^4 + 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b*c^4 - 4*(b^2 - 4*a*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3 - 4*(b^2 - 4*a*c)*a^2*b*c^3)*abs(a))*arctan(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt((2*a*c + b*c + sqrt(-4*(a*c + b*c + c^2)*a*c + (2*a*c + b*c)^2))/(a*c)))/((3*a^5*b^2*c^2 + 5*a^4*b^3*c^2 + a^3*b^4*c^2 - a^2*b^5*c^2 - 12*a^6*c^3 - 20*a^5*b*c^3 + 3*a^4*b^2*c^3 + 10*a^3*b^3*c^3 - a^2*b^4*c^3 - 28*a^5*c^4 - 24*a^4*b*c^4 + 8*a^3*b^2*c^4 - 16*a^4*c^5)*abs(c)) - 1/8*((2*a^2*b^4 - 16*a^3*b^2*c + 32*a^4*c^2 + 3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^2 + 2*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^3 - sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^4 - 12*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c - 8*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c + 8*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^2*c - 16*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2 + 8*(b^2 - 4*a*c)*a^3*c)*c^2*abs(a) + 2*(3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^3*b^2*c + 5*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^2*b^3*c + sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a*b^4*c - 2*a^2*b^4*c - sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*b^5*c - 2*a*b^5*c - 12*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^4*c^2 - 20*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^3*b*c^2 + 3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^2*b^2*c^2 + 16*a^3*b^2*c^2 + 10*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a*b^3*c^2 + 16*a^2*b^3*c^2 - sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*b^4*c^2 - 2*a*b^4*c^2 - 28*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^3*c^3 - 32*a^4*c^3 - 24*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^2*b*c^3 - 32*a^3*b*c^3 + 8*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 16*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a^2*b^2*c + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^3*c^2 - 8*(b^2 - 4*a*c)*a^2*b*c^2 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*abs(a)*abs(c) + (4*a^3*b^3*c^2 + 2*a^2*b^4*c^2 - 16*a^4*b*c^3 + 4*a^2*b^3*c^3 - 32*a^4*c^4 - 16*a^3*b*c^4 + 6*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b*c^2 + 7*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^4*c^2 + 12*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c^3 + 22*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c^3 + 4*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^2*c^3 - 2*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^3*c^3 + 16*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*c^4 + 8*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b*c^4 - 4*(b^2 - 4*a*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3 - 4*(b^2 - 4*a*c)*a^2*b*c^3)*abs(a))*arctan(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt((2*a*c + b*c - sqrt(-4*(a*c + b*c + c^2)*a*c + (2*a*c + b*c)^2))/(a*c)))/((3*a^5*b^2*c^2 + 5*a^4*b^3*c^2 + a^3*b^4*c^2 - a^2*b^5*c^2 - 12*a^6*c^3 - 20*a^5*b*c^3 + 3*a^4*b^2*c^3 + 10*a^3*b^3*c^3 - a^2*b^4

```
*c^3 - 28*a^5*c^4 - 24*a^4*b*c^4 + 8*a^3*b^2*c^4 - 16*a^4*c^5)*abs(c))
```

maple [C] time = 0.02, size = 175, normalized size = 0.67

$$\frac{2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)}{c}-\frac{\left(\mathrm{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)^6a+(3a+4b+4c)\mathrm{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)^4+(3a+4b+4c)\mathrm{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)^2+a\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)+\frac{\sqrt{-x^2+1}-1}{x}\right)}{4c\left(\mathrm{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)^7a+3\mathrm{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)^5a+3\mathrm{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)^5b+3\mathrm{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)^3a+4\mathrm{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)^3b+8\mathrm{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)^3c+\mathrm{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)a+\mathrm{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4/c*sum((_R^6*a+(4*c+3*a+4*b)*_R^4+(4*c+3*a+4*b)*_R^2+a)/(_R^7*a+3*_R^5*
a+3*_R^5*b+3*_R^3*a+4*_R^3*b+8*_R^3*c+_R*a+_R*b)*ln(-_R+((-x^2+1)^(1/2)-1)/
x),_R=RootOf(_Z^8*a+(4*a+4*b)*_Z^6+(6*a+8*b+16*c)*_Z^4+(4*a+4*b)*_Z^2+a))+2
/c*arctan(((-x^2+1)^(1/2)-1)/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}\,x^2}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^2/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 1.27, size = 870, normalized size = 3.31

$$-\frac{\mathrm{asin}(x)}{c}-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(2a\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+b\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+b\left(-\frac{b-\sqrt{b^2-4ac}}{2c}\right)^{3/2}+2c\left(-\frac{b-\sqrt{b^2-4ac}}{2c}\right)^{3/2}\right)}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,\left(8ac-2b^2\right)}-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+1\right)1i}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(2a\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+b\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+b\left(-\frac{b+\sqrt{b^2-4ac}}{2c}\right)^{3/2}+2c\left(-\frac{b+\sqrt{b^2-4ac}}{2c}\right)^{3/2}\right)}{\left(8ac-2b^2\right)\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(2a\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+b\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+b\left(-\frac{b-\sqrt{b^2-4ac}}{2c}\right)^{3/2}+2c\left(-\frac{b-\sqrt{b^2-4ac}}{2c}\right)^{3/2}\right)}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,\left(8ac-2b^2\right)}+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(2a\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+b\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+b\left(-\frac{b+\sqrt{b^2-4ac}}{2c}\right)^{3/2}+2c\left(-\frac{b+\sqrt{b^2-4ac}}{2c}\right)^{3/2}\right)}{\left(8ac-2b^2\right)\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c
)^(1/2))/(2*c))^(1/2)))*(2*a*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(
-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(-(b - (b^2 - 4*a*c)^(1/2))/(2*
c))^(3/2) + 2*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(((b - (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2)*(8*a*c - 2*b^2)) - (log((((x*(-(b - (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2
) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(2*
a*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(-(b - (b^2 - 4*a*c)^(1/2))/
(2*c))^(1/2) + b*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + 2*c*(-(b - (b^2
 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2
)*(8*a*c - 2*b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1
)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x
+ (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(2*a*(-(b + (b^2 - 4*a*c)^(1/2
))/(2*c))^(1/2) + b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(-(b + (b^
2 - 4*a*c)^(1/2))/(2*c))^(3/2) + 2*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/
2)))/((8*a*c - 2*b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - asin(x
)/c + (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^
2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 -
 4*a*c)^(1/2))/(2*c))^(1/2)))*(2*a*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)
 + b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(-(b + (b^2 - 4*a*c)^(1/2
))/(2*c))^(3/2) + 2*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/((8*a*c -
2*b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**2*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

3.283 $\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=220

$$\frac{\sqrt{-\sqrt{b^2-4ac}+b+2c}\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{\sqrt{b^2-4ac}+b+2c}\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.29, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1174, 402, 216, 377, 205}

$$\frac{\sqrt{-\sqrt{b^2-4ac}+b+2c}\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{\sqrt{b^2-4ac}+b+2c}\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]
]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[b
 + 2*c + Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])]
)/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 1174

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && !IntegerQ[q]
```

Rubi steps

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{(2c)\int \frac{\sqrt{1-x^2}}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{\sqrt{1-x^2}}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(b+2c-\sqrt{b^2-4ac}\right)\int \frac{1}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(b+2c+\sqrt{b^2-4ac}\right)\int \frac{1}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4a\cdots}\right.}}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(b+2c-\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-b-2c+\sqrt{b^2-4ac}\right)x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)}{\sqrt{b^2-4ac}} - \frac{\left(b+2c+\sqrt{b^2\cdots}\right.}{}$$

$$= \frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{1-x^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{b+2c+\sqrt{\cdots}}}{\sqrt{b+\sqrt{b^2-4\cdots}}}\right.}{\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [B] time = 5.42, size = 2266, normalized size = 10.30

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] -1/2*(Sqrt[2]*(-b - 2*c + Sqrt[b^2 - 4*a*c])*Sqrt[-((b^2 + c*(-2*a + Sqrt[b
^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*Log[-(Sqrt[(-b + Sqrt[b^2 -
 4*a*c])/c]/Sqrt[2]) + x] + Sqrt[2]*(-((-b - 2*c + Sqrt[b^2 - 4*a*c])*Sqrt[
-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*Lo
g[Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]/Sqrt[2] + x]) + (b + 2*c + Sqrt[b^2 - 4*
a*c])*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c])
)/c^2]*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2]) + x] - b*Sqrt[(-b^
2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*Log[Sqrt
[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2] + x] - 2*c*Sqrt[(-b^2 + c*(2*a + Sqr
t[b^2 - 4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*Log[Sqrt[-((b + Sqrt[b^2
 - 4*a*c])/c)]/Sqrt[2] + x] - Sqrt[b^2 - 4*a*c]*Sqrt[(-b^2 + c*(2*a + Sqrt[
b^2 - 4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*Log[Sqrt[-((b + Sqrt[b^2 -
 4*a*c])/c)]/Sqrt[2] + x] + b*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) +
b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*Log[2 - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*
c])/c]*x + Sqrt[2]*Sqrt[(b + 2*c - Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] + 2
*c*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))/
c^2)]*Log[2 - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*x + Sqrt[2]*Sqrt[(b
+ 2*c - Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] - Sqrt[b^2 - 4*a*c]*Sqrt[-((b^
2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*Log[2 -
 Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*x + Sqrt[2]*Sqrt[(b + 2*c - Sqrt[
b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] - b*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*
c]) + b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*Log[2 + Sqrt[2]*Sqrt[(-b + Sqrt[b^2
- 4*a*c])/c]*x + Sqrt[2]*Sqrt[(b + 2*c - Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2
]] - 2*c*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a
*c]))/c^2)]*Log[2 + Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*x + Sqrt[2]*Sq
rt[(b + 2*c - Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] + Sqrt[b^2 - 4*a*c]*Sqrt
[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*L
og[2 + Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*x + Sqrt[2]*Sqrt[(b + 2*c -
 Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] - b*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 -
4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*Log[2 - Sqrt[2]*Sqrt[-((b + Sqrt
[b^2 - 4*a*c])/c)]*x + Sqrt[2]*Sqrt[(b + 2*c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1
```

```
 - x^2]] - 2*c*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqrt[b^2
- 4*a*c]))/c^2]*Log[2 - Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*x + Sqrt
[2]*Sqrt[(b + 2*c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] - Sqrt[b^2 - 4*a*c
]*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c]))/c^
2]*Log[2 - Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*x + Sqrt[2]*Sqrt[(b +
 2*c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] + b*Sqrt[(-b^2 + c*(2*a + Sqrt[
b^2 - 4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*Log[2 + Sqrt[2]*Sqrt[-((b
+ Sqrt[b^2 - 4*a*c])/c)]*x + Sqrt[2]*Sqrt[(b + 2*c + Sqrt[b^2 - 4*a*c])/c]*
Sqrt[1 - x^2]] + 2*c*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqr
t[b^2 - 4*a*c]))/c^2]*Log[2 + Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*x
+ Sqrt[2]*Sqrt[(b + 2*c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] + Sqrt[b^2 -
 4*a*c]*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c
]))/c^2]*Log[2 + Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*x + Sqrt[2]*Sqr
t[(b + 2*c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]]))/(c*Sqrt[b^2 - 4*a*c]*Sq
rt[((b + 2*c - Sqrt[b^2 - 4*a*c])*(-b + Sqrt[b^2 - 4*a*c]))/c^2]*Sqrt[-(((b
 + Sqrt[b^2 - 4*a*c])*(b + 2*c + Sqrt[b^2 - 4*a*c]))/c^2)])
```

IntegrateAlgebraic [C] time = 0.17, size = 155, normalized size = 0.70

$$\frac{1}{2}i\text{RootSum}\left[\#1^4c-4\#1^3b-8\#1^3c+16\#1^2a+20\#1^2b+24\#1^2c-32\#1a-32\#1b-32\#1c+16a+16b+16c\&,\frac{\#1^2\log\left(-\#1-2x^2-2i\sqrt{1-x^2}\,x+2\right)}{\#1^3c-3\#1^2b-6\#1^2c+8\#1a+10\#1b+12\#1c-8a-8b-8c}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] (I/2)*RootSum[16*a + 16*b + 16*c - 32*a*#1 - 32*b*#1 - 32*c*#1 + 16*a*#1^2
+ 20*b*#1^2 + 24*c*#1^2 - 4*b*#1^3 - 8*c*#1^3 + c*#1^4 & , (Log[2 - 2*x^2 -
 (2*I)*x*Sqrt[1 - x^2] - #1]*#1^2)/(-8*a - 8*b - 8*c + 8*a*#1 + 10*b*#1 + 1
2*c*#1 - 3*b*#1^2 - 6*c*#1^2 + c*#1^3) & ]
```

fricas [B] time = 0.93, size = 759, normalized size = 3.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(
a*b^2 - 4*a^2*c))*log(-(x^2 + sqrt(1/2)*((a*b^2 - 4*a^2*c)*sqrt(-x^2 + 1)*x
 - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4
*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqrt(-x^2 + 1) - 1)/x
^2) - 1/2*sqrt(1/2)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3
*c))/(a*b^2 - 4*a^2*c))*log(-(x^2 - sqrt(1/2)*((a*b^2 - 4*a^2*c)*sqrt(-x^2
+ 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b
^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqrt(-x^2 + 1)
- 1)/x^2) - 1/2*sqrt(1/2)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 -
 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(-(x^2 + sqrt(1/2)*((a*b^2 - 4*a^2*c)*sqrt
(-x^2 + 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c)/sqrt
(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqrt(-x^2
 + 1) - 1)/x^2) + 1/2*sqrt(1/2)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c)/sqrt(a^2
*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(-(x^2 - sqrt(1/2)*((a*b^2 - 4*a^2*c
)*sqrt(-x^2 + 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c
)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqr
t(-x^2 + 1) - 1)/x^2)
```

giac [B] time = 5.12, size = 641, normalized size = 2.91

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*b^2 - 8*a^3*c + 3*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*
a)*sqrt(b^2 - 4*a*c)*a^2 + 2*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a
)*sqrt(b^2 - 4*a*c)*a*b - sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*s
qrt(b^2 - 4*a*c)*b^2 + 4*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sq
rt(b^2 - 4*a*c)*a*c - 2*(b^2 - 4*a*c)*a^2)*abs(a)*arctan(-1/2*sqrt(2)*(x/(s
qrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt((2*a + b + sqrt((2*a + b)
^2 - 4*(a + b + c)*a))/a))/(3*a^4*b^2 + 2*a^3*b^3 - a^2*b^4 - 12*a^5*c - 8*
a^4*b*c + 8*a^3*b^2*c - 16*a^4*c^2) + 1/2*(2*a^2*b^2 - 8*a^3*c + 3*sqrt(2)*
sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2 + 2*sqrt(2)*s
qrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b - sqrt(2)*sqrt
(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^2 + 4*sqrt(2)*sqrt(
2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*c - 2*(b^2 - 4*a*c)*
a^2)*abs(a)*arctan(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) -
 1)/x)/sqrt((2*a + b - sqrt((2*a + b)^2 - 4*(a + b + c)*a))/a))/(3*a^4*b^2
+ 2*a^3*b^3 - a^2*b^4 - 12*a^5*c - 8*a^4*b*c + 8*a^3*b^2*c - 16*a^4*c^2)
```

maple [C] time = 0.01, size = 130, normalized size = 0.59

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4*sum((_R^6-_R^4-_R^2+1)/(_R^7*a+3*_R^5*a+3*_R^5*b+3*_R^3*a+4*_R^3*b+8*_
R^3*c+_R*a+_R*b)*ln(-_R+((-x^2+1)^(1/2)-1)/x),_R=RootOf(_Z^8*a+(4*a+4*b)*_Z
^6+(6*a+8*b+16*c)*_Z^4+(4*a+4*b)*_Z^2+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 1.27, size = 989, normalized size = 4.50

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4*a*c
)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + a*b
*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2
))/(2*c))^(1/2) + 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-(b
 + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*a*(4*a*c - b^2)*((b + (b^2 - 4*a*
c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^
(1/2) - 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2
)*1i)/(x - (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b - (b^2 - 4*
a*c)^(1/2))/(2*c))^(1/2) + a*b*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2
*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b - (b^2 - 4*a*c)^
(1/2))/(2*c))^(3/2) + b*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*a*(
(b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2)) + (log((((x*(-(b
```

```
- (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*
c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))
^(1/2)))*(b^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + a*b*(-(b - (b^2 -
4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)
 + 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-(b - (b^2 - 4*a*c
)^(1/2))/(2*c))^(3/2)))/(2*a*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4
*a*c - b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/
((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b
 + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*
c))^(1/2) + a*b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b + (b^
2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(
3/2) + b*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*a*(4*a*c - b^2)*((
b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

3.284 $\int \frac{\sqrt{1-x^2}}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=265

$$-\frac{c\left(\frac{2a+b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{1-x^2}}{ax}$$

Rubi [A] time = 0.78, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1295, 264, 1692, 377, 205}

$$-\frac{c\left(\frac{2a+b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{1-x^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[1 - x^2]/(a*x)) - (c*(1 + (2*a + b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
b + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])
])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (c*(
1 - (2*a + b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*
x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(a*Sqrt[b + Sqrt[b^2 - 4*a
*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1295

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[((f*x)^(m + 2)*(d + e*x^2)^(q - 1)*Simp[b*d - a*e
+ c*d*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
```

```
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{1-x^2}}{x^2\left(a+bx^2+cx^4\right)}dx = \frac{\int \frac{1}{x^2\sqrt{1-x^2}}dx}{a} - \frac{\int \frac{a+b+cx^2}{\sqrt{1-x^2}\left(a+bx^2+cx^4\right)}dx}{a}$$

$$= -\frac{\sqrt{1-x^2}}{ax} - \frac{\int \left(\frac{c+\frac{(2a+b)c}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)} + \frac{c-\frac{(2a+b)c}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\right)dx}{a}$$

$$= -\frac{\sqrt{1-x^2}}{ax} - \frac{\left(c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}dx}{a} - \frac{\left(c\left(1+\frac{2a+b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{}{\sqrt{1}}}{a}$$

$$= -\frac{\sqrt{1-x^2}}{ax} - \frac{\left(c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-b-2c-\sqrt{b^2-4ac}\right)x^2}dx, x, \frac{x}{\sqrt{1-x^2}}\right)}{a} -$$

$$= -\frac{\sqrt{1-x^2}}{ax} - \frac{c\left(1+\frac{2a+b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{1-x^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{b}}\right)}{a\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{b+2}}$$

Mathematica [B] time = 4.95, size = 2661, normalized size = 10.04

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*(4*Sqrt[b^2 - 4*a*c]*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c
+ Sqrt[b^2 - 4*a*c]))/c^2]*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(
c + Sqrt[b^2 - 4*a*c]))/c^2)]*Sqrt[1 - x^2] + Sqrt[2]*(2*a + b + Sqrt[b^2 -
 4*a*c])*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a
*c]))/c^2)]*x*Log[-(Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]/Sqrt[2]) + x] - Sqrt[2
]*(2*a + b + Sqrt[b^2 - 4*a*c])*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c])
+ b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*x*Log[Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]/S
qrt[2] + x] - 2*Sqrt[2]*a*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c
+ Sqrt[b^2 - 4*a*c]))/c^2]*x*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[
2]) + x] - Sqrt[2]*b*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqr
t[b^2 - 4*a*c]))/c^2]*x*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2]) +
 x] + Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) +
b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*x*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]
/Sqrt[2]) + x] + 2*Sqrt[2]*a*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(
-c + Sqrt[b^2 - 4*a*c]))/c^2]*x*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt
[2] + x] + Sqrt[2]*b*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqr
t[b^2 - 4*a*c]))/c^2]*x*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2] + x]
 - Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(
-c + Sqrt[b^2 - 4*a*c]))/c^2]*x*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt
[2] + x] - 2*Sqrt[2]*a*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c +
Sqrt[b^2 - 4*a*c]))/c^2)]*x*Log[2 - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c
]*x + Sqrt[2]*Sqrt[(b + 2*c - Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] - Sqrt[2
]*b*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))
/c^2)]*x*Log[2 - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*x + Sqrt[2]*Sqrt[
(b + 2*c - Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] - Sqrt[2]*Sqrt[b^2 - 4*a*c]
```

```
*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))/c^
2)]*x*Log[2 - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*x + Sqrt[2]*Sqrt[(b
+ 2*c - Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] + 2*Sqrt[2]*a*Sqrt[-((b^2 + c*
(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*x*Log[2 + Sqr
t[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*x + Sqrt[2]*Sqrt[(b + 2*c - Sqrt[b^2
- 4*a*c])/c]*Sqrt[1 - x^2]] + Sqrt[2]*b*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 -
4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*x*Log[2 + Sqrt[2]*Sqrt[(-b + Sqr
t[b^2 - 4*a*c])/c]*x + Sqrt[2]*Sqrt[(b + 2*c - Sqrt[b^2 - 4*a*c])/c]*Sqrt[1
 - x^2]] + Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a
*c]) + b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*x*Log[2 + Sqrt[2]*Sqrt[(-b + Sqrt[b
^2 - 4*a*c])/c]*x + Sqrt[2]*Sqrt[(b + 2*c - Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 -
x^2]] + 2*Sqrt[2]*a*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqrt
[b^2 - 4*a*c]))/c^2]*x*Log[2 - Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*x
 + Sqrt[2]*Sqrt[(b + 2*c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] + Sqrt[2]*b
*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c]))/c^2
]*x*Log[2 - Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*x + Sqrt[2]*Sqrt[(b
+ 2*c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] - Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sq
rt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*x
*Log[2 - Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*x + Sqrt[2]*Sqrt[(b + 2
*c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] - 2*Sqrt[2]*a*Sqrt[(-b^2 + c*(2*a
 + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*x*Log[2 + Sqrt[2]*
Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*x + Sqrt[2]*Sqrt[(b + 2*c + Sqrt[b^2 - 4
*a*c])/c]*Sqrt[1 - x^2]] - Sqrt[2]*b*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c
]) + b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*x*Log[2 + Sqrt[2]*Sqrt[-((b + Sqrt[b^
2 - 4*a*c])/c)]*x + Sqrt[2]*Sqrt[(b + 2*c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 -
x^2]] + Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c])
+ b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*x*Log[2 + Sqrt[2]*Sqrt[-((b + Sqrt[b^2 -
 4*a*c])/c)]*x + Sqrt[2]*Sqrt[(b + 2*c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2
]])/(a*Sqrt[b^2 - 4*a*c]*Sqrt[((b + 2*c - Sqrt[b^2 - 4*a*c])*(-b + Sqrt[b^2
 - 4*a*c]))/c^2]*Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(b + 2*c + Sqrt[b^2 - 4*a*
c]))/c^2)]*x)
```

IntegrateAlgebraic [C] time = 0.69, size = 471, normalized size = 1.78

$$\frac{\text{RootSum}\left[\#1^8a+4\#1^6a+4\#1^6b+6\#1^4a+8\#1^4b+16\#1^4c+4\#1^2a+4\#1^2b+a\&,\frac{\#1^6a\log\left(\#1(-x)+\sqrt{1-x^2}-1\right)+\#1^6(-a)\log(x)+\#1^6b\log\left(\#1(-x)+\sqrt{1-x^2}-1\right)-\#1^6b\log(x)+3\#1^4a\log\left(\#1(-x)+\sqrt{1-x^2}-1\right)-3\#1^4a\log(x)+3\#1^4b\log\left(\#1(-x)+\sqrt{1-x^2}-1\right)-3\#1^4b\log(x)+4\#1^4c\log\left(\#1(-x)+\sqrt{1-x^2}-1\right)-4\#1^4c\log(x)+3\#1^2a\log\left(\#1(-x)+\sqrt{1-x^2}-1\right)-3\#1^2a\log(x)+3\#1^2b\log\left(\#1(-x)+\sqrt{1-x^2}-1\right)-3\#1^2b\log(x)+4\#1^2c\log\left(\#1(-x)+\sqrt{1-x^2}-1\right)-4\#1^2c\log(x)+a\log\left(\#1(-x)+\sqrt{1-x^2}-1\right)+b\log\left(\#1(-x)+\sqrt{1-x^2}-1\right)-a\log(x)-b\log(x)}{\#1^7a+3\#1^5a+3\#1^5b+3\#1^3a+4\#1^3b+8\#1^3c+\#1a+\#1b}\&\right]}{4a}-\frac{\sqrt{1-x^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[1 - x^2]/(a*x)) + RootSum[a + 4*a*#1^2 + 4*b*#1^2 + 6*a*#1^4 + 8*b*#
1^4 + 16*c*#1^4 + 4*a*#1^6 + 4*b*#1^6 + a*#1^8 & , (-(a*Log[x]) - b*Log[x]
+ a*Log[-1 + Sqrt[1 - x^2] - x*#1] + b*Log[-1 + Sqrt[1 - x^2] - x*#1] - 3*a
*Log[x]*#1^2 - 3*b*Log[x]*#1^2 - 4*c*Log[x]*#1^2 + 3*a*Log[-1 + Sqrt[1 - x^
2] - x*#1]*#1^2 + 3*b*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^2 + 4*c*Log[-1 + Sq
rt[1 - x^2] - x*#1]*#1^2 - 3*a*Log[x]*#1^4 - 3*b*Log[x]*#1^4 - 4*c*Log[x]*#
1^4 + 3*a*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^4 + 3*b*Log[-1 + Sqrt[1 - x^2]
- x*#1]*#1^4 + 4*c*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^4 - a*Log[x]*#1^6 - b*
Log[x]*#1^6 + a*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^6 + b*Log[-1 + Sqrt[1 - x
^2] - x*#1]*#1^6)/(a*#1 + b*#1 + 3*a*#1^3 + 4*b*#1^3 + 8*c*#1^3 + 3*a*#1^5
+ 3*b*#1^5 + a*#1^7) & ]/(4*a)
```

fricas [B] time = 1.42, size = 1998, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*a*x*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^
4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^
```

```
2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c
)*x^2 - 2*(a*b + b^2)*c + sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b +
5*a*b^2)*c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^
2)*c)*x - ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4*a^4*b*c)*x
)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 -
 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^4*c)*sq
rt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a
^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^
2) - sqrt(1/2)*a*x*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^
4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^
2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c
)*x^2 - 2*(a*b + b^2)*c - sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b +
5*a*b^2)*c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^
2)*c)*x - ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4*a^4*b*c)*x
)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 -
 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^4*c)*sq
rt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a
^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^
2) + sqrt(1/2)*a*x*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^
4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^
2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c
)*x^2 - 2*(a*b + b^2)*c + sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b +
5*a*b^2)*c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^
2)*c)*x + ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4*a^4*b*c)*x
)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 -
 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^4*c)*sq
rt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a
^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^
2) - sqrt(1/2)*a*x*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^
4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^
2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c
)*x^2 - 2*(a*b + b^2)*c - sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b +
5*a*b^2)*c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^
2)*c)*x + ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4*a^4*b*c)*x
)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 -
 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^4*c)*sq
rt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a
^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^
2) - 2*sqrt(-x^2 + 1))/(a*x)
```

giac [B] time = 5.04, size = 3965, normalized size = 14.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*(4*a^6*b^3 + 6*a^5*b^4 + 2*a^4*b^5 - 16*a^7*b*c - 32*a^6*b^2*c - 12*a^5
*b^3*c + 32*a^7*c^2 + 16*a^6*b*c^2 + 6*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2
- 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^6*b + 13*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^
2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*b^2 + 7*sqrt(2)*sqrt(2*a^2 + a*b - sqrt
(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b^3 - sqrt(2)*sqrt(2*a^2 + a*b - sqr
t(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^4 - sqrt(2)*sqrt(2*a^2 + a*b - sq
rt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^5 - 12*sqrt(2)*sqrt(2*a^2 + a*b
- sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^6*c - 6*sqrt(2)*sqrt(2*a^2 + a*b
 - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*b*c + 12*sqrt(2)*sqrt(2*a^2 +
 a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b^2*c + 6*sqrt(2)*sqrt(2*
a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^3*c - 16*sqrt(2)*s
qrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*c^2 - 8*sqrt(2
)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b*c^2 - 4*(
```

b^2 - 4*a*c)*a^6*b - 6*(b^2 - 4*a*c)*a^5*b^2 - 2*(b^2 - 4*a*c)*a^4*b^3 + 8*(b^2 - 4*a*c)*a^6*c + 4*(b^2 - 4*a*c)*a^5*b*c - (2*a^3*b^4 + 2*a^2*b^5 - 16*a^4*b^2*c - 16*a^3*b^3*c + 32*a^5*c^2 + 32*a^4*b*c^2 + 3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^2 + 5*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^3 + sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^4 - sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^5 - 12*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*c - 20*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b*c + 8*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^3*c - 16*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c^2 - 16*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^3*b^2 - 2*(b^2 - 4*a*c)*a^2*b^3 + 8*(b^2 - 4*a*c)*a^4*c + 8*(b^2 - 4*a*c)*a^3*b*c)*a^2 + 2*(3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^5*b^2 + 5*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^4*b^3 + sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^3*b^4 - 2*a^4*b^4 - sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^2*b^5 - 2*a^3*b^5 - 12*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^6*c - 20*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^5*b*c + 3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^4*b^2*c + 16*a^5*b^2*c + 10*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^3*b^3*c + 16*a^4*b^3*c - sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^2*b^4*c - 2*a^3*b^4*c - 28*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^5*c^2 - 32*a^6*c^2 - 24*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^4*b*c^2 - 32*a^5*b*c^2 + 8*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^3*b^2*c^2 + 16*a^4*b^2*c^2 - 16*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^4*c^3 - 32*a^5*c^3 + 2*(b^2 - 4*a*c)*a^4*b^2 + 2*(b^2 - 4*a*c)*a^3*b^3 - 8*(b^2 - 4*a*c)*a^5*c - 8*(b^2 - 4*a*c)*a^4*b*c + 2*(b^2 - 4*a*c)*a^3*b^2*c - 8*(b^2 - 4*a*c)*a^4*c^2)*abs(a))*arctan(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt((2*a^2 + a*b + sqrt(-4*(a^2 + a*b + a*c)*a^2 + (2*a^2 + a*b)^2))/a^2))/(3*a^8*b^2 + 5*a^7*b^3 + a^6*b^4 - a^5*b^5 - 12*a^9*c - 20*a^8*b*c + 3*a^7*b^2*c + 10*a^6*b^3*c - a^5*b^4*c - 28*a^8*c^2 - 24*a^7*b*c^2 + 8*a^6*b^2*c^2 - 16*a^7*c^3) - 1/8*(4*a^6*b^3 + 6*a^5*b^4 + 2*a^4*b^5 - 16*a^7*b*c - 32*a^6*b^2*c - 12*a^5*b^3*c + 32*a^7*c^2 + 16*a^6*b*c^2 + 6*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^6*b + 13*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*b^2 + 7*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b^3 - sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^4 - sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^5 - 12*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^6*c - 6*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*b*c + 12*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b^2*c + 6*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^3*c - 16*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*c^2 - 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b*c^2 - 4*(b^2 - 4*a*c)*a^6*b - 6*(b^2 - 4*a*c)*a^5*b^2 - 2*(b^2 - 4*a*c)*a^4*b^3 + 8*(b^2 - 4*a*c)*a^6*c + 4*(b^2 - 4*a*c)*a^5*b*c - (2*a^3*b^4 + 2*a^2*b^5 - 16*a^4*b^2*c - 16*a^3*b^3*c + 32*a^5*c^2 + 32*a^4*b*c^2 + 3*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^2 + 5*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^3 + sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^4 - sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^5 - 12*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*c - 20*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b*c + 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^3*c - 16*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c^2 - 16*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^3*b^2 - 2*(b^2 - 4*a*c)*a^2*b^3 + 8*(b^2 - 4*a*c)*a^4*c + 8*(b^2 - 4*a*c)*a^3*b*c)*a^2 - 2*(3*sqrt(2)*sqrt(2*a^2 + a*b + sqrt

```
(b^2 - 4*a*c)*a)*a^5*b^2 + 5*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a
)*a^4*b^3 + sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^3*b^4 + 2*a^4
*b^4 - sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*b^5 + 2*a^3*b^5
- 12*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^6*c - 20*sqrt(2)*sqr
t(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^5*b*c + 3*sqrt(2)*sqrt(2*a^2 + a*b +
 sqrt(b^2 - 4*a*c)*a)*a^4*b^2*c - 16*a^5*b^2*c + 10*sqrt(2)*sqrt(2*a^2 + a*
b + sqrt(b^2 - 4*a*c)*a)*a^3*b^3*c - 16*a^4*b^3*c - sqrt(2)*sqrt(2*a^2 + a*
b + sqrt(b^2 - 4*a*c)*a)*a^2*b^4*c + 2*a^3*b^4*c - 28*sqrt(2)*sqrt(2*a^2 +
a*b + sqrt(b^2 - 4*a*c)*a)*a^5*c^2 + 32*a^6*c^2 - 24*sqrt(2)*sqrt(2*a^2 + a
*b + sqrt(b^2 - 4*a*c)*a)*a^4*b*c^2 + 32*a^5*b*c^2 + 8*sqrt(2)*sqrt(2*a^2 +
 a*b + sqrt(b^2 - 4*a*c)*a)*a^3*b^2*c^2 - 16*a^4*b^2*c^2 - 16*sqrt(2)*sqrt(
2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^4*c^3 + 32*a^5*c^3 - 2*(b^2 - 4*a*c)*a
^4*b^2 - 2*(b^2 - 4*a*c)*a^3*b^3 + 8*(b^2 - 4*a*c)*a^5*c + 8*(b^2 - 4*a*c)*
a^4*b*c - 2*(b^2 - 4*a*c)*a^3*b^2*c + 8*(b^2 - 4*a*c)*a^4*c^2)*abs(a))*arct
an(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt((2*a
^2 + a*b - sqrt(-4*(a^2 + a*b + a*c)*a^2 + (2*a^2 + a*b)^2))/a^2))/(3*a^8*b
^2 + 5*a^7*b^3 + a^6*b^4 - a^5*b^5 - 12*a^9*c - 20*a^8*b*c + 3*a^7*b^2*c +
10*a^6*b^3*c - a^5*b^4*c - 28*a^8*c^2 - 24*a^7*b*c^2 + 8*a^6*b^2*c^2 - 16*a
^7*c^3) + 1/2*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/a
```

maple [C] time = 0.03, size = 217, normalized size = 0.82

$$-\frac{\sqrt{-x^2+1}\,x}{a}-\frac{\arcsin(x)}{a}-\frac{2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)}{a}+\frac{\left(\left(a+b\right)\mathrm{RootOf}\left(\_Z^8a+\left(4a+4b\right)\_Z^6+\left(6a+8b+16c\right)\_Z^4+\left(4a+4b\right)\_Z^2+a\right)^6+\left(3a+3b+4c\right)\mathrm{RootOf}\left(\_Z^8a+\left(4a+4b\right)\_Z^6+\left(6a+8b+16c\right)\_Z^4+\left(4a+4b\right)\_Z^2+a\right)^4+\left(3a+3b+4c\right)\mathrm{RootOf}\left(\_Z^8a+\left(4a+4b\right)\_Z^6+\left(6a+8b+16c\right)\_Z^4+\left(4a+4b\right)\_Z^2+a\right)^2+a+b\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8a+\left(4a+4b\right)\_Z^6+\left(6a+8b+16c\right)\_Z^4+\left(4a+4b\right)\_Z^2+a\right)+\frac{\sqrt{-x^2+1}-1}{x}\right)}{4a\left(\mathrm{RootOf}\left(\_Z^8a+\left(4a+4b\right)\_Z^6+\left(6a+8b+16c\right)\_Z^4+\left(4a+4b\right)\_Z^2+a\right)^7a+3\,\mathrm{RootOf}\left(\_Z^8a+\left(4a+4b\right)\_Z^6+\left(6a+8b+16c\right)\_Z^4+\left(4a+4b\right)\_Z^2+a\right)^5a+3\,\mathrm{RootOf}\left(\_Z^8a+\left(4a+4b\right)\_Z^6+\left(6a+8b+16c\right)\_Z^4+\left(4a+4b\right)\_Z^2+a\right)^5b+4\,\mathrm{RootOf}\left(\_Z^8a+\left(4a+4b\right)\_Z^6+\left(6a+8b+16c\right)\_Z^4+\left(4a+4b\right)\_Z^2+a\right)^3a+4\,\mathrm{RootOf}\left(\_Z^8a+\left(4a+4b\right)\_Z^6+\left(6a+8b+16c\right)\_Z^4+\left(4a+4b\right)\_Z^2+a\right)^3b+8\,\mathrm{RootOf}\left(\_Z^8a+\left(4a+4b\right)\_Z^6+\left(6a+8b+16c\right)\_Z^4+\left(4a+4b\right)\_Z^2+a\right)^3c+\mathrm{RootOf}\left(\_Z^8a+\left(4a+4b\right)\_Z^6+\left(6a+8b+16c\right)\_Z^4+\left(4a+4b\right)\_Z^2+a\right)a+\mathrm{RootOf}\left(\_Z^8a+\left(4a+4b\right)\_Z^6+\left(6a+8b+16c\right)\_Z^4+\left(4a+4b\right)\_Z^2+a\right)b\right)}-\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/a/x*(-x^2+1)^(3/2)-1/a*x*(-x^2+1)^(1/2)-1/a*arcsin(x)+1/4/a*sum(((a+b)*_
R^6+(3*a+3*b+4*c)*_R^4+(3*a+3*b+4*c)*_R^2+a+b)/(_R^7*a+3*_R^5*a+3*_R^5*b+3*
_R^3*a+4*_R^3*b+8*_R^3*c+_R*a+_R*b)*ln(-_R+((-x^2+1)^(1/2)-1)/x),_R=RootOf(
_Z^8*a+(4*a+4*b)*_Z^6+(6*a+8*b+16*c)*_Z^4+(4*a+4*b)*_Z^2+a))-2/a*arctan(((-
x^2+1)^(1/2)-1)/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}}{\left(cx^4+bx^2+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x^2), x)
```

mupad [B] time = 1.21, size = 1234, normalized size = 4.66

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(x^2*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 - 4*a*c
)^(1/2))/(2*c))^(1/2)))*(b^3*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + a*b
^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a^2*c*(-(b + (b^2 - 4*a*c)^
(1/2))/(2*c))^(1/2) - 2*a*c^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b^
2*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) - 3*a*b*c*(-(b + (b^2 - 4*a*c)
^(1/2))/(2*c))^(1/2) + a*b*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*
a^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (1 - x^2)^
```

```
(1/2)/(a*x) + (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/(
(b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b
- (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^3*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c
))^(1/2) + a*b^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a^2*c*(-(b -
(b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*
c))^(3/2) + b^2*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) - 3*a*b*c*(-(b -
 (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + a*b*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c
))^(3/2)))/(2*a^2*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2)
) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2
- 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4
*a*c)^(1/2))/(2*c))^(1/2)))*(b^3*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) +
 a*b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a^2*c*(-(b + (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2) - 2*a*c^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)
+ b^2*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) - 3*a*b*c*(-(b + (b^2 - 4*
a*c)^(1/2))/(2*c))^(1/2) + a*b*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))
/(2*a^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((
((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(
1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2
))/(2*c))^(1/2)))*(b^3*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + a*b^2*(-(
b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a^2*c*(-(b - (b^2 - 4*a*c)^(1/2))
/(2*c))^(1/2) - 2*a*c^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b^2*c*(-
(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) - 3*a*b*c*(-(b - (b^2 - 4*a*c)^(1/2)
)/(2*c))^(1/2) + a*b*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*a^2*((
b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)}}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(x**2*(a + b*x**2 + c*x**4)), x)
```

3.285 $\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx$

Optimal. Leaf size=96

$$\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,x}{\sqrt{1-x^2}}\right)-\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\,x}{\sqrt{1-x^2}}\right)-\sin^{-1}(x)$$

Rubi [A] time = 0.20, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1293, 216, 1692, 377, 207, 203}

$$\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,x}{\sqrt{1-x^2}}\right)-\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\,x}{\sqrt{1-x^2}}\right)-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 - x^2])/(-1 + x^2 + x^4),x]
```

```
[Out] -ArcSin[x] + Sqrt[(2 + Sqrt[5])/5]*ArcTan[(Sqrt[(1 + Sqrt[5])/2]*x)/Sqrt[1
- x^2]] - Sqrt[(-2 + Sqrt[5])/5]*ArcTanh[(Sqrt[(-1 + Sqrt[5])/2]*x)/Sqrt[1
- x^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1293

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a
*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
```

```
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx &= -\int \frac{1}{\sqrt{1-x^2}}\,dx - \int \frac{1-2x^2}{\sqrt{1-x^2}\left(-1+x^2+x^4\right)}\,dx\\
&= -\sin^{-1}(x) - \int \left(\frac{-2+\frac{4}{\sqrt{5}}}{\sqrt{1-x^2}\left(1-\sqrt{5}+2x^2\right)} + \frac{-2-\frac{4}{\sqrt{5}}}{\sqrt{1-x^2}\left(1+\sqrt{5}+2x^2\right)}\right) dx\\
&= -\sin^{-1}(x) + \frac{1}{5}\left(2\left(5-2\sqrt{5}\right)\right)\int \frac{1}{\sqrt{1-x^2}\left(1-\sqrt{5}+2x^2\right)}\,dx + \frac{1}{5}\left(2\left(5+2\sqrt{5}\right)\right)\int \frac{}{\sqrt{1-x^2}}\\
&= -\sin^{-1}(x) + \frac{1}{5}\left(2\left(5-2\sqrt{5}\right)\right)\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{5}-\left(-3+\sqrt{5}\right)x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right) + \frac{1}{5}\left(2\left(5\right.\right.\\
&= -\sin^{-1}(x) + \sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,x}{\sqrt{1-x^2}}\right) - \sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(-1+\right.}}{\sqrt{1-x}}\right.
\end{aligned}$$

Mathematica [C] time = 0.50, size = 743, normalized size = 7.74

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Sqrt[1 - x^2])/(-1 + x^2 + x^4),x]
```

```
[Out] (-2*Sqrt[5]*ArcSin[x] + (-2 + Sqrt[5])*Sqrt[2 + Sqrt[5]]*Log[-Sqrt[(-1 + Sq
rt[5])/2] + x] + 2*Sqrt[2 + Sqrt[5]]*Log[Sqrt[(-1 + Sqrt[5])/2] + x] - Sqrt
[5*(2 + Sqrt[5])]*Log[Sqrt[(-1 + Sqrt[5])/2] + x] - (2*I)*Sqrt[-2 + Sqrt[5]
]*Log[(-I)*Sqrt[(1 + Sqrt[5])/2] + x] - I*Sqrt[5*(-2 + Sqrt[5])]*Log[(-I)*S
qrt[(1 + Sqrt[5])/2] + x] + (2*I)*Sqrt[-2 + Sqrt[5]]*Log[I*Sqrt[(1 + Sqrt[5
])/2] + x] + I*Sqrt[5*(-2 + Sqrt[5])]*Log[I*Sqrt[(1 + Sqrt[5])/2] + x] + (2
*I)*Sqrt[-2 + Sqrt[5]]*Log[2 - I*Sqrt[2*(1 + Sqrt[5])]*x + Sqrt[2*(3 + Sqrt
[5])]*Sqrt[1 - x^2]] + I*Sqrt[5*(-2 + Sqrt[5])]*Log[2 - I*Sqrt[2*(1 + Sqrt[
5])]*x + Sqrt[2*(3 + Sqrt[5])]*Sqrt[1 - x^2]] - (2*I)*Sqrt[-2 + Sqrt[5]]*Lo
g[2 + I*Sqrt[2*(1 + Sqrt[5])]*x + Sqrt[2*(3 + Sqrt[5])]*Sqrt[1 - x^2]] - I*
Sqrt[5*(-2 + Sqrt[5])]*Log[2 + I*Sqrt[2*(1 + Sqrt[5])]*x + Sqrt[2*(3 + Sqrt
[5])]*Sqrt[1 - x^2]] + 2*Sqrt[2 + Sqrt[5]]*Log[2 - Sqrt[2*(-1 + Sqrt[5])]*x
 + Sqrt[2]*Sqrt[(-3 + Sqrt[5])*(-1 + x^2)]] - Sqrt[5*(2 + Sqrt[5])]*Log[2 -
 Sqrt[2*(-1 + Sqrt[5])]*x + Sqrt[2]*Sqrt[(-3 + Sqrt[5])*(-1 + x^2)]] - 2*Sq
rt[2 + Sqrt[5]]*Log[2 + Sqrt[2*(-1 + Sqrt[5])]*x + Sqrt[2]*Sqrt[(-3 + Sqrt[
5])*(-1 + x^2)]] + Sqrt[5*(2 + Sqrt[5])]*Log[2 + Sqrt[2*(-1 + Sqrt[5])]*x +
 Sqrt[2]*Sqrt[(-3 + Sqrt[5])*(-1 + x^2)]])/(2*Sqrt[5])
```

IntegrateAlgebraic [B] time = 0.59, size = 202, normalized size = 2.10

$$-2\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}-1}\right)+\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\sqrt{5}-2}\,x}{\sqrt{1-x^2}-1}\right)+\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{5}}\,x}{\sqrt{1-x^2}-1}\right)+\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{\sqrt{\sqrt{5}-2}\,x}{\sqrt{1-x^2}-1}\right)-\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{\sqrt{2+\sqrt{5}}\,x}{\sqrt{1-x^2}-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[1 - x^2])/(-1 + x^2 + x^4),x]
```

```
[Out] -2*ArcTan[x/(-1 + Sqrt[1 - x^2])] + Sqrt[(2 + Sqrt[5])/5]*ArcTan[(Sqrt[-2 +
 Sqrt[5]]*x)/(-1 + Sqrt[1 - x^2])] + Sqrt[(2 + Sqrt[5])/5]*ArcTan[(Sqrt[2 +
```

```
Sqrt[5]]*x)/(-1 + Sqrt[1 - x^2])] + Sqrt[(-2 + Sqrt[5])/5]*ArcTanh[(Sqrt[-
2 + Sqrt[5]]*x)/(-1 + Sqrt[1 - x^2])] - Sqrt[(-2 + Sqrt[5])/5]*ArcTanh[(Sqr
t[2 + Sqrt[5]]*x)/(-1 + Sqrt[1 - x^2])]
```

fricas [B] time = 1.07, size = 290, normalized size = 3.02

$$\frac{2}{5}\sqrt{5}\sqrt{\sqrt{5}+2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{-x^2+1}(\sqrt{5}-3)+\sqrt{5}-3\right)\sqrt{\sqrt{5}+2}\sqrt{\frac{x^4-4x^2-\sqrt{5}(x^4-2x^2)-2(\sqrt{5}x^2-x^2+2)\sqrt{-x^2+1}+4}{x^4}}+2\sqrt{-x^2+1}\sqrt{\sqrt{5}+2}(\sqrt{5}-3)}{4x}\right)+\frac{1}{10}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(-\frac{2x^2+\left(\sqrt{-x^2+1}(\sqrt{5}x+x)-\sqrt{5}x-x\right)\sqrt{\sqrt{5}-2}+2\sqrt{-x^2+1}-2}{x^2}\right)-\frac{1}{10}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(-\frac{2x^2-\left(\sqrt{-x^2+1}(\sqrt{5}x+x)-\sqrt{5}x-x\right)\sqrt{\sqrt{5}-2}+2\sqrt{-x^2+1}-2}{x^2}\right)+2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(5)*sqrt(sqrt(5) + 2)*arctan(1/4*(sqrt(2)*(sqrt(-x^2 + 1)*(sqrt(5)
- 3) + sqrt(5) - 3)*sqrt(sqrt(5) + 2)*sqrt((x^4 - 4*x^2 - sqrt(5)*(x^4 - 2*
x^2) - 2*(sqrt(5)*x^2 - x^2 + 2)*sqrt(-x^2 + 1) + 4)/x^4) + 2*sqrt(-x^2 + 1
)*sqrt(sqrt(5) + 2)*(sqrt(5) - 3))/x) + 1/10*sqrt(5)*sqrt(sqrt(5) - 2)*log(
-(2*x^2 + (sqrt(-x^2 + 1)*(sqrt(5)*x + x) - sqrt(5)*x - x)*sqrt(sqrt(5) - 2
) + 2*sqrt(-x^2 + 1) - 2)/x^2) - 1/10*sqrt(5)*sqrt(sqrt(5) - 2)*log(-(2*x^2
 - (sqrt(-x^2 + 1)*(sqrt(5)*x + x) - sqrt(5)*x - x)*sqrt(sqrt(5) - 2) + 2*s
qrt(-x^2 + 1) - 2)/x^2) + 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

giac [B] time = 0.72, size = 209, normalized size = 2.18

$$-\frac{1}{2}\pi\operatorname{sgn}(x)-\frac{1}{5}\sqrt{5\sqrt{5}+10}\arctan\left(-\frac{\frac{x}{\sqrt{-x^2+1}-1}-\frac{\sqrt{-x^2+1}-1}{x}}{\sqrt{2\sqrt{5}+2}}\right)-\frac{1}{10}\sqrt{5\sqrt{5}-10}\log\left(\left|\sqrt{2\sqrt{5}-2}-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}\right|\right)+\frac{1}{10}\sqrt{5\sqrt{5}-10}\log\left(\left|-\sqrt{2\sqrt{5}-2}-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}\right|\right)-\arctan\left(-\frac{x\left(\frac{(\sqrt{-x^2+1}-1)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x, algorithm="giac")
```

```
[Out] -1/2*pi*sgn(x) - 1/5*sqrt(5*sqrt(5) + 10)*arctan(-(x/(sqrt(-x^2 + 1) - 1) -
 (sqrt(-x^2 + 1) - 1)/x)/sqrt(2*sqrt(5) + 2)) - 1/10*sqrt(5*sqrt(5) - 10)*l
og(abs(sqrt(2*sqrt(5) - 2) - x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/
x)) + 1/10*sqrt(5*sqrt(5) - 10)*log(abs(-sqrt(2*sqrt(5) - 2) - x/(sqrt(-x^2
 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x)) - arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)
^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))
```

maple [B] time = 0.09, size = 160, normalized size = 1.67

$$-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{-x^2+1}-1}{\sqrt{2+\sqrt{5}}\,x}\right)}{5\sqrt{2+\sqrt{5}}}+\frac{\sqrt{\sqrt{5}-2}\,\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{-x^2+1}-1}{\sqrt{\sqrt{5}-2}\,x}\right)}{5}+2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\frac{\sqrt{2+\sqrt{5}}\,\sqrt{5}\,\arctan\left(\frac{\sqrt{-x^2+1}-1}{\sqrt{2+\sqrt{5}}\,x}\right)}{5}-\frac{\sqrt{5}\,\arctan\left(\frac{\sqrt{-x^2+1}-1}{\sqrt{\sqrt{5}-2}\,x}\right)}{5\sqrt{\sqrt{5}-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x)
```

```
[Out] -1/5*(2+5^(1/2))^(1/2)*5^(1/2)*arctan(((-x^2+1)^(1/2)-1)/x/(2+5^(1/2))^(1/2
))+1/5*(5^(1/2)-2)^(1/2)*5^(1/2)*arctanh(((-x^2+1)^(1/2)-1)/x/(5^(1/2)-2)^(
1/2))-1/5*5^(1/2)/(2+5^(1/2))^(1/2)*arctanh(((-x^2+1)^(1/2)-1)/x/(2+5^(1/2)
)^(1/2))-1/5*5^(1/2)/(5^(1/2)-2)^(1/2)*arctan(((-x^2+1)^(1/2)-1)/x/(5^(1/2)
-2)^(1/2))+2*arctan(((-x^2+1)^(1/2)-1)/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}\,x^2}{x^4+x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^2/(x^4 + x^2 - 1), x)
```

mupad [B] time = 1.50, size = 383, normalized size = 3.99

$$-\mathrm{asin}(x)-\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}-1\right)1\mathrm{i}}{\sqrt{\frac{3}{2}-\frac{\sqrt{5}}{2}}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\right)\left(\sqrt{5}-2\right)}{\left(2\,\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{3}{2}-\frac{\sqrt{5}}{2}}}+\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}-1\right)1\mathrm{i}}{\sqrt{\frac{\sqrt{5}}{2}+\frac{3}{2}}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}}\right)\left(\sqrt{5}+2\right)}{\left(2\,\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{\sqrt{5}}{2}+\frac{3}{2}}}+\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+1\right)1\mathrm{i}}{\sqrt{\frac{3}{2}-\frac{\sqrt{5}}{2}}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\right)\left(\sqrt{5}-2\right)}{\left(2\,\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{3}{2}-\frac{\sqrt{5}}{2}}}-\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+1\right)1\mathrm{i}}{\sqrt{\frac{\sqrt{5}}{2}+\frac{3}{2}}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}}\right)\left(\sqrt{5}+2\right)}{\left(2\,\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{\sqrt{5}}{2}+\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - x^2)^(1/2))/(x^2 + x^4 - 1),x)
```

```
[Out] (log((((x*(- 5^(1/2)/2 - 1/2)^(1/2) - 1)*1i)/(5^(1/2)/2 + 3/2)^(1/2) - (1 -
 x^2)^(1/2)*1i)/(x - (- 5^(1/2)/2 - 1/2)^(1/2)))*(5^(1/2) + 2))/((2*(- 5^(1
/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2))*(5^(1/2)/2 + 3/2)^(1/2))
- (log((((x*(5^(1/2)/2 - 1/2)^(1/2) - 1)*1i)/(3/2 - 5^(1/2)/2)^(1/2) - (1 -
 x^2)^(1/2)*1i)/(x - (5^(1/2)/2 - 1/2)^(1/2)))*(5^(1/2) - 2))/((2*(5^(1/2)/
2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(3/2 - 5^(1/2)/2)^(1/2)) - asin
(x) + (log((((x*(5^(1/2)/2 - 1/2)^(1/2) + 1)*1i)/(3/2 - 5^(1/2)/2)^(1/2) +
(1 - x^2)^(1/2)*1i)/(x + (5^(1/2)/2 - 1/2)^(1/2)))*(5^(1/2) - 2))/((2*(5^(1
/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(3/2 - 5^(1/2)/2)^(1/2)) -
(log((((x*(- 5^(1/2)/2 - 1/2)^(1/2) + 1)*1i)/(5^(1/2)/2 + 3/2)^(1/2) + (1 -
 x^2)^(1/2)*1i)/(x + (- 5^(1/2)/2 - 1/2)^(1/2)))*(5^(1/2) + 2))/((2*(- 5^(1
/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2))*(5^(1/2)/2 + 3/2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-\left(x-1\right)\left(x+1\right)}}{x^4+x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**2+1)**(1/2)/(x**4+x**2-1),x)
```

```
[Out] Integral(x**2*sqrt(-(x - 1)*(x + 1))/(x**4 + x**2 - 1), x)
```

3.286 $\int \frac{x^8}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=479

$$-\frac{\left(-\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c^3\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{c^3\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

Rubi [A] time = 1.86, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1303, 217, 206, 321, 1692, 377, 205}

$$-\frac{\left(-\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c^3\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{c^3\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\left(b^2-ac\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^3\sqrt{e}}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2e^{3/2}}-\frac{bx\sqrt{d+ex^2}}{2c^2e}+\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8ce^{5/2}}-\frac{3dx\sqrt{d+ex^2}}{8ce^2}+\frac{x^3\sqrt{d+ex^2}}{4ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-3*d*x*Sqrt[d + e*x^2])/(8*c*e^2) - (b*x*Sqrt[d + e*x^2])/(2*c^2*e) + (x^3
*Sqrt[d + e*x^2])/(4*c*e) - ((b^3 - 2*a*b*c - (b^4 - 4*a*b^2*c + 2*a^2*c^2)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqr
t[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^3*Sqrt[b - Sqrt[b^2 - 4*a*c]
]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((b^3 - 2*a*b*c + (b^4 - 4*a*b
^2*c + 2*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^3*Sqrt[b +
Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + (3*d^2*ArcTan
h[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*c*e^(5/2)) + (b*d*ArcTanh[(Sqrt[e]*x)/Sq
rt[d + e*x^2]])/(2*c^2*e^(3/2)) + ((b^2 - a*c)*ArcTanh[(Sqrt[e]*x)/Sqrt[d +
 e*x^2]])/(c^3*Sqrt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^8}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{b^2-ac}{c^3\sqrt{d+ex^2}} - \frac{bx^2}{c^2\sqrt{d+ex^2}} + \frac{x^4}{c\sqrt{d+ex^2}} - \frac{a\left(b^2-ac\right)+b\left(b^2-2ac\right)x^2}{c^3\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right) d$$

$$= -\frac{\int \frac{a\left(b^2-ac\right)+b\left(b^2-2ac\right)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{c^3} - \frac{b\int \frac{x^2}{\sqrt{d+ex^2}}\,dx}{c^2} + \frac{\int \frac{x^4}{\sqrt{d+ex^2}}\,dx}{c} + \frac{\left(b^2-ac\right)\int \frac{1}{\sqrt{d+ex^2}}}{c^3}$$

$$= -\frac{bx\sqrt{d+ex^2}}{2c^2e} + \frac{x^3\sqrt{d+ex^2}}{4ce} - \frac{\int \left(\frac{b\left(b^2-2ac\right)+\frac{-b^4+4ab^2c-2a^2c^2}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{b\left(b^2-2ac\right)-\frac{-b^4+4ab^2c-2}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d}}\right.}{c^3}$$

$$= -\frac{3dx\sqrt{d+ex^2}}{8ce^2} - \frac{bx\sqrt{d+ex^2}}{2c^2e} + \frac{x^3\sqrt{d+ex^2}}{4ce} + \frac{\left(b^2-ac\right)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c^3\sqrt{e}} - $$

$$= -\frac{3dx\sqrt{d+ex^2}}{8ce^2} - \frac{bx\sqrt{d+ex^2}}{2c^2e} + \frac{x^3\sqrt{d+ex^2}}{4ce} + \frac{bd\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2c^2e^{3/2}} + \frac{\left(b^2-a\right.}{}$$

$$= -\frac{3dx\sqrt{d+ex^2}}{8ce^2} - \frac{bx\sqrt{d+ex^2}}{2c^2e} + \frac{x^3\sqrt{d+ex^2}}{4ce} - \frac{\left(b^3-2abc-\frac{b^4-4ab^2c+2a^2c^2}{\sqrt{b^2-4ac}}\right)\tan}{c^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd}}$$

Mathematica [A] time = 1.87, size = 461, normalized size = 0.96

$$\frac{-\frac{8\left(-\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} - \frac{8\left(\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} + \frac{8\left(b^2-ac\right)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}} + \frac{4bcd\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{e^{3/2}} - \frac{4bcx\sqrt{d+ex^2}}{e} + \frac{3c^2d\left(d\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\sqrt{e}\,x\sqrt{d+ex^2}\right)}{e^{5/2}} + \frac{2c^2x^3\sqrt{d+ex^2}}{e}}{8c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-4*b*c*x*Sqrt[d + e*x^2])/e + (2*c^2*x^3*Sqrt[d + e*x^2])/e - (8*(b^3 - 2
*a*b*c - (b^4 - 4*a*b^2*c + 2*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*
```

```
d - b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x
^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e
]) - (8*(b^3 - 2*a*b*c + (b^4 - 4*a*b^2*c + 2*a^2*c^2)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*
c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e]) + (4*b*c*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/e^(3/2)
+ (8*(b^2 - a*c)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[e] + (3*c^2*d*(
-(Sqrt[e]*x*Sqrt[d + e*x^2]) + d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]))/e^(
5/2))/(8*c^3)
```

IntegrateAlgebraic [C]   time = 60.19, size = 538, normalized size = 1.12

$$\frac{\sqrt{e}\,\mathrm{RootSum}\left[\#1^8c+4\#1^6be-4\#1^6cd+16\#1^4ae^2-8\#1^4bde+6\#1^4cd^2+4\#1^2bd^2e-4\#1^2cd^3+cd^4\&,\frac{-2\#1^4abc\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+\#1^4b^3\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-4\#1^2a^2ce\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+4\#1^2ab^2e\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+4\#1^2abcd\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2\#1^2b^3d\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2abcd^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+b^3d^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)}{\#1^6c+3\#1^4be-3\#1^4cd+8\#1^2ae^2-4\#1^2bde+3\#1^2cd^2+bd^2e-cd^3}\&\right]}{2c^3}+\frac{\log\left(\sqrt{d+ex^2}-\sqrt{e}x\right)\left(8ace^2-8b^2e^2-4bcde-3c^2d^2\right)}{8c^3e^{5/2}}+\frac{\sqrt{d+ex^2}\left(-4bex-3cdx+2cex^3\right)}{8c^2e^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[d + e*x^2]*(-3*c*d*x - 4*b*e*x + 2*c*e*x^3))/(8*c^2*e^2) + ((-3*c^2*d
^2 - 4*b*c*d*e - 8*b^2*e^2 + 8*a*c*e^2)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]]
)/(8*c^3*e^(5/2)) + (Sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2
+ 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c
*#1^8 & , (b^3*d^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - 2*a*b*c*d^2*L
og[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - 2*b^3*d*Log[-(Sqrt[e]*x) + Sqrt[d
 + e*x^2] - #1]*#1^2 + 4*a*b*c*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#
1^2 + 4*a*b^2*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 - 4*a^2*c*e*L
og[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + b^3*Log[-(Sqrt[e]*x) + Sqrt[
d + e*x^2] - #1]*#1^4 - 2*a*b*c*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1
^4)/(-(c*d^3) + b*d^2*e + 3*c*d^2*#1^2 - 4*b*d*e*#1^2 + 8*a*e^2*#1^2 - 3*c*
d*#1^4 + 3*b*e*#1^4 + c*#1^6) & ])/(2*c^3)
```

fricas [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A]   time = 2.03, size = 105, normalized size = 0.22

$$\frac{1}{8}\sqrt{x^2e+d}\left(\frac{2x^2e^{(-1)}}{c}-\frac{\left(3c^5de+4bc^4e^2\right)e^{(-3)}}{c^6}\right)x-\frac{\left(3c^2d^2+4bcde+8b^2e^2-8ace^2\right)e^{\left(-\frac{5}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{16c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(x^2*e + d)*(2*x^2*e^(-1)/c - (3*c^5*d*e + 4*b*c^4*e^2)*e^(-3)/c^6)
*x - 1/16*(3*c^2*d^2 + 4*b*c*d*e + 8*b^2*e^2 - 8*a*c*e^2)*e^(-5/2)*log((x*e
^(1/2) - sqrt(x^2*e + d))^2)/c^3
```

maple [C]   time = 0.03, size = 377, normalized size = 0.79

$$\frac{\sqrt{ex^2+d}\,x^3}{4ce}-\frac{a\ln\left(\sqrt{e}x+\sqrt{ex^2+d}\right)}{c^2\sqrt{e}}+\frac{b^2\ln\left(\sqrt{e}x+\sqrt{ex^2+d}\right)}{c^3\sqrt{e}}+\frac{bd\ln\left(\sqrt{e}x+\sqrt{ex^2+d}\right)}{2c^2e^{\frac{3}{2}}}+\frac{3d^2\ln\left(\sqrt{e}x+\sqrt{ex^2+d}\right)}{8ce^{\frac{5}{2}}}-\frac{\sqrt{ex^2+d}\,bx}{2c^2e}-\frac{3\sqrt{ex^2+d}\,dx}{8ce^2}-\frac{\sqrt{e}\left(2abcd^2-b^3d^2+\left(2ac-b^2\right)\mathrm{RootOf}\left(\_Z^4c+\left(4be-4cd\right)\_Z^3+\left(16ae^2-8bde+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z+cd^4\right)^2b+2\left(2a^2ce-2ab^2e-2abcd+b^3d\right)\mathrm{RootOf}\left(\_Z^4c+\left(4be-4cd\right)\_Z^3+\left(16ae^2-8bde+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z+cd^4\right)\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4c+\left(4be-4cd\right)\_Z^3+\left(16ae^2-8bde+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z+cd^4\right)+\left(-\sqrt{e}x+\sqrt{ex^2+d}\right)^2\right)}{2c^3\left(\mathrm{RootOf}\left(\dots\right)^3c+3\mathrm{RootOf}\left(\dots\right)^2be-3\mathrm{RootOf}\left(\dots\right)^2cd+8\mathrm{RootOf}\left(\dots\right)ae^2-4\mathrm{RootOf}\left(\dots\right)bde+3\mathrm{RootOf}\left(\dots\right)cd^2+bd^2e-cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] 1/4*x^3*(e*x^2+d)^(1/2)/c/e-3/8*d*x*(e*x^2+d)^(1/2)/c/e^2+3/8/c*d^2/e^(5/2)
*ln(e^(1/2)*x+(e*x^2+d)^(1/2))-1/2*b*x*(e*x^2+d)^(1/2)/c^2/e+1/2/c^2*b*d/e^
(3/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))-1/c^2*a*ln(e^(1/2)*x+(e*x^2+d)^(1/2))/e
^(1/2)+1/c^3*b^2*ln(e^(1/2)*x+(e*x^2+d)^(1/2))/e^(1/2)-1/2/c^3*e^(1/2)*sum(
(b*(2*a*c-b^2)*_R^2+2*(2*a^2*c*e-2*a*b^2*e-2*a*b*c*d+b^3*d)*_R+2*a*b*c*d^2-
b^3*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d
^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(
4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(cx^4 + bx^2 + a\right)\sqrt{ex^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(x^8/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^8}{\sqrt{e\,x^2 + d}\,\left(c\,x^4 + b\,x^2 + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(x^8/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{d + ex^2}\,\left(a + bx^2 + cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
[Out] Integral(x**8/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.287 $\int \frac{x^6}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=366

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{b\tanh^{-1}}{c^2\ldots}$$

Rubi [A] time = 1.17, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1303, 217, 206, 321, 1692, 377, 205}

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2ce^{3/2}}+\frac{x\sqrt{d+ex^2}}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(2*c*e) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2
 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e]) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^
2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e]) - (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2
*c*e^(3/2)) - (b*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(c^2*Sqrt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^6}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx = \int \left(-\frac{b}{c^2\sqrt{d+ex^2}}+\frac{x^2}{c\sqrt{d+ex^2}}+\frac{ab+\left(b^2-ac\right)x^2}{c^2\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right)dx$$

$$= \frac{\int \frac{ab+\left(b^2-ac\right)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{c^2} - \frac{b\int \frac{1}{\sqrt{d+ex^2}}\,dx}{c^2} + \frac{\int \frac{x^2}{\sqrt{d+ex^2}}\,dx}{c}$$

$$= \frac{x\sqrt{d+ex^2}}{2ce} + \frac{\int \left(\frac{b^2-ac+\frac{b\left(-b^2+3ac\right)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}+\frac{b^2-ac-\frac{b\left(-b^2+3ac\right)}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{c^2} - \frac{b\,\mathrm{Subs}\cdots}{\cdots}$$

$$= \frac{x\sqrt{d+ex^2}}{2ce} - \frac{b\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}} + \frac{\left(b^2-ac-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+e\cdots}}\cdots}{c^2}$$

$$= \frac{x\sqrt{d+ex^2}}{2ce} - \frac{d\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2ce^{3/2}} - \frac{b\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}} + \frac{\left(b^2-ac-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}\right)\mathrm{S}\cdots}{\cdots}$$

$$= \frac{x\sqrt{d+ex^2}}{2ce} + \frac{\left(b^2-ac-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\left(b^2-ac+\frac{b(b\cdots}{\sqrt{b\cdots}}\cdots\right.}{c^2\sqrt{b+\sqrt{b^2\cdots}}}$$

Mathematica [A] time = 1.03, size = 355, normalized size = 0.97

$$\frac{\frac{2\left(-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{2\left(\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}-\frac{cd\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}+\frac{cx\sqrt{d+ex^2}}{e}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((c*x*Sqrt[d + e*x^2])/e + (2*(b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2
- 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b
```

```
+ Sqrt[b^2 - 4*a*c])*e]) + (2*(b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2
- 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e]) - (c*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/e^(3/2
) - (2*b*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[e])/(2*c^2)
```

IntegrateAlgebraic [C] time = 45.10, size = 458, normalized size = 1.25

$$-\frac{\sqrt{e}\,\text{RootSum}\left[\#1^8c+4\#1^6be-4\#1^6cd+16\#1^4ae^2-8\#1^4bde+6\#1^4cd^2+4\#1^2bd^2e-4\#1^2cd^3+cd^4\&,\frac{-\#1^4ac\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+\#1^4b^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+4\#1^2abe\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+2\#1^2acd\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2\#1^2b^2d\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-acd^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+b^2d^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)}{\#1^6c+3\#1^4be-3\#1^4cd+8\#1^2ae^2-4\#1^2bde+3\#1^2cd^2+bd^2e-cd^3}\&\right]}{2c^2}+\frac{(2be+cd)\log\left(\sqrt{d+ex^2}-\sqrt{e}x\right)}{2c^2e^{3/2}}+\frac{x\sqrt{d+ex^2}}{2ce}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(2*c*e) + ((c*d + 2*b*e)*Log[-(Sqrt[e]*x) + Sqrt[d + e*
x^2]])/(2*c^2*e^(3/2)) - (Sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*
#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1^
6 + c*#1^8 & , (b^2*d^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - a*c*d^2*
Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - 2*b^2*d*Log[-(Sqrt[e]*x) + Sqrt[
d + e*x^2] - #1]*#1^2 + 2*a*c*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1
^2 + 4*a*b*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + b^2*Log[-(Sqrt
[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4 - a*c*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2
] - #1]*#1^4)/(-(c*d^3) + b*d^2*e + 3*c*d^2*#1^2 - 4*b*d*e*#1^2 + 8*a*e^2*#
1^2 - 3*c*d*#1^4 + 3*b*e*#1^4 + c*#1^6) & ])/(2*c^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 2.14, size = 55, normalized size = 0.15

$$\frac{\sqrt{x^2e+d}\,xe^{(-1)}}{2\,c}+\frac{(cd+2\,be)e^{\left(-\frac{3}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2*e + d)*x*e^(-1)/c + 1/4*(c*d + 2*b*e)*e^(-3/2)*log((x*e^(1/2)
- sqrt(x^2*e + d))^2)/c^2
```

maple [C] time = 0.03, size = 269, normalized size = 0.73

$$-\frac{b\ln\left(\sqrt{e}\,x+\sqrt{e\,x^2+d}\right)}{c^2\sqrt{e}}-\frac{d\ln\left(\sqrt{e}\,x+\sqrt{e\,x^2+d}\right)}{2c\,e^{\frac{3}{2}}}+\frac{\sqrt{e}\,\left(a c\,d^2-b^2d^2+\left(ac-b^2\right)\text{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\,\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)^2+2\left(-2abe-acd+b^2d\right)\text{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\,\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)\right)\ln\left(-\text{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\,\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)+\left(-\sqrt{e}\,x+\sqrt{e\,x^2+d}\right)^2\right)}{2c^2\left(\text{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\,\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)^3c+3\,\text{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\,\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)^2be-3\,\text{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\,\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)^2cd+8\,\text{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\,\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)a\,e^2-4\,\text{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\,\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)bde+3\,\text{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\,\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)c\,d^2+b\,d^2e-c\,d^3\right)}+\frac{\sqrt{e\,x^2+d}\;x}{2ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] 1/2*x*(e*x^2+d)^(1/2)/c/e-1/2/c*d/e^(3/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))-1/c
^2*b*ln(e^(1/2)*x+(e*x^2+d)^(1/2))/e^(1/2)+1/2/c^2*e^(1/2)*sum(((a*c-b^2)*_
R^2+2*(-2*a*b*e-a*c*d+b^2*d)*_R+a*c*d^2-b^2*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*
c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x
^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e
+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(cx^4 + bx^2 + a)\sqrt{ex^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(x^6/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\sqrt{e\,x^2 + d}\,\left(c\,x^4 + b\,x^2 + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(x^6/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{d + ex^2}\,\left(a + bx^2 + cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
[Out] Integral(x**6/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.288 $\int \frac{x^4}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=298

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

Rubi [A] time = 0.72, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1303, 217, 206, 1692, 377, 205}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2
 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b
- Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])) - ((b + (b^2
 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + ArcTanh[(Sqrt[e]*x)/Sqr
t[d + e*x^2]]/(c*Sqrt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
```

```
*c, 0] && !IntegerQ[q] && IntegerQ[m]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{1}{c\sqrt{d+ex^2}} - \frac{a+bx^2}{c\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right)dx$$

$$= \frac{\int \frac{1}{\sqrt{d+ex^2}}\,dx}{c} - \frac{\int \frac{a+bx^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{c}$$

$$= -\frac{\int \left(\frac{b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{b-\frac{-b^2+2ac}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{c} + \frac{\mathrm{Subst}\left(\int \frac{1}{1-ex^2}\,dx,\right.}{c}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\mathrm{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx,\right.}{c}$$

$$= -\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-}}{\sqrt{b+}}\right)}{c\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(\right.}}$$

Mathematica [A] time = 0.59, size = 292, normalized size = 0.98

$$\frac{-\frac{\left(\frac{2ac-b^2}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} - \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-(((b + (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - b*e + Sqrt[
b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b
- Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e])) - ((b + (b^
2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 -
4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + ArcTanh[(Sqrt[e]*x)/Sqrt
[d + e*x^2]]/Sqrt[e])/c
```

IntegrateAlgebraic [C] time = 30.97, size = 320, normalized size = 1.07

$$\frac{\sqrt{e}\,\text{RootSum}\left[\#1^8c+4\#1^6be-4\#1^6cd+16\#1^4ae^2-8\#1^4bde+6\#1^4cd^2+4\#1^2bd^2e-4\#1^2cd^3+cd^4\&,\frac{\#1^4b\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}\,x\right)+4\#1^2ae\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}\,x\right)-2\#1^2bd\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}\,x\right)+bd^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}\,x\right)}{\#1^6c+3\#1^4be-3\#1^4cd+8\#1^2ae^2-4\#1^2bde+3\#1^2cd^2+bd^2e-cd^3}\&\right]}{2c}-\frac{\log\left(\sqrt{d+ex^2}-\sqrt{e}\,x\right)}{c\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]]/(c*Sqrt[e])) + (Sqrt[e]*RootSum[c*d^4
 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#
1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (b*d^2*Log[-(Sqrt[e]*x) + Sqrt[d
 + e*x^2] - #1] - 2*b*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 4*a
*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + b*Log[-(Sqrt[e]*x) + Sqr
t[d + e*x^2] - #1]*#1^4)/(-(c*d^3) + b*d^2*e + 3*c*d^2*#1^2 - 4*b*d*e*#1^2
+ 8*a*e^2*#1^2 - 3*c*d*#1^4 + 3*b*e*#1^4 + c*#1^6) & ])/(2*c)
```

fricas [B] time = 51.77, size = 11094, normalized size = 37.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(1/2)*c*e*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b^2*
c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^
2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c
)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4
- 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a
^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*
c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*log((2*a^3*b*d*e + ((a*b^2*c^3 - 4
*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)
*d*e^2)*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3
- a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e +
(b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d
*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b
*e^2 + (a*b^3 - a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 + 2*sqrt(1/2)
*sqrt(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c^2 - 5*a
*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d
*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^
2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*
(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 -
 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) + ((b^
5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*e
+ (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c
)*e - ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4
*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b
^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e
 + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5
)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*
c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2) - sqrt(1/2)*c*e*
sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b^2*c^3 - 4*a*c^4)*d^2 -
 (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2
 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4
*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^
2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c
^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^
2 - 4*a^2*c^3)*e^2))*log((2*a^3*b*d*e + ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b
^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((a^
2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^
2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c
```

```
^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4
- 4*a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a*b^3 - a^2*
b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((
b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c
^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2
- 4*a^3*b*c^3)*e^3)*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 -
 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6
)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^
2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) + ((b^5 - 5*a*b^3*c + 4*a^
2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*e + (a^2*b^3 - 4*a^3*b
*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b^2*c^3 - 4*
a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt(
(a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(
(b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^
2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c
^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e
 + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2) - sqrt(1/2)*c*e*sqrt(-((b^3 - 3*a*b*
c)*d - (a*b^2 - 2*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^
3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c
+ a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3
*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(
a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3
 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))
*log((2*a^3*b*d*e - ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3
)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2
*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4
 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*
e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) -
 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a*b^3 - a^2*b*c)*d^2 - (5*a^2*b^
2 - 4*a^3*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^
4 + 8*a^2*c^5)*d^3 - (b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4
*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*
x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)
*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 -
 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^
2*b^2*c^4 - 4*a^3*c^5)*e^4)) - ((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^2 - (2*a*
b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*e + (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((
b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^
2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4
- 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*
d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d
^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)
))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^
2*c^3)*e^2)))/x^2) + sqrt(1/2)*c*e*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^
2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2
- 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(
a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^
3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*
c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b
^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*log((2*a^3*b*d*e -
((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c
^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*
d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*
b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 -
 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)
*d^2 + (4*a^3*b*e^2 + (a*b^3 - a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^
2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 -
 (b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3
 + 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*sqrt((a^2*b^2*e^2
```

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+ (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*
a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2
*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^
5)*e^4)) - ((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c +
4*a^3*c^2)*d*e + (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (
a*b^2 - 2*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e +
 (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^
2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4
*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^
4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c
^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2) +
 2*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d))/(c*e), 1/4*(sqr
t(1/2)*c*e*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b^2*c^3 - 4*a
*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((
a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((
b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2
*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^
4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e
+ (a*b^2*c^2 - 4*a^2*c^3)*e^2))*log((2*a^3*b*d*e + ((a*b^2*c^3 - 4*a^2*c^4)
*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x
^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c
)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4
- 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a
^2*b^2*c^4 - 4*a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a
*b^3 - a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x
^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c^2 - 5*a*b^3*c^3
+ 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d*e^2 - (a
^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2
*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5
- 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3
*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) + ((b^5 - 5*a*b
^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*e + (a^2*b^
3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b
^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)
*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*
b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c
^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 +
 (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a
*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2) - sqrt(1/2)*c*e*sqrt(-((b
^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2
 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 -
 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d
^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^
2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4))
)/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2
*c^3)*e^2))*log((2*a^3*b*d*e + ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 -
4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((a^2*b^2*e^2
 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4
*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^
2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c
^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a*b^3 - a^2*b*c)*d^2
- (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^4*c^3 -
 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e
 + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b
*c^3)*e^3)*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3
 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e +
 (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*
d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) + ((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*
d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*e + (a^2*b^3 - 4*a^3*b*c)*e^2)*
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x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b^2*c^3 - 4*a*c^4)*d^
2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*
e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6
 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8
*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^
3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2
*c^2 - 4*a^2*c^3)*e^2)))/x^2) - sqrt(1/2)*c*e*sqrt(-((b^3 - 3*a*b*c)*d - (a
*b^2 - 2*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e +
(a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2
)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*
a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4
 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^
4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*log((2*a
^3*b*d*e - ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e +
 (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c
+ a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3
*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(
a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) - 2*(a^2*b
^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a*b^3 - a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3
*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2
*c^5)*d^3 - (b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*
a^2*b^2*c^3 + 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*sqrt((a
^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b
^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*
c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4
 - 4*a^3*c^5)*e^4)) - ((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a
^2*b^2*c + 4*a^3*c^2)*d*e + (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a
*b*c)*d - (a*b^2 - 2*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b
*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2
*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(
b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 -
2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*
c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^
2)))/x^2) + sqrt(1/2)*c*e*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e +
((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c
^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a
^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^
4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^
3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 -
4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*log((2*a^3*b*d*e - ((a*b^2*c
^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^
3*c^3)*d*e^2)*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(
a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^
3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*
c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4
*a^3*b*e^2 + (a*b^3 - a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 - 2*sqr
t(1/2)*sqrt(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c^2
 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*
c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*sqrt((a^2*b^2*e^2 + (b^4 -
2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^
4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2
*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4))
- ((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2
)*d*e + (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2
*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c
^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 -
2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)
*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2
*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 -
```

```
(b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2) - 4*sqrt(-
e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/(c*e)]
```

giac [A] time = 2.05, size = 27, normalized size = 0.09

$$-\frac{e^{\left(-\frac{1}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*e^(-1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c
```

maple [C] time = 0.02, size = 200, normalized size = 0.67

$$\frac{\sqrt{e}\left(\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^2b+bd^2+2(2ae-bd)\,\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)+\left(-\sqrt{e}\,x+\sqrt{ex^2+d}\right)^2\right)}{2c\left(\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^3c+3\,\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^2be-3\,\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^2cd+8\,\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)ae^2-4\,\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)bde+3\,\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)cd^2+bd^2e-cd^3\right)}+\frac{\ln\left(\sqrt{e}\,x+\sqrt{ex^2+d}\right)}{c\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] 1/c*ln(e^(1/2)*x+(e*x^2+d)^(1/2))/e^(1/2)+1/2/c*e^(1/2)*sum((_R^2*b+2*(2*a*
e-b*d)*_R+b*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c
*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c
+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^
3)*_Z))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\left(cx^4+bx^2+a\right)\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^4}{\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int(x^4/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\sqrt{d+ex^2}\,\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.289 $\int \frac{x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=240

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Rubi [A] time = 0.30, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1303, 377, 205}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(Sqrt[b^2 - 4*a*c]*S
qrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*Arc
Tan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]
]*Sqrt[d + e*x^2])]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}dx = \int \left(\frac{1-\frac{b}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{1+\frac{b}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right.$$

$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}dx + \left(1+\frac{b}{\sqrt{b^2-4a}}\right.$$

$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\right.$$

$$= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\right.}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b\right.}}$$

Mathematica [A] time = 0.48, size = 227, normalized size = 0.95

$$\frac{\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c
]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/Sqrt[2*c*d + (-b +
Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])
])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])/Sqrt[b^2 - 4*a*c]
```

IntegrateAlgebraic [C] time = 9.82, size = 250, normalized size = 1.04

$$\frac{1}{2}\sqrt{e}\,\text{RootSum}\left[\#1^8c+4\#1^6be-4\#1^6cd+16\#1^4ae^2-8\#1^4bde+6\#1^4cd^2+4\#1^2bd^2e-4\#1^2cd^3+cd^4\&,\frac{\#1^4\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2\#1^2d\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+d^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)}{\#1^6(-c)-3\#1^4be+3\#1^4cd-8\#1^2ae^2+4\#1^2bde-3\#1^2cd^2-bd^2e+cd^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b
*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (d^2*Log[-
(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - 2*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2
] - #1]*#1^2 + Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4)/(c*d^3 - b*d^
2*e - 3*c*d^2*#1^2 + 4*b*d*e*#1^2 - 8*a*e^2*#1^2 + 3*c*d*#1^4 - 3*b*e*#1^4
- c*#1^6) & ])/2
```

fricas [B] time = 12.71, size = 3395, normalized size = 14.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)
*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c
- 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a
^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 -
4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log((((b^2*c - 4*a*c^2)*d^3 - (b^3 -
 4*a*b*c)*d^2*e + (a*b^2 - 4*a^2*c)*d*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^
4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2
*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x^2 + 2*a*d^2 - (b*d
^2 - 4*a*d*e)*x^2 + 2*sqrt(1/2)*((b^2 - 4*a*c)*d^2*x - ((b^3*c - 4*a*b*c^2)
*d^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e + 3*(a*b^3 - 4*a^2*b*c)*d*e^2 -
2*(a^2*b^2 - 4*a^3*c)*e^3)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4
*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*
b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x)*sqrt(e*x^2 + d)*sqrt(-(b*d - 2*a*
e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*s
qrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a
*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^
3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c
)*e^2)))/x^2) - 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 -
 (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)
*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2
- 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^
2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log((((b^2*c - 4*a*c
^2)*d^3 - (b^3 - 4*a*b*c)*d^2*e + (a*b^2 - 4*a^2*c)*d*e^2)*sqrt(d^2/((b^2*c
^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*
c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x^2
+ 2*a*d^2 - (b*d^2 - 4*a*d*e)*x^2 - 2*sqrt(1/2)*((b^2 - 4*a*c)*d^2*x - ((b^
3*c - 4*a*b*c^2)*d^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e + 3*(a*b^3 - 4*a
^2*b*c)*d*e^2 - 2*(a^2*b^2 - 4*a^3*c)*e^3)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^
4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2
*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x)*sqrt(e*x^2 + d)*s
qrt(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 -
 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^
3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 +
 (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e +
(a*b^2 - 4*a^2*c)*e^2)))/x^2) - 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e - ((b^2*c
- 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^
2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a
^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/
((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log(
-(((b^2*c - 4*a*c^2)*d^3 - (b^3 - 4*a*b*c)*d^2*e + (a*b^2 - 4*a^2*c)*d*e^2)
*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2
*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*
a^3*c)*e^4))*x^2 - 2*a*d^2 + (b*d^2 - 4*a*d*e)*x^2 + 2*sqrt(1/2)*((b^2 - 4*
a*c)*d^2*x + ((b^3*c - 4*a*b*c^2)*d^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e
 + 3*(a*b^3 - 4*a^2*b*c)*d*e^2 - 2*(a^2*b^2 - 4*a^3*c)*e^3)*sqrt(d^2/((b^2*
c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2
*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x)*
sqrt(e*x^2 + d)*sqrt(-(b*d - 2*a*e - ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*
c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*
c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4
*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3
- 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)))/x^2) + 1/4*sqrt(1/2)*sqrt(-(b*d -
 2*a*e - ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e
^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4
- 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 -
 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*
a^2*c)*e^2))*log(-(((b^2*c - 4*a*c^2)*d^3 - (b^3 - 4*a*b*c)*d^2*e + (a*b^2
- 4*a^2*c)*d*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)
*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^
```

```
3 + (a^2*b^2 - 4*a^3*c)*e^4))*x^2 - 2*a*d^2 + (b*d^2 - 4*a*d*e)*x^2 - 2*sqr
t(1/2)*((b^2 - 4*a*c)*d^2*x + ((b^3*c - 4*a*b*c^2)*d^3 - (b^4 - 2*a*b^2*c -
 8*a^2*c^2)*d^2*e + 3*(a*b^3 - 4*a^2*b*c)*d*e^2 - 2*(a^2*b^2 - 4*a^3*c)*e^3
)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 -
2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4
*a^3*c)*e^4))*x)*sqrt(e*x^2 + d)*sqrt(-(b*d - 2*a*e - ((b^2*c - 4*a*c^2)*d^
2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c
^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e
^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a
*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)))/x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b,c]=[44,93,-37]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a,b,c]=[-72,-7,6]Evaluation time: 0.44Unable to divide, perhaps due
to rounding error%%%{18446744069414584320,[4,7,8,2,3,14,2]%%%}+%%%{-2147483
648,[3,8,10,8,3,10,1]%%%}+%%%{-12884901888,[3,8,10,7,2,12,2]%%%}+%%%{463856
467968,[3,8,10,6,1,14,3]%%%}+%%%{1924145348608,[3,8,10,5,0,16,4]%%%}+%%%{53
6870912,[3,8,9,8,5,10,0]%%%}+%%%{20401094656,[3,8,9,7,4,12,1]%%%}+%%%{-1503
23855360,[3,8,9,6,3,14,2]%%%}+%%%{-3135326126080,[3,8,9,5,2,16,3]%%%}+%%%{-
4672924418048,[3,8,9,4,1,18,4]%%%}+%%%{6047313952768,[3,8,9,3,0,20,5]%%%}+%
%%{-4294967296,[3,8,8,7,6,12,0]%%%}+%%%{-42412802048,[3,8,8,6,5,14,1]%%%}+%
%%{1046898278400,[3,8,8,5,4,16,2]%%%}+%%%{6210522710016,[3,8,8,4,3,18,3]%%%
}+%%%{-1786706395136,[3,8,8,3,2,20,4]%%%}+%%%{-11544872091648,[3,8,8,2,1,22
,5]%%%}+%%%{4398046511104,[3,8,8,1,0,24,6]%%%}+%%%{12750684160,[3,8,7,6,7,1
4,0]%%%}+%%%{-23890755584,[3,8,7,5,6,16,1]%%%}+%%%{-2103460233216,[3,8,7,4,
5,18,2]%%%}+%%%{-3324304687104,[3,8,7,3,4,20,3]%%%}+%%%{9758165696512,[3,8,
7,2,3,22,4]%%%}+%%%{1649267441664,[3,8,7,1,2,24,5]%%%}+%%%{-4398046511104,[
3,8,7,0,1,26,6]%%%}+%%%{-17985175552,[3,8,6,5,8,16,0]%%%}+%%%{161866579968,
[3,8,6,4,7,18,1]%%%}+%%%{1586990415872,[3,8,6,3,6,20,2]%%%}+%%%{-1795296329
728,[3,8,6,2,5,22,3]%%%}+%%%{-4123168604160,[3,8,6,1,4,24,4]%%%}+%%%{384829
0697216,[3,8,6,0,3,26,5]%%%}+%%%{12213813248,[3,8,5,4,9,18,0]%%%}+%%%{-1717
98691840,[3,8,5,3,8,20,1]%%%}+%%%{-212600881152,[3,8,5,2,7,22,2]%%%}+%%%{14
77468749824,[3,8,5,1,6,24,3]%%%}+%%%{-1099511627776,[3,8,5,0,5,26,4]%%%}+%%
%{-3221225472,[3,8,4,3,10,20,0]%%%}+%%%{57982058496,[3,8,4,2,9,22,1]%%%}+%%
%{-154618822656,[3,8,4,1,8,24,2]%%%}+%%%{103079215104,[3,8,4,0,7,26,3]%%%}+
%%%{1048576,[3,6,10,4,2,4,0]%%%}+%%%{8388608,[3,6,10,3,1,6,1]%%%}+%%%{16777
216,[3,6,10,2,0,8,2]%%%}+%%%{-5242880,[3,6,9,3,3,6,0]%%%}+%%%{-29360128,[3,
6,9,2,2,8,1]%%%}+%%%{-33554432,[3,6,9,1,1,10,2]%%%}+%%%{9699328,[3,6,8,2,4,
8,0]%%%}+%%%{33554432,[3,6,8,1,3,10,1]%%%}+%%%{16777216,[3,6,8,0,2,12,2]%%%
}+%%%{-7864320,[3,6,7,1,5,10,0]%%%}+%%%{-12582912,[3,6,7,0,4,12,1]%%%}+%%%{
2359296,[3,6,6,0,6,12,0]%%%}+%%%{536870912,[2,7,10,6,2,8,1]%%%}+%%%{6710886
400,[2,7,10,5,1,10,2]%%%}+%%%{18253611008,[2,7,10,4,0,12,3]%%%}+%%%{-134217
728,[2,7,9,6,4,8,0]%%%}+%%%{-5502926848,[2,7,9,5,3,10,1]%%%}+%%%{-369098752
00,[2,7,9,4,2,12,2]%%%}+%%%{-42949672960,[2,7,9,3,1,14,3]%%%}+%%%{429496729
60,[2,7,9,2,0,16,4]%%%}+%%%{956301312,[2,7,8,5,5,10,0]%%%}+%%%{18656264192,
[2,7,8,4,4,12,1]%%%}+%%%{64961380352,[2,7,8,3,3,14,2]%%%}+%%%{-8589934592,[
2,7,8,2,2,16,3]%%%}+%%%{-85899345920,[2,7,8,1,1,18,4]%%%}+%%%{-2642411520,[
2,7,7,4,6,12,0]%%%}+%%%{-27783069696,[2,7,7,3,5,14,1]%%%}+%%%{-33957085184,
[2,7,7,2,4,16,2]%%%}+%%%{73014444032,[2,7,7,1,3,18,3]%%%}+%%%{42949672960,[
2,7,7,0,2,20,4]%%%}+%%%{3556769792,[2,7,6,3,7,14,0]%%%}+%%%{17716740096,[2,
```

7,6,2,6,16,1]%%%}+%%%{-12884901888,[2,7,6,1,5,18,2]%%%}+%%%{-39728447488,[2
,7,6,0,4,20,3]%%%}+%%%{-2340421632,[2,7,5,2,8,16,0]%%%}+%%%{-2415919104,[2,
7,5,1,7,18,1]%%%}+%%%{12079595520,[2,7,5,0,6,20,2]%%%}+%%%{603979776,[2,7,4
,1,9,18,0]%%%}+%%%{-1207959552,[2,7,4,0,8,20,1]%%%}+%%%{2147483648,[1,8,10,
9,3,10,1]%%%}+%%%{38654705664,[1,8,10,8,2,12,2]%%%}+%%%{51539607552,[1,8,10
,7,1,14,3]%%%}+%%%{-274877906944,[1,8,10,6,0,16,4]%%%}+%%%{-536870912,[1,8,
9,9,5,10,0]%%%}+%%%{-26843545600,[1,8,9,8,4,12,1]%%%}+%%%{-188978561024,[1,
8,9,7,3,14,2]%%%}+%%%{146028888064,[1,8,9,6,2,16,3]%%%}+%%%{962072674304,[1
,8,9,5,1,18,4]%%%}+%%%{-549755813888,[1,8,9,4,0,20,5]%%%}+%%%{4294967296,[1
,8,8,8,6,12,0]%%%}+%%%{95026151424,[1,8,8,7,5,14,1]%%%}+%%%{239444426752,[1
,8,8,6,4,16,2]%%%}+%%%{-858993459200,[1,8,8,5,3,18,3]%%%}+%%%{-618475290624
,[1,8,8,4,2,20,4]%%%}+%%%{1099511627776,[1,8,8,3,1,22,5]%%%}+%%%{-127506841
60,[1,8,7,7,7,14,0]%%%}+%%%{-136633647104,[1,8,7,6,6,16,1]%%%}+%%%{62277025
792,[1,8,7,5,5,18,2]%%%}+%%%{936302870528,[1,8,7,4,4,20,3]%%%}+%%%{-5497558
13888,[1,8,7,3,3,22,4]%%%}+%%%{-549755813888,[1,8,7,2,2,24,5]%%%}+%%%{17985
175552,[1,8,6,6,8,16,0]%%%}+%%%{71940702208,[1,8,6,5,7,18,1]%%%}+%%%{-26736
1714176,[1,8,6,4,6,20,2]%%%}+%%%{-137438953472,[1,8,6,3,5,22,3]%%%}+%%%{481
036337152,[1,8,6,2,4,24,4]%%%}+%%%{-12213813248,[1,8,5,5,9,18,0]%%%}+%%%{72
47757312,[1,8,5,4,8,20,1]%%%}+%%%{103079215104,[1,8,5,3,7,22,2]%%%}+%%%{-13
7438953472,[1,8,5,2,6,24,3]%%%}+%%%{3221225472,[1,8,4,4,10,20,0]%%%}+%%%{-1
2884901888,[1,8,4,3,9,22,1]%%%}+%%%{12884901888,[1,8,4,2,8,24,2]%%%}+%%%{-1
048576,[1,6,10,5,2,4,0]%%%}+%%%{-8388608,[1,6,10,4,1,6,1]%%%}+%%%{-16777216
,[1,6,10,3,0,8,2]%%%}+%%%{8388608,[1,6,9,4,3,6,0]%%%}+%%%{62914560,[1,6,9,3
,2,8,1]%%%}+%%%{150994944,[1,6,9,2,1,10,2]%%%}+%%%{134217728,[1,6,9,1,0,12,
3]%%%}+%%%{-26476544,[1,6,8,3,4,8,0]%%%}+%%%{-163577856,[1,6,8,2,3,10,1]%%%
}+%%%{-301989888,[1,6,8,1,2,12,2]%%%}+%%%{-134217728,[1,6,8,0,1,14,3]%%%}+%
%%{41156608,[1,6,7,2,5,10,0]%%%}+%%%{178257920,[1,6,7,1,4,12,1]%%%}+%%%{167
772160,[1,6,7,0,3,14,2]%%%}+%%%{-31457280,[1,6,6,1,6,12,0]%%%}+%%%{-6920601
6,[1,6,6,0,5,14,1]%%%}+%%%{9437184,[1,6,5,0,7,14,0]%%%}+%%%{-402653184,[0,7
,10,7,2,8,1]%%%}+%%%{-5637144576,[0,7,10,6,1,10,2]%%%}+%%%{-16106127360,[0,
7,10,5,0,12,3]%%%}+%%%{100663296,[0,7,9,7,4,8,0]%%%}+%%%{4160749568,[0,7,9,
6,3,10,1]%%%}+%%%{30198988800,[0,7,9,5,2,12,2]%%%}+%%%{28991029248,[0,7,9,4
,1,14,3]%%%}+%%%{-68719476736,[0,7,9,3,0,16,4]%%%}+%%%{-687865856,[0,7,8,6,
5,10,0]%%%}+%%%{-13925089280,[0,7,8,5,4,12,1]%%%}+%%%{-48184164352,[0,7,8,4
,3,14,2]%%%}+%%%{49392123904,[0,7,8,3,2,16,3]%%%}+%%%{120259084288,[0,7,8,2
,1,18,4]%%%}+%%%{-68719476736,[0,7,8,1,0,20,5]%%%}+%%%{1845493760,[0,7,7,5,
6,12,0]%%%}+%%%{19964887040,[0,7,7,4,5,14,1]%%%}+%%%{11542724608,[0,7,7,3,4
,16,2]%%%}+%%%{-113816633344,[0,7,7,2,3,18,3]%%%}+%%%{8589934592,[0,7,7,1,2
,20,4]%%%}+%%%{68719476736,[0,7,7,0,1,22,5]%%%}+%%%{-2432696320,[0,7,6,4,7,
14,0]%%%}+%%%{-11207180288,[0,7,6,3,6,16,1]%%%}+%%%{28185722880,[0,7,6,2,5,
18,2]%%%}+%%%{34359738368,[0,7,6,1,4,20,3]%%%}+%%%{-60129542144,[0,7,6,0,3,
22,4]%%%}+%%%{1577058304,[0,7,5,3,8,16,0]%%%}+%%%{-201326592,[0,7,5,2,7,18,
1]%%%}+%%%{-14495514624,[0,7,5,1,6,20,2]%%%}+%%%{17179869184,[0,7,5,0,5,22,
3]%%%}+%%%{-402653184,[0,7,4,2,9,18,0]%%%}+%%%{1610612736,[0,7,4,1,8,20,1]%
%%}+%%%{-1610612736,[0,7,4,0,7,22,2]%%%} / %%%{-1024,[0,3,4,2,1,2,0]%%%}+%%
%{-4096,[0,3,4,1,0,4,1]%%%}+%%%{2560,[0,3,3,1,2,4,0]%%%}+%%%{4096,[0,3,3,0,
1,6,1]%%%}+%%%{-1536,[0,3,2,0,3,6,0]%%%} Error: Bad Argument Value

maple [C] time = 0.02, size = 161, normalized size = 0.67

$$-\frac{\sqrt{e}\,\left(\mathrm{RootOf}\left(\_Z^{4} c+c\,d^{4}+\left(4 b e-4 c d\right)\_Z^{3}+\left(16 a\,e^{2}-8 d e b+6 c\,d^{2}\right)\_Z^{2}+\left(4 b\,d^{2} e-4 c\,d^{3}\right)\_Z\right)^{2}-2\,\mathrm{RootOf}\left(\_Z^{4} c+c\,d^{4}+\left(4 b e-4 c d\right)\_Z^{3}+\left(16 a\,e^{2}-8 d e b+6 c\,d^{2}\right)\_Z^{2}+\left(4 b\,d^{2} e-4 c\,d^{3}\right)\_Z\right) d+d^{2}\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{4} c+c\,d^{4}+\left(4 b e-4 c d\right)\_Z^{3}+\left(16 a\,e^{2}-8 d e b+6 c\,d^{2}\right)\_Z^{2}+\left(4 b\,d^{2} e-4 c\,d^{3}\right)\_Z\right)+\left(-\sqrt{e}\,x+\sqrt{e\,x^{2}+d}\right)^{2}\right)}{2\left(\mathrm{RootOf}\left(\_Z^{4} c+c\,d^{4}+\left(4 b e-4 c d\right)\_Z^{3}+\left(16 a\,e^{2}-8 d e b+6 c\,d^{2}\right)\_Z^{2}+\left(4 b\,d^{2} e-4 c\,d^{3}\right)\_Z\right)^{3} c+3\,\mathrm{RootOf}\left(\_Z^{4} c+c\,d^{4}+\left(4 b e-4 c d\right)\_Z^{3}+\left(16 a\,e^{2}-8 d e b+6 c\,d^{2}\right)\_Z^{2}+\left(4 b\,d^{2} e-4 c\,d^{3}\right)\_Z\right)^{2} b e-3\,\mathrm{RootOf}\left(\_Z^{4} c+c\,d^{4}+\left(4 b e-4 c d\right)\_Z^{3}+\left(16 a\,e^{2}-8 d e b+6 c\,d^{2}\right)\_Z^{2}+\left(4 b\,d^{2} e-4 c\,d^{3}\right)\_Z\right)^{2} c d+8\,\mathrm{RootOf}\left(\_Z^{4} c+c\,d^{4}+\left(4 b e-4 c d\right)\_Z^{3}+\left(16 a\,e^{2}-8 d e b+6 c\,d^{2}\right)\_Z^{2}+\left(4 b\,d^{2} e-4 c\,d^{3}\right)\_Z\right) a\,e^{2}-4\,\mathrm{RootOf}\left(\_Z^{4} c+c\,d^{4}+\left(4 b e-4 c d\right)\_Z^{3}+\left(16 a\,e^{2}-8 d e b+6 c\,d^{2}\right)\_Z^{2}+\left(4 b\,d^{2} e-4 c\,d^{3}\right)\_Z\right) b d e+3\,\mathrm{RootOf}\left(\_Z^{4} c+c\,d^{4}+\left(4 b e-4 c d\right)\_Z^{3}+\left(16 a\,e^{2}-8 d e b+6 c\,d^{2}\right)\_Z^{2}+\left(4 b\,d^{2} e-4 c\,d^{3}\right)\_Z\right) c\,d^{2}+b\,d^{2} e-c\,d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)

[Out] -1/2*e^(1/2)*sum((_R^2-2*_R*d+d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2
-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2
),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2
+(4*b*d^2*e-4*c*d^3)*_Z))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(cx^4+bx^2+a\right)\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(x^2/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{d+ex^2}\,\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
[Out] Integral(x**2/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.290 $\int \frac{1}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=243

$$\frac{2c\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{2c\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

Rubi [A] time = 0.18, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1174, 377, 205}

$$\frac{2c\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{2c\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*c*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2
- 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]
*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (2*c*ArcTan[(Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1174

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx = \frac{(2c)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{(2c)\,\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}} - \frac{(2c)\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{2c\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{2c\,\text{ta}\cdots}{\sqrt{b^2-4ac}\,\sqrt{b+\cdots}}$$

<u>**Mathematica**</u> **[A]** time = 0.41, size = 229, normalized size = 0.94

$$\frac{2c\left(\frac{\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} - \frac{\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*c*(ArcTan[(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2
 - 4*a*c]]*Sqrt[d + e*x^2])]/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b
+ Sqrt[b^2 - 4*a*c])*e]) - ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*
x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]/(Sqrt[b + Sqrt[b^2 - 4*a*
c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])))/Sqrt[b^2 - 4*a*c]
```

<u>**IntegrateAlgebraic**</u> **[C]** time = 0.20, size = 179, normalized size = 0.74

$$-2e^{3/2}\text{RootSum}\left[\#1^4c+4\#1^3be-4\#1^3cd+16\#1^2ae^2-8\#1^2bde+6\#1^2cd^2+4\#1bd^2e-4\#1cd^3+cd^4\&, \frac{\#1\log\left(-\#1-2\sqrt{e}\,x\sqrt{d+ex^2}+d+2ex^2\right)}{\#1^3c+3\#1^2be-3\#1^2cd+8\#1ae^2-4\#1bde+3\#1cd^2+bd^2e-cd^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -2*e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 4*b*d^2*e*#1 + 6*c*d^2*#1^2 - 8*b*d
*e*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + 4*b*e*#1^3 + c*#1^4 & , (Log[d + 2*e
*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1)/(-(c*d^3) + b*d^2*e + 3*c*d^2*
#1 - 4*b*d*e*#1 + 8*a*e^2*#1 - 3*c*d*#1^2 + 3*b*e*#1^2 + c*#1^3) & ]
```

<u>**fricas**</u> **[B]** time = 30.48, size = 4557, normalized size = 18.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)*d^2 -
 (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d
*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*
```

d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*log(-(2*a*c^2*d^2 - 2*a*b*c*d*e + ((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2*b*c^2)*d^2*e + (a^2*b^2*c - 4*a^3*c^2)*d*e^2)*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) - (b*c^2*d^2 + 4*a*b*c*e^2 - (b^2*c + 4*a*c^2)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^2 - 4*a^3*c^3)*d^3 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d*e^2 - (a^3*b^3 - 4*a^4*b*c)*e^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) - ((a*b^2*c - 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)))/x^2) - 1/4*sqrt(1/2)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*log(-(2*a*c^2*d^2 - 2*a*b*c*d*e + ((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2*b*c^2)*d^2*e + (a^2*b^2*c - 4*a^3*c^2)*d*e^2)*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) - (b*c^2*d^2 + 4*a*b*c*e^2 - (b^2*c + 4*a*c^2)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^2 - 4*a^3*c^3)*d^3 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d*e^2 - (a^3*b^3 - 4*a^4*b*c)*e^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) - ((a*b^2*c - 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)))/x^2) - 1/4*sqrt(1/2)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*log(-(2*a*c^2*d^2 - 2*a*b*c*d*e - ((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2*b*c^2)*d^2*e + (a^2*b^2*c - 4*a^3*c^2)*d*e^2)*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) - (b*c^2*d^2 + 4*a*b*c*e^2 - (b^2*c + 4*a*c^2)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^2 - 4*a^3*c^3)*d^3 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d*e^2 - (a^3*b^3 - 4*a^4*b*c)*e^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) + ((a*b^2*c - 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + ((a*b^2*

```
c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqr
t((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b
^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2
*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2
*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)))/x^2) + 1/4
*sqrt(1/2)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + ((a*b^2*c - 4*a^2*c^2)*d^2 - (a
*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e
+ b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3
*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*
d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*
a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*log(-(2*a*c^2*d^2 - 2*a*b*c*d*e -
((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2*b*c^2)*d^2*e + (a^2*b^2*c -
 4*a^3*c^2)*d*e^2)*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 -
 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*
c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c
)*e^4)) - (b*c^2*d^2 + 4*a*b*c*e^2 - (b^2*c + 4*a*c^2)*d*e)*x^2 - 2*sqrt(1/
2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^2 - 4*a^3*c^3)*d^3 - 3*(a^2*b^3*c - 4*a^3
*b*c^2)*d^2*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d*e^2 - (a^3*b^3 - 4*a^
4*b*c)*e^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^
3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4
*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) +
 ((a*b^2*c - 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*x)*sqrt(-(b*c*d - (b
^2 - 2*a*c)*e + ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2
*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4
*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c
- 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*
e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*
a^3*c)*e^2)))/x^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.02, size = 151, normalized size = 0.62

$$-\frac{2 e^{\frac{3}{2}} \mathit{RootOf}\left(\_Z^{4} c+c d^{4}+\left(4 b e-4 c d\right) \_Z^{3}+\left(16 a e^{2}-8 d e b+6 c d^{2}\right) \_Z^{2}+\left(4 b d^{2} e-4 c d^{3}\right) \_Z\right) \ln\left(-\mathit{RootOf}\left(\_Z^{4} c+c d^{4}+\left(4 b e-4 c d\right) \_Z^{3}+\left(16 a e^{2}-8 d e b+6 c d^{2}\right) \_Z^{2}+\left(4 b d^{2} e-4 c d^{3}\right) \_Z\right)+\left(-\sqrt{e}\, x+\sqrt{e x^{2}+d}\right)^{2}\right)}{\mathit{RootOf}\left(\_Z^{4} c+c d^{4}+\left(4 b e-4 c d\right) \_Z^{3}+\left(16 a e^{2}-8 d e b+6 c d^{2}\right) \_Z^{2}+\left(4 b d^{2} e-4 c d^{3}\right) \_Z\right)^{3} c+3 \mathit{RootOf}\left(\_Z^{4} c+c d^{4}+\left(4 b e-4 c d\right) \_Z^{3}+\left(16 a e^{2}-8 d e b+6 c d^{2}\right) \_Z^{2}+\left(4 b d^{2} e-4 c d^{3}\right) \_Z\right)^{2} b e-3 \mathit{RootOf}\left(\_Z^{4} c+c d^{4}+\left(4 b e-4 c d\right) \_Z^{3}+\left(16 a e^{2}-8 d e b+6 c d^{2}\right) \_Z^{2}+\left(4 b d^{2} e-4 c d^{3}\right) \_Z\right)^{2} c d+8 \mathit{RootOf}\left(\_Z^{4} c+c d^{4}+\left(4 b e-4 c d\right) \_Z^{3}+\left(16 a e^{2}-8 d e b+6 c d^{2}\right) \_Z^{2}+\left(4 b d^{2} e-4 c d^{3}\right) \_Z\right) a e^{2}-4 \mathit{RootOf}\left(\_Z^{4} c+c d^{4}+\left(4 b e-4 c d\right) \_Z^{3}+\left(16 a e^{2}-8 d e b+6 c d^{2}\right) \_Z^{2}+\left(4 b d^{2} e-4 c d^{3}\right) \_Z\right) b d e+3 \mathit{RootOf}\left(\_Z^{4} c+c d^{4}+\left(4 b e-4 c d\right) \_Z^{3}+\left(16 a e^{2}-8 d e b+6 c d^{2}\right) \_Z^{2}+\left(4 b d^{2} e-4 c d^{3}\right) \_Z\right) c d^{2}+b d^{2} e-c d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] -2*e^(3/2)*sum(_R/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*
c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*
c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d
^3)*_Z))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c x^{4}+b x^{2}+a\right) \sqrt{e x^{2}+d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(1/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
[Out] Integral(1/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.291 $\int \frac{1}{x^2\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=280

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d+ex^2}}{adx}$$

Rubi [A] time = 0.60, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1303, 264, 1692, 377, 205}

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d+ex^2}}{adx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[d + e*x^2]/(a*d*x)) - (c*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x
^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e]) - (c*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt[b + Sqrt
[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
```

```
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{1}{ax^2\sqrt{d+ex^2}}+\frac{-b-cx^2}{a\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right)dx$$

$$= \frac{\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{a}+\frac{\int \frac{-b-cx^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{adx}+\frac{\int \left(\frac{-c-\frac{bc}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}+\frac{-c+\frac{bc}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{adx}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4a}}\right.\right.}{}$$

$$= -\frac{\sqrt{d+ex^2}}{adx}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx,\right.}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{adx}-\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}-\frac{c\left(1-\frac{b}{\sqrt{b^2-4}}\right.}{a\sqrt{b+\sqrt{b^2-}}}$$

Mathematica [A] time = 1.01, size = 271, normalized size = 0.97

$$-\frac{\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{d+ex^2}}{dx}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[d + e*x^2]/(d*x) + (c*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d
- b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2
])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e])
 + (c*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2
- 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/a)
```

IntegrateAlgebraic [C] time = 36.31, size = 312, normalized size = 1.11

$$-\frac{\sqrt{e}\,\text{RootSum}\left[\#1^8c+4\#1^6be-4\#1^6cd+16\#1^4ae^2-8\#1^4bde+6\#1^4cd^2+4\#1^2bd^2e-4\#1^2cd^3+cd^4\&,\frac{\#1^4c\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+4\#1^2be\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2\#1^2cd\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+cd^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)}{\#1^6(-c)-3\#1^4be+3\#1^4cd-8\#1^2ae^2+4\#1^2bde-3\#1^2cd^2-bd^2e+cd^3}\&\right]}{2a}-\frac{\sqrt{d+ex^2}}{adx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[d + e*x^2]/(a*d*x)) - (Sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^
2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e
*#1^6 + c*#1^8 & , (c*d^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - 2*c*d*
Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 4*b*e*Log[-(Sqrt[e]*x) + Sq
rt[d + e*x^2] - #1]*#1^2 + c*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4)
/(c*d^3 - b*d^2*e - 3*c*d^2*#1^2 + 4*b*d*e*#1^2 - 8*a*e^2*#1^2 + 3*c*d*#1^4
 - 3*b*e*#1^4 - c*#1^6) & ])/(2*a)
```

fricas [B] time = 59.58, size = 6431, normalized size = 22.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*a*d*x*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a
^2*c^2)*e - ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4
*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c
 - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/
((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b
^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^
8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c
)*d*e + (a^4*b^2 - 4*a^5*c)*e^2))*log((((a^3*b^2*c^3 - 4*a^4*c^4)*d^3 - (a^
3*b^3*c^2 - 4*a^4*b*c^3)*d^2*e + (a^4*b^2*c^2 - 4*a^5*c^3)*d*e^2)*x^2*sqrt(
((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c
^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)
*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c
^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)) + 2
*(a*b^2*c^3 - a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 2*a^2*b*c^3)*d*e - ((b^3*c^3 -
a*b*c^4)*d^2 - (b^4*c^2 + 2*a*b^2*c^3 - 4*a^2*c^4)*d*e + 4*(a*b^3*c^2 - 2*a
^2*b*c^3)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3*c^2 - 4*a^5*b*c
^3)*d^3 - 2*(a^4*b^4*c - 5*a^5*b^2*c^2 + 4*a^6*c^3)*d^2*e + (a^4*b^5 - 5*a^
5*b^3*c + 4*a^6*b*c^2)*d*e^2 - (a^5*b^4 - 6*a^6*b^2*c + 8*a^7*c^2)*e^3)*x*s
qrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2
*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*
c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a
^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4))
 + ((a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d^2 - (2*a*b^5*c - 11*a^2*b^3*c
^2 + 12*a^3*b*c^3)*d*e + (a*b^6 - 6*a^2*b^4*c + 8*a^3*b^2*c^2)*e^2)*x)*sqrt
(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - ((a^3*b^2*c -
4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(
((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c
^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)
*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c
^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)))/((
a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c
)*e^2)))/x^2) - sqrt(1/2)*a*d*x*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b
^2*c + 2*a^2*c^2)*e - ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*
d*e + (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2
- 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*
c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*
e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d
*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 -
 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2))*log((((a^3*b^2*c^3 - 4*a^4*c^4)
*d^3 - (a^3*b^3*c^2 - 4*a^4*b*c^3)*d^2*e + (a^4*b^2*c^2 - 4*a^5*c^3)*d*e^2)
*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 +
 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 -
4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c
 - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)
*e^4)) + 2*(a*b^2*c^3 - a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 2*a^2*b*c^3)*d*e - ((
```

```
b^3*c^3 - a*b*c^4)*d^2 - (b^4*c^2 + 2*a*b^2*c^3 - 4*a^2*c^4)*d*e + 4*(a*b^3
*c^2 - 2*a^2*b*c^3)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3*c^2 -
 4*a^5*b*c^3)*d^3 - 2*(a^4*b^4*c - 5*a^5*b^2*c^2 + 4*a^6*c^3)*d^2*e + (a^4*
b^5 - 5*a^5*b^3*c + 4*a^6*b*c^2)*d*e^2 - (a^5*b^4 - 6*a^6*b^2*c + 8*a^7*c^2
)*e^3)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c
^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^
2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b
^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^
9*c)*e^4)) + ((a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d^2 - (2*a*b^5*c - 11
*a^2*b^3*c^2 + 12*a^3*b*c^3)*d*e + (a*b^6 - 6*a^2*b^4*c + 8*a^3*b^2*c^2)*e^
2)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - ((a^
3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*
e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 +
 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 -
4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c
 - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)
*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2
 - 4*a^5*c)*e^2)))/x^2) + sqrt(1/2)*a*d*x*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b
^4 - 4*a*b^2*c + 2*a^2*c^2)*e + ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4
*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2
*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c +
4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b
*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*
a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 -
(a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2))*log(-(((a^3*b^2*c^3 -
 4*a^4*c^4)*d^3 - (a^3*b^3*c^2 - 4*a^4*b*c^3)*d^2*e + (a^4*b^2*c^2 - 4*a^5*
c^3)*d*e^2)*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*
a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6
*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 -
2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2
 - 4*a^9*c)*e^4)) - 2*(a*b^2*c^3 - a^2*c^4)*d^2 + 2*(a*b^3*c^2 - 2*a^2*b*c^
3)*d*e + ((b^3*c^3 - a*b*c^4)*d^2 - (b^4*c^2 + 2*a*b^2*c^3 - 4*a^2*c^4)*d*e
 + 4*(a*b^3*c^2 - 2*a^2*b*c^3)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^
4*b^3*c^2 - 4*a^5*b*c^3)*d^3 - 2*(a^4*b^4*c - 5*a^5*b^2*c^2 + 4*a^6*c^3)*d^
2*e + (a^4*b^5 - 5*a^5*b^3*c + 4*a^6*b*c^2)*d*e^2 - (a^5*b^4 - 6*a^6*b^2*c
+ 8*a^7*c^2)*e^3)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c
- 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(
(a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^
4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8
*b^2 - 4*a^9*c)*e^4)) - ((a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d^2 - (2*a
*b^5*c - 11*a^2*b^3*c^2 + 12*a^3*b*c^3)*d*e + (a*b^6 - 6*a^2*b^4*c + 8*a^3*
b^2*c^2)*e^2)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^
2)*e + ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2
- 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*
a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6
*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 -
2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2
 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e
 + (a^4*b^2 - 4*a^5*c)*e^2)))/x^2) - sqrt(1/2)*a*d*x*sqrt(-((b^3*c - 3*a*b*
c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e + ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (
a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^
2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4
*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*
c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a
^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*
c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2))*log(-(((a^
3*b^2*c^3 - 4*a^4*c^4)*d^3 - (a^3*b^3*c^2 - 4*a^4*b*c^3)*d^2*e + (a^4*b^2*c
^2 - 4*a^5*c^3)*d*e^2)*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*
(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)
```

```
*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e +
(a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3
 + (a^8*b^2 - 4*a^9*c)*e^4)) - 2*(a*b^2*c^3 - a^2*c^4)*d^2 + 2*(a*b^3*c^2 -
 2*a^2*b*c^3)*d*e + ((b^3*c^3 - a*b*c^4)*d^2 - (b^4*c^2 + 2*a*b^2*c^3 - 4*a
^2*c^4)*d*e + 4*(a*b^3*c^2 - 2*a^2*b*c^3)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2
 + d)*(((a^4*b^3*c^2 - 4*a^5*b*c^3)*d^3 - 2*(a^4*b^4*c - 5*a^5*b^2*c^2 + 4*
a^6*c^3)*d^2*e + (a^4*b^5 - 5*a^5*b^3*c + 4*a^6*b*c^2)*d*e^2 - (a^5*b^4 - 6
*a^6*b^2*c + 8*a^7*c^2)*e^3)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2
- 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*
c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*
e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d
*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)) - ((a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)
*d^2 - (2*a*b^5*c - 11*a^2*b^3*c^2 + 12*a^3*b*c^3)*d*e + (a*b^6 - 6*a^2*b^4
*c + 8*a^3*b^2*c^2)*e^2)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c
 + 2*a^2*c^2)*e + ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e
+ (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*
(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)
*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e +
(a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3
 + (a^8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a
^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)))/x^2) + 4*sqrt(e*x^2 + d))/(a*d*x)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**C**] time = 0.02, size = 197, normalized size = 0.70

$$\frac{\sqrt{e}\left(\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)^2c+cd^2+2(2be-cd)\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)+\left(-\sqrt{e}\,x+\sqrt{ex^2+d}\right)^2\right)}{2a\left(\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)^3c+3\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)^2be-3\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)^2cd+8\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)ae^2-4\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)bde+3\mathrm{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+(16ae^2-8deb+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z\right)cd^2+bd^2e-cd^3\right)}-\frac{\sqrt{ex^2+d}}{adx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] -(e*x^2+d)^(1/2)/a/d/x+1/2/a*e^(1/2)*sum((c*_R^2+2*(2*b*e-c*d)*_R+c*d^2)/(_
R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)
*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d
)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2+a\right)\sqrt{ex^2+d}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int(1/(x^2*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.292 $\int \frac{1}{x^4\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=341

$$\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{b\sqrt{d+ex^2}}{a^2dx}+$$

Rubi [A] time = 0.74, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1303, 271, 264, 1692, 377, 205}

$$\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{b\sqrt{d+ex^2}}{a^2dx}+\frac{2e\sqrt{d+ex^2}}{3ad^2x}-\frac{\sqrt{d+ex^2}}{3adx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -Sqrt[d + e*x^2]/(3*a*d*x^3) + (b*Sqrt[d + e*x^2])/(a^2*d*x) + (2*e*Sqrt[d
+ e*x^2])/(3*a*d^2*x) + (c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[
d + e*x^2])])/(a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 -
 4*a*c])*e]) + (c*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2
])])/(a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
```

```
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}dx = \int \left(\frac{1}{ax^4\sqrt{d+ex^2}} - \frac{b}{a^2x^2\sqrt{d+ex^2}} + \frac{b^2-ac+bcx^2}{a^2\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right)dx$$

$$= \frac{\int \frac{b^2-ac+bcx^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}dx}{a^2} + \frac{\int \frac{1}{x^4\sqrt{d+ex^2}}dx}{a} - \frac{b\int \frac{1}{x^2\sqrt{d+ex^2}}dx}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{3adx^3} + \frac{b\sqrt{d+ex^2}}{a^2dx} + \frac{\int \left(\frac{bc+\frac{c\left(b^2-2ac\right)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{bc-\frac{c\left(b^2-2ac\right)}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+\ldots}}\right.}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{3adx^3} + \frac{b\sqrt{d+ex^2}}{a^2dx} + \frac{2e\sqrt{d+ex^2}}{3ad^2x} + \frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2\ldots\right.}}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{3adx^3} + \frac{b\sqrt{d+ex^2}}{a^2dx} + \frac{2e\sqrt{d+ex^2}}{3ad^2x} + \frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{}{b+\sqrt{b^2\ldots}}\right.}{}$$

$$= -\frac{\sqrt{d+ex^2}}{3adx^3} + \frac{b\sqrt{d+ex^2}}{a^2dx} + \frac{2e\sqrt{d+ex^2}}{3ad^2x} + \frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b\ldots\right.}}{\sqrt{b-\sqrt{b^2\ldots}}}\right.}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\ldots\right.}}$$

Mathematica [A] time = 0.69, size = 320, normalized size = 0.94

$$\frac{\frac{3c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} + \frac{3c\left(\frac{2ac-b^2}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} - \frac{a\left(d-2ex^2\right)\sqrt{d+ex^2}}{d^2x^3} + \frac{3b\sqrt{d+ex^2}}{dx}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((3*b*Sqrt[d + e*x^2])/(d*x) - (a*(d - 2*e*x^2)*Sqrt[d + e*x^2])/(d^2*x^3)
+ (3*c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - b*e + Sqr
t[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(Sqrt[
b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (3*c*(b
+ (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(Sqrt[b + Sqrt[
b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/(3*a^2)
```

IntegrateAlgebraic [C] time = 49.09, size = 371, normalized size = 1.09

$$\frac{\sqrt{d+ex^2}\left(-ad+2aex^2+3bdx^2\right)}{3a^2d^2x^3}-\frac{\sqrt{e}\,\text{RootSum}\left[\#1^8c+4\#1^6be-4\#1^6cd+16\#1^4ae^2-8\#1^4bde+6\#1^4cd^2+4\#1^2bd^2e-4\#1^2cd^3+cd^4\&,\frac{\#1^4bc\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-4\#1^2ace\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+4\#1^2b^2e\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2\#1^2bcd\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+bcd^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)}{\#1^6c+3\#1^4be-3\#1^4cd+8\#1^2ae^2-4\#1^2bde+3\#1^2cd^2+bd^2e-cd^3}\&\right]}{2a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[d + e*x^2]*(-(a*d) + 3*b*d*x^2 + 2*a*e*x^2))/(3*a^2*d^2*x^3) - (Sqrt[
e]*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#
1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (b*c*d^2*Log[-(S
qrt[e]*x) + Sqrt[d + e*x^2] - #1] - 2*b*c*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x
^2] - #1]*#1^2 + 4*b^2*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 - 4*
a*c*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + b*c*Log[-(Sqrt[e]*x)
+ Sqrt[d + e*x^2] - #1]*#1^4)/(-(c*d^3) + b*d^2*e + 3*c*d^2*#1^2 - 4*b*d*e*
#1^2 + 8*a*e^2*#1^2 - 3*c*d*#1^4 + 3*b*e*#1^4 + c*#1^6) & ])/(2*a^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.03, size = 248, normalized size = 0.73

$$-\frac{\sqrt{e}\left(\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^2bc+bcd^2+2\left(-2ace+2b^2e-bcd\right)\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)\right)\ln\left(-\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)+\left(-\sqrt{e}\,x+\sqrt{ex^2+d}\right)^2\right)}{2a^2\left(\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^3c+3\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^2be-3\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^2cd+8\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)ae^2-4\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)bde+3\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)cd^2+bd^2e-cd^3\right)}+\frac{2\sqrt{ex^2+d}\,e}{3ad^2x}+\frac{\sqrt{ex^2+d}\,b}{a^2dx}-\frac{\sqrt{ex^2+d}}{3adx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] b*(e*x^2+d)^(1/2)/a^2/d/x-1/2/a^2*e^(1/2)*sum((b*c*_R^2+2*(-2*a*c*e+2*b^2*e
-b*c*d)*_R+b*c*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_
R*c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^
4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c
*d^3)*_Z))-1/3*(e*x^2+d)^(1/2)/a/d/x^3+2/3*e*(e*x^2+d)^(1/2)/a/d^2/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2+a\right)\sqrt{ex^2+d}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int(1/(x^4*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.293 $\int \frac{1}{x^6\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=443

$$-\frac{\left(b^2-ac\right)\sqrt{d+ex^2}}{a^3dx}-\frac{c\left(\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd\cdots}}{\sqrt{\sqrt{b^2\cdots}}}\right.}{a^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2\cdots}\right.}}$$

Rubi [A] time = 1.43, antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1303, 271, 264, 1692, 377, 205}

$$-\frac{\left(b^2-ac\right)\sqrt{d+ex^2}}{a^3dx}-\frac{c\left(\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2be\sqrt{d+ex^2}}{3a^2d^2x}+\frac{b\sqrt{d+ex^2}}{3a^2dx^3}-\frac{8e^2\sqrt{d+ex^2}}{15ad^3x}+\frac{4e\sqrt{d+ex^2}}{15ad^2x^3}-\frac{\sqrt{d+ex^2}}{5adx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -Sqrt[d + e*x^2]/(5*a*d*x^5) + (b*Sqrt[d + e*x^2])/(3*a^2*d*x^3) + (4*e*Sqr
t[d + e*x^2])/(15*a*d^2*x^3) - ((b^2 - a*c)*Sqrt[d + e*x^2])/(a^3*d*x) - (2
*b*e*Sqrt[d + e*x^2])/(3*a^2*d^2*x) - (8*e^2*Sqrt[d + e*x^2])/(15*a*d^3*x)
- (c*(b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2]
)])/(a^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
]) - (c*(b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x
^2])])/(a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{1}{ax^6\sqrt{d+ex^2}} - \frac{b}{a^2x^4\sqrt{d+ex^2}} + \frac{b^2-ac}{a^3x^2\sqrt{d+ex^2}} + \frac{-b\left(b^2-2ac\right)-c\left(b^2\right.}{a^3\sqrt{d+ex^2}\left(a+bx^2\right.}\right.$$

$$= \frac{\int \frac{-b\left(b^2-2ac\right)-c\left(b^2-ac\right)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{a^3} + \frac{\int \frac{1}{x^6\sqrt{d+ex^2}}\,dx}{a} - \frac{b\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx}{a^2} + \frac{\left(b^2-ac\right)\int \frac{}{x^2}}{a^3}$$

$$= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2dx^3} - \frac{\left(b^2-ac\right)\sqrt{d+ex^2}}{a^3dx} + \frac{\int\left(\frac{-\frac{bc\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-c\left(b^2-ac\right)}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right.}{}$$

$$= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2dx^3} + \frac{4e\sqrt{d+ex^2}}{15ad^2x^3} - \frac{\left(b^2-ac\right)\sqrt{d+ex^2}}{a^3dx} - \frac{2be\sqrt{d+e}}{3a^2d^2x}$$

$$= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2dx^3} + \frac{4e\sqrt{d+ex^2}}{15ad^2x^3} - \frac{\left(b^2-ac\right)\sqrt{d+ex^2}}{a^3dx} - \frac{2be\sqrt{d+e}}{3a^2d^2x}$$

$$= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2dx^3} + \frac{4e\sqrt{d+ex^2}}{15ad^2x^3} - \frac{\left(b^2-ac\right)\sqrt{d+ex^2}}{a^3dx} - \frac{2be\sqrt{d+e}}{3a^2d^2x}$$

Mathematica [A] time = 1.64, size = 383, normalized size = 0.86

$$-\frac{\frac{a^2\sqrt{d+ex^2}\left(3d^2-4dex^2+8e^2x^4\right)}{d^3x^5} + \frac{15\left(b^2-ac\right)\sqrt{d+ex^2}}{dx} + \frac{15c\left(\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} + \frac{15c\left(-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} - \frac{5ab\left(d-2ex^2\right)\sqrt{d+ex^2}}{d^2x^3}}{15a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/15*((15*(b^2 - a*c)*Sqrt[d + e*x^2])/(d*x) - (5*a*b*(d - 2*e*x^2)*Sqrt[d
 + e*x^2])/(d^2*x^3) + (a^2*Sqrt[d + e*x^2]*(3*d^2 - 4*d*e*x^2 + 8*e^2*x^4)
)/(d^3*x^5) + (15*c*(b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*
Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2
 - 4*a*c])*e]) + (15*c*(b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c
```

```
]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e]))/a^3
```

IntegrateAlgebraic [C] time = 65.03, size = 534, normalized size = 1.21

$$\frac{\sqrt{e}\,\text{RootSum}\left[\#1^8c+4\#1^6be-4\#1^6cd+16\#1^4ae^2-8\#1^4bde+6\#1^4cd^2+4\#1^2bd^2e-4\#1^2cd^3+cd^4\&,\frac{\#1^4(-a)c^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+\#1^4b^2c\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-8\#1^2abce\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+2\#1^2ac^2d\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+4\#1^2b^3e\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2\#1^2b^2cd\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-ac^2d^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+b^2cd^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)}{\#1^6c+3\#1^4be-3\#1^4cd+8\#1^2ae^2-4\#1^2bde+3\#1^2cd^2+bd^2e-cd^3}\&\right]}{2a^3}+\frac{\sqrt{d+ex^2}\left(-3a^2d^2+4a^2dex^2-8a^2e^2x^4+5abd^2x^2-10abdex^4+15acd^2x^4-15b^2d^2x^4\right)}{15a^3d^3x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^6*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[d + e*x^2]*(-3*a^2*d^2 + 5*a*b*d^2*x^2 + 4*a^2*d*e*x^2 - 15*b^2*d^2*x
^4 + 15*a*c*d^2*x^4 - 10*a*b*d*e*x^4 - 8*a^2*e^2*x^4))/(15*a^3*d^3*x^5) + (
Sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*
d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (b^2*c*d^2*
Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - a*c^2*d^2*Log[-(Sqrt[e]*x) + Sqr
t[d + e*x^2] - #1] - 2*b^2*c*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^
2 + 2*a*c^2*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 4*b^3*e*Log[-
(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 - 8*a*b*c*e*Log[-(Sqrt[e]*x) + Sqr
t[d + e*x^2] - #1]*#1^2 + b^2*c*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1
^4 - a*c^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4)/(-(c*d^3) + b*d^2
*e + 3*c*d^2*#1^2 - 4*b*d*e*#1^2 + 8*a*e^2*#1^2 - 3*c*d*#1^4 + 3*b*e*#1^4 +
 c*#1^6) & ])/(2*a^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.04, size = 350, normalized size = 0.79

$$-\frac{\sqrt{e}\left(ac^2d^2-b^2cd^2+\left(ac-b^2\right)\text{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^2c+2\left(4abce-ac^2d-2b^3e+b^2cd\right)\text{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)\right)\ln\left(-\text{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)+\left(-\sqrt{e}\,x+\sqrt{ex^2+d}\right)^2\right)}{2a^3\left(\text{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^3c+3\,\text{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^2be-3\,\text{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^2cd+8\,\text{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)ae^2-4\,\text{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)bde+3\,\text{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)cd^2+bd^2e-cd^3\right)}-\frac{8\sqrt{ex^2+d}\,e^2}{15ad^3x}-\frac{2\sqrt{ex^2+d}\,be}{3a^2d^2x}+\frac{4\sqrt{ex^2+d}\,e}{15ad^2x^3}+\frac{\sqrt{ex^2+d}\,b}{3a^2dx^3}-\frac{\left(-ac+b^2\right)\sqrt{ex^2+d}}{a^3dx}-\frac{\sqrt{ex^2+d}}{5adx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] -(-a*c+b^2)*(e*x^2+d)^(1/2)/a^3/d/x-1/2/a^3*e^(1/2)*sum((c*(a*c-b^2)*_R^2+2
*(4*a*b*c*e-a*c^2*d-2*b^3*e+b^2*c*d)*_R+a*c^2*d^2-b^2*c*d^2)/(_R^3*c+3*_R^2
*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^
(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a
*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))+1/3*b*(e*x^2+d)^(1/2)/a
^2/d/x^3-2/3*b*e*(e*x^2+d)^(1/2)/a^2/d^2/x-1/5*(e*x^2+d)^(1/2)/a/d/x^5+4/15
*e*(e*x^2+d)^(1/2)/a/d^2/x^3-8/15*e^2*(e*x^2+d)^(1/2)/a/d^3/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2+a\right)\sqrt{ex^2+d}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6\,\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int(1/(x^6*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{6}\sqrt{d+ex^{2}}\left(a+bx^{2}+cx^{4}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/(x**6*sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.294 $\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=350

$$\frac{2\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}}+\frac{2\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}}-\frac{}{e\sqrt{d+}}$$

Rubi [A] time = 4.33, antiderivative size = 507, normalized size of antiderivative = 1.45, number of steps used = 14, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1297, 288, 217, 206, 1692, 377, 205}

$$\frac{\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}+\frac{\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(ae^2-bde+cd^2\right)}-\frac{d^2x}{e\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}\left(ae^2-bde+cd^2\right)}-\frac{(bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d^2*x)/(e*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2])) + ((b^2*d - a*c*d -
a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*S
qrt[d + e*x^2])])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2
 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d - a*c*d - a*b*e + (b^3*d -
 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]
)/(c*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c
*d^2 - b*d*e + a*e^2)) + (d^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(e^(3/2
)*(c*d^2 - b*d*e + a*e^2)) - ((b*d - a*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^
2]])/(c*Sqrt[e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1297

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(d^2*f^4)/(c*d^2 - b*d*e + a*e^2), Int[(f
*x)^(m - 4)*(d + e*x^2)^q, x], x] - Dist[f^4/(c*d^2 - b*d*e + a*e^2), Int[(
(f*x)^(m - 4)*(d + e*x^2)^(q + 1)*Simp[a*d + (b*d - a*e)*x^2, x])/(a + b*x^
2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[q] && LtQ[q, -1] && GtQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^6}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx = -\frac{\int \frac{x^2\left(ad+(bd-ae)x^2\right)}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{cd^2-bde+ae^2}+\frac{d^2\int \frac{x^2}{\left(d+ex^2\right)^{3/2}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}-\frac{\int \left(\frac{bd-ae}{c\sqrt{d+ex^2}}-\frac{a(bd-ae)+\left(b^2d-acd-abe\right)x^2}{c\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right)dx}{cd^2-bde+ae^2}+$$

$$= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}+\frac{\int \frac{a(bd-ae)+\left(b^2d-acd-abe\right)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{c\left(cd^2-bde+ae^2\right)}+\frac{d^2\,\mathrm{Subst}\left(\int\right.}{e\left(cd^2\right.}$$

$$= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{e^{3/2}\left(cd^2-bde+ae^2\right)}+\frac{\int\left(\frac{b^2d-acd-abe+\frac{-b^3d}{}}{\left(b-\sqrt{b^2-4ac}+2\right.}\right.}{}$$

$$= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{e^{3/2}\left(cd^2-bde+ae^2\right)}-\frac{(bd-ae)\tanh^{-1}\left(\right.}{c\sqrt{e}\left(cd^2-bde\right.}$$

$$= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{e^{3/2}\left(cd^2-bde+ae^2\right)}-\frac{(bd-ae)\tanh^{-1}\left(\right.}{c\sqrt{e}\left(cd^2-bde\right.}$$

$$= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}+\frac{\left(b^2d-acd-abe-\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\tan}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right.}}$$

Mathematica [B] time = 11.26, size = 10968, normalized size = 31.34

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]

[Out] Result too large to show

IntegrateAlgebraic [C] time = 84.95, size = 593, normalized size = 1.69

$$-\frac{\sqrt{e}\,\text{RootSum}\left[\#1^8c+4\#1^6be-4\#1^6cd+16\#1^4ae^2-8\#1^4bde+6\#1^4cd^2+4\#1^2bd^2e-4\#1^2cd^3+cd^4\&,\frac{-\#1^4abe\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-\#1^4acd\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+\#1^4b^2d\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-4\#1^2a^2e^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+6\#1^2abde\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+2\#1^2acd^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2\#1^2b^2d^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-abd^2e\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-acd^3\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+b^2d^3\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)}{\#1^6c+3\#1^4be-3\#1^4cd+8\#1^2ae^2-4\#1^2bde+3\#1^2cd^2+bd^2e-cd^3}\&\right]}{2c\left(ae^2-bde+cd^2\right)}-\frac{d^2x}{c\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)}-\frac{\log\left(\sqrt{d+ex^2}-\sqrt{e}x\right)}{ce^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]

[Out] -((d^2*x)/(e*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2])) - Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]]/(c*e^(3/2)) - (Sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (b^2*d^3*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - a*c*d^3*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - a*b*d^2*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - 2*b^2*d^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 2*a*c*d^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 6*a*b*d*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 - 4*a^2*e^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + b^2*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4 - a*c*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4 - a*b*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4)/(-(c*d^3) + b*d^2*e + 3*c*d^2*#1^2 - 4*b*d*e*#1^2 + 8*a*e^2*#1^2 - 3*c*d*#1^4 + 3*b*e*#1^4 + c*#1^6) &])/(2*c*(c*d^2 - b*d*e + a*e^2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 2.25, size = 75, normalized size = 0.21

$$-\frac{c^2d^2x}{\left(c^3d^2e-bc^2de^2+ac^2e^3\right)\sqrt{x^2e+d}}-\frac{e^{\left(-\frac{3}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] -c^2*d^2*x/((c^3*d^2*e - b*c^2*d*e^2 + a*c^2*e^3)*sqrt(x^2*e + d)) - 1/2*e^(-3/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c

maple [C] time = 0.04, size = 480, normalized size = 1.37

$$\frac{8abe^{\frac{3}{2}}}{\left(4ae^2-4deb+4cd^2\right)\left(2ex^2-2\sqrt{ex^2+d}\,\sqrt{e}\,x+2d\right)c^2}+\frac{8ad\sqrt{e}}{\left(4ae^2-4deb+4cd^2\right)\left(2ex^2-2\sqrt{ex^2+d}\,\sqrt{e}\,x+2d\right)c}-\frac{8b^2d\sqrt{e}}{\left(4ae^2-4deb+4cd^2\right)\left(2ex^2-2\sqrt{ex^2+d}\,\sqrt{e}\,x+2d\right)c^2}+\frac{2\sqrt{e}\left(abd^2e+acd^3-b^2d^3+\left(abe+acd-b^2d\right)\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^2+2\left(2a^2e^2-3abde-acd^2+b^2d^2\right)\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)\right)\ln\left(-\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)+\left(-\sqrt{e}\,x+\sqrt{ex^2+d}\right)^2\right)}{\left(4ae^2-4deb+4cd^2\right)c\left(\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^3c+3\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^2be-3\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^2cd+8\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)ae^2-4\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)bde+3\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)cd^2+bd^2e-cd^3\right)}-\frac{bx}{\sqrt{ex^2+d}\,c^2d}-\frac{x}{\sqrt{ex^2+d}\,ce}+\frac{\ln\left(\sqrt{e}\,x+\sqrt{ex^2+d}\right)}{ce^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)

[Out] -1/c*x/e/(e*x^2+d)^(1/2)+1/c/e^(3/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))-1/c^2*b*x/d/(e*x^2+d)^(1/2)+2/c*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)*sum(((a*b*e+a*c*d-b^2*d)*_R^2+2*(2*a^2*e^2-3*a*b*d*e-a*c*d^2+b^2*d^2)*_R+a*b*d^2*e+a*c*d^3-b^2*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e+

```
2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4
*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))+8
/c^2*e^(3/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^(1/2)*(e*x^2+d)^(1/2)*x
+2*d)*a*b+8/c*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^(1/2)*(e*x^2+d
)^(1/2)*x+2*d)*a*d-8/c^2*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^(1/
2)*(e*x^2+d)^(1/2)*x+2*d)*b^2*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(cx^4+bx^2+a)(ex^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(x^6/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{(e\,x^2+d)^{3/2}\,(c\,x^4+b\,x^2+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(d+ex^2)^{\frac{3}{2}}\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x**6/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

3.295 $\int \frac{x^4}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=360

$$-\frac{\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,\left(ae^2-bde+cd^2\right)}-\frac{\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,\left(ae^2-bde+cd^2\right)}+\frac{dx}{\sqrt{d+ex^2}\,\left(ae^2-bde+cd^2\right)}$$

Rubi [A] time = 1.27, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1297, 191, 1692, 377, 205}

$$-\frac{\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,\left(ae^2-bde+cd^2\right)}-\frac{\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,\left(ae^2-bde+cd^2\right)}+\frac{dx}{\sqrt{d+ex^2}\,\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (d*x)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - ((b*d - a*e - (b^2*d - 2*
a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^
2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)
) - ((b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d
+ e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1297

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(d^2*f^4)/(c*d^2 - b*d*e + a*e^2), Int[(f
*x)^(m - 4)*(d + e*x^2)^q, x], x] - Dist[f^4/(c*d^2 - b*d*e + a*e^2), Int[(
(f*x)^(m - 4)*(d + e*x^2)^(q + 1)*Simp[a*d + (b*d - a*e)*x^2, x])/(a + b*x^
2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[q] && LtQ[q, -1] && GtQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx = -\frac{\int \frac{ad+(bd-ae)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{cd^2-bde+ae^2}+\frac{d^2\int \frac{1}{\left(d+ex^2\right)^{3/2}}\,dx}{cd^2-bde+ae^2}$$

$$=\frac{dx}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}-\frac{\int \left(\frac{bd-ae+\frac{-b^2d+2acd+abe}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}+\frac{bd-ae-\frac{-b^2d+2acd+a}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{}}\right.}{cd^2-bde+ae^2}$$

$$=\frac{dx}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}-\frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+e}}}{cd^2-bde+ae^2}$$

$$=\frac{dx}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}-\frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{b-\sqrt{b^2-4ac}-(-}\right.}{cd^2-bde+ae}$$

$$=\frac{dx}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}-\frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-(b-}}{\sqrt{b-\sqrt{b^2-4}}}\right.}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e\,(}}$$

Mathematica [C] time = 7.90, size = 2162, normalized size = 6.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] x/(c*d*Sqrt[d + e*x^2]) - ((b + (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(45*Sqr
t[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d +
 e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*Sqrt[-(((-
b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2)
)/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[-(((2*c*d
 + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))
]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*
(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/d - (45*(2*c*d + (-b + Sqrt[b^2 - 4
*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(
-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2
)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[-(((2*c*d +
 (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]]
)/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) + 4*(-(((2*c*d + (-b + Sqrt[b^2
- 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5/2)*Sqrt[((-b
+ Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]*H
ypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d
*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))] + (4*e*x^2*(-(((2*c*d + (-b + Sqrt[b
^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5/2)*Sqrt[((
```

```
-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))
]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)
/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))])/d))/(15*c*(b - Sqrt[b^2 - 4*a*c]
)*d*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c
] - 2*c*x^2))))^(3/2)*(1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]))*Sqrt[d + e*x
^2]*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2))]) - ((b - (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(45*Sqrt[-(((b +
Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d
^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*Sqrt[-(((b + Sqrt[b^2
 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]] - (30*e*x^2*Ar
cSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2))]])/d + (45*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sq
rt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2))]])/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) - (30*e*(-2*c*d + (b + Sqr
t[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2
)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)) + 4*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))] + (4*e*x
^2*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))])/d))/(15*c*(b + S
qrt[b^2 - 4*a*c])*d*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^2)))^(3/2)*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))*Sq
rt[d + e*x^2]*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2))])
```

IntegrateAlgebraic [C] time = 55.59, size = 405, normalized size = 1.12

$$\frac{\sqrt{e}\,\text{RootSum}\left[\#1^8c+4\#1^6be-4\#1^6cd+16\#1^4ae^2-8\#1^4bde+6\#1^4cd^2+4\#1^2bd^2e-4\#1^2cd^3+cd^4\&,\frac{-\#1^4ae\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+\#1^4bd\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+6\#1^2ade\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2\#1^2bd^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-ad^2e\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+bd^3\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)}{\#1^6c+3\#1^4be-3\#1^4cd+8\#1^2ae^2-4\#1^2bde+3\#1^2cd^2+bd^2e-cd^3}\&\right]}{2\left(ae^2-bde+cd^2\right)}+\frac{dx}{\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (d*x)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) + (Sqrt[e]*RootSum[c*d^4 -
4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4
 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (b*d^3*Log[-(Sqrt[e]*x) + Sqrt[d +
e*x^2] - #1] - a*d^2*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - 2*b*d^2*L
og[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 6*a*d*e*Log[-(Sqrt[e]*x) + S
qrt[d + e*x^2] - #1]*#1^2 + b*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1
^4 - a*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4)/(-(c*d^3) + b*d^2*e
 + 3*c*d^2*#1^2 - 4*b*d*e*#1^2 + 8*a*e^2*#1^2 - 3*c*d*#1^4 + 3*b*e*#1^4 + c
*#1^6) & ])/(2*(c*d^2 - b*d*e + a*e^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b,c]=[44,93,-37]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a,b,c]=[-72,-7,6]Evaluation time: 0.6Unable to divide, perhaps due t
o rounding error%%%{-2147483648,[3,8,10,8,3,10,1]%%%}+%%%{-12884901888,[3,8
,10,7,2,12,2]%%%}+%%%{463856467968,[3,8,10,6,1,14,3]%%%}+%%%{1924145348608,
[3,8,10,5,0,16,4]%%%}+%%%{536870912,[3,8,9,8,5,10,0]%%%}+%%%{20401094656,[3
,8,9,7,4,12,1]%%%}+%%%{-150323855360,[3,8,9,6,3,14,2]%%%}+%%%{-313532612608
0,[3,8,9,5,2,16,3]%%%}+%%%{-4672924418048,[3,8,9,4,1,18,4]%%%}+%%%{60473139
52768,[3,8,9,3,0,20,5]%%%}+%%%{-4294967296,[3,8,8,7,6,12,0]%%%}+%%%{-424128
02048,[3,8,8,6,5,14,1]%%%}+%%%{1046898278400,[3,8,8,5,4,16,2]%%%}+%%%{62105
22710016,[3,8,8,4,3,18,3]%%%}+%%%{-1786706395136,[3,8,8,3,2,20,4]%%%}+%%%{-
11544872091648,[3,8,8,2,1,22,5]%%%}+%%%{4398046511104,[3,8,8,1,0,24,6]%%%}+
%%%{12750684160,[3,8,7,6,7,14,0]%%%}+%%%{-23890755584,[3,8,7,5,6,16,1]%%%}+
%%%{-2103460233216,[3,8,7,4,5,18,2]%%%}+%%%{-3324304687104,[3,8,7,3,4,20,3]
%%%}+%%%{9758165696512,[3,8,7,2,3,22,4]%%%}+%%%{1649267441664,[3,8,7,1,2,24
,5]%%%}+%%%{-4398046511104,[3,8,7,0,1,26,6]%%%}+%%%{-17985175552,[3,8,6,5,8
,16,0]%%%}+%%%{161866579968,[3,8,6,4,7,18,1]%%%}+%%%{1586990415872,[3,8,6,3
,6,20,2]%%%}+%%%{-1795296329728,[3,8,6,2,5,22,3]%%%}+%%%{-4123168604160,[3,
8,6,1,4,24,4]%%%}+%%%{3848290697216,[3,8,6,0,3,26,5]%%%}+%%%{12213813248,[3
,8,5,4,9,18,0]%%%}+%%%{-171798691840,[3,8,5,3,8,20,1]%%%}+%%%{-212600881152
,[3,8,5,2,7,22,2]%%%}+%%%{1477468749824,[3,8,5,1,6,24,3]%%%}+%%%{-109951162
7776,[3,8,5,0,5,26,4]%%%}+%%%{-3221225472,[3,8,4,3,10,20,0]%%%}+%%%{5798205
8496,[3,8,4,2,9,22,1]%%%}+%%%{-154618822656,[3,8,4,1,8,24,2]%%%}+%%%{103079
215104,[3,8,4,0,7,26,3]%%%}+%%%{1048576,[3,6,10,4,2,4,0]%%%}+%%%{8388608,[3
,6,10,3,1,6,1]%%%}+%%%{16777216,[3,6,10,2,0,8,2]%%%}+%%%{-5242880,[3,6,9,3,
3,6,0]%%%}+%%%{-29360128,[3,6,9,2,2,8,1]%%%}+%%%{-33554432,[3,6,9,1,1,10,2]
%%%}+%%%{9699328,[3,6,8,2,4,8,0]%%%}+%%%{33554432,[3,6,8,1,3,10,1]%%%}+%%%{
16777216,[3,6,8,0,2,12,2]%%%}+%%%{-7864320,[3,6,7,1,5,10,0]%%%}+%%%{-125829
12,[3,6,7,0,4,12,1]%%%}+%%%{2359296,[3,6,6,0,6,12,0]%%%}+%%%{-536870912,[2,
7,10,6,2,8,1]%%%}+%%%{18446744062703697920,[2,7,10,5,1,10,2]%%%}+%%%{-18253
611008,[2,7,10,4,0,12,3]%%%}+%%%{134217728,[2,7,9,6,4,8,0]%%%}+%%%{55029268
48,[2,7,9,5,3,10,1]%%%}+%%%{36909875200,[2,7,9,4,2,12,2]%%%}+%%%{4294967296
0,[2,7,9,3,1,14,3]%%%}+%%%{-42949672960,[2,7,9,2,0,16,4]%%%}+%%%{-956301312
,[2,7,8,5,5,10,0]%%%}+%%%{-18656264192,[2,7,8,4,4,12,1]%%%}+%%%{-6496138035
2,[2,7,8,3,3,14,2]%%%}+%%%{8589934592,[2,7,8,2,2,16,3]%%%}+%%%{85899345920,
[2,7,8,1,1,18,4]%%%}+%%%{2642411520,[2,7,7,4,6,12,0]%%%}+%%%{27783069696,[2
,7,7,3,5,14,1]%%%}+%%%{33957085184,[2,7,7,2,4,16,2]%%%}+%%%{-73014444032,[2
,7,7,1,3,18,3]%%%}+%%%{-42949672960,[2,7,7,0,2,20,4]%%%}+%%%{-3556769792,[2
,7,6,3,7,14,0]%%%}+%%%{-17716740096,[2,7,6,2,6,16,1]%%%}+%%%{12884901888,[2
,7,6,1,5,18,2]%%%}+%%%{39728447488,[2,7,6,0,4,20,3]%%%}+%%%{2340421632,[2,7
,5,2,8,16,0]%%%}+%%%{2415919104,[2,7,5,1,7,18,1]%%%}+%%%{-12079595520,[2,7,
5,0,6,20,2]%%%}+%%%{-603979776,[2,7,4,1,9,18,0]%%%}+%%%{1207959552,[2,7,4,0
,8,20,1]%%%}+%%%{2147483648,[1,8,10,9,3,10,1]%%%}+%%%{38654705664,[1,8,10,8
,2,12,2]%%%}+%%%{51539607552,[1,8,10,7,1,14,3]%%%}+%%%{-274877906944,[1,8,1
0,6,0,16,4]%%%}+%%%{-536870912,[1,8,9,9,5,10,0]%%%}+%%%{-26843545600,[1,8,9
,8,4,12,1]%%%}+%%%{-188978561024,[1,8,9,7,3,14,2]%%%}+%%%{146028888064,[1,8
,9,6,2,16,3]%%%}+%%%{962072674304,[1,8,9,5,1,18,4]%%%}+%%%{-549755813888,[1
,8,9,4,0,20,5]%%%}+%%%{4294967296,[1,8,8,8,6,12,0]%%%}+%%%{95026151424,[1,8
,8,7,5,14,1]%%%}+%%%{239444426752,[1,8,8,6,4,16,2]%%%}+%%%{-858993459200,[1
,8,8,5,3,18,3]%%%}+%%%{-618475290624,[1,8,8,4,2,20,4]%%%}+%%%{1099511627776
,[1,8,8,3,1,22,5]%%%}+%%%{-12750684160,[1,8,7,7,7,14,0]%%%}+%%%{-1366336471
04,[1,8,7,6,6,16,1]%%%}+%%%{62277025792,[1,8,7,5,5,18,2]%%%}+%%%{9363028705
28,[1,8,7,4,4,20,3]%%%}+%%%{-549755813888,[1,8,7,3,3,22,4]%%%}+%%%{-5497558

```
13888,[1,8,7,2,2,24,5]%%%}+%%%{17985175552,[1,8,6,6,8,16,0]%%%}+%%%{7194070
2208,[1,8,6,5,7,18,1]%%%}+%%%{-267361714176,[1,8,6,4,6,20,2]%%%}+%%%{-13743
8953472,[1,8,6,3,5,22,3]%%%}+%%%{481036337152,[1,8,6,2,4,24,4]%%%}+%%%{-122
13813248,[1,8,5,5,9,18,0]%%%}+%%%{7247757312,[1,8,5,4,8,20,1]%%%}+%%%{10307
9215104,[1,8,5,3,7,22,2]%%%}+%%%{-137438953472,[1,8,5,2,6,24,3]%%%}+%%%{322
1225472,[1,8,4,4,10,20,0]%%%}+%%%{-12884901888,[1,8,4,3,9,22,1]%%%}+%%%{128
84901888,[1,8,4,2,8,24,2]%%%}+%%%{-1048576,[1,6,10,5,2,4,0]%%%}+%%%{-838860
8,[1,6,10,4,1,6,1]%%%}+%%%{-16777216,[1,6,10,3,0,8,2]%%%}+%%%{8388608,[1,6,
9,4,3,6,0]%%%}+%%%{62914560,[1,6,9,3,2,8,1]%%%}+%%%{150994944,[1,6,9,2,1,10
,2]%%%}+%%%{134217728,[1,6,9,1,0,12,3]%%%}+%%%{-26476544,[1,6,8,3,4,8,0]%%%
}+%%%{-163577856,[1,6,8,2,3,10,1]%%%}+%%%{-301989888,[1,6,8,1,2,12,2]%%%}+%
%%{-134217728,[1,6,8,0,1,14,3]%%%}+%%%{41156608,[1,6,7,2,5,10,0]%%%}+%%%{17
8257920,[1,6,7,1,4,12,1]%%%}+%%%{167772160,[1,6,7,0,3,14,2]%%%}+%%%{-314572
80,[1,6,6,1,6,12,0]%%%}+%%%{-69206016,[1,6,6,0,5,14,1]%%%}+%%%{9437184,[1,6
,5,0,7,14,0]%%%}+%%%{402653184,[0,7,10,7,2,8,1]%%%}+%%%{5637144576,[0,7,10,
6,1,10,2]%%%}+%%%{16106127360,[0,7,10,5,0,12,3]%%%}+%%%{-100663296,[0,7,9,7
,4,8,0]%%%}+%%%{-4160749568,[0,7,9,6,3,10,1]%%%}+%%%{-30198988800,[0,7,9,5,
2,12,2]%%%}+%%%{-28991029248,[0,7,9,4,1,14,3]%%%}+%%%{68719476736,[0,7,9,3,
0,16,4]%%%}+%%%{687865856,[0,7,8,6,5,10,0]%%%}+%%%{13925089280,[0,7,8,5,4,1
2,1]%%%}+%%%{48184164352,[0,7,8,4,3,14,2]%%%}+%%%{-49392123904,[0,7,8,3,2,1
6,3]%%%}+%%%{-120259084288,[0,7,8,2,1,18,4]%%%}+%%%{68719476736,[0,7,8,1,0,
20,5]%%%}+%%%{-1845493760,[0,7,7,5,6,12,0]%%%}+%%%{-19964887040,[0,7,7,4,5,
14,1]%%%}+%%%{-11542724608,[0,7,7,3,4,16,2]%%%}+%%%{113816633344,[0,7,7,2,3
,18,3]%%%}+%%%{-8589934592,[0,7,7,1,2,20,4]%%%}+%%%{-68719476736,[0,7,7,0,1
,22,5]%%%}+%%%{2432696320,[0,7,6,4,7,14,0]%%%}+%%%{11207180288,[0,7,6,3,6,1
6,1]%%%}+%%%{-28185722880,[0,7,6,2,5,18,2]%%%}+%%%{-34359738368,[0,7,6,1,4,
20,3]%%%}+%%%{60129542144,[0,7,6,0,3,22,4]%%%}+%%%{-1577058304,[0,7,5,3,8,1
6,0]%%%}+%%%{201326592,[0,7,5,2,7,18,1]%%%}+%%%{14495514624,[0,7,5,1,6,20,2
]%%%}+%%%{-17179869184,[0,7,5,0,5,22,3]%%%}+%%%{402653184,[0,7,4,2,9,18,0]%
%%}+%%%{-1610612736,[0,7,4,1,8,20,1]%%%}+%%%{1610612736,[0,7,4,0,7,22,2]%%%
} / %%%{1024,[0,3,4,2,1,2,0]%%%}+%%%{4096,[0,3,4,1,0,4,1]%%%}+%%%{-2560,[0,
3,3,1,2,4,0]%%%}+%%%{-4096,[0,3,3,0,1,6,1]%%%}+%%%{1536,[0,3,2,0,3,6,0]%%%}
 Error: Bad Argument Value
```

maple [C] time = 0.03, size = 338, normalized size = 0.94

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/c*x/d/(e*x^2+d)^(1/2)-2*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)*sum(((a*e-b*d)*
_R^2+2*d*(-3*a*e+b*d)*_R+a*d^2*e-b*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*
a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/
2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)
*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))-8/c*e^(3/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x
^2-2*(e*x^2+d)^(1/2)*e^(1/2)*x+2*d)*a+8/c*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)
/(2*e*x^2-2*(e*x^2+d)^(1/2)*e^(1/2)*x+2*d)*b*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(cx^4+bx^2+a\right)\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(e\,x^2+d\right)^{3/2}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int(x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(d+ex^2\right)^{\frac{3}{2}}\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**4/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

3.296 $\int \frac{x^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=333

$$\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}+\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4\cdots}\right.}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{\cdots}}\right.}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,(\cdots}$$

Rubi [A] time = 0.66, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1299, 191, 1692, 377, 205}

$$\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}+\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(ae^2-bde+cd^2\right)}-\frac{ex}{\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((e*x)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2])) + (c*(d - (b*d - 2*a*e)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt
[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqr
t[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) + (c*(d + (b*
d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 -
4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1299

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> -Dist[(d*e*f^2)/(c*d^2 - b*d*e + a*e^2), Int[(
f*x)^(m - 2)*(d + e*x^2)^q, x], x] + Dist[f^2/(c*d^2 - b*d*e + a*e^2), Int[
((f*x)^(m - 2)*(d + e*x^2)^(q + 1)*Simp[a*e + c*d*x^2, x])/(a + b*x^2 + c*x
^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !Int
egerQ[q] && LtQ[q, -1] && GtQ[m, 1] && LeQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
```

```
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx = \frac{\int \frac{ae+cdx^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{cd^2-bde+ae^2} - \frac{(de)\int \frac{1}{\left(d+ex^2\right)^{3/2}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{ex}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{\int \left(\frac{cd+\frac{c(-bd+2ae)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{cd-\frac{c(-bd+2ae)}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\right.}{cd^2-bde+ae^2}$$

$$= -\frac{ex}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,d}{cd^2-bde+ae^2}$$

$$= -\frac{ex}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\right.}\right.}{cd^2-bde+ae^2}$$

$$= -\frac{ex}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2}\right.}}{\sqrt{b-\sqrt{b^2-4ac}}}\right.}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

Mathematica [C] time = 6.72, size = 2119, normalized size = 6.36

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((1 - b/Sqrt[b^2 - 4*a*c])*x*(45*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d +
(-b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] +
2*c*x^2)^2))] + (30*e*x^2*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + S
qrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2
)^2))])/d - 45*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*
(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d +
(-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])
/d - (45*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (
-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/
(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*
a*c])*e)*x^4*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-
b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^
2)) + 4*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4
*a*c] - 2*c*x^2))))^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-
b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d +
 (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]
+ (4*e*x^2*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2
- 4*a*c] - 2*c*x^2))))^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d
*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*
d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))
)])/d))/(15*(b - Sqrt[b^2 - 4*a*c])*d*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])
```

```
*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(3/2)*(1 - (2*c*x^2)/(-b
+ Sqrt[b^2 - 4*a*c]))*Sqrt[d + e*x^2]*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e
*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]) + ((1 + b/Sqrt[b^2 - 4*a*c]
)*x*(45*Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e
)*x^2*(d + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*
Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d
 + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[
((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2))]] - (30*e*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d
*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/d + (45*(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) - (30
*e*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d^2*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2)) + 4*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)
/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])
*(d + e*x^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2
, 7/2, ((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2))] + (4*e*x^2*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x
^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
))])/d))/(15*(b + Sqrt[b^2 - 4*a*c])*d*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(3/2)*(1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]))*Sqrt[d + e*x^2]*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))
/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))])
```

IntegrateAlgebraic [C] time = 49.22, size = 343, normalized size = 1.03

$$\frac{\sqrt{e}\,\text{RootSum}\left[\#1^8 c+4\#1^6 be-4\#1^6 cd+16\#1^4 ae^2-8\#1^4 bde+6\#1^4 cd^2+4\#1^2 bd^2 e-4\#1^2 cd^3+cd^4\&,\frac{\#1^4 cd\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+4\#1^2 ae^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2\#1^2 cd^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+cd^3\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)}{\#1^6(-c)-3\#1^4 be+3\#1^4 cd-8\#1^2 ae^2+4\#1^2 bde-3\#1^2 cd^2-bd^2 e+cd^3}\&\right]}{2\left(ae^2-bde+cd^2\right)}-\frac{ex}{\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((e*x)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2])) + (Sqrt[e]*RootSum[c*d^4
 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#
1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (c*d^3*Log[-(Sqrt[e]*x) + Sqrt[d
 + e*x^2] - #1] - 2*c*d^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 4
*a*e^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + c*d*Log[-(Sqrt[e]*x)
 + Sqrt[d + e*x^2] - #1]*#1^4)/(c*d^3 - b*d^2*e - 3*c*d^2*#1^2 + 4*b*d*e*#1
^2 - 8*a*e^2*#1^2 + 3*c*d*#1^4 - 3*b*e*#1^4 - c*#1^6) & ])/(2*(c*d^2 - b*d*
e + a*e^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b,c]=[44,93,-37]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a,b,c]=[-72,-7,6]Evaluation time: 0.57Unable to divide, perhaps due
to rounding error%%%{-2147483648,[3,8,10,8,3,10,1]%%%}+%%%{-12884901888,[3,
8,10,7,2,12,2]%%%}+%%%{463856467968,[3,8,10,6,1,14,3]%%%}+%%%{1924145348608
,[3,8,10,5,0,16,4]%%%}+%%%{536870912,[3,8,9,8,5,10,0]%%%}+%%%{20401094656,[
3,8,9,7,4,12,1]%%%}+%%%{-150323855360,[3,8,9,6,3,14,2]%%%}+%%%{-31353261260
80,[3,8,9,5,2,16,3]%%%}+%%%{-4672924418048,[3,8,9,4,1,18,4]%%%}+%%%{6047313
952768,[3,8,9,3,0,20,5]%%%}+%%%{-4294967296,[3,8,8,7,6,12,0]%%%}+%%%{-42412
802048,[3,8,8,6,5,14,1]%%%}+%%%{1046898278400,[3,8,8,5,4,16,2]%%%}+%%%{6210
522710016,[3,8,8,4,3,18,3]%%%}+%%%{-1786706395136,[3,8,8,3,2,20,4]%%%}+%%%{
-11544872091648,[3,8,8,2,1,22,5]%%%}+%%%{4398046511104,[3,8,8,1,0,24,6]%%%}
+%%%{12750684160,[3,8,7,6,7,14,0]%%%}+%%%{-23890755584,[3,8,7,5,6,16,1]%%%}
+%%%{-2103460233216,[3,8,7,4,5,18,2]%%%}+%%%{-3324304687104,[3,8,7,3,4,20,3
]%%%}+%%%{9758165696512,[3,8,7,2,3,22,4]%%%}+%%%{1649267441664,[3,8,7,1,2,2
4,5]%%%}+%%%{-4398046511104,[3,8,7,0,1,26,6]%%%}+%%%{-17985175552,[3,8,6,5,
8,16,0]%%%}+%%%{161866579968,[3,8,6,4,7,18,1]%%%}+%%%{1586990415872,[3,8,6,
3,6,20,2]%%%}+%%%{-1795296329728,[3,8,6,2,5,22,3]%%%}+%%%{-4123168604160,[3
,8,6,1,4,24,4]%%%}+%%%{3848290697216,[3,8,6,0,3,26,5]%%%}+%%%{12213813248,[
3,8,5,4,9,18,0]%%%}+%%%{-171798691840,[3,8,5,3,8,20,1]%%%}+%%%{-21260088115
2,[3,8,5,2,7,22,2]%%%}+%%%{1477468749824,[3,8,5,1,6,24,3]%%%}+%%%{-10995116
27776,[3,8,5,0,5,26,4]%%%}+%%%{-3221225472,[3,8,4,3,10,20,0]%%%}+%%%{579820
58496,[3,8,4,2,9,22,1]%%%}+%%%{-154618822656,[3,8,4,1,8,24,2]%%%}+%%%{10307
9215104,[3,8,4,0,7,26,3]%%%}+%%%{1048576,[3,6,10,4,2,4,0]%%%}+%%%{8388608,[
3,6,10,3,1,6,1]%%%}+%%%{16777216,[3,6,10,2,0,8,2]%%%}+%%%{-5242880,[3,6,9,3
,3,6,0]%%%}+%%%{-29360128,[3,6,9,2,2,8,1]%%%}+%%%{-33554432,[3,6,9,1,1,10,2
]%%%}+%%%{9699328,[3,6,8,2,4,8,0]%%%}+%%%{33554432,[3,6,8,1,3,10,1]%%%}+%%%
{16777216,[3,6,8,0,2,12,2]%%%}+%%%{-7864320,[3,6,7,1,5,10,0]%%%}+%%%{-12582
912,[3,6,7,0,4,12,1]%%%}+%%%{2359296,[3,6,6,0,6,12,0]%%%}+%%%{-536870912,[2
,7,10,6,2,8,1]%%%}+%%%{18446744062703697920,[2,7,10,5,1,10,2]%%%}+%%%{-1825
3611008,[2,7,10,4,0,12,3]%%%}+%%%{134217728,[2,7,9,6,4,8,0]%%%}+%%%{5502926
848,[2,7,9,5,3,10,1]%%%}+%%%{36909875200,[2,7,9,4,2,12,2]%%%}+%%%{429496729
60,[2,7,9,3,1,14,3]%%%}+%%%{-42949672960,[2,7,9,2,0,16,4]%%%}+%%%{-95630131
2,[2,7,8,5,5,10,0]%%%}+%%%{-18656264192,[2,7,8,4,4,12,1]%%%}+%%%{-649613803
52,[2,7,8,3,3,14,2]%%%}+%%%{8589934592,[2,7,8,2,2,16,3]%%%}+%%%{85899345920
,[2,7,8,1,1,18,4]%%%}+%%%{2642411520,[2,7,7,4,6,12,0]%%%}+%%%{27783069696,[
2,7,7,3,5,14,1]%%%}+%%%{33957085184,[2,7,7,2,4,16,2]%%%}+%%%{-73014444032,[
2,7,7,1,3,18,3]%%%}+%%%{-42949672960,[2,7,7,0,2,20,4]%%%}+%%%{-3556769792,[
2,7,6,3,7,14,0]%%%}+%%%{-17716740096,[2,7,6,2,6,16,1]%%%}+%%%{12884901888,[
2,7,6,1,5,18,2]%%%}+%%%{39728447488,[2,7,6,0,4,20,3]%%%}+%%%{2340421632,[2,
7,5,2,8,16,0]%%%}+%%%{2415919104,[2,7,5,1,7,18,1]%%%}+%%%{-12079595520,[2,7
,5,0,6,20,2]%%%}+%%%{-603979776,[2,7,4,1,9,18,0]%%%}+%%%{1207959552,[2,7,4,
0,8,20,1]%%%}+%%%{2147483648,[1,8,10,9,3,10,1]%%%}+%%%{38654705664,[1,8,10,
8,2,12,2]%%%}+%%%{51539607552,[1,8,10,7,1,14,3]%%%}+%%%{-274877906944,[1,8,
10,6,0,16,4]%%%}+%%%{-536870912,[1,8,9,9,5,10,0]%%%}+%%%{-26843545600,[1,8,
9,8,4,12,1]%%%}+%%%{-188978561024,[1,8,9,7,3,14,2]%%%}+%%%{146028888064,[1,
8,9,6,2,16,3]%%%}+%%%{962072674304,[1,8,9,5,1,18,4]%%%}+%%%{-549755813888,[
1,8,9,4,0,20,5]%%%}+%%%{4294967296,[1,8,8,8,6,12,0]%%%}+%%%{95026151424,[1,
8,8,7,5,14,1]%%%}+%%%{239444426752,[1,8,8,6,4,16,2]%%%}+%%%{-858993459200,[
1,8,8,5,3,18,3]%%%}+%%%{-618475290624,[1,8,8,4,2,20,4]%%%}+%%%{109951162777
6,[1,8,8,3,1,22,5]%%%}+%%%{-12750684160,[1,8,7,7,7,14,0]%%%}+%%%{-136633647
104,[1,8,7,6,6,16,1]%%%}+%%%{62277025792,[1,8,7,5,5,18,2]%%%}+%%%{936302870
528,[1,8,7,4,4,20,3]%%%}+%%%{-549755813888,[1,8,7,3,3,22,4]%%%}+%%%{-549755
813888,[1,8,7,2,2,24,5]%%%}+%%%{17985175552,[1,8,6,6,8,16,0]%%%}+%%%{719407
02208,[1,8,6,5,7,18,1]%%%}+%%%{-267361714176,[1,8,6,4,6,20,2]%%%}+%%%{-1374
38953472,[1,8,6,3,5,22,3]%%%}+%%%{481036337152,[1,8,6,2,4,24,4]%%%}+%%%{-12
```

```
213813248,[1,8,5,5,9,18,0]%%%}+%%%{7247757312,[1,8,5,4,8,20,1]%%%}+%%%{1030
79215104,[1,8,5,3,7,22,2]%%%}+%%%{-137438953472,[1,8,5,2,6,24,3]%%%}+%%%{32
21225472,[1,8,4,4,10,20,0]%%%}+%%%{-12884901888,[1,8,4,3,9,22,1]%%%}+%%%{12
884901888,[1,8,4,2,8,24,2]%%%}+%%%{-1048576,[1,6,10,5,2,4,0]%%%}+%%%{-83886
08,[1,6,10,4,1,6,1]%%%}+%%%{-16777216,[1,6,10,3,0,8,2]%%%}+%%%{8388608,[1,6
,9,4,3,6,0]%%%}+%%%{62914560,[1,6,9,3,2,8,1]%%%}+%%%{150994944,[1,6,9,2,1,1
0,2]%%%}+%%%{134217728,[1,6,9,1,0,12,3]%%%}+%%%{-26476544,[1,6,8,3,4,8,0]%%
%}+%%%{-163577856,[1,6,8,2,3,10,1]%%%}+%%%{-301989888,[1,6,8,1,2,12,2]%%%}+
%%%{-134217728,[1,6,8,0,1,14,3]%%%}+%%%{41156608,[1,6,7,2,5,10,0]%%%}+%%%{1
78257920,[1,6,7,1,4,12,1]%%%}+%%%{167772160,[1,6,7,0,3,14,2]%%%}+%%%{-31457
280,[1,6,6,1,6,12,0]%%%}+%%%{-69206016,[1,6,6,0,5,14,1]%%%}+%%%{9437184,[1,
6,5,0,7,14,0]%%%}+%%%{402653184,[0,7,10,7,2,8,1]%%%}+%%%{5637144576,[0,7,10
,6,1,10,2]%%%}+%%%{16106127360,[0,7,10,5,0,12,3]%%%}+%%%{-100663296,[0,7,9,
7,4,8,0]%%%}+%%%{-4160749568,[0,7,9,6,3,10,1]%%%}+%%%{-30198988800,[0,7,9,5
,2,12,2]%%%}+%%%{-28991029248,[0,7,9,4,1,14,3]%%%}+%%%{68719476736,[0,7,9,3
,0,16,4]%%%}+%%%{687865856,[0,7,8,6,5,10,0]%%%}+%%%{13925089280,[0,7,8,5,4,
12,1]%%%}+%%%{48184164352,[0,7,8,4,3,14,2]%%%}+%%%{-49392123904,[0,7,8,3,2,
16,3]%%%}+%%%{-120259084288,[0,7,8,2,1,18,4]%%%}+%%%{68719476736,[0,7,8,1,0
,20,5]%%%}+%%%{-1845493760,[0,7,7,5,6,12,0]%%%}+%%%{-19964887040,[0,7,7,4,5
,14,1]%%%}+%%%{-11542724608,[0,7,7,3,4,16,2]%%%}+%%%{113816633344,[0,7,7,2,
3,18,3]%%%}+%%%{-8589934592,[0,7,7,1,2,20,4]%%%}+%%%{-68719476736,[0,7,7,0,
1,22,5]%%%}+%%%{2432696320,[0,7,6,4,7,14,0]%%%}+%%%{11207180288,[0,7,6,3,6,
16,1]%%%}+%%%{-28185722880,[0,7,6,2,5,18,2]%%%}+%%%{-34359738368,[0,7,6,1,4
,20,3]%%%}+%%%{60129542144,[0,7,6,0,3,22,4]%%%}+%%%{-1577058304,[0,7,5,3,8,
16,0]%%%}+%%%{201326592,[0,7,5,2,7,18,1]%%%}+%%%{14495514624,[0,7,5,1,6,20,
2]%%%}+%%%{-17179869184,[0,7,5,0,5,22,3]%%%}+%%%{402653184,[0,7,4,2,9,18,0]
%%%}+%%%{-1610612736,[0,7,4,1,8,20,1]%%%}+%%%{1610612736,[0,7,4,0,7,22,2]%%
%} / %%%{1024,[0,3,4,2,1,2,0]%%%}+%%%{4096,[0,3,4,1,0,4,1]%%%}+%%%{-2560,[0
,3,3,1,2,4,0]%%%}+%%%{-4096,[0,3,3,0,1,6,1]%%%}+%%%{1536,[0,3,2,0,3,6,0]%%%
} Error: Bad Argument Value
```

maple [C] time = 0.03, size = 252, normalized size = 0.76

$$-\frac{8d\sqrt{e}}{\left(4ae^2-4deb+4cd^2\right)\left(2ex^2-2\sqrt{ex^2+d}\,\sqrt{e}\,x+2d\right)}-\frac{2\sqrt{e}\left(\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^2cd+cd^3+2\left(2ae^2-cd^2\right)\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)\right)\ln\left(-\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)+\left(-\sqrt{e}\,x+\sqrt{ex^2+d}\right)^2\right)}{\left(4ae^2-4deb+4cd^2\right)\left(\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^3c+3\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^2be-3\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)^2cd+8\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)ae^2-4\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)bde+3\operatorname{RootOf}\left(\_Z^4c+cd^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z\right)cd^2+bd^2e-cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -2*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)*sum((_R^2*c*d+2*(2*a*e^2-c*d^2)*_R+c*d
^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-
c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e
-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))-8*e^(
1/2)*d/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*(e*x^2+d)^(1/2)*e^(1/2)*x+2*d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(cx^4+bx^2+a\right)\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{{\left(e\,x^2+d\right)}^{3/2}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int(x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(d+ex^2\right)^{\frac{3}{2}}\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**2/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

3.297 $\int \frac{1}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=341

$$-\frac{c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}-\frac{c\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-\cdots}\right.}}{\sqrt{\sqrt{b^2-4ac}+b}\,\cdots}\right.}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\cdots}$$

Rubi [A] time = 0.77, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1172, 191, 1692, 377, 205}

$$-\frac{c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}-\frac{c\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(ae^2-bde+cd^2\right)}+\frac{e^2x}{d\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (e^2*x)/(d*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - (c*(e - (2*c*d - b*e)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqr
t[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sq
rt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) - (c*(e + (2
*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 -
 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x^2)^q, x], x] + Dist[1
/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x^2)^(q + 1)*(c*d - b*e - c*e*x^2))/(
a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q] && LtQ[q, -1]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
```

```
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx = \frac{\int \frac{cd-be-cex^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{cd^2-bde+ae^2} + \frac{e^2\int \frac{1}{\left(d+ex^2\right)^{3/2}}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{e^2x}{d\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{\int \left(\frac{-ce-\frac{c(-2cd+be)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{-ce+\frac{c(-2cd+b}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\right.}{cd^2-bde+ae^2}$$

$$= \frac{e^2x}{d\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{\left(c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{e^2x}{d\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{\left(c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\right.}\right.}{cd^2-bde+ae^2}$$

$$= \frac{e^2x}{d\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2\right.}}}{\sqrt{b-\sqrt{b^2-4ac}}}\right.}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

Mathematica [C] time = 7.15, size = 2112, normalized size = 6.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*c*x*(45*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*
x^2*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x
^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[
Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c
] - 2*c*x^2)))]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/d - (45*(2*c*d + (-b
+ Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c]
)*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d*(-b + Sqrt[b^2 - 4*
a*c] - 2*c*x^2)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sq
rt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2)))]])/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) + 4*(-(((2*c*d + (
-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5
/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*
c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))] + (4*e*x^2*(-(((2*c*d
+ (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))
^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*
c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4
*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))])/d))/(15*Sqrt[b^2 -
 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*
```

```
x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(3/2)*(1 - (2*c*x^2)/(-b + Sq
rt[b^2 - 4*a*c]))*Sqrt[d + e*x^2]*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2
))/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]) - (2*c*x*(45*Sqrt[-(((b + Sqrt[
b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*Sqrt[-(((b + Sqrt[b^2 - 4*
a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 -
4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]] - (30*e*x^2*ArcSin[
Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2))]])/d + (45*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[((
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
))]])/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) - (30*e*(-2*c*d + (b + Sqrt[b^2
 - 4*a*c])*e)*x^4*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
)) + 4*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))] + (4*e*x^2*((
(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))])/d))/(15*Sqrt[b^2 - 4*
a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(
d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(3/2)*(1 + (2*c*x^2)/(b + Sqrt[b^2 -
4*a*c]))*Sqrt[d + e*x^2]*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2))])
```

IntegrateAlgebraic [C] time = 62.48, size = 342, normalized size = 1.00

$$\frac{e^2x}{d\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)}-\frac{e^{3/2}\text{RootSum}\left[\#1^8c+4\#1^6be-4\#1^6cd+16\#1^4ae^2-8\#1^4bde+6\#1^4cd^2+4\#1^2bd^2e-4\#1^2cd^3+cd^4\&,\frac{\#1^4c\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+4\#1^2be\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-6\#1^2cd\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+cd^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)}{\#1^6(-c)-3\#1^4be+3\#1^4cd-8\#1^2ae^2+4\#1^2bde-3\#1^2cd^2-bd^2e+cd^3}\&\right]}{2\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (e^2*x)/(d*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - (e^(3/2)*RootSum[c*d^
4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*
#1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (c*d^2*Log[-(Sqrt[e]*x) + Sqrt[
d + e*x^2] - #1] - 6*c*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 4*
b*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + c*Log[-(Sqrt[e]*x) + Sq
rt[d + e*x^2] - #1]*#1^4)/(c*d^3 - b*d^2*e - 3*c*d^2*#1^2 + 4*b*d*e*#1^2 -
8*a*e^2*#1^2 + 3*c*d*#1^4 - 3*b*e*#1^4 - c*#1^6) & ])/(2*(c*d^2 - b*d*e + a
*e^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b,c]=[44,93,-37]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a,b,c]=[-72,-7,6]Evaluation time: 0.58Unable to divide, perhaps due
to rounding error%%%{-2147483648,[3,8,10,8,3,10,1]%%%}+%%%{-12884901888,[3,
8,10,7,2,12,2]%%%}+%%%{463856467968,[3,8,10,6,1,14,3]%%%}+%%%{1924145348608
,[3,8,10,5,0,16,4]%%%}+%%%{536870912,[3,8,9,8,5,10,0]%%%}+%%%{20401094656,[
3,8,9,7,4,12,1]%%%}+%%%{-150323855360,[3,8,9,6,3,14,2]%%%}+%%%{-31353261260
80,[3,8,9,5,2,16,3]%%%}+%%%{-4672924418048,[3,8,9,4,1,18,4]%%%}+%%%{6047313
952768,[3,8,9,3,0,20,5]%%%}+%%%{-4294967296,[3,8,8,7,6,12,0]%%%}+%%%{-42412
802048,[3,8,8,6,5,14,1]%%%}+%%%{1046898278400,[3,8,8,5,4,16,2]%%%}+%%%{6210
522710016,[3,8,8,4,3,18,3]%%%}+%%%{-1786706395136,[3,8,8,3,2,20,4]%%%}+%%%{
-11544872091648,[3,8,8,2,1,22,5]%%%}+%%%{4398046511104,[3,8,8,1,0,24,6]%%%}
+%%%{12750684160,[3,8,7,6,7,14,0]%%%}+%%%{-23890755584,[3,8,7,5,6,16,1]%%%}
+%%%{-2103460233216,[3,8,7,4,5,18,2]%%%}+%%%{-3324304687104,[3,8,7,3,4,20,3
]%%%}+%%%{9758165696512,[3,8,7,2,3,22,4]%%%}+%%%{1649267441664,[3,8,7,1,2,2
4,5]%%%}+%%%{-4398046511104,[3,8,7,0,1,26,6]%%%}+%%%{-17985175552,[3,8,6,5,
8,16,0]%%%}+%%%{161866579968,[3,8,6,4,7,18,1]%%%}+%%%{1586990415872,[3,8,6,
3,6,20,2]%%%}+%%%{-1795296329728,[3,8,6,2,5,22,3]%%%}+%%%{-4123168604160,[3
,8,6,1,4,24,4]%%%}+%%%{3848290697216,[3,8,6,0,3,26,5]%%%}+%%%{12213813248,[
3,8,5,4,9,18,0]%%%}+%%%{-171798691840,[3,8,5,3,8,20,1]%%%}+%%%{-21260088115
2,[3,8,5,2,7,22,2]%%%}+%%%{1477468749824,[3,8,5,1,6,24,3]%%%}+%%%{-10995116
27776,[3,8,5,0,5,26,4]%%%}+%%%{-3221225472,[3,8,4,3,10,20,0]%%%}+%%%{579820
58496,[3,8,4,2,9,22,1]%%%}+%%%{-154618822656,[3,8,4,1,8,24,2]%%%}+%%%{10307
9215104,[3,8,4,0,7,26,3]%%%}+%%%{1048576,[3,6,10,4,2,4,0]%%%}+%%%{8388608,[
3,6,10,3,1,6,1]%%%}+%%%{16777216,[3,6,10,2,0,8,2]%%%}+%%%{-5242880,[3,6,9,3
,3,6,0]%%%}+%%%{-29360128,[3,6,9,2,2,8,1]%%%}+%%%{-33554432,[3,6,9,1,1,10,2
]%%%}+%%%{9699328,[3,6,8,2,4,8,0]%%%}+%%%{33554432,[3,6,8,1,3,10,1]%%%}+%%%
{16777216,[3,6,8,0,2,12,2]%%%}+%%%{-7864320,[3,6,7,1,5,10,0]%%%}+%%%{-12582
912,[3,6,7,0,4,12,1]%%%}+%%%{2359296,[3,6,6,0,6,12,0]%%%}+%%%{-536870912,[2
,7,10,6,2,8,1]%%%}+%%%{18446744062703697920,[2,7,10,5,1,10,2]%%%}+%%%{-1825
3611008,[2,7,10,4,0,12,3]%%%}+%%%{134217728,[2,7,9,6,4,8,0]%%%}+%%%{5502926
848,[2,7,9,5,3,10,1]%%%}+%%%{36909875200,[2,7,9,4,2,12,2]%%%}+%%%{429496729
60,[2,7,9,3,1,14,3]%%%}+%%%{-42949672960,[2,7,9,2,0,16,4]%%%}+%%%{-95630131
2,[2,7,8,5,5,10,0]%%%}+%%%{-18656264192,[2,7,8,4,4,12,1]%%%}+%%%{-649613803
52,[2,7,8,3,3,14,2]%%%}+%%%{8589934592,[2,7,8,2,2,16,3]%%%}+%%%{85899345920
,[2,7,8,1,1,18,4]%%%}+%%%{2642411520,[2,7,7,4,6,12,0]%%%}+%%%{27783069696,[
2,7,7,3,5,14,1]%%%}+%%%{33957085184,[2,7,7,2,4,16,2]%%%}+%%%{-73014444032,[
2,7,7,1,3,18,3]%%%}+%%%{-42949672960,[2,7,7,0,2,20,4]%%%}+%%%{-3556769792,[
2,7,6,3,7,14,0]%%%}+%%%{-17716740096,[2,7,6,2,6,16,1]%%%}+%%%{12884901888,[
2,7,6,1,5,18,2]%%%}+%%%{39728447488,[2,7,6,0,4,20,3]%%%}+%%%{2340421632,[2,
7,5,2,8,16,0]%%%}+%%%{2415919104,[2,7,5,1,7,18,1]%%%}+%%%{-12079595520,[2,7
,5,0,6,20,2]%%%}+%%%{-603979776,[2,7,4,1,9,18,0]%%%}+%%%{1207959552,[2,7,4,
0,8,20,1]%%%}+%%%{2147483648,[1,8,10,9,3,10,1]%%%}+%%%{38654705664,[1,8,10,
8,2,12,2]%%%}+%%%{51539607552,[1,8,10,7,1,14,3]%%%}+%%%{-274877906944,[1,8,
10,6,0,16,4]%%%}+%%%{-536870912,[1,8,9,9,5,10,0]%%%}+%%%{-26843545600,[1,8,
9,8,4,12,1]%%%}+%%%{-188978561024,[1,8,9,7,3,14,2]%%%}+%%%{146028888064,[1,
8,9,6,2,16,3]%%%}+%%%{962072674304,[1,8,9,5,1,18,4]%%%}+%%%{-549755813888,[
1,8,9,4,0,20,5]%%%}+%%%{4294967296,[1,8,8,8,6,12,0]%%%}+%%%{95026151424,[1,
8,8,7,5,14,1]%%%}+%%%{239444426752,[1,8,8,6,4,16,2]%%%}+%%%{-858993459200,[
1,8,8,5,3,18,3]%%%}+%%%{-618475290624,[1,8,8,4,2,20,4]%%%}+%%%{109951162777
6,[1,8,8,3,1,22,5]%%%}+%%%{-12750684160,[1,8,7,7,7,14,0]%%%}+%%%{-136633647
104,[1,8,7,6,6,16,1]%%%}+%%%{62277025792,[1,8,7,5,5,18,2]%%%}+%%%{936302870
528,[1,8,7,4,4,20,3]%%%}+%%%{-549755813888,[1,8,7,3,3,22,4]%%%}+%%%{-549755
813888,[1,8,7,2,2,24,5]%%%}+%%%{17985175552,[1,8,6,6,8,16,0]%%%}+%%%{719407
02208,[1,8,6,5,7,18,1]%%%}+%%%{-267361714176,[1,8,6,4,6,20,2]%%%}+%%%{-1374
38953472,[1,8,6,3,5,22,3]%%%}+%%%{481036337152,[1,8,6,2,4,24,4]%%%}+%%%{-12
```

```
213813248,[1,8,5,5,9,18,0]%%%}+%%%{7247757312,[1,8,5,4,8,20,1]%%%}+%%%{1030
79215104,[1,8,5,3,7,22,2]%%%}+%%%{-137438953472,[1,8,5,2,6,24,3]%%%}+%%%{32
21225472,[1,8,4,4,10,20,0]%%%}+%%%{-12884901888,[1,8,4,3,9,22,1]%%%}+%%%{12
884901888,[1,8,4,2,8,24,2]%%%}+%%%{-1048576,[1,6,10,5,2,4,0]%%%}+%%%{-83886
08,[1,6,10,4,1,6,1]%%%}+%%%{-16777216,[1,6,10,3,0,8,2]%%%}+%%%{8388608,[1,6
,9,4,3,6,0]%%%}+%%%{62914560,[1,6,9,3,2,8,1]%%%}+%%%{150994944,[1,6,9,2,1,1
0,2]%%%}+%%%{134217728,[1,6,9,1,0,12,3]%%%}+%%%{-26476544,[1,6,8,3,4,8,0]%%
%}+%%%{-163577856,[1,6,8,2,3,10,1]%%%}+%%%{-301989888,[1,6,8,1,2,12,2]%%%}+
%%%{-134217728,[1,6,8,0,1,14,3]%%%}+%%%{41156608,[1,6,7,2,5,10,0]%%%}+%%%{1
78257920,[1,6,7,1,4,12,1]%%%}+%%%{167772160,[1,6,7,0,3,14,2]%%%}+%%%{-31457
280,[1,6,6,1,6,12,0]%%%}+%%%{-69206016,[1,6,6,0,5,14,1]%%%}+%%%{9437184,[1,
6,5,0,7,14,0]%%%}+%%%{402653184,[0,7,10,7,2,8,1]%%%}+%%%{5637144576,[0,7,10
,6,1,10,2]%%%}+%%%{16106127360,[0,7,10,5,0,12,3]%%%}+%%%{-100663296,[0,7,9,
7,4,8,0]%%%}+%%%{-4160749568,[0,7,9,6,3,10,1]%%%}+%%%{-30198988800,[0,7,9,5
,2,12,2]%%%}+%%%{-28991029248,[0,7,9,4,1,14,3]%%%}+%%%{68719476736,[0,7,9,3
,0,16,4]%%%}+%%%{687865856,[0,7,8,6,5,10,0]%%%}+%%%{13925089280,[0,7,8,5,4,
12,1]%%%}+%%%{48184164352,[0,7,8,4,3,14,2]%%%}+%%%{-49392123904,[0,7,8,3,2,
16,3]%%%}+%%%{-120259084288,[0,7,8,2,1,18,4]%%%}+%%%{68719476736,[0,7,8,1,0
,20,5]%%%}+%%%{-1845493760,[0,7,7,5,6,12,0]%%%}+%%%{-19964887040,[0,7,7,4,5
,14,1]%%%}+%%%{-11542724608,[0,7,7,3,4,16,2]%%%}+%%%{113816633344,[0,7,7,2,
3,18,3]%%%}+%%%{-8589934592,[0,7,7,1,2,20,4]%%%}+%%%{-68719476736,[0,7,7,0,
1,22,5]%%%}+%%%{2432696320,[0,7,6,4,7,14,0]%%%}+%%%{11207180288,[0,7,6,3,6,
16,1]%%%}+%%%{-28185722880,[0,7,6,2,5,18,2]%%%}+%%%{-34359738368,[0,7,6,1,4
,20,3]%%%}+%%%{60129542144,[0,7,6,0,3,22,4]%%%}+%%%{-1577058304,[0,7,5,3,8,
16,0]%%%}+%%%{201326592,[0,7,5,2,7,18,1]%%%}+%%%{14495514624,[0,7,5,1,6,20,
2]%%%}+%%%{-17179869184,[0,7,5,0,5,22,3]%%%}+%%%{402653184,[0,7,4,2,9,18,0]
%%%}+%%%{-1610612736,[0,7,4,1,8,20,1]%%%}+%%%{1610612736,[0,7,4,0,7,22,2]%%
%} / %%%{1024,[0,3,4,2,1,2,0]%%%}+%%%{4096,[0,3,4,1,0,4,1]%%%}+%%%{-2560,[0
,3,3,1,2,4,0]%%%}+%%%{-4096,[0,3,3,0,1,6,1]%%%}+%%%{1536,[0,3,2,0,3,6,0]%%%
} Error: Bad Argument Value
```

maple [C] time = 0.02, size = 246, normalized size = 0.72

$$\frac{8e^{\frac{3}{2}}\left(\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)^2c+c\,d^2+2(2be-3cd)\,\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)+\left(-\sqrt{e}\,x+\sqrt{e\,x^2+d}\right)^2\right)}{\left(16a\,e^2-16deb+16c\,d^2\right)\left(\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)^3c+3\,\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)^2be-3\,\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)^2cd+8\,\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)a\,e^2-4\,\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)bde+3\,\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)c\,d^2+b\,d^2e-c\,d^3\right)}+\frac{32e^{\frac{3}{2}}}{\left(16a\,e^2-16deb+16c\,d^2\right)\left(2e\,x^2-2\sqrt{e\,x^2+d}\,\sqrt{e}\,x+2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 8*e^(3/2)/(16*a*e^2-16*b*d*e+16*c*d^2)*sum((_R^2*c+2*(2*b*e-3*c*d)*_R+c*d^2
)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*
d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4
*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))+32*e^(3
/2)/(16*a*e^2-16*b*d*e+16*c*d^2)/(2*e*x^2-2*(e*x^2+d)^(1/2)*e^(1/2)*x+2*d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2+a\right)\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(e\,x^2+d\right)}^{3/2}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int(1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex^2\right)^{\frac{3}{2}} \left(a + bx^2 + cx^4\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(1/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

3.298 $\int \frac{1}{x^2\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=339

$$-\frac{2c^2\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\,\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}}-\frac{2c^2\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\,\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}}+\frac{}{ad\sqrt{d+e\cdots}}$$

Rubi [A] time = 2.84, antiderivative size = 462, normalized size of antiderivative = 1.36, number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1301, 271, 191, 6728, 264, 1692, 377, 205}

$$-\frac{c\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,\left(ae^2-bde+cd^2\right)}-\frac{c\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,\left(ae^2-bde+cd^2\right)}-\frac{2e^3x}{d^2\sqrt{d+ex^2}\,\left(ae^2-bde+cd^2\right)}-\frac{e^2}{dx\sqrt{d+ex^2}\,\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{d+ex^2}\,(cd-be)}{adx\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(e^2/(d*(c*d^2 - b*d*e + a*e^2)*x*Sqrt[d + e*x^2])) - (2*e^3*x)/(d^2*(c*d^
2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - ((c*d - b*e)*Sqrt[d + e*x^2])/(a*d*(c
*d^2 - b*d*e + a*e^2)*x) - (c*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - S
qrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*
c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) - (c*(c*d - b*e -
 (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sq
rt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 -
b*d*e + a*e^2))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^m*(
d + e*x^2)^q, x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((f*x)^m*(d + e*
x^2)^(q + 1)*Simp[c*d - b*e - c*e*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && L
tQ[q, -1]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx=\frac{\int \frac{cd-be-cex^2}{x^2\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{cd^2-bde+ae^2}+\frac{e^2\int \frac{1}{x^2\left(d+ex^2\right)^{3/2}}\,dx}{cd^2-bde+ae^2}$$

$$=-\frac{e^2}{d\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}}+\frac{\int \left(\frac{cd-be}{ax^2\sqrt{d+ex^2}}+\frac{-bcd+b^2e-ace-c(cd-be)x^2}{a\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right) \cdots}{cd^2-bde+ae^2}$$

$$=-\frac{e^2}{d\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}}-\frac{2e^3x}{d^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}+\frac{\int \frac{-bc\cdots}{\sqrt{\cdots}}}{a\,(\cdots}$$

$$=-\frac{e^2}{d\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}}-\frac{2e^3x}{d^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}-\frac{(cd\cdots}{ad\,(c\cdots}$$

$$=-\frac{e^2}{d\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}}-\frac{2e^3x}{d^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}-\frac{(cd\cdots}{ad\,(c\cdots}$$

$$=-\frac{e^2}{d\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}}-\frac{2e^3x}{d^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}-\frac{(cd\cdots}{ad\,(c\cdots}$$

$$=-\frac{e^2}{d\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}}-\frac{2e^3x}{d^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}-\frac{(cd\cdots}{ad\,(c\cdots}$$

Mathematica [C] time = 6.77, size = 2158, normalized size = 6.37

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d + 2*e*x^2)/(a*d^2*x*Sqrt[d + e*x^2])) - ((c + (b*c)/Sqrt[b^2 - 4*a*c])
*x*(45*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e
)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*
Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2*(
d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt
[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] -
2*c*x^2)))]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e
)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/d - (45*(2*c*d + (-b + Sq
rt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)
*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d*(-b + Sqrt[b^2 - 4*a*c]
 - 2*c*x^2)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[-
(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*
c*x^2)))]])/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) + 4*(-(((2*c*d + (-b +
 Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5/2)*
Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*
c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*
e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))] + (4*e*x^2*(-(((2*c*d + (-
b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5/
2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] -
 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))])/d))/(15*a*(b - Sqrt[b^
2 - 4*a*c])*d*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b
^2 - 4*a*c] - 2*c*x^2))))^(3/2)*(1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]))*Sq
rt[d + e*x^2]*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2
 - 4*a*c] - 2*c*x^2))]) + ((-c + (b*c)/Sqrt[b^2 - 4*a*c])*x*(45*Sqrt[-(((b
+ Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/
(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*Sqrt[-(((b + Sqrt[b
^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]] - (30*e*x^2*
ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2))]])/d + (45*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[
Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2))]])/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) - (30*e*(-2*c*d + (b + S
qrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x
^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d^2*(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)) + 4*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))] + (4*e
*x^2*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))])/d))/(15*a*(b +
 Sqrt[b^2 - 4*a*c])*d*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2)))^(3/2)*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))*
Sqrt[d + e*x^2]*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2))])
```

IntegrateAlgebraic [C] time = 109.35, size = 546, normalized size = 1.61

$$\frac{-ade^2-2ae^3x^2+bd^2e+bde^2x^2-cd^3-cd^2ex^2}{ad^2x\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{e}\,\mathrm{RootSum}\left[\#1^8c+4\#1^6be-4\#1^6cd+16\#1^4ae^2-8\#1^4bde+6\#1^4cd^2+4\#1^2bd^2e-4\#1^2cd^3+cd^4\&,\frac{-\#1^4bce\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+\#1^4c^2d\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+4\#1^2ace^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-4\#1^2b^2e^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+6\#1^2bcde\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-2\#1^2c^2d^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-bcd^2e\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+c^2d^3\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)}{\#1^6(-c)-3\#1^4be+3\#1^4cd-8\#1^2ae^2+4\#1^2bde-3\#1^2cd^2-bd^2e+cd^3}\&\right]}{2a\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-(c*d^3) + b*d^2*e - a*d*e^2 - c*d^2*e*x^2 + b*d*e^2*x^2 - 2*a*e^3*x^2)/(a
*d^2*(c*d^2 - b*d*e + a*e^2)*x*Sqrt[d + e*x^2]) - (Sqrt[e]*RootSum[c*d^4 -
4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4
 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (c^2*d^3*Log[-(Sqrt[e]*x) + Sqrt[d
+ e*x^2] - #1] - b*c*d^2*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - 2*c^2
*d^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 6*b*c*d*e*Log[-(Sqrt[e
]*x) + Sqrt[d + e*x^2] - #1]*#1^2 - 4*b^2*e^2*Log[-(Sqrt[e]*x) + Sqrt[d + e
*x^2] - #1]*#1^2 + 4*a*c*e^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2
+ c^2*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4 - b*c*e*Log[-(Sqrt[e]
*x) + Sqrt[d + e*x^2] - #1]*#1^4)/(c*d^3 - b*d^2*e - 3*c*d^2*#1^2 + 4*b*d*e
*#1^2 - 8*a*e^2*#1^2 + 3*c*d*#1^4 - 3*b*e*#1^4 - c*#1^6) & ])/(2*a*(c*d^2 -
 b*d*e + a*e^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.04, size = 387, normalized size = 1.14

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/a/d/x/(e*x^2+d)^(1/2)-2/a*e/d^2*x/(e*x^2+d)^(1/2)-2/a*e^(1/2)/(4*a*e^2-4
*b*d*e+4*c*d^2)*sum((c*(b*e-c*d)*_R^2+2*(-2*a*c*e^2+2*b^2*e^2-3*b*c*d*e+c^2
*d^2)*_R+b*c*d^2*e-c^2*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b
*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=Ro
otOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d
^2*e-4*c*d^3)*_Z))-8/a*e^(3/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*(e*x^2+
d)^(1/2)*e^(1/2)*x+2*d)*b+8/a*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*
(e*x^2+d)^(1/2)*e^(1/2)*x+2*d)*c*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4 + bx^2 + a\right)\left(ex^2 + d\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(e\,x^2+d\right)}^{3/2}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(d + e x^{2}\right)^{\frac{3}{2}} \left(a + b x^{2} + c x^{4}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(1/(x**2*(d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

3.299 $\int \frac{1}{x^4(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=419

$$\frac{2c^2\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}}+\frac{2c^2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}}-\frac{ex\left(ace\right.}{a^2d\sqrt{d+ex^2}}$$

Rubi [A] time = 5.57, antiderivative size = 647, normalized size of antiderivative = 1.54, number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1301, 271, 191, 6728, 264, 1692, 377, 205}

$$\frac{c\left(\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}+\frac{c\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{d+ex^2}\left(ace+b^2(-e)+bcd\right)}{a^2dx\left(ae^2-bde+cd^2\right)}+\frac{8e^4x}{3d^3\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)}+\frac{4e^3}{3d^2x\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)}-\frac{e^2}{3dx^3\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)}+\frac{2e\sqrt{d+ex^2}(cd-be)}{3ad^2x\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{d+ex^2}(cd-be)}{3adx^3\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -e^2/(3*d*(c*d^2 - b*d*e + a*e^2)*x^3*Sqrt[d + e*x^2]) + (4*e^3)/(3*d^2*(c*
d^2 - b*d*e + a*e^2)*x*Sqrt[d + e*x^2]) + (8*e^4*x)/(3*d^3*(c*d^2 - b*d*e +
 a*e^2)*Sqrt[d + e*x^2]) - ((c*d - b*e)*Sqrt[d + e*x^2])/(3*a*d*(c*d^2 - b*
d*e + a*e^2)*x^3) + (2*e*(c*d - b*e)*Sqrt[d + e*x^2])/(3*a*d^2*(c*d^2 - b*d
*e + a*e^2)*x) + ((b*c*d - b^2*e + a*c*e)*Sqrt[d + e*x^2])/(a^2*d*(c*d^2 -
b*d*e + a*e^2)*x) + (c*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*
e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b - Sqr
t[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*
e^2)) + (c*(b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*
e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(S
qrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b + Sqrt[b^2 - 4*a*
c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^m*(
d + e*x^2)^q, x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((f*x)^m*(d + e*
x^2)^(q + 1)*Simp[c*d - b*e - c*e*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && L
tQ[q, -1]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}dx = \frac{\int \frac{cd-be-cex^2}{x^4\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}dx}{cd^2-bde+ae^2} + \frac{e^2\int \frac{1}{x^4\left(d+ex^2\right)^{3/2}}dx}{cd^2-bde+ae^2}$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{\int \left(\frac{cd-be}{ax^4\sqrt{d+ex^2}} + \frac{-bcd+b^2e-ace}{a^2x^2\sqrt{d+ex^2}} + \frac{b^2cd-ac}{\ldots}\right.}{cd^2-bde+\ldots}$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{4e^3}{3d^2\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} + \ldots$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{4e^3}{3d^2\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} + \ldots$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{4e^3}{3d^2\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} + \ldots$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{4e^3}{3d^2\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} + \ldots$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{4e^3}{3d^2\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} + \ldots$$

Mathematica [C] time = 6.80, size = 2218, normalized size = 5.29

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (b*(d + 2*e*x^2))/(a^2*d^2*x*Sqrt[d + e*x^2]) - (d^2 - 4*d*e*x^2 - 8*e^2*x^
4)/(3*a*d^3*x^3*Sqrt[d + e*x^2]) + ((b*c + (c*(b^2 - 2*a*c))/Sqrt[b^2 - 4*a
*c])*x*(45*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*
x^2*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x
^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[
Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c
] - 2*c*x^2)))]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/d - (45*(2*c*d + (-b
+ Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c]
)*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d*(-b + Sqrt[b^2 - 4*
a*c] - 2*c*x^2)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sq
rt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2)))]])/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) + 4*(-(((2*c*d + (
-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5
/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*
```

```
c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))] + (4*e*x^2*(-(((2*c*d
+ (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))
^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*
c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4
*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))])/d))/(15*a^2*(b - S
qrt[b^2 - 4*a*c])*d*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b +
Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(3/2)*(1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c
]))*Sqrt[d + e*x^2]*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sq
rt[b^2 - 4*a*c] - 2*c*x^2))]) + ((b*c - (c*(b^2 - 2*a*c))/Sqrt[b^2 - 4*a*c]
)*x*(45*Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e
)*x^2*(d + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*
Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d
 + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[
((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2))]] - (30*e*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d
*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/d + (45*(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) - (30
*e*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d^2*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2)) + 4*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)
/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])
*(d + e*x^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2
, 7/2, ((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2))] + (4*e*x^2*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x
^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
))])/d))/(15*a^2*(b + Sqrt[b^2 - 4*a*c])*d*(((2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(3/2)*(1 + (2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c]))*Sqrt[d + e*x^2]*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x
^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))])
```

IntegrateAlgebraic [C] time = 144.27, size = 762, normalized size = 1.82

$$\frac{\sqrt{e}\,\text{RootSum}\left[\#1^8c+4\#1^6be-4\#1^6cd+16\#1^4ae^2-8\#1^4bde+6\#1^4cd^2+4\#1^2bd^2e-4\#1^2cd^3+cd^4\&,\frac{-\#1^4ac^2e\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+\#1^4b^2ce\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+\#1^4(-b)c^2d\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-8\#1^2abce^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+6\#1^2ac^2de\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+4\#1^2b^3e^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-6\#1^2b^2cde\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+2\#1^2bc^2d^2\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-ac^2d^2e\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)+b^2cd^2e\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)-bc^2d^3\log\left(-\#1+\sqrt{d+ex^2}-\sqrt{e}x\right)}{\#1^6c+3\#1^4be-3\#1^4cd+8\#1^2ae^2-4\#1^2bde+3\#1^2cd^2+bd^2e-cd^3}\&\right]}{2a^2\left(ae^2-bde+cd^2\right)}+\frac{-a^2d^2e^2+4a^2de^3x^2+8a^2e^4x^4+abd^3e-abd^2e^2x^2-2abde^3x^4-acd^4+4acd^3ex^2+5acd^2e^2x^4-3b^2d^3ex^2-3b^2d^2e^2x^4+3bcd^4x^2+3bcd^3ex^4}{3a^2d^3x^3\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-(a*c*d^4) + a*b*d^3*e - a^2*d^2*e^2 + 3*b*c*d^4*x^2 - 3*b^2*d^3*e*x^2 + 4
*a*c*d^3*e*x^2 - a*b*d^2*e^2*x^2 + 4*a^2*d*e^3*x^2 + 3*b*c*d^3*e*x^4 - 3*b^
2*d^2*e^2*x^4 + 5*a*c*d^2*e^2*x^4 - 2*a*b*d*e^3*x^4 + 8*a^2*e^4*x^4)/(3*a^2
*d^3*(c*d^2 - b*d*e + a*e^2)*x^3*Sqrt[d + e*x^2]) + (Sqrt[e]*RootSum[c*d^4
- 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1
^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (-(b*c^2*d^3*Log[-(Sqrt[e]*x) + S
qrt[d + e*x^2] - #1]) + b^2*c*d^2*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1
] - a*c^2*d^2*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] + 2*b*c^2*d^2*Log[
-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 - 6*b^2*c*d*e*Log[-(Sqrt[e]*x) +
Sqrt[d + e*x^2] - #1]*#1^2 + 6*a*c^2*d*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]
 - #1]*#1^2 + 4*b^3*e^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 - 8*a
*b*c*e^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 - b*c^2*d*Log[-(Sqrt
[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4 + b^2*c*e*Log[-(Sqrt[e]*x) + Sqrt[d + e
*x^2] - #1]*#1^4 - a*c^2*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4)/(
-(c*d^3) + b*d^2*e + 3*c*d^2*#1^2 - 4*b*d*e*#1^2 + 8*a*e^2*#1^2 - 3*c*d*#1^
4 + 3*b*e*#1^4 + c*#1^6) & ])/(2*a^2*(c*d^2 - b*d*e + a*e^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.04, size = 541, normalized size = 1.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/a^2*b/d/x/(e*x^2+d)^(1/2)+2/a^2*b*e/d^2*x/(e*x^2+d)^(1/2)-2/a^2*e^(1/2)/(
4*a*e^2-4*b*d*e+4*c*d^2)*sum((c*(a*c*e-b^2*e+b*c*d)*_R^2+2*(4*a*b*c*e^2-3*a
*c^2*d*e-2*b^3*e^2+3*b^2*c*d*e-b*c^2*d^2)*_R+a*c^2*d^2*e-b^2*d^2*e*c+b*c^2*
d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e
-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*
e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))-8/a*
e^(3/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*(e*x^2+d)^(1/2)*e^(1/2)*x+2*d)
*c+8/a^2*e^(3/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*(e*x^2+d)^(1/2)*e^(1/
2)*x+2*d)*b^2-8/a^2*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*(e*x^2+d)^
(1/2)*e^(1/2)*x+2*d)*b*c*d-1/3/a/d/x^3/(e*x^2+d)^(1/2)+4/3/a*e/d^2/x/(e*x^2
+d)^(1/2)+8/3/a*e^2/d^3*x/(e*x^2+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2+a\right)\left(ex^2+d\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,{\left(e\,x^2+d\right)}^{3/2}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int(1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4}\left(d+ex^{2}\right)^{\frac{3}{2}}\left(a+bx^{2}+cx^{4}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(1/(x**4*(d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

3.300 $\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=40

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{1-c^4x^4}}{cx\sqrt{\frac{1}{c^2x^2}+1}}\right)}{c}$$

Rubi [A] time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1448, 1252, 848, 63, 208}

$$-\frac{x\sqrt{\frac{1}{c^2x^2}+1}\,\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 1/(c^2*x^2)]/Sqrt[1 - c^4*x^4],x]
[Out] -((Sqrt[1 + 1/(c^2*x^2)]*x*ArcTanh[Sqrt[1 - c^2*x^2]])/Sqrt[1 + c^2*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1448

```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sym
bol] :> Dist[(e^IntPart[q]*(d + e*x^mn)^FracPart[q])/(x^(mn*FracPart[q])*(1
 + d/(x^mn*e))^FracPart[q]), Int[x^(mn*q)*(1 + d/(x^mn*e))^q*(a + c*x^n2)^p
, x], x] /; FreeQ[{a, c, d, e, mn, p, q}, x] && EqQ[n2, -2*mn] &&  !Integer
Q[p] &&  !IntegerQ[q] && PosQ[n2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx &= \frac{\left(\sqrt{1+\frac{1}{c^2x^2}}\,x\right)\int \frac{\sqrt{1+c^2x^2}}{x\sqrt{1-c^4x^4}}\,dx}{\sqrt{1+c^2x^2}} \\
&= \frac{\left(\sqrt{1+\frac{1}{c^2x^2}}\,x\right)\text{Subst}\left(\int \frac{\sqrt{1+c^2x}}{x\sqrt{1-c^4x^2}}\,dx,x,x^2\right)}{2\sqrt{1+c^2x^2}} \\
&= \frac{\left(\sqrt{1+\frac{1}{c^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{2\sqrt{1+c^2x^2}} \\
&= -\frac{\left(\sqrt{1+\frac{1}{c^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx,x,\sqrt{1-c^2x^2}\right)}{c^2\sqrt{1+c^2x^2}} \\
&= -\frac{\sqrt{1+\frac{1}{c^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{\sqrt{1+c^2x^2}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 44, normalized size = 1.10

$$
-\frac{x\sqrt{\frac{1}{c^2x^2}+1}\,\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{\sqrt{c^2x^2+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 1/(c^2*x^2)]/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -((Sqrt[1 + 1/(c^2*x^2)]*x*ArcTanh[Sqrt[1 - c^2*x^2]])/Sqrt[1 + c^2*x^2])
```

IntegrateAlgebraic [B] time = 1.09, size = 107, normalized size = 2.68

$$
\frac{cx\sqrt{\frac{1}{c^2x^2}+1}\left(\frac{\log\left(\frac{\sqrt{1-c^4x^4}}{\sqrt{c^2x^2+1}}-1\right)}{2c}-\frac{\log\left(\frac{c\sqrt{1-c^4x^4}}{\sqrt{c^2x^2+1}}+c\right)}{2c}\right)}{\sqrt{c^2x^2+1}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + 1/(c^2*x^2)]/Sqrt[1 - c^4*x^4],x]
```

```
[Out] (c*Sqrt[1 + 1/(c^2*x^2)]*x*(Log[-1 + Sqrt[1 - c^4*x^4]/Sqrt[1 + c^2*x^2]]/(
2*c) - Log[c + (c*Sqrt[1 - c^4*x^4])/Sqrt[1 + c^2*x^2]]/(2*c)))/Sqrt[1 + c^
2*x^2]
```

fricas [B] time = 0.94, size = 120, normalized size = 3.00

$$
-\frac{\log\left(\frac{c^2x^2+\sqrt{-c^4x^4+1}\,cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{c^2x^2+1}\right)-\log\left(-\frac{c^2x^2-\sqrt{-c^4x^4+1}\,cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{c^2x^2+1}\right)}{2\,c}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 1)/(c^2*x^2 + 1)) - log(-(c^2*x^2 - sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 +
 1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)))/c
```

giac [A] time = 0.22, size = 42, normalized size = 1.05

$$-\frac{\left(\log\left(\sqrt{-c^2x^2+1}+1\right)-\log\left(-\sqrt{-c^2x^2+1}+1\right)\right)|c|}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(log(sqrt(-c^2*x^2 + 1) + 1) - log(-sqrt(-c^2*x^2 + 1) + 1))*abs(c)/c^
2
```

maple [C] time = 0.06, size = 101, normalized size = 2.52

$$-\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,\sqrt{-c^4x^4+1}\,x\,\mathrm{csgn}\left(\frac{1}{c}\right)\ln\left(\frac{2\sqrt{-\frac{c^2x^2-1}{c^2}}\,c\,\mathrm{csgn}\left(\frac{1}{c}\right)+2}{c^2x}\right)}{\left(c^2x^2+1\right)\sqrt{-\frac{c^2x^2-1}{c^2}}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] -((c^2*x^2+1)/c^2/x^2)^(1/2)*x*(-c^4*x^4+1)^(1/2)*csgn(1/c)*ln(2*(csgn(1/c)
*c*(-1/c^2*(c^2*x^2-1))^(1/2)+1)/x/c^2)/(c^2*x^2+1)/(-1/c^2*(c^2*x^2-1))^(1
/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{c^2x^2}+1}}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(1/(c^2*x^2) + 1)/sqrt(-c^4*x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{c^2\,x^2}+1}}{\sqrt{1-c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(c^2*x^2) + 1)^(1/2)/(1 - c^4*x^4)^(1/2),x)
```

```
[Out] int((1/(c^2*x^2) + 1)^(1/2)/(1 - c^4*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{-\left(cx-1\right)\left(cx+1\right)\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/c**2/x**2)**(1/2)/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(1 + 1/(c**2*x**2))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))
, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
```

```
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
```

```
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
```

```
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
```

```
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
```

```
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
```

```
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```